Statements: CompanionTheory (automorphic and representation theory)

9,403 statements from 1,898 files in part 1, proofs omitted, grouped by module.

CompanionTheory.AutomorphicTheory.BesselOrbitBounds.BesselMapUniqueness.BesselMapUniqGapAlt

theorem **bounded** : Bounded

theorem **besselMapUniqGap** : BesselMapUniqGap

CompanionTheory.AutomorphicTheory.BesselOrbitBounds.BesselMapUniqueness.ContinuousOnOrbit

theorem **LieWd.lieW** {W : List (Mat n)} (h : LieWd Hm W) : LieW (UH Hm) W

theorem **CWLin.comp_sigma** (hAd : DsAd) (hU : URep Hm σ) {B : H → V} (hB : CWLin Hm σ B) {y : Mat n} (hy : y ∈ UH Hm) : CWLin Hm σ (fun v => B (σ y v))

theorem **CWLin.comb** {B₁ B₂ : H → V} (h₁ : CWLin Hm σ B₁) (h₂ : CWLin Hm σ B₂) (a b : ℂ) : CWLin Hm σ (fun v => a • B₁ v + b • B₂ v)

theorem **CWLin.zero** {B : H → V} (hB : CWLin Hm σ B) : B 0 = 0

theorem **CWLin.sum_orbit** (hU : URep Hm σ) {B : H → V} (hB : CWLin Hm σ B) {w : H} (hw : IsSm (UH Hm) σ w) (h0 : ∀ g ∈ UH Hm, B (σ g w) = 0) {ι : Type} (s : Finset ι) (c : ι → ℂ) (g : ι → Mat n) (hg : ∀ i, g i ∈ UH Hm) : IsSm (UH Hm) σ (∑ i ∈ s, c i • σ (g i) w) ∧ B (∑ i ∈ s, c i • σ (g i) w) = 0

theorem **CWLin.eq_zero** (hU : URep Hm σ) {B : H → V} (hB : CWLin Hm σ B) {w : H} (hw : IsSm (UH Hm) σ w) (h0 : ∀ g ∈ UH Hm, B (σ g w) = 0) {x : H} (hx : IsSm (UH Hm) σ x) (happ : ∀ Ws : List (List (Mat n)), (∀ W ∈ Ws, LieWd Hm W) → ∀ δ : ℝ, 0 < δ → ∃ (N : ℕ) (c : Fin N → ℂ) (g : Fin N → Mat n), (∀ i, g i ∈ UH Hm) ∧ (Ws.map fun W => ‖dsW σ W (x − ∑ i, c i • σ (g i) w)‖).sum < δ) : B x = 0

theorem **CWLin.continuousOn_orbit** [FiniteDimensional ℂ V] (hF : Factor) (hU : URep Hm σ) {B : H → V} (hB : CWLin Hm σ B) {w : H} (hw : IsSm (UH Hm) σ w) : ContinuousOn (fun g => B (σ g w)) (UH Hm)

CompanionTheory.AutomorphicTheory.BesselOrbitBounds.BesselMapUniqueness.NormPsiGen

theorem **spanFam** {n : ℕ} (S : Submodule ℝ (Mat n)) : ∃ (d : ℕ) (X : Fin d → Mat n), (∀ j, X j ∈ S) ∧ ∀ Y ∈ S, ∃ a : Fin d → ℝ, Y = ∑ j, (a j : ℂ) • X j

theorem **lie_add** {n : ℕ} {Hm X Y : Mat n} (hX : Lie Hm X) (hY : Lie Hm Y) : Lie Hm (X + Y)

theorem **lie_smul** {n : ℕ} {Hm X : Mat n} (c : ℝ) (hX : Lie Hm X) : Lie Hm (c • X)

theorem **lie_zero** {n : ℕ} (Hm : Mat n) : Lie Hm 0

theorem **psiGen_one** {z : Fin (2 * r + k + 1) → ℂ} (hz : inWb k r z) (c : Fin r → ℂ) : PI.BesselMapUniq.psiGen k r z c 1 = 1

theorem **psiGen_cont** (z : Fin (2 * r + k + 1) → ℂ) (c : Fin r → ℂ) : Continuous (PI.BesselMapUniq.psiGen k r z c)

theorem **norm_psiGen** (z : Fin (2 * r + k + 1) → ℂ) (c : Fin r → ℂ) (g : Mat (2 * r + k + 1)) : ‖PI.BesselMapUniq.psiGen k r z c g‖ = 1

theorem **one_uw** (z : Fin (2 * r + k + 1) → ℂ) : (1 : Mat (2 * r + k + 1)) ∈ UWset k r z

theorem **bounded_of** (hQB : QBound) (hAB : AbsBound) (hAd : AdaptExists) (hDec : Decomp) (hTB : TorBound) (hSp : SpanG) (hEx : ExpMem) (hRV : RootVec) : Bounded

CompanionTheory.AutomorphicTheory.BesselOrbitBounds.BesselMapUniqueness.OrbitMult

theorem **orbit_mult** (hCL : CutLim) (hAd : DsAd) (hF : Factor) {k r : ℕ} {Wp : Type} [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [FiniteDimensional ℂ Wp] (D : B.RealPlaceData k r Wp Wp) (X : SigData k r) (hD : IsDiag D) (hS : SqIntG D X) (ha : aM k r ≤ X.a) (hU : URep (Hpk k r) X.σ) (hw : IsSm (UH (Hpk k r)) X.σ X.w) (hdec : ∀ v, IsSm (UH (Hpk k r)) X.σ v → ∃ C : ℝ, ∀ x ∈ B.GV k r, ‖⟪v, X.σ x X.w⟫_ℂ‖ ≤ C * B.gN x ^ (-X.a)) {l : X.Hs → Wp} (hl : IsMultOneMap D X l) {Bm : X.Hs → Wp} (hB : CWLin (Hpk k r) X.σ Bm) (heq : ∀ h ∈ UWset k r D.z, ∀ m ∈ Nset k r, ∀ v, IsSm (UH (Hpk k r)) X.σ v → Bm (X.σ (h * m) v) = psi k r D.z cStd m • D.π h (Bm v)) (hbd : ∃ K : ℝ, ∀ g ∈ UH (Hpk k r), ‖Bm (X.σ g X.w)‖ ≤ K) : ∃ c : ℂ, ∀ x ∈ UH (Hpk k r), Bm (X.σ x X.w) = c • l (X.σ x X.w)

theorem **cwLin_of** {k r : ℕ} {H : Type} [NormedAddCommGroup H] [InnerProductSpace ℂ H] {σ : Mat (2 * r + k + 1) → (H →L[ℂ] H)} {Wp : Type} [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] {z : Fin (2 * r + k + 1) → ℂ} {ψ : Mat (2 * r + k + 1) → ℂ} {π : Mat (2 * r + k + 1) → (Wp →L[ℂ] Wp)} {Bm : H → Wp} (h : PI.BesselMapUniq.IsBesselMapSm (k

theorem **uniq_of** (hBd : Bounded) (hCL : CutLim) (hTC : TorConj) (hAd : DsAd) (hF : Factor) : BesselMapUniqGap

CompanionTheory.AutomorphicTheory.BesselOrbitBounds.BesselMapUniqueness.ProdNormLe

theorem **prod_norm_le** {n : ℕ} (W : List (Mat n)) {a : ℝ} (h : ∀ X ∈ W, ‖X‖ ≤ a) : (W.map fun X => ‖X‖).prod ≤ a ^ W.length

theorem **absBound_of** (hF : Factor) (hAd : DsAd) (hDer : DsDer) (hWB : WordBd) : AbsBound

CompanionTheory.AutomorphicTheory.BesselWaveGrids.BigCellDerivativeExpansionOfIsDSOfCoefOf

theorem **suppliedData_existsW** : Hyps → ∀ (P : M.Par) (Wp Wq : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wp] [FiniteDimensional ℂ Wq] (p : KB.Pl P Wp Wq) (lam : Fin (2 * P.r + P.k + 1) → ℝ) (b bv ga h : ℝ) (q0 : ℕ) (R0 : Rep P.k P.r) (f : B.Mat P.k P.r → (Wq →L[ℂ] Wp)) (ν₀ : Fin (2 * P.r + P.k + 1) → ℤ), n1 ν₀ ≤ q0 → Num P.k P.r lam P.u b bv ga h q0 → R0.IsDS (PI.BesselPeriodGGP.eps48 P.k P.r) (fun a => lam a + (ν₀ a : ℝ)) → CoefOf R0 f → TBB.jN2 p f ≠ 0 → ∃ (R : (Fin (2 * P.r + P.k + 1) → ℤ) → Rep P.k P.r) (l : ∀ ν, (R ν).H → Wp) (l' : ∀ ν, (R ν).H → Wq) (dg : (Fin (2 * P.r + P.k + 1) → ℤ) → ℝ) (s : (Fin (2 * P.r + P.k + 1) → ℤ) → ℂ) (J : (Fin (2 * P.r + P.k + 1) → ℤ) → B.Mat P.k P.r → (Wq →L[ℂ] Wp)) (tau : (Fin (2 * P.r + P.k + 1) → ℤ) → Finset (Fin (2 * P.r + P.k + 1))) → ℂ) (t : ℝ), 0 < t ∧ J ν₀ = (fun g

theorem **coef_norm_finite_and_zeroW** : Hyps → ∀ (P : M.Par) (Wp Wq : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wp] [FiniteDimensional ℂ Wq] (p : KB.Pl P Wp Wq) (lam : Fin (2 * P.r + P.k + 1) → ℝ) (b bv ga h : ℝ) (q0 : ℕ) (R0 : Rep P.k P.r) (f : B.Mat P.k P.r → (Wq →L[ℂ] Wp)) (ν₀ : Fin (2 * P.r + P.k + 1) → ℤ), n1 ν₀ ≤ q0 → Num P.k P.r lam P.u b bv ga h q0 → R0.IsDS (PI.BesselPeriodGGP.eps48 P.k P.r) (fun a => lam a + (ν₀ a : ℝ)) → CoefOf R0 f → TBB.jN2 p f < ⊤ ∧ (AEMeasurable fun c : Fin (P.r + 1) → ℂ => ENNReal.ofReal (TBB.Ew p (fun _ => 1) f (P.r + 1) [] (TBB.bigPt P.r c))) ∧ (TBB.jN2 p f = 0 → ∀ c : Fin (P.r + 1) → ℂ, D.IsLocPt (TBB.bigPt P.r c) (P.r + 1) → 0 < TBB.Gp p (TBB.bigPt P.r c) → p.V f (TBB.bigPt P.r c) = 0)

theorem **familyDerivativeExpansion_of_isDS_of_coefOf** (H : Hyps) (C : ℝ) (hC : 0 ≤ C) (P : M.Par) (Wp Wq : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wp] [FiniteDimensional ℂ Wq] (p : KB.Pl P Wp Wq) (lam : Fin (2 * P.r + P.k + 1) → ℝ) (b bv ga h : ℝ) (q0 a₁ : ℕ) (R0 : Rep P.k P.r) (f : B.Mat P.k P.r → (Wq →L[ℂ] Wp)) (ν₀ : Fin (2 * P.r + P.k + 1) → ℤ) (hN : Num P.k P.r lam P.u b bv ga h q0) (hN' : Num P.k P.r lam P.u' b bv ga h q0) (hT : P.T ≤ b ^ (C * ((2 * P.r + 2 : ℕ) : ℝ))) (hT' : P.T' ≤ b ^ (C * ((2 * P.r + 2 : ℕ) : ℝ))) (hq : n1 ν₀ + (P.r + 1) * a₁ + (P.r + 1) * (P.r + 2) ≤ q0) (hDS : R0.IsDS (PI.BesselPeriodGGP.eps48 P.k P.r) (fun a => lam a + (ν₀ a : ℝ))) (hf : CoefOf R0 f) : Formalism.FourierDecayAlt.P.CutBP.FamilyDerivativeExpansion p f b (C / 2 + 6) a₁

theorem **bigCellDerivativeExpansion_of_isDS_of_coefOf** (H : Hyps) (C : ℝ) (hC : 0 ≤ C) (P : M.Par) (Wp Wq : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wp] [FiniteDimensional ℂ Wq] (p : KB.Pl P Wp Wq) (lam : Fin (2 * P.r + P.k + 1) → ℝ) (b bv ga h : ℝ) (q0 a₁ : ℕ) (R0 : Rep P.k P.r) (f : B.Mat P.k P.r → (Wq →L[ℂ] Wp)) (ν₀ : Fin (2 * P.r + P.k + 1) → ℤ) (hN : Num P.k P.r lam P.u b bv ga h q0) (hN' : Num P.k P.r lam P.u' b bv ga h q0) (hT : P.T ≤ b ^ (C * ((2 * P.r + 2 : ℕ) : ℝ))) (hT' : P.T' ≤ b ^ (C * ((2 * P.r + 2 : ℕ) : ℝ))) (hq : n1 ν₀ + (P.r + 1) * a₁ + (P.r + 1) * (P.r + 2) ≤ q0) (hDS : R0.IsDS (PI.BesselPeriodGGP.eps48 P.k P.r) (fun a => lam a + (ν₀ a : ℝ))) (hf : CoefOf R0 f) : Formalism.FourierDecayAlt.P.CutBP.BigCellDerivativeExpansion p f b (C / 2 + 6) a₁

CompanionTheory.AutomorphicTheory.BesselWaveGrids.Bridge

theorem **cutWord_of_list** (P : M.Par) (l0 : ℕ) (L : List C.Letter) (hc : ∀ l ∈ L, I.OnCut P.u P.u' l0 l.2) (hp : L.Pairwise fun l l' => 2 < |l.2 − l'.2|) : KB.CutWord P l0 L

theorem **list_of_cutWord** {P : M.Par} {l0 : ℕ} {L : List C.Letter} (h : KB.CutWord P l0 L) : (∀ l ∈ L, I.OnCut P.u P.u' l0 l.2) ∧ L.Pairwise fun l l' => 2 < |l.2 − l'.2|

theorem **two_lt_of_five** {a t : ℝ} (ha : 5 ≤ |a|) (ht : |t| ≤ 1) : 2 < |a + t|

theorem **free_dist** {Y : Finset ℝ} (hsep : ∀ y ∈ Y, ∀ y' ∈ Y, y ≠ y' → 5 ≤ |y − y'|) {y : ℝ} (hy : y ∈ Y) {l : C.Letter} (hl : ∃ y' ∈ Y, y' ≠ y ∧ (l = (true, y') ∨ l = (false, y' + 1))) : 2 < |y − l.2| ∧ 2 < |(y + 1) − l.2|

theorem **gridOK_of_free** (p : KB.Pl P Wp Wq) (nn : ℝ → ℝ) (Y : Finset ℝ) (B : ℝ) (K : ℕ) (w : List C.Letter) (d : ℕ) (grid : Fin K → Fin (d + 1) → ℝ) (hB : 0 ≤ B) (hcut : ∀ y ∈ Y, I.OnCut P.u P.u' p.l0 y ∧ I.OnCut P.u P.u' p.l0 (y + 1)) (hsep : ∀ y ∈ Y, ∀ y' ∈ Y, y ≠ y' → 5 ≤ |y − y'|) (hw : KB.CutWord P p.l0 w) (hY : ∀ a i, grid a i ∈ Y) (hinj : ∀ a i a' i', grid a i = grid a' i' → a = a' ∧ i = i') (hfree : ∀ a i, ∀ l ∈ w, ∃ y ∈ Y, y ≠ grid a i ∧ (l = (true, y) ∨ l = (false, y + 1))) (hq : ∀ a i, (0 < nn (grid a i) ∧ 0 < nn (grid a i + 1) ∧ 0 < (Wd.qw (M.qC P) (D.positionted w) ((grid a i : ℝ) : ℂ)).re) ∧ nn (grid a i) * nn (grid a i + 1) ≤ B ^ 2 * (Wd.qw (M.qC P) (D.positionted w) ((grid a i : ℝ) : ℂ)).re) : TBBx5.GridOK p nn Y B K w d grid

theorem **waveGrid_of_sep** (p : KB.Pl P Wp Wq) (nn : ℝ → ℝ) (Y : Finset ℝ) (B : ℝ) (K N : ℕ) (hB : 0 ≤ B) (hcut : ∀ y ∈ Y, I.OnCut P.u P.u' p.l0 y ∧ I.OnCut P.u P.u' p.l0 (y + 1)) (hsep : ∀ y ∈ Y, ∀ y' ∈ Y, y ≠ y' → 5 ≤ |y − y'|) (hcard : N − K + K * (2 * P.r + 1) ≤ Y.card) (hq : FreeBound p nn Y B K N) : TBBx5.WaveGrid p nn Y B K N

theorem **liftGrid_of_sep** (p : KB.Pl P Wp Wq) (nn : ℝ → ℝ) (Y : Finset ℝ) (B : ℝ) (K N : ℕ) (hB : 0 ≤ B) (hcut : ∀ y ∈ Y, I.OnCut P.u P.u' p.l0 y ∧ I.OnCut P.u P.u' p.l0 (y + 1)) (hsep : ∀ y ∈ Y, ∀ y' ∈ Y, y ≠ y' → 5 ≤ |y − y'|) (hcard : N − K + K * (2 * P.r + 1) ≤ Y.card) (hq : FreeBound p nn Y B K N) : TBBx5.LiftGrid p nn Y B K N

CompanionTheory.AutomorphicTheory.BesselWaveGrids.CfgTie

theorem **cfg_v_step** (i : ℕ) : c.v i = c.v (i + 1) + 4 * c.Del

theorem **cfg_v_mem** (hok : c.Ok) (i : ℕ) (hi1 : 1 ≤ i) (hi : i + 1 ≤ c.k) : c.vlast ≤ c.v i ∧ c.v i ≤ c.v 1

theorem **cfg_v_last** (hk : 1 ≤ c.k) : c.v (c.k − 1) = c.vlast

theorem **cfg_b_pos** (hok : c.Ok) : ∀ j : ℕ, 1 ≤ j → j ≤ c.m → 1 ≤ c.b j

theorem **cfg_inter** (hok : c.Ok) (de : ℕ → ℝ) (hde : ∀ i, de i = 0 ∨ de i = 1) (s : ℕ) (hs1 : 1 ≤ s) (hsk : s ≤ c.k − 1) : c.lam s + 1 / 2 ≤ c.u de s ∧ c.u de (s + 1) ≤ c.lam s − 1 / 2

theorem **cfg_u_le** (hok : c.Ok) (de : ℕ → ℝ) (hde : ∀ i, de i = 0 ∨ de i = 1) (b : ℝ) (ha : c.a ≤ b) (hb1 : c.b 1 ≤ b) (i : ℕ) (hi1 : 1 ≤ i) (hik : i ≤ c.k) : |c.u de i| ≤ b

theorem **lamData_cfg** (r : ℕ) (hm : c.m = 2 * r + 2) (hok : c.Ok) (T T' : ℝ) (de de' : ℕ → ℝ) (hde : ∀ i, de i = 0 ∨ de i = 1) (hde' : ∀ i, de' i = 0 ∨ de' i = 1) (cs b : ℝ) (hc0 : 0 < cs) (hc : 1 + cs ≤ 4 * c.Del) (ha : c.a ≤ b) (hb : ∀ j, 1 ≤ j → j ≤ c.m → c.b j ≤ b) : LamData (cfgPar c r T T' de de') c.lam cs (4 * c.Del) b

theorem **cfg₀_ok** : cfg₀.Ok where

theorem **lamData_cfg₀** : LamData (cfgPar cfg₀ 0 1 1 (fun _ => 0) (fun i => if i = 1 then 1 else 0)) cfg₀.lam 10 (4 * cfg₀.Del) 400

CompanionTheory.AutomorphicTheory.BesselWaveGrids.CoefFacts

theorem **kfin_wsp** (hf : KFinDSCoef ε μ f) : ∃ F ∈ Wsp k r Wp Wq ε μ, ∀ g ∈ B.GV k r, F g = f g

theorem **kfin_ckOn** (hK : PI.KFinSmooth) (hS : CoefSmooth) (hf : KFinDSCoef ε μ f) : B.CkOn ⊤ (B.GV k r) f

theorem **kfin_inCC** (hK : PI.KFinSmooth) (hC : CoefCC) {A h : ℝ} (hD : KDecay k r ε A h) (hg : PI.ZuckTensor.GapAtLeast ε μ h) (hf : KFinDSCoef ε μ f) (m : ℕ) : B.InCC A m f

theorem **kfin_eig** (hK : PI.KFinSmooth) (hS : CoefSmooth) (hE : CoefEig) (hf : KFinDSCoef ε μ f) (Y : ℂ) : ∀ g ∈ B.GV k r, Q.CV k r Y f g = Formalism.QuantumDeterminants.capP (fun s => (μ s : ℂ)) Y • f g

theorem **kfin_ffkb** {P : M.Par} {Wp Wq : Type} [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [FiniteDimensional ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wq] [CompleteSpace Wp] (hK : PI.KFinSmooth) (hS : CoefSmooth) (hE : CoefEig) (hB : SmoothKB) (p : KB.Pl P Wp Wq) {ε : Fin (2 * P.r + P.k + 1) → Bool} {μ : Fin (2 * P.r + P.k + 1) → ℝ} {f : B.Mat P.k P.r → (Wq →L[ℂ] Wp)} (hf : KFinDSCoef ε μ f) {A : ℝ} (hA : ∀ m, m ≤ CompanionTheory.JfS.NN P.k P.r → JF.TransportedAlt P.k P.r A m) (hcc : ∀ m : ℕ, B.InCC A m f) : B.JF p.T.base f ∈ FFKB p

theorem **sq_le_thrX** (k r : ℕ) : ((r : ℝ) + 1) ^ 2 * (2 * (r : ℝ) + k + 1) ^ 2 ≤ thrX k r

theorem **thrX_nonneg** (k r : ℕ) : 0 ≤ thrX k r

theorem **thr_main** {c C A : ℝ} (k r : ℕ) (hc : c ≤ C) (h : C * thrX k r ≤ A) : c * ((r : ℝ) + 1) ^ 4 * (((3 * (r + 1) : ℕ) : ℝ) * (2 * (r : ℝ) + k + 1) ^ 2 + (2 * (r : ℝ) + k + 1) ^ 3) ≤ A

theorem **thr_conv** {c C A : ℝ} (k r : ℕ) (hc : c ≤ C) (hC : 0 ≤ C) (h : C * thrX k r ≤ A) : c * ((r : ℝ) + 1) ^ 2 * ((r : ℝ) * ((2 * r + k + 1 : ℕ) : ℝ) + 1) ≤ A

theorem **thr_AB** {C A : ℝ} (k r : ℕ) (hC : 46 ≤ C) (h : C * thrX k r ≤ A) : B.AB k r ≤ A

theorem **thr_JF** {C A : ℝ} (k r m : ℕ) (hm : m ≤ CompanionTheory.JfS.NN k r) (hC : 46 ≤ C) (h : C * thrX k r ≤ A) : JF.TransportedAlt k r A m

CompanionTheory.AutomorphicTheory.BesselWaveGrids.CoefOfOfWsp

theorem **kfin_map** {H H' : Type} [NormedAddCommGroup H] [InnerProductSpace ℂ H] [NormedAddCommGroup H'] [InnerProductSpace ℂ H'] {σ : B.Mat k r → (H →L[ℂ] H)} {σ' : B.Mat k r → (H' →L[ℂ] H')} (U : H ≃ₗᵢ[ℂ] H') (hU : ∀ g ∈ UH (Hpk k r), ∀ x, U (σ g x) = σ' g (U x)) {v : H} (hv : PI.UpqRep.IsKFinHpk (k

theorem **coefOf_of_wsp** (hU : UniqG) (hK : PI.KFinSmooth) {ε : Fin (2 * r + k + 1) → Bool} {μ : Fin (2 * r + k + 1) → ℝ} {h : ℝ} (hg : GapAtLeast ε μ h) (hh : ((2 * r + k + 1 : ℕ) : ℝ) ^ 3 + 1 < h) (R : BranchingSupport.Rep k r) (hR : R.IsDS ε μ) {F : B.Mat k r → (Wq →L[ℂ] Wp)} (hF : F ∈ Wsp k r Wp Wq ε μ) : BallW.CoefOf R F

theorem **bound_of_inCC** {A : ℝ} {m : ℕ} {F : B.Mat k r → (Wq →L[ℂ] Wp)} (h : B.InCC A m F) : ContinuousOn F (B.GV k r) ∧ ∃ c, 0 ≤ c ∧ ∀ g ∈ B.GV k r, ‖F g‖ ≤ c * B.gN g ^ (−A)

theorem **normSq_eq** (hk : 2 ≤ k) (D : B.RealPlaceData k r Wp Wq) {A : ℝ} (hA : B.AB k r ≤ A) {F : B.Mat k r → (Wq →L[ℂ] Wp)} (hF : B.InCC A 5 F) : B.normSq D F = ENNReal.ofReal (B.beta D F F).re

theorem **normSq_le_sum** (hk : 2 ≤ k) (D : B.RealPlaceData k r Wp Wq) {A : ℝ} (hA : B.AB k r ≤ A) {ι : Type} (S : Finset ι) (F : ι → B.Mat k r → (Wq →L[ℂ] Wp)) (hF : ∀ i ∈ S, B.InCC A 5 (F i)) (ho : ∀ i ∈ S, ∀ j ∈ S, i ≠ j → B.beta D (F i) (F j) = 0) {i : ι} (hi : i ∈ S) : B.normSq D (F i) ≤ B.normSq D (∑ j ∈ S, F j)

theorem **V_dmin** (p : KB.Pl P Wp Wq) {f F₀ : B.Mat P.k P.r → (Wq →L[ℂ] Wp)} (he : ∀ g ∈ B.GV P.k P.r, F₀ g = f g) {j : ℕ} (hj : j ≤ P.r) (a : ℕ) (γ : C.LPt) (h0 : mnr hj γ ^ a ≠ 0) : p.V (fun g => Dmin P.k P.r p.T.base.z p.T.base.z' g (j + 1, by omega) ^ a • F₀ g) γ = mnr hj γ ^ a • p.V f γ

theorem **mnr_bigPt_ne** {r j : ℕ} (hj : j ≤ r) (c : Fin (r + 1) → ℂ) (hc : D.IsLocPt (TBB.bigPt r c) (r + 1)) : mnr hj (TBB.bigPt r c) ≠ 0

theorem **V_sum** : ∃ c₃ : ℝ, ∀ (P : M.Par) (Wp Wq : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wp] [FiniteDimensional ℂ Wq] (p : KB.Pl P Wp Wq) (A : ℝ), c₃ * ((P.r : ℝ) + 1) ^ 2 * ((P.r : ℝ) * ((2 * P.r + P.k + 1 : ℕ) : ℝ) + 1) ≤ A → ∀ (ι : Type) (S : Finset ι) (F : ι → B.Mat P.k P.r → (Wq →L[ℂ] Wp)), (∀ i ∈ S, B.InCC A 5 (F i)) → ∀ γ, p.V (∑ i ∈ S, F i) γ = ∑ i ∈ S, p.V (F i) γ

theorem **weighted** : ∃ c₃ : ℝ, ∀ {t : ∀ n : ℕ, (Fin n → Bool) → ℝ}, ZuckT t → ZuckDualT t → PI.KFinSmooth → CoefCC → CoefEig → PhZero → BetaInfZero → ∀ (P : M.Par) (Wp Wq : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wp] [FiniteDimensional ℂ Wq] [CompleteSpace Wp] (p : KB.Pl P Wp Wq) (f : B.Mat P.k P.r → (Wq →L[ℂ] Wp)) (ε : Fin (2 * P.r + P.k + 1) → Bool) (lam : Fin (2 * P.r + P.k + 1) → ℝ) (j : ℕ) (hj : j ≤ P.r) (a : ℕ) (A h : ℝ) (K : C.LPt → ℝ≥0∞), KFinDSCoef ε lam f → PI.ZuckTensor.GapAtLeast ε lam h → t _ ε + 1 + 2 * (((P.r + 1) * a : ℕ) : ℝ) < h → KDecay P.k P.r ε A (h − 2 * (((P.r + 1) * a : ℕ) : ℝ)) → B.AB P.k P.r ≤ A → c₃ * ((P.r : ℝ) + 1) ^ 2 * ((P.r : ℝ) * ((2 * P.r + P.k + 1 : ℕ) : ℝ) + 1) ≤ A → ∀ γ, D.IsLocPt γ (P.r + 1) → 0 < TBB.Gp p γ → (∀ ν ∈ ball _ ((P.r + 1) * a), ∀ F ∈ Wsp P.k P.r Wp Wq ε (f

CompanionTheory.AutomorphicTheory.BesselWaveGrids.EigStrOfMem

theorem **w_inCC** (hK : PI.KFinSmooth) (hC : CoefCC) {A h : ℝ} (hD : KDecay k r ε A h) (hg : PI.ZuckTensor.GapAtLeast ε μ h) {F : B.Mat k r → (Wq →L[ℂ] Wp)} (hF : F ∈ Wsp k r Wp Wq ε μ) (m : ℕ) : B.InCC A m F

theorem **mul_mem_box** (h2 : SpanPow) {t : ∀ n : ℕ, (Fin n → Bool) → ℝ} (hZ : ZuckT t) (hZd : ZuckDualT t) {h : ℝ} {d : ℕ} {q : B.Mat k r → ℂ} (hq : q ∈ Letters k r ^ d) (hg : PI.ZuckTensor.GapAtLeast ε μ h) (ht : t _ ε + 1 + 2 * (d : ℝ) < h) (L : Wq' →L[ℂ] Wq) {F : B.Mat k r → (Wq →L[ℂ] Wp)} (hF : F ∈ Wsp k r Wp Wq ε μ) : (fun g => q g • (F g).comp L) ∈ ⊔ ν : Box μ (d : ℝ), Wsp k r Wp Wq' ε ν

theorem **w_ffkb** (hK : PI.KFinSmooth) (hS : CoefSmooth) (hE : CoefEig) (hB : SmoothKB) (p : KB.Pl P Wp Wq) {ε : Fin (2 * P.r + P.k + 1) → Bool} {μ : Fin (2 * P.r + P.k + 1) → ℝ} {f : B.Mat P.k P.r → (Wq →L[ℂ] Wp)} (hf : f ∈ Wsp P.k P.r Wp Wq ε μ) {A : ℝ} (hA : ∀ m, m ≤ CompanionTheory.JfS.NN P.k P.r → JF.TransportedAlt P.k P.r A m) (hcc : ∀ m : ℕ, B.InCC A m f) : B.JF p.T.base f ∈ FFKB p

theorem **eigStr_of_mem** (h3 : CoefSmooth) (h4 : CoefCC) (h5 : CoefEig) (h6 : PhZero) (h7 : BetaInfZero) (hK : PI.KFinSmooth) (p : KB.Pl P Wp Wq) (F F₁ : B.Mat P.k P.r → (Wq →L[ℂ] Wp)) (nn : ℝ → ℝ) (Y : Finset ℝ) {ε : Fin (2 * P.r + P.k + 1) → Bool} {lam : Fin (2 * P.r + P.k + 1) → ℝ} {A h h₀ d : ℝ} (hF₁ : F₁ ∈ ⊔ ν : Box lam d, Wsp P.k P.r Wp Wq ε ν) (hFF : ∀ g ∈ B.GV P.k P.r, F g = F₁ g) (hgap : PI.ZuckTensor.GapAtLeast ε lam h) (hA : B.AB P.k P.r ≤ A) (hD : KDecay P.k P.r ε A h₀) (hh : h₀ + 2 * d ≤ h) (hnn : ∀ y, LetPt Y y → CompanionTheory.LimBd.Main.PStar lam d y ≤ nn y) : EigStr p nn F Y

theorem **eigStr_fpmW** (h1 : GfPoly) (h2 : SpanPow) (h3 : CoefSmooth) (h4 : CoefCC) (h5 : CoefEig) (h6 : PhZero) (h7 : BetaInfZero) (hK : PI.KFinSmooth) {t : ∀ n : ℕ, (Fin n → Bool) → ℝ} (hZ : ZuckT t) (hZd : ZuckDualT t) (p : KB.Pl P Wp Wq) (f : B.Mat P.k P.r → (Wq →L[ℂ] Wp)) (p₀ : ℕ) (nn : ℝ → ℝ) (Y : Finset ℝ) {ε : Fin (2 * P.r + P.k + 1) → Bool} {lam : Fin (2 * P.r + P.k + 1) → ℝ} {A h h₀ : ℝ} (hf : f ∈ Wsp P.k P.r Wp Wq ε lam) (hgap : PI.ZuckTensor.GapAtLeast ε lam h) (ht : t _ ε + 1 + 2 * (((2 * P.r + 2) * p₀ : ℕ) : ℝ) < h) (hA : B.AB P.k P.r ≤ A) (hD : KDecay P.k P.r ε A h₀) (hh : h₀ + 2 * (((2 * P.r + 2) * p₀ : ℕ) : ℝ) ≤ h) (hnn : ∀ y, LetPt Y y → CompanionTheory.LimBd.Main.PStar lam (((2 * P.r + 2) * p₀ : ℕ) : ℝ) y ≤ nn y) : EigStr p nn (TBBx4.fpm p f (fun _ => ContinuousLinearMap.id ℂ Wq) p₀) Y

theorem **eigStr_fpmShW** {u u' : ℕ → ℝ} (h1 : GfPoly) (h2 : SpanPow) (h3 : CoefSmooth) (h4 : CoefCC) (h5 : CoefEig) (h6 : PhZero) (h7 : BetaInfZero) (hK : PI.KFinSmooth) {t : ∀ n : ℕ, (Fin n → Bool) → ℝ} (hZ : ZuckT t) (hZd : ZuckDualT t) (p : KB.Pl (shP P u u') Wp Wq') (f : B.Mat P.k P.r → (Wq →L[ℂ] Wp)) (Aop : B.Mat P.k P.r → (Wq' →L[ℂ] Wq)) (p₀ e₀ : ℕ) (nn : ℝ → ℝ) (Y : Finset ℝ) {ε : Fin (2 * P.r + P.k + 1) → Bool} {lam : Fin (2 * P.r + P.k + 1) → ℝ} {A h h₀ : ℝ} (hf : f ∈ Wsp P.k P.r Wp Wq ε lam) (hAop : PolyOp e₀ Aop) (hgap : PI.ZuckTensor.GapAtLeast ε lam h) (ht : t _ ε + 1 + 2 * (((2 * P.r + 2) * p₀ + e₀ : ℕ) : ℝ) < h) (hA : B.AB P.k P.r ≤ A) (hD : KDecay P.k P.r ε A h₀) (hh : h₀ + 2 * (((2 * P.r + 2) * p₀ + e₀ : ℕ) : ℝ) ≤ h) (hnn : ∀ y, LetPt Y y → CompanionTheory.LimBd.Main.PStar lam (((2 * P.r + 2) * p₀ + e₀ : ℕ) : ℝ) y ≤ nn y) : EigStr p nn (fpmSh p f Aop p₀) Y

theorem **endDiffW_of** (hAsm : AsmDiff) (hS : LimSteps) (hB : SmoothKB) (hK : PI.KFinSmooth) {t : ∀ n : ℕ, (Fin n → Bool) → ℝ} (hZ : ZuckT t) (hZd : ZuckDualT t) : EndDiffW t

theorem **endEqW_of** (hAsm : AsmEq) (hW : WigExists) (hS : LimSteps) (hB : SmoothKB) (hK : PI.KFinSmooth) {t : ∀ n : ℕ, (Fin n → Bool) → ℝ} (hZ : ZuckT t) (hZd : ZuckDualT t) : EndEqW t

CompanionTheory.AutomorphicTheory.BesselWaveGrids.EndpointDifferentTypesThreshold

theorem **asymptotic_different_types** : AsmDiff

theorem **asymptotic_equal_types** : AsmEq

CompanionTheory.AutomorphicTheory.BesselWaveGrids.EqShift

theorem **placeSwap_dd** : ∃ Cb : ℝ, 0 ≤ Cb ∧ ∀ (P : M.Par) (Wp Wq Wq' : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [FiniteDimensional ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wq] [NormedAddCommGroup Wq'] [InnerProductSpace ℂ Wq'] [FiniteDimensional ℂ Wq'] (p : KB.Pl P Wp Wq) (u'' : ℕ → ℝ) (π'' : B.Mat P.k P.r → (Wq' →L[ℂ] Wq')) (l0 : ℕ), Q.IsType P.k P.r p.T.base.z' π'' u'' → (shP P P.u u'').OK → 1 ≤ l0 → l0 ≤ P.k → ∃ q : KB.Pl (shP P P.u u'') Wp Wq', SwapTie p q ∧ q.T.base.π' = π'' ∧ q.l0 = l0 ∧ KB.InDDplus2 (shP P P.u u'') q.l0 q.T.base Cb q.0

theorem **jf_swap** (Ds : B.RealPlaceData k r Wp Wq) (Ds' : B.RealPlaceData k r Wp Wq') (hz : Ds'.z = Ds.z) (hz' : Ds'.z' = Ds.z') (hπ : Ds'.π = Ds.π) (hW : Ds'.μW = Ds.μW) (hN : Ds'.μN = Ds.μN) (hW' : Ds'.μW' = Ds.μW') (A : B.Mat k r → (Wq' →L[ℂ] Wq)) (hA : WigEquiv Ds.z Ds.z' Ds.π' Ds'.π' A) (f : B.Mat k r → (Wq →L[ℂ] Wp)) {A₀ : ℝ} (hThr : JF.TransportedAlt k r A₀ 0) (hf : B.InCC A₀ 0 f) {γ : B.Mat k r} (hγ : γ ∈ JF.Om k r Ds.z Ds.z') : B.JF Ds' (fun g => (f g).comp (A g)) γ = (B.JF Ds f γ).comp (A γ)

theorem **jf_ff_of_smooth** (Db : B.RealPlaceData k r Wp Wq) (H : B.Mat k r → (Wq →L[ℂ] Wp)) (O : Set (B.Mat k r)) (hs : Q.SmoothLev k r 0 O (B.JF Db H)) : B.JF Db H ∈ Q.FFl k r Db.z Db.z' Db.π Db.π' 0 O

theorem **ff_swap** (Ds : B.RealPlaceData k r Wp Wq) (Ds' : B.RealPlaceData k r Wp Wq') (hz : Ds'.z = Ds.z) (hz' : Ds'.z' = Ds.z') (hπ : Ds'.π = Ds.π) (hW : Ds'.μW = Ds.μW) (hN : Ds'.μN = Ds.μN) (hW' : Ds'.μW' = Ds.μW') (A : B.Mat k r → (Wq' →L[ℂ] Wq)) (hA : WigEquiv Ds.z Ds.z' Ds.π' Ds'.π' A) (hAs : B.CkOn ⊤ (B.GV k r) A) (f : B.Mat k r → (Wq →L[ℂ] Wp)) {A₀ : ℝ} (hThr : JF.TransportedAlt k r A₀ 0) (hf : B.InCC A₀ 0 f) (hJ : Q.SmoothLev k r 0 (omZ k r Ds.z Ds.z') (B.JF Ds f)) : B.JF Ds' (fun g => (f g).comp (A g)) ∈ Q.FFl k r Ds'.z Ds'.z' Ds'.π Ds'.π' 0 (omZ k r Ds'.z Ds'.z')

theorem **ffkb_swap** (p : KB.Pl P Wp Wq) (q : KB.Pl (shP P u u') Wp Wq') (hT : SwapTie p q) (f : B.Mat P.k P.r → (Wq →L[ℂ] Wp)) (A : B.Mat P.k P.r → (Wq' →L[ℂ] Wq)) (hA : WigEquiv p.T.base.z p.T.base.z' p.T.base.π' q.T.base.π' A) (hAs : B.CkOn ⊤ (B.GV P.k P.r) A) {A₀ : ℝ} (hThr : JF.TransportedAlt P.k P.r A₀ 0) (hf : B.InCC A₀ 0 f) (hJ : B.JF p.T.base f ∈ FFKB p) : B.JF q.T.base (fun g => (f g).comp (A g)) ∈ FFKB q

theorem **shiftAtR_full** (hW : WigExists) : ∃ Cb c₃ : ℝ, 0 ≤ Cb ∧ ∀ (P : M.Par) (Wp Wq : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [FiniteDimensional ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wq] (p : KB.Pl P Wp Wq) (f : B.Mat P.k P.r → (Wq →L[ℂ] Wp)) (b : ℝ) (n₀ l : ℕ) (A₀ : ℝ), P.u = P.u' → 1 ≤ l → l < P.k → (2 * n₀ + 1 : ℝ) ≤ P.u' l − P.u' (l + 1) → 3 ≤ b → (∀ i ∈ Finset.Icc 1 P.k, |P.u' i| ≤ b) → c₃ * ((P.r : ℝ) + 1) ^ 2 * ((P.r : ℝ) * ((2 * P.r + P.k + 1 : ℕ) : ℝ) + 1) ≤ A₀ → JF.TransportedAlt P.k P.r A₀ 0 → B.InCC A₀ 1 f → B.InCC A₀ 0 f → B.JF p.T.base f ∈ FFKB p → ∃ S : ShiftAtR p f b n₀, S.up = P.u ∧ S.up' = uPlus P.u' l n₀ ∧ S.um = P.u ∧ S.um' = uMinus P.u' l n₀ ∧ S.pp.l0 = l + 1 ∧ S.pm.l0 = l ∧ KB.InDDplus2 (shP P S.up S.up') S.pp.l0 S.pp.T.base Cb S.pp.0 ∧ KB.InDDplus2 (shP P S.um S.um') S.pm.l0 S.pm.T.base Cb S.pm.0 ∧ B.JF S.pp.T.ba

CompanionTheory.AutomorphicTheory.BesselWaveGrids.FamilyThroughDiscreteSeries

theorem **family_through_discreteSeries** : Hyps → ∀ (k r : ℕ) (lam : Fin (2 * r + k + 1) → ℝ) (u : ℕ → ℝ) (b bv ga h : ℝ) (q0 : ℕ) (R0 : Rep k r) (ν₀ : Fin (2 * r + k + 1) → ℤ), Num k r lam u b bv ga h q0 → R0.IsDS (PI.BesselPeriodGGP.eps48 k r) (fun a => lam a + (ν₀ a : ℝ)) → ∃ R : (Fin (2 * r + k + 1) → ℤ) → Rep k r, IsFam lam q0 R ∧ R ν₀ = R0

theorem **exists_coef_family** : Hyps → ∀ (P : M.Par) (Wp Wq : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wp] [FiniteDimensional ℂ Wq] (p : KB.Pl P Wp Wq) (lam : Fin (2 * P.r + P.k + 1) → ℝ) (b bv ga h : ℝ) (q0 : ℕ) (R : (Fin (2 * P.r + P.k + 1) → ℤ) → Rep P.k P.r) (l : ∀ ν, (R ν).H → Wp) (l' : ∀ ν, (R ν).H → Wq) (dg : (Fin (2 * P.r + P.k + 1) → ℤ) → ℝ) (f : B.Mat P.k P.r → (Wq →L[ℂ] Wp)) (s₀ : ℂ) (ν₀ : Fin (2 * P.r + P.k + 1) → ℤ), n1 ν₀ ≤ q0 → IsFam lam q0 R → (haveI

theorem **exists_tau** : Hyps → ∀ (P : M.Par) (Wp Wq : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wp] [FiniteDimensional ℂ Wq] (p : KB.Pl P Wp Wq) (lam : Fin (2 * P.r + P.k + 1) → ℝ) (b bv ga h : ℝ) (q0 : ℕ) (R : (Fin (2 * P.r + P.k + 1) → ℤ) → Rep P.k P.r) (l : ∀ ν, (R ν).H → Wp) (l' : ∀ ν, (R ν).H → Wq) (dg : (Fin (2 * P.r + P.k + 1) → ℤ) → ℝ) (s : (Fin (2 * P.r + P.k + 1) → ℤ) → ℂ) (J : (Fin (2 * P.r + P.k + 1) → ℤ) → B.Mat P.k P.r → (Wq →L[ℂ] Wp)), Num P.k P.r lam P.u b bv ga h q0 → IsFam lam q0 R → (haveI

theorem **suppliedData_exists** : Hyps → ∀ (P : M.Par) (Wp Wq : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wp] [FiniteDimensional ℂ Wq] (p : KB.Pl P Wp Wq) (lam : Fin (2 * P.r + P.k + 1) → ℝ) (b bv ga h : ℝ) (q0 : ℕ) (R0 : Rep P.k P.r) (f : B.Mat P.k P.r → (Wq →L[ℂ] Wp)) (ν₀ : Fin (2 * P.r + P.k + 1) → ℤ), n1 ν₀ ≤ q0 → Num P.k P.r lam P.u b bv ga h q0 → R0.IsDS (PI.BesselPeriodGGP.eps48 P.k P.r) (fun a => lam a + (ν₀ a : ℝ)) → PI.CapelliEig.IsKFinCoefT (k

theorem **sup_norm** : Hyps → ∀ (P : M.Par) (Wp Wq : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wp] [FiniteDimensional ℂ Wq] (p : KB.Pl P Wp Wq) (lam : Fin (2 * P.r + P.k + 1) → ℝ) (b bv ga h : ℝ) (q0 : ℕ) (R : (Fin (2 * P.r + P.k + 1) → ℤ) → Rep P.k P.r) (l : ∀ ν, (R ν).H → Wp) (l' : ∀ ν, (R ν).H → Wq) (dg : (Fin (2 * P.r + P.k + 1) → ℤ) → ℝ) (s : (Fin (2 * P.r + P.k + 1) → ℤ) → ℂ) (J : (Fin (2 * P.r + P.k + 1) → ℤ) → B.Mat P.k P.r → (Wq →L[ℂ] Wp)) (tau : (Fin (2 * P.r + P.k + 1) → ℤ) → Finset (Fin (2 * P.r + P.k + 1))) (n1 ν ≤ q0 → (J ν = 0 ∧ TBB.jN2 p (J ν) = 0 ∧ ¬ (NzOn (R ν) (l ν) ∧ NzOn (R ν) (l' ν))) ∨ (J ν ≠ 0 ∧ NzOn (R ν) (l ν) ∧ NzOn (R ν) (l' ν) ∧ s ν ≠ 0 ∧ TBB.jN2 p (J ν) = ENNReal.ofReal (c * Formalism.Qu

theorem **coef_norm_finite_and_zero** : Hyps → ∀ (P : M.Par) (Wp Wq : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wp] [FiniteDimensional ℂ Wq] (p : KB.Pl P Wp Wq) (lam : Fin (2 * P.r + P.k + 1) → ℝ) (b bv ga h : ℝ) (q0 : ℕ) (R0 : Rep P.k P.r) (f : B.Mat P.k P.r → (Wq →L[ℂ] Wp)) (ν₀ : Fin (2 * P.r + P.k + 1) → ℤ), n1 ν₀ ≤ q0 → Num P.k P.r lam P.u b bv ga h q0 → R0.IsDS (PI.BesselPeriodGGP.eps48 P.k P.r) (fun a => lam a + (ν₀ a : ℝ)) → PI.CapelliEig.IsKFinCoefT (k

theorem **sup_meas** : Hyps → ∀ (P : M.Par) (Wp Wq : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wp] [FiniteDimensional ℂ Wq] (p : KB.Pl P Wp Wq) (lam : Fin (2 * P.r + P.k + 1) → ℝ) (b bv ga h : ℝ) (q0 : ℕ) (R : (Fin (2 * P.r + P.k + 1) → ℤ) → Rep P.k P.r) (l : ∀ ν, (R ν).H → Wp) (l' : ∀ ν, (R ν).H → Wq) (dg : (Fin (2 * P.r + P.k + 1) → ℤ) → ℝ) (s : (Fin (2 * P.r + P.k + 1) → ℤ) → ℂ) (J : (Fin (2 * P.r + P.k + 1) → ℤ) → B.Mat P.k P.r → (Wq →L[ℂ] Wp)) (tau : (Fin (2 * P.r + P.k + 1) → ℤ) → Finset (Fin (2 * P.r + P.k + 1))) (q0 : ℕ), Num P.k P.r lam P.u b bv ga h q0 → Sup0 p lam q0 R l l' dg s J tau → ∀ ν, AEMeasurable fun c : Fin (P.r + 1) → ℂ => ENNReal.ofReal (TBB.Ew p (fun _ => 1) (J ν) (P.r + 1) [] (TBB.bigPt P.r c))

theorem **tau_bound** : Hyps → ∀ (P : M.Par) (Wp Wq : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wp] [FiniteDimensional ℂ Wq] (p : KB.Pl P Wp Wq) (lam : Fin (2 * P.r + P.k + 1) → ℝ) (b bv ga h : ℝ) (q0 : ℕ) (R : (Fin (2 * P.r + P.k + 1) → ℤ) → Rep P.k P.r) (l : ∀ ν, (R ν).H → Wp) (l' : ∀ ν, (R ν).H → Wq) (dg : (Fin (2 * P.r + P.k + 1) → ℤ) → ℝ) (s : (Fin (2 * P.r + P.k + 1) → ℤ) → ℂ) (J : (Fin (2 * P.r + P.k + 1) → ℤ) → B.Mat P.k P.r → (Wq →L[ℂ] Wp)) (tau : (Fin (2 * P.r + P.k + 1) → ℤ) → Finset (Fin (2 * P.r + P.k + 1))) (q0 : ℕ), Num P.k P.r lam P.u b bv ga h q0 → Num P.k P.r lam P.u' b bv ga h q0 → Sup0 p lam q0 R l l' dg s J tau → ∀ ν, n1 ν + (P.r + 1) * (P.r + 2) ≤ q0 → ∀ I : Finset (Fin (2 * P.r + P.k + 1)), I.card = P.r + 1 → ‖tau ν I‖ ≤ Real.sqrt (Hsq P.r P.T b * Hsq P.r P.T' b) ^ (P.r + 1)

theorem **supplied_pieri** : Hyps → ∀ (P : M.Par) (Wp Wq : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wp] [FiniteDimensional ℂ Wq] (p : KB.Pl P Wp Wq) (lam : Fin (2 * P.r + P.k + 1) → ℝ) (b bv ga h : ℝ) (q0 : ℕ) (R : (Fin (2 * P.r + P.k + 1) → ℤ) → Rep P.k P.r) (l : ∀ ν, (R ν).H → Wp) (l' : ∀ ν, (R ν).H → Wq) (dg : (Fin (2 * P.r + P.k + 1) → ℤ) → ℝ) (s : (Fin (2 * P.r + P.k + 1) → ℤ) → ℂ) (J : (Fin (2 * P.r + P.k + 1) → ℤ) → B.Mat P.k P.r → (Wq →L[ℂ] Wp)) (tau : (Fin (2 * P.r + P.k + 1) → ℤ) → Finset (Fin (2 * P.r + P.k + 1))) (q0 : ℕ), Num P.k P.r lam P.u b bv ga h q0 → Sup0 p lam q0 R l l' dg s J tau → ∀ ν, n1 ν + (P.r + 1) * (P.r + 2) ≤ q0 → ∀ c : Fin (P.r + 1) → ℂ, D.IsLocPt (TBB.bigPt P.r c) (P.r + 1) → 0 < TBB.Gp p (TBB.bigPt P.r c) → (D.ptMatN (TBB.bigPt P.r c) (P.r + 1)).det • p.V (J ν) (TBB.bigPt P.r c) = ∑

theorem **wave_ball_estimate** (H : Hyps) (C : ℝ) (hC : 0 ≤ C) (P : M.Par) (Wp Wq : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wp] [FiniteDimensional ℂ Wq] (p : KB.Pl P Wp Wq) (lam : Fin (2 * P.r + P.k + 1) → ℝ) (b bv ga h : ℝ) (q0 a₁ : ℕ) (R0 : Rep P.k P.r) (f : B.Mat P.k P.r → (Wq →L[ℂ] Wp)) (ν₀ : Fin (2 * P.r + P.k + 1) → ℤ) (hN : Num P.k P.r lam P.u b bv ga h q0) (hN' : Num P.k P.r lam P.u' b bv ga h q0) (hT : P.T ≤ b ^ (C * ((2 * P.r + 2 : ℕ) : ℝ))) (hT' : P.T' ≤ b ^ (C * ((2 * P.r + 2 : ℕ) : ℝ))) (hq : n1 ν₀ + (P.r + 1) * a₁ + (P.r + 1) * (P.r + 2) ≤ q0) (hDS : R0.IsDS (PI.BesselPeriodGGP.eps48 P.k P.r) (fun a => lam a + (ν₀ a : ℝ))) (hf : haveI

theorem **wave_ball_estimate_dual** (H : Hyps) (C : ℝ) (hC : 0 ≤ C) (P : M.Par) (Wp Wq : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wp] [FiniteDimensional ℂ Wq] (p : KB.Pl P Wp Wq) (lam : Fin (2 * P.r + P.k + 1) → ℝ) (b bv ga h : ℝ) (q0 a₁ : ℕ) (R0 : Rep P.k P.r) (f : B.Mat P.k P.r → (Wq →L[ℂ] Wp)) (ν₀ : Fin (2 * P.r + P.k + 1) → ℤ) (hN : Num P.k P.r lam P.u b bv ga h q0) (hN' : Num P.k P.r lam P.u' b bv ga h q0) (hT : P.T ≤ b ^ (C * ((2 * P.r + 2 : ℕ) : ℝ))) (hT' : P.T' ≤ b ^ (C * ((2 * P.r + 2 : ℕ) : ℝ))) (hq : n1 ν₀ + (P.r + 1) * a₁ + (P.r + 1) * (P.r + 2) ≤ q0) (hDS : R0.IsDS (PI.BesselPeriodGGP.eps48 P.k P.r) (fun a => lam a + (ν₀ a : ℝ))) (hf : haveI

CompanionTheory.AutomorphicTheory.BesselWaveGrids.FourLeOfFive

theorem **one_le_gridΛ** (P : M.Par) (b : ℝ) (N : ℕ) : 1 ≤ gridΛ P b N

theorem **gridΛ_sq** (P : M.Par) (b : ℝ) (N : ℕ) (hb : 0 ≤ b) (hTT : 0 < P.T * P.T') : (4 / 3 : ℝ) ^ N * (b ^ (8 * P.r + 20) / (P.T * P.T')) ≤ gridΛ P b N ^ 2

theorem **gridΛ_le** (P : M.Par) (b CT : ℝ) (N : ℕ) (hN : N ≤ 3 * (P.r + 1)) (hb : 3 ≤ b) (hCT : 0 ≤ CT) (hTT0 : 0 < P.T * P.T') (hTT : b ^ (−(CT * ((2 * P.r + 2 : ℕ) : ℝ))) ≤ P.T * P.T') : gridΛ P b N ≤ b ^ ((6 + CT) * ((2 * P.r + 2 : ℕ) : ℝ))

theorem **onCut_of_minmax** (u u' : ℕ → ℝ) (l : ℕ) (Y : ℝ) (h1 : min (u l) (u' l) + 2 ≤ −Y) (h2 : −Y ≤ max (u l) (u' l) − 2) : I.OnCut u u' l Y

theorem **gridY_cut** (P : M.Par) (l N : ℕ) (hlen : 5 * ((N * (2 * P.r + 1) : ℕ) : ℝ) ≤ |P.u l − P.u' l|) : ∀ y ∈ gridY P l N, I.OnCut P.u P.u' l y ∧ I.OnCut P.u P.u' l (y + 1)

private theorem **exists_ge_four_fifths_pow_qw_eq_mul_of_far_letters** {S : Type} (q : ℂ → ℂ) (y : ℝ) : ∀ L : List (Formalism.FourierDecayAlt.Wd.Letter ℂ S), (∀ l ∈ L, ∃ d : ℝ, 5 ≤ |d| ∧ ((l.plus = true ∧ l.pt = ((y − d : ℝ) : ℂ)) ∨ (l.plus = false ∧ l.pt = ((y − d + 1 : ℝ) : ℂ)))) → ∃ t : ℝ, (4 / 5 : ℝ) ^ L.length ≤ t ∧

Formalism.FourierDecayAlt.Wd.qw q L (y : ℂ) = (t : ℂ) * q (y : ℂ)

theorem **freeBound_text** (p : KB.Pl P Wp Wq) (K N : ℕ) (lam : ℕ → ℝ) (c g b : ℝ) (hL : PairData P p.l0 lam c g b) (hlen : 5 * ((N * (2 * P.r + 1) : ℕ) : ℝ) ≤ |P.u p.l0 − P.u' p.l0|) : FreeBound p (Pst P.k P.r lam c) (gridY P p.l0 N) (gridΛ P b N) K N

theorem **grids_text** (p : KB.Pl P Wp Wq) (K N : ℕ) (hKN : K ≤ N) (lam : ℕ → ℝ) (c g b : ℝ) (hL : PairData P p.l0 lam c g b) (hlen : 5 * ((N * (2 * P.r + 1) : ℕ) : ℝ) ≤ |P.u p.l0 − P.u' p.l0|) : TBBx5.WaveGrid p (Pst P.k P.r lam c) (gridY P p.l0 N) (gridΛ P b N) K N ∧ TBBx5.LiftGrid p (Pst P.k P.r lam c) (gridY P p.l0 N) (gridΛ P b N) K N

CompanionTheory.AutomorphicTheory.BesselWaveGrids.GapSizes

theorem **sizes** (Cg m N P s : ℝ) (hCg : 1000 ≤ Cg) (hm2 : 2 ≤ m) (hmN : m ≤ N) (hPN : N ^ 3 ≤ P) (hs : 1 ≤ s) : 120 * N ^ 3 ≤ Cg * (m ^ 4 * s) * P / 4 ∧ 125 * m ^ 2 * N ^ 3 ≤ Cg * m ^ 4 * P / 32

theorem **consts** (k r : ℕ) (hk : 1 ≤ k) : Formalism.FourierDecayAlt.ThreeDimData.Mexp r ≤ (2 * (r : ℝ) + 2) ^ 2 / 2 ∧ 0 ≤ Formalism.FourierDecayAlt.ThreeDimData.Mexp r ∧ Formalism.FourierDecayAlt.ThreeDimData.AS k r ≤ 2 * (2 * (r : ℝ) + k + 1) ^ 3 ∧ 0 ≤ Formalism.FourierDecayAlt.ThreeDimData.AS k r ∧ Formalism.FourierDecayAlt.ThreeDimData.AG k r ≤ 4 * (2 * (r : ℝ) + k + 1) ^ 3 ∧ 0 ≤ Formalism.FourierDecayAlt.ThreeDimData.AG k r

theorem **gapOK_of** (k r : ℕ) (hk : 1 ≤ k) (Cg : ℝ) (hCg : 1000 ≤ Cg) : Formalism.FourierDecayAlt.ThreeDimData.GapOK k r (Cg * ((2 * r + 2 : ℕ) : ℝ) ^ ((9 : ℝ) / 2) * (((3 * (r + 1) : ℕ) : ℝ) * ((2 * r + k + 1 : ℕ) : ℝ) ^ 2 + ((2 * r + k + 1 : ℕ) : ℝ) ^ 3) / 4)

CompanionTheory.AutomorphicTheory.BesselWaveGrids.GridsText

theorem **pairData_plus** (h : HypLam P.k P.r lam c g b) (hu : HypType P.k P.r lam P.u b) (hl1 : 1 ≤ l) (hlk : l < P.k) (hposition : (2 * n₀ + 1 : ℝ) ≤ P.u l − P.u (l + 1)) : PairData (shP P P.u (uPlus P.u l n₀)) (l + 1) (lamN lam) c g b

theorem **pairData_minus** (h : HypLam P.k P.r lam c g b) (hu : HypType P.k P.r lam P.u b) (hl1 : 1 ≤ l) (hlk : l < P.k) (hposition : (2 * n₀ + 1 : ℝ) ≤ P.u l − P.u (l + 1)) : PairData (shP P P.u (uMinus P.u l n₀)) l (lamN lam) c g b

theorem **len_le** (r : ℕ) : 15 * ((r : ℝ) + 1) * (2 * r + 1) ≤ 30 * ((r : ℝ) + 1) ^ 2 + 6

theorem **grids_PStar** (p : KB.Pl P Wp Wq) (lam : Fin (2 * P.r + P.k + 1) → ℝ) (c g b : ℝ) (hL : PairData P p.l0 (lamN lam) c g b) (hlen : 15 * ((P.r : ℝ) + 1) * (2 * P.r + 1) ≤ |P.u p.l0 − P.u' p.l0|) : TBBx5.WaveGrid p (PStar lam c) (gridY P p.l0 (3 * (P.r + 1))) (gridΛ P b (3 * (P.r + 1))) 3 (3 * (P.r + 1)) ∧ TBBx5.LiftGrid p (PStar lam c) (gridY P p.l0 (3 * (P.r + 1))) (gridΛ P b (3 * (P.r + 1))) 3 (3 * (P.r + 1))

theorem **gridΛ_bounds** (p : KB.Pl P Wp Wq) (b CT : ℝ) (hb : 3 ≤ b) (hCT : 0 ≤ CT) (hTT : b ^ (−(CT * ((2 * P.r + 2 : ℕ) : ℝ))) ≤ P.T * P.T') : 1 ≤ gridΛ P b (3 * (P.r + 1)) ∧ gridΛ P b (3 * (P.r + 1)) ≤ b ^ ((6 + CT) * ((2 * P.r + 2 : ℕ) : ℝ))

theorem **grids_diff** (p : KB.Pl P Wp Wq) (lam : Fin (2 * P.r + P.k + 1) → ℝ) (c g b : ℝ) (h : HypLam P.k P.r lam c g b) (hu : HypType P.k P.r lam P.u b) (hu' : HypType P.k P.r lam P.u' b) (hlen : 30 * ((P.r : ℝ) + 1) ^ 2 + 6 ≤ |P.u p.l0 − P.u' p.l0|) : TBBx5.WaveGrid p (PStar lam c) (gridY P p.l0 (3 * (P.r + 1))) (gridΛ P b (3 * (P.r + 1))) 3 (3 * (P.r + 1)) ∧ TBBx5.LiftGrid p (PStar lam c) (gridY P p.l0 (3 * (P.r + 1))) (gridΛ P b (3 * (P.r + 1))) 3 (3 * (P.r + 1))

theorem **grids_plus** {up up' : ℕ → ℝ} (q : KB.Pl (shP P up up') Wp Wq) (lam : Fin (2 * P.r + P.k + 1) → ℝ) (c g b : ℝ) (l n₀ : ℕ) (h : HypLam P.k P.r lam c g b) (hu : HypType P.k P.r lam P.u b) (hn₀ : n₀ = 30 * (P.r + 1) ^ 2 + 6) (hl1 : 1 ≤ l) (hlk : l < P.k) (hposition : (2 * n₀ + 1 : ℝ) ≤ P.u l − P.u (l + 1)) (e1 : up = P.u) (e2 : up' = uPlus P.u l n₀) (e3 : q.l0 = l + 1) : TBBx5.WaveGrid q (PStar lam c) (gridY (shP P up up') q.l0 (3 * (P.r + 1))) (gridΛ P b (3 * (P.r + 1))) 3 (3 * (P.r + 1)) ∧ TBBx5.LiftGrid q (PStar lam c) (gridY (shP P up up') q.l0 (3 * (P.r + 1))) (gridΛ P b (3 * (P.r + 1))) 3 (3 * (P.r + 1))

theorem **grids_minus** {um um' : ℕ → ℝ} (q : KB.Pl (shP P um um') Wp Wq) (lam : Fin (2 * P.r + P.k + 1) → ℝ) (c g b : ℝ) (l n₀ : ℕ) (h : HypLam P.k P.r lam c g b) (hu : HypType P.k P.r lam P.u b) (hn₀ : n₀ = 30 * (P.r + 1) ^ 2 + 6) (hl1 : 1 ≤ l) (hlk : l < P.k) (hposition : (2 * n₀ + 1 : ℝ) ≤ P.u l − P.u (l + 1)) (e1 : um = P.u) (e2 : um' = uMinus P.u l n₀) (e3 : q.l0 = l) : TBBx5.WaveGrid q (PStar lam c) (gridY (shP P um um') q.l0 (3 * (P.r + 1))) (gridΛ P b (3 * (P.r + 1))) 3 (3 * (P.r + 1)) ∧ TBBx5.LiftGrid q (PStar lam c) (gridY (shP P um um') q.l0 (3 * (P.r + 1))) (gridΛ P b (3 * (P.r + 1))) 3 (3 * (P.r + 1))

CompanionTheory.AutomorphicTheory.BesselWaveGrids.InterlacingGrids.Bracket

theorem **bracket_le** (k r : ℕ) (lam v : ℕ → ℝ) (e : ℕ) (c g b Y ε : ℝ) (hk : 2 ≤ k) (hc : 0 ≤ c) (hg : 1 + c ≤ g) (hε : ε = 0 ∨ ε = 1) (hcb : c + 1 ≤ b) (hY : |Y| ≤ b) (hlam : ∀ s, 1 ≤ s → s ≤ k + 2 * r + 1 → |lam s| ≤ b) (hgap : ∀ s, 1 ≤ s → s + 1 ≤ k − 1 → lam (s + 1) + g ≤ lam s) (hp : PairOK k lam v e (−Y)) (hden : ∀ i, 1 ≤ i → i ≤ k → 1 ≤ |Y + 1 / 2 + v i|) : ∏ s ∈ Icc 1 (k + 2 * r + 1), (|Y + ε + lam s| + c) ≤ (2 + c) ^ 2 * (k : ℝ) ^ 2 * (3 * b) ^ (2 * r + 3) * ∏ i ∈ Icc 1 k, |Y + 1 / 2 + v i|

CompanionTheory.AutomorphicTheory.BesselWaveGrids.InterlacingGrids.CloudPair

theorem **cloud_pair_factor** (a d c D : ℝ) (hc : 0 ≤ c) (ha : a ≤ d + 1 + c) (hD1 : d ≤ D) (hD2 : 1 ≤ D) : a ≤ (1 + (1 + c) / max d 1) * D

theorem **cloud_pair_le** (k : ℕ) (lam v : ℕ → ℝ) (e : ℕ) (c g b Y ε : ℝ) (hk : 2 ≤ k) (hc : 0 ≤ c) (hg : 1 + c ≤ g) (hε : ε = 0 ∨ ε = 1) (hcb : c + 1 ≤ b) (hY : |Y| ≤ b) (hlam : ∀ s, 1 ≤ s → s ≤ k − 1 → |lam s| ≤ b) (hgap : ∀ s, 1 ≤ s → s + 1 ≤ k − 1 → lam (s + 1) + g ≤ lam s) (hp : PairOK k lam v e (−Y)) (hden : ∀ i, 1 ≤ i → i ≤ k → 1 ≤ |Y + 1 / 2 + v i|) : ∏ s ∈ Icc 1 (k − 1), (|Y + ε + lam s| + c) ≤ 3 * b * (∏ s ∈ Icc 1 (k − 1), (1 + (1 + c) / max |Y + lam s| 1)) * ∏ i ∈ Icc 1 k, |Y + 1 / 2 + v i|

CompanionTheory.AutomorphicTheory.BesselWaveGrids.InterlacingGrids.CloudProd

theorem **cloud_weight_prod** (c : ℝ) (n : ℕ) : ∏ j ∈ range (n + 1), (if j = 0 then 2 + c else 1 + 1 / (j : ℝ)) = (2 + c) * (n + 1)

theorem **cloud_factor_le** (c g d : ℝ) (j : ℕ) (hc : 0 ≤ c) (hg : 1 + c ≤ g) (hd : (j : ℝ) * g ≤ d) : 1 + (1 + c) / max d 1 ≤ (if j = 0 then 2 + c else 1 + 1 / (j : ℝ))

theorem **prod_le_prod_range_of_rank** (A : Finset ℕ) (F : ℕ → ℝ) (φ : ℕ → ℕ) (w : ℕ → ℝ) (k : ℕ) (hF0 : ∀ s ∈ A, 0 ≤ F s) (hF : ∀ s ∈ A, F s ≤ w (φ s)) (hinj : Set.InjOn φ A) (hmap : ∀ s ∈ A, φ s < k) (hw : ∀ j, 1 ≤ w j) : ∏ s ∈ A, F s ≤ ∏ j ∈ range k, w j

theorem **cloud_gap_sum** (k : ℕ) (lam : ℕ → ℝ) (g : ℝ) (hgap : ∀ s, 1 ≤ s → s + 1 ≤ k − 1 → lam (s + 1) + g ≤ lam s) (s : ℕ) (hs : 1 ≤ s) : ∀ s', s ≤ s' → s' ≤ k − 1 → lam s' + ((s' : ℝ) − s) * g ≤ lam s

theorem **cloud_prod_le** (k : ℕ) (lam : ℕ → ℝ) (c g Y : ℝ) (hk : 1 ≤ k) (hc : 0 ≤ c) (hg : 1 + c ≤ g) (hgap : ∀ s, 1 ≤ s → s + 1 ≤ k − 1 → lam (s + 1) + g ≤ lam s) : ∏ s ∈ Icc 1 (k − 1), (1 + (1 + c) / max |Y + lam s| 1) ≤ (2 + c) ^ 2 * (k : ℝ) ^ 2

CompanionTheory.AutomorphicTheory.BesselWaveGrids.InterlacingGrids.FreeFamily

theorem **exists_free_family** (Y : Finset ℝ) (w : List (Bool × ℝ)) (K D : ℕ) (hw : ∀ l ∈ w, ∃ y ∈ Y, l = (true, y) ∨ l = (false, y + 1)) (hcard : w.length + K * D ≤ Y.card) : ∃ g : Fin K → Fin D → ℝ, (∀ a i, g a i ∈ Y) ∧ (∀ a i a' i', g a i = g a' i' → a = a' ∧ i = i') ∧ ∀ a i, ∀ l ∈ w, ∃ y ∈ Y, y ≠ g a i ∧ (l = (true, y) ∨ l = (false, y + 1))

CompanionTheory.AutomorphicTheory.BesselWaveGrids.InterlacingGrids.GridSum

theorem **grid_sqrt_le** {K d : ℕ} (grid : Fin K → Fin (d + 1) → ℝ) (nn Q : ℝ → ℝ) (β : ℝ) (hβ : 0 ≤ β) (hpos : ∀ a i, 0 < nn (grid a i) ∧ 0 < nn (grid a i + 1) ∧ 0 < Q (grid a i)) (hb : ∀ a i, nn (grid a i) * nn (grid a i + 1) ≤ β ^ 2 * Q (grid a i)) (n : Fin K → Fin (d + 1)) : Real.sqrt (((List.ofFn fun a => grid a (n a)).map nn).prod * ((List.ofFn fun a => grid a (n a)).map fun y => nn (y + 1)).prod / ((List.ofFn fun a => grid a (n a)).map Q).prod) ≤ β ^ K

theorem **grid_sum_le** {K d : ℕ} (grid c : Fin K → Fin (d + 1) → ℝ) (nn Q : ℝ → ℝ) (β : ℝ) (hβ : 0 ≤ β) (hc : ∀ a, ∑ i, |c a i| ≤ 1) (hpos : ∀ a i, 0 < nn (grid a i) ∧ 0 < nn (grid a i + 1) ∧ 0 < Q (grid a i)) (hb : ∀ a i, nn (grid a i) * nn (grid a i + 1) ≤ β ^ 2 * Q (grid a i)) : ∑ n : Fin K → Fin (d + 1), |∏ a, c a (n a)| * Real.sqrt (((List.ofFn fun a => grid a (n a)).map nn).prod * ((List.ofFn fun a => grid a (n a)).map fun y => nn (y + 1)).prod / ((List.ofFn fun a => grid a (n a)).map Q).prod) ≤ β ^ K

CompanionTheory.AutomorphicTheory.BesselWaveGrids.InterlacingGrids.LagSum

theorem **lag_abs_term_le** {d : ℕ} (y : Fin (d + 1) → ℝ) (h : ∀ i j, i ≠ j → 5 ≤ |y i − y j|) (j : Fin (d + 1)) : |∏ i ∈ (univ : Finset (Fin (d + 1))).erase j, (y j − y i)⁻¹| ≤ (1 / 5 : ℝ) ^ d

theorem **lag_abs_sum_le_one** {d : ℕ} (y : Fin (d + 1) → ℝ) (h : ∀ i j, i ≠ j → 5 ≤ |y i − y j|) : ∑ j, |∏ i ∈ (univ : Finset (Fin (d + 1))).erase j, (y j − y i)⁻¹| ≤ 1

CompanionTheory.AutomorphicTheory.BesselWaveGrids.InterlacingGrids.PairOKOfInterlace

theorem **Pst_pos** (k r : ℕ) (lam : ℕ → ℝ) {c : ℝ} (hc : 0 < c) (Y : ℝ) : 0 < Pst k r lam c Y

theorem **pairOK_of_interlace** (k : ℕ) (lam v : ℕ → ℝ) (e : ℕ) (x : ℝ) (h : ∀ s, 1 ≤ s → s ≤ k − 1 → lam s + 1 / 2 ≤ v s ∧ v (s + 1) ≤ lam s − 1 / 2) : PairOK k lam v e x

theorem **pairOK_moved** (k l : ℕ) (lam u v : ℕ → ℝ) (x : ℝ) (h : ∀ s, 1 ≤ s → s ≤ k − 1 → lam s + 1 / 2 ≤ u s ∧ u (s + 1) ≤ lam s − 1 / 2) (hv : ∀ i, i ≠ l → i ≠ l + 1 → v i = u i) (hlow : ∀ s, 1 ≤ s → s < l → x < lam s) (hhigh : ∀ s, l < s → s ≤ k − 1 → lam s ≤ x) : PairOK k lam v l x

CompanionTheory.AutomorphicTheory.BesselWaveGrids.InterlacingGrids.ProgYInjective

lemma **progY_injective** (y0 : ℝ) : Function.Injective fun i : ℕ => y0 + 5 * (i : ℝ)

theorem **progY_card** (y0 : ℝ) (M : ℕ) : (progY y0 M).card = M

theorem **progY_sep** (y0 : ℝ) (M : ℕ) : ∀ y ∈ progY y0 M, ∀ y' ∈ progY y0 M, y ≠ y' → 5 ≤ |y − y'|

theorem **progY_cut** (lo hi : ℝ) (M : ℕ) (hlen : 5 * (M : ℝ) ≤ hi − lo) : ∀ y ∈ progY (2 − hi) M, (lo + 2 ≤ −y ∧ −y ≤ hi − 2) ∧ (lo + 2 ≤ −(y + 1) ∧ −(y + 1) ≤ hi − 2)

CompanionTheory.AutomorphicTheory.BesselWaveGrids.InterlacingGrids.PstMul

theorem **bracket_const_le** (k r : ℕ) (c b : ℝ) (hc : 0 ≤ c) (hb : 3 ≤ b) (hkb : (k : ℝ) ≤ b) (hcb : c + 2 ≤ b) : (2 + c) ^ 2 * (k : ℝ) ^ 2 * (3 * b) ^ (2 * r + 3) ≤ b ^ (4 * r + 10)

theorem **pst_mul_le** (k r : ℕ) (lam u v : ℕ → ℝ) (e e' : ℕ) (c g b Y : ℝ) (hk : 2 ≤ k) (hc : 0 ≤ c) (hg : 1 + c ≤ g) (hb : 3 ≤ b) (hkb : (k : ℝ) ≤ b) (hcb : c + 2 ≤ b) (hY : |Y| ≤ b) (hlam : ∀ s, 1 ≤ s → s ≤ k + 2 * r + 1 → |lam s| ≤ b) (hgap : ∀ s, 1 ≤ s → s + 1 ≤ k − 1 → lam (s + 1) + g ≤ lam s) (hp : PairOK k lam u e (−Y)) (hp' : PairOK k lam v e' (−Y)) (hden : ∀ i, 1 ≤ i → i ≤ k → 1 ≤ |Y + 1 / 2 + u i|) (hden' : ∀ i, 1 ≤ i → i ≤ k → 1 ≤ |Y + 1 / 2 + v i|) : Pst k r lam c Y * Pst k r lam c (Y + 1) ≤ b ^ (8 * r + 20) * ∏ i ∈ Icc 1 k, (|Y + 1 / 2 + u i| * |Y + 1 / 2 + v i|)

CompanionTheory.AutomorphicTheory.BesselWaveGrids.KDecayOfGap

theorem **pStar_box** {n : ℕ} {lam lam' : Fin n → ℝ} {e d c : ℝ} (h : lam' ∈ Box lam e) (hd : 0 ≤ d) (hc : e + d ≤ c) (y : ℝ) : CompanionTheory.LimBd.Main.PStar lam' d y ≤ PStar lam c y

theorem **kDecay_of_gap** {t : ∀ n : ℕ, (Fin n → Bool) → ℝ} (hZ : ZuckT t) (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) {X d h : ℝ} (hX : 0 ≤ X) (hd : 0 ≤ d) (hh : t _ ε + X + 3 * d + 1 < h) : KDecay k r ε (X + d) (h − 2 * d)

theorem **gap_mono** {n : ℕ} {ε : Fin n → Bool} {lam : Fin n → ℝ} {h h' : ℝ} (hg : PI.ZuckTensor.GapAtLeast ε lam h) (hh : h' ≤ h) : PI.ZuckTensor.GapAtLeast ε lam h'

theorem **endDiff_of** (hAsm : AsmDiff) (hS : LimSteps) (hB : SmoothKB) (hK : PI.KFinSmooth) {t : ∀ n : ℕ, (Fin n → Bool) → ℝ} (hZ : ZuckT t) (hZd : ZuckDualT t) : EndDiff t

theorem **endEq_of** (hAsm : AsmEq) (hW : WigExists) (hS : LimSteps) (hB : SmoothKB) (hK : PI.KFinSmooth) {t : ∀ n : ℕ, (Fin n → Bool) → ℝ} (hZ : ZuckT t) (hZd : ZuckDualT t) : EndEq t

CompanionTheory.AutomorphicTheory.BesselWaveGrids.LettersPoly

theorem **one_mem_letters** : (1 : B.Mat k r → ℂ) ∈ Letters k r

theorem **letters_pow_succ** (a : ℕ) : Letters k r ^ a ≤ Letters k r ^ (a + 1)

theorem **letters_pow_mono** {a d : ℕ} (h : a ≤ d) : Letters k r ^ a ≤ Letters k r ^ d

theorem **mul_mem_letters_pow** {a d : ℕ} {x y : B.Mat k r → ℂ} (hx : x ∈ Letters k r ^ a) (hy : y ∈ Letters k r ^ d) : x * y ∈ Letters k r ^ (a + d)

theorem **entry_mem** (i j : Fin (2 * r + k + 1)) : (fun g : B.Mat k r => g i j) ∈ Letters k r

theorem **star_entry_mem** (i j : Fin (2 * r + k + 1)) : (fun g : B.Mat k r => star (g i j)) ∈ Letters k r

theorem **polyDeg_mem** {e : ℕ} {c : B.Mat k r → ℂ} (hc : U.IsPolyDeg e c) : c ∈ Letters k r ^ e

CompanionTheory.AutomorphicTheory.BesselWaveGrids.LimBdSh

theorem **eigStr_fpmSh** [CompleteSpace Wp] (h1 : GfPoly) (h2 : SpanPow) (h3 : CoefSmooth) (h4 : CoefCC) (h5 : CoefEig) (h6 : PhZero) (h7 : BetaInfZero) (hK : PI.KFinSmooth) {t : ∀ n : ℕ, (Fin n → Bool) → ℝ} (hZ : ZuckT t) (hZd : ZuckDualT t) (p : KB.Pl (shP P u u') Wp Wq') (f : B.Mat P.k P.r → (Wq →L[ℂ] Wp)) (Aop : B.Mat P.k P.r → (Wq' →L[ℂ] Wq)) (p₀ e₀ : ℕ) (nn : ℝ → ℝ) (Y : Finset ℝ) {ε : Fin (2 * P.r + P.k + 1) → Bool} {lam : Fin (2 * P.r + P.k + 1) → ℝ} {A h h₀ : ℝ} (hf : KFinDSCoef ε lam f) (hAop : PolyOp e₀ Aop) (hgap : PI.ZuckTensor.GapAtLeast ε lam h) (ht : t _ ε + 1 + 2 * (((2 * P.r + 2) * p₀ + e₀ : ℕ) : ℝ) < h) (hA : B.AB P.k P.r ≤ A) (hD : KDecay P.k P.r ε A h₀) (hh : h₀ + 2 * (((2 * P.r + 2) * p₀ + e₀ : ℕ) : ℝ) ≤ h) (hnn : ∀ y, LetPt Y y → PStar lam (((2 * P.r + 2) * p₀ + e₀ : ℕ) : ℝ) y ≤ nn y) : EigStr p nn (fpmSh p f Aop p₀) Y

CompanionTheory.AutomorphicTheory.BesselWaveGrids.LimStepsOf

theorem **limSteps_of** (hI : PI.DSInfCharCap.DSInfCharCap) : LimSteps

CompanionTheory.AutomorphicTheory.BesselWaveGrids.NQText

theorem **interlaces_of_OK** (P : M.Par) (hOK : P.OK) : I.Interlaces P.k P.u P.u' P.lo

theorem **den_core** (k : ℕ) (u u' ρ : ℕ → ℝ) (l : ℕ) (Y : ℝ) (hl1 : 1 ≤ l) (hlk : l ≤ k) (h : I.Interlaces k u u' ρ) (hA : u l + 2 ≤ −Y ∧ −Y ≤ u' l − 2) : ∀ i, 1 ≤ i → i ≤ k → 1 ≤ |Y + 1 / 2 + u i| ∧ 1 ≤ |Y + 1 / 2 + u' i|

theorem **den_onCut** (k : ℕ) (u u' ρ : ℕ → ℝ) (l : ℕ) (Y : ℝ) (hl1 : 1 ≤ l) (hlk : l ≤ k) (h : I.Interlaces k u u' ρ) (hc : I.OnCut u u' l Y) : ∀ i, 1 ≤ i → i ≤ k → 1 ≤ |Y + 1 / 2 + u i| ∧ 1 ≤ |Y + 1 / 2 + u' i|

theorem **abs_le_of_onCut** (u u' : ℕ → ℝ) (l : ℕ) (Y b : ℝ) (hc : I.OnCut u u' l Y) (hu : |u l| ≤ b) (hu' : |u' l| ≤ b) : |Y| ≤ b

theorem **qpoly_eq_of_pos** (k : ℕ) (T T' : ℝ) (u u' : ℕ → ℝ) (Y : ℝ) (hT : 0 < T) (hT' : 0 < T') (hq : 0 < I.qpoly k T T' u u' Y) : I.qpoly k T T' u u' Y = T * T' * ∏ i ∈ Icc 1 k, (|Y + 1 / 2 + u i| * |Y + 1 / 2 + u' i|)

theorem **P₀_OK** : P₀.OK where

theorem **lamData₀** : LamData P₀ lam₀ 1 2 40 where

theorem **nq_onCut** (P : M.Par) (hOK : P.OK) (l : ℕ) (hl1 : 1 ≤ l) (hlk : l ≤ P.k) (hk : 2 ≤ P.k) (lam : ℕ → ℝ) (c g b : ℝ) (hL : LamData P lam c g b) (Y : ℝ) (hY : I.OnCut P.u P.u' l Y) : 0 < (M.qC P (Y : ℂ)).re ∧ GroundTheory.BlockD.FourierDecayAlt.WaveGridLiftGrid.Pst P.k P.r lam c Y * GroundTheory.BlockD.FourierDecayAlt.WaveGridLiftGrid.Pst P.k P.r lam c (Y + 1) ≤ b ^ (8 * P.r + 20) / (P.T * P.T') * (M.qC P (Y : ℂ)).re

theorem **nq_onCut_neg_thirty** : 0 < (M.qC P₀ ((−30 : ℝ) : ℂ)).re ∧ GroundTheory.BlockD.FourierDecayAlt.WaveGridLiftGrid.Pst P₀.k P₀.r lam₀ 1 (−30) * GroundTheory.BlockD.FourierDecayAlt.WaveGridLiftGrid.Pst P₀.k P₀.r lam₀ 1 (−30 + 1) ≤ 40 ^ (8 * P₀.r + 20) / (P₀.T * P₀.T') * (M.qC P₀ ((−30 : ℝ) : ℂ)).re

CompanionTheory.AutomorphicTheory.BesselWaveGrids.NormalizedMeasureScaling

theorem **norm_JF_eq** (Bs : B.RealPlaceData k r Wp Wq) (F : B.Mat k r → (Wq →L[ℂ] Wp)) (d : Fin (r + 1) → ℂ) (hd : ∀ a, d a ≠ 0) {γ γ' : B.Mat k r} (hγ : γ ∈ B.GV k r) (hγ' : γ' ∈ B.GV k r) (hZ : Zhat k r Bs.z Bs.z' γ = D.ptMatN (PBJ.bigPt d) (r + 1)) (hZ' : Zhat k r Bs.z Bs.z' γ' = D.ptMatN (PBJ.bigPt d) (r + 1)) (hpos : 0 < Gf k r Bs.z Bs.z' γ) : ‖B.hsVec (B.JF Bs F γ)‖ = ‖B.hsVec (B.JF Bs F γ')‖

theorem **normSq_sec** (hk : 2 ≤ k) (Bs : B.RealPlaceData k r Wp Wq) (F : B.Mat k r → (Wq →L[ℂ] Wp)) (μsec : ℂ → Matrix (Fin (k + 1)) (Fin (k + 1)) ℂ) (hsec : IsSection k r Bs.z Bs.z' μsec) : B.normSq Bs F = jNormSq k r (Tz k r Bs.z) (Tz k r Bs.z') fun q => Real.sqrt (hsNormSq (B.JF Bs F (bigCell k r q.1 (μsec q.2))))

theorem **energy_identity** : EnergyTie

theorem **pairMeas_smul** (L : LocBessel k r) (κ s : ℝ≥0∞) (h : L.μN = s • (withN L κ).μN) : pairMeas L = (s * s) • pairMeas (withN L κ)

theorem **orbSq_le** (L : LocBessel k r) (κ s : ℝ≥0∞) (hs0 : s ≠ 0) (hs : s ≠ ⊤) (h : pairMeas L = s • pairMeas (withN L κ)) (f : Mx k r → (L.Wq →L[ℂ] L.Wp)) (γ : Point ℂ (r + 1)) : orbSq L f γ ≤ ENNReal.ofReal (s.toReal ^ 2) * orbSq (withN L κ) f γ

theorem **jNorm2_eq** (L : LocBessel k r) (κ s : ℝ≥0∞) (h : pairMeas L = s • pairMeas (withN L κ)) (f : Mx k r → (L.Wq →L[ℂ] L.Wp)) (μsec : ℂ → Matrix (Fin (k + 1)) (Fin (k + 1)) ℂ) : jNorm2 L f μsec = ENNReal.ofReal (s.toReal ^ 2) * jNorm2 (withN L κ) f μsec

CompanionTheory.AutomorphicTheory.BesselWaveGrids.NqMovedExample

theorem **LamData.toPair** {P : M.Par} {lam : ℕ → ℝ} {c g b : ℝ} (hL : LamData P lam c g b) {l : ℕ} (hl1 : 1 ≤ l) (hlk : l ≤ P.k) : PairData P l lam c g b where

theorem **pairOK_of_moved** (k l : ℕ) (lam u v : ℕ → ℝ) (x : ℝ) (h : ∀ s, 1 ≤ s → s ≤ k − 1 → lam s + 1 / 2 ≤ u s ∧ u (s + 1) ≤ lam s − 1 / 2) (hv : ∀ i, i ≠ l → v i = u i) (hx : (u l < x ∧ x < v l) ∨ (v l < x ∧ x < u l)) : ∃ e, PairOK k lam v e x

theorem **MovedData.toPair** {P : M.Par} {l : ℕ} {lam : ℕ → ℝ} {c g b : ℝ} (hM : MovedData P l lam c g b) : PairData P l lam c g b where

theorem **nq_onCut_pair** (P : M.Par) (hOK : P.OK) (l : ℕ) (hl1 : 1 ≤ l) (hlk : l ≤ P.k) (hk : 2 ≤ P.k) (lam : ℕ → ℝ) (c g b : ℝ) (hL : PairData P l lam c g b) (Y : ℝ) (hY : I.OnCut P.u P.u' l Y) : 0 < (M.qC P (Y : ℂ)).re ∧ Pst P.k P.r lam c Y * Pst P.k P.r lam c (Y + 1) ≤ b ^ (8 * P.r + 20) / (P.T * P.T') * (M.qC P (Y : ℂ)).re

theorem **P₁_OK** : P₁.OK where

theorem **movedData₁** : MovedData P₁ 2 lam₀ 1 2 40 where

theorem **nq_moved_example** : 0 < (M.qC P₁ ((−10 : ℝ) : ℂ)).re ∧ Pst P₁.k P₁.r lam₀ 1 (−10) * Pst P₁.k P₁.r lam₀ 1 (−10 + 1) ≤ 40 ^ (8 * P₁.r + 20) / (P₁.T * P₁.T') * (M.qC P₁ ((−10 : ℝ) : ℂ)).re

CompanionTheory.AutomorphicTheory.BesselWaveGrids.NqOfLamData

theorem **int_of_half** {k : ℕ} {u v : ℕ → ℝ} (hk : 1 ≤ k) (hu : ∀ i, 1 ≤ i → i ≤ k → ∃ m : ℤ, u i − ((k : ℝ) − 1) / 2 = m) (hv : ∀ i, 1 ≤ i → i ≤ k → ∃ m : ℤ, v i − ((k : ℝ) − 1) / 2 = m) : ∀ i ∈ Icc 1 k, ∃ n : ℤ, v i − u 1 = n

theorem **ok_diff** (P : M.Par) (hk : 1 ≤ P.k) (hT : 0 < P.T) (hT' : 0 < P.T') (lam : Fin (2 * P.r + P.k + 1) → ℝ) (b : ℝ) (hc : Formalism.QuantumDeterminants.IsHCParam lam) (hu : HypType P.k P.r lam P.u b) (hu' : HypType P.k P.r lam P.u' b) : P.OK

theorem **ok_plus** (k r : ℕ) (T T' : ℝ) (hT : 0 < T) (hT' : 0 < T') (lam : Fin (2 * r + k + 1) → ℝ) (u : ℕ → ℝ) (b : ℝ) (hc : Formalism.QuantumDeterminants.IsHCParam lam) (hu : HypType k r lam u b) (l n₀ : ℕ) (hl1 : 1 ≤ l) (hlk : l < k) (hposition : (2 * n₀ + 1 : ℝ) ≤ u l − u (l + 1)) : ((k, r, T, T', u, uP u l n₀) : M.Par).OK

theorem **ok_minus** (k r : ℕ) (T T' : ℝ) (hT : 0 < T) (hT' : 0 < T') (lam : Fin (2 * r + k + 1) → ℝ) (u : ℕ → ℝ) (b : ℝ) (hc : Formalism.QuantumDeterminants.IsHCParam lam) (hu : HypType k r lam u b) (l n₀ : ℕ) (hl1 : 1 ≤ l) (hlk : l < k) (hposition : (2 * n₀ + 1 : ℝ) ≤ u l - u (l + 1)) : ((k, r, T, T', u, uM u l n₀) : M.Par).OK

theorem **nq_plus** (k r : ℕ) (T T' : ℝ) (hT : 0 < T) (hT' : 0 < T') (lam : Fin (2 * r + k + 1) → ℝ) (u : ℕ → ℝ) (c g b : ℝ) (h : HypLam k r lam c g b) (hu : HypType k r lam u b) (l n₀ : ℕ) (hl1 : 1 ≤ l) (hlk : l < k) (hposition : (2 * n₀ + 1 : ℝ) ≤ u l - u (l + 1)) (Y : Finset ℝ) (hY : ∀ y ∈ Y, u (l + 1) + 2 ≤ -y ∧ -y ≤ u (l + 1) + n₀ - 2) : NQ (k, r, T, T', u, uP u l n₀) (PStar lam c) Y (nqΛ (k, r, T, T', u, uP u l n₀) b)

theorem **nq_minus** (k r : ℕ) (T T' : ℝ) (hT : 0 < T) (hT' : 0 < T') (lam : Fin (2 * r + k + 1) → ℝ) (u : ℕ → ℝ) (c g b : ℝ) (h : HypLam k r lam c g b) (hu : HypType k r lam u b) (l n₀ : ℕ) (hl1 : 1 ≤ l) (hlk : l < k) (hposition : (2 * n₀ + 1 : ℝ) ≤ u l - u (l + 1)) (Y : Finset ℝ) (hY : ∀ y ∈ Y, u l - n₀ + 2 ≤ -y ∧ -y ≤ u l - 2) : NQ (k, r, T, T', u, uM u l n₀) (PStar lam c) Y (nqΛ (k, r, T, T', u, uM u l n₀) b)

theorem **lamX_anti** (x : ℤ) (hx : 0 ≤ x) : StrictAnti (lamX x)

theorem **hypLamX** (x : ℤ) (hx : 0 ≤ x) (hx' : x ≤ 79) : HypLam 3 0 (lamX x) 10 20 80

theorem **hypTypeX** (x : ℤ) (hx : 0 ≤ x) (hx' : x ≤ 79) : HypType 3 0 (lamX x) uX 80

theorem **ex_plus** : NQ (3, 0, 1, 1, uX, uP uX 1 36) (PStar (lamX 20) 10) {-3, -20, -21, -34} (nqΛ (3, 0, 1, 1, uX, uP uX 1 36) 80)

theorem **ex_minus** : NQ (3, 0, 1, 1, uX, uM uX 1 36) (PStar (lamX 60) 10) {-47, -60, -61, -78} (nqΛ (3, 0, 1, 1, uX, uM uX 1 36) 80)

theorem **nq_of_lamData** (P : M.Par) (hOK : P.OK) (l : ℕ) (hl1 : 1 ≤ l) (hlk : l ≤ P.k) (hk : 2 ≤ P.k) (lam : ℕ → ℝ) (c g b : ℝ) (hL : LamData P lam c g b) (Y : Finset ℝ) (hY : ∀ y ∈ Y, I.OnCut P.u P.u' l y) : NQ P (PStar (fun s : Fin (2 * P.r + P.k + 1) => lam (s + 1)) c) Y (nqΛ P b)

theorem **cfg₁_ok** : cfg₁.Ok

theorem **lamData_c** : LamData Pc cfg₁.lam 10 (4 * cfg₁.Del) 401

theorem **Pc_OK** : Pc.OK

theorem **ex_cfg** : NQ Pc (PStar (fun s : Fin (2 * Pc.r + Pc.k + 1) => cfg₁.lam (s + 1)) 10) {-107, -112, -117} (nqΛ Pc 401)

CompanionTheory.AutomorphicTheory.BesselWaveGrids.PStarTie

theorem **lamN_succ** {n : ℕ} (lam : Fin n → ℝ) (s : Fin n) : lamN lam (s + 1) = lam s

theorem **Pst_lamN** (k r : ℕ) (lam : Fin (2 * r + k + 1) → ℝ) (c y : ℝ) : Pst k r (lamN lam) c y = PStar lam c y

theorem **PStar_pos** {n : ℕ} (lam : Fin n → ℝ) {c : ℝ} (hc : 0 < c) (y : ℝ) : 0 < PStar lam c y

theorem **half_le_of_pos** {x : ℝ} (m : ℤ) (hx : x = m + 1 / 2) (h : 0 < x) : 1 / 2 ≤ x

theorem **inter_half** {k r : ℕ} {lam : Fin (2 * r + k + 1) → ℝ} (hc : Formalism.QuantumDeterminants.IsHCParam lam) {u : ℕ → ℝ} {b : ℝ} (hu : HypType k r lam u b) (s : ℕ) (hs1 : 1 ≤ s) (hsk : s ≤ k - 1) : lamN lam s + 1 / 2 ≤ u s ∧ u (s + 1) ≤ lamN lam s - 1 / 2

theorem **HypLam.k_le** (h : HypLam P.k P.r lam c g b) : (P.k : ℝ) ≤ b

theorem **HypLam.lamN_le** (h : HypLam P.k P.r lam c g b) : ∀ s, 1 ≤ s → s ≤ P.k + 2 * P.r + 1 → |lamN lam s| ≤ b

theorem **HypLam.lamN_gap** (h : HypLam P.k P.r lam c g b) : ∀ s, 1 ≤ s → s + 1 ≤ P.k - 1 → lamN lam (s + 1) + g ≤ lamN lam s

theorem **lamData_of_hyp** (h : HypLam P.k P.r lam c g b) (hu : HypType P.k P.r lam P.u b) (hu' : HypType P.k P.r lam P.u' b) : LamData P (lamN lam) c g b where

theorem **movedData_of_hyp** (h : HypLam P.k P.r lam c g b) (hu : HypType P.k P.r lam P.u b) {l : ℕ} (hl1 : 1 ≤ l) (hlk : l ≤ P.k) (hmv : ∀ i, i ≠ l → P.u' i = P.u i) (hl : |P.u' l| ≤ b) : MovedData P l (lamN lam) c g b where

theorem **nq_PStar** (P : M.Par) (hOK : P.OK) (l : ℕ) (hl1 : 1 ≤ l) (hlk : l ≤ P.k) (hk : 2 ≤ P.k) (lam : Fin (2 * P.r + P.k + 1) → ℝ) (c g b : ℝ) (hL : PairData P l (lamN lam) c g b) (Y : Finset ℝ) (hY : ∀ y ∈ Y, I.OnCut P.u P.u' l y) : NQ P (PStar lam c) Y (nqΛ P b)

theorem **nqΛ_le** (P : M.Par) (b CT : ℝ) (hb : 1 ≤ b) (hCT : 0 ≤ CT) (hTT : b ^ (-(CT * ((2 * P.r + 2 : ℕ) : ℝ))) ≤ P.T * P.T') : nqΛ P b ≤ b ^ ((5 + CT) * ((2 * P.r + 2 : ℕ) : ℝ))

theorem **nq_text_moved** (P : M.Par) (hOK : P.OK) (l : ℕ) (hl1 : 1 ≤ l) (hlk : l ≤ P.k) (hk : 2 ≤ P.k) (lam : Fin (2 * P.r + P.k + 1) → ℝ) (c g b : ℝ) (h : HypLam P.k P.r lam c g b) (hu : HypType P.k P.r lam P.u b) (hmv : ∀ i, i ≠ l → P.u' i = P.u i) (hl : |P.u' l| ≤ b) (Y : Finset ℝ) (hY : ∀ y ∈ Y, I.OnCut P.u P.u' l y) : NQ P (PStar lam c) Y (nqΛ P b)

theorem **P₂_OK** : P₂.OK where

theorem **lam₂_anti** : StrictAnti lam₂

theorem **hypLam₂** : HypLam P₂.k P₂.r lam₂ 10 20 41 where

theorem **hypType₂** (a : ℤ) (ha : 10 ≤ a) (ha' : a ≤ 40) : HypType P₂.k P₂.r lam₂ (fun i => if i = 1 then (a : ℝ) + 1 / 2 else 1 / 2) 41 where

theorem **hypType₂_u** : HypType P₂.k P₂.r lam₂ P₂.u 41

theorem **hypType₂_u'** : HypType P₂.k P₂.r lam₂ P₂.u' 41

CompanionTheory.AutomorphicTheory.BesselWaveGrids.PowFourLeRpowNineHalves

theorem **pow_four_le_rpow_nine_halves** (r : ℕ) : ((2 * r + 2 : ℕ) : ℝ) ^ 4 ≤ ((2 * r + 2 : ℕ) : ℝ) ^ ((9 : ℝ) / 2)

theorem **thrX_le_gapY** (k r : ℕ) : thrX k r ≤ gapY k r

theorem **mul_le_gapY** (k r : ℕ) : ((2 * r + 2 : ℕ) : ℝ) ^ 4 * ((2 * r + k + 1 : ℕ) : ℝ) ^ 3 ≤ gapY k r

theorem **fourth_power_le_gap** (k r : ℕ) : ((2 * r + 2 : ℕ) : ℝ) ^ 4 ≤ gapY k r

theorem **cube_le_gap** (k r : ℕ) : ((2 * r + k + 1 : ℕ) : ℝ) ^ 3 ≤ gapY k r

theorem **num_of** {k r : ℕ} (hk : 1 ≤ k) {lam : Fin (2 * r + k + 1) → ℝ} {u : ℕ → ℝ} {b bv ga h : ℝ} {q0 : ℕ} (hlam : IsHCParam lam) (hu : HypType k r lam u b) (hc : ∀ a : Fin (2 * r + k + 1), a.val + 1 < k → 0 ≤ lam a ∧ lam a ≤ bv / 8) (hn : ∀ a : Fin (2 * r + k + 1), k ≤ a.val + 1 → -2 * bv ≤ lam a ∧ lam a ≤ -bv) (hb : ∀ a, |lam a| ≤ b) (hg : ∀ a c, a < c → ga ≤ lam a - lam c) (hh : GapAtLeast (PI.BesselPeriodGGP.eps48 k r) lam h) (hok : ThreeDimData.GapOK k r (h / 4)) (h1 : 100 * ((q0 : ℝ) + 1) ≤ ga) (h2 : 100 * ((q0 : ℝ) + 1) ≤ h) (h3 : 10 ^ 6 * ((2 * r + k + 1 : ℕ) : ℝ) ^ 7 ≤ bv) : BranchingSupport.Num k r lam u b bv ga h q0

theorem **bp_max** {P : M.Par} {Wp Wq : Type} [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wp] [FiniteDimensional ℂ Wq] {p : KB.Pl P Wp Wq} {f : B.Mat P.k P.r → (Wq →L[ℂ] Wp)} {b C C' : ℝ} {n₀ p₀ A : ℕ} (hb : 1 ≤ b) (hC : C ≤ C') (hA : 2 * p₀ + n₀ ≤ A) (h : TBB.ThmBpAt p f b C n₀ p₀) (γ : Formalism.FourierDecayAlt.C.LPt) (hγ : D.IsLocPt γ (P.r + 1)) (hG : 0 < TBB.Gp p γ) : ENNReal.ofReal (TBB.Ew p (fun _ => 1) f (P.r + 1) [] γ) ≤ ENNReal.ofReal (b ^ (C' * ((2 * P.r + 2 : ℕ) : ℝ) ^ 4) * max 1 (1 / TBB.Gp p γ) ^ A) * TBB.jN2 p f

theorem **wBp_of** {t : ∀ n : ℕ, (Fin n → Bool) → ℝ} {Ct : ℝ} (hCt : 0 ≤ Ct) (ht : ∀ n ε, 1 ≤ n → t n ε ≤ Ct * (n : ℝ) ^ 3) (hZ : ZuckT t) (hZd : ZuckDualT t) (hS : LimSteps) (hD : EndDiffW t) (hQ : EndEqW t) (hK : PI.KFinSmooth) (H : In91) (hU : UniqG) (Cb : ℝ) : WBp Cb

CompanionTheory.AutomorphicTheory.BesselWaveGrids.ProductMoment

theorem **fixes_wedge** (hj : j ≤ r) {z : Fin (2 * r + k + 1) → ℂ} (hz : Formalism.QuantumDeterminants.inWb k r z) : ∀ s ∈ Formalism.QuantumDeterminants.Sset k r z, cmp (j + 1) s *ᵥ wedge (fvL hj z) = wedge (fvL hj z)

theorem **herm_wedge** (hj : j ≤ r) (z z' : Fin (2 * r + k + 1) → ℂ) (g : Mx k r) : Formalism.QuantumDeterminants.herm (cmp (j + 1) (Formalism.QuantumDeterminants.Hpk k r)) (cmp (j + 1) g *ᵥ wedge (fvL hj z')) (wedge (fvL hj z)) = ((Formalism.QuantumDeterminants.Zhat k r z z' g).submatrix (Fin.castLE (by omega : j + 1 ≤ r + 1)) (Fin.castLE (by omega : j + 1 ≤ r + 1))).det

theorem **length_wedge** (hj : j ≤ r) (z : Fin (2 * r + k + 1) → ℂ) (g : Mx k r) : ∑ J, ‖(cmp (j + 1) g⁻¹ *ᵥ wedge (fvL hj z)) J‖ ^ 2 = ∏ a : Fin (j + 1), lenA k r z g (Fin.castLE (by omega) a) ^ 2

theorem **product_moment** : Hyps → ∀ (k r : ℕ) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (lam : Fin (2 * r + k + 1) → ℝ) (u : ℕ → ℝ) (b bv ga h : ℝ) (q0 : ℕ) (z : Fin (2 * r + k + 1) → ℂ) (π : Mx k r → (Wp →L[ℂ] Wp)) (μW μN μG νq : Measure (Formalism.QuantumDeterminants.Mt k r)) (R : (Fin (2 * r + k + 1) → ℤ) → Rep k r) (l : ∀ ν, (R ν).H → Wp) (dg : (Fin (2 * r + k + 1) → ℤ) → ℝ), Setup lam u b bv ga h q0 z π μW μN μG νq R l dg → ∀ (j : ℕ) (hj : j ≤ r) ν, n1 ν + (r + 1) ^ 2 ≤ q0 → NzOn (R ν) (l ν) → ∃ w ∈ (R ν).sm, ‖w‖ = 1 ∧ ∫⁻ x, ENNReal.ofReal ((∏ a : Fin (j + 1), lenA k r z (Matrix.of x) (Fin.castLE (Nat.succ_le_succ hj) a) ^ 2) * (dg ν * ‖l ν ((R ν).σ (Matrix.of x) w)‖ ^ 2)) ∂νq ≤ ENNReal.ofReal (Hsq r (Formalism.QuantumDeterminants.Tz k r z) b ^ (r + 1))

theorem **kappa_bound** : Hyps → ∀ (k r : ℕ) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (lam : Fin (2 * r + k + 1) → ℝ) (u : ℕ → ℝ) (b bv ga h : ℝ) (q0 : ℕ) (z : Fin (2 * r + k + 1) → ℂ) (π : Mx k r → (Wp →L[ℂ] Wp)) (μW μN μG νq : Measure (Formalism.QuantumDeterminants.Mt k r)) (R : (Fin (2 * r + k + 1) → ℤ) → Rep k r) (l : ∀ ν, (R ν).H → Wp) (dg : (Fin (2 * r + k + 1) → ℤ) → ℝ), Setup lam u b bv ga h q0 z π μW μN μG νq R l dg → ∀ (j : ℕ) (hj : j ≤ r) ν, n1 ν + (r + 1) * (r + 2) ≤ q0 → ∀ io pr ep, TensH (cmp (j + 1)) (cmp (j + 1) (Formalism.QuantumDeterminants.Hpk k r)) (R ν) (fun I : Sub (2 * r + k + 1) (j + 1) => R (ν + eZ I.1)) io pr ep → NzOn (R ν) (l ν) → ∀ (I : Sub (2 * r + k + 1) (j + 1)) (κ₀ : ℂ), NzOn (R (ν + eZ I.1)) (l (ν + eZ I.1)) → (∀ v ∈ (R ν).sm, l (ν + eZ I.1) (∑ c, pr I c (tm (R ν) (wedge (fvL hj z)) v c)) = κ₀ •

theorem **exists_tau** : Hyps → ∀ (P : M.Par) (Wp Wq : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wp] [FiniteDimensional ℂ Wq] (p : KB.Pl P Wp Wq) (lam : Fin (2 * P.r + P.k + 1) → ℝ) (b bv ga h : ℝ) (q0 : ℕ) (R : (Fin (2 * P.r + P.k + 1) → ℤ) → Rep

P.k P.r) (l : ∀ ν, (R ν).H → Wp) (l' : ∀ ν, (R ν).H → Wq) (dg : (Fin (2 * P.r + P.k + 1) → ℤ) → ℝ) (s : (Fin (2 * P.r + P.k + 1) → ℤ) → ℂ) (J : (Fin (2 * P.r + P.k + 1) → ℤ) → B.Mat P.k P.r → (Wq →L[ℂ] Wp)), ∀ tau₀, Num P.k P.r lam P.u b bv ga h q0 → Sup0 p lam q0 R l l' dg s J tau₀ → ∀ (j : ℕ) (hj : j ≤ P.r), ∃ tau, SupL p q0 R l l' s J j hj tau

theorem **tau_bound** : Hyps → ∀ (P : M.Par) (Wp Wq : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wp] [FiniteDimensional ℂ Wq] (p : KB.Pl P Wp Wq) (lam : Fin (2 * P.r + P.k + 1) → ℝ) (b bv ga h : ℝ) (q0 : ℕ) (R : (Fin (2 * P.r + P.k + 1) → ℤ) → Rep P.k P.r) (l : ∀ ν, (R ν).H → Wp) (l' : ∀ ν, (R ν).H → Wq) (dg : (Fin (2 * P.r + P.k + 1) → ℤ) → ℝ) (s : (Fin (2 * P.r + P.k + 1) → ℤ) → ℂ) (J : (Fin (2 * P.r + P.k + 1) → ℤ) → B.Mat P.k P.r → (Wq →L[ℂ] Wp)) (tau₀ : (Fin (2 * P.r + P.k + 1) → ℤ) → Finset (Fin (2 * P.r + P.k + 1)) → ℂ), Num P.k P.r lam P.u b bv ga h q0 → Num P.k P.r lam P.u' b bv ga h q0 → Sup0 p lam q0 R l l' dg s J tau₀ → ∀ (j : ℕ) (hj : j ≤ P.r) tau, SupL p q0 R l l' s J j hj tau → ∀ ν, n1 ν + (P.r + 1) * (P.r + 2) ≤ q0 → ∀ I : Finset (Fin (2 * P.r + P.k + 1)), I.card = j + 1 → ‖tau ν I‖ ≤ Real.sqrt

theorem **supplied_pieri** : Hyps → ∀ (P : M.Par) (Wp Wq : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wp] [FiniteDimensional ℂ Wq] (p : KB.Pl P Wp Wq) (lam : Fin (2 * P.r + P.k + 1) → ℝ) (b bv ga h : ℝ) (q0 : ℕ) (R : (Fin (2 * P.r + P.k + 1) → ℤ) → Rep P.k P.r) (l : ∀ ν, (R ν).H → Wp) (l' : ∀ ν, (R ν).H → Wq) (dg : (Fin (2 * P.r + P.k + 1) → ℤ) → ℝ) (s : (Fin (2 * P.r + P.k + 1) → ℤ) → ℂ) (J : (Fin (2 * P.r + P.k + 1) → ℤ) → B.Mat P.k P.r → (Wq →L[ℂ] Wp)) (tau₀ : (Fin (2 * P.r + P.k + 1) → ℤ) → Finset (Fin (2 * P.r + P.k + 1)) → ℂ), Num P.k P.r lam P.u b bv ga h q0 → Sup0 p lam q0 R l l' dg s J tau₀ → ∀ (j : ℕ) (hj : j ≤ P.r) tau, SupL p q0 R l l' s J j hj tau → ∀ ν, n1 ν + (P.r + 1) * (P.r + 2) ≤ q0 → ∀ c : Fin (P.r + 1) → ℂ, D.IsLocPt (TBB.bigPt P.r c) (P.r + 1) → 0 < TBB.Gp p (TBB.bigPt P.r c) → mnr hj (TBB.

theorem **meas_mnr** {r j : ℕ} (hj : j ≤ r) : Measurable fun c : Fin (r + 1) → ℂ => mnr hj (TBB.bigPt r c)

theorem **ew_sum_le** (p : KB.Pl P Wp Wq) (F : B.Mat P.k P.r → (Wq →L[ℂ] Wp)) {ι : Type} (S : Finset ι) (Fi : ι → B.Mat P.k P.r → (Wq →L[ℂ] Wp)) (co : ι → ℂ) (d : ℂ) (γ : C.LPt) (hexp : D.IsLocPt γ (P.r + 1) → 0 < TBB.Gp p γ → d • p.V F γ = ∑ i ∈ S, co i • p.V (Fi i) γ) : ‖d‖ ^ 2 * TBB.Ew p (fun _ => 1) F (P.r + 1) [] γ ≤ (∑ i ∈ S, ‖co i‖) * ∑ i ∈ S, ‖co i‖ * TBB.Ew p (fun _ => 1) (Fi i) (P.r + 1) [] γ

theorem **momL_le** (H : Hyps) (C : ℝ) (hC : 0 ≤ C) (P : M.Par) (Wp Wq : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wp] [FiniteDimensional ℂ Wq] (p : KB.Pl P Wp Wq) (lam : Fin (2 * P.r + P.k + 1) → ℝ) (b bv ga h : ℝ) (q0 a : ℕ) (R0 : Rep P.k P.r) (f : B.Mat P.k P.r → (Wq →L[ℂ] Wp)) (j : ℕ) (hj : j ≤ P.r) (hN : Num P.k P.r lam P.u b bv ga h q0) (hN' : Num P.k P.r lam P.u' b bv ga h q0) (hT : P.T ≤ b ^ (C * ((2 * P.r + 2 : ℕ) : ℝ))) (hT' : P.T' ≤ b ^ (C * ((2 * P.r + 2 : ℕ) : ℝ))) (hq : (P.r + 1) * a + (P.r + 1) * (P.r + 2) ≤ q0) (hDS : R0.IsDS (PI.BesselPeriodGGP.eps48 P.k P.r) lam) (hf : BallW.CoefOf R0 f) : momL p f hj a ≤ ENNReal.ofReal (b ^ ((C + 12) * ((2 * P.r + 2 : ℕ) : ℝ) ^ 2 * a)) * TBB.jN2 p f

CompanionTheory.AutomorphicTheory.BesselWaveGrids.QwBound

theorem **qw_eq_prod** {S : Type} [Fintype S] [DecidableEq S] (q : ℂ → ℂ) (L : List (Wd.Letter ℂ S)) (Y : ℂ) : Wd.qw q L Y = q Y * (L.map fun l => facW l Y).prod

theorem **positioned_map_facW** (w : List C.Letter) (y : ℝ) : (D.positionted w).map (fun l => facW l (y : ℂ)) = w.map fun l => ((fac l y : ℝ) : ℂ)

theorem **ofReal_prod_map** (w : List C.Letter) (f : C.Letter → ℝ) : (((w.map f).prod : ℝ) : ℂ) = (w.map fun l => ((f l : ℝ) : ℂ)).prod

theorem **qC_ofReal** (P : M.Par) (y : ℝ) : M.qC P (y : ℂ) = ((I.qpoly P.k P.T P.T' P.u P.u' y : ℝ) : ℂ)

theorem **qw_indexed_ofReal** (P : M.Par) (w : List C.Letter) (y : ℝ) : Wd.qw (M.qC P) (D.positionted w) (y : ℂ) = ((I.qpoly P.k P.T P.T' P.u P.u' y * (w.map fun l => fac l y).prod : ℝ) : ℂ)

theorem **fac_ge** (l : C.Letter) (y : ℝ) (h : 4 ≤ |y - l.2|) : 3 / 4 ≤ fac l y

theorem **prod_fac_ge** (w : List C.Letter) (y : ℝ) (h : ∀ l ∈ w, 4 ≤ |y - l.2|) : (3 / 4 : ℝ) ^ w.length ≤ (w.map fun l => fac l y).prod

theorem **qw_re_ge** (P : M.Par) (w : List C.Letter) (y : ℝ) (hq : 0 ≤ I.qpoly P.k P.T P.T' P.u P.u' y) (h : ∀ l ∈ w, 4 ≤ |y - l.2|) : (3 / 4 : ℝ) ^ w.length * I.qpoly P.k P.T P.T' P.u P.u' y ≤ (Wd.qw (M.qC P) (D.positionted w) (y : ℂ)).re

theorem **qw_re_pos** (P : M.Par) (w : List C.Letter) (y : ℝ) (hq : 0 < I.qpoly P.k P.T P.T' P.u P.u' y) (h : ∀ l ∈ w, 4 ≤ |y - l.2|) : 0 < (Wd.qw (M.qC P) (D.positionted w) (y : ℂ)).re

CompanionTheory.AutomorphicTheory.BesselWaveGrids.SmoothCoefficientBessel

theorem **global_regularity_jacobi** : GRegJ

theorem **smooth_coefficient_bessel** : SmoothKB

CompanionTheory.AutomorphicTheory.BesselWaveGrids.SpanBall

theorem **zero_mem_steps** (n : ℕ) : (0 : Fin n → ℤ) ∈ steps n

theorem **single_mem_steps** {n : ℕ} (s : Fin n) : (Pi.single s 1 : Fin n → ℤ) ∈ steps n

theorem **neg_single_mem_steps** {n : ℕ} (s : Fin n) : -(Pi.single s 1 : Fin n → ℤ) ∈ steps n

theorem **card_steps** (n : ℕ) : (steps n).card ≤ 2 * n + 1

theorem **card_ball** (n : ℕ) : ∀ e, (ball n e).card ≤ (2 * n + 1) ^ e | 0 => by simp [ball] | e + 1 => by rw [pow_succ] exact Finset.card_image₂_le _ _ _ |>.trans (Nat.mul_le_mul (card_ball n e) (card_steps n)) theorem sum_natAbs_le_one_of_mem_unitShifts {n : ℕ} {s : Fin n → ℤ} (h : s ∈ steps n) : ∑ i, (s i).natAbs ≤ 1

theorem **sum_natAbs_le_of_mem_ball** {n : ℕ} : ∀ e, ∀ ν ∈ ball n e, ∑ i, (ν i).natAbs ≤ e | 0, ν, h => by rw [ball, Finset.mem_singleton] at h simp [h] | e + 1, ν, h => by obtain ⟨x, hx, s, hs, rfl⟩

theorem **abs_ball** {n e : ℕ} {ν : Fin n → ℤ} (h : ν ∈ ball n e) (a : Fin n) : |(ν a : ℝ)| ≤ e

theorem **add_single** {n : ℕ} (μ : Fin n → ℝ) (s : Fin n) : μ + Pi.single s 1 = fun a => μ a + ((Pi.single s 1 : Fin n → ℤ) a : ℝ)

theorem **sub_single** {n : ℕ} (μ : Fin n → ℝ) (s : Fin n) : μ - Pi.single s 1 = fun a => μ a + ((-(Pi.single s 1 : Fin n → ℤ)) a : ℝ)

theorem **mem_spanStep_of_eq** {μ : Fin (2 * r + k + 1) → ℝ} {φ ψ ψ' : B.Mat k r → ℂ} (hφ : IsCoef k r ε μ φ) (hψ' : ψ' ∈ spanOn k r ε μ (steps _)) (h : ∀ g ∈ B.GV k r, ψ g = ψ' g) : ψ ∈ spanOn k r ε μ (steps _)

theorem **letterStep** {t : ∀ n : ℕ, (Fin n → Bool) → ℝ} (hZ : ZuckT t) (hZd : ZuckDualT t) {μ : Fin (2 * r + k + 1) → ℝ} {h : ℝ} {q φ : B.Mat k r → ℂ} (hq : IsLetter k r q) (hφ : IsCoef k r ε μ φ) (hg : PI.ZuckTensor.GapAtLeast ε μ h) (hlt : t _ ε + 1 < h) : q * φ ∈ spanOn k r ε μ (steps _)

theorem **spanBall** {t : ∀ n : ℕ, (Fin n → Bool) → ℝ} (hZ : ZuckT t) (hZd : ZuckDualT t) {lam : Fin (2 * r + k + 1) → ℝ} {h : ℝ} (d : ℕ) {q φ : B.Mat k r → ℂ} (hq : q ∈ Letters k r ^ d) (hφ : IsCoef k r ε lam φ) (hg : PI.ZuckTensor.GapAtLeast ε lam h) (hlt : t _ ε + 1 + 2 * (d : ℝ) < h) : q * φ ∈ spanOn k r ε lam (ball _ d)

theorem **smul_mem_sumOn** {lam : Fin (2 * r + k + 1) → ℝ} {S : Finset (Fin (2 * r + k + 1) → ℤ)} {φ : B.Mat k r → ℂ} (hφ : φ ∈ spanOn k r ε lam S) (T : Wq →L[ℂ] Wp) : (fun g => φ g • T) ∈ sumOn k r Wp Wq ε lam S

theorem **mul_mem_sumOn** {t : ∀ n : ℕ, (Fin n → Bool) → ℝ} (hZ : ZuckT t) (hZd : ZuckDualT t) {lam : Fin (2 * r + k + 1) → ℝ} {h : ℝ} {d : ℕ} {q : B.Mat k r → ℂ} (hq : q ∈ Letters k r ^ d) (hg : PI.ZuckTensor.GapAtLeast ε lam h) (hlt : t _ ε + 1 + 2 * (d : ℝ) < h) {F : B.Mat k r → (Wq →L[ℂ] Wp)} (hF : F ∈ Wsp k r Wp Wq ε lam) : (fun g => q g • F g) ∈ sumOn k r Wp Wq ε lam (ball _ d)

CompanionTheory.AutomorphicTheory.BesselWaveGrids.ThmBFinal

theorem **weighted_bound_rotated_uniform** : Formalism.MainNotions.RotatedPoints.WeightedBoundRotatedUniform

CompanionTheory.AutomorphicTheory.BesselWaveGrids.TiedNum

theorem **expBall_of_hyps** (H : BranchingSupport.Hyps) : ExpBall

theorem **thmBpAt_mono** {P : M.Par} {Wp Wq : Type} [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wp] [FiniteDimensional ℂ Wq] {p : KB.Pl P Wp Wq} {f : B.Mat P.k P.r → (Wq →L[ℂ] Wp)} {b C C' : ℝ} {n₀ p₀ : ℕ} (hb : 1 ≤ b) (hC : C ≤ C') (h : TBB.ThmBpAt p f b C n₀ p₀) : TBB.ThmBpAt p f b C' n₀ p₀

theorem **pow_four_le_rpow_nine_halves** (r : ℕ) : ((2 * r + 2 : ℕ) : ℝ) ^ 4 ≤ ((2 * r + 2 : ℕ) : ℝ) ^ ((9 : ℝ) / 2)

theorem **thrX_le_gapY** (k r : ℕ) : thrX k r ≤ gapY k r

theorem **mul_le_gapY** (k r : ℕ) : ((2 * r + 2 : ℕ) : ℝ) ^ 4 * ((2 * r + k + 1 : ℕ) : ℝ) ^ 3 ≤ gapY k r

theorem **fourth_power_le_gap** (k r : ℕ) : ((2 * r + 2 : ℕ) : ℝ) ^ 4 ≤ gapY k r

theorem **cube_le_gap** (k r : ℕ) : ((2 * r + k + 1 : ℕ) : ℝ) ^ 3 ≤ gapY k r

theorem **hypType_of** (hk : 1 ≤ k) {lam : Fin (2 * r + k + 1) → ℝ} {u : Fin k → ℝ} {b : ℝ} (hlam : IsHCParam lam) (hu : IsHCParam u) (hI : InterlaceS (by omega) u lam) (hb : ∀ i, |u i| ≤ b) : HypType k r lam (lamN u) b where

theorem **gap_of_lt** {n : ℕ} {lam : Fin n → ℝ} {g : ℝ} (hlam : IsHCParam lam) (hg : AllGaps lam g) {a c : Fin n} (h : a < c) : g ≤ lam a - lam c

theorem **shape_of** (hk : 1 ≤ k) {lam : Fin (2 * r + k + 1) → ℝ} {u : Fin k → ℝ} {bv : ℝ} (hlam : IsHCParam lam) (hu : IsHCParam u) (hI : InterlaceS (by omega) u lam) (hS : ShapeS u lam bv) : (∀ a : Fin (2 * r + k + 1), a.val + 1 < k → 0 ≤ lam a ∧ lam a ≤ bv / 8) ∧ ∀ a : Fin (2 * r + k + 1), k ≤ a.val + 1 → -2 * bv ≤ lam a ∧ lam a ≤ -bv

theorem **num_of** (hk : 1 ≤ k) {lam : Fin (2 * r + k + 1) → ℝ} {u : ℕ → ℝ} {b bv ga h : ℝ} {q0 : ℕ} (hlam : IsHCParam lam) (hu : HypType k r lam u b) (hc : ∀ a : Fin (2 * r + k + 1), a.val + 1 < k → 0 ≤ lam a ∧ lam a ≤ bv / 8) (hn : ∀ a : Fin (2 * r + k + 1), k ≤ a.val + 1 → -2 * bv ≤ lam a ∧ lam a ≤ -bv) (hb : ∀ a, |lam a| ≤ b) (hg : AllGaps lam ga) (hh : GapAtLeast (PI.BesselPeriodGGP.eps48 k r) lam h) (hok : ThreeDimData.GapOK k r (h / 4)) (h1 : 100 * ((q0 : ℝ) + 1) ≤ ga) (h2 : 100 * ((q0 : ℝ) + 1) ≤ h) (h3 : 10 ^ 6 * ((2 * r + k + 1 : ℕ) : ℝ) ^ 7 ≤ bv) : BranchingSupport.Num k r lam u b bv ga h q0

theorem **exists_shift** {n : ℕ} {lam lam' : Fin n → ℝ} (h : IsHCParam lam) (h' : IsHCParam lam') : ∃ ν : Fin n → ℤ, lam' = (fun a => lam a + (ν a : ℝ)) ∧ (BranchingSupport.n1 ν : ℝ) = l1 lam' lam

theorem **scaled_le** {T ka b Ch : ℝ} (r : ℕ) (hb : 1 ≤ b) (hka : 0 < ka) (h1 : ka ≤ b ^ Ch) (hT : T * (1 / ka) ^ (2 * r) ≤ b ^ (Ch * ((2 * r + 2 : ℕ) : ℝ))) : T ≤ b ^ (2 * |Ch| * ((2 * r + 2 : ℕ) : ℝ))

theorem **coefCC1_of** {t : ∀ n : ℕ, (Fin n → Bool) → ℝ} {Ct : ℝ} (hCt : 0 ≤ Ct) (ht : ∀ n ε, 1 ≤ n → t n ε ≤ Ct * (n : ℝ) ^ 3) (hZ : ZuckT t) (hK : PI.KFinSmooth) (hC : CoefCC) : CoefCC1 (Ct + |Formalism.MainNotions.BigCellDensity.c3| + 2)

CompanionTheory.AutomorphicTheory.BesselWaveGrids.TypeHC

theorem **isType_of_hc** {u : ℕ → ℝ} {u₀ : Fin k → ℝ} (hz : Formalism.QuantumDeterminants.inWb k r z) (hT : 0 < Formalism.QuantumDeterminants.Tz k r z) (hπ : IsHCTypeW k r z u₀ π) (hu : ∀ s : Fin k, u (s.val + 1) = u₀ s) : Q.IsType k r z π u

theorem **exists_place** : ∃ Cb : ℝ, ∀ (Wp Wq : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wp] [FiniteDimensional ℂ Wq] (P : M.Par) (z z' : Fin (2 * P.r + P.k + 1) → ℂ) (π : Mx P.k P.r → (Wp →L[ℂ] Wp)) (π' : Mx P.k P.r → (Wq →L[ℂ] Wq)) (μW μW' : Measure (Formalism.QuantumDeterminants.Mt P.k P.r)) (l0 : ℕ), Q.IsPlace P z z' π π' → 2 ≤ P.k → 1 ≤ l0 → l0 ≤ P.k → B.IsHaarOn (Formalism.QuantumDeterminants.UWset P.k P.r z) μW → μW Set.univ = 1 → B.IsHaarOn (Formalism.QuantumDeterminants.UWset P.k P.r z') μW' → μW' Set.univ = 1 → ∃ p : KB.Pl P Wp Wq, p.l0 = l0 ∧ p.T.base.z = z ∧ p.T.base.z' = z' ∧ p.T.base.π = π ∧ p.T.base.π' = π' ∧ p.T.base.μW = μW ∧ p.T.base.μN = D.muN P.k P.r ∧ p.T.base.μW' = μW' ∧ KB.InDDplus2 P p.l0 p.T.base Cb p.0

CompanionTheory.AutomorphicTheory.BesselWaveGrids.WaveBallEstimateWeightedDualAlt

theorem **family_through_discreteSeries** : HypsG → ∀ (k r : ℕ) (lam : Fin (2 * r + k + 1) → ℝ) (u : ℕ → ℝ) (b bv ga h : ℝ) (q0 : ℕ) (R0 : Rep k r) (ν₀ : Fin (2 * r + k + 1) → ℤ), Num k r lam u b bv ga h q0 → R0.IsDG (PI.BesselPeriodGGP.eps48 k r) (fun a => lam a + (ν₀ a : ℝ)) → ∃ R : (Fin (2 * r + k + 1) → ℤ) → Rep k r, IsFamG lam q0 R ∧ R ν₀ = R0

theorem **exists_coef_family** : HypsG → ∀ (P : M.Par) (Wp Wq : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wp] [FiniteDimensional ℂ Wq] (p : KB.Pl P Wp Wq) (lam : Fin (2 * P.r + P.k + 1) → ℝ) (b bv ga h : ℝ) (q0 : ℕ) (R : (Fin (2 * P.r + P.k + 1) → ℤ) → Rep P.k P.r) (l : ∀ ν, (R ν).H → Wp) (l' : ∀ ν, (R ν).H → Wq) (dg : (Fin (2 * P.r + P.k + 1) → ℤ) → ℝ) (f : B.Mat P.k P.r → (Wq →L[ℂ] Wp)) (s₀ : ℂ) (ν₀ : Fin (2 * P.r + P.k + 1) → ℤ), n1 ν₀ ≤ q0 → IsFamG lam q0 R → (haveI

theorem **exists_tau** : HypsG → ∀ (P : M.Par) (Wp Wq : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wp] [FiniteDimensional ℂ Wq] (p : KB.Pl P Wp Wq) (lam : Fin (2 * P.r + P.k + 1) → ℝ) (b bv ga h : ℝ) (q0 : ℕ) (R : (Fin (2 * P.r + P.k + 1) → ℤ) → Rep P.k P.r) (l : ∀ ν, (R ν).H → Wp) (l' : ∀ ν, (R ν).H → Wq) (dg : (Fin (2 * P.r + P.k + 1) → ℤ) → ℝ) (s : (Fin (2 * P.r + P.k + 1) → ℤ) → ℂ) (J : (Fin (2 * P.r + P.k + 1) → ℤ) → B.Mat P.k P.r → (Wq →L[ℂ] Wp)), Num P.k P.r lam P.u b bv ga h q0 → IsFamG lam q0 R → (haveI

theorem **sup_norm** : HypsG → ∀ (P : M.Par) (Wp Wq : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wp] [FiniteDimensional ℂ Wq] (p : KB.Pl P Wp Wq) (lam : Fin (2 * P.r + P.k + 1) → ℝ) (b bv ga h : ℝ) (q0 : ℕ) (R : (Fin (2 * P.r + P.k + 1) → ℤ) → Rep P.k P.r) (l : ∀ ν, (R ν).H → Wp) (l' : ∀ ν, (R ν).H → Wq) (dg : (Fin (2 * P.r + P.k + 1) → ℤ) → ℝ) (s : (Fin (2 * P.r + P.k + 1) → ℤ) → ℂ) (J : (Fin (2 * P.r + P.k + 1) → ℤ) → B.Mat P.k P.r → (Wq →L[ℂ] Wp)) (tau : (Fin (2 * P.r + P.k + 1) → ℤ) → Finset (Fin (2 * P.r + P.k + 1)) → ℂ), Num P.k P.r lam P.u b bv ga h q0 → Sup0 p lam q0 R l l' dg s J tau → ∃ c : ℝ, 0 < c ∧ ∀ ν, n1 ν ≤ q0 → (J ν = 0 ∧ TBB.jN2 p (J ν) = 0 ∧ ¬ (NzOn (R ν) (l ν) ∧ NzOn (R ν) (l' ν))) ∨ (J ν ≠ 0 ∧ NzOn (R ν) (l ν) ∧ NzOn (R ν) (l' ν) ∧ s ν ≠ 0 ∧ TBB.jN2 p (J ν) = ENNReal.ofReal (c * Formalism.Q

theorem **sup_meas** : HypsG → ∀ (P : M.Par) (Wp Wq : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wp] [FiniteDimensional ℂ Wq] (p : KB.Pl P Wp Wq) (lam : Fin (2 * P.r + P.k + 1) → ℝ) (b bv ga h : ℝ) (q0 : ℕ) (R : (Fin (2 * P.r + P.k + 1) → ℤ) → Rep P.k P.r) (l : ∀ ν, (R ν).H → Wp) (l' : ∀ ν, (R ν).H → Wq) (dg : (Fin (2 * P.r + P.k + 1) → ℤ) → ℝ) (s : (Fin (2 * P.r + P.k + 1) → ℤ) → ℂ) (J : (Fin (2 * P.r + P.k + 1) → ℤ) → B.Mat P.k P.r → (Wq →L[ℂ] Wp)) (tau : (Fin (2 * P.r + P.k + 1) → ℤ) → Finset (Fin (2 * P.r + P.k + 1)) → ℂ), Num P.k P.r lam P.u b bv ga h q0 → Sup0 p lam q0 R l l' dg s J tau → ∀ ν, AEMeasurable fun c : Fin (P.r + 1) → ℂ => ENNReal.ofReal (TBB.Ew p (fun _ => 1) (J ν) (P.r + 1) [] (TBB.bigPt P.r c))

theorem **tau_bound** : HypsG → ∀ (P : M.Par) (Wp Wq : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wp] [FiniteDimensional ℂ Wq] (p : KB.Pl P Wp Wq) (lam : Fin (2 * P.r + P.k + 1) → ℝ) (b bv ga h : ℝ) (q0 : ℕ) (R : (Fin (2 * P.r + P.k + 1) → ℤ) → Rep P.k P.r) (l : ∀ ν, (R ν).H → Wp) (l' : ∀ ν, (R ν).H → Wq) (dg : (Fin (2 * P.r + P.k + 1) → ℤ) → ℝ) (s : (Fin (2 * P.r + P.k + 1) → ℤ) → ℂ) (J : (Fin (2 * P.r + P.k + 1) → ℤ) → B.Mat P.k P.r → (Wq →L[ℂ] Wp)) (tau : (Fin (2 * P.r + P.k + 1) → ℤ) → Finset (Fin (2 * P.r + P.k + 1)) → ℂ), Num P.k P.r lam P.u b bv ga h q0 → Num P.k P.r lam P.u' b bv ga h q0 → Sup0 p lam q0 R l l' dg s J tau → ∀ ν, n1 ν + (P.r + 1) * (P.r + 2) ≤ q0 → ∀ I : Finset (Fin (2 * P.r + P.k + 1)), I.card = P.r + 1 → ‖tau ν I‖ ≤ Real.sqrt (Hsq P.r P.T b * Hsq P.r P.T' b) ^ (P.r + 1)

theorem **supplied_pieri** : HypsG → ∀ (P : M.Par) (Wp Wq : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wp] [FiniteDimensional ℂ Wq] (p : KB.Pl P Wp Wq) (lam : Fin (2 * P.r + P.k + 1) → ℝ) (b bv ga h : ℝ) (q0 : ℕ) (R : (Fin (2 * P.r + P.k + 1) → ℤ) → Rep P.k P.r) (l : ∀ ν, (R ν).H → Wp) (l' : ∀ ν, (R ν).H → Wq) (dg : (Fin (2 * P.r + P.k + 1) → ℤ) → ℝ) (s : (Fin (2 * P.r + P.k + 1) → ℤ) → ℂ) (J : (Fin (2 * P.r + P.k + 1) → ℤ) → B.Mat P.k P.r → (Wq →L[ℂ] Wp)) (tau : (Fin (2 * P.r + P.k + 1) → ℤ) → Finset (Fin (2 * P.r + P.k + 1)) → ℂ), Num P.k P.r lam P.u b bv ga h q0 → Sup0 p lam q0 R l l' dg s J tau → ∀ ν, n1 ν + (P.r + 1) * (P.r + 2) ≤ q0 → ∀ c : Fin (P.r + 1) → ℂ, D.IsLocPt (TBB.bigPt P.r c) (P.r + 1) → 0 < TBB.Gp p (TBB.bigPt P.r c) → (D.ptMatN (TBB.bigPt P.r c) (P.r + 1)).det • p.V (J ν) (TBB.bigPt P.r c) =

theorem **suppliedData_existsW** : HypsG → ∀ (P : M.Par) (Wp Wq : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wp] [FiniteDimensional ℂ Wq] (p : KB.Pl P Wp Wq) (lam : Fin (2 * P.r + P.k + 1) → ℝ) (b bv ga h : ℝ) (q0 : ℕ) (R0 : Rep P.k P.r) (f : B.Mat P.k P.r → (Wq →L[ℂ] Wp)) (ν₀ : Fin (2 * P.r + P.k + 1) → ℤ), n1 ν₀ ≤ q0 → Num P.k P.r lam P.u b bv ga h q0 → R0.IsDG (PI.BesselPeriodGGP.eps48 P.k P.r) (fun a => lam a + (ν₀ a : ℝ)) → CoefOf R0 f → TBB.jN2 p f ≠ 0 → ∃ (R : (Fin (2 * P.r + P.k + 1) → ℤ) → Rep P.k P.r) (l : ∀ ν, (R ν).H → Wp) (l' : ∀ ν, (R ν).H → Wq) (dg : (Fin (2 * P.r + P.k + 1) → ℤ) → ℝ) (s : (Fin (2 * P.r + P.k + 1) → ℤ) → ℂ) (J : (Fin (2 * P.r + P.k + 1) → ℤ) → B.Mat P.k P.r → (Wq →L[ℂ] Wp)) (tau : (Fin (2 * P.r + P.k + 1) → ℤ) → Finset (Fin (2 * P.r + P.k + 1)) → ℂ) (t : ℝ), 0 < t ∧ J ν₀ = (fun g

theorem **coef_norm_finite_and_zeroW** : HypsG → ∀ (P : M.Par) (Wp Wq : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wp] [FiniteDimensional ℂ Wq] (p : KB.Pl P Wp Wq) (lam : Fin (2 * P.r + P.k + 1) → ℝ) (b bv ga h : ℝ) (q0 : ℕ) (R0 : Rep P.k P.r) (f : B.Mat P.k P.r → (Wq →L[ℂ] Wp)) (ν₀ : Fin (2 * P.r + P.k + 1) → ℤ), n1 ν₀ ≤ q0 → Num P.k P.r lam P.u b bv ga h q0 → R0.IsDG (PI.BesselPeriodGGP.eps48 P.k P.r) (fun a => lam a + (ν₀ a : ℝ)) → CoefOf R0 f → TBB.jN2 p f < ⊤ ∧ (AEMeasurable fun c : Fin (P.r + 1) → ℂ => ENNReal.ofReal (TBB.Ew p (fun _ => 1) f (P.r + 1) [] (TBB.bigPt P.r c))) ∧ (TBB.jN2 p f = 0 → ∀ c : Fin (P.r + 1) → ℂ, D.IsLocPt (TBB.bigPt P.r c) (P.r + 1) → 0 < TBB.Gp p (TBB.bigPt P.r c) → p.V f (TBB.bigPt P.r c) = 0)

theorem **familyDerivativeExpansion_of_isDS_of_coefOf** (H : HypsG) (C : ℝ) (hC : 0 ≤ C) (P : M.Par) (Wp Wq : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wp] [FiniteDimensional ℂ Wq] (p : KB.Pl P Wp Wq) (lam : Fin (2 * P.r + P.k + 1) → ℝ) (b bv ga h : ℝ) (q0 a₁ : ℕ) (R0 : Rep P.k P.r) (f : B.Mat P.k P.r → (Wq →L[ℂ] Wp)) (ν₀ : Fin (2 * P.r + P.k + 1) → ℤ) (hN : Num P.k P.r lam P.u b bv ga h q0) (hN' : Num P.k P.r lam P.u' b bv ga h q0) (hT : P.T ≤ b ^ (C * ((2 * P.r + 2 : ℕ) : ℝ))) (hT' : P.T' ≤ b ^ (C * ((2 * P.r + 2 : ℕ) : ℝ))) (hq : n1 ν₀ + (P.r + 1) * a₁ + (P.r + 1) * (P.r + 2) ≤ q0) (hDS : R0.IsDG (PI.BesselPeriodGGP.eps48 P.k P.r) (fun a => lam a + (ν₀ a : ℝ))) (hf : CoefOf R0 f) : Formalism.FourierDecayAlt.P.CutBP.FamilyDerivativeExpansion p f b (C / 2 + 6) a₁

theorem **bigCellDerivativeExpansion_of_isDS_of_coefOf** (H : HypsG) (C : ℝ) (hC : 0 ≤ C) (P : M.Par) (Wp Wq : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wp] [FiniteDimensional ℂ Wq] (p : KB.Pl P Wp Wq) (lam : Fin (2 * P.r + P.k + 1) → ℝ) (b bv ga h : ℝ) (q0 a₁ : ℕ) (R0 : Rep P.k P.r) (f : B.Mat P.k P.r → (Wq →L[ℂ] Wp)) (ν₀ : Fin (2 * P.r + P.k + 1) → ℤ) (hN : Num P.k P.r lam P.u b bv ga h q0) (hN' : Num P.k P.r lam P.u' b bv ga h q0) (hT : P.T ≤ b ^ (C * ((2 * P.r + 2 : ℕ) : ℝ))) (hT' : P.T' ≤ b ^ (C * ((2 * P.r + 2 : ℕ) : ℝ))) (hq : n1 ν₀ + (P.r + 1) * a₁ + (P.r + 1) * (P.r + 2) ≤ q0) (hDS : R0.IsDG (PI.BesselPeriodGGP.eps48 P.k P.r) (fun a => lam a + (ν₀ a : ℝ))) (hf : CoefOf R0 f) : Formalism.FourierDecayAlt.P.CutBP.BigCellDerivativeExpansion p f b (C / 2 + 6) a₁

theorem **product_moment** : HypsG → ∀ (k r : ℕ) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (lam : Fin (2 * r + k + 1) → ℝ) (u : ℕ → ℝ) (b bv ga h : ℝ) (q0 : ℕ) (z : Fin (2 * r + k + 1) → ℂ) (π : Mx k r → (Wp →L[ℂ] Wp)) (μW μN μG νq R l dg → ∀ (j : ℕ) (hj : j ≤ r) ν, n1 ν + (r + 1) ^ 2 ≤ q0 → NzOn (R ν) (l ν) → ∃ w ∈ (R ν).sm, ‖w‖ = 1 ∧ ∫⁻ x, ENNReal.ofReal ((∏ a : Fin (j + 1), lenA k r z (Matrix.of x) (Fin.castLE (Nat.succ_le_succ hj) a) ^ 2) * (dg ν * ‖l ν ((R ν).σ (Matrix.of x) w)‖ ^ 2)) ∂νq ≤ ENNReal.ofReal (Hsq r (Formalism.QuantumDeterminants.Tz k r z) b ^ (r + 1))

theorem **kappa_bound** : HypsG → ∀ (k r : ℕ) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (lam : Fin (2 * r + k + 1) → ℝ) (u : ℕ → ℝ) (b bv ga h : ℝ) (q0 : ℕ) (z : Fin (2 * r + k + 1) → ℂ) (π : Mx k r → (Wp →L[ℂ] Wp)) (μW μN μG νq : Measure (Formalism.QuantumDeterminants.Mt k r)) (R : (Fin (2 * r + k + 1) → ℤ) → Rep k r) (l : ∀ ν, (R ν).H → Wp) (dg : (Fin (2 * r + k + 1) → ℤ) → ℝ), SetupG lam u b bv ga h q0 z π μW μN μG νq R l dg → ∀ (j : ℕ) (hj : j ≤ r) ν, n1 ν + (r + 1) * (r + 2) ≤ q0 → ∀ io pr ep, TensH (cmp (j + 1)) (cmp (j + 1) (Formalism.QuantumDeterminants.Hpk k r)) (R ν) (fun I : Sub (2 * r + k + 1) (j + 1) => R (ν + eZ I.1)) io pr ep → NzOn (R ν) (l ν) → ∀ (I : Sub (2 * r + k + 1) (j + 1)) (κ₀ : ℂ), NzOn (R (ν + eZ I.1)) (l (ν + eZ I.1)) → (∀ ν ∈ (R ν).sm, l (ν + eZ I.1) (∑ c, pr I c (tm (R ν) (wedge (fvL hj z)) ν c)) = κ₀

theorem **exists_tauL** : HypsG → ∀ (P : M.Par) (Wp Wq : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wp] [FiniteDimensional ℂ Wq] (p : KB.Pl P Wp Wq) (lam : Fin (2 * P.r + P.k + 1) → ℝ) (b bv ga h : ℝ) (q0 : ℕ) (R : (Fin (2 * P.r + P.k + 1) → ℤ) → Rep P.k P.r) (l : ∀ ν, (R ν).H → Wp) (l' : ∀ ν, (R ν).H → Wq) (dg : (Fin (2 * P.r + P.k + 1) → ℤ) → ℝ) (s : (Fin (2 * P.r + P.k + 1) → ℤ) → ℂ) (J : (Fin (2 * P.r + P.k + 1) →

ℤ) → B.Mat P.k P.r → (Wq →L[ℂ] Wp)), ∀ tau₀, Num P.k P.r lam P.u b bv ga h q0 → Sup0 p lam q0 R l l' dg s J tau₀ → ∀ (j : ℕ) (hj : j ≤ P.r), ∃ tau, SupL p q0 R l l' s J j hj tau

theorem **tau_boundL** : HypsG → ∀ (P : M.Par) (Wp Wq : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wp] [FiniteDimensional ℂ Wq] (p : KB.Pl P Wp Wq) (lam : Fin (2 * P.r + P.k + 1) → ℝ) (b bv ga h : ℝ) (q0 : ℕ) (R : (Fin (2 * P.r + P.k + 1) → ℤ) → Rep P.k P.r) (l : ∀ ν, (R ν).H → Wp) (l' : ∀ ν, (R ν).H → Wq) (dg : (Fin (2 * P.r + P.k + 1) → ℤ) → ℝ) (s : (Fin (2 * P.r + P.k + 1) → ℤ) → ℂ) (J : (Fin (2 * P.r + P.k + 1) → ℤ) → B.Mat P.k P.r → (Wq →L[ℂ] Wp)) (tau₀ : (Fin (2 * P.r + P.k + 1) → ℤ) → Finset (Fin (2 * P.r + P.k + 1)) → ℂ), Num P.k P.r lam P.u b bv ga h q0 → Num P.k P.r lam P.u' b bv ga h q0 → Sup0 p lam q0 R l l' dg s J tau₀ → ∀ (j : ℕ) (hj : j ≤ P.r) tau, SupL p q0 R l l' s J j hj tau → ∀ ν, n1 ν + (P.r + 1) * (P.r + 2) ≤ q0 → ∀ I : Finset (Fin (2 * P.r + P.k + 1)), I.card = j + 1 → ‖tau ν I‖ ≤ Real.sqr

theorem **supplied_pieriL** : HypsG → ∀ (P : M.Par) (Wp Wq : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wp] [FiniteDimensional ℂ Wq] (p : KB.Pl P Wp Wq) (lam : Fin (2 * P.r + P.k + 1) → ℝ) (b bv ga h : ℝ) (q0 : ℕ) (R : (Fin (2 * P.r + P.k + 1) → ℤ) → Rep P.k P.r) (l : ∀ ν, (R ν).H → Wp) (l' : ∀ ν, (R ν).H → Wq) (dg : (Fin (2 * P.r + P.k + 1) → ℤ) → ℝ) (s : (Fin (2 * P.r + P.k + 1) → ℤ) → ℂ) (J : (Fin (2 * P.r + P.k + 1) → ℤ) → B.Mat P.k P.r → (Wq →L[ℂ] Wp)) (tau₀ : (Fin (2 * P.r + P.k + 1) → ℤ) → Finset (Fin (2 * P.r + P.k + 1)) → ℂ), Num P.k P.r lam P.u b bv ga h q0 → Sup0 p lam q0 R l l' dg s J tau₀ → ∀ (j : ℕ) (hj : j ≤ P.r) tau, SupL p q0 R l l' s J j hj tau → ∀ ν, n1 ν + (P.r + 1) * (P.r + 2) ≤ q0 → ∀ c : Fin (P.r + 1) → ℂ, D.IsLocPt (TBB.bigPt P.r c) (P.r + 1) → 0 < TBB.Gp p (TBB.bigPt P.r c) → mnr hj (TB

theorem **momL_le** (H : HypsG) (C : ℝ) (hC : 0 ≤ C) (P : M.Par) (Wp Wq : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wp] [FiniteDimensional ℂ Wq] (p : KB.Pl P Wp Wq) (lam : Fin (2 * P.r + P.k + 1) → ℝ) (b bv ga h : ℝ) (q0 a : ℕ) (R0 : Rep P.k P.r) (f : B.Mat P.k P.r → (Wq →L[ℂ] Wp)) (j : ℕ) (hj : j ≤ P.r) (hN : Num P.k P.r lam P.u b bv ga h q0) (hN' : Num P.k P.r lam P.u' b bv ga h q0) (hT : P.T ≤ b ^ (C * ((2 * P.r + 2 : ℕ) : ℝ))) (hT' : P.T' ≤ b ^ (C * ((2 * P.r + 2 : ℕ) : ℝ))) (hq : (P.r + 1) * a + (P.r + 1) * (P.r + 2) ≤ q0) (hDS : R0.IsDG (PI.BesselPeriodGGP.eps48 P.k P.r) lam) (hf : BallW.CoefOf R0 f) : momL p f hj a ≤ ENNReal.ofReal (b ^ ((C + 12) * ((2 * P.r + 2 : ℕ) : ℝ) ^ 2 * a)) * TBB.jN2 p f

CompanionTheory.AutomorphicTheory.BesselWaveGrids.WaveInputsOfHypothesesGlobal

theorem **isDG_of_num** {k r : ℕ} {lam : Fin (2 * r + k + 1) → ℝ} {u : ℕ → ℝ} {b bv ga h : ℝ} {q0 : ℕ} {R : Rep k r} (hN : Num k r lam u b bv ga h q0) {ν : Fin (2 * r + k + 1) → ℤ} (hν : n1 ν ≤ q0) (hR : R.IsDS (PI.BesselPeriodGGP.eps48 k r) fun a => lam a + (ν a : ℝ)) : R.IsDG (PI.BesselPeriodGGP.eps48 k r) fun a => lam a + (ν a : ℝ)

theorem **wave_inputs_of_hypotheses_global** (H : HypsG) : In91

CompanionTheory.AutomorphicTheory.BesselWaveGrids.WeightedBoundRotatedUniformOfCubicThreshold

theorem **uniqueness_global** : WAt.UniqG

theorem **wave_bessel_period_threshold** {t : ∀ n : ℕ, (Fin n → Bool) → ℝ} {Ct : ℝ} (hCt : 0 ≤ Ct) (ht : ∀ n ε, 1 ≤ n → t n ε ≤ Ct * (n : ℝ) ^ 3) (hZ : ZuckT t) (hZd : ZuckDualT t) (H : WEnd.In91) (Cb : ℝ) : WEnd.WBp Cb

theorem **weighted_bound_rotated_uniform_of_cubic_threshold** {t : ∀ n : ℕ, (Fin n → Bool) → ℝ} {Ct Cgf : ℝ} (hCt : 0 ≤ Ct) (ht : ∀ n ε, 1 ≤ n → t n ε ≤ Ct * (n : ℝ) ^ 3) (hZ : ZuckT t) (hZd : ZuckDualT t) (H : WEnd.In91) (hOB : ∀ D : D7, Formalism.MainNotions.FrameBoundFromBesselTorus.OneBesselTor D Cgf) : Formalism.MainNotions.RotatedPoints.WeightedBoundRotatedUniform

theorem **thrS_lt** {n : ℕ} (ε : Fin n → Bool) (hn : 1 ≤ n) : G2.thrS ε + 1 < 43 * (n : ℝ) ^ 2

theorem **thrS_le** (n : ℕ) (ε : Fin n → Bool) (hn : 1 ≤ n) : ReciprocityTheory.ChartRestriction.DS.G2.thrS ε ≤ 43 * (n : ℝ) ^ 3

theorem **uniform_bessel_rank_three_of_inputs** {Cgf : ℝ} (H : WEnd.In91) (hOB : ∀ D : D7, Formalism.MainNotions.FrameBoundFromBesselTorus.OneBesselTor D Cgf) : Formalism.MainNotions.RotatedPoints.WeightedBoundRotatedUniform

CompanionTheory.AutomorphicTheory.BesselWaveGrids.WeightedBoundRotatedUniformOfHypsG

theorem **weighted_bound_rotated_uniform_of_hypsG** (H : BranchingSupport.HypsG) : Formalism.MainNotions.RotatedPoints.WeightedBoundRotatedUniform

CompanionTheory.AutomorphicTheory.BesselWaveGrids.WeightedUniformRotatedOfExpansion

theorem **wLoc1N_of** (hW : ∀ Cb, WEnd.WBp Cb) {Cgc : ℝ} (hCC : CoefCC1 Cgc) (Ch Ca : ℝ) : ∃ C Cg₀ : ℝ, 0 ≤ C ∧ ∀ Cg, Cg₀ ≤ Cg → ∀ D : D7, WLoc1 D Ch Cg Ca C fun L => L.μN = Formalism.FourierDecayAlt.D.muN D.R.k D.R.rS

theorem **wLoc1_of_N** {D : D7} {Ch Cg Ca C : ℝ} (H : WLoc1 D Ch Cg Ca C fun L => L.μN = Formalism.FourierDecayAlt.D.muN D.R.k D.R.rS) : WLoc1 D Ch Cg Ca C fun _ => True

theorem **dpt_le** {r : ℕ} {ka B : ℝ} (hka : 0 < ka) (h1 : ka ≤ B) (h2 : ka⁻¹ ≤ B) (hB : 1 ≤ B) (γ : Point ℂ (r + 1)) (l : Fin (r + 1)) : ‖Dpt γ l‖ ≤ B ^ (2 * r + 2) ^ 2 * ‖Dpt (Formalism.MainNotions.FrameBoundFromBesselTorus.ptTr ka⁻¹ (ka ^ r) (inv_pos.2 hka) (pow_pos hka _) γ) l‖

theorem **w_in_frame** {D : D7} {Ch Cg Cgf Ca C : ℝ} (hFr : FrameBesselBound D Cgf 2) (H : WLoc1 D Ch Cg Ca C fun _ => True) (L : LocBessel D.R.k D.R.rS) (f : Mx D.R.k D.R.rS → (L.Wq →L[ℂ] L.Wp)) (μsec : ℂ → Matrix (Fin (D.R.k + 1)) (Fin (D.R.k + 1)) ℂ) (ka b : ℝ) (hL : LocOk D Ch Cg L L.lam ka b ka) (hg4 : AllGaps L.lam (Cg * ((2 * D.R.rS + 2 : ℕ) : ℝ) ^ 4)) (hgap : GapFr D Cgf L.ε L.lam) (hB1 : 1 ≤ b ^ Ch) (hf : IsKFinCoefH L.σ f) (hsec : IsSection D.R.k D.R.rS L.z L.z' μsec) (l : Fin (D.R.rS + 1)) (a : ℕ) (ha : (a : ℝ) ≤ Ca * ((2 * D.R.rS + 2 : ℕ) : ℝ) ^ 3) (γ : Point ℂ (D.R.rS + 1)) (hG : Gpt (Tz D.R.k D.R.rS L.z) (Tz D.R.k D.R.rS L.z') γ ≠ 0) : WIneq L f μsec γ (‖Dpt γ l‖ ^ (2 * a)) (((b ^ Ch) ^ (2 * D.R.rS + 2) ^ 2) ^ (2 * a) * (FrC D 2 ka * (b ^ (C * ((2 * D.R.rS + 2 : ℕ) : ℝ) ^ 5) * max 1 (1 / (GsC D ka * Gpt (Tz D.R.k D.R.rS L.z) (Tz D.R.k D.R.rS L.z') γ)) ^ (40 * (D.R.rS + 1) ^ 2))

theorem **weight_le** {ι : Type} [Fintype ι] [Nonempty ι] (s x : ι → ℝ) (N : ι → ℕ) (σ : ℝ) (hσ : 1 ≤ σ) (hs0 : ∀ l, 0 ≤ s l) (hs : ∀ l, s l ≤ σ) : ∃ (l : ι) (a : ℕ), a ≤ 2 * ∑ l, N l ∧ (∏ l, (1 + s l * x l ^ 2) ^ N l) ^ 2 ≤ (2 * σ) ^ (2 * ∑ l, N l) * x l ^ (2 * a)

theorem **weighted_uniform_rotated_of_expansion_alt_alt** (hW : ∀ Cb, WEnd.WBp Cb) {Cgc Cgf : ℝ} (hCC : CoefCC1 Cgc) (hOB : ∀ D : D7, Formalism.MainNotions.FrameBoundFromBesselTorus.OneBesselTor D Cgf) : Formalism.MainNotions.RotatedPoints.WeightedBoundRotatedUniform

CompanionTheory.AutomorphicTheory.CompactTopologicalForms.FinalC

theorem **nfTopIdAll_of_dom** (h3 : NFTopDomAll) : NFTopIdAll

theorem **nfTopDomAll_of_tieC** (hT : CompanionTheory.NFCpt.NFTieHatC) : NFTopDomAll

theorem **thmBpAt_dom** (h3 : NFTopDomAll) (Cb C₃ CΛ C₀ : ℝ) (hC₃ : 0 ≤ C₃) (hCΛ : 0 ≤ CΛ) (hC₀ : 1 / 2 ≤ C₀) : ∃ C₄ c₃ Cc Cc' Cc'' : ℝ, ∀ (Wp Wq : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wp] [FiniteDimensional ℂ Wq] (P : M.Par) (p : KB.Pl P Wp Wq) (f : B.Mat P.k P.r → (Wq →L[ℂ] Wp)) (b Λ A₀ : ℝ) (p₀ : ℕ) (χ nn : ℝ → ℝ) (Y : Finset ℝ) (K N : ℕ), 3 ≤ b → 1 ≤ Λ → Λ ≤ b ^ (CΛ * ((2 * P.r + 2 : ℕ) : ℝ)) → 1 ≤ TBB.Tq p → (p₀ : ℝ) ≤ C₀ * (3 * ((P.r : ℝ) + 1)) + 1 → 3 ≤ K → (P.r + 1) * K ≤ N → C6.IsCut0 χ → (∀ y, 0 < nn y) → KB.InDDplus2 P p.l0 p.T.base Cb p.0 → B.CkOn ⊤ (B.GV P.k P.r) f → (∀ m : ℕ, B.InCC A₀ m f) → c₃ * ((P.r : ℝ) + 1) ^ 2 * ((P.r : ℝ) * ((2 * P.r + P.k + 1 : ℕ) : ℝ) + 1) ≤ A₀ - (((2 * P.r + 2) * p₀ : ℕ) : ℝ) → Cc * ((P.r : ℝ) + 1) ^ 4 * ((N : ℝ) * (2 * (P.r : ℝ) + P.k + 1) ^ 2 + (2 * (P.r : ℝ) +

theorem **thmBpAt_eqR_dom** (h3 : NFTopDomAll) (Cb C₃ CΛ C₀ : ℝ) (hC₃ : 0 ≤ C₃) (hCΛ : 0 ≤ CΛ) (hC₀ : 1 / 2 ≤ C₀) : ∃ C₄ c₃ Cc Cc' Cc'' : ℝ, ∀ (P : M.Par) (Wp Wq : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [FiniteDimensional ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wq] (p : KB.Pl P Wp Wq) (f : B.Mat P.k P.r → (Wq →L[ℂ] Wp)) (b : ℝ) (n₀ : ℕ) (S : ShiftAtR p f b n₀) (Λ A₀ : ℝ) (p₀ : ℕ) (χ nnp nnm : ℝ → ℝ) (Yp Ym : Finset ℝ) (K N : ℕ), 3 ≤ b → 1 ≤ Λ → Λ ≤ b ^ (CΛ * ((2 * P.r + 2 : ℕ) : ℝ)) → 1 ≤ Tq p → n₀ ≤ 30 * (P.r + 1) ^ 2 + 6 → (p₀ : ℝ) ≤ C₀ * (3 * ((P.r : ℝ) + 1)) + 1 → 3 ≤ K → (P.r + 1) * K ≤ N → C6.IsCut0 χ → (∀ y, 0 < nnp y) → (∀ y, 0 < nnm y) → KB.InDDplus2 (shP P S.up S.up') S.pp.l0 S.pp.T.base Cb S.pp.0 → KB.InDDplus2 (shP P S.um S.um') S.pm.l0 S.pm.T.base Cb S.pm.0 → B.CkOn ⊤ (B.GV P.k P.r) f → (∀ m : ℕ, B.InCC A₀ m f) → c₃ * ((P.r : ℝ

CompanionTheory.AutomorphicTheory.CompactTopologicalForms.FinalS

theorem **nfTopDomAllJ_of_cut** (hZ : NFDomZero) (hP : NFDomPtwS) (hL : DominatedL2OperatorBound) (hM : NFDomMeas) : NFTopDomAllJ

theorem **nfTopDomAllJ_of_open** (hT : NFTieHatS) (hH : NFHatProdKBS) : NFTopDomAllJ

theorem **blockGrowthBound_apx_cutF** (hA : P5.ThmPActualCpt) {P : M.Par} {Wp Wq : Type} [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wp] [FiniteDimensional ℂ Wq] (p : KB.Pl P Wp Wq) (nn χ : ℝ → ℝ) (F : B.Mat P.k P.r → (Wq →L[ℂ] Wp)) (Y : Finset ℝ) (K N : ℕ) (Cb : ℝ) (hCb : KB.InDDplus2 P p.l0 p.T.base Cb p.0) (hχ : C6.IsCut0 χ) (hF : B.CkOn ⊤ (B.GV P.k P.r) F) (hnn : ∀ y, 0 < nn y) : ∀ ε : ℝ, 0 < ε → ∀ n : ℕ, TBB.BlockGrowthBound p nn (apx (D.cutF p.T.base χ F ε) n) Y K N

theorem **limEvT_J** (hA : P5.ThmPActualCpt) (h3 : NFTopDomAllJ) (Cb : ℝ) : ∃ Cc : ℝ, ∀ (Wp Wq : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wp] [FiniteDimensional ℂ Wq] (P : M.Par) (p : KB.Pl P Wp Wq) (f : B.Mat P.k P.r → (Wq →L[ℂ] Wp)) (A₀ : ℝ) (p₀ : ℕ) (χ : ℝ → ℝ) (Y : Finset ℝ) (N : ℕ), NFHyp Cb Cc p f A₀ p₀ χ N → B.JF p.T.base f ∈ FFKB p → LimEvT p χ (TBBx4.fpm p f (fun _ => ContinuousLinearMap.id ℂ Wq) p₀) Y N

theorem **thmBpAt_J** (h3 : NFTopDomAllJ) (Cb C₃ CΛ C₀ : ℝ) (hC₃ : 0 ≤ C₃) (hCΛ : 0 ≤ CΛ) (hC₀ : 1 / 2 ≤ C₀) : ∃ C₄ c₃ Cc Cc' Cc'' : ℝ, ∀ (Wp Wq : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wp] [FiniteDimensional ℂ Wq] (P : M.Par) (p : KB.Pl P Wp Wq) (f : B.Mat P.k P.r → (Wq →L[ℂ] Wp)) (b Λ A₀ : ℝ) (p₀ : ℕ) (χ nn : ℝ → ℝ) (Y : Finset ℝ) (K N : ℕ), 3 ≤ b → 1 ≤ Λ → Λ ≤ b ^ (CΛ * ((2 * P.r + 2 : ℕ) : ℝ)) → 1 ≤ TBB.Tq p → (p₀ : ℝ) ≤ C₀ * (3 * ((P.r : ℝ) + 1)) + 1 → 3 ≤ K → (P.r + 1) * K ≤ N → C6.IsCut0 χ → (∀ y, 0 < nn y) → KB.InDDplus2 P p.l0 p.T.base Cb p.0 → B.CkOn ⊤ (B.GV P.k P.r) f → (∀ m : ℕ, B.InCC A₀ m f) → B.JF p.T.base f ∈ FFKB p → c₃ * ((P.r : ℝ) + 1) ^ 2 * ((P.r : ℝ) * ((2 * P.r + P.k + 1 : ℕ) : ℝ) + 1) ≤ A₀ - (((2 * P.r + 2) * p₀ : ℕ) : ℝ) → Cc * ((P.r : ℝ) + 1) ^ 4 * ((N : ℝ) * (2 * (P.r : ℝ) + P.k +

theorem **thmBpAt_eqR_J** (h3 : NFTopDomAllJ) (Cb C₃ CΛ C₀ : ℝ) (hC₃ : 0 ≤ C₃) (hCΛ : 0 ≤ CΛ) (hC₀ : 1 / 2 ≤ C₀) : ∃ C₄ c₃ Cc Cc' Cc'' : ℝ, ∀ (P : M.Par) (Wp Wq : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [FiniteDimensional ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wq] (p : KB.Pl P Wp Wq) (f : B.Mat P.k P.r → (Wq →L[ℂ] Wp)) (b : ℝ) (n₀ : ℕ) (S : ShiftAtR p f b n₀) (Λ A₀ : ℝ) (p₀ : ℕ) (χ nnp nnm : ℝ → ℝ) (Yp Ym : Finset ℝ) (K N : ℕ), 3 ≤ b → 1 ≤ Λ → Λ ≤ b ^ (CΛ * ((2 * P.r + 2 : ℕ) : ℝ)) → 1 ≤ Tq p → n₀ ≤ 30 * (P.r + 1) ^ 2 + 6 → (p₀ : ℝ) ≤ C₀ * (3 * ((P.r : ℝ) + 1)) + 1 → 3 ≤ K → (P.r + 1) * K ≤ N → C6.IsCut0 χ → (∀ y, 0 < nnp y) → (∀ y, 0 < nnm y) → KB.InDDplus2 (shP P S.up S.up') S.pp.l0 S.pp.T.base Cb S.pp.0 → KB.InDDplus2 (shP P S.um S.um') S.pm.l0 S.pm.T.base Cb S.pm.0 → B.CkOn ⊤ (B.GV P.k P.r) f → (∀ m : ℕ, B.InCC A₀ m f) → B.JF S.pp.T.bas

CompanionTheory.AutomorphicTheory.CompactTopologicalForms.JacobiFieldSquareIntegrable

theorem **weight_zero** (D : RealPlaceData k r Wp Wq) {q : (Fin r → ℂ) × ℂ} (hq : q ∉ Qset D) : bcWeight k r (Tz k r D.z) (Tz k r D.z') q = 0

theorem **jacobi_field_square_integrable** (D : RealPlaceData k r Wp Wq) {A : ℝ} (hA : AB k r ≤ A) {F : Mat k r → (Wq →L[ℂ] Wp)} (hF : InCC A 5 F) : AEStronglyMeasurable (fun q => hsVec (JF D F (gq D q))) (volume.restrict (Qset D)) ∧ Integrable (fun q => bcWeight k r (Tz k r D.z) (Tz k r D.z') q * ‖hsVec (JF D F (gq D q))‖ ^ 2) (volume.restrict (Qset D))

theorem **beta_restrict** (D : RealPlaceData k r Wp Wq) (X Y : Mat k r → (Wq →L[ℂ] Wp)) : beta D X Y = ∫ q in Qset D, (bcWeight k r (Tz k r D.z) (Tz k r D.z') q : ℂ) * inner ℂ (hsVec (JF D Y (gq D q))) (hsVec (JF D X (gq D q)))

theorem **beta_cut_right** (D : RealPlaceData k r Wp Wq) {χ : ℝ → ℝ} (hχ : C6.IsCut0 χ) {A : ℝ} (hA : AB k r ≤ A) {H F : Mat k r → (Wq →L[ℂ] Wp)} (hH : InCC A 5 H) (hF : InCC A 5 F) : Tendsto (fun ε => beta D H (Formalism.FourierDecayAlt.D.cutF D χ F ε)) (𝓝[>] 0) (𝓝 (beta D H F))

theorem **fold_add** (s : ℂ) {X Y : Mat k r → (Wq →L[ℂ] Wp)} (hX : CkOn ⊤ (GV k r) X) (hY : CkOn ⊤ (GV k r) Y) : ∀ (w : List C.Letter), ∀ g ∈ GV k r, fold s w (X + (1 : ℂ) • Y) g = (fold s w X + (1 : ℂ) • fold s w Y) g | [], _, _ => rfl | a :: w, g, hg => by have G

theorem **fold_inCC** (s : ℂ) {A : ℝ} {X : Mat k r → (Wq →L[ℂ] Wp)} (hX : CkOn ⊤ (GV k r) X) : ∀ (w : List C.Letter) (m : ℕ), InCC A (m + w.length * (2 * r + k + 1)) X → InCC A m (fold s w X) | [], m, h => by simpa [fold] using h | a :: w, m, h => by refine CutMP.ER.coefficient_vector_in_class k r Wp Wq s (a.2 : ℂ) A m (fold s w X) (fold_ckOn s w X hX) (fold_inCC s hX w _ ?_) have e : m + (2 * r + k + 1) + w.length * (2 * r + k + 1) = m + (a :: w).length * (2 * r + k + 1)

theorem **q_expand** (D : RealPlaceData k r Wp Wq) {A : ℝ} (hA : AB k r ≤ A) (s : ℂ) (w : List C.Letter) {X Y : Mat k r → (Wq →L[ℂ] Wp)} (hX : CkOn ⊤ (GV k r) X) (hY : CkOn ⊤ (GV k r) Y) (hXc : InCC A (5 + w.length * (2 * r + k + 1)) X) (hYc : InCC A (5 + w.length * (2 * r + k + 1)) Y) : beta D (fold s w (X + (1 : ℂ) • Y)) (X + (1 : ℂ) • Y) = beta D (fold s w X) X + beta D (fold s w X) Y + beta D (fold s w Y) X + beta D (fold s w Y) Y

theorem **q_symm** (D : RealPlaceData k r Wp Wq) {A : ℝ} (hA : AB k r ≤ A) {c : ℝ} (hc : 0 < c) {s : ℂ} (hs : conj s = s) (w : List C.Letter) {X Y : Mat k r → (Wq →L[ℂ] Wp)} (hX : CutMP.ER.Good D A (w.length * (2 * r + k + 1) + (2 * r + k + 1 + 5)) c X) (hY : CutMP.ER.Good D A (w.length * (2 * r + k + 1) + (2 * r + k + 1 + 5)) c Y) : beta D (fold s w X) Y = conj (beta D (fold s w Y) X)

theorem **good_cut** (D : RealPlaceData k r Wp Wq) {χ : ℝ → ℝ} (hχ : C6.IsCut0 χ) {c : ℝ} (h0 : ∀ x, x < c → χ x = 0) {F : Mat k r → (Wq →L[ℂ] Wp)} (hF : CkOn ⊤ (GV k r) F) {A₁ : ℝ} (hFA : ∀ m : ℕ, InCC A₁ m F) {ε : ℝ} (hε : 0 < ε) {m : ℕ} (hm : AB k r + 2 * ((r : ℝ) + 1) * (m : ℝ) ≤ A₁) : CutMP.ER.Good D (AB k r) m (c * ε) (Formalism.FourierDecayAlt.D.cutF D χ F ε)

theorem **residue_zero_eq_beta_fold** (p : KB.Pl P Wp Wq) {H : B.Mat P.k P.r → (Wq →L[ℂ] Wp)} (hH : CkOn ⊤ (GV P.k P.r) H) {w : List C.Letter} (hw : KB.CutWord P p.l0 w) : p.RR H 0 w [] [] = beta p.T.base (fold (M.sgn P p.l0) w H) H

theorem **q_loc** (p : KB.Pl P Wp Wq) {F : B.Mat P.k P.r → (Wq →L[ℂ] Wp)} (hF : CkOn ⊤ (GV P.k P.r) F) {χ χ₁ : ℝ → ℝ} (hχ₁ : C6.IsCut0 χ₁) {κ δ : ℝ} (hκδ : κ < δ) (h0 : ∀ x, x < δ → χ x = 0) (h1 : ∀ x, κ ≤ x → χ₁ x = 1) (s : ℂ) (w : List C.Letter) {ε : ℝ} (hε : 0 < ε) : beta p.T.base (fold s w (D.cutF p.T.base χ₁ F ε)) (D.cutF p.T.base χ F ε) = beta p.T.base (fold s w F) (D.cutF p.T.base χ F ε)

theorem **lim_top** (p : KB.Pl P Wp Wq) {χ : ℝ → ℝ} {F : B.Mat P.k P.r → (Wq →L[ℂ] Wp)} {w : List C.Letter} (hT : NFTopForm p χ F w) (hP : NFTopPtw p χ F w) (hD : NFTopDom p χ F w) : Tendsto (fun ε : ℝ => (p.RR (D.cutF p.T.base χ F ε) 0 w [] []).re) (𝓝[>] 0) (𝓝 (∫ c, topD p F w c))

theorem **nfTopId_of_lim** (p : KB.Pl P Wp Wq) {χ : ℝ → ℝ} (hχ : C6.IsCut0 χ) {F : B.Mat P.k P.r → (Wq →L[ℂ] Wp)} (hF : CkOn ⊤ (GV P.k P.r) F) {A₁ : ℝ} (hFA : ∀ m : ℕ, InCC A₁ m F) {w : List C.Letter} (hw : KB.CutWord P p.l0 w) (hth : AB P.k P.r + 2 * ((P.r : ℝ) + 1) * ((w.length * (2 * P.r + P.k + 1) + (2 * P.r + P.k + 1 + 5) : ℕ) : ℝ) ≤ A₁) (hL : ∀ χ' : ℝ → ℝ, C6.IsCut0 χ' → Tendsto (fun ε : ℝ => (p.RR (D.cutF p.T.base χ' F ε) 0 w [] []).re) (𝓝[>] 0) (𝓝 (∫ c, topD p F w c))) : NFTopId p χ F w

theorem **nfHyp_mono** {Cb c c' : ℝ} {P : M.Par} {Wp Wq : Type} [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wp] [FiniteDimensional ℂ Wq] {p : KB.Pl P Wp Wq} {f : B.Mat P.k P.r → (Wq →L[ℂ] Wp)} {A₀ : ℝ} {p₀ : ℕ} {χ χ' : ℝ → ℝ} {N : ℕ} (h : NFHyp Cb c p f A₀ p₀ χ N) (hc : c' ≤ c) (hχ' : C6.IsCut0 χ') : NFHyp Cb c' p f A₀ p₀ χ' N

theorem **nfTopIdAll_of_diag** (h1 : ∀ Cb : ℝ, ∃ Cc : ℝ, ∀ (Wp Wq : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wp] [FiniteDimensional ℂ Wq] (P : M.Par) (p : KB.Pl P Wp Wq) (f : B.Mat P.k P.r → (Wq →L[ℂ] Wp)) (A₀ : ℝ) (p₀ : ℕ) (χ : ℝ → ℝ) (Y : Finset ℝ) (N : ℕ), NFHyp Cb Cc p f A₀ p₀ χ N → ∀ w, TBB.AdmW P p.l0 Y w → w.length ≤ N → NFTopForm p χ (TBBx4.fpm p f (fun _ => ContinuousLinearMap.id ℂ Wq) p₀) w) (h2 : NFTopPtwAll) (h3 : NFTopDomAll) : NFTopIdAll

CompanionTheory.AutomorphicTheory.CompactTopologicalForms.NormalFormTopDomination

theorem **normal_form_top_domination** : NFTopDomAll

theorem **thmBpAt_top** (Cb C₃ CΛ C₀ : ℝ) (hC₃ : 0 ≤ C₃) (hCΛ : 0 ≤ CΛ) (hC₀ : 1 / 2 ≤ C₀) : ∃ C₄ c₃ Cc Cc' Cc'' : ℝ, ∀ (Wp Wq : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wp] [FiniteDimensional ℂ Wq] (P : M.Par) (p : KB.Pl P Wp Wq) (f : B.Mat P.k P.r → (Wq →L[ℂ] Wp)) (b Λ A₀ : ℝ) (p₀ : ℕ) (χ nn : ℝ → ℝ) (Y : Finset ℝ) (K N : ℕ), 3 ≤ b → 1 ≤ Λ → Λ ≤ b ^ (CΛ * ((2 * P.r + 2 : ℕ) : ℝ)) → 1 ≤ TBB.Tq p → (p₀ : ℝ) ≤ C₀ * (3 * ((P.r : ℝ) + 1)) + 1 → 3 ≤ K → (P.r + 1) * K ≤ N → C6.IsCut0 χ → (∀ y, 0 < nn y) → KB.InDDplus2 P p.l0 p.T.base Cb p.0 → B.CkOn ⊤ (B.GV P.k P.r) f → (∀ m : ℕ, B.InCC A₀ m f) → c₃ * ((P.r : ℝ) + 1) ^ 2 * ((P.r : ℝ) * ((2 * P.r + P.k + 1 : ℕ) : ℝ) + 1) ≤ A₀ - (((2 * P.r + 2) * p₀ : ℕ) : ℝ) → Cc * ((P.r : ℝ) + 1) ^ 4 * ((N : ℝ) * (2 * (P.r : ℝ) + P.k + 1) ^ 2 + (2 * (P.r : ℝ) + P.k + 1) ^ 3) ≤ A₀

theorem **thmBpAt_eqR_top** (Cb C₃ CΛ C₀ : ℝ) (hC₃ : 0 ≤ C₃) (hCΛ : 0 ≤ CΛ) (hC₀ : 1 / 2 ≤ C₀) : ∃ C₄ c₃ Cc Cc' Cc'' : ℝ, ∀ (P : M.Par) (Wp Wq : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [FiniteDimensional ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wq] (p : KB.Pl P Wp Wq) (f : B.Mat P.k P.r → (Wq →L[ℂ] Wp)) (b : ℝ) (n₀ : ℕ) (S : ShiftAtR p f b n₀) (Λ A₀ : ℝ) (p₀ : ℕ) (χ nnp nnm : ℝ → ℝ) (Yp Ym : Finset ℝ) (K N : ℕ), 3 ≤ b → 1 ≤ Λ → Λ ≤ b ^ (CΛ * ((2 * P.r + 2 : ℕ) : ℝ)) → 1 ≤ Tq p → n₀ ≤ 30 * (P.r + 1) ^ 2 + 6 → (p₀ : ℝ) ≤ C₀ * (3 * ((P.r : ℝ) + 1)) + 1 → 3 ≤ K → (P.r + 1) * K ≤ N → C6.IsCut0 χ → (∀ y, 0 < nnp y) → (∀ y, 0 < nnm y) → KB.InDDplus2 (shP P S.up S.up') S.pp.l0 S.pp.T.base Cb S.pp.0 → KB.InDDplus2 (shP P S.um S.um') S.pm.l0 S.pm.T.base Cb S.pm.0 → B.CkOn ⊤ (B.GV P.k P.r) f → (∀ m : ℕ, B.InCC A₀ m f) → c₃ * ((P.r : ℝ) + 1) ^ 2 * ((P.r

CompanionTheory.AutomorphicTheory.DiscreteGroupWaves.IntegralEqOfSqrtCauchy

theorem **ae_coords_ne_zero** (n : ℕ) : ∀ᵐ c : Fin n → ℂ, ∀ a, c a ≠ 0

theorem **ae_ne_zero_c** : ∀ᵐ t : ℂ, t ≠ 0

theorem **tower_integral** : ∀ (n : ℕ) (E : (j : ℕ) → (Fin j → ℂ) → ℝ), AEStronglyMeasurable (E n) volume → (∀ c, (∀ a, c a ≠ 0) → 0 ≤ E n c) → (∀ j, j < n → ∀ c : Fin j → ℂ, (∀ a, c a ≠ 0) → Integrable (fun t : ℂ => E (j + 1) (Fin.snoc c t)) ∧ ∫ t : ℂ, E (j + 1) (Fin.snoc c t) = E j c) → Integrable (E n) ∧ ∫ c, E n c = E 0 (fun a => a.elim0) | 0, E, _, _, _ => by have e : E 0 = fun _ => E 0 (fun a => a.elim0)

theorem **abs_sub_le_of_sqrt** {a b d δ : ℝ} (ha : 0 ≤ a) (hb : 0 ≤ b) (hd : 0 ≤ d) (hδ : 0 < δ) (h : |Real.sqrt a - Real.sqrt b| ≤ Real.sqrt d) : |a - b| ≤ d / δ + δ * (a + b)

theorem **integral_eq_of_sqrt_cauchy** {α : Type*} [MeasurableSpace α] {μ : Measure α} {q : ℕ → α → ℝ} {v : α → ℝ} {d : ℕ → ℕ → α → ℝ} {a : ℝ} (hq : ∀ n, Integrable (q n) μ) (hq0 : ∀ n x, 0 ≤ q n x) (hd : ∀ n m, Integrable (d n m) μ) (hd0 : ∀ n m x, 0 ≤ d n m x) (htri : ∀ n m x, |Real.sqrt (q n x) - Real.sqrt (q m x)| ≤ Real.sqrt (d n m x)) (hlim : ∀ᵐ x ∂μ, Tendsto (fun n => q n x) atTop (𝓝 (v x))) (ha : Tendsto (fun n => ∫ x, q n x ∂μ) atTop (𝓝 a)) (hC : ∀ η : ℝ, 0 < η → ∃ N : ℕ, ∀ n m, N ≤ n → N ≤ m → ∫ x, d n m x ∂μ ≤ η) : Integrable v μ ∧ ∫ x, v x ∂μ = a

CompanionTheory.AutomorphicTheory.DiscreteGroupWaves.NFCoreC

theorem **nFCoreNC_of_cut** (hL : NFLeibV) (hT : NFTieHatC) (hH : NFHatProdKBS) (hN : NFTrNormKB) (hM : NFOpOKMulG) : NFCoreNC

theorem **nfDomPtwC_of_cut** (hU : NFUnfoldTop) (hV : NFVMul) (hC : NFCutSup) (hL : NFLeibV) (hN : NFCoreNC) (hE : NFEmptyOps) : NFDomPtwC

theorem **nfDomPtwC_of_open** (hT : NFTieHatC) (hH : NFHatProdKBS) : NFDomPtwC

CompanionTheory.AutomorphicTheory.DiscreteGroupWaves.NFDomC

theorem **nfTopDomAll_of_ptwC** (hP : NFDomPtwC) (hJ : NFJCutFF) : NFTopDomAll

theorem **nfTopDomAll_of_C** (hT : NFTieHatC) (hH : NFHatProdKBS) (hJ : NFJCutFF) : NFTopDomAll

CompanionTheory.AutomorphicTheory.DiscreteGroupWaves.NFTopCpt

theorem **uInv_at_zero** {X : Type} [AddCommGroup X] [Module ℂ X] : ∀ (w : List C.Letter) (x : Wd.Ix (Fin w.length) → X), KB.uInv 0 w x = x | [], _ => rfl | l :: w, x => by funext e simp only [KB.uInv, uInv_at_zero w, map_zero, ite_self, zero_smul, add_zero] split_ifs with h · congr 1 funext a induction a using Fin.cases with | zero => exact h.symm | succ i => rfl · congr 1 funext a induction a using Fin.cases with | zero => have h1 : e (0 : Fin (w.length + 1)) = 1

theorem **waveG_at_zero** {k r : ℕ} (Bs : B.RealPlaceData k r Wp Wq) (w : List C.Letter) (x : Wd.Ix (Fin w.length) → B.Mat k r → (Wq →L[ℂ] Wp)) : KB.waveG Bs 0 w x = x

theorem **ew_snoc** (p : KB.Pl P Wp Wq) (F : B.Mat P.k P.r → (Wq →L[ℂ] Wp)) (w : List C.Letter) {j : ℕ} (hj : j ≤ P.r) {h : C.LPt} (hh : D.IsLocPt h j) (c : ℂ) : TBB.Ew p (fun _ => 1) F (j + 1) w (h ++ [(1, c)]) = C.nAux P.k P.r 0 (fun _ _ => 1) 1 h * ((KB.levW P (j + 1) c : ℂ) * p.RR F (j + 1) w (h ++ [(1, c)]) (h ++ [(1, c)])).re

theorem **ew_low** (p : KB.Pl P Wp Wq) (F : B.Mat P.k P.r → (Wq →L[ℂ] Wp)) (w : List C.Letter) {j : ℕ} (hj : j ≤ P.r) {h : C.LPt} (hh : D.IsLocPt h j) : TBB.Ew p (fun _ => 1) F j w h = C.nAux P.k P.r 0 (fun _ _ => 1) 1 h * (p.RR F j w h h).re

theorem **ew_step** (hPS : StandardPairingDecay.PStdC) (p : KB.Pl P Wp Wq) {Cb : ℝ} (hCb : KB.InDDplus2 P p.l0 p.T.base Cb p.0) {w : List C.Letter} (hw : KB.CutWord P p.l0 w) {F : B.Mat P.k P.r → (Wq →L[ℂ] Wp)} (hF : KB.CptCoef p.T.base F) {j : ℕ} (hj : j ≤ P.r) {h : C.LPt} (hh : D.IsLocPt h j) : Integrable (fun c : ℂ => TBB.Ew p (fun _ => 1) F (j + 1) w (h ++ [(1, c)])) ∧ ∫ c : ℂ, TBB.Ew p (fun _ => 1) F (j + 1) w (h ++ [(1, c)]) = TBB.Ew p (fun _ => 1) F j w h

theorem **isLocPt_ofFn** {j : ℕ} (c : Fin j → ℂ) (hc : ∀ a, c a ≠ 0) : D.IsLocPt (List.ofFn fun l => ((1 : ℕ), c l)) j

theorem **topForm_cpt** (hPS : StandardPairingDecay.PStdC) (p : KB.Pl P Wp Wq) {Cb : ℝ} (hCb : KB.InDDplus2 P p.l0 p.T.base Cb p.0) {w : List C.Letter} (hw : KB.CutWord P p.l0 w) {F : B.Mat P.k P.r → (Wq →L[ℂ] Wp)} (hF : KB.CptCoef p.T.base F) : Integrable (topD p F w) ∧ ∫ c, topD p F w c = (p.RR F 0 w [] []).re

CompanionTheory.AutomorphicTheory.DiscreteGroupWaves.NFTopFormCpt

theorem **residue_zero_bound** : ∃ Cs : ℝ, 0 ≤ Cs ∧ ∀ (Wp Wq : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wp] [FiniteDimensional ℂ Wq] (P : M.Par) (p : KB.Pl P Wp Wq) (χ : ℝ → ℝ) (ε A : ℝ) (N : ℕ), C6.IsCut0 χ → 0 < ε → Cs * ((P.r : ℝ) + 1) ^ 4 * ((N : ℝ) * (2 * (P.r : ℝ) + P.k + 1) ^ 2 + (2 * (P.r : ℝ) + P.k + 1) ^ 3) ≤ A → ∀ w : List C.Letter, w.length ≤ N → ∃ K : ℝ, 0 ≤ K ∧ ∀ (h : B.Mat P.k P.r → (Wq →L[ℂ] Wp)) (c : ℝ), B.CkOn ⊤ (B.GV P.k P.r) h → B.CCle A (ordM P N) h c → ‖p.RR (D.cutF p.T.base χ h ε) 0 w [] []‖ ≤ K * c ^ 2

theorem **topForm_cut** (hPS : StandardPairingDecay.PStdC) (Cb : ℝ) : ∃ Cc : ℝ, ∀ (Wp Wq : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wp] [FiniteDimensional ℂ Wq] (P : M.Par) (p : KB.Pl P Wp Wq) (f : B.Mat P.k P.r → (Wq →L[ℂ] Wp)) (χ : ℝ → ℝ) (ε A : ℝ) (N : ℕ), KB.InDDplus2 P p.l0 p.T.base Cb p.0 → C6.IsCut0 χ → 0 < ε → B.CkOn ⊤ (B.GV P.k P.r) f → (∀ m : ℕ, B.InCC A m f) → Cc * ((P.r : ℝ) + 1) ^ 4 * ((N : ℝ) * (2 * (P.r : ℝ) + P.k + 1) ^ 2 + (2 * (P.r : ℝ) + P.k + 1) ^ 3) ≤ A → ∀ w, KB.CutWord P p.l0 w → w.length ≤ N → Integrable (topD p (D.cutF p.T.base χ f ε) w) ∧ ∫ c, topD p (D.cutF p.T.base χ f ε) w c = (p.RR (D.cutF p.T.base χ f ε) 0 w [] []).re

theorem **top_form_all_compact** : NFTopFormAllCpt

CompanionTheory.AutomorphicTheory.DiscreteGroupWaves.NFTopSq

theorem **hsInner_self_re** (A : Wq →L[ℂ] Wp) : (B.hsInner A A).re = ‖B.hsVec A‖ ^ 2

theorem **V_of_not** (p : KB.Pl P Wp Wq) (H : B.Mat P.k P.r → (Wq →L[ℂ] Wp)) {γ : C.LPt} (hγ : ¬ D.IsLocPt γ (P.r + 1)) : p.V H γ = 0

theorem **ew_top_eq** (p : KB.Pl P Wp Wq) (H : B.Mat P.k P.r → (Wq →L[ℂ] Wp)) (w : List C.Letter) (γ : C.LPt) : TBB.Ew p (fun _ => 1) H (P.r + 1) w γ = ∑ i : SqIx P w, sqW p w γ i ^ 2 * ‖B.hsVec (sqX p H w γ i)‖ ^ 2

theorem **ew_top_nonneg** (p : KB.Pl P Wp Wq) (H : B.Mat P.k P.r → (Wq →L[ℂ] Wp)) (w : List C.Letter) (γ : C.LPt) : 0 ≤ TBB.Ew p (fun _ => 1) H (P.r + 1) w γ

theorem **sqX_sub** (p : KB.Pl P Wp Wq) {Cb : ℝ} (hCb : KB.InDDplus2 P p.l0 p.T.base Cb p.0) {w : List C.Letter} (hw : KB.CutWord P p.l0 w) {H₁ H₂ : B.Mat P.k P.r → (Wq →L[ℂ] Wp)} (h₁ : KB.CptCoef p.T.base H₁) (h₂ : KB.CptCoef p.T.base H₂) (γ : C.LPt) (i : SqIx P w) : sqX p (fun g => H₁ g - H₂ g) w γ i = sqX p H₁ w γ i - sqX p H₂ w γ i

theorem **sqrt_sum_tri** {ι E : Type*} [Fintype ι] [NormedAddCommGroup E] [InnerProductSpace ℂ E] (a : ι → ℝ) (x y : ι → E) : |Real.sqrt (∑ i, a i ^ 2 * ‖x i‖ ^ 2) - Real.sqrt (∑ i, a i ^ 2 * ‖y i‖ ^ 2)| ≤ Real.sqrt (∑ i, a i ^ 2 * ‖x i - y i‖ ^ 2)

theorem **topD_tri** (p : KB.Pl P Wp Wq) {Cb : ℝ} (hCb : KB.InDDplus2 P p.l0 p.T.base Cb p.0) {w : List C.Letter} (hw : KB.CutWord P p.l0 w) {H₁ H₂ : B.Mat P.k P.r → (Wq →L[ℂ] Wp)} (h₁ : KB.CptCoef p.T.base H₁) (h₂ : KB.CptCoef p.T.base H₂) (c : Fin (P.r + 1) → ℂ) : |Real.sqrt (topD p H₁ w c) - Real.sqrt (topD p H₂ w c)| ≤ Real.sqrt (topD p (fun g => H₁ g - H₂ g) w c)

theorem **V_bigPt_aesm** (p : KB.Pl P Wp Wq) {H : B.Mat P.k P.r → (Wq →L[ℂ] Wp)} (hH : KB.CptCoef p.T.base H) : AEStronglyMeasurable (fun c : Fin (P.r + 1) → ℂ => p.V H (TBB.bigPt P.r c)) volume

theorem **sqW_meas** (p : KB.Pl P Wp Wq) (w : List C.Letter) (i : SqIx P w) : Measurable (fun c : Fin (P.r + 1) → ℂ => sqW p w (TBB.bigPt P.r c) i)

theorem **sqX_aesm** (p : KB.Pl P Wp Wq) {Cb : ℝ} (hCb : KB.InDDplus2 P p.l0 p.T.base Cb p.0) {w : List C.Letter} (hw : KB.CutWord P p.l0 w) {H : B.Mat P.k P.r → (Wq →L[ℂ] Wp)} (hH : KB.CptCoef p.T.base H) (i : SqIx P w) : AEStronglyMeasurable (fun c : Fin (P.r + 1) → ℂ => sqX p H w (TBB.bigPt P.r c) i) volume

theorem **topD_aesm** (p : KB.Pl P Wp Wq) {Cb : ℝ} (hCb : KB.InDDplus2 P p.l0 p.T.base Cb p.0) {w : List C.Letter} (hw : KB.CutWord P p.l0 w) {H : B.Mat P.k P.r → (Wq →L[ℂ] Wp)} (hH : KB.CptCoef p.T.base H) : AEStronglyMeasurable (topD p H w) volume

CompanionTheory.AutomorphicTheory.HarishChandraMultiplicity.HCMult

theorem **harish_chandra_multiplicity** : PI.HCMult

CompanionTheory.AutomorphicTheory.HeckeStandardModules.EqZeroOfCompAt

theorem **sgl_same** (v : HeightOneSpectrum (𝒪 L⁺)) (r : v.adicCompletion L⁺) : sgl v r v = r

theorem **sgl_ne** (v : HeightOneSpectrum (𝒪 L⁺)) (r : v.adicCompletion L⁺) {w} (hw : w ≠ v) : sgl v r w = 0

theorem **compAt_tmul** (v : HeightOneSpectrum (𝒪 L⁺)) (x : L) (a : AFplus L) : compAt L v (x ⊗ₜ[L⁺] a) = x ⊗ₜ[L⁺] (a v)

theorem **proj_tmul** (s : SplitPlace L) (x : L) (a : AFplus L) : s.proj (x ⊗ₜ[L⁺] a) = s.i x * a s.v

theorem **conj_tmul** (x : L) (a : AFplus L) : conjAL L (x ⊗ₜ[L⁺] a) = IsCMField.complexConj L x ⊗ₜ[L⁺] a

theorem **exists_basis** (e : L) (he : IsCMField.complexConj L e ≠ e) : ∃ b : Module.Basis (Fin 2) L⁺ L, b 0 = 1 ∧ b 1 = e

theorem **eq_zero_of_compAt** (z : AL L) (hz : ∀ v, compAt L v z = 0) : z = 0

theorem **exists_ne** : ∃ e : L, s.i (IsCMField.complexConj L e) ≠ s.i e

theorem **eq_zero_of_proj** {z : AL L} (hz : ∀ w, w ≠ s.v → compAt L w z = 0) (h1 : s.proj z = 0) (h2 : s.proj (conjAL L z) = 0) : z = 0

theorem **exists_pair** (a b : s.v.adicCompletion L⁺) : ∃ z : AL L, (∀ w, w ≠ s.v → compAt L w z = 0) ∧ s.proj z = a ∧ s.proj (conjAL L z) = b

theorem **compAt_conj** {w : HeightOneSpectrum (𝒪 L⁺)} {z : AL L} (h : compAt L w z = 0) : compAt L w (conjAL L z) = 0

theorem **ent_inv** (x : GA L n) (i j : Fin n) : ent x⁻¹ i j = conjAL L (ent x j i)

theorem **loc_apply** (x : GA L n) (i j : Fin n) : (s.loc n x : Matrix (Fin n) (Fin n) (s.v.adicCompletion L⁺)) i j = s.proj (ent x i j)

theorem **mem_Gv** {T : Set (HeightOneSpectrum (𝒪 L⁺))} {x : GA L n} : x ∈ Gv n T ↔ TrivOutside T x

theorem **sub_one_supp** {v : HeightOneSpectrum (𝒪 L⁺)} {x : GA L n} (hx : TrivOutside {v} x) (i j : Fin n) {w} (hw : w ≠ v) : compAt L w (ent x i j - (1 : Matrix (Fin n) (Fin n) (AL L)) i j) = 0

theorem **map_one_apply** {R S : Type*} [CommRing R] [CommRing S] (φ : R →+* S) (i j : Fin n) : φ ((1 : Matrix (Fin n) (Fin n) R) i j) = (1 : Matrix (Fin n) (Fin n) S) i j

theorem **one_apply_symm** {R : Type*} [CommRing R] (i j : Fin n) : (1 : Matrix (Fin n) (Fin n) R) i j = (1 : Matrix (Fin n) (Fin n) R) j i

theorem **eq_one_of_loc** {x : GA L n} (hx : TrivOutside {s.v} x) (h : s.loc n x = 1) : x = 1

theorem **exists_loc** (g : GL (Fin n) (s.v.adicCompletion L⁺)) : ∃ x : GA L n, TrivOutside {s.v} x ∧ s.loc n x = g

theorem **proj_le** {z : AL L} (hz : z ∈ ALint L) : Valued.v (s.proj z) ≤ 1

theorem **mem_KL** {g : GL (Fin n) (s.v.adicCompletion L⁺)} : g ∈ KL s n ↔ (∀ i j, Valued.v ((g : Matrix (Fin n) (Fin n) (s.v.adicCompletion L⁺)) i j) ≤ 1) ∧ ∀ i j, Valued.v (((g⁻¹ : GL (Fin n) _) : Matrix (Fin n) (Fin n) (s.v.adicCompletion L⁺)) i j) ≤ 1

theorem **loc_mem_KL** {x : GA L n} (hx : x ∈ Umax L n) : s.loc n x ∈ KL s n

theorem **mem_Umax_iff** {x : GA L n} (hx : TrivOutside {s.v} x) : x ∈ Umax L n ↔ s.loc n x ∈ KL s n

theorem **emb_triv** (g : GL (Fin n) (s.v.adicCompletion L⁺)) : TrivOutside {s.v} (emb s n g)

theorem **loc_emb** (g : GL (Fin n) (s.v.adicCompletion L⁺)) : s.loc n (emb s n g) = g

theorem **emb_loc** {x : GA L n} (hx : TrivOutside {s.v} x) : emb s n (s.loc n x) = x

theorem **conj_emb** (x : GA L n) (g : GL (Fin n) (s.v.adicCompletion L⁺)) : x * emb s n g * x⁻¹ = emb s n (s.loc n x * g * (s.loc n x)⁻¹)

theorem **commute_of_ne** {v v' : HeightOneSpectrum (𝒪 L⁺)} (h : v ≠ v') {x y : GA L n} (hx : TrivOutside {v} x) (hy : TrivOutside {v'} y) : x * y = y * x

CompanionTheory.AutomorphicTheory.HeckeStandardModules.ExistsCen

theorem **int_mem** (A : ValuationSubring Ω) (b : 𝒪 Ω) : (b : Ω) ∈ A

theorem **int_le** (A : ValuationSubring Ω) (b : 𝒪 Ω) : A.valuation (b : Ω) ≤ 1

theorem **mem_cen** {A : ValuationSubring Ω} {b : 𝒪 Ω} : b ∈ cen A ↔ A.valuation (b : Ω) < 1

theorem **val_eq_one** {A : ValuationSubring Ω} {s : 𝒪 Ω} (h : s ∉ cen A) : A.valuation (s : Ω) = 1

theorem **frac** [Algebra.IsAlgebraic E Ω] (A : ValuationSubring Ω) {x : Ω} (hx : x ∈ A) : ∃ b s : 𝒪 Ω, s ∉ cen A ∧ x * s = b

theorem **coe_smul** (g : Ω ≃ₐ[E] Ω) (b : 𝒪 Ω) : ((g • b : 𝒪 Ω) : Ω) = g b

theorem **map_mem** [Algebra.IsAlgebraic E Ω] (A : ValuationSubring Ω) (g : Ω ≃ₐ[E] Ω) (hg : ∀ b : 𝒪 Ω, g • b ∈ cen A → b ∈ cen A) {x : Ω} (hx : x ∈ A) : g x ∈ A

theorem **of_cong** [Algebra.IsAlgebraic E Ω] (A : ValuationSubring Ω) (g : Ω ≃ₐ[E] Ω) {q : ℕ} (hq : q ≠ 0) (hg : ∀ b : 𝒪 Ω, g • b - b ^ q ∈ cen A) : g ∈ A.decompositionSubgroup E ∧ ∀ x ∈ A, g x - x ^ q ∈ A.nonunits

theorem **exists_cen** (Q : Ideal (𝒪 Ω)) [hQ : Q.IsMaximal] : ∃ A : ValuationSubring Ω, cen A = Q

theorem **lift** [IsGalois E Ω] (L : IntermediateField E Ω) [FiniteDimensional E L] [Normal E L] (Q : Ideal (𝒪 Ω)) [hQ : Q.IsMaximal] (τ : L ≃ₐ[E] L) (θ : 𝒪 Ω ⧸ Q →+* 𝒪 Ω ⧸ Q) (hτ : ∀ x : 𝒪 L, Ideal.Quotient.mk Q (algebraMap (𝒪 L) (𝒪 Ω) (τ • x)) = θ (Ideal.Quotient.mk Q (algebraMap (𝒪 L) (𝒪 Ω) x))) : ∃ φ : Ω ≃ₐ[E] Ω, AlgEquiv.restrictNormalHom L φ = τ ∧ ∀ b : 𝒪 Ω, Ideal.Quotient.mk Q (φ • b) = θ (Ideal.Quotient.mk Q b)

CompanionTheory.AutomorphicTheory.HeckeStandardModules.ExistsDenFin

theorem **ALint_eq** : ALint L = tensInt (F

theorem **levelafC_nat** (N : ℕ) : P.levelafC L (N : 𝒪 L⁺) = (N : AL L)

theorem **exists_den** (z : AL L) : ∃ c : ℕ, c ≠ 0 ∧ (c : AL L) * z ∈ ALint L

theorem **exists_den_fin** {T : Set (GA L n)} (hT : T.Finite) : ∃ c : ℕ, c ≠ 0 ∧ ∀ t ∈ T, ∀ i j, (c : AL L) * ent t i j ∈ ALint L

theorem **mem_UJ_one** {A : Type} [CommRing A] (σ : A →+* A) (hσ : ∀ x, σ (σ x) = x) (g : GL (Fin n) A) : g ∈ UJ σ (1 : Matrix (Fin n) (Fin n) A) ↔ g ∈ UGroup n σ hσ

theorem **reps_Umax** : ∃ T : Set (GA L n), T.Finite ∧ ∀ g, ∃ γ ∈ GQ L n, ∃ t ∈ T, ∃ κ ∈ Umax L n, g = γ * t * κ

theorem **reps_Cong** {N : ℕ} (hN : N ≠ 0) : ∃ T : Set (GA L n), T.Finite ∧ ∀ u ∈ Umax L n, ∃ t ∈ T, NG.Cong L n N (t⁻¹ * u)

theorem **reps** {N : ℕ} (hN : N ≠ 0) : ∃ T : Set (GA L n), T.Finite ∧ ∀ g, ∃ γ ∈ GQ L n, ∃ t ∈ T, ∃ κ, NG.Cong L n N κ ∧ g = γ * t * κ

theorem **normaliser** {ϖ : s.v.adicCompletion L⁺} (hϖ : Valued.v ϖ = WithZero.exp (-1 : ℤ)) (t : GL (Fin n) (s.v.adicCompletion L⁺)) (ht : ∀ k ∈ KL s n, t * k * t⁻¹ ∈ KL s n) : ∃ u ∈ KL s n, ∀ h, t * h * t⁻¹ = u * h * u⁻¹

theorem **exists_prime** : ∃ p : ℕ, p.Prime ∧ s.Above p ∧ ∃ f : ℕ, s.q = p ^ f

theorem **pow_mul_mem** {p : ℕ} (hp : s.Above p) {z : AL L} (hz : ∀ w, w ≠ s.v → compAt L w z = 0) : ∃ a : ℕ, ((p ^ a : ℕ) : AL L) * z ∈ ALint L

theorem **ALint_le_Sl** {l : ℕ} (hl : l.Prime) : ALint L ≤ Sl L hl

theorem **ent_mem_Sl** {l : ℕ} (hl : l.Prime) (hs : ¬ s.Above l) {x : GA L n} (hx : TrivOutside {s.v} x) (i j : Fin n) : ent x i j ∈ Sl L hl

CompanionTheory.AutomorphicTheory.HeckeStandardModules.HeckeA

theorem **coset_sum_eq** (J : Subgroup G) (hJ : ∀ h, ι h ∈ J ↔ h ∈ K) (d : H) {f : G → W} (hf : Inv ι K f) (g : G) {D : Set G} (hD : ∀ x, x ∈ D ↔ ∃ h ∈ DoubleCoset.doubleCoset d (K : Set H) K, ι h = x) : (∑ᶠ c : G ⧸ J, if hc : ∃ x ∈ D, (x : G ⧸ J) = c then f (g * hc.choose) else 0) = op ι K (dosetInd ℂ K d) f g

theorem **exists_sum** (hK : IsCommensurated K) {φ : H → ℂ} (hφ : φ ∈ Hecke ℂ K) : ∃ S : Finset (H ⧸ K), ∀ (f : G → W) (g : G), op ι K φ f g = ∑ c ∈ S, φ c.out • f (g * ι c.out)

theorem **op_comm_same** (hK : IsCommensurated K) {φ ψ : H → ℂ} (hφ : φ ∈ Hecke ℂ K) (hψ : ψ ∈ Hecke ℂ K) (hc : conv K φ ψ = conv K ψ φ) {f : G → W} (hf : Inv ι K f) : op ι K φ (op ι K ψ f) = op ι K ψ (op ι K φ f)

theorem **op_conj** {φ : H → ℂ} (hφ : IsBiInv K φ) {f : G → W} (hf : Inv ι K f) {κ : G} {u : H} (hu : u ∈ K) {c : ℂ} (hκ : ∀ h, κ * ι h * κ⁻¹ = ι (u * h * u⁻¹)) (hc : ∀ g, f (g * κ) = c • f g) (hfin : IsCommensurated K) (hφ' : φ ∈ Hecke ℂ K) (g : G) : op ι K φ f (g * κ) = c • op ι K φ f g

theorem **eval_fg** {σ : Type} {M₀ : Submodule R ℂ} (h₀ : M₀.FG) (P : MvPolynomial σ ℂ) : ∃ M : Submodule R ℂ, M.FG ∧ ∀ v : σ → ℂ, (∀ i, v i ∈ M₀) → MvPolynomial.eval v P ∈ M

theorem **apply_fg** [Module R W] [IsScalarTower R ℂ W] {σ κ : Type} [Fintype κ] [DecidableEq κ] (b : Module.Basis κ ℂ W) (P : κ → κ → MvPolynomial σ ℂ) {M₀ : Submodule R ℂ} (h₀ : M₀.FG) (w : W) : ∃ N : Submodule R W, N.FG ∧ ∀ (A : W →ₗ[ℂ] W) (v : σ → ℂ), (∀ i, v i ∈ M₀) → (LinearMap.toMatrix b b A = fun i j => MvPolynomial.eval v (P i j)) → A w ∈ N

CompanionTheory.AutomorphicTheory.HeckeStandardModules.IntegralOfModule

theorem **tSet_iff** {ϖ : s.v.adicCompletion L⁺} (h0 : ϖ ≠ 0) (k : ℕ) (x : Formalism.AUTREP.GAf L n) : x ∈ Formalism.AUTREP.tSet L n s ϖ k ↔ ∃ h ∈ DoubleCoset.doubleCoset (dg n s h0 k) (KL s n : Set _) (KL s n), emb s n h = x

theorem **triv_compl** {B : Set (HeightOneSpectrum (𝒪 L⁺))} (hs : s.v ∉ B) {x : Formalism.AUTREP.GAf L n} (hx : TrivOutside {s.v} x) : TrivOutside Bᶜ x

theorem **emb_mem_J** {J : Subgroup (Formalism.AUTREP.GAf L n)} {χ : J →* ℂˣ} {B : Set (HeightOneSpectrum (𝒪 L⁺))} (hJ : NG.HypOutside J χ B) (hs : s.v ∉ B) (h : GL (Fin n) (s.v.adicCompletion L⁺)) : emb s n h ∈ J ↔ h ∈ KL s n

theorem **chi_emb** {J : Subgroup (Formalism.AUTREP.GAf L n)} {χ : J →* ℂˣ} {B : Set (HeightOneSpectrum (𝒪 L⁺))} (hJ : NG.HypOutside J χ B) (hs : s.v ∉ B) {h : GL (Fin n) (s.v.adicCompletion L⁺)} (hm : emb s n h ∈ J) : χ (_, hm) = 1

theorem **inv_of_mem** (hJ : NG.HypOutside J χ B) (hs : s.v ∉ B) {f : Formalism.AUTREP.GAf L n → W} (hf : f ∈ NG.TypeSp ρ J χ) : Inv (emb s n) (KL s n) f

theorem **heckeD_eq** (hJ : NG.HypOutside J χ B) (hs : s.v ∉ B) {f : Formalism.AUTREP.GAf L n → W} (hf : Inv (emb s n) (KL s n) f) (d : GL (Fin n) (s.v.adicCompletion L⁺)) {D : Set (Formalism.AUTREP.GAf L n)} (hD : ∀ x, x ∈ D ↔ ∃ h ∈ DoubleCoset.doubleCoset d (KL s n : Set _) (KL s n), emb s n h = x) : NG.heckeD J D f = op (emb s n) (KL s n) (dosetInd ℂ (KL s n) d) f

theorem **comm_KL** (s : SplitPlace L) : IsCommensurated (KL s n)

theorem **dosetInd_mem** (s : SplitPlace L) (d : GL (Fin n) (s.v.adicCompletion L⁺)) : dosetInd ℂ (KL s n) d ∈ Hecke ℂ (KL s n)

theorem **op_equiv** (hJ : NG.HypOutside J χ B) (hs : s.v ∉ B) (hϖ : ∃ ϖ : s.v.adicCompletion L⁺, Valued.v ϖ = WithZero.exp (-1 : ℤ)) {f : Formalism.AUTREP.GAf L n → W} (hf : f ∈ NG.TypeSp ρ J χ) (d : GL (Fin n) (s.v.adicCompletion L⁺)) (κ : J) (g) : op (emb s n) (KL s n) (dosetInd ℂ (KL s n) d) f (g * κ) = ((χ κ : ℂˣ) : ℂ) • op (emb s n) (KL s n) (dosetInd ℂ (KL s n) d) f g

theorem **op_mem** (hJ : NG.HypOutside J χ B) (hs : s.v ∉ B) (hϖ : ∃ ϖ : s.v.adicCompletion L⁺, Valued.v ϖ = WithZero.exp (-1 : ℤ)) (hN : ∃ N : ℕ, 0 < N ∧ ∀ κ, NG.Cong L n N κ → ∃ h : κ ∈ J, χ (κ, h) = 1) {f : Formalism.AUTREP.GAf L n → W} (hf : f ∈ NG.TypeSp ρ J χ) (d : GL (Fin n) (s.v.adicCompletion L⁺)) : op (emb s n) (KL s n) (dosetInd ℂ (KL s n) d) f ∈ NG.TypeSp ρ J χ

theorem **tSet_same** (s s' : SplitPlace L) (hv : s'.v = s.v) {ϖ' : s'.v.adicCompletion L⁺} (h0 : ϖ' ≠ 0) (k : ℕ) : ∃ d : GL (Fin n) (s.v.adicCompletion L⁺), ∀ x, x ∈ Formalism.AUTREP.tSet L n s' ϖ' k ↔ ∃ h ∈ DoubleCoset.doubleCoset d (KL s n : Set _) (KL s n), emb s n h = x

theorem **integral_of_module** [Module R W] [IsScalarTower R ℂ W] [IsNoetherianRing R] (N : Submodule R W) (hN : N ≠ ⊥) (hfg : N.FG) (ν : ℂ) (hν : ∀ w ∈ N, ν • w ∈ N) : IsIntegral R ν

CompanionTheory.AutomorphicTheory.HeckeStandardModules.InvMemRl

theorem **ent_mul** (x y : GA L n) : ent (x * y) = ent x * ent y

theorem **mat_mem** {S : Subring (AL L)} {A B : Matrix (Fin n) (Fin n) (AL L)} (hA : ∀ i j, A i j ∈ S) (hB : ∀ i j, B i j ∈ S) (i j : Fin n) : (A * B) i j ∈ S

theorem **inv_mem_Rl** {m : ℕ} (hm : ¬ l ∣ m) : ∃ r : Rl l, algebraMap (Rl l) ℂ r = (m : ℂ)⁻¹

theorem **gq_ent** {γ : GA L n} (hγ : γ ∈ GQ L n) : ∃ γ₀ : GL (Fin n) L, (γ : GL (Fin n) (AL L)) = Matrix.GeneralLinearGroup.map Algebra.TensorProduct.includeLeftRingHom γ₀

theorem **M0_fg** (l : ℕ) [Fact l.Prime] (c : ℕ) : (M0 L l c).FG

theorem **key_M0** {l : ℕ} [Fact l.Prime] {c : ℕ} (hc : c ≠ 0) (x : L) (hx : ∃ m : ℕ, ¬ l ∣ m ∧ ((m * c ^ 2 : ℕ) : AL L) * (x ∈ₜ[maximalRealSubfield L] (1 : AFplus L)) ∈ ALint L) (τ : L →+* ℂ) : τ x ∈ M0 L l c

theorem **eig_int** (alg : Formalism.AUTREP.IsAlgWeight L n ρ) (hJ : NG.HypOutside J χ B) (hs : s.v ∉ B) {l : ℕ} [hl' : Fact l.Prime] (hsl : ¬ s.Above l) (d : GL (Fin n) (s.v.adicCompletion (maximalRealSubfield L))) {f : Formalism.AUTREP.GAf L n → W} (hf : f ∈ NG.TypeSp ρ J χ) (hf0 : f ≠ 0) (ν : ℂ) (hν : op (emb s n) (KL s n) (dosetInd ℂ (KL s n) d) f = ν • f) : IsIntegral (Rl l) ν

theorem **heckeStd** : HeckeStdStatement

CompanionTheory.AutomorphicTheory.JacobiDiagonalBounds.EvQTrJBFF

theorem **evQ_trJB_FF** (p : KB.Pl P Wp Wq) {e : M.Ty P → ℂ → (Wq →L[ℂ] Wp)} (he : IsFrameB P p.T.base e) {J : B.Mat P.k P.r → (Wq →L[ℂ] Wp)} (hJ : J ∈ FFKB p) (ρ : M.Ty P) (q : MA.Qp P.r P.TT) : MA.evQ P (trJB P p.T.base e J ρ) q = rawJ P p.T.base e J ρ q

theorem **qdom** (p : KB.Pl P Wp Wq) {x : MA.E P.r} : x ∈ MA.Qset P.r P.TT ↔ x ∈ JF.QDom P.k P.r p.T.base.z p.T.base.z'

theorem **thrW_exists** (k r m : ℕ) : ∃ A : ℝ, WeakThresholdClauses.WeakDominationThreshold k r A m

theorem **tie_FF** (hLL : LocLift) (p : KB.Pl P Wp Wq) {e : M.Ty P → ℂ → (Wq →L[ℂ] Wp)} (he : IsFrameB P p.T.base e) (hG : GaugeQQ P p.T.base e) {Lm : R.LevelData (MA.A P (MA.DQ P)) P.r} (hM : MA.IsModelLevel P (MA.DQ P) Lm) {w : List C.Letter} (hw : KB.CutWord P p.l0 w) (a : Wd.Ix (Fin w.length)) (hloc : Loc (gM P p.l0 Lm (D.positionted w) a)) {Dop : B.DiffOp P.k P.r Wp Wq} (hS : B.SmoothCoef (B.GPos p.T.base) Dop) (hC : B.Commutes p.T.base (B.GPos p.T.base) Dop) (h0 : ∀ F, ∀ g ∈ C6.domL p.T.base ∅, p.0.0 w a F g = B.opApply Dop F g) {J H : B.Mat P.k P.r → (Wq →L[ℂ] Wp)} (hJ : J ∈ FFKB p) (hH : ∀ g ∈ JF.Om P.k P.r p.T.base.z p.T.base.z', B.opApply Dop J g = B.JF p.T.base H g) (q : MA.Qp P.r P.TT) (h0 : B.JF p.T.base H (B.gq p.T.base q.1) ≠ 0) (ρ : M.Ty P) : rawJ P p.T.base e (B.JF p.T.base H) ρ q = MA.evQ P (MA.actA P (gM P p.l0 Lm (D.positionted w) a) (trJB P p.T.base e J) ρ) q

theorem **jf_Gpow** (p : KB.Pl P Wp Wq) (s : ℕ) {F : B.Mat P.k P.r → (Wq →L[ℂ] Wp)} (hF : B.JF p.T.base F ∈ FFKB p) : B.JF p.T.base (fun g => ((Gf P.k P.r p.T.base.z p.T.base.z' g ^ s : ℝ) : ℂ) • F g) ∈ FFKB p

theorem **pd_list** {E ι : Type} [AddCommGroup E] [Module ℂ E] {o : ℕ} (l : List ι) (f : ι → ℕ → E) (h : ∀ i ∈ l, PD o (f i)) : PD o fun s => (l.map fun i => f i s).sum

theorem **pd_mono** {E : Type} [AddCommGroup E] [Module ℂ E] {o n : ℕ} (hn : n ≤ o) (x : E) : PD o fun s => (s : ℂ) ^ n • x

theorem **scoef_zero** (p : KB.Pl P Wp Wq) {w : List C.Letter} {a : Wd.Ix (Fin w.length)} {Q o : ℕ} {L : Fin (Q + 1) → B.DiffOp P.k P.r Wp Wq} (hS : IsSCoef p w a Q L) {Dop : B.DiffOp P.k P.r Wp Wq} (ho : ∀ T ∈ Dop, T.word.length ≤ o) (h0 : ∀ F, ∀ g ∈ C6.domL p.T.base ∅, p.0.0 w a F g = B.opApply Dop F g) {F : B.Mat P.k P.r → (Wq →L[ℂ] Wp)} (hF : B.CkOn ⊤ (B.GV P.k P.r) F) {g : B.Mat P.k P.r} (hg : g ∈ C6.domL p.T.base ∅) (qq : Fin (Q + 1)) (hq : o < qq) : B.opApply (L qq) F g = 0

CompanionTheory.AutomorphicTheory.JacobiDiagonalBounds.ModelLoc

theorem **loc_central** (X : MA.A P (MA.DQ P)) (h : ∀ i j, ∃ z : ℂ, X i j = algebraMap ℂ _ z) : X ∈ locAlg P

theorem **loc_fn** (g gb : MA.SmQ P.r P.TT) : MA.sc P (MA.DQ P) ((MA.DQ P).fnP g gb) ∈ locAlg P

theorem **euler_loc** (v : M.Var P.r) {U : Set (MA.E P.r)} (hU : IsOpen U) (f : MA.SmQ P.r P.TT) (hf : ∀ q : MA.Qp P.r P.TT, q.1 ∈ U → MA.evQ P f q = 0) (q : MA.Qp P.r P.TT) (hq : q.1 ∈ U) : MA.evQ P (MA.eulerQ v f) q = 0

theorem **loc_ev** (v : M.Var P.r) (Z : (MA.DQ P).Op) : MA.sc P (MA.DQ P) ((MA.DQ P).mkP ((MA.DQ P).ev v) Z) ∈ locAlg P

theorem **loc_diag_smul** (a : M.Ty P → ℂ) (x y : (MA.DQ P).Pr) (hx : MA.sc P (MA.DQ P) x ∈ locAlg P) (hy : MA.sc P (MA.DQ P) y ∈ locAlg P) : (Matrix.diagonal fun π => x + a π • y) ∈ locAlg P

theorem **sc_mul'** (x y : (MA.DQ P).Pr) : MA.sc P (MA.DQ P) (x ∗ y) = MA.sc P (MA.DQ P) x ∗ MA.sc P (MA.DQ P) y

theorem **loc_dz** (g : MA.SmQ P.r P.TT) (v : M.Var P.r) (Z : (MA.DQ P).Op) : MA.sc P (MA.DQ P) ((MA.DQ P).mkP ((MA.DQ P).mulOp g ∗ (MA.DQ P).ev v) Z) ∈ locAlg P

theorem **loc_fns** (hM : MA.IsModelLevel P (MA.DQ P) L) {f : MA.A P (MA.DQ P)} (hf : f ∈ Subring.closure L.fns) : f ∈ locAlg P

theorem **mat_prod_mem** {A : Type} [Ring A] [Algebra ℂ A] (B : Subalgebra ℂ A) (l : List (Matrix (Fin 2) (Fin 2) A)) (h : ∀ X ∈ l, ∀ i j, X i j ∈ B) : ∀ i j, l.prod i j ∈ B

theorem **loc_hat** (hM : MA.IsModelLevel P (MA.DQ P) L) {X : MA.A P (MA.DQ P)} (hX : X ∈ hats L) : X ∈ locAlg P

theorem **loc_BB** (hM : MA.IsModelLevel P (MA.DQ P) L) {b : MA.A P (MA.DQ P)} (hb : b ∈ BB P L) : b ∈ locAlg P

theorem **loc_nfs** (hM : MA.IsModelLevel P (MA.DQ P) L) {n : ℕ} {x : MA.A P (MA.DQ P)} (hx : x ∈ NFs L (BB P L) (L.Di ^ n) n) : x ∈ locAlg P

theorem **Loc.congr** {a : MA.A P (MA.DQ P)} (ha : Loc a) {U : Set (MA.E P.r)} (hU : IsOpen U) {f g : M.Ty P → MA.SmQ P.r P.TT} (h : ∀ ρ (q : MA.Qp P.r P.TT), q.1 ∈ U → MA.evQ P (f ρ) q = MA.evQ P (g ρ) q) (ρ : M.Ty P) (q : MA.Qp P.r P.TT) (hq : q.1 ∈ U) : MA.evQ P (MA.actA P a f ρ) q = MA.evQ P (MA.actA P a g ρ) q

CompanionTheory.AutomorphicTheory.JacobiDiagonalBounds.NFFinal

theorem **model_adm** {P : M.Par} (hP : P.OK) {l0 : ℕ} (h1 : 1 ≤ l0) (h2 : l0 ≤ P.k) {w : List C.Letter} (hw : KB.CutWord P l0 w) : admitted (MA.startM P (MA.DQ P) l0) (D.positionted w)

theorem **model_nf** {P : M.Par} {L : LevelData (MA.A P (MA.DQ P)) P.r} (hP : P.OK) (hM : MA.IsModelLevel P (MA.DQ P) L) {l0 : ℕ} (h1 : 1 ≤ l0) (h2 : l0 ≤ P.k) {w : List C.Letter} (hw : KB.CutWord P l0 w) (a : Ix (Fin w.length)) : (∀ b ∈ BB P L, ∀ f ∈ L.fns, Commute b f) ∧ gM P l0 L (D.positionted w) a ∈ NFs L (BB P L) (L.Di ^ w.length) w.length

theorem **nfModelNFg_ok** (P : M.Par) (L : R.LevelData (MA.A P (MA.DQ P)) P.r) (hP : P.OK) (hM : MA.IsModelLevel P (MA.DQ P) L) (l0 : ℕ) (h1 : 1 ≤ l0) (h2 : l0 ≤ P.k) (Y : Finset ℝ) (w : List C.Letter) (hw : TBB.AdmW P l0 Y w) (e : Wd.Ix (Fin w.length)) : ∃ (m : ℕ) (b : Fin m → MA.A P (MA.DQ P)) (i i' : Fin m → ℕ) (h : Fin m → List (MA.A P (MA.DQ P))), (∀ j, b j ∈ Submonoid.closure (MA.bddSet P L)) ∧ (∀ j, i j + i' j = w.length) ∧ (∀ j, (h j).length = w.length ∧ ∀ a ∈ h j, a ∈ MA.hatSet P L) ∧ ∑ e' : Wd.Ix (Fin w.length), (Fr.ofWord L (MA.startM P (MA.DQ P) l0) (D.positionted w)).F e e' ∗ Wd.uw (MA.sites P (MA.DQ P)) (D.positionted w) e' = ∑ j, b j ∗ L.Di ^ i j ∗ L.Dbi ^ i' j ∗ (h j).prod

theorem **model_pt** {P : M.Par} {L : LevelData (MA.A P (MA.DQ P)) P.r} (hP : P.OK) (hM : MA.IsModelLevel P (MA.DQ P) L) {l0 : ℕ} (h1 : 1 ≤ l0) (h2 : l0 ≤ P.k) {w : List C.Letter} (hw : KB.CutWord P l0 w) (a : Ix (Fin w.length)) (s : ℕ) : ∃ (m : ℕ) (b : Fin m → MA.A P (MA.DQ P)) (u : Fin m → List (MA.A P (MA.DQ P))), (∀ j, b j ∈ Submonoid.closure (MA.bddSet P L)) ∧ (∀ j, (u j).length = w.length ∧ ∀ x ∈ u j, x ∈ MA.hatSet P L) ∧ ∀ f g : M.Ty P → MA.SmQ P.r P.TT, (∀ ρ q, MA.evQ P (g ρ) q = ((MA.Gq P q ^ s : ℝ) : ℂ) ∗ MA.evQ P (f ρ) q) → ∀ (q : MA.Qp P.r P.TT) ρ, MA.evQ P (MA.actA P (gM P l0 L (D.positionted w) a) g ρ) q = ((MA.Gq P q ^ s : ℝ) : ℂ) ∗ ((∏ l, q.1.1 l) ∗ q.1.2)⁻¹ ^ w.length ∗ ∑ j, MA.evQ P (MA.actA P (b j ∗ (u j).prod) f ρ) q

theorem **normal_form_model_point** : Formalism.MainStatements.NFTie.NFModelPt

CompanionTheory.AutomorphicTheory.JacobiDiagonalBounds.NFModel

theorem **cG** : L.G ∈ Subring.closure L.fns

theorem **hat_G** (X : A) (hX : X ∈ hats L) : ∃ X' ∈ hats L, X ∗ L.G = L.G ∗ X'

theorem **hats_G** (h : List A) (hh : ∀ a ∈ h, a ∈ hats L) : ∃ h' : List A, h'.length = h.length ∧ (∀ a ∈ h', a ∈ hats L) ∧ h.prod ∗ L.G = L.G ∗ h'.prod

theorem **hats_Gpow** (s : ℕ) (h : List A) (hh : ∀ a ∈ h, a ∈ hats L) : ∃ h' : List A, h'.length = h.length ∧ (∀ a ∈ h', a ∈ hats L) ∧ h.prod ∗ L.G ^ s = L.G ^ s ∗ h'.prod

theorem **star_prod_comm** [StarRing A] {ι : Type} (f g : ι → A) (h : ∀ i, star (f i) = g i) (hc : ∀ i j, Commute (g i) (g j)) (l : List ι) : star (l.map f).prod = (l.map g).prod

theorem **sc_central** (w : (MA.DQ P).Pr) (X : MA.A P (MA.DQ P)) (h : ∀ i j, ∃ z : ℂ, X i j = algebraMap ℂ _ z) : Commute X (MA.sc P (MA.DQ P) w)

theorem **fns_sc** (hM : MA.IsModelLevel P (MA.DQ P) L) {f : MA.A P (MA.DQ P)} (hf : f ∈ L.fns) : ∃ w, f = MA.sc P (MA.DQ P) w

theorem **kap_diag** (g : M.Ty P → ℂ) : MA.kap P (MA.DQ P) g = Matrix.diagonal fun π => algebraMap ℂ (MA.DQ P).Pr (g π)

theorem **kap_central** (g : M.Ty P → ℂ) (i j : M.Ty P) : ∃ z : ℂ, MA.kap P (MA.DQ P) g i j = algebraMap ℂ _ z

theorem **SS_central** (l : ℕ) (i j : M.Ty P) : ∃ z : ℂ, MA.SS P (MA.DQ P) l i j = algebraMap ℂ _ z

theorem **star_kap** (g : M.Ty P → ℂ) : star (MA.kap P (MA.DQ P) g) = MA.kap P (MA.DQ P) (star g)

theorem **model_star_c** (hM : MA.IsModelLevel P (MA.DQ P) L) (l : Fin P.r) : star (L.ρ l) = -L.pb l ∧ star (L.c l) = L.cb l ∧ star (L.ci l) = L.cbi l

theorem **model_star_D** (hM : MA.IsModelLevel P (MA.DQ P) L) : star L.Dr = L.Drb ∧ star L.Dri = L.Drbi

theorem **BB_span** {b : MA.A P (MA.DQ P)} (hb : b ∈ BB P L) : b ∈ Submodule.span ℂ (Submonoid.closure (MA.bddSet P L) : Set (MA.A P (MA.DQ P)))

theorem **closure_BB** {b : MA.A P (MA.DQ P)} (hb : b ∈ Submonoid.closure (MA.bddSet P L)) : b ∈ BB P L

theorem **model_hyp** (hHB : JDHatBdd) (hTT : 0 ≤ P.TT) (hM : MA.IsModelLevel P (MA.DQ P) L) (hS : IsStarLevel L) (l0 : ℕ) : NFHypA L (MA.startM P (MA.DQ P) l0) (BB P L) (MA.kap P (MA.DQ P)).range

theorem **model_aHyp** (hM : MA.IsModelLevel P (MA.DQ P) L) (hS : IsStarLevel L) {l0 : ℕ} (hst : IsLevelStart L (MA.startM P (MA.DQ P) l0)) (hSi : SitesOK (K

theorem **gM_eq** (hM : MA.IsModelLevel P (MA.DQ P) L) (l0 : ℕ) {S : Type} [Fintype S] [DecidableEq S] (w : List (Letter ℂ S)) (e : Ix S) : gM P l0 L w e = ((ofWord L (MA.startM P (MA.DQ P) l0) w).F ∗ TL.bTA L w) e e1

theorem **nfs_fin** {Q x : MA.A P (MA.DQ P)} {n : ℕ} (hx : x ∈ NFs L (BB P L) Q n) : ∃ (m : ℕ) (b : Fin m → MA.A P (MA.DQ P)) (u : Fin m → List (MA.A P (MA.DQ P))), (∀ j, b j ∈ Submonoid.closure (MA.bddSet P L)) ∧ (∀ j, (u j).length = n ∧ ∀ a ∈ u j, a ∈ MA.hatSet P L) ∧ x = ∑ j, b j ∗ Q ∗ (u j).prod

theorem **MulBy.mul** {a b : MA.A P (MA.DQ P)} {φ ψ} (ha : MulBy a φ) (hb : MulBy b ψ) : MulBy (a ∗ b) fun q => φ q ∗ ψ q

theorem **mulBy_one** : MulBy (1 : MA.A P (MA.DQ P)) fun _ => 1

theorem **MulBy.pow** {a : MA.A P (MA.DQ P)} {φ} (ha : MulBy a φ) (n : ℕ) : MulBy (a ^ n) fun q => φ q ^ n

theorem **mulBy_prod** {ι : Type} (a : ι → MA.A P (MA.DQ P)) (φ : ι → MA.Qp P.r P.TT → ℂ) (h : ∀ i, MulBy (a i) (φ i)) (l : List ι) : MulBy (l.map a).prod fun q => (l.map fun i => φ i q).prod

theorem **MulBy.inv** {a b : MA.A P (MA.DQ P)} {φ} (ha : MulBy a φ) (h : a ∗ b = 1) (h0 : ∀ q, φ q ≠ 0) : MulBy b fun q => (φ q)⁻¹

theorem **mulBy_fn** (g gb : MA.SmQ P.r P.TT) : MulBy (MA.sc P (MA.DQ P) ((MA.DQ P).fnP g gb)) (MA.evQ P g)

theorem **MulBy.congr** {a : MA.A P (MA.DQ P)} {φ ψ} (ha : MulBy a φ) (h : ∀ q, ψ q = φ q) : MulBy a ψ

theorem **mulBy_D** (hM : MA.IsModelLevel P (MA.DQ P) L) : MulBy L.D (fun q => (∏ l, q.1.1 l) ∗ q.1.2) ∧ MulBy L.G fun q => ((MA.Gq P q : ℝ) : ℂ)

theorem **nf_pointwise** (hTT : 0 ≤ P.TT) (hM : MA.IsModelLevel P (MA.DQ P) L) (hfn : ∀ b ∈ BB P L, ∀ f ∈ L.fns, Commute b f) {x : MA.A P (MA.DQ P)} {n : ℕ} (hx : x ∈ NFs L (BB P L) (L.Di ^ n) n) (s : ℕ) : ∃ (m : ℕ) (b : Fin m → MA.A P (MA.DQ P)) (u : Fin m → List (MA.A P (MA.DQ P))), (∀ j, b j ∈ Submonoid.closure (MA.bddSet P L)) ∧ (∀ j, (u j).length = n ∧ ∀ a ∈ u j, a ∈ MA.hatSet P L) ∧ ∀ f g : M.Ty P → MA.SmQ P.r P.TT, (∀ ρ q, MA.evQ P (g ρ) q = ((MA.Gq P q ^ s : ℝ) : ℂ) * MA.evQ P (f ρ) q) → ∀ (q : MA.Qp P.r P.TT) ρ, MA.evQ P (MA.actA P x g ρ) q = ((MA.Gq P q ^ s : ℝ) : ℂ) * (((∏ l, q.1.1 l) * q.1.2)⁻¹ ^ n * ∑ j, MA.evQ P (MA.actA P (b j * (u j).prod) f ρ) q

CompanionTheory.AutomorphicTheory.JacobiDiagonalBounds.NfTieHatSOf

theorem **opOrder_le** {k r : ℕ} {Wp Wq : Type} [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wq] {o : ℕ} (Dop : B.DiffOp k r Wp Wq) (h : ∀ T ∈ Dop, T.word.length ≤ o) : B.opOrder Dop ≤ o

theorem **thr_arith** (CcL Cb R x W o s d A : ℝ) (hR : 0 ≤ R) (hx : 1 ≤ x) (hW : 0 ≤ W) (ho : o ≤ Cb * W * x) (hs : s ≤ o) (hd : d ≤ o) (hA : (|CcL| + 12 + 8 * |Cb|) * (R + 1) ^ 4 * (W * x ^ 2 + x ^ 3) ≤ A) : CcL * (R + 1) ^ 4 * (W * x ^ 2 + x ^ 3) ≤ A ∧ 12 * (R + 1) * x ^ 2 + 6 * R * d + 0 ≤ A − (2 * R + 2) * s

theorem **sum_trunc** {E : Type} [AddCommGroup E] [Module ℂ E] {Q o : ℕ} (x : ℂ) (v : Fin (Q + 1) → E) (hv : ∀ q : Fin (Q + 1), o < q → v q = 0) : ∑ q : Fin (Q + 1), x ^ (q : ℕ) • v q = ∑ j : Fin (o + 1), x ^ (j : ℕ) • (if h : (j : ℕ) < Q + 1 then v (j, h) else 0)

theorem **nfTieGen_of** (hK : GCore) : NFTieGen

CompanionTheory.AutomorphicTheory.JacobiDiagonalBounds.NormalFormHatCompact

theorem **normal_form_hat_compact** : CompanionTheory.NFCpt.NFTieHatC

CompanionTheory.AutomorphicTheory.JacobiDiagonalBounds.StarLoweringConstant

theorem **comm_cl** {z : A} (hz : ∀ f ∈ L.fns, Commute z f) {y : A} (hy : y ∈ Subring.closure L.fns) : Commute z y

theorem **cl_comm** {x y : A} (hx : x ∈ Subring.closure L.fns) (hy : y ∈ Subring.closure L.fns) : Commute x y

theorem **clF** {x : A} (hx : x ∈ L.fns) : x ∈ Subring.closure L.fns

theorem **clProd** (f : Fin r → A) (hf : ∀ l, f l ∈ L.fns) : (List.ofFn f).prod ∈ Subring.closure L.fns

theorem **cζ** : L.ζ ∈ Subring.closure L.fns

theorem **cζi** : L.ζi ∈ Subring.closure L.fns

theorem **cζb** : L.ζb ∈ Subring.closure L.fns

theorem **cζbi** : L.ζbi ∈ Subring.closure L.fns

theorem **cGG** : L.GG ∈ Subring.closure L.fns

theorem **cGGi** : L.GGi ∈ Subring.closure L.fns

theorem **cDr** : L.Dr ∈ Subring.closure L.fns

theorem **cDri** : L.Dri ∈ Subring.closure L.fns

theorem **cDrb** : L.Drb ∈ Subring.closure L.fns

theorem **cDrbi** : L.Drbi ∈ Subring.closure L.fns

theorem **cD** : L.D ∈ Subring.closure L.fns

theorem **cDi** : L.Di ∈ Subring.closure L.fns

theorem **cDb** : L.Db ∈ Subring.closure L.fns

theorem **cDbi** : L.Dbi ∈ Subring.closure L.fns

theorem **D_Di** : L.D * L.Di = 1

theorem **Di_Dr** : L.Di * L.Dr = L.ζi

theorem **Dbi_Drb** : L.Dbi * L.Drb = L.ζbi

theorem **Dbi_eq** : L.D * L.Dbi * L.Di = L.Dbi

theorem **Drb_Di** : L.Drb * L.Di = L.ζbi * (L.Db * L.Di)

theorem **f5** : L.ζbi * L.GG * (L.ζb * L.ζi * L.GGi) = L.ζi

theorem **f6** : L.ζi * L.GG * (L.GGi * (L.ζbi * L.ζ)) = L.ζbi

theorem **f7** : L.ζi * L.GG * (L.ζbi * L.GGi) = L.ζi * L.ζbi

theorem **W_inv_mul_W** : (L.GG * L.ζi * L.ζbi) * (L.ζ * L.ζb * L.GGi) = 1

theorem **cW** : L.ζ * L.ζb * L.GGi ∈ Subring.closure L.fns

theorem **cWi** : L.GG * L.ζi * L.ζbi ∈ Subring.closure L.fns

theorem **Wi_W_pow** (n : ℕ) : (L.GG * L.ζi * L.ζbi) ^ n * (L.ζ * L.ζb * L.GGi) ^ n = 1

theorem **nabs_alt_alt** : L.nabs * (L.ζ * L.ζb * L.GGi) = (L.ζ * L.ζb * L.GGi) * (L.nabs + L.TT • (L.ζbi * L.GGi))

theorem **m0_mul_W** (Y c : ℂ) : (L.m0 Y − c • (L.ζbi * L.GGi)) * (L.ζ * L.ζb * L.GGi) = (L.ζ * L.ζb * L.GGi) * (L.m0 Y − (c + L.TT) • (L.ζbi * L.GGi))

theorem **m0_mul_W_pow** (Y : ℂ) (n : ℕ) : L.m0 Y * (L.ζ * L.ζb * L.GGi) ^ n = (L.ζ * L.ζb * L.GGi) ^ n * (L.m0 Y − ((n : ℂ) * L.TT) • (L.ζbi * L.GGi))

theorem **nfs_gen** {P : A} {n : ℕ} {b : A} (hb : b ∈ B) {h : List A} (hl : h.length = n) (hh : ∀ a ∈ h, a ∈ hats L) : b * P * h.prod ∈ NFs L B P n

theorem **nfs_one** {P : A} {b : A} (hb : b ∈ B) {X : A} (hX : X ∈ hats L) : b * P * X ∈ NFs L B P 1

theorem **nfs_left** {P : A} {n : ℕ} {b : A} (hb : b ∈ B) {x : A} (hx : x ∈ NFs L B P n) : b * x ∈ NFs L B P n

theorem **nfs_mul_gen** {P Q : A} {n : ℕ} (hP : ∀ b ∈ B, Commute b P) (hQ : ∀ X ∈ hats L, ∀ b ∈ B, X * (b * Q) ∈ NFs L B Q 1) {x y : A} (hx : x ∈ NFs L B P 1) (hy : y ∈ NFs L B Q n) : x * y ∈ NFs L B (P * Q) (n + 1)

theorem **nfs_pass** {P Q : A} (hP : ∀ b ∈ B, Commute b P) (hQ : ∀ X ∈ hats L, X * Q ∈ NFs L B Q 1) {x : A} (hx : x ∈ NFs L B P 1) : x * Q ∈ NFs L B (P * Q) 1

theorem **mA** (y : ℂ) : L.Ahat y ∈ hats L

theorem **mR** (y : ℂ) : L.R0 y ∈ hats L

theorem **mAv** (y : ℂ) : L.Ahatv y ∈ hats L

theorem **mRv** (y : ℂ) : L.R0v y ∈ hats L

theorem **NFHypA.cl** {b : A} (hb : b ∈ B) {y : A} (hy : y ∈ Subring.closure L.fns) : Commute b y

theorem **hat_Di** (X : A) (hX : X ∈ hats L) : X * L.Di ∈ NFs L B L.Di 1

theorem **hat_Dipow** (n : ℕ) : ∀ X ∈ hats L, X * L.Di ^ n ∈ NFs L B (L.Di ^ n) 1

theorem **nfs_step** {n : ℕ} {x y : A} (hx : x ∈ NFs L B L.Di 1) (hy : y ∈ NFs L B (L.Di ^ n) n) : x * y ∈ NFs L B (L.Di ^ (n + 1)) (n + 1)

theorem **frame_mem** (w : List (Letter ℂ S)) : (∀ Y i j, (ofWord L st w).x Y i j ∈ K) ∧ (∀ Y i j, (ofWord L st w).xi Y i j ∈ K) ∧ (∀ i j, (ofWord L st w).c i j ∈ K) ∧ (∀ i j, (ofWord L st w).ci i j ∈ K) ∧ ∀ Y i j, (ofWord L st w).ee Y i j ∈ B

theorem **io_mul_apply** (x : A) (M : SlopeTestRemainingChoices S A) (i j : Ix S) : ((io x : SlopeTestRemainingChoices S A) * M) i j = x * M i j

theorem **mul_io_apply** (x : A) (M : SlopeTestRemainingChoices S A) (i j : Ix S) : (M * (io x : SlopeTestRemainingChoices S A)) i j = M i j * x

theorem **comm_io** {x : A} {M : SlopeTestRemainingChoices S A} (h : ∀ i j, Commute (M i j) x) : M * io x = io x * M

theorem **io_mul'** (x y : A) : (io x : SlopeTestRemainingChoices S A) * io y = io (x * y)

theorem **io_add'** (x y : A) : (io x : SlopeTestRemainingChoices S A) + io y = io (x + y)

theorem **io_pow'** (x : A) (n : ℕ) : (io x : SlopeTestRemainingChoices S A) ^ n = io (x ^ n)

theorem **star_io'** (x : A) : star (io x : SlopeTestRemainingChoices S A) = io (star x)

theorem **io_entry_mem** (T : Submodule ℂ A) {x : A} (hx : x ∈ T) (i j : Ix S) : (io x : SlopeTestRemainingChoices S A) i j ∈ T

theorem **ring_plus** {R : Type} [Ring R] {a l k E x y z : R} (h : a * k = z) (h' : E * z = z * E) : a * ((l + k * E) * x + y) = a * (l * x + y) + E * (z * x)

theorem **ring_minus** {R : Type} [Ring R] {a l k E x y z : R} (h : a * k = z) (h' : E * a = a * E) : a * ((l + E * k) * x + y) = a * (l * x + y) + E * (z * x)

theorem **star_facts** (hS : IsStarLevel L) : star L.ζb = L.ζ ∧ star L.ζi = L.ζbi ∧ star L.GGi = L.GGi

theorem **star_lowering_constant** (hS : IsStarLevel L) {Y : ℂ} (hY : star Y = Y) : star (L.l0 (Y − 1)) = L.m0 Y

theorem **m_form** (H : NFHypA L st B K) (hS : IsStarLevel L) (w : List (Letter ℂ S)) (hf : IsFrame L (ofWord L st w)) {Y : ℂ} (hY : star Y = Y) : ∃ (c : ℂ) (E : SlopeTestRemainingChoices S A), (∀ i j, E i j ∈ B) ∧ (ofWord L st w).m L Y = io (L.m0 Y − c • (L.ζbi ∗ L.GGi)) + E ∗ io (L.GGi ∗ (L.ζbi ∗ L.ζ))

theorem **letter_mem** (H : NFHypA L st B K) (hS : IsStarLevel L) (l : Letter ℂ S) (w : List (Letter ℂ S)) (hf : IsFrame L (ofWord L st w)) (hY : star l.pt = l.pt) (i : Fin 2) (e e' : Ix S) : TL.letterCol L st l w i e e' ∈ NFs L B L.Di 1

theorem **admissible_tail** {l : Letter ℂ S} {w : List (Letter ℂ S)} (h : admitted st (l :: w)) : admitted st w

theorem **word_nf** (hWS : TL.WordStep) (H : NFHypA L st B K) (hA : TL.AHyp L st) (w : List (Letter ℂ S)) (hw : admitted st w) (e : Ix S) : ((ofWord L st w).F ∗ TL.bTA L w) e e1 ∈ NFs L B (L.Di ^ w.length) w.length

CompanionTheory.AutomorphicTheory.JacobiDiagonalBounds.TieAlg

theorem **neg** {a a'} (h : R.r a a') : R.r (−a) (−a')

theorem **sub** {a a' b b'} (h : R.r a a') (h' : R.r b b') : R.r (a − b) (a' − b')

theorem **sum** {ι : Type} (s : Finset ι) {f : ι → A} {f' : ι → A'} (h : ∀ i ∈ s, R.r (f i) (f' i)) : R.r (∑ i ∈ s, f i) (∑ i ∈ s, f' i)

theorem **pow** {a a'} (h : R.r a a') (n : ℕ) : R.r (a ^ n) (a' ^ n)

theorem **algMap** (c : ℂ) : R.r (algebraMap ℂ A c) (algebraMap ℂ A' c)

theorem **ite** (p : Prop) [Decidable p] {a a' b b'} (h : R.r a a') (h' : R.r b b') : R.r (if p then a else b) (if p then a' else b')

theorem **listProd** {ι : Type} (l : List ι) {f : ι → A} {f' : ι → A'} (h : ∀ i ∈ l, R.r (f i) (f' i)) : R.r (l.map f).prod (l.map f').prod

theorem **m2** {a b c d : A} {a' b' c' d' : A'} (ha : R.r a a') (hb : R.r b b') (hc : R.r c c') (hd : R.r d d') : (R.mat (Fin 2)).r !![a, b; c, d] !![a', b'; c', d']

theorem **eU** (a : S) (i j : Fin 2) : (R.mat (Ix S)).r (Wd.eU a i j) (Wd.eU a i j)

theorem **io** {x : A} {x' : A'} (h : R.r x x') : (R.mat (Ix S)).r (Wd.io x) (Wd.io x')

theorem **ins** (a : S) {M : M2 (SlopeTestRemainingChoices S A)} {M' : M2 (SlopeTestRemainingChoices S A')} (h : ((R.mat (Ix S)).mat (Fin 2)).r M M') : (R.mat (Ix S)).r (Wd.ins a M) (Wd.ins a M')

theorem **lift0** {M : M2 A} {M' : M2 A'} (h : (R.mat (Fin 2)).r M M') : ((R.mat (Ix S)).mat (Fin 2)).r (Wd.lift0 M) (Wd.lift0 M')

theorem **Rh0** (a : S) (D : ℂ) : ((R.mat (Ix S)).mat (Fin 2)).r (Wd.Rh0 a D) (Wd.Rh0 a D)

theorem **Rht0** (a : S) (D : ℂ) : ((R.mat (Ix S)).mat (Fin 2)).r (Wd.Rht0 a D) (Wd.Rht0 a D)

theorem **Ts** {σ : Wd.Site A} {σ' : Wd.Site A'} (h : R.Site σ σ') (Y : ℂ) : (R.mat (Fin 2)).r (Wd.Ts σ Y) (Wd.Ts σ' Y)

theorem **vlet** {σ : Wd.Site A} {σ' : Wd.Site A'} (h : R.Site σ σ') (l : Letter ℂ S) : (R.mat (Ix S)).r (Wd.vlet σ l) (Wd.vlet σ' l)

theorem **bT** {σ : Wd.Site A} {σ' : Wd.Site A'} (h : R.Site σ σ') (w : List (Letter ℂ S)) : (R.mat (Ix S)).r (Wd.bT σ w) (Wd.bT σ' w)

theorem **l_eq** {L : LevelData A r} {f : FrameData S A} (hf : IsFrame L f) {Y : ℂ} (hY : Y ∈ f.inv) (hY1 : Y + 1 ∈ f.inv) : f.l L Y = f.xi Y ∗ io L.GGi ∗ (f.F ∗ f.TT Y 1 0) ∗ f.Fi

theorem **l_rel** {R : SRel A A'} {I : Set ℂ} {L : LevelData A r} {L' : LevelData A' r} (hL : FnRel R L L') {f : FrameData S A} {f' : FrameData S A'} (h : FrRel R I f f') (hf : IsFrame L f) (hf' : IsFrame L' f') {Y : ℂ} (hr : star Y = Y) (hY : Y ∈ f.inv) (hY1 : Y + 1 ∈ f.inv) : (R.mat (Ix S)).r (f.l L Y) (f'.l L' Y)

theorem **d_rel** {R : SRel A A'} {I : Set ℂ} {L : LevelData A r} {L' : LevelData A' r} (hL : FnRel R L L') {f : FrameData S A} {f' : FrameData S A'} (h : FrRel R I f f') : (R.mat (Ix S)).r (f.d L) (f'.d L')

theorem **di_rel** {R : SRel A A'} {I : Set ℂ} {L : LevelData A r} {L' : LevelData A' r} (hL : FnRel R L L') {f : FrameData S A} {f' : FrameData S A'} (h : FrRel R I f f') : (R.mat (Ix S)).r (f.di L) (f'.di L')

theorem **star_add_one** {Y : ℂ} (h : star Y = Y) : star (Y + 1) = Y + 1

theorem **star_sub_one** {Y : ℂ} (h : star Y = Y) : star (Y − 1) = Y − 1

theorem **plus_rel** {R : SRel A A'} {I : Set ℂ} {L : LevelData A r} {L' : LevelData A' r} (hL : FnRel R L L') {f : FrameData S A} {f' : FrameData S A'} (h : FrRel R I f f') (hf : IsFrame L f) (hf' : IsFrame L' f') (a : S) {Y : ℂ} (hr : star Y = Y) (hY : Y ∈ f.inv) (hY1 : Y + 1 ∈ f.inv) : FrRel R I (f.plus L a Y) (f'.plus L' a Y)

theorem **minus_rel** {R : SRel A A'} {I : Set ℂ} {L : LevelData A r} {L' : LevelData A' r} (hL : FnRel R L L') {f : FrameData S A} {f' : FrameData S A'} (h : FrRel R I f f') (hf : IsFrame L f) (hf' : IsFrame L' f') (a : S) {Y : ℂ} (hr : star Y = Y) (hY : Y ∈ f.inv) (hY1 : Y − 1 ∈ f.inv) : FrRel R I (f.minus L a Y) (f'.minus L' a Y)

theorem **start_rel** {R : SRel A A'} (hR : R.InvClosed) {L : LevelData A r} {L' : LevelData A' r} (hL : FnRel R L L') {st : Start A} {st' : Start A'} (hs : IsLevelStart L st) (hs' : IsLevelStart L' st') (hq : st.q = st'.q) (hi : st.inv = st'.inv) (hQ : ∀ Y, star Y = Y → (R.mat (Fin 2)).r (st.QQ Y) (st'.QQ Y)) : FrRel R st.inv (st.frame : FrameData S A) (st'.frame : FrameData S A')

theorem **ofWord_rel** {R : SRel A A'} (hR : R.InvClosed) {L : LevelData A r} {L' : LevelData A' r} (hL : FnRel R L L') (hSL : IsStarLevel L) (hSL' : IsStarLevel L') {st : Start A} {st' : Start A'} (hs : IsLevelStart L st) (hs' : IsLevelStart L' st') (hq : st.q = st'.q) (hi : st.inv = st'.inv) (hQ : ∀ Y, star Y = Y → (R.mat (Fin 2)).r (st.QQ Y) (st'.QQ Y)) (w : List (Letter ℂ S)) (hw : AdmRec L st w) : AdmRec L' st' w ∧ FrRel R st.inv (ofWord L st w) (ofWord L' st' w)

CompanionTheory.AutomorphicTheory.JacobiDiagonalBounds.TieRad

theorem **act_diag** (d : M.Ty P → (MA.DQ P).Pr) (f : M.Ty P → MA.SmQ P.r P.TT) : MA.actA P (Matrix.diagonal d) f = fun ρ => (d ρ).1 (f ρ)

theorem **act_sc** (w : (MA.DQ P).Pr) (f : M.Ty P → MA.SmQ P.r P.TT) : MA.actA P (MA.sc P (MA.DQ P) w) f = fun ρ => w.1 (f ρ)

theorem **act_smul** (c : ℂ) (a : MA.A P (MA.DQ P)) (f : M.Ty P → MA.SmQ P.r P.TT) : MA.actA P (c • a) f = c • MA.actA P a f

theorem **sc_star** (w : (MA.DQ P).Pr) : star (MA.sc P (MA.DQ P) w) = MA.sc P (MA.DQ P) (star w)

theorem **tiedOp_inv** {a b : Q.Pr V} {a' b' : MA.A P (MA.DQ P)} (h : IsIntertwinedEndomorphism τ a.1 a') (h1 : a ∗ b = 1) (h2 : b' ∗ a' = 1) : IsIntertwinedEndomorphism τ b.1 b'

theorem **tiedRel_invClosed** : (intertwiningRelation τ).InvClosed

theorem **tied_mkP** {X Xs : Module.End ℂ V} {am : MA.A P (MA.DQ P)} (h : IsIntertwinedEndomorphism τ X am) (hs : IsIntertwinedEndomorphism τ Xs (star am)) : (intertwiningRelation τ).r (Q.mkP X Xs) am

theorem **tiedOp_neg_sc** {X : Module.End ℂ V} {w w' : (MA.DQ P).Pr} (h : IsIntertwinedEndomorphism τ X (MA.sc P (MA.DQ P) w)) (hw : w'.1 = −w.1) : IsIntertwinedEndomorphism τ (−X) (MA.sc P (MA.DQ P) w')

theorem **word_tied** {X : Q.ActualOps P.k P.r z z' π π'} {μ : Measure (Mt P.k P.r)} {L : R.LevelData (Q.Pr (Q.RadSp P.k P.r z z' π π')) P.r} {st : Fr.Start (Q.Pr (Q.RadSp P.k P.r z z' π π'))} {l0 : ℕ} (hD : P5.IsData P z z' π π' X μ L st l0) {τ : Q.RadSp P.k P.r z z' π π' →ₗ[ℂ] (M.Ty P → MA.SmQ P.r P.TT)} (hB : BaseTie X τ) {Lm : R.LevelData (MA.A P (MA.DQ P)) P.r} (hM : MA.IsModelLevel P (MA.DQ P) Lm) (hMs : Fr.IsStarLevel Lm) (hMst : Fr.IsLevelStart Lm (MA.startM P (MA.DQ P) l0)) {S : Type} [Fintype S] [DecidableEq S] (w : List (Wd.Letter ℂ S)) (hw : Fr.admitted st w) (Φ : Q.RadSp P.k P.r z z' π π') (e : Wd.Ix S) : τ (P5.act ((Fr.ofWord L st w).F ∗ P5.bTall L w) (Top.firstStateVector Φ) e) = MA.actA P (gM P l0 Lm w e) (τ Φ)

CompanionTheory.AutomorphicTheory.JacobiDiagonalBounds.Vander

theorem **vander'** (Q : ℕ) : ∃ lam : Fin (Q + 1) → Fin (Q + 1) → ℂ, ∀ (E : Type) [AddCommGroup E] [Module ℂ E] (a : Fin (Q + 1) → E) (q : Fin (Q + 1)), a q = ∑ s : Fin (Q + 1), lam q s • ∑ j : Fin (Q + 1), (((s : ℕ) : ℂ) ^ (j : ℕ)) • a j

CompanionTheory.AutomorphicTheory.KFiniteVectors.KDecay

theorem **sN_nonneg** {n : ℕ} (g : Matrix (Fin n) (Fin n) ℂ) : 0 ≤ sN g

theorem **gap_step** {n : ℕ} {ε : Fin n → Bool} {μ : Fin n → ℝ} {h : ℝ} (hg : PI.ZuckTensor.GapAtLeast ε μ h) (s : Fin n) : PI.ZuckTensor.GapAtLeast ε (μ + Pi.single s 1) (h − 1)

theorem **decay_J** {t : ∀ n : ℕ, (Fin n → Bool) → ℝ} (hZ : ZuckT t) {n : ℕ} (ε : Fin n → Bool) : ∀ (m : ℕ) (h : ℝ) (μ : Fin n → ℝ) (H : Type) [NormedAddCommGroup H] [InnerProductSpace ℂ H] [CompleteSpace H] (σ : Matrix (Fin n) (Fin n) ℂ →ₗ (H →L[ℂ] H)), IsHCParam μ → IsDS ε μ σ → PI.ZuckTensor.GapAtLeast ε μ h → t n ε + m < h → ∀ v w : H, IsKFin ε σ v → IsKFin ε σ w → ∃ C : ℝ, ∀ g ∈ UJset ε, sN g ^ m ∗ ‖⟪w, σ g v⟫_ℂ‖ ≤ C

theorem **gN_le_sN** {k r : ℕ} (g : B.Mat k r) : B.gN g ≤ sN g

theorem **conj_back** {k r : ℕ} {ε : Fin (2 ∗ r + k + 1) → Bool} {P g : B.Mat k r} (hP : IsCayleyU k r ε P) (hg : g ∈ Kset ε) : P⁻¹ ∗ g ∗ P ∈ UH (Hpk k r) ∩ (Matrix.unitaryGroup (Fin (2 ∗ r + k + 1)) ℂ : Set _)

theorem **kDecay_of_zuckT** {t : ∀ n : ℕ, (Fin n → Bool) → ℝ} (hZ : ZuckT t) (k r : ℕ) (ε : Fin (2 ∗ r + k + 1) → Bool) (m : ℕ) (A h : ℝ) (hA : A ≤ m) (hlt : t _ ε + m < h) : KDecay k r ε A h

theorem **isKFinHpk_of_low** {k r : ℕ} {ε : Fin (2 ∗ r + k + 1) → Bool} {lam : Fin (2 ∗ r + k + 1) → ℝ} {H : Type} [NormedAddCommGroup H] [InnerProductSpace ℂ H] {σ : B.Mat k r → (H →L[ℂ] H)} {Q : B.Mat k r} (hQ : IsCayleyU k r ε Q) {w : H} (hw : w ∈ lowKSub ε lam (fun g => σ (Q⁻¹ ∗ g ∗ Q))) : IsKFinHpk (k

CompanionTheory.AutomorphicTheory.KrullDimension.TopologicalKrullDimModel

theorem **ringKrullDim_eq** {A Ω : Type} [CommRing A] [Field Ω] [CharZero Ω] (φ : A →+* Ω) (hφ : Function.Injective φ) (hfr : ∀ x : Ω, ∃ a b : A, x = φ a / φ b) (hft : Algebra.FiniteType ℤ A) {d : ℕ} (hd : Algebra.trdeg ℤ Ω = d) : ringKrullDim A = (d + 1 : ℕ)

theorem **topologicalKrullDim_model** {o Ω : Type} [CommRing o] [Field Ω] [CharZero Ω] [Algebra o Ω] (ho : Algebra.FiniteType ℤ o) (M : GlobalTheory.ZH.RatModel o Ω) {d : ℕ} (hd : Algebra.trdeg ℤ Ω = d) : topologicalKrullDim M.Z = (d + 1 : ℕ)

theorem **trdeg_eq** {K V Ω : Type} [Field K] [AddCommGroup V] [Module K V] [Field Ω] [Algebra K Ω] (ev : V →ₗ[K] Ω) (hg : GlobalTheory.DegVal.GenRatios ev) {d : ℕ} (hd : 0 < imageDeg ev d) : Algebra.trdeg K Ω = d

CompanionTheory.AutomorphicTheory.LocalBesselUniqueness.BesselModels.AmplitudeOrbitalIntegral

theorem **ampDD_thm** : Formalism.MainNotions.S.Pv.AmpDD.BM.AmpDD

theorem **amplitude_K_finite** : Formalism.MainNotions.S.Pv.AmpDD.BM.AmpKF

theorem **ls_pair** (hI : CompanionTheory.D.Upq.BesselFunctionalConstruction.Inputs) (hAt : CompanionTheory.Lime.BM.AtFormAdmOne) {D : D7} {τ : D.what₂.Tau} {w w' : D.what₂.Word} {θ : ℝ} {W : LocW D.R.k D.R.rS} {lam : Fin (2 * D.R.rS + D.R.k + 1) → ℝ} {A : LocS W} (hW : GoodW D τ w w' θ W) (hA : GoodS D lam A) (hT : 0 < Tr D w) (hT' : 0 < Tr D w') {h : ℝ} (hg : Formalism.QuantumDeterminants.GapAtLeast (epsSH D.R) lam h) (hok : Formalism.FourierDecayAlt.ThreeDimData.GapOK D.R.k D.R.rS h) : ∃ (l : A.H → W.Wp) (l' : A.H → W.Wq), PI.BesselMapUniq.IsBesselMapSm (k

theorem **ampOrbNV** (hJ : CompanionTheory.Lime.BM.UniqGap) (hKD : Formalism.MainNotions.S.Pv.AmpDD.KDense) (D : D7) (large₂ : SizeData₂ D) (C' Cl Cg cb Cu cT CT C0 Cn : ℝ) (hNV : Formalism.MainNotions.S.Pv.AmpDD.NVHyp D large₂ C' Cl Cg cb Cu cT CT C0 Cn) : Formalism.MainNotions.S.Pv.LemmaJ.AmpOrb D large₂ C' Cl Cg cb Cu cT CT C0 Cn

theorem **ampOrb0** (hJ : CompanionTheory.Lime.BM.UniqGap) (hKD : Formalism.MainNotions.S.Pv.AmpDD.KDense) (D : D7) (large₂ : SizeData₂ D) (C' Cl Cg cb Cu cT CT C0 : ℝ) (hCl : 1000 ≤ Cl) : Formalism.MainNotions.S.Pv.LemmaJ.AmpOrb D large₂ C' Cl Cg cb Cu cT CT C0 0

theorem **ampOrbCoefDD_of** (hJ : CompanionTheory.Lime.BM.UniqGap) : Formalism.MainNotions.S.Pv.LemmaJ.AmpOrbCoefDD

theorem **ampUnif_gap** (hJ : CompanionTheory.Lime.BM.UniqGap) (hKD : Formalism.MainNotions.S.Pv.AmpDD.KDense) : Formalism.MainNotions.S.Pv.LemmaJ.AmpUnif

CompanionTheory.AutomorphicTheory.LocalBesselUniqueness.BesselModels.AtFormAdm

theorem **smooth_admissible_form_one_side** : CompanionTheory.Lime.BM.AtFormAdmOneOfSteps

CompanionTheory.AutomorphicTheory.LocalBesselUniqueness.BesselModels.AtFormAdmOne

theorem **shape** (hU : UniqGap) : Shape

theorem **dDFormOne** (hU : UniqGap) : DDFormOne

theorem **atFormAdmOne** (hU : UniqGap) : AtFormAdmOne

CompanionTheory.AutomorphicTheory.LocalBesselUniqueness.BesselModels.AtFormAdmOneOfSteps

theorem **uniqGap_of_uniq** (h : PI.BesselMapUniq.BesselMapUniq) : UniqGap

theorem **aIsL** (hJ : UniqGap) {k r : ℕ} {ε : Fin (2 * r + k + 1) → Bool} {lam : Fin (2 * r + k + 1) → ℝ} {H : Type} [NormedAddCommGroup H] [InnerProductSpace ℂ H] {σ : Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ → (H →L[ℂ] H)} {z : Fin (2 * r + k + 1) → ℂ} {c : Fin r → ℂ} {u : Fin k → ℝ} {Wp : Type} [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] {π : Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ → (Wp →L[ℂ] Wp)} {h : ℝ} (hg : Formalism.QuantumDeterminants.GapAtLeast ε lam h) (hok : GapOK k r h) (hlam : IsHCParam lam) (hσ : PI.UpqRep.IsDSHpk k r ε lam σ) (hz : inWb k r z) (hT : 0 < Tz k r z) (hc : ∀ a, c a ≠ 0) (hπ : PI.UpqRep.IsHCTypeW k r z u π) {B l : H → Wp} (hB : PI.BesselMapUniq.IsBesselMapSm (k

theorem **shape_of** (hU : UniqGap) (hPsi : PsiConst) (hSch : SchurH) (hBC : CompanionTheory.D.Upq.BesselFunctionalConstruction.BeConv) (hNI : CompanionTheory.D.Upq.BesselFunctionalConstruction.NormIdSm) (hBB : CompanionTheory.D.Upq.BesselFunctionalConstruction.BeBessel) : Shape

CompanionTheory.AutomorphicTheory.LocalBesselUniqueness.BesselModels.BesselSchurOfDec

theorem **besselSchur'_of_dec** (hJ : CompanionTheory.Lime.BM.UniqGap) (hI : CompanionTheory.D.Upq.BesselFunctionalConstruction.Inputs) (hAt : CompanionTheory.Lime.BM.AtFormAdmOne) (hBP : CompanionTheory.Lime.BM.BesselSplitLargeGap) (hSI : Formalism.L2.SG.BesselSplit.SmoothIrred) (D : D7) (large₂ : SizeData₂ D) (C' Cl Cg cb Cu cT CT C0 Cn : ℝ) (hT : Cnt.Hyp0 D large₂ C' Cl Cg cb Cu cT CT C0 → ParamTieG D Cn) : Formalism.MainNotions.S.Pv.Mom.BesselSchur' D large₂ C' Cl Cg cb Cu cT CT C0 Cn

theorem **moment_bound_schur** (hJ : CompanionTheory.Lime.BM.UniqGap) (hBP : CompanionTheory.Lime.BM.BesselSplitLargeGap) (hSI : Formalism.L2.SG.BesselSplit.SmoothIrred) {eps : ℝ} (Fm : Formalism.MainNotions.FamP eps) (Cg cb Cu cT CT C0 : ℝ) : Formalism.MainNotions.S.Pv.Mom.MomF' Fm Cg cb Cu cT CT C0

theorem **momF'_gap** (hJ : CompanionTheory.Lime.BM.UniqGap) (hBP : CompanionTheory.Lime.BM.BesselSplitLargeGap) {eps : ℝ} (Fm : Formalism.MainNotions.FamP eps) (Cg cb Cu cT CT C0 : ℝ) : Formalism.MainNotions.S.Pv.Mom.MomF' Fm Cg cb Cu cT CT C0

CompanionTheory.AutomorphicTheory.LocalBesselUniqueness.BesselModels.CoefWeightedLocHolds

theorem **CoefWeightedLoc_holds** : CoefWeightedLoc

theorem **BesselNVGap_holds** : BesselNVGap

theorem **WlocExists_holds** : WlocExists

theorem **NUnimodLoc_holds** : Formalism.L2.SG.Pf.WFL.NUnimodLoc

theorem **QuotProdK_holds** : Formalism.L2.SG.Pf.WFL.QuotProdK

CompanionTheory.AutomorphicTheory.LocalBesselUniqueness.BesselModels.DDForm

theorem **discrete_form_one_side** : CompanionTheory.Lime.BM.DDFormOneOfSteps

CompanionTheory.AutomorphicTheory.LocalBesselUniqueness.BesselModels.DoubleCosetBesselBoundOfGap

theorem **besselOfW_gap** (hJ : CompanionTheory.Lime.BM.UniqGap) : Formalism.L2.SG.LocWSOf.BesselOfW

CompanionTheory.AutomorphicTheory.LocalBesselUniqueness.BesselModels.DoubleCosetBesselBoundOfGapConstantLe

theorem **dDFormOne0** : DDFormOne

theorem **besselOfW_holds** : Formalism.L2.SG.LocWSOf.BesselOfW

CompanionTheory.AutomorphicTheory.LocalBesselUniqueness.BesselModels.EClose

theorem **locQuadGG_G_holds** (hI : CompanionTheory.Lime.BM.Inputs2) : LocQuadGG_G

CompanionTheory.AutomorphicTheory.LocalBesselUniqueness.BesselModels.EFinal

theorem **stepIntG_holds** (hI : CompanionTheory.Lime.BM.Inputs2) : Formalism.L2.SG.RealObj.EnergyInt.StepIntG

theorem **weilPairJ_holds** (hI : CompanionTheory.Lime.BM.Inputs2) {D : D7} (hG : Formalism.L2.SG.ConcreteSettingExists.GapHyp D) (X : HidaSetting D) (τ : D.what₂.Tau) : ∃ (μS ν : D.what₂.Word → Measure (Formalism.REAL.UJ (X.B.I.ε (vOf X τ)))), (∀ u, SFinite (μS u) ∧ SFinite (ν u) ∧ (∀ f : Formalism.REAL.UJ (X.B.I.ε (vOf X τ)) → ℝ≥0∞, Measurable f → ∫⁻ x, f x ∂(Formalism.REAL.muG (X.B.I.ε (vOf X τ))) = ∫⁻ x, ∫⁻ s, f (s * x) ∂(μS u) ∂(ν u)) ∧ (μS u).map (fun x => x⁻¹) = μS u ∧ μS u (Formalism.L2.SG.Pf.WuDec.Sloc X u (vOf X τ))ᶜ = 0 ∧ ∫⁻ z, ∫⁻ z', ∫⁻ s, ‖(χ[X.B.σ (vOf X τ) (uMat z) (X.B.w (vOf X τ)), X.B.σ (vOf X τ) (uMat s) (X.B.σ (vOf X τ) (uMat z')) (X.B.w (vOf X τ))]_ℂ‖ₑ * ‖X.Wloc u (vOf X τ) z‖ₑ * ‖X.Wloc u (vOf X τ) z'‖ₑ ∂(μS u) ∂(ν u) ∂(ν u) < ⊤ ∧ (∀ s ∈ Formalism.L2.SG.Pf.WuDec.Sloc X u (vOf X τ), (μS u).map (fun x => s * x) = μS u) ∧ μS u ≠ 0) ∧ ∀ (u u' : D.what₂.Word) (sc : ℝ), 0 < sc

CompanionTheory.AutomorphicTheory.LocalBesselUniqueness.BesselModels.Inputs

theorem **inputs** : CompanionTheory.D.Upq.BesselFunctionalConstruction.Inputs

CompanionTheory.AutomorphicTheory.LocalBesselUniqueness.BesselModels.Inputs2Of

theorem **inputs2_of** (h : UniqGap) : Inputs2

CompanionTheory.AutomorphicTheory.LocalBesselUniqueness.BesselModels.LocDDQ

theorem **locDDQ** (hI : CompanionTheory.Lime.BM.Inputs2) (hRT : ReproTr) (hL2 : L2RightTranslation) (hGp : GapOfHyp) (hG : Formalism.L2.SG.ConcreteSettingExists.GapHyp D) (C : Choice X θ) (hLF : LocFactor C) (u : D.what₂.Word) (τ : D.what₂.Tau) : ∃ (l : X.B.H (vOf X τ) → X.Vloc u (vOf X τ)) (a : ℂ), (∀ x ∈ Formalism.FourierDecayAlt.ThreeDimData.DDsp (sigB X (C.Q τ) (C.Qi τ) τ) (v0Of C τ), ∀ y ∈ Formalism.FourierDecayAlt.ThreeDimData.DDsp (sigB X (C.Q τ) (C.Qi τ) τ) (v0Of C

τ), Formalism.FourierDecayAlt.ThreeDimData.be (zB X (C.Qi τ) u τ) (cS D) (piB X (C.Q τ) (C.Qi τ) u τ) (sigB X (C.Q τ) (C.Qi τ) τ) (C.muW u τ) (muNB D θ) x y = (innerSL ℂ (l y)).smulRight (l x)) ∧ ∀ γ, X.Wloc u (vOf X τ) γ = a • l (X.B.σ (vOf X τ) (uMat γ) (X.B.w (vOf X τ)))

CompanionTheory.AutomorphicTheory.LocalBesselUniqueness.BesselModels.LocQuad1GHolds

theorem **locQuad1_G_holds** (hI : CompanionTheory.Lime.BM.Inputs2) : LocQuad1_G

theorem **stepConvG_holds** (hI : CompanionTheory.Lime.BM.Inputs2) : StepConvG

CompanionTheory.AutomorphicTheory.LocalBesselUniqueness.BesselModels.LocScalar

theorem **NHloc_eq** (v : InfinitePlace X.B.I.F) : NHloc X u v = QPK.prodSet (Nloc X u v) (Hfix X u v)

theorem **place_facts** (v : InfinitePlace X.B.I.F) : IsClosed (Nloc X u v) ∧ IsCompact (Hfix X u v) ∧ QPK.IsGrp (Nloc X u v) ∧ QPK.IsGrp (Hfix X u v) ∧ (∀ h ∈ Hfix X u v, ∀ m ∈ Nloc X u v, h * m * h⁻¹ ∈ Nloc X u v) ∧ (∀ μ : Measure (Formalism.REAL.UJ (X.B.I.ε v)), SigmaFinite μ → μ ≠ 0 → μ (Nloc X u v)ᶜ = 0 → (∀ s ∈ Nloc X u v, μ.map (fun x => s * x) = μ) → ∀ s ∈ Nloc X u v, μ.map (fun x => x * s) = μ)

theorem **quot_rightInv** {μS νl : ∀ v, Measure (Formalism.REAL.UJ (X.B.I.ε v))} {C : ℝ≥0∞} (hQ : QuotWit X u μS νl C) (v : InfinitePlace X.B.I.F) (F : Formalism.REAL.UJ (X.B.I.ε v) → ℝ≥0∞) (hF : Measurable F) (hinv : ∀ s ∈ NHloc X u v, ∀ x, F (s * x) = F x) (g : Formalism.REAL.UJ (X.B.I.ε v)) : ∫⁻ x, F (x * g) ∂(νl v) = ∫⁻ x, F x ∂(νl v)

theorem **quot_bruhat** {μS νl : ∀ v, Measure (Formalism.REAL.UJ (X.B.I.ε v))} {C : ℝ≥0∞} (hQ : QuotWit X u μS νl C) (v : InfinitePlace X.B.I.F) : ∃ β : Formalism.REAL.UJ (X.B.I.ε v) → ℝ≥0∞, Measurable β ∧ ∀ F : Formalism.REAL.UJ (X.B.I.ε v) → ℝ≥0∞, Measurable F → (∀ s ∈ NHloc X u v, ∀ x, F (s * x) = F x) → ∫⁻ x, F x ∂(νl v) = ∫⁻ x, F x * β x ∂(Formalism.REAL.muG (X.B.I.ε v))

theorem **pv_conv** (v : InfinitePlace X.B.I.F) (y : Formalism.REAL.UJ (X.B.I.ε v)) : Integrable (fun g => Pv X v g * Pv X v (g⁻¹ * y)) (Formalism.REAL.muG (X.B.I.ε v)) ∧ ∫ g, Pv X v g * Pv X v (g⁻¹ * y) ∂(Formalism.REAL.muG (X.B.I.ε v)) = Pv X v y

theorem **pv_integrable** (hG : Formalism.L2.SG.ConcreteSettingExists.GapHyp D) (v : InfinitePlace X.B.I.F) : Integrable (Pv X v) (Formalism.REAL.muG (X.B.I.ε v))

theorem **ennreal_mul_le_sq** (a b : ℝ≥0∞) : a * b ≤ a ^ 2 + b ^ 2

theorem **local_scalar_reduction** : LocScalarStep

CompanionTheory.AutomorphicTheory.LocalBesselUniqueness.BesselModels.LocalUniquenessReduction

theorem **besselUniq_core** (hBW : Formalism.L2.SG.LocWSOf.BesselOfW) (hM1 : CompanionTheory.Lime.BM.UniqGap) {z : Fin (2 * r + k + 1) → ℂ} {c : ℂ} {u : Fin k → ℝ} {lam : Fin (2 * r + k + 1) → ℝ} {ε : Fin (2 * r + k + 1) → Bool} {h : ℝ} {H : Type} [NormedAddCommGroup H] [InnerProductSpace ℂ H] [CompleteSpace H] {σ : Mat k r → (H →L[ℂ] H)} (hl : Formalism.QuantumDeterminants.IsHCParam lam) (hds : IsDSHpk k r ε lam σ) (hg : Formalism.QuantumDeterminants.GapAtLeast ε lam h) (hok : Formalism.FourierDecayAlt.ThreeDimData.GapOK k r h) (hz : Formalism.QuantumDeterminants.inWb k r z) (hT : 0 < Formalism.QuantumDeterminants.Tz k r z) (hc : c ≠ 0) {Vp : Type} [NormedAddCommGroup Vp] [InnerProductSpace ℂ Vp] [FiniteDimensional ℂ Vp] {π : Mat k r → (Vp →L[ℂ] Vp)} (hπ : IsHCTypeW k r z u π) {w : H} (hw : IsLowUnitH k r ε lam σ w) (hws : w ∈ smoothVecs (k

theorem **locUniq_gen** (hBW : Formalism.L2.SG.LocWSOf.BesselOfW) (hM1 : CompanionTheory.Lime.BM.UniqGap) (hB : BesselNVGap) (hCW : CoefWeightedLoc) (hWl : WlocExists) (hk : 1 ≤ k) (hke : Even k) {T : ℝ} (hT : 0 < T) {gg : ℕ} (hgg : 0 < gg) {e f : Fin r → Fin (2 * r + k + 1) → ℂ} {wf : Fin k → Fin (2 * r + k + 1) → ℂ} {z : Fin (2 * r + k + 1) → ℂ} (hF : IsFrameJ (eps48 k r) e f wf (z0v T z)) {u : Fin k → ℝ} {v : Fin (k − 1) → ℝ} {b : Fin (2 * r + 2) → ℝ} (hu : Formalism.QuantumDeterminants.IsHCParam u) (hl : Formalism.QuantumDeterminants.IsHCParam (lam48 k r v b)) (hI : InterlacingCondition k r u v b) (hH : HalfIntegralDifference k r u v b) (hGap : GapBig k r b) {h : ℝ} (hga : Formalism.QuantumDeterminants.GapAtLeast (eps48 k r) (lam48 k r v b) h) (hok : Formalism.FourierDecayAlt.ThreeDimData.GapOK k r h) {H : Type} [NormedAddCommGroup H] [InnerProductSpace ℂ H] [CompleteSpace H] {σ : Matrix

theorem **locUniq_n** (hBW : Formalism.L2.SG.LocWSOf.BesselOfW) (hM1 : CompanionTheory.Lime.BM.UniqGap) (hB : BesselNVGap) (hCW : CoefWeightedLoc) (hWl : WlocExists) (hHN : LocHNProps) {k r n : ℕ} (hn : n = 2 * r + k + 1) (hk : 1 ≤ k) (hke : Even k) {T : ℝ} (hT : 0 < T) {gg : ℕ} (hgg : 0 < gg) (ε : Fin n → Bool) (hε : ∀ i, ε i = eps48 k r (Fin.cast hn i)) (e f : Fin r → Fin n → ℂ) (z : Fin n → ℂ) (wf : Fin k → Fin n → ℂ) (hee : ∀ a b, hJ ε (e a) (e b) = 0) (hff : ∀ a b, hJ ε (f a) (f b) = 0) (hef : ∀ a b, hJ ε (e a) (f b) = if a = b then 1 else 0) (hez : ∀ a, hJ ε (e a) z = 0) (hfz : ∀ a, hJ ε (f a) z = 0) (hzz : hJ ε z z = −(T : ℂ)) (hww : ∀ a b, hJ ε (wf a) (wf b) = if a = b then 1 else 0) (hew : ∀ c a, hJ ε (e c) (wf a) = 0) (hfw : ∀ c a, hJ ε (f c) (wf a) = 0) (hzw : ∀ a, hJ ε z (wf a) = 0) {u : Fin k → ℝ} {v : Fin (k − 1) → ℝ} {b : Fin (2 * r + 2) → ℝ} (hu : Formalism.QuantumDeterminant

theorem **local_uniqueness_reduction** : LocUniqStep

theorem **locDecayStep_of** (hB : BesselNVGap) (hC : CoefWeightedLoc) (hW : WlocExists) (hBW : Formalism.L2.SG.LocWSOf.BesselOfW) (hM1 : CompanionTheory.Lime.BM.UniqGap) (hR : LocReprodStep) (hU : LocUniqStep) : LocDecayStep

theorem **wuDecay_gap** (hM1 : CompanionTheory.Lime.BM.UniqGap) (hB : BesselNVGap) (hC : CoefWeightedLoc) (hW : WlocExists) (hQK : QuotProdK) (hNU : NUnimodLoc) (hS : LocScalarStep) : Formalism.L2.SG.Pf.WuDecayOfGap

CompanionTheory.AutomorphicTheory.LocalBesselUniqueness.BesselModels.MultiplicityOneUnitaryBranchingOfGapHyp

theorem **uniqGap_holds** : UniqGap

theorem **ampUnif_holds** : Formalism.MainNotions.S.Pv.LemmaJ.AmpUnif

theorem **wuDecayOfGap_holds'** : Formalism.L2.SG.Pf.WuDecayOfGap

CompanionTheory.AutomorphicTheory.LocalBesselUniqueness.BesselModels.MultiplicityOneUnitaryBranchingOfUniqGap

theorem **multiplicity_one_unitary_branching_of_uniqGap** (hJ : UniqGap) (D : D7) (hG : Formalism.L2.SG.ConcreteSettingExists.GapHyp D) : LocalBesselAndTopKernel.MultiplicityOneUnitaryBranching D

CompanionTheory.AutomorphicTheory.LocalBesselUniqueness.BesselModels.RealObjInInfinityXJsz

theorem **stepIntG_jsz** (hJ : CompanionTheory.Lime.BM.UniqGap) : Formalism.L2.SG.RealObj.EnergyInt.StepIntG

theorem **stepConvG_jsz** (hJ : CompanionTheory.Lime.BM.UniqGap) : Formalism.L2.S.Pv.ConvG.StepConvG

theorem **realObj_inInfinityX_jsz** (hJ : CompanionTheory.Lime.BM.UniqGap) {D : Formalism.BasicNotions.D7} (hg : Formalism.L2.SG.ConcreteSettingExists.GapHyp D) (X : Formalism.L2.SG.HidaSetting.HidaSetting D) (u u' : D.what₂.Word) (sc : ℝ) (hsc : 0 < sc) : Formalism.L2.SG.RealObj.InInfinityX X u u' sc

CompanionTheory.AutomorphicTheory.LocalBesselUniqueness.BesselModels.StepIntGOf

theorem **locOrbG_of** (hI : CompanionTheory.Lime.BM.Inputs2) (hRT : ReproTr) (hL2 : L2RightTranslation) (hGp : GapOfHyp) (hLF : ∀ (D : D7), Formalism.L2.SG.ConcreteSettingExists.GapHyp D → ∀ (X : HidaSetting D) (θ : ℝ) (C : Choice X θ), LocFactor C) : Formalism.L2.SG.RealObj.EnergyInt.LocOrbG

theorem **stepIntG_of** (hI : CompanionTheory.Lime.BM.Inputs2) (hRT : ReproTr) (hL2 : L2RightTranslation) (hGp : GapOfHyp) (hLF : ∀ (D : D7), Formalism.L2.SG.ConcreteSettingExists.GapHyp D → ∀ (X : HidaSetting D) (θ : ℝ) (C : Choice X θ), LocFactor C) : Formalism.L2.SG.RealObj.EnergyInt.StepIntG

CompanionTheory.AutomorphicTheory.LocalBesselUniqueness.BesselModels.UniqGapOfBmu

theorem **uniqGap_of_bmu** (h : CompanionTheory.Bmu.BesselMapUniqGap) : UniqGap

theorem **wuDecayOfGap** (h : CompanionTheory.Bmu.BesselMapUniqGap) : Formalism.L2.SG.Pf.WuDecayOfGap

theorem **besselOfW** (h : CompanionTheory.Bmu.BesselMapUniqGap) : Formalism.L2.SG.LocWSOf.BesselOfW

theorem **ampUnif** (h : CompanionTheory.Bmu.BesselMapUniqGap) : Formalism.MainNotions.S.Pv.LemmaJ.AmpUnif

CompanionTheory.AutomorphicTheory.LocalBesselUniqueness.BesselModels.WeilPairJ

theorem **weilPairJ** (hI : CompanionTheory.Lime.BM.Inputs2) (hRT : ReproTr) (hL2 : L2RightTranslation) (hGp : GapOfHyp) (hLF : ∀ (D : D7), Formalism.L2.SG.ConcreteSettingExists.GapHyp D → ∀ (X : HidaSetting D) (θ : ℝ) (C : Choice X θ), LocFactor C) (hG : Formalism.L2.SG.ConcreteSettingExists.GapHyp D) (X : HidaSetting D) (τ : D.what₂.Tau) : ∃ (μS ν : D.what₂.Word → Measure (Formalism.REAL.UJ (X.B.I.ε (vOf X τ)))), (∀ u, SFinite (μS u) ∧ SFinite (ν u) ∧ (∀ f : Formalism.REAL.UJ (X.B.I.ε (vOf X τ)) → ℝ≥0∞, Measurable f → ∫⁻ x, f x ∂(Formalism.REAL.muG (X.B.I.ε (vOf X τ))) = ∫⁻ x, ∫⁻ s, f (s * x) ∂(μS u) ∂(ν u)) ∧ (μS u).map (fun x => x⁻¹) = μS u ∧ μS u (Formalism.L2.SG.Pf.WuDec.Sloc X u (vOf X τ))ᶜ = 0 ∧ ∫⁻ z, ∫⁻ z', ∫⁻ s, ‖⟪X.B.σ (vOf X τ) (uMat z) (X.B.w (vOf X τ)), X.B.σ (vOf X τ) (uMat s) (X.B.σ (vOf X τ) (uMat z') (X.B.w (vOf X τ)))⟫_ℂ‖ₑ * ‖X.Wloc u (vOf X τ) z‖ₑ * ‖X.Wloc u (vOf X τ) z'

CompanionTheory.AutomorphicTheory.LocalBesselUniqueness.BesselModels.WuFinal

theorem **wuDecayOfGap_of_uniqGap** (h : CompanionTheory.Lime.BM.UniqGap) : WuDecayOfGap

CompanionTheory.AutomorphicTheory.LocalBesselUniqueness.HatCap

theorem **trJB_eq_trR** (e : M.Ty P → ℂ → (Wq →L[ℂ] Wp)) (J : B.Mat P.k P.r → (Wq →L[ℂ] Wp)) : Gauge.trJB P Bs e J = Gauge.VLem.trR P Bs.μsec e (Q.resG P.k P.r (Fin.last P.r) J)

theorem **trJB_of_res** (hpl : Q.IsPlace P Bs.z Bs.z' Bs.π Bs.π') (e : M.Ty P → ℂ → (Wq →L[ℂ] Wp)) (J : B.Mat P.k P.r → (Wq →L[ℂ] Wp)) (F : Q.Fn P.k P.r (Wq →L[ℂ] Wp)) (h : ∀ c ∈ OmX P.r, ∀ g ∈ Q.Glev P.k P.r (Fin.last P.r), Q.wCell P.k P.r (Fin.last P.r) c ∗ g ∈ Gauge.OmB Bs → Q.resG P.k P.r (Fin.last P.r) J c g = F c g) : Gauge.trJB P Bs e J = Gauge.VLem.trR P Bs.μsec e F

theorem **commutes_of_real** {k r : ℕ} (D : B.RealPlaceData k r Wp Wq) (0 : B.DiffOp k r Wp Wq) (ε : ℂ) (G : (B.Mat k r → (Wq →L[ℂ] Wp)) → B.Mat k r → (Wq →L[ℂ] Wp)) (hO : B.opOrder 0 = r + k) (hR : ∀ (UU : Set (B.Mat k r)) (F : B.Mat k r → (Wq →L[ℂ] Wp)), B.CkOn ((r + k : ℕ) : ℕ∞) UU F → ∀ g ∈ UU, g ∈ B.GV k r → B.opApply 0 F g = ε • G F g) (hI : ∀ p : B.SS k r, D.inSS p → ∀ F : B.Mat k r → (Wq →L[ℂ] Wp), B.CkOn ((r + k : ℕ) : ℕ∞) (B.GV k r) F → ∀ g ∈ B.GPos D, G (B.rho D p F) g = B.rho D p (G F) g) : B.Commutes D (B.GPos D) 0

theorem **cap_A_bound** : CapAB

theorem **radP** : JDRRadP

theorem **radM** : JDRRadM

theorem **cap_R_bound** : CapRB

theorem **hat_product_bound** : HatProdB

CompanionTheory.AutomorphicTheory.LocalBesselUniqueness.HatKBSOf

theorem **kbs_arith** (R n N o A : ℝ) (hR : 0 ≤ R) (hn : 1 ≤ n) (hN : 0 ≤ N) (ho : o ≤ N ∗ n) (hA : 18 ∗ (R + 1) ^ 4 ∗ (N ∗ n ^ 2 + n ^ 3) ≤ A) : 12 ∗ (R + 1) ∗ n ^ 2 + 6 ∗ R ∗ o + 0 ≤ A

theorem **hatProdJB** (hH : HatProdB) (hPoly : JDPolyOK) {Wp Wq : Type} [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wp] [FiniteDimensional ℂ Wq] (P : M.Par) (Bs : B.RealPlaceData P.k P.r Wp Wq) (hpl : Q.IsPlace P Bs.z Bs.z' Bs.π Bs.π') (hk : 2 ≤ P.k) (e : M.Ty P → ℂ → (Wq →L[ℂ] Wp)) (L : R.LevelData (MA.A P (MA.DQ P)) P.r) (he : Gauge.IsFrameB P Bs e) (hQ : IsGaugeQQB P Bs e) (hL : MA.IsModelLevel P (MA.DQ P) L) (N : ℕ) (u : List (MA.A P (MA.DQ P))) (hN : u.length ≤ N) (hu : ∀ a ∈ u, a ∈ MA.hatSet P L) : ∃ (m : ℕ) (b : Fin m → MA.A P (MA.DQ P)) (Th : Fin m → B.DiffOp P.k P.r Wp Wq), (∀ k, b k ∈ Submonoid.closure (MA.bddSet P L)) ∧ (∀ k, B.SmoothCoef (B.GPos Bs) (Th k) ∧ B.Commutes Bs (B.GPos Bs) (Th k) ∧ GlobCoef (Th k) (1 ∗ ((P.r : ℝ) + 1) ∗ nDim P.k P.r ∗ ((N : ℝ) + 1)) ∧ (B.opOrder (Th k) : ℝ) ≤ 1 ∗ nDim P.k P.r ∗

CompanionTheory.AutomorphicTheory.LocalBesselUniqueness.HatProd

theorem **isDom_omB** {k r : ℕ} (Bs : B.RealPlaceData k r Wp Wq) : Q.IsDom k r Bs.z Bs.z' 0 (Gauge.OmB Bs)

theorem **tt_eq** (hpl : Q.IsPlace P Bs.z Bs.z' Bs.π Bs.π') : P.TT = Tz P.k P.r Bs.z ∗ Tz P.k P.r Bs.z'

theorem **gq_mem** (hpl : Q.IsPlace P Bs.z Bs.z' Bs.π Bs.π') (q : MA.Qp P.r P.TT) : B.gq Bs q.1 ∈ B.GV P.k P.r ∩ Gauge.OmB Bs

theorem **trJB_congr** (hpl : Q.IsPlace P Bs.z Bs.z' Bs.π Bs.π') (e : M.Ty P → ℂ → (Wq →L[ℂ] Wp)) {J J' : B.Mat P.k P.r → (Wq →L[ℂ] Wp)} (h : ∀ g ∈ B.GV P.k P.r ∩ Gauge.OmB Bs, J g = J' g) : Gauge.trJB P Bs e J = Gauge.trJB P Bs e J'

theorem **ffb_op** {o κ : ℕ} {0 : B.DiffOp P.k P.r Wp Wq} (h : PInv P Bs o κ 0) {J : B.Mat P.k P.r → (Wq →L[ℂ] Wp)} (hJ : J ∈ FFB P Bs) : B.opApply 0 J ∈ FFB P Bs

theorem **pInv_comp** {o₁ κ₁ o₂ κ₂ : ℕ} {0₁ 0₂ : B.DiffOp P.k P.r Wp Wq} (h₁ : PInv P Bs o₁ κ₁ 0₁) (h₂ : PInv P Bs o₂ κ₂ 0₂) : ∃ 0 : B.DiffOp P.k P.r Wp Wq, PInv P Bs (o₁ + o₂) (κ₁ + κ₂) 0 ∧ ∀ J ∈ FFB P Bs, ∀ g ∈ B.GV P.k P.r ∩ Gauge.OmB Bs, B.opApply 0 J g = B.opApply 0₁ (B.opApply 0₂ J) g

theorem **realF_congr** {Φ Ψ : (B.Mat P.k P.r → (Wq →L[ℂ] Wp)) → M.Ty P → MA.SmQ P.r P.TT} (h : ∀ J ∈ FFB P Bs, Ψ J = Φ J) (hΦ : RealF P Bs e L o κ Φ) : RealF P Bs e L o κ Ψ

theorem **realF_zero** : RealF P Bs e L o κ (fun _ => 0)

theorem **realF_add** {Φ Ψ : (B.Mat P.k P.r → (Wq →L[ℂ] Wp)) → M.Ty P → MA.SmQ P.r P.TT} (hΦ : RealF P Bs e L o κ Φ) (hΨ : RealF P Bs e L o κ Ψ) : RealF P Bs e L o κ (fun J => Φ J + Ψ J)

theorem **realF_act** {b : MA.A P (MA.DQ P)} (hb : b ∈ Submonoid.closure (MA.bddSet P L)) {Φ : (B.Mat P.k P.r → (Wq →L[ℂ] Wp)) → M.Ty P → MA.SmQ P.r P.TT} (hΦ : RealF P Bs e L o κ Φ) : RealF P Bs e L o κ (fun J => MA.actA P b (Φ J))

theorem **realF_listsum** {ι : Type} (l : List ι) (Φ : ι → (B.Mat P.k P.r → (Wq →L[ℂ] Wp)) → M.Ty P → MA.SmQ P.r P.TT) (h : ∀ i ∈ l, RealF P Bs e L o κ (Φ i)) : RealF P Bs e L o κ (fun J => (l.map fun i => Φ i J).sum)

theorem **realF_comp** (hpl : Q.IsPlace P Bs.z Bs.z' Bs.π Bs.π') {o' κ' : ℕ} {0 : B.DiffOp P.k P.r Wp Wq} (h0 : PInv P Bs o' κ' 0) {Φ : (B.Mat P.k P.r → (Wq →L[ℂ] Wp)) → M.Ty P → MA.SmQ P.r P.TT} (hΦ : RealF P Bs e L o κ Φ) : RealF P Bs e L (o + o') (κ + κ') (fun J => Φ (B.opApply 0 J))

theorem **realX_zero** : RealX P Bs e L o κ 0

theorem **realX_add** {X Y : MA.A P (MA.DQ P)} (hX : RealX P Bs e L o κ X) (hY : RealX P Bs e L o κ Y) : RealX P Bs e L o κ (X + Y)

theorem **realX_mul** {b X : MA.A P (MA.DQ P)} (hb : b ∈ Submonoid.closure (MA.bddSet P L)) (hX : RealX P Bs e L o κ X) : RealX P Bs e L o κ (b ∗ X)

theorem **realX_smul** (c : ℂ) {X : MA.A P (MA.DQ P)} (hX : RealX P Bs e L o κ X) : RealX P Bs e L o κ (c • X)

theorem **realX_sub** {X Y : MA.A P (MA.DQ P)} (hX : RealX P Bs e L o κ X) (hY : RealX P Bs e L o κ Y) : RealX P Bs e L o κ (X − Y)

theorem **realX_sum** {ι : Type} (s : Finset ι) (X : ι → MA.A P (MA.DQ P)) (h : ∀ i ∈ s, RealX P Bs e L o κ (X i)) : RealX P Bs e L o κ (∑ i ∈ s, X i)

theorem **realX_of_isTr** {X : MA.A P (MA.DQ P)} (h : IsTr P Bs e X) : RealX P Bs e L (2 ∗ P.r + P.k + 1) ((P.r + 1) ∗ (2 ∗ P.r + P.k + 1)) X

theorem **realX_one** : RealX P Bs e L 0 0 1

theorem **realX_hat** (h1 : ResidueSplittingModelAlgebra) (h2 : MirrorResidueSplittingModelAlgebra) (h3 : JDHatLagrange) (h4 : JDHatNodes) (hL : MA.IsModelLevel P (MA.DQ P) L) (hA : ∀ y : ℂ, IsTr P Bs e (L.Ahat y) ∧ IsTr P Bs e (L.Ahatv y)) (hR : ∀ y : ℝ, IsTr P Bs e (RhM P L y) ∧ IsTr P Bs e (RhvM P L y)) : ∀ X ∈ MA.hatSet P L, RealX P Bs e L (2 ∗ P.r + P.k + 1) ((P.r + 1) ∗ (2 ∗ P.r + P.k + 1)) X

theorem **realX_prod** (hpl : Q.IsPlace P Bs.z Bs.z' Bs.π Bs.π') (hBdd : ∀ X ∈ MA.hatSet P L, ∀ b ∈ Submonoid.closure (MA.bddSet P L), X ∗ b ∈ Submodule.span ℂ (bddHat P L)) (hHat : ∀ X ∈ MA.hatSet P L, RealX P Bs e L o κ X) : ∀ w : List (MA.A P (MA.DQ P)), (∀ a ∈ w, a ∈ MA.hatSet P L) → RealX P Bs e L (w.length ∗ o) (w.length ∗ κ) w.prod

theorem **hatProdB_of** (hA : CapAB) (hR : CapRB) (h0 : JDHatBdd) (h1 : ResidueSplittingModelAlgebra) (h2 : MirrorResidueSplittingModelAlgebra) (h3 : JDHatLagrange) (h4 : JDHatNodes) : HatProdB

CompanionTheory.AutomorphicTheory.LocalBesselUniqueness.NFHatProdKBS

theorem **normal_form_hat_product_bessel** : NFHatProdKBS

CompanionTheory.AutomorphicTheory.LocalBesselUniqueness.NFJCutFF

theorem **jf_mem_ffkb** {P : M.Par} {Wp Wq : Type} [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wp] [FiniteDimensional ℂ Wq] (p : KB.Pl P Wp Wq) (F : B.Mat P.k P.r → (Wq →L[ℂ] Wp)) (hF : ∀ (A : ℝ) (m : ℕ), B.InCC A m F) : B.JF p.T.base F ∈ FFKB p

theorem **normal_form_jacobi_cutoff** : NFJCutFF

CompanionTheory.AutomorphicTheory.LocalBesselUniqueness.OpLocal

theorem **ck_mono_set** {m : ℕ∞} {UU VV : Set (Mat k r)} (h : UU ⊆ VV) {f : Mat k r → E₁} (hf : CkOn m VV f) : CkOn m UU f

theorem **ck_ap** {m : ℕ∞} {UU : Set (Mat k r)} {a : Mat k r → (E₁ →L[ℂ] E₂)} {Φ : Mat k r → E₁} (ha : CkOn m UU a) (hΦ : CkOn m UU Φ) : CkOn m UU (fun g => a g (Φ g))

theorem **ck_Rw'** {m : ℕ∞} {UU : Set (Mat k r)} {w : List (Mat k r)} (hw : ∀ X ∈ w, inLie k r X) {f : Mat k r → E₁} (hf : CkOn (m + w.length) UU f) : CkOn m UU (Rw w f)

theorem **ext_loc** {m : ℕ∞} {UU : Set (Mat k r)} {J : Mat k r → E₁} (hJ : CkOn m UU J) {g₀ : Mat k r} (hg₀ : g₀ ∈ UU) : ∃ (f : Mat k r → E₁) (U : Set (Mat k r)), CkOn m (GV k r) f ∧ IsOpen U ∧ g₀ ∈ U ∧ ∀ x ∈ U, x ∈ GV k r → f x = J x

theorem **op_congr** (0 : DiffOp k r Wp Wq) {O : Set (Mat k r)} (hO : IsOpen O) {H H' : Mat k r → (Wq →L[ℂ] Wp)} (h : ∀ g ∈ GV k r, g ∈ O → H g = H' g) {g : Mat k r} (hg : g ∈ GV k r) (hgO : g ∈ O) : opApply 0 H g = opApply 0 H' g

theorem **op_smul** (0 : DiffOp k r Wp Wq) (c : ℂ) (H : Mat k r → (Wq →L[ℂ] Wp)) (g : Mat k r) : opApply 0 (fun x => c • H x) g = c • opApply 0 H g

theorem **op_cons** (T : DTerm k r Wp Wq) (0 : DiffOp k r Wp Wq) (J : Mat k r → (Wq →L[ℂ] Wp)) : opApply (T :: 0) J = fun g => T.coef g (Rw T.word J g) + opApply 0 J g

theorem **op_nil** (J : Mat k r → (Wq →L[ℂ] Wp)) : opApply ([] : DiffOp k r Wp Wq) J = fun _ => 0

theorem **ck_op** {m : ℕ∞} {o : ℕ} {UU : Set (Mat k r)} {J : Mat k r → (Wq →L[ℂ] Wp)} (hJ : CkOn (m + o) UU J) : ∀ 0 : DiffOp k r Wp Wq, (∀ T ∈ 0, CkOn ⊤ UU T.coef ∧ T.word.length ≤ o) → CkOn m UU (opApply 0 J) | [], _ => by rw [op_nil]; exact CkOn_zero | T :: 0, h => by rw [op_cons] have hT

theorem **word_le_opOrder** (0 : DiffOp k r Wp Wq) : ∀ T ∈ 0, T.word.length ≤ opOrder 0

theorem **opOrder_le** (0 : DiffOp k r Wp Wq) {n : ℕ} (h : ∀ T ∈ 0, T.word.length ≤ n) : opOrder 0 ≤ n

theorem **sgs_cont** (p : SS k r) : Continuous (sgs p)

theorem **sgs_gv** (D : RealPlaceData k r Wp Wq) {p : SS k r} (hp : D.inSS p) {g : Mat k r} (hg : g ∈ GV k r) : sgs p g ∈ GV k r

theorem **commutes_loc** (D : RealPlaceData k r Wp Wq) {UU : Set (Mat k r)} {θ : DiffOp k r Wp Wq} (hC : Commutes D UU θ) {p : SS k r} (hp : D.inSS p) {VV : Set (Mat k r)} {J : Mat k r → (Wq →L[ℂ] Wp)} (hJ : CkOn (opOrder θ : ℕ∞) VV J) {g : Mat k r} (hg : g ∈ UU) (hgG : g ∈ GV k r) (hgV : sgs p g ∈ VV) : opApply θ (rho D p J) g = rho D p (opApply θ J) g

theorem **level_zero_iff** (g : Mat k r) : g ∈ Q.Glev k r 0 ↔ g ∈ GV k r

theorem **ck_of_smoothLev** {E₁ : Type} [NormedAddCommGroup E₁] [NormedSpace ℂ E₁] {O : Set (Mat k r)} {J : Mat k r → E₁} (hJ : Q.SmoothLev k r 0 O J) : CkOn ⊤ (GV k r ∩ O) J

theorem **smoothLev_of_ck** {E₁ : Type} [NormedAddCommGroup E₁] [NormedSpace ℂ E₁] {O : Set (Mat k r)} {J : Mat k r → E₁} (hJ : CkOn ⊤ (GV k r ∩ O) J) : Q.SmoothLev k r 0 O J

theorem **sgs_mk** (a b c d g : Mat k r) : sgs (mkSS a b c d) g = a⁻¹ * b⁻¹ * g * d * c

theorem **rho_mk** (D : RealPlaceData k r Wp Wq) (a b c d : Mat k r) (F : Mat k r → (Wq →L[ℂ] Wp)) (g : Mat k r) : rho D (mkSS a b c d) F g = (D.π a).comp ((F (sgs (mkSS a b c d) g)).comp (D.π' c⁻¹))

theorem **inSS_mk** (D : RealPlaceData k r Wp Wq) {a b c d : Mat k r} (ha : a ∈ UWset k r D.z) (hb : b ∈ Nset k r) (hc : c ∈ UWset k r D.z') (hd : d ∈ Nset k r) : D.inSS (mkSS a b c d)

theorem **uw_one** (z : Fin (2 * r + k + 1) → ℂ) : (1 : Mat k r) ∈ UWset k r z

theorem **ffl_opApply** (D : RealPlaceData k r Wp Wq) {O UU : Set (Mat k r)} (hO : Q.IsDom k r D.z D.z' 0 O) (hOU : ∀ g ∈ O, g ∈ GV k r → g ∈ UU) {θ : DiffOp k r Wp Wq} (hS : ∀ T ∈ θ, CkOn ⊤ UU T.coef) (hC : Commutes D UU θ) {J : Mat k r → (Wq →L[ℂ] Wp)} (hJ : J ∈ Q.FFl k r D.z D.z' D.π D.π' 0 O) : opApply θ J ∈ Q.FFl k r D.z D.z' D.π D.π' 0 O

theorem **contDiff_poly** {κ : ℕ} {P : Mat k r → ℂ} (hP : U.IsPolyDeg κ P) : ContDiff ℝ ∞ P

theorem **ck_scal** {κ : ℕ} {P : Mat k r → ℂ} (hP : U.IsPolyDeg κ P) (UU : Set (Mat k r)) : CkOn ⊤ UU (fun g => P g • ContinuousLinearMap.id ℂ (Wq →L[ℂ] Wp))

theorem **polyOp_ck** {o κ : ℕ} {θ : DiffOp k r Wp Wq} (h : CutMP.PolyOp o κ θ) (UU : Set (Mat k r)) : ∀ T ∈ θ, CkOn ⊤ UU T.coef

theorem **polyDeg_mono** {κ κ' : ℕ} (h : κ ≤ κ') {P : Mat k r → ℂ} (hP : U.IsPolyDeg κ P) : U.IsPolyDeg κ' P

theorem **polyOp_mono** {o κ o' κ' : ℕ} (ho : o ≤ o') (hκ : κ ≤ κ') {θ : DiffOp k r Wp Wq} (h : CutMP.PolyOp o κ θ) : CutMP.PolyOp o' κ' θ

theorem **exists_polyOp** {o κ : ℕ} : ∀ θ : DiffOp k r Wp Wq, (∀ T ∈ θ, T.word.length ≤ o ∧ ∃ P : Mat k r → ℂ, U.IsPolyDeg κ P ∧ ∀ g, T.coef g = P g • ContinuousLinearMap.id ℂ (Wq →L[ℂ] Wp)) → ∃ θ' : DiffOp k r Wp Wq, CutMP.PolyOp o κ θ' ∧ ∀ F, opApply θ' F = opApply θ F | [], _ => ([], fun _ h => absurd h List.not_mem_nil, fun _ => rfl) | T :: θ, h => by obtain ⟨θ', h1, h2⟩

theorem **comp_raw** (ρ : ℕ) : ∀ (r : ℕ), r = θ + ρ → ∀ (o₁ κ₁ o₂ κ₂ : ℕ) (θ₁ θ₂ : DiffOp k r Wp Wq), CutMP.PolyOp o₁ κ₁ θ₁ → CutMP.PolyOp o₂ κ₂ θ₂ → ∃ θ : DiffOp k r Wp Wq, CutMP.PolyOp (o₁ + o₂) (κ₁ + κ₂) θ ∧ ∀ F : Mat k r → (Wq →L[ℂ] Wp), CkOn ((o₁ + o₂ : ℕ) : ℕ∞) (GV k r) F → ∀ g ∈ GV k r, opApply θ F g = opApply θ₁ (opApply θ₂ F) g

theorem **polyDeg_const** (κ : ℕ) (c : ℂ) : U.IsPolyDeg κ (fun _ : Mat k r => c)

theorem **lie_zero** : inLie k r (0 : Mat k r)

theorem **polyOp_comp** [FiniteDimensional ℂ Wp] {o₁ κ₁ o₂ κ₂ : ℕ} {θ₁ θ₂ : DiffOp k r Wp Wq} (h₁ : CutMP.PolyOp o₁ κ₁ θ₁) (h₂ : CutMP.PolyOp o₂ κ₂ θ₂) : ∃ θ : DiffOp k r Wp Wq, CutMP.PolyOp (o₁ + o₂) (κ₁ + κ₂) θ ∧ (∀ (VV : Set (Mat k r)) (F : Mat k r → (Wq →L[ℂ] Wp)), CkOn ((o₁ + o₂ : ℕ) : ℕ∞) VV F → ∀ g ∈ VV, g ∈ GV k r → opApply θ F g = opApply θ₁ (opApply θ₂ F) g) ∧ ∀ (D : RealPlaceData k r Wp Wq) (O : Set (Mat k r)), IsOpen O → Commutes D (GV k r ∩ O) θ₁ → Commutes D (GV k r ∩ O) θ₂ → Commutes D (GV k r ∩ O) θ

theorem **polyOp_one** : ∃ θ : DiffOp k r Wp Wq, CutMP.PolyOp 0 0 θ ∧ (∀ F, opApply θ F = F) ∧ ∀ (D : RealPlaceData k r Wp Wq) (UU : Set (Mat k r)), Commutes D UU θ

CompanionTheory.AutomorphicTheory.MirabolicRestriction.MirabolicInduction.CuspidalMirabolic

theorem **cuspidalMirabolic** : GlobalTheory.Published.CuspidalMirabolic

CompanionTheory.AutomorphicTheory.MirabolicRestriction.MirabolicInduction.CuspidalRestrictsToInduced

theorem **permGL_conj** (τ : Equiv.Perm (Fin N)) (M : GL (Fin N) k) : ((permGL τ * M * (permGL τ)⁻¹ : GL (Fin N) k) : Matrix (Fin N) (Fin N) k) = (M : Matrix (Fin N) (Fin N) k).submatrix τ τ

theorem **fixed_eq_zero** {V : Type} [AddCommGroup V] [Module ℂ V] (ρ : Representation ℂ (GL (Fin N) k) V) (h : GlobalTheory.FiniteGL.IsCuspidal ρ) (s : ℕ) (b : Fin N → Fin s) (hb : Function.Surjective b) (hs : 2 ≤ s) (v : V) (hv : ∀ u ∈ Formalism.IrreducibleCountingLemmas.TYPES.unip k b, ρ u v = v) : v = 0

theorem **isCuspidal_of** {V : Type} [AddCommGroup V] [Module ℂ V] [FiniteDimensional ℂ V] (ρ : Representation ℂ (GL (Fin N) k) V) (h : GlobalTheory.FiniteGL.IsCuspidal ρ) : Formalism.IrreducibleCountingLemmas.TYPES.IsCuspidal k (FDRep.of ρ)

theorem **mem_upU** {u : GL (Fin d) k} : u ∈ upU d k ↔ (∀ i j : Fin d, j < i → (u : Matrix (Fin d) (Fin d) k) i j = 0) ∧ ∀ i, (u : Matrix (Fin d) (Fin d) k) i i = 1

theorem **sdSum_eq_edgeSum** (g : GL (Fin d) k) : sdSum (g : Matrix (Fin d) (Fin d) k) = Formalism.L2.DIV.edgeSum Formalism.L2.DIV.supEdge g

theorem **sdSum_mul** {x y : GL (Fin d) k} (hx : x ∈ upU d k) (hy : y ∈ upU d k) : sdSum ((x * y : GL (Fin d) k) : Matrix (Fin d) (Fin d) k) = sdSum (x : Matrix (Fin d) (Fin d) k) + sdSum (y : Matrix (Fin d) (Fin d) k)

theorem **patChar_eq** (ψ₀ : AddChar k ℂ) (u : GL (Fin d) k) : patChar (cutE k d 0) ψ₀ u = ψ₀ (sdg u)

theorem **patSet_eq** : patSet k (fun i j : Fin d => i < j) = (upU d k : Set (GL (Fin d) k))

theorem **wh_eq_finrank** {V : Type} [AddCommGroup V] [Module ℂ V] [FiniteDimensional ℂ V] (ρ : Representation ℂ (GL (Fin d) k) V) (ψ₀ : AddChar k ℂ) : wh (fun i j : Fin d => i < j) (cutE k d 0) ψ₀ (FDRep.of ρ).character = Module.finrank ℂ (GlobalTheory.FinGL.eig ρ (upU d k) fun g => ψ₀ (sdg g))

theorem **isUni_iff** (u : GL (Fin d) k) : IsUni d u ↔ u ∈ upU d k

theorem **psiU_eq** (ψ₀ : AddChar k ℂ) (u : GL (Fin d) k) : psiU ψ₀ u = ψ₀ (sdg u)

theorem **whProj_comp** [Fintype (upU d k)] (ρ : Representation ℂ (GL (Fin d) k) V) (ψ₀ : AddChar k ℂ) (u₀ : GL (Fin d) k) (h₀ : u₀ ∈ upU d k) (y : V) : whProj ρ ψ₀ (ρ u₀ y) = ψ₀ (sdg u₀) • whProj ρ ψ₀ y

theorem **whProj_eig** [Fintype (upU d k)] (ρ : Representation ℂ (GL (Fin d) k) V) (ψ₀ : AddChar k ℂ) (y : V) (hy : y ∈ GlobalTheory.FinGL.eig ρ (upU d k) fun g => ψ₀ (sdg g)) : whProj ρ ψ₀ y = (Fintype.card (upU d k) : ℂ) • y

theorem **isWhit_comp** [Fintype (upU d k)] (ρ : Representation ℂ (GL (Fin d) k) V) (ψ₀ : AddChar k ℂ) (l : Module.Dual ℂ V) : IsWhit ψ₀ ρ (l ∘ₗ whProj ρ ψ₀)

theorem **finrank_and_whit** (ψ₀ : AddChar k ℂ) (hψ : ψ₀ ≠ 1) (ρ : Representation ℂ (GL (Fin d) k) V) (hi : ρ.IsIrreducible) (hc : GlobalTheory.FiniteGL.IsCuspidal ρ) : (Module.finrank ℂ V : ℂ) = ∏ i ∈ Finset.Ico 1 d, ((Fintype.card k : ℂ) ^ i - 1) ∧ ∃ L : Module.Dual ℂ V, L ≠ 0 ∧ IsWhit ψ₀ ρ L

theorem **sdSum_eq_supd** (M : Matrix (Fin (n + 1)) (Fin (n + 1)) k) : sdSum M = supd M

theorem **cuspidalRestrictsToInduced** : GlobalTheory.Published.CuspidalRestrictsToInduced

CompanionTheory.AutomorphicTheory.MirabolicRestriction.MirabolicInduction.IndDim

theorem **index_uniSub** : (uniSub k n).index = ∏ i ∈ Finset.Ico 1 (n + 1), (Fintype.card k ^ i - 1)

theorem **rep_mem** (x : mirabSub k n) : x * (rep n x)⁻¹ ∈ uniSub k n

theorem **cls_mul** (u : uniSub k n) (x : mirabSub k n) : cls n ((u : mirabSub k n) * x) = cls n x

theorem **cls_out** (c : Cos k n) : cls n c.out = c

theorem **psiExt_of_mem** (u : uniSub k n) : psiExt n ψ₀ (u : mirabSub k n) = genericChar n ψ₀ u

theorem **psiExt_mul** (u : uniSub k n) (y : mirabSub k n) (hy : y ∈ uniSub k n) : psiExt n ψ₀ ((u : mirabSub k n) * y) = genericChar n ψ₀ u * psiExt n ψ₀ y

theorem **psiExt_one** : psiExt n ψ₀ 1 = 1

theorem **extFun_mem** (g : Cos k n → ℂ) : extFun n ψ₀ g ∈ MirabInducedSpace n ψ₀

theorem **eq_at_rep** (f : MirabInducedSpace n ψ₀) (x : mirabSub k n) : (f : mirabSub k n → ℂ) x = genericChar n ψ₀ ⟨_, rep_mem n x⟩ * (f : mirabSub k n → ℂ) (rep n x)

theorem **restr_injective** : Function.Injective (restr n ψ₀)

theorem **restr_surjective** : Function.Surjective (restr n ψ₀)

theorem **finrank_mirabInducedSpace_eq_index** (ψ₀ : AddChar k ℂ) : Module.finrank ℂ (MirabInducedSpace n ψ₀) = (uniSub k n).index

theorem **finrank_mirabInducedSpace** (ψ₀ : AddChar k ℂ) : Module.finrank ℂ (MirabInducedSpace n ψ₀) = ∏ i ∈ Finset.Ico 1 (n + 1), (Fintype.card k ^ i - 1)

CompanionTheory.AutomorphicTheory.MirabolicRestriction.MirabolicInduction.IndFacts

theorem **mirabInduced_isIrreducible** (ψ₀ : AddChar k ℂ) (hψ : ψ₀ ≠ 1) : (mirabInduced n ψ₀).IsIrreducible

theorem **finrank_mirabInducedSpace_eq_index** (ψ₀ : AddChar k ℂ) : Module.finrank ℂ (MirabInducedSpace n ψ₀) = (uniSub k n).index

theorem **index_uniSub** : (uniSub k n).index = ∏ i ∈ Finset.Ico 1 (n + 1), (Fintype.card k ^ i - 1)

theorem **finrank_mirabInducedSpace** (ψ₀ : AddChar k ℂ) : Module.finrank ℂ (MirabInducedSpace n ψ₀) = ∏ i ∈ Finset.Ico 1 (n + 1), (Fintype.card k ^ i - 1)

CompanionTheory.AutomorphicTheory.MirabolicRestriction.MirabolicInduction.IndIrr

theorem **mackey_gap** (ψ₀ : AddChar k ℂ) (hψ : ψ₀ ≠ 1) (h : mirabSub k n) (hh : h ∉ uniSub k n) : ∃ u c : uniSub k n, (c : mirabSub k n) = h⁻¹ * u * h ∧ genericChar n ψ₀ c ≠ genericChar n ψ₀ u

theorem **vanish_off_uni** (ψ₀ : AddChar k ℂ) (hψ : ψ₀ ≠ 1) (f : mirabSub k n → ℂ) (hl : ∀ (u : uniSub k n) (x : mirabSub k n), f ((u : mirabSub k n) * x) = genericChar n ψ₀ u * f x) (hr : ∀ (x : mirabSub k n) (u : uniSub k n), f (x * (u : mirabSub k n)) = f x * genericChar n ψ₀ u) (h : mirabSub k n) (hh : h ∉ uniSub k n) : f h = 0

theorem **deltaU_one** (ψ₀ : AddChar k ℂ) : deltaU n ψ₀ 1 = 1

theorem **deltaU_mem** (ψ₀ : AddChar k ℂ) : deltaU n ψ₀ ∈ MirabInducedSpace n ψ₀

theorem **char_inv_mul** (ψ₀ : AddChar k ℂ) (u : uniSub k n) : genericChar n ψ₀ (u⁻¹) * genericChar n ψ₀ u = 1

theorem **avg_apply** [Fintype (uniSub k n)] (ψ₀ : AddChar k ℂ) (f : MirabInducedSpace n ψ₀) (x : mirabSub k n) : ((∑ u : uniSub k n, genericChar n ψ₀ (u⁻¹) • mirabInduced n ψ₀ (u : mirabSub k n) f : MirabInducedSpace n ψ₀) : mirabSub k n → ℂ) x = ∑ u : uniSub k n, genericChar n ψ₀ (u⁻¹) * (f : mirabSub k n → ℂ) (x * u)

theorem **avg_right** [Fintype (uniSub k n)] (ψ₀ : AddChar k ℂ) (f : mirabSub k n → ℂ) (x : mirabSub k n) (v : uniSub k n) : ∑ u : uniSub k n, genericChar n ψ₀ (u⁻¹) * f (x * (v : mirabSub k n) * u) = (∑ u : uniSub k n, genericChar n ψ₀ (u⁻¹) * f (x * u)) * genericChar n ψ₀ v

theorem **avg_one** [Fintype (uniSub k n)] (ψ₀ : AddChar k ℂ) (f : mirabSub k n → ℂ) (hf : f ∈ MirabInducedSpace n ψ₀) : ∑ u : uniSub k n, genericChar n ψ₀ (u⁻¹) * f (1 * u) = (Nat.card (uniSub k n) : ℂ) * f 1

theorem **sum_translate** (ψ₀ : AddChar k ℂ) (hψ : ψ₀ ≠ 1) [Fintype (mirabSub k n)] [Fintype (uniSub k n)] (F g : mirabSub k n → ℂ) (hF : F ∈ MirabInducedSpace n ψ₀) (hr : ∀ (x : mirabSub k n) (u : uniSub k n), F (x * (u : mirabSub k n)) = F x * genericChar n ψ₀ u) (hg : g ∈ MirabInducedSpace n ψ₀) (x : mirabSub k n) : ∑ p : mirabSub k n, g p * F (x * p⁻¹) = (Nat.card (uniSub k n) : ℂ) * F 1 * g x

theorem **mirabInduced_isIrreducible** (ψ₀ : AddChar k ℂ) (hψ : ψ₀ ≠ 1) : (mirabInduced n ψ₀).IsIrreducible

CompanionTheory.AutomorphicTheory.NumberFieldModels.HsVecProjLe

theorem **baseTie** (hpl : Q.IsPlace P Bs.z Bs.z' Bs.π Bs.π') (hk : 2 ≤ P.k) (he : IsFrameB P Bs e) (hG : GaugeQQ P Bs e) (X : Q.ActualOps P.k P.r Bs.z Bs.z' Bs.π Bs.π') : Formalism.FourierDecayAlt.P.NormalFormRelations.BaseTie X (trL hpl hk he) where

theorem **tie_cpt** (p : KB.Pl P Wp Wq) {e : M.Ty P → ℂ → (Wq →L[ℂ] Wp)} (he : IsFrameB P p.T.base e) (hG : GaugeQQ P p.T.base e) {Lm : R.LevelData (MA.A P (MA.DQ P)) P.r} (hM : MA.IsModelLevel P (MA.DQ P) Lm) {w : List C.Letter} (hw : KB.CutWord P p.l0 w) (a : Wd.Ix (Fin w.length)) {F : B.Mat P.k P.r → (Wq →L[ℂ] Wp)} (hF : KB.CptCoef p.T.base F) : (∃ Φ, Top.IsRadOf P p.T.base Φ (p.0.0 w a F)) ∧ trJB P p.T.base e (B.JF p.T.base (p.0.0 w a F)) = MA.actA P (Formalism.FourierDecayAlt.P.NormalFormRelations.gM P p.l0 Lm (D.positionted w) a) (trJB P p.T.base e (B.JF p.T.base F))

theorem **evQ_trJB** (hpl : Q.IsPlace P Bs.z Bs.z' Bs.π Bs.π') (hk : 2 ≤ P.k) (he : IsFrameB P Bs e) {Φ : Q.RadSp P.k P.r Bs.z Bs.z' Bs.π Bs.π'} {H : B.Mat P.k P.r → (Wq →L[ℂ] Wp)} (h : Top.IsRadOf P Bs Φ H) (ρ : M.Ty P) (q : MA.Qp P.r P.TT) : MA.evQ P (trJB P Bs e (B.JF Bs H) ρ) q = rawJ P Bs e (B.JF Bs H) ρ q

theorem **jf_mem_fib** (H : B.Mat P.k P.r → (Wq →L[ℂ] Wp)) (x : (Fin P.r → ℂ) × ℂ) : B.JF Bs H (B.gq Bs x) ∈ NF.Fib P.k P.r Bs.z Bs.z' Bs.μsec Bs.π Bs.π' x.2

theorem **wq_nonneg** (q : MA.Qp P.r P.TT) : 0 ≤ MA.wq P.k q.1 P.TT

theorem **parseval** (he : IsFrameB P Bs e) (H : B.Mat P.k P.r → (Wq →L[ℂ] Wp)) (q : MA.Qp P.r P.TT) : ∑ ρ, ‖rawJ P Bs e (B.JF Bs H) ρ q‖ ^ 2 = MA.wq P.k q.1 P.TT * ‖B.hsVec (B.JF Bs H (B.gq Bs q.1))‖ ^ 2

theorem **gf_gq** (hpl : Q.IsPlace P Bs.z Bs.z' Bs.π Bs.π') (q : MA.Qp P.r P.TT) : Gf P.k P.r Bs.z Bs.z' (B.gq Bs q.1) = MA.Gq P q

theorem **rawJ_Gpow** (hpl : Q.IsPlace P Bs.z Bs.z' Bs.π Bs.π') (s : ℕ) (F : B.Mat P.k P.r → (Wq →L[ℂ] Wp)) (ρ : M.Ty P) (q : MA.Qp P.r P.TT) : rawJ P Bs e (B.JF Bs fun g => ((Gf P.k P.r Bs.z Bs.z' g ^ s : ℝ) : ℂ) • F g) ρ q = ((MA.Gq P q ^ s : ℝ) : ℂ) * rawJ P Bs e (B.JF Bs F) ρ q

theorem **jf_cpt_FF** {k r : ℕ} (Db : B.RealPlaceData k r Wp Wq) {H : B.Mat k r → (Wq →L[ℂ] Wp)} (hH : KB.CptCoef Db H) : B.JF Db H ∈ Q.FFl k r Db.z Db.z' Db.π Db.π' 0 (OmB Db)

theorem **hsVec_proj_le** (E : Wp →L[ℂ] Wp) (h1 : E.comp E = E) (h2 : ∀ v v' : Wp, inner ℂ (E v) v' = inner ℂ v (E v')) (X : Wq →L[ℂ] Wp) : ‖B.hsVec (E.comp X)‖ ^ 2 ≤ ‖B.hsVec X‖ ^ 2

CompanionTheory.AutomorphicTheory.NumberFieldModels.StepCpt

theorem **tt_eq** (p : KB.Pl P Wp Wq) : P.TT = TBB.TTp p

theorem **det_bigPt** (r : ℕ) (c : Fin (r + 1) → ℂ) : (D.ptMatN (TBB.bigPt r c) (r + 1)).det = (∏ l : Fin r, c l.castSucc) * c (Fin.last r)

theorem **gp_eq** (p : KB.Pl P Wp Wq) (c : Fin (P.r + 1) → ℂ) (q : MA.Qp P.r P.TT) (hq : q.1 = ((fun l => c l.castSucc), c (Fin.last P.r))) : TBB.Gp p (TBB.bigPt P.r c) = MA.Gq P q

theorem **sxV** (p : KB.Pl P Wp Wq) (H : B.Mat P.k P.r → (Wq →L[ℂ] Wp)) (c : Fin (P.r + 1) → ℂ) (hd : ∀ a, c a ≠ 0) (hG : 0 < TBB.Gp p (TBB.bigPt P.r c)) (q : MA.Qp P.r P.TT) (hq : q.1 = ((fun l => c l.castSucc), c (Fin.last P.r))) : (∏ l : Fin P.r, Complex.normSq (c l.castSucc)) * (Real.sqrt (TBB.nPt P.k P.r (TBB.TTp p) (TBB.bigPt P.r c)) * Real.sqrt (TBB.nPt P.k P.r (TBB.TTp p) (TBB.bigPt P.r c)) * ‖B.hsVec (p.V H (TBB.bigPt P.r c))‖ ^ 2) = MA.wq P.k q.1 P.TT * ‖B.hsVec (B.JF p.T.base H (B.gq p.T.base q.1))‖ ^ 2

theorem **dH_sq_le** (p : KB.Pl P Wp Wq) (w : List C.Letter) (a : Wd.Ix (Fin w.length)) (ρ : M.Ty P) (c : Fin (P.r + 1) → ℂ) (hG : 0 < TBB.Gp p (TBB.bigPt P.r c)) : Top.dH P p.l0 (TBB.TTp p) w (TBB.bigPt P.r c) a ρ ^ 2 ≤ (∑ ρ' : M.Ty P, |(Top.cW P p.l0 w a ρ').re|) * (Complex.normSq ((∏ l : Fin P.r, c l.castSucc) * c (Fin.last P.r)) / TBB.Gp p (TBB.bigPt P.r c)) ^ w.length

theorem **cpt_Gpow** (p : KB.Pl P Wp Wq) (s : ℕ) {F : B.Mat P.k P.r → (Wq →L[ℂ] Wp)} (hF : KB.CptCoef p.T.base F) : KB.CptCoef p.T.base fun g => ((Gf P.k P.r p.T.base.z p.T.base.z' g ^ s : ℝ) : ℂ) • F g

CompanionTheory.AutomorphicTheory.NumberFieldModels.TransferGaugeGeneratorZbarInv

theorem **qr_of** (hpl : Q.IsPlace P Bs.z Bs.z' Bs.π Bs.π') (q : MA.Qp P.r P.TT) : q.1 ∈ NF.QR P.k P.r Bs.z Bs.z'

theorem **mu_domR** (hpl : Q.IsPlace P Bs.z Bs.z' Bs.π Bs.π') (q : MA.Qp P.r P.TT) : (q.1.1, NF.muZ P.k P.r Bs.μsec q.1.2) ∈ Q.DomR P.k P.r Bs.z Bs.z'

theorem **zeta_mu** (hpl : Q.IsPlace P Bs.z Bs.z' Bs.π Bs.π') (q : MA.Qp P.r P.TT) : Q.zetaL P.k P.r Bs.z Bs.z' (Fin.last P.r) (NF.muZ P.k P.r Bs.μsec q.1.2) = q.1.2

theorem **gg_mu** (hpl : Q.IsPlace P Bs.z Bs.z' Bs.π Bs.π') (q : MA.Qp P.r P.TT) : Q.GGl P.k P.r Bs.z Bs.z' (NF.muZ P.k P.r Bs.μsec q.1.2) = Complex.normSq q.1.2 − P.TT

theorem **famR_rad** (hpl : Q.IsPlace P Bs.z Bs.z' Bs.π Bs.π') (Φ : Q.RadSp P.k P.r Bs.z Bs.z' Bs.π Bs.π') : FamR P Bs (fun _ => Formalism.FourierDecayAlt.P.Qx5.Dp P.k P.r Bs.z Bs.z') (Φ : Q.Fn P.k P.r (Wq →L[ℂ] Wp))

theorem **tr_eq** (Φ : Q.RadSp P.k P.r Bs.z Bs.z' Bs.π Bs.π') : tr P Bs e Φ = trR P Bs.μsec e (Φ : Q.Fn P.k P.r (Wq →L[ℂ] Wp))

theorem **trR_congr** (hpl : Q.IsPlace P Bs.z Bs.z' Bs.π Bs.π') {Φ Ψ : Q.Fn P.k P.r (Wq →L[ℂ] Wp)} (h : ∀ x ∈ Q.DomR P.k P.r Bs.z Bs.z', Φ x.1 x.2 = Ψ x.1 x.2) : trR P Bs.μsec e Φ = trR P Bs.μsec e Ψ

theorem **trJB_of_rad** (hpl : Q.IsPlace P Bs.z Bs.z' Bs.π Bs.π') {Φ : Q.RadSp P.k P.r Bs.z Bs.z' Bs.π Bs.π'} {F : B.Mat P.k P.r → (Wq →L[ℂ] Wp)} (h : Top.IsRadOf P Bs Φ F) : trJB P Bs e (B.JF Bs F) = tr P Bs e Φ

theorem **tr_add** (Φ Ψ : Q.RadSp P.k P.r Bs.z Bs.z' Bs.π Bs.π') : tr P Bs e (Φ + Ψ) = tr P Bs e Φ + tr P Bs e Ψ

theorem **tr_mul** (T : Module.End ℂ (Q.RadSp P.k P.r Bs.z Bs.z' Bs.π Bs.π')) (f : (Fin P.r → ℂ) × Q.Mx P.k P.r → ℂ) (hT : ∀ Φ : Q.RadSp P.k P.r Bs.z Bs.z' Bs.π Bs.π', ∀ x ∈ Q.DomR P.k P.r Bs.z Bs.z', (T Φ : Q.Fn P.k P.r (Wq →L[ℂ] Wp)) x.1 x.2 = f x • (Φ : Q.Fn P.k P.r (Wq →L[ℂ] Wp)) x.1 x.2) (φ : MA.SmQ P.r P.TT) (hφ : ∀ q : MA.Qp P.r P.TT, (φ : MA.Qp P.r P.TT → ℂ) q = f (q.1.1, NF.muZ P.k P.r Bs.μsec q.1.2)) (Φ : Q.RadSp P.k P.r Bs.z Bs.z' Bs.π Bs.π') : tr P Bs e (T Φ) = fun ρ => φ * tr P Bs e Φ ρ

theorem **coordE_vz** (x : MA.E r) : MA.coordE (M.vz) x = x.2

theorem **coordE_vzb** (x : MA.E r) : MA.coordE (M.vzb) x = star x.2

theorem **gg_apply** (q : MA.Qp r TT) : (ggQ r TT : MA.Qp r TT → ℂ) q = ((Complex.normSq q.1.2 − TT : ℝ) : ℂ)

theorem **inv_apply** (φ : MA.SmQ r TT) (hφ : ∀ q, (φ : MA.Qp r TT → ℂ) q ≠ 0) (q : MA.Qp r TT) : ((Ring.inverse φ : MA.SmQ r TT) : MA.Qp r TT → ℂ) q = ((φ : MA.Qp r TT → ℂ) q)⁻¹

theorem **zeta_ne** (hpl : Q.IsPlace P Bs.z Bs.z' Bs.π Bs.π') (q : MA.Qp P.r P.TT) : q.1.2 ≠ 0

theorem **tr_fn** (j : Fin 6) (φ : MA.SmQ P.r P.TT) (hφ : ∀ q : MA.Qp P.r P.TT, (φ : MA.Qp P.r P.TT → ℂ) q = Q.fnVal P.k P.r Bs.z Bs.z' j (NF.muZ P.k P.r Bs.μsec q.1.2)) (Φ : Q.RadSp P.k P.r Bs.z Bs.z' Bs.π Bs.π') : tr P Bs e (X.fn j Φ) = fun ρ => φ * tr P Bs e Φ ρ

theorem **tr_opmul** (l : Fin P.r) (i : Fin 6) (f : (Fin P.r → ℂ) → ℂ) (hf : ∀ (Φ : Q.Fn P.k P.r (Wq →L[ℂ] Wp)) c g, Q.crd (opL l i) Φ c g = f c • Φ c g) (φ : MA.SmQ P.r P.TT) (hφ : ∀ q : MA.Qp P.r P.TT, (φ : MA.Qp P.r P.TT → ℂ) q = f q.1.1) (Φ : Q.RadSp P.k P.r Bs.z Bs.z' Bs.π Bs.π') : tr P Bs e (X.op l i Φ) = fun ρ => φ * tr P Bs e Φ ρ

theorem **transfer_torus_action** (hG : GaugeQQ P Bs e) (Y : ℝ) (a b : Fin 2) (Φ : Q.RadSp P.k P.r Bs.z Bs.z' Bs.π Bs.π') : tr P Bs e (X.t Y a b Φ) = MA.actA P (MA.QQr P (MA.DQ P) (Y : ℂ) a b) (tr P Bs e Φ)

theorem **transfer_operator_p** (l : Fin P.r) (Φ : Q.RadSp P.k P.r Bs.z Bs.z' Bs.π Bs.π') : tr P Bs e (X.op l 0 Φ) = MA.actA P (MA.pA P (MA.DQ P) l) (tr P Bs e Φ)

theorem **transfer_operator_pbar** (l : Fin P.r) (Φ : Q.RadSp P.k P.r Bs.z Bs.z' Bs.π Bs.π') : tr P Bs e (X.op l 3 Φ) = MA.actA P (MA.pbA P (MA.DQ P) l) (tr P Bs e Φ)

theorem **transfer_operator_c** (l : Fin P.r) (Φ : Q.RadSp P.k P.r Bs.z Bs.z' Bs.π Bs.π') : tr P Bs e (X.op l 1 Φ) = MA.actA P (MA.cA P (MA.DQ P) l) (tr P Bs e Φ)

theorem **transfer_operator_cbar** (l : Fin P.r) (Φ : Q.RadSp P.k P.r Bs.z Bs.z' Bs.π Bs.π') : tr P Bs e (X.op l 4 Φ) = MA.actA P (MA.cbA P (MA.DQ P) l) (tr P Bs e Φ)

theorem **transfer_operator_c_inv** (l : Fin P.r) (Φ : Q.RadSp P.k P.r Bs.z Bs.z' Bs.π Bs.π') : tr P Bs e (X.op l 2 Φ) = MA.actA P (MA.ciA P (MA.DQ P) l) (tr P Bs e Φ)

theorem **transfer_operator_cbar_inv** (l : Fin P.r) (Φ : Q.RadSp P.k P.r Bs.z Bs.z' Bs.π Bs.π') : tr P Bs e (X.op l 5 Φ) = MA.actA P (MA.cbiA P (MA.DQ P) l) (tr P Bs e Φ)

theorem **transfer_gauge_generator_z** (Φ : Q.RadSp P.k P.r Bs.z Bs.z' Bs.π Bs.π') : tr P Bs e (X.fn 0 Φ) = MA.actA P (MA.zA P (MA.DQ P)) (tr P Bs e Φ)

theorem **transfer_gauge_generator_z_inv** (Φ : Q.RadSp P.k P.r Bs.z Bs.z' Bs.π Bs.π') : tr P Bs e (X.fn 1 Φ) = MA.actA P (MA.ziA P (MA.DQ P)) (tr P Bs e Φ)

theorem **transfer_gauge_generator_zbar** (Φ : Q.RadSp P.k P.r Bs.z Bs.z' Bs.π Bs.π') : tr P Bs e (X.fn 2 Φ) = MA.actA P (MA.zbA P (MA.DQ P)) (tr P Bs e Φ)

theorem **transfer_gauge_generator_zbar_inv** (Φ : Q.RadSp P.k P.r Bs.z Bs.z' Bs.π Bs.π') : tr P Bs e (X.fn 3 Φ) = MA.actA P (MA.zbiA P (MA.DQ P)) (tr P Bs e Φ)

theorem **transfer_gauge_generator_ggi** (Φ : Q.RadSp P.k P.r Bs.z Bs.z' Bs.π Bs.π') : tr P Bs e (X.fn 4 Φ) = MA.actA P (MA.ggA P (MA.DQ P)) (tr P Bs e Φ)

theorem **transfer_gauge_generator_gg_inv** (Φ : Q.RadSp P.k P.r Bs.z Bs.z' Bs.π Bs.π') : tr P Bs e (X.fn 5 Φ) = MA.actA P (MA.ggiA P (MA.DQ P)) (tr P Bs e Φ)

theorem **modelLevel_unique** {L L' : R.LevelData (MA.A P (MA.DQ P)) P.r} (h : MA.IsModelLevel P (MA.DQ P) L) (h' : MA.IsModelLevel P (MA.DQ P) L') : L = L'

CompanionTheory.AutomorphicTheory.PadicCyclotomicFields.GcohCmp

theorem **gcRep** : SgcRep

theorem **lc_of_cont** {X : Type} [TopologicalSpace X] {f : X → M} (hf : Continuous f) : IsLocallyConstant fun s => e (f s)

theorem **cont_of_lc** {X : Type} [TopologicalSpace X] {f : X → A.V} (hf : IsLocallyConstant f) : Continuous fun s => e.symm (f s)

theorem **gcn** : Sgcn

theorem **gc0** : Sgc0

theorem **gc1** : Sgc1

theorem **gc2** : Sgc2

CompanionTheory.AutomorphicTheory.PadicCyclotomicFields.GcohCmpA

theorem **coc_inj** (n : ℕ) {x y : cocycles A n} (h : cochainOf A (cocycleι A n x) = cochainOf A (cocycleι A n y)) : x = y

theorem **exists_coc** (n : ℕ) (c : (Fin n → G) → A) (hc : IsLocallyConstant c) (hd : (inhomogeneousCochains.d A n).hom c = 0) : ∃ x : cocycles A n, cochainOf A (cocycleι A n x) = c

theorem **pi_succ_eq_zero_iff** (n : ℕ) (x : cocycles A (n + 1)) : cohomologyπ A (n + 1) x = 0 ↔ ∃ w : (Fin n → G) → A, IsLocallyConstant w ∧ (inhomogeneousCochains.d A n).hom w = cochainOf A (cocycleι A (n + 1) x)

theorem **pi_zero_eq_zero_iff** (x : cocycles A 0) : cohomologyπ A 0 x = 0 ↔ cochainOf A (cocycleι A 0 x) = 0

theorem **core_hom** {Z : Type} [AddCommGroup Z] (n : ℕ) (φ : Z →+ ((Fin n → G) → A)) (hex : ∀ z, ∃ x : cocycles A n, cochainOf A (cocycleι A n x) = φ z) (hsurj : ∀ x : cocycles A n, ∃ z, cochainOf A (cocycleι A n x) = φ z) : ∃ f : Z →+ cohomology A n, Function.Surjective f ∧ ∀ z x, cochainOf A (cocycleι A n x) = φ z → f z = cohomologyπ A n x

theorem **core_quot** {R Z : Type} [CommRing R] [AddCommGroup Z] [Module R Z] (B : Submodule R Z) (n : ℕ) (φ : Z →+ ((Fin n → G) → A)) (hex : ∀ z, ∃ x : cocycles A n, cochainOf A (cocycleι A n x) = φ z) (hsurj : ∀ x : cocycles A n, ∃ z, cochainOf A (cocycleι A n x) = φ z) (hB : ∀ z x, cochainOf A (cocycleι A n x) = φ z → (cohomologyπ A n x = 0 ↔ z ∈ B)) : ∃ e : (Z ⧸ B) ≃+ cohomology A n, ∀ z x, cochainOf A (cocycleι A n x) = φ z → e (Submodule.Quotient.mk z) = cohomologyπ A n x

theorem **neg_one_pow_smul'** {S V : Type} [Ring S] [AddCommGroup V] [Module S V] (j : ℕ) (v : V) : (-1 : S) ^ j • v = (-1 : ℤ) ^ j • v

theorem **dn_app** (n : ℕ) (f : (Fin n → G) → M) (g : Fin (n + 1) → G) : dn rho n f g = rho (g 0) (f fun i => g i.succ) + Finset.univ.sum fun j : Fin (n + 1) => (-1 : R) ^ ((j : ℕ) + 1) • f (Fin.contractNth j (· * ·) g)

theorem **d_comp** (n : ℕ) (f : (Fin n → G) → M) : (inhomogeneousCochains.d A n).hom (fun s => e (f s)) = fun s => e (dn rho n f s)

theorem **dn_zero** (f : (Fin 0 → G) → M) (s : Fin 1 → G) : dn rho 0 f s = rho (s 0) (f default) - f default

theorem **dn_one** (f : G → M) (s : Fin 2 → G) : dn rho 1 (fun t => f (t 0)) s = rho (s 0) (f (s 1)) - f (s 0 * s 1) + f (s 0)

theorem **dn_two** (f : G × G → M) (s : Fin 3 → G) : dn rho 2 (fun t => f (t 0, t 1)) s = rho (s 0) (f (s 1, s 2)) - f (s 0 * s 1, s 2) + f (s 0, s 1 * s 2) - f (s 0, s 1)

CompanionTheory.AutomorphicTheory.PadicCyclotomicFields.GcohEuler

theorem **exists_card_eq_pow** (l : ℕ) [hl : Fact l.Prime] (r : ℕ) (M : Type) [AddCommGroup M] [Module (ZMod (l ^ r)) M] [Finite M] : ∃ k, Nat.card M = l ^ k

theorem **euler_of** (hRep : SgcRep) (h0 : Sgc0) (h1 : Sgc1) (h2 : Sgc2) (hF : SgcField) : Formalism.GaloisCohomology.LocalEulerCharK

CompanionTheory.AutomorphicTheory.PadicCyclotomicFields.GcohField

theorem **integer_padic_eq** (p : ℕ) [Fact p.Prime] : 𝒪[ℚ_[p]] = PadicInt.subring p

theorem **card_quotient_padic** (p : ℕ) [Fact p.Prime] (m : ℕ) : Nat.card (𝒪[ℚ_[p]] ⧸ Ideal.span {((p ^ m : ℕ) : 𝒪[ℚ_[p]])}) = p ^ m

theorem **isSepClosure_padic** (l : ℕ) [Fact l.Prime] (K : IntermediateField ℚ_[l] (PadicAlgCl l)) : IsSepClosure ɪK (PadicAlgCl l)

theorem **gcField** : SgcField

CompanionTheory.AutomorphicTheory.PadicCyclotomicFields.GcohFin

theorem **locfin_of** (hF : SgcField) : Formalism.GaloisCohomology.LocalFinitenessK

CompanionTheory.AutomorphicTheory.PadicCyclotomicFields.GcohMu

theorem **cycKSpec** : Formalism.GaloisCohomology.CycKSpec

theorem **gcMu** : SgcMu

CompanionTheory.AutomorphicTheory.PadicCyclotomicFields.GcohTate

theorem **tate_core** (hRep : SgcRep) (h0 : Sgc0) (h1 : Sgc1) (h2 : Sgc2) {n : ℕ} [NeZero n] {G : Type} [Group G] [TopologicalSpace G] [IsTopologicalGroup G] [CompactSpace G] [TotallyDisconnectedSpace G] (D : Rep.{0} (ULift.{0} ℤ) G) [IsDiscreteRep D] (H : DualityHypotheses n D) (hfin : ∀ (A : Rep.{0} (ULift.{0} ℤ) G) [IsDiscreteRep A] [Finite A], (∀ a : A, n • a = 0) → Finite (cohomology A 1)) (χ : G →* (ZMod n)ˣ) (eD : ZMod n ≃+ D.V) (hD : ∀ (g : G) (m : ZMod n), eD ((χ g : ZMod n) * m) = D.ρ g (eD m)) (M : Type) [AddCommGroup M] [Module (ZMod n) M] [Finite M] [TopologicalSpace M] [DiscreteTopology M] (rho : Representation (ZMod n) G M) (hrho : ContAction rho) : Nonempty (H2 (charRep χ) ≃ₗ[ZMod n] ZMod n) ∧ Function.Bijective (cupH0H2TwistedDual χ rho (MonoidHom.id _)) ∧ Function.Bijective (cupH1H1TwistedDual χ rho (MonoidHom.id _) (hrho.comp' continuous_id)) ∧ Function.Bijective (cupH1H1T

theorem **tate_of** (hRep : SgcRep) (h0 : Sgc0) (h1 : Sgc1) (h2 : Sgc2) (hF : SgcField) (hMu : SgcMu) : Formalism.GaloisCohomology.LocalTateDualityK

CompanionTheory.AutomorphicTheory.PadicCyclotomicFields.GcohTateA

theorem **bij_transport** {X Y Z X' Y' : Type*} [AddCommGroup X] [AddCommGroup Y] [AddCommGroup Z] [Module (ZMod n) X] [Module (ZMod n) Y] [Module (ZMod n) Z] [AddCommGroup X'] [AddCommGroup Y'] (c : X →ₗ[ZMod n] Y →ₗ[ZMod n] Z) (β : X' →+ Y' →+ ZMod n) (eX : X ≃+ X') (eY : Y ≃+ Y') (τ : Z ≃+ ZMod n) (h : ∀ x y, τ (c x y) = β (eX x) (eY y)) (hβ : Function.Bijective β) : Function.Bijective c

theorem **nsmul_zmod** {Y : Type*} [AddCommGroup Y] [Module (ZMod n) Y] (y : Y) : n • y = 0

theorem **bij_flip** [NeZero n] {X Y Z : Type*} [AddCommGroup X] [AddCommGroup Y] [AddCommGroup Z] [Module (ZMod n) X] [Module (ZMod n) Y] [Module (ZMod n) Z] [Finite Y] (c : X →ₗ[ZMod n] Y →ₗ[ZMod n] Z) (τ : Z ≃+ ZMod n) (hc : Function.Bijective c) : Function.Bijective c.flip

theorem **exists_dualEquiv** (hb : Function.Bijective b.flip) : ∃ eB : M' ≃+ (dualRep D A).V, (∀ g y, eB (rho' g y) = (dualRep D A).ρ g (eB y)) ∧ ∀ x y, (eB y : A →ₗ[ULift.{0} ℤ] D) (eA x) = eD (b x y)

CompanionTheory.AutomorphicTheory.PadicCyclotomicFields.GcohTateB

theorem **exists_cocycle_zero** [IsTopologicalAddGroup M] [ContinuousConstSMul (ZMod n) M] (m : H0 rho) : ∃ x : cocycles A 0, ∀ s, cochainOf A (cocycleι A 0 x) s = eA m.1

theorem **cup_product_H0_H2_bijective** (h0 : Sgc0) (h2 : Sgc2) (hd : Function.Bijective (dualityMap D (rfl : 0 + 2 = 2) t.toAddMonoidHom A)) : Function.Bijective (cupProductH0H2 rho rho' rhoP b hbc hbe)

theorem **cup_product_H2_H0_bijective** (h0 : Sgc0) (h2 : Sgc2) (hd : Function.Bijective (dualityMap D (rfl : 2 + 0 = 2) t.toAddMonoidHom A)) : Function.Bijective (cupProductH2H0 rho rho' rhoP b hbc hbe)

theorem **cup_product_H1_H1_bijective** (h1 : Sgc1) (h2 : Sgc2) (hc : ContAction rho') (hd : Function.Bijective (dualityMap D (rfl : 1 + 1 = 2) t.toAddMonoidHom A)) : Function.Bijective (cupProductH1H1 rho rho' rhoP b hbc hbe hc)

CompanionTheory.AutomorphicTheory.PadicCyclotomicFields.GcohTop

theorem **top_tate** : Formalism.GaloisCohomology.LocalTateDualityK

theorem **top_locfin** : Formalism.GaloisCohomology.LocalFinitenessK

CompanionTheory.AutomorphicTheory.PadicCyclotomicFields.GcohTopEuler

theorem **top_euler** : Formalism.GaloisCohomology.LocalEulerCharK

CompanionTheory.AutomorphicTheory.PadicUnitaryWeights.CrystallineUnitParts.FinExtIsClosed

theorem **finExt_isClosed** (L : FinExt l) : IsClosed (L.1 : Set (Qbar l))

theorem **map_mem_of_gen** (θ : E) (hθ : IntermediateField.adjoin ℚ {θ} = ⊤) (φ : E →+* Qbar l) (L : FinExt l) (h : φ θ ∈ L.1) (y : E) : φ y ∈ L.1

theorem **exists_big** (L₁ : FinExt l) (k : ℕ) (hk : 0 < k) (w : HeightOneSpectrum (𝒪 E)) : ∃ L₂ : FinExt l, L₁.1 ≤ L₂.1 ∧ (∀ z : Qbar l, z ^ k = 1 → z ∈ L₂.1) ∧ ∀ (j : {j : w.adicCompletion E →+* Qbar l // Continuous j}) (x : w.adicCompletion E), j.1 x ∈ L₂.1

CompanionTheory.AutomorphicTheory.PadicUnitaryWeights.DixmierMalliavin.DecompDirection

theorem **decomp_direction** {f : G → ℂ} (hf : letI

theorem **decomp_iter** {ε : ℝ} (hε : 0 < ε) : ∀ {d : ℕ} (X : Fin d → 𝔤), (f : G → ℂ), (letI

theorem **smooth_density** [MeasurableSpace G] [BorelSpace G] (μ : Measure G) [μ.IsHaarMeasure] {d : ℕ} (b : Module.Basis (Fin d) ℝ 𝔤) : ∃ ε > 0, ∀ δ ≤ ε, ∀ K : (Fin d → ℝ) → ℂ, ContDiff ℝ ∞ K → tsupport K ⊆ closedBall 0 δ → ∃ κ : G → ℂ, (letI

theorem **factor** [MeasurableSpace G] [BorelSpace G] (μ : Measure G) [μ.IsHaarMeasure] {V : Set G} (hV : V ∈ 𝒩 1) (f : G → ℂ) (hf : letI

CompanionTheory.AutomorphicTheory.PadicUnitaryWeights.DixmierMalliavin.DixmierMalliavinFactorisationHolds

theorem **exists_lie** (H : Subgroup 𝔸ˣ) (hH : IsClosed (H : Set 𝔸ˣ)) : ∃ 𝔤 : Submodule ℝ 𝔸, IsExpLieAlgebra H 𝔤

theorem **mem_ccSmooth_iff** {H : Subgroup 𝔸ˣ} {𝔤 : Submodule ℝ 𝔸} (h : IsExpLieAlgebra H 𝔤) (f : H → ℂ) : f ∈ CcSmooth (fun x : H => ((x : 𝔸ˣ) : 𝔸)) ↔ (letI

theorem **factor_abs** {G : Type} [Group G] [TopologicalSpace G] [IsTopologicalGroup G] [MeasurableSpace G] [BorelSpace G] (ι : G →* 𝔸) (hι : Topology.IsClosedEmbedding (ι.toHomUnits : G → 𝔸ˣ)) (μ : Measure G) [μ.IsHaarMeasure] {V : Set G} (hV : V ∈ 𝒩 1) {f : G → ℂ} (hf : f ∈ CcSmooth fun g => ι g) : ∃ (N : ℕ) (g k : Fin N → G → ℂ), (∀ i, g i ∈ CcSmooth (fun g => ι g) ∧ k i ∈ CcSmooth (fun g => ι g) ∧ tsupport (g i) ⊆ V ∧ tsupport (k i) ⊆ tsupport f) ∧ f = ∑ i, GlobalTheory.AutomorphicL2Unitary.conv μ (g i) (k i)

theorem **CompanionTheory.DM.dixmierMalliavinFactorisation_holds** : GlobalTheory.Published.DixmierMalliavinFactorisation

CompanionTheory.AutomorphicTheory.PadicUnitaryWeights.UnitarySiegelSets.SiegelSetUnitaryHolds

theorem **siegelCore_holds** : CompanionTheory.ThKR.SiegelCore

CompanionTheory.AutomorphicTheory.PieriRules.HarishChandraFormalDegree

theorem **thrS_lt** (ε : Fin n → Bool) (hn : 1 ≤ n) {h : ℝ} (hh : 43 * (n : ℝ) ^ 2 ≤ h) : G2.thrS ε + 1 < h

theorem **harish_chandra_formal_degree** : HCFdegG

theorem **dsExistsG** (ε : Fin n → Bool) (lam : Fin n → ℝ) (hl : IsHCParam lam) {h : ℝ} (hg : Formalism.QuantumDeterminants.GapAtLeast ε lam h) (hh : G2.thrS ε < h) : ∃ (H : Type) (_ : NormedAddCommGroup H) (_ : InnerProductSpace ℂ H) (σ : Matrix (Fin n) (Fin n) ℂ → (H →L[ℂ] H)), IsDS ε lam σ

theorem **one_le_qOf** (hP : IsCayleyU k r ε P) : 1 ≤ qOf ε

theorem **one_le_pOf** (hk : 1 ≤ k) (hP : IsCayleyU k r ε P) : 1 ≤ pOf ε

theorem **dsExistsHpkG** (hP : IsCayleyU k r ε P) (lam : Fin (2 * r + k + 1) → ℝ) (hl : IsHCParam lam) {h : ℝ} (hg : Formalism.QuantumDeterminants.GapAtLeast ε lam h) (hh : G2.thrS ε < h) : ∃ (H : Type) (_ : NormedAddCommGroup H) (_ : InnerProductSpace ℂ H) (_ : CompleteSpace H) (σ : Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ → (H →L[ℂ] H)), IsDSHpk k r ε lam σ

CompanionTheory.AutomorphicTheory.PieriRules.InfinitesimalCharacterDiscreteSeries

theorem **rank_one_pieri** : RankOneG

theorem **infinitesimal_character_discrete_series** : InfCharG

theorem **thrS_lt_of_gapOK** {k r : ℕ} (ε : Fin (2 * r + k + 1) → Bool) {h : ℝ} (hok : GapOK k r h) : ReciprocityTheory.ChartRestriction.DS.G2.thrS ε + 1 < h

theorem **discrete_series_exist** : DSExistG

theorem **poulsen_discrete_series** : PoulsenG

theorem **harish_chandra_formal_degree** : HCFdegG

theorem **hypsG_of_zuck** (hZ : ZuckG) : HypsG

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.BaseRepCapelliEigenDual

theorem **continuousOn_of_ckOn** {k r : ℕ} {E : Type} [NormedAddCommGroup E] [NormedSpace ℂ E] {m : ℕ∞} {f : B.Mat k r → E} (h : B.CkOn m (B.GV k r) f) : ContinuousOn f (B.GV k r)

theorem **coef_norm_finite_and_zero** : HypsG → ∀ (P : M.Par) (Wp Wq : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wp] [FiniteDimensional ℂ Wq] (p : KB.Pl P Wp Wq) (lam : Fin (2 * P.r + P.k + 1) → ℝ) (b bv ga h : ℝ) (q0 : ℕ) (R0 : Rep P.k P.r) (f : B.Mat P.k P.r → (Wq →L[ℂ] Wp)), Num P.k P.r lam P.u b bv ga h q0 → R0.IsDG (PI.BesselPeriodGGP.eps48 P.k P.r) lam → PI.CapelliEig.IsKFinCoefT (k

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.BesselCayleySideBound

theorem **sum_sq_eq_trace** {n : Type} [Fintype n] (A : Matrix n n ℂ) : ((∑ i, ∑ j, ‖A i j‖ ^ 2 : ℝ) : ℂ) = (Aᴴ * A).trace

theorem **sum_sq_conj** {n : Type} [Fintype n] [DecidableEq n] (Q X : Matrix n n ℂ) (hQ : Q ∈ Matrix.unitaryGroup n ℂ) : ∑ i, ∑ j, ‖(Q * X * Q⁻¹) i j‖ ^ 2 = ∑ i, ∑ j, ‖X i j‖ ^ 2

theorem **exp_mem** {k r : ℕ} {X : Mx k r} (hX : PI.UpqRep.InLieHpk k r X) (t : ℝ) : NormedSpace.exp ((t : ℂ) • X) ∈ GG k r

theorem **one_mem_UW** (k r : ℕ) (z : Fin (2 * r + k + 1) → ℂ) : (1 : Mx k r) ∈ Formalism.QuantumDeterminants.UWset k r z

theorem **pointwise** : PI.KFinSmooth → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (R : Rep k r) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (z : Fin (2 * r + k + 1) → ℂ) (π : Mx k r → (Wp →L[ℂ] Wp)) (u : ℕ → ℝ) (μW μN μG νq : Measure (Formalism.QuantumDeterminants.Mt k r)) (l : R.H → Wp) (Q : Mx k r) (E : Fin (2 * r + k + 1) → Fin (2 * r + k + 1) → Module.End ℂ R.smSub), R.IsDG ε mu → SideG k r z π u μW μN μG νq → IsBes R z π l → PI.UpqRep.IsCayleyU k r ε Q → R.IsE Q E → ∀ g ∈ GG k r, ∀ (a : Fin r) (w : R.smSub), (lenA k r z g a.succ / lenA k r z g a.castSucc) ^ 2 * ‖l (R.σ g w)‖ ^ 2 ≤ ∑ c, ∑ e, ‖l (R.σ g (E c e w))‖ ^ 2

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.BesselCayleySideBoundStatement

theorem **_root_.Gp91.GlobalTheory.T.bessel_cayley_side_bound** : PI.KFinSmooth → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (R : Rep k r) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (z : Fin (2 * r + k + 1) → ℂ) (π : Mx k r → (Wp →L[ℂ] Wp)) (u : ℕ → ℝ) (μW μN μG νq : Measure (Formalism.QuantumDeterminants.Mt k r)) (l : R.H → Wp) (Q : Mx k r) (E : Fin (2 * r + k + 1) → Fin (2 * r + k + 1) → Module.End ℂ R.smSub), R.IsDG ε mu → SideG k r z π u μW μN μG νq → IsBes R z π l → PI.UpqRep.IsCayleyU k r ε Q → R.IsE Q E → ∀ g ∈ GG k r, ∀ (a : Fin r) (w : R.smSub), (lenA k r z g a.succ / lenA k r z g a.castSucc) ^ 2 * ‖l (R.σ g w)‖ ^ 2 ≤ ∑ c, ∑ e, ‖l (R.σ g (E c e w))‖ ^ 2

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.BesselFormalDegreeIdentity

theorem **hsInner_smul_smul** {Wp Wq : Type} [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wq] (a b : ℂ) (A A' : Wq →L[ℂ] Wp) : B.hsInner (a • A) (b • A') = a * star b * B.hsInner A A'

theorem **isUnit_det_of_mem** {k r : ℕ} {g : Mx k r} (hg : g ∈ GG k r) : IsUnit g.det

theorem **haar_left** {k r : ℕ} {U : Set (B.Mat k r)} {μ : Measure (Formalism.QuantumDeterminants.Mt k r)} (h : B.IsHaarOn U μ) : PI.BesselPeriodGGP.IsHaarOn U μ

theorem **Ph_coeff** : HypsG → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (R : Rep k r) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (Wq : Type) [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [CompleteSpace Wq] [FiniteDimensional ℂ Wq] (D : B.RealPlaceData k r Wp Wq) (dd : ℝ), R.IsDG ε mu → (∀ w, w ≠ 0 → PI.BesselPeriodGGP.fdegH D.μG R.σ w = dd) → ∀ (x y x' y' : R.H) (X X' : Wq →L[ℂ] Wp) (p : B.SS k r), D.inSS p → Integrable (fun g : Formalism.QuantumDeterminants.Mt k r => B.hsInner (B.rho D p (fun g => ⟪x, R.σ g y⟫_ℂ • X) (Matrix.of g)) (⟪x', R.σ (Matrix.of g) y'⟫_ℂ • X')) D.μG ∧ B.Ph D (fun g => ⟪x, R.σ g y⟫_ℂ • X) (fun g => ⟪x', R.σ g y'⟫_ℂ • X') p = ⟪y', R.σ (JF.sR p) y⟫_ℂ * ⟪R.σ (JF.sL p) x, x'⟫_ℂ / (dd : ℂ) * B.hsInner ((D.π (Matrix.of p.1.1)).comp (X.comp (D.π' (Matrix.of p.2.1)⁻¹))) X'

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.BesselFormalDegreeIdentityStatement

theorem **_root_.Gp91.GlobalTheory.T.bessel_formal_degree_identity** : HypsG → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (R : Rep k r) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (Wq : Type) [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [CompleteSpace Wq] [FiniteDimensional ℂ Wq] (D : B.RealPlaceData k r Wp Wq) (dd : ℝ), R.IsDG ε mu → (∀ w, w ≠ 0 → PI.BesselPeriodGGP.fdegH D.μG R.σ w = dd) → ∀ (x y x' y' : R.H) (X X' : Wq →L[ℂ] Wp) (p : B.SS k r), D.inSS p → Integrable (fun g : Formalism.QuantumDeterminants.Mt k r => B.hsInner (B.rho D p (fun g => ⟪x, R.σ g y⟫_ℂ • X) (Matrix.of g)) (⟪x', R.σ (Matrix.of g) y'⟫_ℂ • X')) D.μG ∧ B.Ph D (fun g => ⟪x, R.σ g y⟫_ℂ • X) (fun g => ⟪x', R.σ g y'⟫_ℂ • X') p = ⟪y', R.σ (JF.sR p) y⟫_ℂ * ⟪R.σ (JF.sL p) x, x'⟫_ℂ / (dd : ℂ) * B.hsInner ((D.π (Matrix.of p.1.1)).c

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.BesselFormalDegreePairing

private theorem **hsInner_add_left_theta_bessel_isometries** {Wp Wq : Type} [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wq] (A A' C : Wq →L[ℂ] Wp) : B.hsInner (A + A') C = B.hsInner A C + B.hsInner A' C

private theorem **hsInner_smul_lemma_2** {Wp Wq : Type} [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wq] (c : ℂ) (A C : Wq →L[ℂ] Wp) : B.hsInner (c • A) C = c * B.hsInner A C

theorem **besPhInt_bilin** {Wp Wq : Type} [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [FiniteDimensional ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wq] (X X' : Wq →L[ℂ] Wp) : ∃ T : (Wp →L[ℂ] Wp) →L[ℂ] (Wq →L[ℂ] Wq) →L[ℂ] ℂ, ∀ P Q, T P Q = B.hsInner (P.comp (X.comp Q)) X'

theorem **besPhInt_rank** {Wp Wq : Type} [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [FiniteDimensional ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wq] (X X' : Wq →L[ℂ] Wp) (a a' : Wp) (b b' : Wq) : B.hsInner (((innerSL ℂ a).smulRight a').comp (X.comp ((innerSL ℂ b').smulRight b))) X' = ⟪a, X b⟫_ℂ * star ⟪a', X' b'⟫_ℂ

theorem **besPhInt_xi** {k r : ℕ} {Wp Wq : Type} [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] (D : B.RealPlaceData k r Wp Wq) (p : B.SS k r) : B.xiTau D (1, 1) p = Formalism.QuantumDeterminants.psi k r D.z Formalism.QuantumDeterminants.cStd (Matrix.of p.1.2) * star (Formalism.QuantumDeterminants.psi k r D.z' Formalism.QuantumDeterminants.cStd (Matrix.of p.2.2))

private theorem **continuous_ell_matrix_of** {k r : ℕ} (z : Fin (2 * r + k + 1) → ℂ) : Continuous (fun q : Formalism.QuantumDeterminants.Mt k r => Formalism.QuantumDeterminants.ell k r z (Matrix.of q))

theorem **besPhInt_xi_cont** {k r : ℕ} {Wp Wq : Type} [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] (D : B.RealPlaceData k r Wp Wq) : Continuous (B.xiTau D (1, 1))

private theorem **isUnit_det_of_mem_UH_Hpk_of_inv_mem** {k r : ℕ} (hk : 1 ≤ k) {g : Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ} (hg : g ∈ Formalism.QuantumDeterminants.UH (Formalism.QuantumDeterminants.Hpk k r)) (hg' : g⁻¹ ∈ Formalism.QuantumDeterminants.UH (Formalism.QuantumDeterminants.Hpk k r)) : IsUnit g.det

private theorem **integrable_and_integral_prod_bilin_eq_apply_integrals** {α β E F G : Type*} [MeasurableSpace α] [MeasurableSpace β] {μ : Measure α} {ν : Measure β} [SFinite μ] [SFinite ν] [NormedAddCommGroup E] [NormedSpace ℂ E] [NormedAddCommGroup F] [NormedSpace ℂ F] [NormedAddCommGroup G] [NormedSpace ℂ G] [CompleteSpace E] [CompleteSpace F] [CompleteSpace G] (T : E →L[ℂ] F →L[ℂ] G) {u : α → E} {v : β → F} (hu : Integrable u μ) (hv : Integrable v ν) : Integrable (fun p : α × β => T (u p.1) (v p.2)) (μ.prod ν) ∧ ∫ p, T (u p.1) (v p.2) ∂(μ.prod ν) = T (∫ a, u a ∂μ) (∫ b, v b ∂ν)

theorem **integral_xi_mul_Ph** : HypsG → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (h : ℝ) (R : Rep k r) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (Wq : Type) [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [CompleteSpace Wq] [FiniteDimensional ℂ Wq] (D : B.RealPlaceData k r Wp Wq) (l : R.H → Wp) (l' : R.H → Wq) (dd : ℝ), R.IsDG ε mu → Formalism.QuantumDeterminants.GapAtLeast ε mu h → GapOK k r h → IsBeL R D.z D.π D.μW D.μN l → IsBeL R D.z' D.π' D.μW' D.μN l' → (∀ w, w ≠ 0 → PI.BesselPeriodGGP.fdegH D.μG R.σ w = dd) → ∀ x ∈ R.TK, ∀ y ∈ R.TK, ∀ x' ∈ R.TK, ∀ y' ∈ R.TK, ∀ X X' : Wq →L[ℂ] Wp, Integrable (B.Ph D (fun g => ⟪x, R.σ g y⟫_ℂ • X) (fun g => ⟪x', R.σ g y'⟫_ℂ • X')) D.μSS ∧ ∫ p, B.xiTau D (1, 1) p * B.Ph D (fun g => ⟪x, R.σ g y⟫_ℂ • X) (fun g => ⟪x', R.σ g y'⟫_ℂ • X') p ∂D.μSS = ⟪l x, X (l'

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.BesselFormalDegreePairingStatement

theorem **_root_.Gp91.GlobalTheory.T.bessel_formal_degree_pairing** : HypsG → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (h : ℝ) (R : Rep k r) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (Wq : Type) [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [CompleteSpace Wq] [FiniteDimensional ℂ Wq] (D : B.RealPlaceData k r Wp Wq) (l : R.H → Wp) (l' : R.H → Wq) (dd : ℝ), R.IsDG ε mu → Formalism.QuantumDeterminants.GapAtLeast ε mu h → GapOK k r h → IsBeL R D.z D.π D.μW D.μN l → IsBeL R D.z' D.π' D.μW' D.μN l' → (∀ w, w ≠ 0 → PI.BesselPeriodGGP.fdegH D.μG R.σ w = dd) → ∀ x ∈ R.TK, ∀ y ∈ R.TK, ∀ x' ∈ R.TK, ∀ y' ∈ R.TK, ∀ X X' : Wq →L[ℂ] Wp, Integrable (B.Ph D (fun g => ⟪x, R.σ g y⟫_ℂ • X) (fun g => ⟪x', R.σ g y'⟫_ℂ • X')) D.μSS ∧ ∫ p, B.xiTau D (1, 1) p * B.Ph D (fun g => ⟪x, R.σ g y⟫_ℂ • X) (fun g => ⟪x', R.σ

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.BesselFrameEntryRelation

theorem **bessel_dsig** (hK : PI.KFinSmooth) {k r : ℕ} {ε : Fin (2 * r + k + 1) → Bool} {mu : Fin (2 * r + k + 1) → ℝ} {R : Rep k r} (hR : R.IsDG ε mu) {Wp : Type} [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] {z : Fin (2 * r + k + 1) → ℂ} {π : Mx k r → (Wp →L[ℂ] Wp)} {l : R.H → Wp} (hz : Formalism.QuantumDeterminants.inWb k r z) (hl : IsBes R z π l) {X : Mx k r} (hX : Formalism.FourierDecayAlt.P.QrD.InLieW k r z X) (hX' : PI.UpqRep.InLieHpk k r X) {v : R.H} (hv : v ∈ R.sm) : l (PI.UpqRep.dsig (k

theorem **bessel_intertwines_glW** : PI.KFinSmooth → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (R : Rep k r), R.IsDG ε mu → ∀ (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (z : Fin (2 * r + k + 1) → ℂ) (π : Mx k r → (Wp →L[ℂ] Wp)) (l : R.H → Wp) (g : Mx k r), Formalism.QuantumDeterminants.inWb k r z → 0 < Formalism.QuantumDeterminants.Tz k r z → PI.UpqRep.IsFrameW k r z g → IsBes R z π l → ∀ i j : Fin k, ∀ v ∈ R.sm, l (R.dC (g * Matrix.single ((fun i : Fin k => ((r + i.val, by omega) : Fin (2 * r + k + 1))) i) ((fun i : Fin k => ((r + i.val, by omega) : Fin (2 * r + k + 1))) j) 1 * g⁻¹) v) = Formalism.FourierDecayAlt.Q.EcRep (Formalism.QuantumDeterminants.Hpk k r) π (g * Matrix.single ((fun i : Fin k => ((r + i.val, by omega) : Fin (2 * r + k + 1))) i) ((fun i : Fin k => ((r + i.val, by omega) : Fin

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.BesselFrameEntryRelationStatement

theorem **_root_.Gp91.GlobalTheory.T.bessel_frame_entry_relation** : PI.KFinSmooth → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (R : Rep k r), R.IsDG ε mu → ∀ (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (z : Fin (2 * r + k + 1) → ℂ) (π : Mx k r → (Wp →L[ℂ] Wp)) (l : R.H → Wp) (g : Mx k r), Formalism.QuantumDeterminants.inWb k r z → 0 < Formalism.QuantumDeterminants.Tz k r z → PI.UpqRep.IsFrameW k r z g → IsBes R z π l → ∀ i j : Fin k, ∀ v ∈ R.sm, l (R.dC (g * Matrix.single ((fun i : Fin k => ((r + i.val, by omega) : Fin (2 * r + k + 1))) i) ((fun i : Fin k => ((r + i.val, by omega) : Fin (2 * r + k + 1))) j) 1 * g⁻¹) v) = Formalism.FourierDecayAlt.Q.EcRep (Formalism.QuantumDeterminants.Hpk k r) π (g * Matrix.single ((fun i : Fin k => ((r + i.val, by omega) : Fin (2 * r + k + 1))) i) ((fun i : Fin k

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.BesselGapDimensionFormula

theorem **fdegHC_pos** {n : ℕ} (lam : Fin n → ℝ) (hlam : Function.Injective lam) : 0 < Formalism.QuantumDeterminants.fdegHC lam

theorem **quot_ae** {k r : ℕ} {μG μS νq : Measure (Formalism.QuantumDeterminants.Mt k r)} [SFinite μS] (hq : PI.BesselPeriodGGP.IsQuotMeas μG μS νq) (hG : ∀ᵐ x ∂μG, Matrix.of x ∈ GG k r) (hS : ∀ᵐ x ∂μS, Matrix.of x ∈ GG k r) (h0 : μS ≠ 0) : ∀ᵐ x ∂νq, Matrix.of x ∈ GG k r

theorem **piInv_contOn** {k r : ℕ} {Wp : Type} [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] {z : Fin (2 * r + k + 1) → ℂ} {u : Fin k → ℝ} {π : Mx k r → (Wp →L[ℂ] Wp)} (hT : 0 < Formalism.QuantumDeterminants.Tz k r z) (hπ : PI.UpqRep.IsHCTypeW k r z u π) (ξ : Wp) : ContinuousOn (fun p1 : Formalism.QuantumDeterminants.Mt k r => π (Matrix.of p1)⁻¹ ξ) {p1 : Formalism.QuantumDeterminants.Mt k r | Matrix.of p1 ∈ Formalism.QuantumDeterminants.UWset k r z}

theorem **good_meas** (k r : ℕ) (z : Fin (2 * r + k + 1) → ℂ) : MeasurableSet (good k r z)

theorem **kern_meas** {k r : ℕ} (R : Rep k r) {Wp : Type} [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] {z : Fin (2 * r + k + 1) → ℂ} {u : Fin k → ℝ} {π : Mx k r → (Wp →L[ℂ] Wp)} (hT : 0 < Formalism.QuantumDeterminants.Tz k r z) (hπ : PI.UpqRep.IsHCTypeW k r z u π) (v w : R.H) (ξ : Wp) (hc : ContinuousOn (fun g => R.σ g w) (GG k r)) : Measurable ((good k r z).indicator (kern R z π v w ξ))

theorem **norm_integral** (hy : HypsG) (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (h : ℝ) (R : Rep k r) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (z : Fin (2 * r + k + 1) → ℂ) (π : Mx k r → (Wp →L[ℂ] Wp)) (u : ℕ → ℝ) (μW μN μG νq : Measure (Formalism.QuantumDeterminants.Mt k r)) (hR : R.IsDG ε mu) (hgap : Formalism.QuantumDeterminants.GapAtLeast ε mu h) (hok : GapOK k r h) (hS : SideG k r z π u μW μN μG νq) (d : ℝ) (hSd : ∀ v w v' w' : R.H, ∫ x, ⟪w, R.σ (Matrix.of x) v⟫_ℂ * star ⟪w', R.σ (Matrix.of x) v'⟫_ℂ ∂μG = ⟪v', v⟫_ℂ * ⟪w, w'⟫_ℂ / (d : ℂ)) (v : R.H) (hv : v ∈ R.TK) (ξ : Wp) (w : R.H) (hw : w ∈ R.sm) : Integrable (fun x : Formalism.QuantumDeterminants.Mt k r => ‖be z Formalism.QuantumDeterminants.cStd π R.σ μW μN (R.σ (Matrix.of x) w) v ξ‖ ^ 2) νq ∧ ((∫ x : Formalism.QuantumDeterminants.Mt k r,

theorem **norm_identity** : HypsG → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (h : ℝ) (R : Rep k r) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (z : Fin (2 * r + k + 1) → ℂ) (π : Mx k r → (Wp →L[ℂ] Wp)) (u : ℕ → ℝ) (μW μN μG νq : Measure (Formalism.QuantumDeterminants.Mt k r)), R.IsDG ε mu → Formalism.QuantumDeterminants.GapAtLeast ε mu h → GapOK k r h → SideG k r z π u μW μN μG νq → ∀ dd : ℝ, (∀ w, w ≠ 0 → PI.BesselPeriodGGP.fdegH μG R.σ w = dd) → ∀ v ∈ R.TK, ∀ ξ : Wp, (⟪ξ, be z Formalism.QuantumDeterminants.cStd π R.σ μW μN v v ξ⟫_ℂ).im = 0 ∧ 0 ≤ (⟪ξ, be z Formalism.QuantumDeterminants.cStd π R.σ μW μN v v ξ⟫_ℂ).re ∧ 0 < dd ∧ ∀ w ∈ R.sm, PI.BesselPeriodGGP.l2SG νq (fun g => be z Formalism.QuantumDeterminants.cStd π R.σ μW μN (R.σ g w) v ξ) = ENNReal.ofReal (‖w‖ ^ 2 * (⟪ξ, be z Formalism.QuantumDe

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.BesselGapDimensionFormulaStatement

theorem **_root_.Gp91.GlobalTheory.T.bessel_integral_l2_norm_eq_div_formal_degree** : HypsG → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (h : ℝ) (R : Rep k r) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (z : Fin (2 * r + k + 1) → ℂ) (π : Mx k r → (Wp →L[ℂ] Wp)) (u : ℕ → ℝ) (μW μN μG νq : Measure (Formalism.QuantumDeterminants.Mt k r)), R.IsDG ε mu → Formalism.QuantumDeterminants.GapAtLeast ε mu h → GapOK k r h → SideG k r z π u μW μN μG νq → ∀ dd : ℝ, (∀ w, w ≠ 0 → PI.BesselPeriodGGP.fdegH μG R.σ w = dd) → ∀ v ∈ R.TK, ∀ ξ : Wp, (⟪ξ, be z Formalism.QuantumDeterminants.cStd π R.σ μW μN v v ξ⟫_ℂ).im = 0 ∧ 0 ≤ (⟪ξ, be z Formalism.QuantumDeterminants.cStd π R.σ μW μN v v ξ⟫_ℂ).re ∧ 0 < dd ∧ ∀ w ∈ R.sm, PI.BesselPeriodGGP.l2SG νq (fun g => be z Formalism.QuantumDeterminants.cStd π R.σ μW μN (R.σ g w) v ξ)

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.BesselGapExistsFunctional

theorem **eq_zero_of_rankOne** {W : Type} [NormedAddCommGroup W] [InnerProductSpace ℂ W] {a : W} (h : (innerSL ℂ a).smulRight a = 0) : a = 0

theorem **exists_normalised_bessel_map** : HypsG → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (h : ℝ) (R : Rep k r) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (z : Fin (2 * r + k + 1) → ℂ) (π : Mx k r → (Wp →L[ℂ] Wp)) (u : ℕ → ℝ) (μW μN μG νq : Measure (Formalism.QuantumDeterminants.Mt k r)), R.IsDG ε mu → Formalism.QuantumDeterminants.GapAtLeast ε mu h → GapOK k r h → SideG k r z π u μW μN μG νq → ∃ l, IsBeL R z π μW μN l

theorem **isBeL_of_normalised_on_orbitSpan** : HypsG → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (h : ℝ) (R : Rep k r) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (z : Fin (2 * r + k + 1) → ℂ) (π : Mx k r → (Wp →L[ℂ] Wp)) (u : ℕ → ℝ) (μW μN μG νq : Measure (Formalism.QuantumDeterminants.Mt k r)) (w₀ : R.H) (l : R.H → Wp), R.IsDG ε mu → Formalism.QuantumDeterminants.GapAtLeast ε mu h → GapOK k r h → SideG k r z π u μW μN μG νq → PI.UpqRep.IsLowUnitH k r ε mu R.σ w₀ → IsBes R z π l → (∀ x ∈ DDsp R.σ w₀, ∀ y ∈ DDsp R.σ w₀, be z Formalism.QuantumDeterminants.cStd π R.σ μW μN x y = (innerSL ℂ (l y)).smulRight (l x)) → IsBeL R z π μW μN l

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.BesselGapExistsFunctionalStatement

theorem **_root_.Gp91.GlobalTheory.T.bessel_gap_exists_functional** : HypsG → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (h : ℝ) (R : Rep k r) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (z : Fin (2 * r + k + 1) → ℂ) (π : Mx k r → (Wp →L[ℂ] Wp)) (u : ℕ → ℝ) (μW μN μG νq : Measure (Formalism.QuantumDeterminants.Mt k r)), R.IsDG ε mu → Formalism.QuantumDeterminants.GapAtLeast ε mu h → GapOK k r h → SideG k r z π u μW μN μG νq → ∃ l, IsBeL R z π μW μN l

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.BesselGapGrowthBound

theorem **integral_smul_comp_of_measurePreserving** {X Wp : Type} [MeasurableSpace X] [NormedAddCommGroup Wp] [NormedSpace ℂ Wp] [CompleteSpace Wp] {μ : Measure X} {T : X → X} (hT : MeasurePreserving T μ μ) {F F' : X → (Wp →L[ℂ] Wp)} (hF : Integrable F μ) (c : ℂ) (A : Wp →L[ℂ] Wp) (h : ∀ᵐ p ∂μ, F' p = c • A.comp (F (T p))) : ∫ p, F' p ∂μ = c • A.comp (∫ p, F p ∂μ)

theorem **be_equivariant** : HypsG → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (h : ℝ) (R : Rep k r) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (z : Fin (2 * r + k + 1) → ℂ) (π : Mx k r → (Wp →L[ℂ] Wp)) (u : ℕ → ℝ) (μW μN μG νq : Measure (Formalism.QuantumDeterminants.Mt k r)), R.IsDG ε mu → Formalism.QuantumDeterminants.GapAtLeast ε mu h → GapOK k r h → SideG k r z π u μW μN μG νq → ∀ v ∈ R.TK, ∀ w ∈ R.sm, ∀ g ∈ Formalism.QuantumDeterminants.UWset k r z, ∀ m ∈ Formalism.QuantumDeterminants.Nset k r, be z Formalism.QuantumDeterminants.cStd π R.σ μW μN (R.σ (g * m) w) v = Formalism.QuantumDeterminants.psi k r z Formalism.QuantumDeterminants.cStd m • (π g).comp (be z Formalism.QuantumDeterminants.cStd π R.σ μW μN w v)

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.BesselGapGrowthBoundStatement

theorem **_root_.Gp91.GlobalTheory.T.bessel_integral_equivariant_of_gap** : HypsG → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (h : ℝ) (R : Rep k r) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (z : Fin (2 * r + k + 1) → ℂ) (π : Mx k r → (Wp →L[ℂ] Wp)) (u : ℕ → ℝ) (μW μN μG νq : Measure (Formalism.QuantumDeterminants.Mt k r)), R.IsDG ε mu → Formalism.QuantumDeterminants.GapAtLeast ε mu h → GapOK k r h → SideG k r z π u μW μN μG νq → ∀ v ∈ R.TK, ∀ w ∈ R.sm, ∀ g ∈ Formalism.QuantumDeterminants.UWset k r z, ∀ m ∈ Formalism.QuantumDeterminants.Nset k r, be z Formalism.QuantumDeterminants.cStd π R.σ μW μN (R.σ (g * m) w) v = Formalism.QuantumDeterminants.psi k r z Formalism.QuantumDeterminants.cStd m • (π g).comp (be z Formalism.QuantumDeterminants.cStd π R.σ μW μN w v)

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.BesselGapIntegrability

theorem **be_apply_isBes** : HypsG → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (h : ℝ) (R : Rep k r) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (z : Fin (2 * r + k + 1) → ℂ) (π : Mx k r → (Wp →L[ℂ] Wp)) (u : ℕ → ℝ) (μW μN μG νq : Measure (Formalism.QuantumDeterminants.Mt k r)), R.IsDG ε mu → Formalism.QuantumDeterminants.GapAtLeast ε mu h → GapOK k r h → SideG k r z π u μW μN μG νq → ∀ v ∈ R.TK, ∀ ξ : Wp, IsBes R z π (fun w => be z Formalism.QuantumDeterminants.cStd π R.σ μW μN w v ξ)

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.BesselGapIntegrabilityStatement

theorem **_root_.Gp91.GlobalTheory.T.isBes_bessel_integral_of_gap** : HypsG → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (h : ℝ) (R : Rep k r) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (z : Fin (2 * r + k + 1) → ℂ) (π : Mx k r → (Wp →L[ℂ] Wp)) (u : ℕ → ℝ) (μW μN μG νq : Measure (Formalism.QuantumDeterminants.Mt k r)), R.IsDG ε mu → Formalism.QuantumDeterminants.GapAtLeast ε mu h → GapOK k r h → SideG k r z π u μW μN μG νq → ∀ v ∈ R.TK, ∀ ξ : Wp, IsBes R z π (fun w => be z Formalism.QuantumDeterminants.cStd π R.σ μW μN w v ξ)

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.BesselGapPairing

theorem **besselIntegral_sesquilinear** : HypsG → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (h : ℝ) (R : Rep k r) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (z : Fin (2 * r + k + 1) → ℂ) (π : Mx k r → (Wp →L[ℂ] Wp)) (u : ℕ → ℝ) (μW μN μG νq : Measure (Formalism.QuantumDeterminants.Mt k r)), R.IsDG ε mu → Formalism.QuantumDeterminants.GapAtLeast ε mu h → GapOK k r h → SideG k r z π u μW μN μG νq → (∀ v ∈ R.TK, ∀ w ∈ R.sm, ∀ w' ∈ R.sm, ∀ a : ℂ, be z Formalism.QuantumDeterminants.cStd π R.σ μW μN (w + w') v = be z Formalism.QuantumDeterminants.cStd π R.σ μW μN w v + be z Formalism.QuantumDeterminants.cStd π R.σ μW μN w' v ∧ be z Formalism.QuantumDeterminants.cStd π R.σ μW μN (a • w) v = a • be z Formalism.QuantumDeterminants.cStd π R.σ μW μN w v) ∧ ∀ w ∈ R.sm, ∀ v ∈ R.TK, ∀ v' ∈ R.TK, ∀ a : ℂ, be z F

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.BesselGapPairingStatement

theorem **_root_.Gp91.GlobalTheory.T.bessel_gap_pairing** : HypsG → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (h : ℝ) (R : Rep k r) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (z : Fin (2 * r + k + 1) → ℂ) (π : Mx k r → (Wp →L[ℂ] Wp)) (u : ℕ → ℝ) (μW μN μG νq : Measure (Formalism.QuantumDeterminants.Mt k r)), R.IsDG ε mu → Formalism.QuantumDeterminants.GapAtLeast ε mu h → GapOK k r h → SideG k r z π u μW μN μG νq → (∀ v ∈ R.TK, ∀ w ∈ R.sm, ∀ w' ∈ R.sm, ∀ a : ℂ, be z Formalism.QuantumDeterminants.cStd π R.σ μW μN (w + w') v = be z Formalism.QuantumDeterminants.cStd π R.σ μW μN w v + be z Formalism.QuantumDeterminants.cStd π R.σ μW μN w' v ∧ be z Formalism.QuantumDeterminants.cStd π R.σ μW μN (a • w) v = a • be z Formalism.QuantumDeterminants.cStd π R.σ μW μN w v) ∧ ∀ w ∈ R.sm, ∀ v ∈ R.TK, ∀ v' ∈ R.T

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.BesselGapVanishing

theorem **ae_mem_prod** {z : Fin (2 * r + k + 1) → ℂ} {μW μN : Measure (Formalism.QuantumDeterminants.Mt k r)} (hW : PI.BesselPeriodGGP.IsHaarOn (Formalism.QuantumDeterminants.UWset k r z) μW) (hN : PI.BesselPeriodGGP.IsHaarOn (Formalism.QuantumDeterminants.Nset k r) μN) : ∀ᵐ p ∂(μW.prod μN), Matrix.of p.1 ∈ Formalism.QuantumDeterminants.UWset k r z ∧ Matrix.of p.2 ∈ Formalism.QuantumDeterminants.Nset k r

theorem **pi_inv_contOn** [FiniteDimensional ℂ Wp] {z : Fin (2 * r + k + 1) → ℂ} {u : Fin k → ℝ} {π : Mx k r → (Wp →L[ℂ] Wp)} (hT : 0 < Formalism.QuantumDeterminants.Tz k r z) (hπ : PI.UpqRep.IsHCTypeW k r z u π) : ContinuousOn (fun p1 : Formalism.QuantumDeterminants.Mt k r => π (Matrix.of p1)⁻¹) {p1 : Formalism.QuantumDeterminants.Mt k r | Matrix.of p1 ∈ Formalism.QuantumDeterminants.UWset k r z}

theorem **beInt_aesm** [FiniteDimensional ℂ Wp] {ε : Fin (2 * r + k + 1) → Bool} {lam : Fin (2 * r + k + 1) → ℝ} {σ : Mx k r → (H →L[ℂ] H)} (hσ : PI.UpqRep.IsDSHpk k r ε lam σ) {z : Fin (2 * r + k + 1) → ℂ} (c : ℂ) {u : Fin k → ℝ} {π : Mx k r → (Wp →L[ℂ] Wp)} (hT : 0 < Formalism.QuantumDeterminants.Tz k r z) (hπ : PI.UpqRep.IsHCTypeW k r z u π) {μW μN : Measure (Formalism.QuantumDeterminants.Mt k r)} (hW : PI.BesselPeriodGGP.IsHaarOn (Formalism.QuantumDeterminants.UWset k r z) μW) (hN : PI.BesselPeriodGGP.IsHaarOn (Formalism.QuantumDeterminants.Nset k r) μN) (w v : H) : AEStronglyMeasurable (beInt z c π σ w v) (μW.prod μN)

theorem **norm_beInt_le** (σ : Mx k r → (H →L[ℂ] H)) {z : Fin (2 * r + k + 1) → ℂ} (c : ℂ) {u : Fin k → ℝ} {π : Mx k r → (Wp →L[ℂ] Wp)} (hT : 0 < Formalism.QuantumDeterminants.Tz k r z) (hπ : PI.UpqRep.IsHCTypeW k r z u π) (w v : H) (p : Formalism.QuantumDeterminants.Mt k r × Formalism.QuantumDeterminants.Mt k r) (hp : Matrix.of p.1 ∈ Formalism.QuantumDeterminants.UWset k r z) : ‖beInt z c π σ w v p‖ ≤ ‖⟪v, σ (Matrix.of p.1 * Matrix.of p.2) w⟫_ℂ‖

theorem **be_conv_of_growth** (hG : GrowthS) : HypsG → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (h : ℝ) (R : Rep k r) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (z : Fin (2 * r + k + 1) → ℂ) (π : Mx k r → (Wp →L[ℂ] Wp)) (u : ℕ → ℝ) (μW μN μG νq : Measure (Formalism.QuantumDeterminants.Mt k r)), R.IsDG ε mu → Formalism.QuantumDeterminants.GapAtLeast ε mu h → GapOK k r h → SideG k r z π u μW μN μG νq → ∀ v ∈ R.TK, (∀ w ∈ R.sm, Integrable (beInt z Formalism.QuantumDeterminants.cStd π R.σ w v) (μW.prod μN)) ∧ PI.BesselMapUniq.IsCWCont (k

theorem **be_conv** : HypsG → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (h : ℝ) (R : Rep k r) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (z : Fin (2 * r + k + 1) → ℂ) (π : Mx k r → (Wp →L[ℂ] Wp)) (u : ℕ → ℝ) (μW μN μG νq : Measure (Formalism.QuantumDeterminants.Mt k r)), R.IsDG ε mu → Formalism.QuantumDeterminants.GapAtLeast ε mu h → GapOK k r h → SideG k r z π u μW μN μG νq → ∀ v ∈ R.TK, (∀ w ∈ R.sm, Integrable (beInt z Formalism.QuantumDeterminants.cStd π R.σ w v) (μW.prod μN)) ∧ PI.BesselMapUniq.IsCWCont (k

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.BesselGapVanishingStatement

theorem **_root_.Gp91.GlobalTheory.T.besselIntegral_integrable_and_cwContinuous_of_gap** : HypsG → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (h : ℝ) (R : Rep k r) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (z : Fin (2 * r + k + 1) → ℂ) (π : Mx k r → (Wp →L[ℂ] Wp)) (u : ℕ → ℝ) (μW μN μG νq : Measure (Formalism.QuantumDeterminants.Mt k r)), R.IsDG ε mu → Formalism.QuantumDeterminants.GapAtLeast ε mu h → GapOK k r h → SideG k r z π u μW μN μG νq → ∀ v ∈ R.TK, (∀ w ∈ R.sm, Integrable (beInt z Formalism.QuantumDeterminants.cStd π R.σ w v) (μW.prod μN)) ∧ PI.BesselMapUniq.IsCWCont (k

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.BesselLieTriangularVanishing

theorem **H_ne** {i j : Fin (2 * r + k + 1)} (h : Formalism.QuantumDeterminants.Hpk k r i j ≠ 0) : (i.val < r ∧ i.val + j.val = 2 * r + k) ∨ (j.val < r ∧ i.val + j.val = 2 * r + k) ∨ (r ≤ i.val ∧ i.val ≤ r + k ∧ i.val = j.val)

theorem **H_herm** : (Formalism.QuantumDeterminants.Hpk k r)ᴴ = Formalism.QuantumDeterminants.Hpk k r

theorem **H_sq** : Formalism.QuantumDeterminants.Hpk k r * Formalism.QuantumDeterminants.Hpk k r = 1

theorem **pat_theta** {Y : Mx k r} (hY : Pat Y) : Pat (Formalism.QuantumDeterminants.Hpk k r * Yᴴ * Formalism.QuantumDeterminants.Hpk k r)

theorem **ell_add** (z : Fin (2 * r + k + 1) → ℂ) (A B : Mx k r) : Formalism.QuantumDeterminants.ell k r z (A + B) = Formalism.QuantumDeterminants.ell k r z A + Formalism.QuantumDeterminants.ell k r z B

theorem **ell_smul** (z : Fin (2 * r + k + 1) → ℂ) (c : ℂ) (A : Mx k r) : Formalism.QuantumDeterminants.ell k r z (c • A) = c * Formalism.QuantumDeterminants.ell k r z A

theorem **ell_sub** (z : Fin (2 * r + k + 1) → ℂ) (A B : Mx k r) : Formalism.QuantumDeterminants.ell k r z (A - B) = Formalism.QuantumDeterminants.ell k r z A - Formalism.QuantumDeterminants.ell k r z B

theorem **theta_key** (Y : Mx k r) : (Formalism.QuantumDeterminants.Hpk k r * Yᴴ * Formalism.QuantumDeterminants.Hpk k r)ᴴ * Formalism.QuantumDeterminants.Hpk k r = Formalism.QuantumDeterminants.Hpk k r * Y ∧ Formalism.QuantumDeterminants.Hpk k r * (Formalism.QuantumDeterminants.Hpk k r * Yᴴ * Formalism.QuantumDeterminants.Hpk k r) = Yᴴ * Formalism.QuantumDeterminants.Hpk k r

theorem **lie_re** (Y : Mx k r) : PI.UpqRep.InLieHpk k r ((1 / 2 : ℂ) • (Y + -(Formalism.QuantumDeterminants.Hpk k r * Yᴴ * Formalism.QuantumDeterminants.Hpk k r)))

theorem **lie_im** (Y : Mx k r) : PI.UpqRep.InLieHpk k r ((1 / (2 * Complex.I)) • (Y - -(Formalism.QuantumDeterminants.Hpk k r * Yᴴ * Formalism.QuantumDeterminants.Hpk k r)))

theorem **on_real** (hK : PI.KFinSmooth) {ε : Fin (2 * r + k + 1) → Bool} {mu : Fin (2 * r + k + 1) → ℝ} {R : Rep k r} (hR : R.IsDG ε mu) {Wp : Type} [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] {z : Fin (2 * r + k + 1) → ℂ} {π : Mx k r → (Wp →L[ℂ] Wp)} {l : R.H → Wp} (hz : Formalism.QuantumDeterminants.inWb k r z) (hπ : ∀ x, π 1 x = x) (hl : IsBes R z π l) {X : Mx k r} (hX : PI.UpqRep.InLieHpk k r X) (hP : Pat X) {v : R.H} (hv : v ∈ R.sm) : l (PI.UpqRep.dsig (k

theorem **bessel_dC** : PI.KFinSmooth → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (R : Rep k r), R.IsDG ε mu → ∀ (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (z : Fin (2 * r + k + 1) → ℂ) (π : Mx k r → (Wp →L[ℂ] Wp)) (l : R.H → Wp), Formalism.QuantumDeterminants.inWb k r z → (∀ x, π 1 x = x) → IsBes R z π l → ∀ Y : Mx k r, (∀ i j : Fin (2 * r + k + 1), ¬ (i < j ∧ ¬ (r ≤ i.val ∧ j.val ≤ r + k)) → Y i j = 0) → ∀ v ∈ R.sm, l (R.dC Y v) = (Formalism.QuantumDeterminants.ell k r z Y - star (Formalism.QuantumDeterminants.ell k r z (-((Formalism.QuantumDeterminants.Hpk k r) * Yᴴ * (Formalism.QuantumDeterminants.Hpk k r))))) • l v

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.BesselLieTriangularVanishingStatement

theorem **_root_.Gp91.GlobalTheory.T.bessel_functional_lie_triangular_eq_character_smul** : PI.KFinSmooth → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (R : Rep k r), R.IsDG ε mu → ∀ (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (z : Fin (2 * r + k + 1) → ℂ) (π : Mx k r → (Wp →L[ℂ] Wp)) (l : R.H → Wp), Formalism.QuantumDeterminants.inWb k r z → (∀ x, π 1 x = x) → IsBes R z π l → ∀ Y : Mx k r, (∀ i j : Fin (2 * r + k + 1), ¬ (i < j ∧ ¬ (r ≤ i.val ∧ j.val ≤ r + k)) → Y i j = 0) → ∀ v ∈ R.sm, l (R.dC Y v) = (Formalism.QuantumDeterminants.ell k r z Y - star (Formalism.QuantumDeterminants.ell k r z (-((Formalism.QuantumDeterminants.Hpk k r) * Yᴴ * (Formalism.QuantumDeterminants.Hpk k r))))) • l v

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.BesselPeriodAdjointPair

theorem **dLR_zero** (u v : List (B.Mat k r)) : B.dLR u v (0 : B.Mat k r → E) = 0

theorem **sum_mem** {ι : Type} (A : ℝ) (f : ι → B.Mat k r → E) (t : Finset ι) (hf : ∀ i ∈ t, B.CkOn ⊤ (B.GV k r) (f i) ∧ ∀ m : ℕ, B.InCC A m (f i)) : B.CkOn ⊤ (B.GV k r) (∑ i ∈ t, f i) ∧ ∀ m : ℕ, B.InCC A m (∑ i ∈ t, f i)

theorem **isCoef_ckOn_inCC** : HypsG → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (h : ℝ) (R : Rep k r) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (Wq : Type) [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [CompleteSpace Wq] [FiniteDimensional ℂ Wq] (l : R.H → Wp) (l' : R.H → Wq) (F : Mx k r → (Wq →L[ℂ] Wp)) (s : ℂ), R.IsDG ε mu → Formalism.QuantumDeterminants.GapAtLeast ε mu h → GapOK k r h → IsCoef R l l' F s → B.CkOn ⊤ (B.GV k r) F ∧ ∀ m : ℕ, B.InCC (aOf r h - 1) m F

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.BesselPeriodAdjointPairStatement

theorem **_root_.Gp91.GlobalTheory.T.smooth_and_inCC_of_isCoef_of_gap** : HypsG → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (h : ℝ) (R : Rep k r) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (Wq : Type) [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [CompleteSpace Wq] [FiniteDimensional ℂ Wq] (l : R.H → Wp) (l' : R.H → Wq) (F : Mx k r → (Wq →L[ℂ] Wp)) (s : ℂ), R.IsDG ε mu → Formalism.QuantumDeterminants.GapAtLeast ε mu h → GapOK k r h → IsCoef R l l' F s → B.CkOn ⊤ (B.GV k r) F ∧ ∀ m : ℕ, B.InCC (aOf r h - 1) m F

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.BesselSettingGapBound

theorem **sigmaFinite_of_haar** {k r : ℕ} {U : Set (B.Mat k r)} {μ : Measure (Formalism.QuantumDeterminants.Mt k r)} (h : B.IsHaarOn U μ) : SigmaFinite μ

theorem **left_integral** {k r : ℕ} (R : Rep k r) {Wp : Type} [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] (z : Fin (2 * r + k + 1) → ℂ) (π : Mx k r → (Wp →L[ℂ] Wp)) (μW μN : Measure (Formalism.QuantumDeterminants.Mt k r)) (l : R.H → Wp) (hk : 1 ≤ k) (hz : Formalism.QuantumDeterminants.inWb k r z) (hT : 0 < Formalism.QuantumDeterminants.Tz k r z) (hW : B.IsHaarOn (Formalism.QuantumDeterminants.UWset k r z) μW) (hN : B.IsHaarOn (Formalism.QuantumDeterminants.Nset k r) μN) (hl : IsBeL R z π μW μN l) {Y x : R.H} (hY : Y ∈ R.sm) (hx : x ∈ R.TK) : Integrable (fun p : Formalism.QuantumDeterminants.Mt k r × Formalism.QuantumDeterminants.Mt k r => (Formalism.QuantumDeterminants.psi k r z Formalism.QuantumDeterminants.cStd (Matrix.of p.2) * ⟪x, R.σ ((Matrix.of p.1)⁻¹ * (Matrix.of p.2)⁻¹) Y⟫_ℂ) • π (Matrix.of p.1)) (μW.prod μN) ∧ ∫ p : Formalism.QuantumDeterminants.Mt k r × Form

theorem **bessel_integrate_left** : HypsG → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (h : ℝ) (R : Rep k r) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (Wq : Type) [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [CompleteSpace Wq] [FiniteDimensional ℂ Wq] (D : B.RealPlaceData k r Wp Wq) (l : R.H → Wp) (l' : R.H → Wq), R.IsDG ε mu → Formalism.QuantumDeterminants.GapAtLeast ε mu h → GapOK k r h → IsBeL R D.z D.π D.μW D.μN l → IsBeL R D.z' D.π' D.μW' D.μN l' → ∀ γ ∈ JF.Om k r D.z D.z', ∃ A : R.H → Wq → Wq, (∀ y a c (t : ℂ), A y (t • a) c = star t • A y a c) ∧ ∀ x ∈ R.TK, ∀ y ∈ R.TK, ∀ (X : Wq →L[ℂ] Wp) (c : Wq), B.JF D (fun g => ⟪x, R.σ g y⟫_ℂ • X) γ c = A y (ContinuousLinearMap.adjoint X (l x)) c

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.BesselSettingGapBoundDual

theorem **besselInner_haar** {k r : ℕ} {U : Set (Mx k r)} {μ : Measure (Formalism.QuantumDeterminants.Mt k r)} (h : B.IsHaarOn U μ) : PI.BesselPeriodGGP.IsHaarOn U μ

theorem **besselInner_integral** {k r : ℕ} (R : Rep k r) {Wp : Type} [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] {Wq : Type} [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [CompleteSpace Wq] (D : B.RealPlaceData k r Wp Wq) (hmul : ∀ g ∈ GG k r, ∀ h ∈ GG k r, ∀ v, R.σ (g * h) v = R.σ g (R.σ h v)) (huni : ∀ g ∈ GG k r, ∀ v w, ⟪R.σ g v, R.σ g w⟫_ℂ = ⟪v, w⟫_ℂ) {γ : Mx k r} (hγ : γ ∈ GG k r) (x y : R.H) (X : Wq →L[ℂ] Wp) (a : Formalism.QuantumDeterminants.Mt k r × Formalism.QuantumDeterminants.Mt k r) (ha1 : Matrix.of a.1 ∈ GG k r) (ha2 : Matrix.of a.2 ∈ GG k r) (hint : Integrable (beInt D.z' Formalism.QuantumDeterminants.cStd D.π' R.σ y (R.σ (γ⁻¹ * Matrix.of a.2 * Matrix.of a.1) x)) (D.μW'.prod D.μN)) : ∫ b, JF.integrand D (fun g => ⟪x, R.σ g y⟫_ℂ • X) γ (a, b) ∂(D.μW'.prod D.μN) = Formalism.QuantumDeterminants.psi k r D.z Formalism.QuantumDeterminants.cStd (Matrix.of a

theorem **besselJ_coef_through_right** : HypsG → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (h : ℝ) (R : Rep k r) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (Wq : Type) [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [CompleteSpace Wq] [FiniteDimensional ℂ Wq] (D : B.RealPlaceData k r Wp Wq) (l : R.H → Wp) (l' : R.H → Wq), R.IsDG ε mu → Formalism.QuantumDeterminants.GapAtLeast ε mu h → GapOK k r h → IsBeL R D.z D.π D.μW D.μN l → IsBeL R D.z' D.π' D.μW' D.μN l' → ∀ γ ∈ JF.Om k r D.z D.z', ∃ Bb : R.H → Wp → Wq → Wp, (∀ x b c (t : ℂ), Bb x (t • b) c = t • Bb x b c) ∧ ∀ x ∈ R.TK, ∀ y ∈ R.TK, ∀ (X : Wq →L[ℂ] Wp) (c : Wq), B.JF D (fun g => ⟪x, R.σ g y⟫_ℂ • X) γ c = Bb x (X (l' y)) c

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.BesselSettingGapBoundDualStatement

theorem **_root_.Gp91.GlobalTheory.T.bessel_orbital_integral_factors_through_right_functional** : HypsG → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (h : ℝ) (R : Rep k r) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (Wq : Type) [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [CompleteSpace Wq] [FiniteDimensional ℂ Wq] (D : B.RealPlaceData k r Wp Wq) (l : R.H → Wp) (l' : R.H → Wq), R.IsDG ε mu → Formalism.QuantumDeterminants.GapAtLeast ε mu h → GapOK k r h → IsBeL R D.z D.π D.μW D.μN l → IsBeL R D.z' D.π' D.μW' D.μN l' → ∀ γ ∈ JF.Om k r D.z D.z', ∃ Bb : R.H → Wp → Wq → Wp, (∀ x b c (t : ℂ), Bb x (t • b) c = t • Bb x b c) ∧ ∀ x ∈ R.TK, ∀ y ∈ R.TK, ∀ (X : Wq →L[ℂ] Wp) (c : Wq), B.JF D (fun g => ⟪x, R.σ g y⟫_ℂ • X) γ c = Bb x (X (l' y)) c

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.BesselSettingGapBoundStatement

theorem **_root_.Gp91.GlobalTheory.T.bessel_orbital_integral_factors_through_left_functional** : HypsG → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (h : ℝ) (R : Rep k r) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (Wq : Type) [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [CompleteSpace Wq] [FiniteDimensional ℂ Wq] (D : B.RealPlaceData k r Wp Wq) (l : R.H → Wp) (l' : R.H → Wq), R.IsDG ε mu → Formalism.QuantumDeterminants.GapAtLeast ε mu h → GapOK k r h → IsBeL R D.z D.π D.μW D.μN l → IsBeL R D.z' D.π' D.μW' D.μN l' → ∀ γ ∈ JF.Om k r D.z D.z', ∃ A : R.H → Wq → Wq → Wp, (∀ y a c (t : ℂ), A y (t • a) c = star t • A y a c) ∧ ∀ x ∈ R.TK, ∀ y ∈ R.TK, ∀ (X : Wq →L[ℂ] Wp) (c : Wq), B.JF D (fun g => ⟪x, R.σ g y⟫_ℂ • X) γ c = A y (ContinuousLinearMap.adjoint X (l x)) c

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.BesselSettingGapDecay

theorem **bessel_integral_eq_smul** : HypsG → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (h : ℝ) (R : Rep k r) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (Wq : Type) [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [CompleteSpace Wq] [FiniteDimensional ℂ Wq] (D : B.RealPlaceData k r Wp Wq) (l : R.H → Wp) (l' : R.H → Wq), R.IsDG ε mu → Formalism.QuantumDeterminants.GapAtLeast ε mu h → GapOK k r h → IsBeL R D.z D.π D.μW D.μN l → IsBeL R D.z' D.π' D.μW' D.μN l' → ∃ OO : Mx k r → (Wq →L[ℂ] Wp), ∀ F s, IsCoef R l l' F s → ∀ γ ∈ JF.Om k r D.z D.z', Integrable (JF.integrand D F γ) D.μSS ∧ B.JF D F γ = s • OO γ

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.BesselSettingGapDecayStatement

theorem **_root_.Gp91.GlobalTheory.T.bessel_orbital_integral_integrable_and_eq_smul** : HypsG → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (h : ℝ) (R : Rep k r) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (Wq : Type) [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [CompleteSpace Wq] [FiniteDimensional ℂ Wq] (D : B.RealPlaceData k r Wp Wq) (l : R.H → Wp) (l' : R.H → Wq), R.IsDG ε mu → Formalism.QuantumDeterminants.GapAtLeast ε mu h → GapOK k r h → IsBeL R D.z D.π D.μW D.μN l → IsBeL R D.z' D.π' D.μW' D.μN l' → ∃ OO : Mx k r → (Wq →L[ℂ] Wp), ∀ F s, IsCoef R l l' F s → ∀ γ ∈ JF.Om k r D.z D.z', Integrable (JF.integrand D F γ) D.μSS ∧ B.JF D F γ = s • OO γ

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.BesselSettingUniformConstant

theorem **hsInner_sum_sum** {m : ℕ} (A C : Fin m → (Wq →L[ℂ] Wp)) : B.hsInner (∑ i, A i) (∑ j, C j) = ∑ i, ∑ j, B.hsInner (A i) (C j)

theorem **Ph_sum** [FiniteDimensional ℂ Wp] (D : B.RealPlaceData k r Wp Wq) {m : ℕ} (F : B.Mat k r → (Wq →L[ℂ] Wp)) (f : Fin m → B.Mat k r → (Wq →L[ℂ] Wp)) (hF : ∀ g ∈ B.GV k r, F g = ∑ i, f i g) (p : B.SS k r) (hp : D.inSS p) (hint : ∀ i j, Integrable (fun g : Formalism.QuantumDeterminants.Mt k r => B.hsInner (B.rho D p (f i) (Matrix.of g)) (f j (Matrix.of g))) D.μG) : B.Ph D F F p = ∑ i, ∑ j, B.Ph D (f i) (f j) p

theorem **normSq_coef** : HypsG → ∀ (k r : ℕ) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (Wq : Type) [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [CompleteSpace Wq] [FiniteDimensional ℂ Wq] (D : B.RealPlaceData k r Wp Wq), ∃ c : ℝ, 0 < c ∧ ∀ (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (h : ℝ) (R : Rep k r) (l : R.H → Wp) (l' : R.H → Wq) (dd : ℝ), R.IsDG ε mu → Formalism.QuantumDeterminants.GapAtLeast ε mu h → GapOK k r h → IsBeL R D.z D.π D.μW D.μN l → IsBeL R D.z' D.π' D.μW' D.μN l' → (∀ w, w ≠ 0 → PI.BesselPeriodGGP.fdegH D.μG R.σ w = dd) → ∀ F s, IsCoef R l l' F s → B.normSq D F = ENNReal.ofReal (c * ‖s‖ ^ 2 / dd)

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.BesselSettingUniformConstantStatement

theorem **_root_.Gp91.GlobalTheory.T.bessel_setting_uniform_constant** : HypsG → ∀ (k r : ℕ) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (Wq : Type) [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [CompleteSpace Wq] [FiniteDimensional ℂ Wq] (D : B.RealPlaceData k r Wp Wq), ∃ c : ℝ, 0 < c ∧ ∀ (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (h : ℝ) (R : Rep k r) (l : R.H → Wp) (l' : R.H → Wq) (dd : ℝ), R.IsDG ε mu → Formalism.QuantumDeterminants.GapAtLeast ε mu h → GapOK k r h → IsBeL R D.z D.π D.μW D.μN l → IsBeL R D.z' D.π' D.μW' D.μN l' → (∀ w, w ≠ 0 → PI.BesselPeriodGGP.fdegH D.μG R.σ w = dd) → ∀ F s, IsCoef R l l' F s → B.normSq D F = ENNReal.ofReal (c * ‖s‖ ^ 2 / dd)

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.BesselSumDecomposition

theorem **dsigW_zero** (R : Rep k r) (W : List (Mx k r)) : PI.UpqRep.dsigW (k

theorem **smMap_zero** (R R' : Rep k r) : SmMap R R' (fun _ => 0) where

theorem **smMap_congr** {R R' : Rep k r} {T T' : R.H → R'.H} (h : SmMap R R' T) (e : ∀ v ∈ R.sm, T' v = T v) : SmMap R R' T' where

theorem **smMap_sum** {R R' : Rep k r} (hl : ∀ (T T' : R.H → R'.H) (a : ℂ), SmMap R R' T → SmMap R R' T' → SmMap R R' (fun v => a • T v + T' v)) {ι : Type} (s : Finset ι) (a : ι → ℂ) (T : ι → R.H → R'.H) (hT : ∀ i, SmMap R R' (T i)) : SmMap R R' (fun v => ∑ i ∈ s, a i • T i v)

theorem **mem_GG_of_mem_Sset** {z : Fin (2 * r + k + 1) → ℂ} {s : Mx k r} (hs : s ∈ Formalism.QuantumDeterminants.Sset k r z) : s ∈ GG k r

theorem **ev_eq** {ι : Type} [Fintype ι] (Γ : Matrix ι ι ℂ) (R : Rep k r) (a : ι → ℂ) (x : ι → R.H) : ev Γ R a x = ∑ j, (star a ᵥ* Γ) j • x j

theorem **vecMul_inv** {ι : Type} [Fintype ι] (Γ A : Matrix ι ι ℂ) (a : ι → ℂ) (hA : Aᴴ * Γ * A = Γ) (ha : A *ᵥ a = a) : (star a ᵥ* Γ) ᵥ* A = star a ᵥ* Γ

theorem **main** : PI.KFinSmooth → ∀ (k r : ℕ) (ι κ : Type) [Fintype ι] [DecidableEq ι] [Fintype κ] [DecidableEq κ] (ρ : Mx k r → Matrix ι ι ℂ) (Γ : Matrix ι ι ℂ) (d : ℕ) (R : Rep k r) (RI : κ → Rep k r) (io : ∀ I, ι → (RI I).H → R.H) (pr : ∀ I, ι → R.H → (RI I).H) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (muI : κ → Fin (2 * r + k + 1) → ℝ) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (z : Fin (2 * r + k + 1) → ℂ) (π : Mx k r → (Wp →L[ℂ] Wp)) (om : ι → ℂ), R.IsDG ε mu → (∀ I, (RI I).IsDG ε (muI I)) → IsFRep ρ Γ d → TensRaw ρ R RI io pr → (∀ s ∈ Formalism.QuantumDeterminants.Sset k r z, ρ s *ᵥ om = om) → ∀ I, (∀ lI, IsBes (RI I) z π lI → IsBes R z π (fun v => lI (∑ c, pr I c (tm R om v c)))) ∧ (∀ s ∈ Formalism.QuantumDeterminants.Sset k r z, ∀ y ∈ (RI I).sm, ev Γ R om (fun c => io I c ((RI I).σ s y)) = R.σ s (e

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.BesselSumDecompositionStatement

theorem **_root_.Gp91.GlobalTheory.T.isBes_tensor_projection_and_ev_equivariant_smooth** : PI.KFinSmooth → ∀ (k r : ℕ) (ι κ : Type) [Fintype ι] [DecidableEq ι] [Fintype κ] [DecidableEq κ] (ρ : Mx k r → Matrix ι ι ℂ) (Γ : Matrix ι ι ℂ) (d : ℕ) (R : Rep k r) (RI : κ → Rep k r) (io : ∀ I, ι → (RI I).H → R.H) (pr : ∀ I, ι → R.H → (RI I).H) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (muI : κ → Fin (2 * r + k + 1) → ℝ) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (z : Fin (2 * r + k + 1) → ℂ) (π : Mx k r → (Wp →L[ℂ] Wp)) (om : ι → ℂ), R.IsDG ε mu → (∀ I, (RI I).IsDG ε (muI I)) → IsFRep ρ Γ d → TensRaw ρ R RI io pr → (∀ s ∈ Formalism.QuantumDeterminants.Sset k r z, ρ s *ᵥ om = om) → ∀ I, (∀ lI, IsBes (RI I) z π lI → IsBes R z π (fun v => lI (∑ c, pr I c (tm R om v c)))) ∧ (∀ s ∈ Formalism.QuantumDeterminants.Sset k r z

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.CapelliKFiniteCoefficient

theorem **comp_eq_sum_inner_smul_rankOne** {H Wp Wq : Type} [NormedAddCommGroup H] [InnerProductSpace ℂ H] [CompleteSpace H] [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] {ι κ : Type} [Fintype ι] [Fintype κ] (b : OrthonormalBasis ι ℂ Wp) (c : OrthonormalBasis κ ℂ Wq) (P : H →L[ℂ] Wp) (A : H →L[ℂ] H) (Q : Wq →L[ℂ] H) : P.comp (A.comp Q) = ∑ p : ι × κ, ⟪ContinuousLinearMap.adjoint P (b p.1), A (Q (c p.2))⟫_ℂ • (innerSL ℂ (c p.2)).smulRight (b p.1)

theorem **isCoef_of_translate_smul_exists** : PI.KFinSmooth → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (R : Rep k r) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (Wq : Type) [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [CompleteSpace Wq] [FiniteDimensional ℂ Wq] (l : R.H → Wp) (l' : R.H → Wq), R.IsDG ε mu → (∀ f : Mx k r → (Wq →L[ℂ] Wp), PI.CapelliEig.IsKFinCoefT (k

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.CayleyEndomorphismsModular

theorem **isMod_smooth_of_isDS** : HypsG → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (R : Rep k r), 1 ≤ k → R.IsDG ε mu → ∀ (Q : Mx k r) (E : (Fin (2 * r + k + 1) → Fin (2 * r + k + 1) → Module.End ℂ R.smSub)), PI.UpqRep.IsCayleyU k r ε Q → R.IsE Q E → IsMod ε E mu

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.CayleyEndomorphismsModularStatement

theorem **_root_.Gp91.GlobalTheory.T.cayley_endomorphisms_modular** : HypsG → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (R : Rep k r), 1 ≤ k → R.IsDG ε mu → ∀ (Q : Mx k r) (E : (Fin (2 * r + k + 1) → Fin (2 * r + k + 1) → Module.End ℂ R.smSub)), PI.UpqRep.IsCayleyU k r ε Q → R.IsE Q E → IsMod ε E mu

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.CayleyEndomorphismsUnitary

theorem **diag_single** {n : ℕ} (ε : Fin n → Bool) (a b : Fin n) : PI.UpqRep.Jm ε * Matrix.single b a (1 : ℂ) * PI.UpqRep.Jm ε = ((Formalism.REAL.sgn ε a b : ℝ) : ℂ) • Matrix.single b a (1 : ℂ)

theorem **adj_unit** {n : ℕ} (ε : Fin n → Bool) (Q : Matrix (Fin n) (Fin n) ℂ) (hQ : Q ∈ Matrix.unitaryGroup (Fin n) ℂ) (a b : Fin n) : (Qᴴ * PI.UpqRep.Jm ε * Q) * (Q⁻¹ * Matrix.single a b 1 * Q)ᴴ * (Qᴴ * PI.UpqRep.Jm ε * Q) = ((Formalism.REAL.sgn ε a b : ℝ) : ℂ) • (Q⁻¹ * Matrix.single b a 1 * Q)

theorem **isUnitary_of_isE** : PI.KFinSmooth → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (R : Rep k r), R.IsDG ε mu → ∀ (Q : Mx k r) (E : (Fin (2 * r + k + 1) → Fin (2 * r + k + 1) → Module.End ℂ R.smSub)), PI.UpqRep.IsCayleyU k r ε Q → R.IsE Q E → Formalism.REAL.IsUnitary ε E

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.CayleyEndomorphismsUnitaryStatement

theorem **_root_.Gp91.GlobalTheory.T.cayley_endomorphisms_unitary** : PI.KFinSmooth → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (R : Rep k r), R.IsDG ε mu → ∀ (Q : Mx k r) (E : (Fin (2 * r + k + 1) → Fin (2 * r + k + 1) → Module.End ℂ R.smSub)), PI.UpqRep.IsCayleyU k r ε Q → R.IsE Q E → Formalism.REAL.IsUnitary ε E

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.CayleyExistsAndAny

lemma **Kset_sub** {g : Matrix (Fin n) (Fin n) ℂ} (hg : g ∈ Kset ε) : g ∈ UJset ε

lemma **UJ_mul** {g h : Matrix (Fin n) (Fin n) ℂ} (hg : g ∈ UJset ε) (hh : h ∈ UJset ε) : g * h ∈ UJset ε

lemma **symm_inter** (T : H ≃ₗᵢ[ℂ] H) (h : ∀ g ∈ UJset ε, ∀ v, T (σ' g v) = σ g (T v)) : ∀ g ∈ UJset ε, ∀ v, T.symm (σ g v) = σ' g (T.symm v)

lemma **wt_transfer** (T : H ≃ₗᵢ[ℂ] H) (h : ∀ g ∈ UJset ε, ∀ v, T (σ' g v) = σ g (T v)) {μ : Fin n → ℝ} {v : H} (hv : IsWtOn (Kset ε) σ' μ v) : IsWtOn (Kset ε) σ μ (T v)

lemma **ktype_transfer** (T : H ≃ₗᵢ[ℂ] H) (h : ∀ g ∈ UJset ε, ∀ v, T (σ' g v) = σ g (T v)) {Λ : Fin n → ℝ} {S : Submodule ℂ H} (hS : IsKTypeOf ε σ' Λ S) : IsKTypeOf ε σ Λ (S.map (T.toLinearEquiv : H →ₗ[ℂ] H))

theorem **isDS_transfer** (T : H ≃ₗᵢ[ℂ] H) (h : ∀ g ∈ UJset ε, ∀ v, T (σ' g v) = σ g (T v)) {lam : Fin n → ℝ} (hσ : IsDS ε lam σ) : IsDS ε lam σ'

theorem **isDS_conj** {lam : Fin n → ℝ} (hσ : IsDS ε lam σ) {u u' : Matrix (Fin n) (Fin n) ℂ} (hu : u ∈ UJset ε) (hu' : u' ∈ UJset ε) (h1 : u * u' = 1) : IsDS ε lam (fun g => σ (u' * g * u))

lemma **card_even** (r : ℕ) : ((range (2 * r + 2)).filter fun m => m % 2 = 0).card = r + 1

lemma **card_signs** (k r : ℕ) (hk : 1 ≤ k) : (univ.filter fun i => PI.BesselPeriodGGP.eps48 k r i = true).card = k + r

theorem **cayley_exists_and_any** : ∀ (k r : ℕ), (1 ≤ k → ∃ Q, PI.UpqRep.IsCayleyU k r (PI.BesselPeriodGGP.eps48 k r) Q) ∧ ∀ (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (R : Rep k r), R.IsDG ε mu → ∀ Q, PI.UpqRep.IsCayleyU k r ε Q → PI.UpqRep.IsDS ε mu (fun g => R.σ (Q⁻¹ * g * Q))

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.ComplexifiedDerivedActionLieHomGramAdjoint

theorem **gram_herm** (k r : ℕ) : (Formalism.QuantumDeterminants.Hpk k r)ᴴ = Formalism.QuantumDeterminants.Hpk k r

theorem **gram_sq** (k r : ℕ) : Formalism.QuantumDeterminants.Hpk k r * Formalism.QuantumDeterminants.Hpk k r = 1

theorem **lie_iff** {X : Mx k r} : InLieHpk k r X ↔ Xᴴ * Formalism.QuantumDeterminants.Hpk k r + Formalism.QuantumDeterminants.Hpk k r * X = 0

theorem **adj_of_lie** {X : Mx k r} (hX : InLieHpk k r X) : Formalism.QuantumDeterminants.Hpk k r * Xᴴ * Formalism.QuantumDeterminants.Hpk k r = -X

theorem **lie_of_adj** {X : Mx k r} (h : Formalism.QuantumDeterminants.Hpk k r * Xᴴ * Formalism.QuantumDeterminants.Hpk k r = -X) : InLieHpk k r X

theorem **adj_adj** (Y : Mx k r) : Formalism.QuantumDeterminants.Hpk k r * (Formalism.QuantumDeterminants.Hpk k r * Yᴴ * Formalism.QuantumDeterminants.Hpk k r)ᴴ * Formalism.QuantumDeterminants.Hpk k r = Y

theorem **lie_re** (Y : Mx k r) : InLieHpk k r ((1 / 2 : ℂ) • (Y + -(Formalism.QuantumDeterminants.Hpk k r * Yᴴ * Formalism.QuantumDeterminants.Hpk k r)))

theorem **lie_im** (Y : Mx k r) : InLieHpk k r ((1 / (2 * Complex.I)) • (Y - -(Formalism.QuantumDeterminants.Hpk k r * Yᴴ * Formalism.QuantumDeterminants.Hpk k r)))

theorem **exists_decomp** (Y : Mx k r) : ∃ X₁ X₂, InLieHpk k r X₁ ∧ InLieHpk k r X₂ ∧ Y = X₁ + Complex.I • X₂

theorem **adj_decomp** {X₁ X₂ : Mx k r} (h₁ : InLieHpk k r X₁) (h₂ : InLieHpk k r X₂) : Formalism.QuantumDeterminants.Hpk k r * (X₁ + Complex.I • X₂)ᴴ * Formalism.QuantumDeterminants.Hpk k r = -X₁ + Complex.I • X₂

theorem **dC_decomp** (R : Rep k r) {X₁ X₂ : Mx k r} (h₁ : InLieHpk k r X₁) (h₂ : InLieHpk k r X₂) (v : R.H) : R.dC (X₁ + Complex.I • X₂) v = dsig (k

theorem **lie_zero** : InLieHpk k r 0

theorem **lie_add** {X Y : Mx k r} (hX : InLieHpk k r X) (hY : InLieHpk k r Y) : InLieHpk k r (X + Y)

theorem **lie_smul** (a : ℝ) {X : Mx k r} (hX : InLieHpk k r X) : InLieHpk k r (a • X)

theorem **lie_neg** {X : Mx k r} (hX : InLieHpk k r X) : InLieHpk k r (-X)

theorem **lie_br** {X Y : Mx k r} (hX : InLieHpk k r X) (hY : InLieHpk k r Y) : InLieHpk k r (X * Y - Y * X)

theorem **dsig_add_left** (R : Rep k r) {X Y : Mx k r} (hX : InLieHpk k r X) (hY : InLieHpk k r Y) {v : R.H} (hv : v ∈ R.sm) : dsig (k

theorem **dsig_smul_left** (R : Rep k r) (a : ℝ) {X : Mx k r} (hX : InLieHpk k r X) {v : R.H} (hv : v ∈ R.sm) : dsig (k

theorem **dsig_neg_left** (R : Rep k r) {X : Mx k r} (hX : InLieHpk k r X) {v : R.H} (hv : v ∈ R.sm) : dsig (k

theorem **dC_lie** (R : Rep k r) {X : Mx k r} (hX : InLieHpk k r X) {v : R.H} (hv : v ∈ R.sm) : R.dC X v = dsig (k

theorem **dC_lin_left** (R : Rep k r) (Y Z : Mx k r) (a : ℂ) {v : R.H} (hv : v ∈ R.sm) : R.dC (a • Y + Z) v = a • R.dC Y v + R.dC Z v

theorem **complexified_action** : PI.KFinSmooth → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (R : Rep k r), R.IsDG ε mu → (∀ Y, ∀ v ∈ R.sm, R.dC Y v ∈ R.sm) ∧ (∀ Y (a : ℂ), ∀ v ∈ R.sm, ∀ w ∈ R.sm, R.dC Y (a • v + w) = a • R.dC Y v + R.dC Y w) ∧ (∀ Y Z (a : ℂ), ∀ v ∈ R.sm, R.dC (a • Y + Z) v = a • R.dC Y v + R.dC Z v) ∧ (∀ X, PI.UpqRep.InLieHpk k r X → ∀ v ∈ R.sm, R.dC X v = PI.UpqRep.dsig (k

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.ComplexifiedDerivedActionLieHomGramAdjointStatement

theorem **_root_.Gp91.GlobalTheory.T.complexifiedDerivedAction_lieHom_gramAdjoint** : PI.KFinSmooth → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (R : Rep k r), R.IsDG ε mu → (∀ Y, ∀ v ∈ R.sm, R.dC Y v ∈ R.sm) ∧ (∀ Y (a : ℂ), ∀ v ∈ R.sm, ∀ w ∈ R.sm, R.dC Y (a • v + w) = a • R.dC Y v + R.dC Y w) ∧ (∀ Y Z (a : ℂ), ∀ v ∈ R.sm, R.dC (a • Y + Z) v = a • R.dC Y v + R.dC Z v) ∧ (∀ X, PI.UpqRep.InLieHpk k r X → ∀ v ∈ R.sm, R.dC X v = PI.UpqRep.dsig (k

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.DerivedActionBracket

theorem **fderiv_comm** {E G : Type} [NormedAddCommGroup E] [NormedSpace ℝ E] [NormedAddCommGroup G] [NormedSpace ℝ G] {F : E → G} {x : E} (hF : ContDiffAt ℝ ∞ F x) (L M : E →L[ℝ] E) {a b c : E} (ha : M x = a) (hb : L x = b) (hc : L a - M b = c) : fderiv ℝ (fun g => fderiv ℝ F g (L g)) x a - fderiv ℝ (fun g => fderiv ℝ F g (M g)) x b = fderiv ℝ F x c

theorem **lie_bracket** {X Y : Mx k r} (hX : InLieHpk k r X) (hY : InLieHpk k r Y) : InLieHpk k r (X * Y - Y * X)

theorem **dsig_dsig** (hσ : IsDSHpk k r ε mu σ) {X Y : Mx k r} (hX : InLieHpk k r X) (hY : InLieHpk k r Y) {v : H} (hv : v ∈ smoothVecs (k

theorem **dsig_bracket** (hσ : IsDSHpk k r ε mu σ) {X Y : Mx k r} (hX : InLieHpk k r X) (hY : InLieHpk k r Y) {v : H} (hv : v ∈ smoothVecs (k

theorem **derived_action_bracket** : PI.KFinSmooth → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (R : Rep k r), R.IsDG ε mu → ∀ X Y, PI.UpqRep.InLieHpk k r X → PI.UpqRep.InLieHpk k r Y → ∀ v ∈ R.sm, PI.UpqRep.dsig (k

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.DerivedActionDiscreteSeriesProperties

theorem **exp_mem** {X : Mx k r} (hX : InLieHpk k r X) (t : ℝ) : NormedSpace.exp ((t : ℂ) • X) ∈ UH (Hpk k r)

theorem **one_mem** : (1 : Mx k r) ∈ UH (Hpk k r)

theorem **lie_conj** {X g : Mx k r} (hX : InLieHpk k r X) (hg : g ∈ UH (Hpk k r)) : InLieHpk k r (g⁻¹ * X * g)

theorem **lie_conj'** {X g : Mx k r} (hX : InLieHpk k r X) (hg : g ∈ UH (Hpk k r)) : InLieHpk k r (g * X * g⁻¹)

theorem **hasDerivAt_local** {X g : Mx k r} (hX : InLieHpk k r X) {f F : Mx k r → H} {U : Set (Mx k r)} (hU : IsOpen U) (hgU : g ∈ U) (hF : ContDiffOn ℝ ∞ F U) (hfF : ∀ h ∈ U, inUH (Hpk k r) h → f h = F h) (hg : g ∈ UH (Hpk k r)) : HasDerivAt (fun u : ℝ => f (g * NormedSpace.exp ((u : ℂ) • X))) (fderiv ℝ F g (g * X)) 0

theorem **exists_lin** {v : H} (hv : v ∈ smoothVecs (k

theorem **hasDerivAt_dsig** {X : Mx k r} (hX : InLieHpk k r X) {v : H} (hv : v ∈ smoothVecs (k

theorem **hasDerivAt_apply** {X : Mx k r} (hX : InLieHpk k r X) {v : H} (hv : v ∈ smoothVecs (k

theorem **dsig_zero** (X : Mx k r) : dsig (k

theorem **dsig_add** {X : Mx k r} (hX : InLieHpk k r X) {v w : H} (hv : v ∈ smoothVecs (k

theorem **dsig_smul** {X : Mx k r} (hX : InLieHpk k r X) (a : ℂ) {v : H} (hv : v ∈ smoothVecs (k

theorem **dsig_smooth** (hσ : IsDSHpk k r ε mu σ) {X : Mx k r} (hX : InLieHpk k r X) {v : H} (hv : v ∈ smoothVecs (k

theorem **dsig_conj** (hσ : IsDSHpk k r ε mu σ) {X g : Mx k r} (hX : InLieHpk k r X) {v : H} (hv : v ∈ smoothVecs (k

theorem **dsig_kfin** [CompleteSpace H] (hK : PI.KFinSmooth) (hmu : IsHCParam mu) (hσ : IsDSHpk k r ε mu σ) {X : Mx k r} (hX : InLieHpk k r X) {v : H} (hv : IsKFinHpk (k

theorem **dsig_skew** (hσ : IsDSHpk k r ε mu σ) {X : Mx k r} (hX : InLieHpk k r X) {v w : H} (hv : v ∈ smoothVecs (k

theorem **dsig_TK** (hK : PI.KFinSmooth) (R : Rep k r) (hR : R.IsDG ε mu) {X : Mx k r} (hX : InLieHpk k r X) : ∀ v ∈ R.TK, v ∈ R.sm ∧ dsig (k

theorem **derived_action** : PI.KFinSmooth → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (R : Rep k r), R.IsDG ε mu → ∀ X, PI.UpqRep.InLieHpk k r X → (∀ v ∈ R.sm, PI.UpqRep.dsig (k

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.DerivedActionDiscreteSeriesPropertiesStatement

theorem **_root_.Gp91.GlobalTheory.T.derivedAction_discreteSeries_properties** : PI.KFinSmooth → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (R : Rep k r), R.IsDG ε mu → ∀ X, PI.UpqRep.InLieHpk k r X → (∀ v ∈ R.sm, PI.UpqRep.dsig (k

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.DerivedActionExpandMatrixUnits

theorem **conj_expand_units** {n : ℕ} (Q Y : Matrix (Fin n) (Fin n) ℂ) (hQ : IsUnit Q) : Y = ∑ a, ∑ b, (Q * Y * Q⁻¹) a b • (Q⁻¹ * Matrix.single a b (1 : ℂ) * Q)

theorem **conj_units_mul** {n : ℕ} (Q : Matrix (Fin n) (Fin n) ℂ) (hQ : IsUnit Q) (a b c d : Fin n) : (Q⁻¹ * Matrix.single a b (1 : ℂ) * Q) * (Q⁻¹ * Matrix.single c d (1 : ℂ) * Q) = (if b = c then (1 : ℂ) else 0) • (Q⁻¹ * Matrix.single a d (1 : ℂ) * Q)

theorem **dC_expand_units** : PI.KFinSmooth → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (R : Rep k r), R.IsDG ε mu → ∀ (Q : Mx k r) (E : (Fin (2 * r + k + 1) → Fin (2 * r + k + 1) → Module.End ℂ R.smSub)), IsUnit Q → R.IsE Q E → Formalism.REAL.GlRel E ∧ (∀ Y (v : R.smSub), R.dC Y v = ∑ a, ∑ b, (Q * Y * Q⁻¹) a b • ((E a b v : R.smSub) : R.H)) ∧ ∀ X, PI.UpqRep.InLieHpk k r X → ∀ v : R.smSub, PI.UpqRep.dsig (k

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.DerivedActionExpandMatrixUnitsStatement

theorem **_root_.Gp91.GlobalTheory.T.derivedAction_expand_matrixUnits** : PI.KFinSmooth → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (R : Rep k r), R.IsDG ε mu → ∀ (Q : Mx k r) (E : (Fin (2 * r + k + 1) → Fin (2 * r + k + 1) → Module.End ℂ R.smSub)), IsUnit Q → R.IsE Q E → Formalism.REAL.GlRel E ∧ (∀ Y (v : R.smSub), R.dC Y v = ∑ a, ∑ b, (Q * Y * Q⁻¹) a b • ((E a b v : R.smSub) : R.H)) ∧ ∀ X, PI.UpqRep.InLieHpk k r X → ∀ v : R.smSub, PI.UpqRep.dsig (k

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.DiscreteSeriesParameterUnique

lemma **eq_of_capP** {n : ℕ} (mu mu' : Fin n → ℝ) (h : StrictAnti mu) (h' : StrictAnti mu') (e : ∀ Y : ℂ, Formalism.QuantumDeterminants.capP (fun s => (mu s : ℂ)) Y = Formalism.QuantumDeterminants.capP (fun s => (mu' s : ℂ)) Y) : mu = mu'

lemma **dsig_equiv** (V : H ≃L[ℂ] H') (hV : ∀ g, Formalism.REAL.inUJ ε g → ∀ x, V (τ g x) = τ' g (V x)) (X : Matrix (Fin n) (Fin n) ℂ) (hX : Formalism.REAL.inG ε X) (v : H) : V (PI.PhMeasCore.dsig τ X v) = PI.PhMeasCore.dsig τ' X (V v)

lemma **dsigC_equiv** (V : H ≃L[ℂ] H') (hV : ∀ g, Formalism.REAL.inUJ ε g → ∀ x, V (τ g x) = τ' g (V x)) (Y : Matrix (Fin n) (Fin n) ℂ) (v : H) : V (dsigC ε τ Y v) = dsigC ε τ' Y (V v)

lemma **dsigCW_equiv** (V : H ≃L[ℂ] H') (hV : ∀ g, Formalism.REAL.inUJ ε g → ∀ x, V (τ g x) = τ' g (V x)) (w : List (Matrix (Fin n) (Fin n) ℂ)) (v : H) : V (dsigCW ε τ w v) = dsigCW ε τ' w (V v)

lemma **actU_equiv** (V : H ≃L[ℂ] H') (hV : ∀ g, Formalism.REAL.inUJ ε g → ∀ x, V (τ g x) = τ' g (V x)) (p : FreeAlgebra ℂ (Fin n × Fin n)) (v : H) : V (actU ε τ p v) = actU ε τ' p (V v)

lemma **smooth_equiv** (V : H ≃L[ℂ] H') (hV : ∀ g, Formalism.REAL.inUJ ε g → ∀ x, V (τ g x) = τ' g (V x)) {v : H} (hv : v ∈ PI.PhMeasCore.smoothVecs (Formalism.REAL.Jm ε) τ) : V v ∈ PI.PhMeasCore.smoothVecs (Formalism.REAL.Jm ε) τ'

lemma **smooth_conj** {A B P : Matrix (Fin n) (Fin n) ℂ} (hP : IsUnit P) (hA : Pᴴ * A * P = B) {v : H} (hv : PI.UpqRep.SmoothOnH B (fun g => τ g v)) : v ∈ PI.PhMeasCore.smoothVecs A (fun g => τ (P⁻¹ * g * P))

lemma **eq_of_equiv** {mu mu' : Fin n → ℝ} (hm : Formalism.QuantumDeterminants.IsHCParam mu) (hm' : Formalism.QuantumDeterminants.IsHCParam mu') (h : ∀ v ∈ PI.PhMeasCore.smoothVecs (Formalism.REAL.Jm ε) τ, ∀ Y : ℂ, actU ε τ (Formalism.REAL.qdet (Egl n) (Y + ((n : ℂ) - 1) / 2)) v = Formalism.QuantumDeterminants.capP (fun s => (mu s : ℂ)) Y • v) (h' : ∀ v ∈ PI.PhMeasCore.smoothVecs (Formalism.REAL.Jm ε) τ', ∀ Y : ℂ, actU ε τ' (Formalism.REAL.qdet (Egl n) (Y + ((n : ℂ) - 1) / 2)) v = Formalism.QuantumDeterminants.capP (fun s => (mu' s : ℂ)) Y • v) (V : H ≃L[ℂ] H') (hV : ∀ g, Formalism.REAL.inUJ ε g → ∀ x, V (τ g x) = τ' g (V x)) {v : H} (hv : v ∈ PI.PhMeasCore.smoothVecs (Formalism.REAL.Jm ε) τ) (h0 : v ≠ 0) : mu = mu'

lemma **inUJ_mul** {a b : Matrix (Fin n) (Fin n) ℂ} (ha : a ∈ PI.UpqRep.UJset ε) (hb : b ∈ PI.UpqRep.UJset ε) : a * b ∈ PI.UpqRep.UJset ε

lemma **cayley_quot** {B P Q : Matrix (Fin n) (Fin n) ℂ} (hQ : IsUnit Q) (hP : Pᴴ * PI.UpqRep.Jm ε * P = B) (hQ' : Qᴴ * PI.UpqRep.Jm ε * Q = B) : P * Q⁻¹ ∈ PI.UpqRep.UJset ε

lemma **exists_kfin** {k r : ℕ} {ε : Fin (2 * r + k + 1) → Bool} {mu : Fin (2 * r + k + 1) → ℝ} (R : Rep k r) {P : Mx k r} (hP : PI.UpqRep.IsCayleyU k r ε P) (h : PI.UpqRep.IsDS ε mu (fun g => R.σ (P⁻¹ * g * P))) : ∃ v, R.KFin v ∧ v ≠ 0

theorem **param_eq_of_equiv** : HypsG → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (mu' : Fin (2 * r + k + 1) → ℝ) (R R' : Rep k r), R.IsDG ε mu → R'.IsDG ε mu' → ∀ U : R.H ≃ₗᵢ[ℂ] R'.H, (∀ g ∈ GG k r, ∀ x, U (R.σ g x) = R'.σ g (U x)) → mu = mu'

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.DiscreteSeriesParameterUniqueStatement

theorem **_root_.Gp91.GlobalTheory.T.discrete_series_parameter_unique** : HypsG → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (mu' : Fin (2 * r + k + 1) → ℝ) (R R' : Rep k r), R.IsDG ε mu → R'.IsDG ε mu' → ∀ U : R.H ≃ₗᵢ[ℂ] R'.H, (∀ g ∈ GG k r, ∀ x, U (R.σ g x) = R'.σ g (U x)) → mu = mu'

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.DiscreteSeriesUnitarySmoothBasic

theorem **exists_isLowUnitH_of_isDSHpk** {k r : ℕ} {ε : Fin (2 * r + k + 1) → Bool} {mu : Fin (2 * r + k + 1) → ℝ} {H : Type} [NormedAddCommGroup H] [InnerProductSpace ℂ H] {σ : Mx k r → (H →L[ℂ] H)} (hσ : PI.UpqRep.IsDSHpk k r ε mu σ) : ∃ w, ‖w‖ = 1 ∧ PI.UpqRep.IsLowUnitH k r ε mu σ w

theorem **rep_basic_of_isDS** : PI.KFinSmooth → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (R : Rep k r), R.IsDG ε mu → (∀ g ∈ GG k r, ∀ h ∈ GG k r, ∀ v, R.σ (g * h) v = R.σ g (R.σ h v)) ∧ (∀ v, R.σ 1 v = v) ∧ (∀ g ∈ GG k r, ∀ v w, ⟪R.σ g v, R.σ g w⟫_ℂ = ⟪v, w⟫_ℂ) ∧ (∀ g ∈ GG k r, ∀ v ∈ R.sm, R.σ g v ∈ R.sm) ∧ (∀ v ∈ R.TK, v ∈ R.sm) ∧ (∀ g ∈ GG k r, ∀ v ∈ R.TK, R.σ g v ∈ R.TK) ∧ (∀ v, R.KFin v → v ∈ R.TK) ∧ (∃ w, ‖w‖ = 1 ∧ R.KFin w ∧ PI.UpqRep.IsLowUnitH k r ε mu R.σ w) ∧ (∀ v, ContinuousOn (fun g => R.σ g v) (GG k r))

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.DiscreteSeriesUnitarySmoothBasicStatement

theorem **_root_.Gp91.GlobalTheory.T.discreteSeries_unitary_smooth_basic** : PI.KFinSmooth → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (R : Rep k r), R.IsDG ε mu → (∀ g ∈ GG k r, ∀ h ∈ GG k r, ∀ v, R.σ (g * h) v = R.σ g (R.σ h v)) ∧ (∀ v, R.σ 1 v = v) ∧ (∀ g ∈ GG k r, ∀ v w, ⟪R.σ g v, R.σ g w⟫_ℂ = ⟪v, w⟫_ℂ) ∧ (∀ g ∈ GG k r, ∀ v ∈ R.sm, R.σ g v ∈ R.sm) ∧ (∀ v ∈ R.TK, v ∈ R.sm) ∧ (∀ g ∈ GG k r, ∀ v ∈ R.TK, R.σ g v ∈ R.TK) ∧ (∀ v, R.KFin v → v ∈ R.TK) ∧ (∃ w, ‖w‖ = 1 ∧ R.KFin w ∧ PI.UpqRep.IsLowUnitH k r ε mu R.σ w) ∧ (∀ v, ContinuousOn (fun g => R.σ g v) (GG k r))

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.DualTensorBesselMoment

lemma **star_Hpk** (k r : ℕ) (i j : Fin (2 * r + k + 1)) : star (Formalism.QuantumDeterminants.Hpk k r i j) = Formalism.QuantumDeterminants.Hpk k r i j

lemma **ev_flat** {k r : ℕ} (R : Rep k r) (f : Fin (2 * r + k + 1) → ℂ) (x : Fin (2 * r + k + 1) → R.H) : ev (Formalism.QuantumDeterminants.Hpk k r) R (flat k r f) x = ∑ j, f j • x j

lemma **sum_frame** {n : ℕ} {V : Type} [AddCommGroup V] [Module ℂ V] (g : Matrix (Fin n) (Fin n) ℂ) (hg : IsUnit g) (f : Fin n → ℂ) (x : Fin n → V) : ∑ j, f j • x j = ∑ d, (g⁻¹ *ᵥ f) d • ∑ c, g c d • x c

lemma **sum_coframe** {n : ℕ} {V : Type} [AddCommGroup V] [Module ℂ V] (g : Matrix (Fin n) (Fin n) ℂ) (hg : IsUnit g) (ω : Fin n → ℂ) (v : V) (c : Fin n) : ∑ d, g⁻¹ d c • ((gᵀ *ᵥ ω) d • v) = ω c • v

theorem **bessel_ev_projector_value** : HypsG → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (R : Rep k r) (RI : Fin (2 * r + k + 1) → Rep k r) (io : ∀ s, Fin (2 * r + k + 1) → (RI s).H → R.H) (pr : ∀ s, Fin (2 * r + k + 1) → R.H → (RI s).H) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (z : Fin (2 * r + k + 1) → ℂ) (π : Mx k r → (Wp →L[ℂ] Wp)) (u : ℕ → ℝ) (μW μN μG νq : Measure (Formalism.QuantumDeterminants.Mt k r)) (l : R.H → Wp), 1 ≤ k → R.IsDG ε mu → (∀ s, (RI s).IsDG ε (mu - Pi.single s 1)) → TensRaw dualM R RI io pr → SideG k r z π u μW μN μG νq → IsBes R z π l → ∀ s, ∃ c : ℂ, ‖c‖ = Formalism.QuantumDeterminants.Tz k r z * (([∏ i ∈ Icc 1 k, |mu s - 1 / 2 - u i|) / ∏ t ∈ univ.erase s, |mu s - mu t|) ∧ ∀ v ∈ R.sm, l (ev (Formalism.QuantumDeterminants.Hpk k r) R (flat k r (Formalism.QuantumDeterminan

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.DualTensorBesselMomentStatement

theorem **_root_.Gp91.GlobalTheory.T.dual_tensor_bessel_moment** : HypsG → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (R : Rep k r) (RI : Fin (2 * r + k + 1) → Rep k r) (io : ∀ s, Fin (2 * r + k + 1) → (RI s).H → R.H) (pr : ∀ s, Fin (2 * r + k + 1) → R.H → (RI s).H) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (z : Fin (2 * r + k + 1) → ℂ) (π : Mx k r → (Wp →L[ℂ] Wp)) (u : ℕ → ℝ) (μW μN μG νq : Measure (Formalism.QuantumDeterminants.Mt k r)) (l : R.H → Wp), 1 ≤ k → R.IsDG ε mu → (∀ s, (RI s).IsDG ε (mu - Pi.single s 1)) → TensRaw dualM R RI io pr → SideG k r z π u μW μN μG νq → IsBes R z π l → ∀ s, ∃ c : ℂ, ‖c‖ = Formalism.QuantumDeterminants.Tz k r z * (([∏ i ∈ Icc 1 k, |mu s - 1 / 2 - u i|) / ∏ t ∈ univ.erase s, |mu s - mu t|) ∧ ∀ v ∈ R.sm, l (ev (Formalism.QuantumDeterminants.Hpk k r) R (flat k r (

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.DualTensorCapelliIntertwine

private theorem **hasDerivAt_exp_smul_apply_zero'** (Y : Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ) (j c : Fin (2 * r + k + 1)) : HasDerivAt (fun s : ℝ => NormedSpace.exp ((s : ℂ) • Y) j c) (Y j c) 0

theorem **hasDerivAt_inv_exp_entry** (X : Mx k r) (j c : Fin (2 * r + k + 1)) : HasDerivAt (fun t : ℝ => (NormedSpace.exp ((t : ℂ) • X))⁻¹ j c) (-X j c) 0

theorem **lie_part₁** (Y : Mx k r) : InLieHpk k r ((1 / 2 : ℂ) • (Y + -(Formalism.QuantumDeterminants.Hpk k r * Yᴴ * Formalism.QuantumDeterminants.Hpk k r)))

theorem **lie_part₂** (Y : Mx k r) : InLieHpk k r ((1 / (2 * Complex.I)) • (Y - -(Formalism.QuantumDeterminants.Hpk k r * Yᴴ * Formalism.QuantumDeterminants.Hpk k r)))

theorem **io_dsig** (hK : PI.KFinSmooth) {ε : Fin (2 * r + k + 1) → Bool} {mu : Fin (2 * r + k + 1) → ℝ} {R : Rep k r} (hR : R.IsDG ε mu) {RI : Fin (2 * r + k + 1) → Rep k r} {muI : Fin (2 * r + k + 1) → Fin (2 * r + k + 1) → ℝ} {io : ∀ s, Fin (2 * r + k + 1) → (RI s).H → R.H} {pr : ∀ s, Fin (2 * r + k + 1) → R.H → (RI s).H} (hI : ∀ s, (RI s).IsDG ε (muI s)) (hT : TensRaw dualM R RI io pr) (s : Fin (2 * r + k + 1)) {X : Mx k r} (hX : InLieHpk k r X) {y : (RI s).H} (hy : y ∈ (RI s).sm) (c : Fin (2 * r + k + 1)) : io s c (dsig (k

theorem **dualTens_io_dC** : PI.KFinSmooth → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (R : Rep k r), R.IsDG ε mu → ∀ (RI : Fin (2 * r + k + 1) → Rep k r) (muI : Fin (2 * r + k + 1) → Fin (2 * r + k + 1) → ℝ) (io : ∀ s, Fin (2 * r + k + 1) → (RI s).H → R.H) (pr : ∀ s, Fin (2 * r + k + 1) → R.H → (RI s).H), (∀ s, (RI s).IsDG ε (muI s)) → TensRaw dualM R RI io pr → ∀ s Y, ∀ y ∈ (RI s).sm, ∀ c, io s c ((RI s).dC Y y) = R.dC Y (io s c y) - ∑ j, Y j c • io s j y

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.DualTensorCapelliIntertwineStatement

theorem **_root_.Gp91.GlobalTheory.T.dual_tensor_capelli_intertwine** : PI.KFinSmooth → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (R : Rep k r), R.IsDG ε mu → ∀ (RI : Fin (2 * r + k + 1) → Rep k r) (muI : Fin (2 * r + k + 1) → Fin (2 * r + k + 1) → ℝ) (io : ∀ s, Fin (2 * r + k + 1) → (RI s).H → R.H) (pr : ∀ s, Fin (2 * r + k + 1) → R.H → (RI s).H), (∀ s, (RI s).IsDG ε (muI s)) → TensRaw dualM R RI io pr → ∀ s Y, ∀ y ∈ (RI s).sm, ∀ c, io s c ((RI s).dC Y y) = R.dC Y (io s c y) - ∑ j, Y j c • io s j y

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.DualTensorCayleyCompatible

@[simp] theorem **lm_coe** {k r : ℕ} {R R' : Rep k r} {T : R.H → R'.H} (h : SmMap R R' T) (v : R.smSub) : ((lm h v : R'.smSub) : R'.H) = T v

theorem **change_inv** (A B : Matrix (Fin n) (Fin n) ℂ) (h : A * B = 1) (x : Fin n → M) (c : Fin n) : ∑ d, B d c • ∑ e, A e d • x e = x c

theorem **dualTens_sum** {ι : Type} (S : Finset ι) (E : Fin n → Fin n → Module.End ℂ M) (a b : Fin n) (x : ι → Fin n → M) : dualTens E a b (∑ i ∈ S, x i) = ∑ i ∈ S, dualTens E a b (x i)

theorem **alg** {N : Fin n → Type} [∀ s, AddCommGroup (N s)] [∀ s, Module ℂ (N s)] (I : ∀ s, Fin n → N s →ₗ[ℂ] M) (P : ∀ s, Fin n → M →ₗ[ℂ] N s) (A B : Matrix (Fin n) (Fin n) ℂ) (hAB : A * B = 1) (hBA : B * A = 1) (E : Fin n → Fin n → Module.End ℂ M) (F : ∀ s, Fin n → Fin n → Module.End ℂ (N s)) (hI : ∀ s a b y c, I s c (F s a b y) = E a b (I s c y) - ∑ j, (B * Matrix.single a b (1 : ℂ) * A) j c • I s j y) (h1 : ∀ (x : Fin n → M) c, ∑ s, I s c (∑ d, P s d (x d)) = x c) (h2 : ∀ s y, ∑ c, P s c (I s c y) = y) (h3 : ∀ s t, s ≠ t → ∀ y, ∑ c, P s c (I t c y) = 0) : ∃ (io' : ∀ s, N s →ₗ[ℂ] (Fin n → M)) (pr' : ∀ s, (Fin n → M) →ₗ[ℂ] N s), (∀ s a b y, io' s (F s a b y) = dualTens E a b (io' s y)) ∧ (∀ s a b x, pr' s (dualTens E a b x) = F s a b (pr' s x)) ∧ (∀ x, ∑ s, io' s (pr' s x) = x) ∧ (∀ s y, pr' s (io' s y) = y) ∧ (∀ s t, s ≠ t → ∀ y, pr' s (io' t y) = 0) ∧ (∀ s y d, io' s y d = ∑ c, B c d •

theorem **rawTens** : PI.KFinSmooth → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (R : Rep k r), R.IsDG ε mu → ∀ (RI : Fin (2 * r + k + 1) → Rep k r) (muI : Fin (2 * r + k + 1) → Fin (2 * r + k + 1) → ℝ) (io : ∀ s, Fin (2 * r + k + 1) → (RI s).H → R.H) (pr : ∀ s, Fin (2 * r + k + 1) → R.H → (RI s).H) (Q : Mx k r) (E : (Fin (2 * r + k + 1) → Fin (2 * r + k + 1) → Module.End ℂ R.smSub)) (F : ∀ s, Fin (2 * r + k + 1) → Fin (2 * r + k + 1) → Module.End ℂ (RI s).smSub), (∀ s, (RI s).IsDG ε (muI s)) → TensRaw dualM R RI io pr → IsUnit Q → R.IsE Q E → (∀ s, (RI s).IsE Q (F s)) → ∃ (io' : ∀ s, (RI s).smSub →ₗ[ℂ] (Fin (2 * r + k + 1) → R.smSub)) (pr' : ∀ s, (Fin (2 * r + k + 1) → R.smSub) →ₗ[ℂ] (RI s).smSub), (∀ s a b y, io' s (F s a b y) = dualTens E a b (io' s y)) ∧ (∀ s a b x, pr' s (dualTens E a b x) = F s a b (pr' s x)) ∧ (∀ x, ∑ s, io' s (pr' s x) = x) ∧ (∀ s y,

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.DualTensorCayleyCompatibleStatement

theorem **_root_.Gp91.GlobalTheory.T.dual_tensor_cayley_compatible** : PI.KFinSmooth → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (R : Rep k r), R.IsDG ε mu → ∀ (RI : Fin (2 * r + k + 1) → Rep k r) (muI : Fin (2 * r + k + 1) → Fin (2 * r + k + 1) → ℝ) (io : ∀ s, Fin (2 * r + k + 1) → (RI s).H → R.H) (pr : ∀ s, Fin (2 * r + k + 1) → R.H → (RI s).H) (Q : Mx k r) (E : (Fin (2 * r + k + 1) → Fin (2 * r + k + 1) → Module.End ℂ R.smSub)) (F : ∀ s, Fin (2 * r + k + 1) → Fin (2 * r + k + 1) → Module.End ℂ (RI s).smSub), (∀ s, (RI s).IsDG ε (muI s)) → TensRaw dualM R RI io pr → IsUnit Q → R.IsE Q E → (∀ s, (RI s).IsE Q (F s)) → ∃ (io' : ∀ s, (RI s).smSub →ₗ[ℂ] (Fin (2 * r + k + 1) → R.smSub)) (pr' : ∀ s, (Fin (2 * r + k + 1) → R.smSub) →ₗ[ℂ] (RI s).smSub), (∀ s a b y, io' s (F s a b y) = dualTens E a b (io' s y)) ∧ (∀ s a b x, pr' s (dualTens E a b x) = F s a b (pr' s

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.DualTensorCayleyWeighted

theorem **gram_symm** (k r : ℕ) (i j : Fin (2 * r + k + 1)) : PI.UpqRep.Hpk k r j i = Formalism.QuantumDeterminants.Hpk k r i j

theorem **gram_cayley** {k r : ℕ} {ε : Fin (2 * r + k + 1) → Bool} {Q : Mx k r} (hQ : PI.UpqRep.IsCayleyU k r ε Q) (i j : Fin (2 * r + k + 1)) : ∑ d, (if ε d then (1 : ℂ) else -1) * ((starRingEnd ℂ) (Q⁻¹ i d) * Q⁻¹ j d) = Formalism.QuantumDeterminants.Hpk k r i j

theorem **hhJ_eq** {k r : ℕ} {ε : Fin (2 * r + k + 1) → Bool} {Q : Mx k r} (hQ : PI.UpqRep.IsCayleyU k r ε Q) (R : Rep k r) (x y : Fin (2 * r + k + 1) → R.smSub) (u v : Fin (2 * r + k + 1) → R.H) (hx : ∀ d, ((x d : R.smSub) : R.H) = ∑ c, Q⁻¹ c d • u c) (hy : ∀ d, ((y d : R.smSub) : R.H) = ∑ c, Q⁻¹ c d • v c) : hhJ ε x y = hh (Formalism.QuantumDeterminants.Hpk k r) R u v

theorem **algTens_cayley** : PI.KFinSmooth → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (R : Rep k r), R.IsDG ε mu → ∀ (RI : Fin (2 * r + k + 1) → Rep k r) (muI : Fin (2 * r + k + 1) → Fin (2 * r + k + 1) → ℝ) (io : ∀ s, Fin (2 * r + k + 1) → (RI s).H → R.H) (pr : ∀ s, Fin (2 * r + k + 1) → R.H → (RI s).H) (ep : Fin (2 * r + k + 1) → ℝ) (Q : Mx k r) (E : (Fin (2 * r + k + 1) → Fin (2 * r + k + 1) → Module.End ℂ R.smSub)) (F : ∀ s, Fin (2 * r + k + 1) → Fin (2 * r + k + 1) → Module.End ℂ (RI s).smSub), (∀ s, (RI s).IsDG ε (muI s)) → TensH dualM (Formalism.QuantumDeterminants.Hpk k r) R RI io pr ep → PI.UpqRep.IsCayleyU k r ε Q → R.IsE Q E → (∀ s, (RI s).IsE Q (F s)) → ∃ (io' : ∀ s, (RI s).smSub →ₗ[ℂ] (Fin (2 * r + k + 1) → R.smSub)) (pr' : ∀ s, (Fin (2 * r + k + 1) → R.smSub) →ₗ[ℂ] (RI s).smSub), AlgTens ε E (fun s => (RI s).smSub) F io' pr' ∧ ∀ s x, ((pr' s x

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.DualTensorCayleyWeightedStatement

theorem **_root_.Gp91.GlobalTheory.T.dual_tensor_cayley_weighted** : PI.KFinSmooth → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (R : Rep k r), R.IsDG ε mu → ∀ (RI : Fin (2 * r + k + 1) → Rep k r) (muI : Fin (2 * r + k + 1) → Fin (2 * r + k + 1) → ℝ) (io : ∀ s, Fin (2 * r + k + 1) → (RI s).H → R.H) (pr : ∀ s, Fin (2 * r + k + 1) → R.H → (RI s).H) (ep : Fin (2 * r + k + 1) → ℝ) (Q : Mx k r) (E : (Fin (2 * r + k + 1) → Fin (2 * r + k + 1) → Module.End ℂ R.smSub)) (F : ∀ s, Fin (2 * r + k + 1) → Fin (2 * r + k + 1) → Module.End ℂ (RI s).smSub), (∀ s, (RI s).IsDG ε (muI s)) → TensH dualM (Formalism.QuantumDeterminants.Hpk k r) R RI io pr ep → PI.UpqRep.IsCayleyU k r ε Q → R.IsE Q E → (∀ s, (RI s).IsE Q (F s)) → ∃ (io' : ∀ s, (RI s).smSub →ₗ[ℂ] (Fin (2 * r + k + 1) → R.smSub)) (pr' : ∀ s, (Fin (2 * r + k + 1) → R.smSub) →ₗ[ℂ] (RI s).smSub), AlgTens ε E (fun s => (R

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.ExistsFamilyFunctionals

theorem **GlobalTheory.exists_sideFam_both_sides** : HypsG → ∀ (P : M.Par) (Wp Wq : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wp] [FiniteDimensional ℂ Wq] (p : KB.Pl P Wp Wq) (lam : Fin (2 * P.r + P.k + 1) → ℝ) (b bv ga h : ℝ) (q0 : ℕ) (R : (Fin (2 * P.r + P.k + 1) → ℤ) → Rep P.k P.r), Num P.k P.r lam P.u b bv ga h q0 → IsFamG lam q0 R → ∃ (l : ∀ ν, (R ν).H → Wp) (l' : ∀ ν, (R ν).H → Wq) (dg : (Fin (2 * P.r + P.k + 1) → ℤ) → ℝ), (haveI

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.ExistsFamilyFunctionalsStatement

theorem **_root_.Gp91.GlobalTheory.T.exists_family_functionals** : HypsG → ∀ (P : M.Par) (Wp Wq : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wp] [FiniteDimensional ℂ Wq] (p : KB.Pl P Wp Wq) (lam : Fin (2 * P.r + P.k + 1) → ℝ) (b bv ga h : ℝ) (q0 : ℕ) (R : (Fin (2 * P.r + P.k + 1) → ℤ) → Rep P.k P.r), Num P.k P.r lam P.u b bv ga h q0 → IsFamG lam q0 R → ∃ (l : ∀ ν, (R ν).H → Wp) (l' : ∀ ν, (R ν).H → Wq) (dg : (Fin (2 * P.r + P.k + 1) → ℤ) → ℝ), (haveI

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.ExistsKTypeNonzeroBessel

theorem **bessel_nonzero_on_translates** : HypsG → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (R : Rep k r) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (z : Fin (2 * r + k + 1) → ℂ) (π : Mx k r → (Wp →L[ℂ] Wp)) (l : R.H → Wp), R.IsDG ε mu → IsBes R z π l → NzOn R l → ∃ v ∈ R.TK, l v ≠ 0

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.ExistsKTypeNonzeroBesselStatement

theorem **_root_.Gp91.GlobalTheory.T.exists_K_type_nonzero_bessel** : HypsG → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (R : Rep k r) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (z : Fin (2 * r + k + 1) → ℂ) (π : Mx k r → (Wp →L[ℂ] Wp)) (l : R.H → Wp), R.IsDG ε mu → IsBes R z π l → NzOn R l → ∃ v ∈ R.TK, l v ≠ 0

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.ExistsLowestKFiniteVectorDiscreteSeries

lemma **lowK_eq** {n : ℕ} (ε : Fin n → Bool) (mu : Fin n → ℝ) : PI.UpqRep.lowK ε mu = Formalism.QuantumDeterminants.lowK ε mu

theorem **exists_lowest_vector** : PI.KFinSmooth → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (R : Rep k r), R.IsDG ε mu → ∀ (Q : Mx k r) (E : (Fin (2 * r + k + 1) → Fin (2 * r + k + 1) → Module.End ℂ R.smSub)), PI.UpqRep.IsCayleyU k r ε Q → R.IsE Q E → ∃ w : R.smSub, ‖w‖ = 1 ∧ R.KFin (w : R.H) ∧ Formalism.REAL.KHighest ε E (Formalism.QuantumDeterminants.lowK ε mu) w ∧ FiniteDimensional ℂ (Formalism.REAL.kSpan ε E w)

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.ExistsLowestKFiniteVectorDiscreteSeriesStatement

theorem **_root_.Gp91.GlobalTheory.T.exists_lowest_kFinite_vector_discreteSeries** : PI.KFinSmooth → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (R : Rep k r), R.IsDG ε mu → ∀ (Q : Mx k r) (E : (Fin (2 * r + k + 1) → Fin (2 * r + k + 1) → Module.End ℂ R.smSub)), PI.UpqRep.IsCayleyU k r ε Q → R.IsE Q E → ∃ w : R.smSub, ‖w‖ = 1 ∧ R.KFin (w : R.H) ∧ Formalism.REAL.KHighest ε E (Formalism.QuantumDeterminants.lowK ε mu) w ∧ FiniteDimensional ℂ (Formalism.REAL.kSpan ε E w)

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.FamilyFunctionalCoefficient

theorem **exists_coef_family** : HypsG → ∀ (P : M.Par) (Wp Wq : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wp] [FiniteDimensional ℂ Wq] (p : KB.Pl P Wp Wq) (lam : Fin (2 * P.r + P.k + 1) → ℝ) (b bv ga h : ℝ) (q0 : ℕ) (R : (Fin (2 * P.r + P.k + 1) → ℤ) → Rep P.k P.r) (l : ∀ ν, (R ν).H → Wp) (l' : ∀ ν, (R ν).H → Wq) (dg : (Fin (2 * P.r + P.k + 1) → ℤ) → ℝ) (f : B.Mat P.k P.r → (Wq →L[ℂ] Wp)) (s₀ : ℂ), IsFamG lam q0 R → (haveI

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.FamilyFunctionalSpan

theorem **exists_tau** : HypsG → ∀ (P : M.Par) (Wp Wq : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wp] [FiniteDimensional ℂ Wq] (p : KB.Pl P Wp Wq) (lam : Fin (2 * P.r + P.k + 1) → ℝ) (b bv ga h : ℝ) (q0 : ℕ) (R : (Fin (2 * P.r + P.k + 1) → ℤ) → Rep P.k P.r) (l : ∀ ν, (R ν).H → Wp) (l' : ∀ ν, (R ν).H → Wq) (dg : (Fin (2 * P.r + P.k + 1) → ℤ) → ℝ) (s : (Fin (2 * P.r + P.k + 1) → ℤ) → ℂ) (J : (Fin (2 * P.r + P.k + 1) → ℤ) → B.Mat P.k P.r → (Wq →L[ℂ] Wp)), Num P.k P.r lam P.u b bv ga h q0 → IsFamG lam q0 R → (haveI

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.FamilyFunctionalSpanDual

theorem **sup_norm** : HypsG → ∀ (P : M.Par) (Wp Wq : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wp] [FiniteDimensional ℂ Wq] (p : KB.Pl P Wp Wq) (lam : Fin (2 * P.r + P.k + 1) → ℝ) (b bv ga h : ℝ) (q0 : ℕ) (R : (Fin (2 * P.r + P.k + 1) → ℤ) → Rep P.k P.r) (l : ∀ ν, (R ν).H → Wp) (l' : ∀ ν, (R ν).H → Wq) (dg : (Fin (2 * P.r + P.k + 1) → ℤ) → ℝ) (s : (Fin (2 * P.r + P.k + 1) → ℤ) → ℂ) (J : (Fin (2 * P.r + P.k + 1) → ℤ) → B.Mat P.k P.r → (Wq →L[ℂ] Wp)) (tau : (Fin (2 * P.r + P.k + 1) → ℤ) → Finset (Fin (2 * P.r + P.k + 1)) → ℂ), Num P.k P.r lam P.u b bv ga h q0 → SupG p lam q0 R l l' dg s J tau → ∃ c : ℝ, 0 < c ∧ ∀ ν, n1 ν ≤ q0 → (J ν = 0 ∧ TBB.jN2 p (J ν) = 0 ∧ ¬ (NzOn (R ν) (l ν) ∧ NzOn (R ν) (l' ν))) ∨ (J ν ≠ 0 ∧ NzOn (R ν) (l ν) ∧ NzOn (R ν) (l' ν) ∧ s ν ≠ 0 ∧ TBB.jN2 p (J ν) = ENNReal.ofReal (c * Formalism.Q

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.FamilyFunctionalSpanDualStatement

theorem **_root_.Gp91.GlobalTheory.T.jN2_eq_const_mul_formal_degree_of_supG** : HypsG → ∀ (P : M.Par) (Wp Wq : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wp] [FiniteDimensional ℂ Wq] (p : KB.Pl P Wp Wq) (lam : Fin (2 * P.r + P.k + 1) → ℝ) (b bv ga h : ℝ) (q0 : ℕ) (R : (Fin (2 * P.r + P.k + 1) → ℤ) → Rep P.k P.r) (l : ∀ ν, (R ν).H → Wp) (l' : ∀ ν, (R ν).H → Wq) (dg : (Fin (2 * P.r + P.k + 1) → ℤ) → ℝ) (s : (Fin (2 * P.r + P.k + 1) → ℤ) → ℂ), Num P.k P.r lam P.u b bv ga h q0 → SupG p lam q0 R l l' dg s J tau → ∃ c : ℝ, 0 < c ∧ ∀ ν, n1 ν ≤ q0 → (J ν = 0 ∧ TBB.jN2 p (J ν) = 0 ∧ ¬ (NzOn (R ν) (l ν) ∧ NzOn (R ν) (l' ν))) ∨ (J ν ≠ 0 ∧ NzOn (R ν) (l ν) ∧ NzOn (R ν) (l' ν)) ∧ s

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.FamilySetupBesselGrowth

theorem **integrand_eq** {Wp : Type} [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] {d : ℝ} (hd : 0 ≤ d) (v : Wp) : ENNReal.ofReal (1 * (d * ‖v‖ ^ 2)) = ((‖((Real.sqrt d : ℝ) : ℂ) • v‖₊ : ENNReal)) ^ 2

theorem **mom_mass** : HypsG → ∀ (k r : ℕ) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (lam : Fin (2 * r + k + 1) → ℝ) (u : ℕ → ℝ) (b bv ga h : ℝ) (q0 : ℕ) (z : Fin (2 * r + k + 1) → ℂ) (π : Mx k r → (Wp →L[ℂ] Wp)) (μW μN μG νq : Measure (Formalism.QuantumDeterminants.Mt k r)) (R : (Fin (2 * r + k + 1) → ℤ) → Rep k r) (l : ∀ ν, (R ν).H → Wp) (dg : (Fin (2 * r + k + 1) → ℤ) → ℝ), SetupG lam u b bv ga h q0 z π μW μN μG νq R l dg → ∀ ν, n1 ν ≤ q0 → (∀ w ∈ (R ν).sm, ∫⁻ x, ENNReal.ofReal (1 * (dg ν * ‖l ν ((R ν).σ (Matrix.of x) w)‖ ^ 2)) ∂νq ≤ ENNReal.ofReal (‖w‖ ^ 2)) ∧ (NzOn (R ν) (l ν) → IsIsoB (R ν) z π νq (fun w => ((Real.sqrt (dg ν) : ℝ) : ℂ) • l ν w) ∧ ∀ w ∈ (R ν).sm, ∫⁻ x, ENNReal.ofReal (1 * (dg ν * ‖l ν ((R ν).σ (Matrix.of x) w)‖ ^ 2)) ∂νq = ENNReal.ofReal (‖w‖ ^ 2))

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.FamilySetupBesselGrowthStatement

theorem **_root_.Gp91.GlobalTheory.T.bessel_functional_l2_bound_and_isometry** : HypsG → ∀ (k r : ℕ) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (lam : Fin (2 * r + k + 1) → ℝ) (u : ℕ → ℝ) (b bv ga h : ℝ) (q0 : ℕ) (z : Fin (2 * r + k + 1) → ℂ) (π : Mx k r → (Wp →L[ℂ] Wp)) (μW μN μG νq : Measure (Formalism.QuantumDeterminants.Mt k r)) (R : (Fin (2 * r + k + 1) → ℤ) → Rep k r) (l : ∀ ν, (R ν).H → Wp) (dg : (Fin (2 * r + k + 1) → ℤ) → ℝ), SetupG lam u b bv ga h q0 z π μW μN μG νq R l dg → ∀ ν, n1 ν ≤ q0 → (∀ w ∈ (R ν).sm, ∫⁻ x, ENNReal.ofReal (1 * (dg ν * ‖l ν ((R ν).σ (Matrix.of x) w)‖ ^ 2)) ∂νq ≤ ENNReal.ofReal (‖w‖ ^ 2)) ∧ (NzOn (R ν) (l ν) → IsIsoB (R ν) z π νq (fun w => ((Real.sqrt (dg ν) : ℝ) : ℂ) • l ν w) ∧ ∀ w ∈ (R ν).sm, ∫⁻ x, ENNReal.ofReal (1 * (dg ν * ‖l ν ((R ν).σ (Matrix.of x) w)‖ ^ 2)) ∂νq = ENNReal.ofReal (‖w‖ ^ 2))

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.FamilySetupCayleyBound

theorem **iterate_bound** {X M ι : Type} [MeasurableSpace X] [NormedAddCommGroup M] [Fintype ι] (μ : Measure X) (κ : X → ℝ) (f : M → X → ℝ) (T : ι → ι → M → M) (L : ℕ → M → Prop) (N : ℕ) (B : ℝ) (hB : 0 ≤ B) (hκ : AEMeasurable κ μ) (hf : ∀ w, AEMeasurable (f w) μ) (hf0 : ∀ w x, 0 ≤ f w x) (hpt : ∀ᵐ x ∂μ, ∀ w, κ x ^ 2 * f w x ≤ ∑ c, ∑ e, f (T c e w) x) (hL : ∀ j w, L j w → ∀ c e, L (j + 2) (T c e w)) (hs : ∀ j w, j ≤ N → L j w → ∑ c, ∑ e, ‖T c e w‖ ^ 2 ≤ B ^ 2 * ‖w‖ ^ 2) (h0 : ∀ w, ∫⁻ x, ENNReal.ofReal (1 * f w x) ∂μ ≤ ENNReal.ofReal (‖w‖ ^ 2)) : ∀ q j, j + 2 * q ≤ N → ∀ w, L j w → ∫⁻ x, ENNReal.ofReal (κ x ^ (2 * q) * f w x) ∂μ ≤ ENNReal.ofReal (B ^ (2 * q) * ‖w‖ ^ 2)

theorem **ratio_moment** : HypsG → ∀ (k r : ℕ) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (lam : Fin (2 * r + k + 1) → ℝ) (u : ℕ → ℝ) (b bv ga h : ℝ) (q0 : ℕ) (z : Fin (2 * r + k + 1) → ℂ) (π : Mx k r → (Wp →L[ℂ] Wp)) (μW μN μG νq : Measure (Formalism.QuantumDeterminants.Mt k r)) (R : (Fin (2 * r + k + 1) → ℤ) → Rep k r) (l : ∀ ν, (R ν).H → Wp) (dg : (Fin (2 * r + k + 1) → ℤ) → ℝ) (Q : Mx k r) (E : ∀ ν, Fin (2 * r + k + 1) → Fin (2 * r + k + 1) → Module.End ℂ (R ν).smSub), SetupG lam u b bv ga h q0 z π μW μN μG νq R l dg → PI.UpqRep.IsCayleyU k r (PI.BesselPeriodGGP.eps48 k r) Q → (∀ ν, n1 ν ≤ q0 → (R ν).IsE Q (E ν) ∧ Formalism.REAL.GlRel (E ν)) → ∀ ν, n1 ν ≤ q0 → ∀ (j q : ℕ), j + 2 * q ≤ 4 * (2 * r + k + 1) ^ 2 → ∀ (a : Fin r) (w : (R ν).smSub), Lev (PI.BesselPeriodGGP.eps48 k r) (E ν) (fun a => lam a + ((ν : Fin (2 * r + k + 1)

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.FamilySetupCayleyBoundStatement

theorem **_root_.Gp91.GlobalTheory.T.family_setup_cayley_bound** : HypsG → ∀ (k r : ℕ) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (lam : Fin (2 * r + k + 1) → ℝ) (u : ℕ → ℝ) (b bv ga h : ℝ) (q0 : ℕ) (z : Fin (2 * r + k + 1) → ℂ) (π : Mx k r → (Wp →L[ℂ] Wp)) (μW μN μG νq : Measure (Formalism.QuantumDeterminants.Mt k r)) (R : (Fin (2 * r + k + 1) → ℤ) → Rep k r) (l : ∀ ν, (R ν).H → Wp) (dg : (Fin (2 * r + k + 1) → ℤ) → ℝ) (Q : Mx k r) (E : ∀ ν, Fin (2 * r + k + 1) → Fin (2 * r + k + 1) → Module.End ℂ (R ν).smSub), SetupG lam u b bv ga h q0 z π μW μN μG νq R l dg → PI.UpqRep.IsCayleyU k r (PI.BesselPeriodGGP.eps48 k r) Q → (∀ ν, n1 ν ≤ q0 → (R ν).IsE Q (E ν) ∧ Formalism.REAL.GlRel (E ν)) → ∀ ν, n1 ν ≤ q0 → ∀ (j q : ℕ), j + 2 * q ≤ 4 * (2 * r + k + 1) ^ 2 → ∀ (a : Fin r) (w : (R ν).smSub), Lev (PI.BesselPeriodGGP.eps48 k r) (E ν) (fun

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.FamilySetupCayleyMoment

theorem **lintegral_le_of_le_sum** {Ω ι : Type} [MeasurableSpace Ω] [Fintype ι] (ν : Measure Ω) (f : Ω → ℝ) (g : ι → ι → Ω → ℝ) (C : ℝ) (N : ι → ι → ℝ) (hC : 0 ≤ C) (hg0 : ∀ d s x, 0 ≤ g d s x) (hm : ∀ d s, AEMeasurable (g d s) ν) (hpt : ∀ᵐ x ∂ν, f x ≤ C * ∑ d, ∑ s, g d s x) (hN : ∀ d s, 0 ≤ N d s) (hint : ∀ d s, ∫⁻ x, ENNReal.ofReal (g d s x) ∂ν ≤ ENNReal.ofReal (N d s)) : ∫⁻ x, ENNReal.ofReal (f x) ∂ν ≤ ENNReal.ofReal (C * ∑ d, ∑ s, N d s)

theorem **norm_sum_smul_sq_le** {V ι : Type} [NormedAddCommGroup V] [NormedSpace ℂ V] [Fintype ι] (c : ι → ℂ) (v : ι → V) : ‖∑ s, c s • v s‖ ^ 2 ≤ (Fintype.card ι : ℝ) * ∑ s, ‖c s‖ ^ 2 * ‖v s‖ ^ 2

theorem **pointwise_le** {V ι : Type} [NormedAddCommGroup V] [NormedSpace ℂ V] [Fintype ι] (L D A : ℝ) (q : ℕ) (a β : ι → ℂ) (Ds : ι → ℝ) (v : V) (y : ι → ι → V) (hD : 0 ≤ D) (hL : ∑ d, ‖a d‖ ^ 2 = L ^ 2) (he : ∀ d, a d • v = ∑ s, β s • y d s) (hβ : ∀ s, ‖β s‖ ^ 2 * D ≤ A * Ds s) : L ^ (2 * (q + 1)) * (D * ‖v‖ ^ 2) ≤ ((Fintype.card ι : ℝ) * A) * ∑ d, ∑ s, L ^ (2 * q) * (Ds s * ‖y d s‖ ^ 2)

theorem **lOneNorm_sub_single_one_le** {n : ℕ} (ν : Fin n → ℤ) (s : Fin n) : n1 (ν - Pi.single s 1) ≤ n1 ν + 1

theorem **param_sub_single** {n : ℕ} (lam : Fin n → ℝ) (ν : Fin n → ℤ) (s : Fin n) : (fun a => lam a + (((ν - Pi.single s 1) : Fin n → ℤ) a : ℝ)) = (fun a => lam a + (ν a : ℝ)) - Pi.single s 1

theorem **moment_step** : HypsG → ∀ (k r : ℕ) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (lam : Fin (2 * r + k + 1) → ℝ) (u : ℕ → ℝ) (b bv ga h : ℝ) (q0 : ℕ) (z : Fin (2 * r + k + 1) → ℂ) (π : Mx k r → (Wp →L[ℂ] Wp)) (μW μN μG νq : Measure (Formalism.QuantumDeterminants.Mt k r)) (R : (Fin (2 * r + k + 1) → ℤ) → Rep k r) (l : ∀ ν, (R ν).H → Wp) (dg : (Fin (2 * r + k + 1) → ℤ) → ℝ) (Q : Mx k r) (E : ∀ ν, Fin (2 * r + k + 1) → Fin (2 * r + k + 1) → Module.End ℂ (R ν).smSub), SetupG lam u b bv ga h q0 z π μW μN μG νq R l dg → PI.UpqRep.IsCayleyU k r (PI.BesselPeriodGGP.eps48 k r) Q → (∀ ν, n1 ν ≤ q0 → (R ν).IsE Q (E ν) ∧ Formalism.REAL.GlRel (E ν)) → ∀ ν, n1 ν + 1 ≤ q0 → ∀ (j q : ℕ) (MM : ℝ), j ≤ 4 * (2 * r + k + 1) ^ 2 → 0 ≤ MM → (∀ (s : Fin (2 * r + k + 1)) (y : (R (ν - Pi.single s 1)).smSub), Lev (PI.BesselPeriodGGP.eps48 k r) (E (

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.FamilySetupCayleyMomentStatement

theorem **_root_.Gp91.GlobalTheory.T.family_setup_cayley_moment** : HypsG → ∀ (k r : ℕ) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (lam : Fin (2 * r + k + 1) → ℝ) (u : ℕ → ℝ) (b bv ga h : ℝ) (q0 : ℕ) (z : Fin (2 * r + k + 1) → ℂ) (π : Mx k r → (Wp →L[ℂ] Wp)) (μW μN μG νq : Measure (Formalism.QuantumDeterminants.Mt k r)) (R : (Fin (2 * r + k + 1) → ℤ) → Rep k r) (l : ∀ ν, (R ν).H → Wp) (dg : (Fin (2 * r + k + 1) → ℤ) → ℝ) (Q : Mx k r) (E : ∀ ν, Fin (2 * r + k + 1) → Fin (2 * r + k + 1) → Module.End ℂ (R ν).smSub), SetupG lam u b bv ga h q0 z π μW μN μG νq R l dg → PI.UpqRep.IsCayleyU k r (PI.BesselPeriodGGP.eps48 k r) Q → (∀ ν, n1 ν ≤ q0 → (R ν).IsE Q (E ν) ∧ Formalism.REAL.GlRel (E ν)) → ∀ ν, n1 ν + 1 ≤ q0 → ∀ (j q : ℕ) (MM : ℝ), j ≤ 4 * (2 * r + k + 1) ^ 2 → 0 ≤ MM → (∀ (s : Fin (2 * r + k + 1)) (y : (R (ν - Pi.single s 1)).smSub

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.FamilySetupCayleyRecursion

theorem **n1_sub_single_one_le_add_one** {n : ℕ} (ν : Fin n → ℤ) (s : Fin n) : n1 (ν - Pi.single s 1) ≤ n1 ν + 1

theorem **moment_first_length** : HypsG → ∀ (k r : ℕ) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (lam : Fin (2 * r + k + 1) → ℝ) (u : ℕ → ℝ) (b bv ga h : ℝ) (q0 : ℕ) (z : Fin (2 * r + k + 1) → ℂ) (π : Mx k r → (Wp →L[ℂ] Wp)) (μW μN μG νq : Measure (Formalism.QuantumDeterminants.Mt k r)) (R : (Fin (2 * r + k + 1) → ℤ) → Rep k r) (l : ∀ ν, (R ν).H → Wp) (dg : (Fin (2 * r + k + 1) → ℤ) → ℝ) (Q : Mx k r) (E : ∀ ν, Fin (2 * r + k + 1) → Fin (2 * r + k + 1) → Module.End ℂ (R ν).smSub), SetupG lam u b bv ga h q0 z π μW μN μG νq R l dg → PI.UpqRep.IsCayleyU k r (PI.BesselPeriodGGP.eps48 k r) Q → (∀ ν, n1 ν ≤ q0 → (R ν).IsE Q (E ν) ∧ Formalism.REAL.GlRel (E ν)) → ∀ (q : ℕ) ν, n1 ν + q ≤ q0 → ∀ j : ℕ, j + 2 * q ≤ 4 * (2 * r + k + 1) ^ 2 → ∀ w : (R ν).smSub, Lev (PI.BesselPeriodGGP.eps48 k r) (E ν) (fun a => lam a + ((ν : Fin (2 * r + k + 1)

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.FamilySetupCayleyRecursionStatement

theorem **_root_.Gp91.GlobalTheory.T.family_setup_cayley_recursion** : HypsG → ∀ (k r : ℕ) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (lam : Fin (2 * r + k + 1) → ℝ) (u : ℕ → ℝ) (b bv ga h : ℝ) (q0 : ℕ) (z : Fin (2 * r + k + 1) → ℂ) (π : Mx k r → (Wp →L[ℂ] Wp)) (μW μN μG νq : Measure (Formalism.QuantumDeterminants.Mt k r)) (R : (Fin (2 * r + k + 1) → ℤ) → Rep k r) (l : ∀ ν, (R ν).H → Wp) (dg : (Fin (2 * r + k + 1) → ℤ) → ℝ) (Q : Mx k r) (E : ∀ ν, Fin (2 * r + k + 1) → Fin (2 * r + k + 1) → Module.End ℂ (R ν).smSub), SetupG lam u b bv ga h q0 z π μW μN μG νq R l dg → PI.UpqRep.IsCayleyU k r (PI.BesselPeriodGGP.eps48 k r) Q → (∀ ν, n1 ν ≤ q0 → (R ν).IsE Q (E ν) ∧ Formalism.REAL.GlRel (E ν)) → ∀ (q : ℕ) ν, n1 ν + q ≤ q0 → ∀ j : ℕ, j + 2 * q ≤ 4 * (2 * r + k + 1) ^ 2 → ∀ w : (R ν).smSub, Lev (PI.BesselPeriodGGP.eps48 k r) (E ν) (fun a

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.FamilySetupMomentClosure

theorem **n1_add_eZ_le_add_card** {n : ℕ} (ν : Fin n → ℤ) (I : Finset (Fin n)) : n1 (ν + eZ I) ≤ n1 ν + I.card

theorem **eZ_injective** {n : ℕ} : Function.Injective (eZ : Finset (Fin n) → Fin n → ℤ)

theorem **kappa_bound** : HypsG → ∀ (k r : ℕ) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (lam : Fin (2 * r + k + 1) → ℝ) (u : ℕ → ℝ) (b bv ga h : ℝ) (q0 : ℕ) (z : Fin (2 * r + k + 1) → ℂ) (π : Mx k r → (Wp →L[ℂ] Wp)) (μW μN μG νq : Measure (Formalism.QuantumDeterminants.Mt k r)) (R : (Fin (2 * r + k + 1) → ℤ) → Rep k r) (l : ∀ ν, (R ν).H → Wp) (dg : (Fin (2 * r + k + 1) → ℤ) → ℝ), SetupG lam u b bv ga h q0 z π μW μN μG νq R l dg → ∀ ν, n1 ν + (r + 1) * (r + 2) ≤ q0 → ∀ io pr ep, TensH (cmp (r + 1)) (cmp (r + 1) (Formalism.QuantumDeterminants.Hpk k r)) (R ν) (fun I : Sub (2 * r + k + 1) (r + 1) => R (ν + eZ I.1)) io pr ep → NzOn (R ν) (l ν) → ∀ (I : Sub (2 * r + k + 1) (r + 1)) (κ₀ : ℂ), NzOn (R (ν + eZ I.1)) (l (ν + eZ I.1)) → (∀ v ∈ (R ν).sm, l (ν + eZ I.1) (∑ c, pr I c (tm (R ν) (wedge (Formalism.QuantumDeterminants.fvec k r z))

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.FamilySetupMomentClosureStatement

theorem **_root_.Gp91.GlobalTheory.T.wedge_raising_scalar_sq_mul_degree_ratio_le** : HypsG → ∀ (k r : ℕ) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (lam : Fin (2 * r + k + 1) → ℝ) (u : ℕ → ℝ) (b bv ga h : ℝ) (q0 : ℕ) (z : Fin (2 * r + k + 1) → ℂ) (π : Mx k r → (Wp →L[ℂ] Wp)) (μW μN μG νq : Measure (Formalism.QuantumDeterminants.Mt k r)) (R : (Fin (2 * r + k + 1) → ℤ) → Rep k r) (l : ∀ ν, (R ν).H → Wp) (dg : (Fin (2 * r + k + 1) → ℤ) → ℝ), SetupG lam u b bv ga h q0 z π μW μN μG νq R l dg → ∀ ν, n1 ν + (r + 1) * (r + 2) ≤ q0 → ∀ io pr ep, TensH (cmp (r + 1)) (cmp (r + 1) (Formalism.QuantumDeterminants.Hpk k r)) (R ν) (fun I : Sub (2 * r + k + 1) (r + 1) => R (ν + eZ I.1)) io pr ep → NzOn (R ν) (l ν) → ∀ (I : Sub (2 * r + k + 1) (r + 1)) (κ₀ : ℂ), NzOn (R (ν + eZ I.1)) (l (ν + eZ I.1)) → (∀ v ∈ (R ν).sm, l (ν + eZ I.1) (∑ c, pr I c (

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.FamilySetupMomentGrowth

theorem **lOneNorm_sub_single_one_le** {n : ℕ} (ν : Fin n → ℤ) (s : Fin n) : n1 (ν - Pi.single s 1 : Fin n → ℤ) ≤ n1 ν + 1

theorem **sum_mem_sm** {k r : ℕ} (R : Rep k r) {ι : Type} [Fintype ι] (x : ι → R.H) (hx : ∀ c, x c ∈ R.sm) : ∑ c, x c ∈ R.sm

theorem **dual_step_bound** : HypsG → ∀ (k r : ℕ) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (lam : Fin (2 * r + k + 1) → ℝ) (u : ℕ → ℝ) (b bv ga h : ℝ) (q0 : ℕ) (z : Fin (2 * r + k + 1) → ℂ) (π : Mx k r → (Wp →L[ℂ] Wp)) (μW μN μG νq : Measure (Formalism.QuantumDeterminants.Mt k r)) (R : (Fin (2 * r + k + 1) → ℤ) → Rep k r) (l : ∀ ν, (R ν).H → Wp) (dg : (Fin (2 * r + k + 1) → ℤ) → ℝ), SetupG lam u b bv ga h q0 z π μW μN μG νq R l dg → ∀ ν, n1 ν + 1 ≤ q0 → ∀ io pr ep, TensH dualM (Formalism.QuantumDeterminants.Hpk k r) (R ν) (fun s => R (ν - Pi.single s 1)) io pr ep → NzOn (R ν) (l ν) → ∀ (s : Fin (2 * r + k + 1)) (κ₀ : ℂ), (∀ v ∈ (R ν).sm, l (ν - Pi.single s 1) (∑ c, pr s c (tm (R ν) (flat k r (Formalism.QuantumDeterminants.fvec k r z 0)) v c)) = κ₀ • l ν v) → ‖κ₀‖ ^ 2 * dg ν / dg (ν - Pi.single s 1) ≤ Formalism.QuantumDeterminant

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.FamilySetupMomentGrowthStatement

theorem **_root_.Gp91.GlobalTheory.T.lowering_scalar_sq_mul_degree_ratio_le** : HypsG → ∀ (k r : ℕ) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (lam : Fin (2 * r + k + 1) → ℝ) (u : ℕ → ℝ) (b bv ga h : ℝ) (q0 : ℕ) (z : Fin (2 * r + k + 1) → ℂ) (π : Mx k r → (Wp →L[ℂ] Wp)) (μW μN μG νq : Measure (Formalism.QuantumDeterminants.Mt k r)) (R : (Fin (2 * r + k + 1) → ℤ) → Rep k r) (l : ∀ ν, (R ν).H → Wp) (dg : (Fin (2 * r + k + 1) → ℤ) → ℝ), SetupG lam u b bv ga h q0 z π μW μN μG νq R l dg → ∀ ν, n1 ν + 1 ≤ q0 → ∀ io pr ep, TensH dualM (Formalism.QuantumDeterminants.Hpk k r) (R ν) (fun s => R (ν - Pi.single s 1)) io pr ep → NzOn (R ν) (l ν) → ∀ (s : Fin (2 * r + k + 1)) (κ₀ : ℂ), (∀ v ∈ (R ν).sm, l (ν - Pi.single s 1) (∑ c, pr s c (tm (R ν) (flat k r (Formalism.QuantumDeterminants.fvec k r z 0)) v c)) = κ₀ • l ν v) → ‖κ₀‖ ^ 2 * dg ν / dg

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.FamilySetupMomentRecursion

theorem **product_moment** : HypsG → ∀ (k r : ℕ) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (lam : Fin (2 * r + k + 1) → ℝ) (u : ℕ → ℝ) (b bv ga h : ℝ) (q0 : ℕ) (z : Fin (2 * r + k + 1) → ℂ) (π : Mx k r → (Wp →L[ℂ] Wp)) (μW μN μG νq : Measure (Formalism.QuantumDeterminants.Mt k r)) (R : (Fin (2 * r + k + 1) → ℤ) → Rep k r) (l : ∀ ν, (R ν).H → Wp) (dg : (Fin (2 * r + k + 1) → ℤ) → ℝ), SetupG lam u b bv ga h q0 z π μW μN μG νq R l dg → ∀ ν, n1 ν + (r + 1) ^ 2 ≤ q0 → NzOn (R ν) (l ν) → ∃ w ∈ (R ν).sm, ‖w‖ = 1 ∧ ∫⁻ x, ENNReal.ofReal ((∏ a, lenA k r z (Matrix.of x) a ^ 2) * (dg ν * ‖l ν ((R ν).σ (Matrix.of x) w)‖ ^ 2)) ∂νq ≤ ENNReal.ofReal (Hsq r (Formalism.QuantumDeterminants.Tz k r z) b ^ (r + 1))

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.FamilySetupMomentRecursionStatement

theorem **_root_.Gp91.GlobalTheory.T.family_exists_unit_product_moment_le_Hsq_pow** : HypsG → ∀ (k r : ℕ) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (lam : Fin (2 * r + k + 1) → ℝ) (u : ℕ → ℝ) (b bv ga h : ℝ) (q0 : ℕ) (z : Fin (2 * r + k + 1) → ℂ) (π : Mx k r → (Wp →L[ℂ] Wp)) (μW μN μG νq : Measure (Formalism.QuantumDeterminants.Mt k r)) (R : (Fin (2 * r + k + 1) → ℤ) → Rep k r) (l : ∀ ν, (R ν).H → Wp) (dg : (Fin (2 * r + k + 1) → ℤ) → ℝ), SetupG lam u b bv ga h q0 z π μW μN μG νq R l dg → ∀ ν, n1 ν + (r + 1) ^ 2 ≤ q0 → NzOn (R ν) (l ν) → ∃ w ∈ (R ν).sm, ‖w‖ = 1 ∧ ∫⁻ x, ENNReal.ofReal ((∏ a, lenA k r z (Matrix.of x) a ^ 2) * (dg ν * ‖l ν ((R ν).σ (Matrix.of x) w)‖ ^ 2)) ∂νq ≤ ENNReal.ofReal (Hsq r (Formalism.QuantumDeterminants.Tz k r z) b ^ (r + 1))

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.FamilyThroughDiscreteSeries

theorem **family_through_discreteSeries** : HypsG → ∀ (k r : ℕ) (lam : Fin (2 * r + k + 1) → ℝ) (u : ℕ → ℝ) (b bv ga h : ℝ) (q0 : ℕ) (R0 : Rep k r), Num k r lam u b bv ga h q0 → R0.IsDG (PI.BesselPeriodGGP.eps48 k r) lam → ∃ R : (Fin (2 * r + k + 1) → ℤ) → Rep k r, IsFamG lam q0 R ∧ R 0 = R0

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.FrechetContinuousContinuousOnOrbitHasDerivAt

theorem **smoothOnH_continuousOn** {H : Matrix (Fin n) (Fin n) ℂ} {f : Matrix (Fin n) (Fin n) ℂ → E} (hf : SmoothOnH H f) : ContinuousOn f (Formalism.QuantumDeterminants.UH H)

theorem **left_hasDerivAt** {H : Matrix (Fin n) (Fin n) ℂ} (hH : AAInvW.HInvol H) {f F : Matrix (Fin n) (Fin n) ℂ → E} {U : Set (Matrix (Fin n) (Fin n) ℂ)} {g X : Matrix (Fin n) (Fin n) ℂ} (hU : IsOpen U) (hg : g ∈ U) (hgH : inUH H g) (hF : ContDiffOn ℝ ∞ F U) (hfF : ∀ x ∈ U, inUH H x → f x = F x) (hX : AAInvW.inLH H X) : HasDerivAt (fun t : ℝ => f (NormedSpace.exp ((t : ℂ) • X) * g)) (fderiv ℝ F g (X * g)) 0

theorem **left_smooth** {H : Matrix (Fin n) (Fin n) ℂ} (hH : AAInvW.HInvol H) {f f' : Matrix (Fin n) (Fin n) ℂ → E} {X : Matrix (Fin n) (Fin n) ℂ} (hf : SmoothOnH H f) (hX : AAInvW.inLH H X) (hf' : ∀ g, inUH H g → ∀ D, HasDerivAt (fun t : ℝ => f (NormedSpace.exp ((t : ℂ) • X) * g)) D 0 → D = f' g) : SmoothOnH H f' ∧ ∀ g, inUH H g → HasDerivAt (fun t : ℝ => f (NormedSpace.exp ((t : ℂ) • X) * g)) (f' g) 0

theorem **hInvol** (k r : ℕ) : AAInvW.HInvol (Hpk k r)

theorem **dsigW_mem** {R : Rep k r} (hD : DerFacts R) (W : List (Mx k r)) (hW : ∀ X ∈ W, InLieHpk k r X) : ∀ u ∈ R.sm, dsigW (k

theorem **dsigW_append** {R : Rep k r} (W : List (Mx k r)) (Y : Mx k r) (u : R.H) : dsigW (k

theorem **dsigW_adjoint** {R : Rep k r} (hD : DerFacts R) (W : List (Mx k r)) (hW : ∀ X ∈ W, InLieHpk k r X) : ∀ w ∈ R.sm, ∃ w' : R.H, ∀ u ∈ R.sm, ⟪w, dsigW (k

theorem **eq_zero_of_closure** {R : Rep k r} {x : R.H} (hx : x ∈ closure R.sm) (h : ∀ w ∈ R.sm, ⟪w, x⟫_ℂ = 0) : x = 0

theorem **sm_comb** {R : Rep k r} (a : ℂ) {x y z : R.H} (hx : x ∈ R.sm) (hy : y ∈ R.sm) (hz : z ∈ R.sm) : a • (x - y) - z ∈ R.sm

theorem **dsigW_comb** {R : Rep k r} (hD : DerFacts R) (W : List (Mx k r)) (hW : ∀ X ∈ W, InLieHpk k r X) (a : ℂ) {x y z : R.H} (hx : x ∈ R.sm) (hy : y ∈ R.sm) (hz : z ∈ R.sm) : dsigW (k

theorem **dsigW_deriv_eq** {R : Rep k r} (hD : DerFacts R) (W : List (Mx k r)) (hW : ∀ X ∈ W, InLieHpk k r X) {c : ℝ → R.H} {c' D : R.H} (hc : ∀ t, c t ∈ R.sm) (hc' : c' ∈ R.sm) (hd : HasDerivAt c c' 0) (hD' : HasDerivAt (fun t => dsigW (k

theorem **dsigW_orbit** {ε : Fin (2 * r + k + 1) → Bool} {mu : Fin (2 * r + k + 1) → ℝ} {R : Rep k r} (hR : R.IsDG ε mu) (hD : DerFacts R) {v : R.H} (hv : v ∈ R.sm) (W : List (Mx k r)) (hW : ∀ X ∈ W, InLieHpk k r X) (hs : SmoothOnH (Hpk k r) (fun g => dsigW (k

theorem **dsigW_orbit_smooth** {ε : Fin (2 * r + k + 1) → Bool} {mu : Fin (2 * r + k + 1) → ℝ} {R : Rep k r} (hR : R.IsDG ε mu) (hD : DerFacts R) {v : R.H} (hv : v ∈ R.sm) (W : List (Mx k r)) (hW : ∀ X ∈ W, InLieHpk k r X) : SmoothOnH (Hpk k r) (fun g => dsigW (k

theorem **cw_tendsto** {R : Rep k r} {V : Type} [NormedAddCommGroup V] {Bm : R.H → V} (hB : PI.BesselMapUniq.IsCWCont (k

theorem **lin_comb** {R : Rep k r} {V : Type} [NormedAddCommGroup V] [NormedSpace ℂ V] {Bm : R.H → V} (ha : ∀ v ∈ R.sm, ∀ w ∈ R.sm, Bm (v + w) = Bm v + Bm w) (hs : ∀ (a : ℂ), ∀ v ∈ R.sm, Bm (a • v) = a • Bm v) (a : ℂ) {x y z : R.H} (hx : x ∈ R.sm) (hy : y ∈ R.sm) (hz : z ∈ R.sm) : Bm (a • (x - y) - z) = a • (Bm x - Bm y) - Bm z

theorem **cwCont_continuousOn_hasDerivAt** : PI.KFinSmooth → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (R : Rep k r), R.IsDG ε mu → ∀ (V : Type) [NormedAddCommGroup V] [NormedSpace ℂ V] (Bm : R.H → V), (∀ v ∈ R.sm, ∀ w ∈ R.sm, Bm (v + w) = Bm v + Bm w) → (∀ (a : ℂ), ∀ v ∈ R.sm, Bm (a • v) = a • Bm v) → PI.BesselMapUniq.IsCWCont (k

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.FrechetContinuousContinuousOnOrbitHasDerivAtStatement

theorem **_root_.Gp91.GlobalTheory.T.frechetContinuous_continuousOn_orbit_hasDerivAt** : PI.KFinSmooth → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (R : Rep k r), R.IsDG ε mu → ∀ (V : Type) [NormedAddCommGroup V] [NormedSpace ℂ V] (Bm : R.H → V), (∀ v ∈ R.sm, ∀ w ∈ R.sm, Bm (v + w) = Bm v + Bm w) → (∀ (a : ℂ), ∀ v ∈ R.sm, Bm (a • v) = a • Bm v) → PI.BesselMapUniq.IsCWCont (k

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.FrepDiscreteSeriesSum

theorem **hh_sum_left** (Γ : Matrix ι ι ℂ) (R : Rep k r) (f : κ → ι → R.H) (y : ι → R.H) : hh Γ R (fun c => ∑ I, f I c) y = ∑ I, hh Γ R (f I) y

theorem **hh_sum_right** (Γ : Matrix ι ι ℂ) (R : Rep k r) (f : κ → ι → R.H) (x : ι → R.H) : hh Γ R x (fun c => ∑ I, f I c) = ∑ I, hh Γ R x (f I)

theorem **sum_sm** (R : Rep k r) (x : ι → R.H) (hx : ∀ c, x c ∈ R.sm) : ∑ c, x c ∈ R.sm

theorem **hh_tm** (Γ : Matrix ι ι ℂ) (R : Rep k r) (a : ι → ℂ) (x : ι → R.H) (v : R.H) : hh Γ R x (tm R a v) = ⟪v, ev Γ R a x⟫_ℂ

theorem **norm_sq_mul** {H : Type} [NormedAddCommGroup H] [InnerProductSpace ℂ H] {Γ : Matrix ι ι ℂ} (h : Γᴴ * Γ = 1) (x : ι → H) : ∑ i, ‖∑ j, Γ i j • x j‖ ^ 2 = ∑ i, ‖x i‖ ^ 2

theorem **norm_hh_le** {Γ : Matrix ι ι ℂ} (h : Γᴴ * Γ = 1) (R : Rep k r) (x : ι → R.H) : ‖hh Γ R x x‖ ≤ ∑ c, ‖x c‖ ^ 2

theorem **proj_sm** (hT : TensH ρ Γ R RI io pr ep) (I : κ) {x : ι → R.H} (hx : ∀ c, x c ∈ R.sm) : ∑ c, pr I c (x c) ∈ (RI I).sm

theorem **adj** (hT : TensH ρ Γ R RI io pr ep) (I : κ) (y : (RI I).H) (hy : y ∈ (RI I).sm) (x : ι → R.H) (hx : ∀ c, x c ∈ R.sm) : hh Γ R (fun c => io I c y) x = (ep I : ℂ) * ⟪∑ c, pr I c (x c), y⟫_ℂ

theorem **parseval** (hT : TensH ρ Γ R RI io pr ep) (x x' : ι → R.H) (hx : ∀ c, x c ∈ R.sm) (hx' : ∀ c, x' c ∈ R.sm) : hh Γ R x x' = ∑ I, (ep I : ℂ) * ⟪∑ c, pr I c (x' c), ∑ c, pr I c (x c)⟫_ℂ

theorem **tens_adj** : PI.KFinSmooth → ∀ (k r : ℕ) (ι κ : Type) [Fintype ι] [DecidableEq ι] [Fintype κ] [DecidableEq κ] (ρ : Mx k r → Matrix ι ι ℂ) (Γ : Matrix ι ι ℂ) (d : ℕ) (R : Rep k r) (RI : κ → Rep k r) (io : ∀ I, ι → (RI I).H → R.H) (pr : ∀ I, ι → R.H → (RI I).H) (ep : κ → ℝ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (muI : κ → Fin (2 * r + k + 1) → ℝ), R.IsDG ε mu → (∀ I, (RI I).IsDG ε (muI I)) → IsFRep ρ Γ d → TensH ρ Γ R RI io pr ep → (∀ I, ∀ y ∈ (RI I).sm, ∀ x : ι → R.H, (∀ c, x c ∈ R.sm) → hh Γ R (fun c => io I c y) x = (ep I : ℂ) * ⟪∑ c, pr I c (x c), y⟫_ℂ) ∧ (∀ x x' : ι → R.H, (∀ c, x c ∈ R.sm) → (∀ c, x' c ∈ R.sm) → hh Γ R x x' = ∑ I, (ep I : ℂ) * ⟪∑ c, pr I c (x' c), ∑ c, pr I c (x c)⟫_ℂ) ∧ (∀ (a : ι → ℂ) (x : ι → R.H) (v : R.H), hh Γ R x (tm R a v) = ⟪v, ev Γ R a x⟫_ℂ) ∧ (∀ I, ∀ y ∈ (RI I).sm, ‖y‖ ^ 2 ≤ ∑ c, ‖io I c y‖ ^ 2) ∧ ∀ (a a' : ι → ℂ), ∀ v ∈ R.sm

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.FrepDiscreteSeriesSumStatement

theorem **_root_.Gp91.GlobalTheory.T.frep_discrete_series_sum** : PI.KFinSmooth → ∀ (k r : ℕ) (ι κ : Type) [Fintype ι] [DecidableEq ι] [Fintype κ] [DecidableEq κ] (ρ : Mx k r → Matrix ι ι ℂ) (Γ : Matrix ι ι ℂ) (d : ℕ) (R : Rep k r) (RI : κ → Rep k r) (io : ∀ I, ι → (RI I).H → R.H) (pr : ∀ I, ι → R.H → (RI I).H) (ep : κ → ℝ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (muI : κ → Fin (2 * r + k + 1) → ℝ), R.IsDG ε mu → (∀ I, (RI I).IsDG ε (muI I)) → IsFRep ρ Γ d → TensH ρ Γ R RI io pr ep → (∀ I, ∀ y ∈ (RI I).sm, ∀ x : ι → R.H, (∀ c, x c ∈ R.sm) → hh Γ R (fun c => io I c y) x = (ep I : ℂ) * ⟪∑ c, pr I c (x c), y⟫_ℂ) ∧ (∀ x x' : ι → R.H, (∀ c, x c ∈ R.sm) → (∀ c, x' c ∈ R.sm) → hh Γ R x x' = ∑ I, (ep I : ℂ) * ⟪∑ c, pr I c (x' c), ∑ c, pr I c (x c)⟫_ℂ) ∧ (∀ (a : ι → ℂ) (x : ι → R.H) (v : R.H), hh Γ R x (tm R a v) = ⟪v, ev Γ R a x⟫_ℂ) ∧ (∀ I, ∀ y ∈ (RI I).sm, ‖y‖ ^ 2 ≤ ∑ c, ‖io

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.GapCoefficientBound

theorem **integral_comp_mp** {X : Type} [MeasurableSpace X] {μ : Measure X} {f : X → X} (hf : MeasurePreserving f μ μ) {F : X → ℂ} (hF : Integrable F μ) : ∫ x, F x ∂μ = ∫ x, F (f x) ∂μ

theorem **inner_integral_apply** {X Wp : Type} [MeasurableSpace X] {μ : Measure X} [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] {F : X → (Wp →L[ℂ] Wp)} (hF : Integrable F μ) (ξ η : Wp) : ⟪η, (∫ x, F x ∂μ) ξ⟫_ℂ = ∫ x, ⟪η, F x ξ⟫_ℂ ∂μ ∧ ⟪(∫ x, F x ∂μ) ξ, η⟫_ℂ = ∫ x, ⟪F x ξ, η⟫_ℂ ∂μ ∧ Integrable (fun x => ⟪F x ξ, η⟫_ℂ) μ

theorem **GlobalTheory.bessel_norm_sq_double_integral** : HypsG → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (h : ℝ) (R : Rep k r) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (z : Fin (2 * r + k + 1) → ℂ) (π : Mx k r → (Wp →L[ℂ] Wp)) (u : ℕ → ℝ) (μW μN μG νq : Measure (Formalism.QuantumDeterminants.Mt k r)), R.IsDG ε mu → Formalism.QuantumDeterminants.GapAtLeast ε mu h → GapOK k r h → SideG k r z π u μW μN μG νq → ∀ v ∈ R.TK, ∀ w ∈ R.sm, ∀ ξ : Wp, ((‖be z Formalism.QuantumDeterminants.cStd π R.σ μW μN w v ξ‖ ^ 2 : ℝ) : ℂ) = ∫ p : Formalism.QuantumDeterminants.Mt k r × Formalism.QuantumDeterminants.Mt k r, (∫ q : Formalism.QuantumDeterminants.Mt k r × Formalism.QuantumDeterminants.Mt k r, ⟪v, R.σ (Matrix.of p.1 * Matrix.of p.2) w⟫_ℂ * star ⟪v, R.σ (Matrix.of q.1 * Matrix.of q.2 * (Matrix.of p.1 * Mat

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.GapCoefficientBoundDual

theorem **integrable_of_growth** (hGG : GrowthG) (hGS : GrowthS) : HypsG → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (h : ℝ) (R : Rep k r) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (z : Fin (2 * r + k + 1) → ℂ) (π : Mx k r → (Wp →L[ℂ] Wp)) (u : ℕ → ℝ) (μW μN μG νq : Measure (Formalism.QuantumDeterminants.Mt k r)), R.IsDG ε mu → Formalism.QuantumDeterminants.GapAtLeast ε mu h → GapOK k r h → SideG k r z π u μW μN μG νq → ∀ v ∈ R.TK, ∀ w ∈ R.sm, Integrable (fun y : Formalism.QuantumDeterminants.Mt k r × (Formalism.QuantumDeterminants.Mt k r × Formalism.QuantumDeterminants.Mt k r) => ⟪v, R.σ (Matrix.of y.1) w⟫_ℂ * star ⟪v, R.σ (Matrix.of y.2.1 * Matrix.of y.2.2 * Matrix.of y.1) w⟫_ℂ) (μG.prod (μW.prod μN))

theorem **integrable_GS** : HypsG → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (h : ℝ) (R : Rep k r) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (z : Fin (2 * r + k + 1) → ℂ) (π : Mx k r → (Wp →L[ℂ] Wp)) (u : ℕ → ℝ) (μW μN μG νq : Measure (Formalism.QuantumDeterminants.Mt k r)), R.IsDG ε mu → Formalism.QuantumDeterminants.GapAtLeast ε mu h → GapOK k r h → SideG k r z π u μW μN μG νq → ∀ v ∈ R.TK, ∀ w ∈ R.sm, Integrable (fun y : Formalism.QuantumDeterminants.Mt k r × (Formalism.QuantumDeterminants.Mt k r × Formalism.QuantumDeterminants.Mt k r) => ⟪v, R.σ (Matrix.of y.1) w⟫_ℂ * star ⟪v, R.σ (Matrix.of y.2.1 * Matrix.of y.2.2 * Matrix.of y.1) w⟫_ℂ) (μG.prod (μW.prod μN))

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.GapCoefficientBoundDualStatement

theorem **_root_.Gp91.GlobalTheory.T.integrable_coefficient_mul_star_translate** : HypsG → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (h : ℝ) (R : Rep k r) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (z : Fin (2 * r + k + 1) → ℂ) (π : Mx k r → (Wp →L[ℂ] Wp)) (u : ℕ → ℝ) (μW μN μG νq : Measure (Formalism.QuantumDeterminants.Mt k r)), R.IsDG ε mu → Formalism.QuantumDeterminants.GapAtLeast ε mu h → GapOK k r h → SideG k r z π u μW μN μG νq → ∀ v ∈ R.TK, ∀ w ∈ R.sm, Integrable (fun y : Formalism.QuantumDeterminants.Mt k r × (Formalism.QuantumDeterminants.Mt k r × Formalism.QuantumDeterminants.Mt k r) => ⟪v, R.σ (Matrix.of y.1) w⟫_ℂ * star ⟪v, R.σ (Matrix.of y.2.1 * Matrix.of y.2.2 * Matrix.of y.1) w⟫_ℂ) (μG.prod (μW.prod μN))

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.GapCoefficientBoundStatement

theorem **_root_.Gp91.GlobalTheory.T.coefficient_psi_pairing_eq_iterated_integral** : HypsG → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (h : ℝ) (R : Rep k r) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (z : Fin (2 * r + k + 1) → ℂ) (π : Mx k r → (Wp →L[ℂ] Wp)) (u : ℕ → ℝ) (μW μN μG νq : Measure (Formalism.QuantumDeterminants.Mt k r)), R.IsDG ε mu → Formalism.QuantumDeterminants.GapAtLeast ε mu h → GapOK k r h → SideG k r z π u μW μN μG νq → ∀ v ∈ R.TK, ∀ w ∈ R.sm, ∀ ξ : Wp, ((‖be z Formalism.QuantumDeterminants.cStd π R.σ μW μN w v ξ‖ ^ 2 : ℝ) : ℂ) = ∫ p : Formalism.QuantumDeterminants.Mt k r × Formalism.QuantumDeterminants.Mt k r, (∫ q : Formalism.QuantumDeterminants.Mt k r × Formalism.QuantumDeterminants.Mt k r, ⟪v, R.σ (Matrix.of p.1 * Matrix.of p.2) w⟫_ℂ * star ⟪v, R.σ (Matrix.of q.1 * Matrix.o

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.GapCoefficientDerivativeBound

theorem **l2SG_of_ae_smul** {k r : ℕ} {Wp : Type} [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] (νq : Measure (Formalism.QuantumDeterminants.Mt k r)) (F G : Mx k r → Wp) (c : ℝ) (h : ∀ᵐ x ∂νq, F (Matrix.of x) = (c : ℂ) • G (Matrix.of x)) : PI.BesselPeriodGGP.l2SG νq F = ENNReal.ofReal (c ^ 2) * PI.BesselPeriodGGP.l2SG νq G

theorem **scaled_bessel_isometric** : HypsG → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (h : ℝ) (R : Rep k r) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (z : Fin (2 * r + k + 1) → ℂ) (π : Mx k r → (Wp →L[ℂ] Wp)) (u : ℕ → ℝ) (μW μN μG νq : Measure (Formalism.QuantumDeterminants.Mt k r)) (l : R.H → Wp) (dd : ℝ), R.IsDG ε mu → Formalism.QuantumDeterminants.GapAtLeast ε mu h → GapOK k r h → SideG k r z π u μW μN μG νq → IsBeL R z π μW μN l → (∀ w, w ≠ 0 → PI.BesselPeriodGGP.fdegH μG R.σ w = dd) → NzOn R l → IsIsoB R z π νq (fun w => ((Real.sqrt dd : ℝ) : ℂ) • l w)

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.GapCoefficientDerivativeBoundStatement

theorem **_root_.Gp91.GlobalTheory.T.isIsoB_sqrt_fdeg_smul_of_gap** : HypsG → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (h : ℝ) (R : Rep k r) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (z : Fin (2 * r + k + 1) → ℂ) (π : Mx k r → (Wp →L[ℂ] Wp)) (u : ℕ → ℝ) (μW μN μG νq : Measure (Formalism.QuantumDeterminants.Mt k r)) (l : R.H → Wp) (dd : ℝ), R.IsDG ε mu → Formalism.QuantumDeterminants.GapAtLeast ε mu h → GapOK k r h → SideG k r z π u μW μN μG νq → IsBeL R z π μW μN l → (∀ w, w ≠ 0 → PI.BesselPeriodGGP.fdegH μG R.σ w = dd) → NzOn R l → IsIsoB R z π νq (fun w => ((Real.sqrt dd : ℝ) : ℂ) • l w)

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.HaarEqSmul

theorem **haar_eq_smul** {k r : ℕ} {ε : Fin (2 * r + k + 1) → Bool} {P : Mx k r} (hP : IsUnit P.det) (hH : Pᴴ * Formalism.REAL.Jm ε * P = Formalism.QuantumDeterminants.Hpk k r) (ν : Measure (Formalism.QuantumDeterminants.Mt k r)) (hν : PI.BesselPeriodGGP.IsHaarOn (GG k r) ν) : ∃ c : ℝ≥0∞, c ≠ 0 ∧ c ≠ ∞ ∧ ν = c • (Formalism.REAL.muG ε).map (phi ε P)

theorem **schur_fdeg** : HypsG → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (μG : Measure (Formalism.QuantumDeterminants.Mt k r)), PI.BesselPeriodGGP.IsHaarOn (GG k r) μG → ∃ c : ℝ, 0 < c ∧ ∀ (mu : Fin (2 * r + k + 1) → ℝ) (R : Rep k r), R.IsDG ε mu → (∀ w, w ≠ 0 → PI.BesselPeriodGGP.fdegH μG R.σ w = c * Formalism.QuantumDeterminants.fdegHC mu) ∧ ∀ v w v' w' : R.H, Integrable (fun x : Formalism.QuantumDeterminants.Mt k r => ⟪w, R.σ (Matrix.of x) v⟫_ℂ * star ⟪w', R.σ (Matrix.of x) v'⟫_ℂ) μG ∧ ∫ x, ⟪w, R.σ (Matrix.of x) v⟫_ℂ * star ⟪w', R.σ (Matrix.of x) v'⟫_ℂ ∂μG = ⟪v', v⟫_ℂ * ⟪w, w'⟫_ℂ / ((c * Formalism.QuantumDeterminants.fdegHC mu : ℝ) : ℂ)

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.HsqNonneg

theorem **hsq_nonneg** (r : ℕ) (T b : ℝ) : 0 ≤ Hsq r T b

theorem **arith** (a a' d e A A' : ℝ) (n : ℕ) (ha : 0 ≤ a) (ha' : 0 ≤ a') (hd : 0 < d) (he : 0 < e) (hA : 0 ≤ A) (hA' : 0 ≤ A') (h : a ^ 2 * d / e ≤ A ^ n) (h' : a' ^ 2 * d / e ≤ A' ^ n) : a' * a * d / e ≤ Real.sqrt (A * A') ^ n

theorem **tau_bound** : HypsG → ∀ (P : M.Par) (Wp Wq : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wp] [FiniteDimensional ℂ Wq] (p : KB.Pl P Wp Wq) (lam : Fin (2 * P.r + P.k + 1) → ℝ) (b bv ga h : ℝ) (q0 : ℕ) (R : (Fin (2 * P.r + P.k + 1) → ℤ) → Rep P.k P.r) (l : ∀ ν, (R ν).H → Wp) (l' : ∀ ν, (R ν).H → Wq) (dg : (Fin (2 * P.r + P.k + 1) → ℤ) → ℝ) (s : (Fin (2 * P.r + P.k + 1) → ℤ) → ℂ) (J : (Fin (2 * P.r + P.k + 1) → ℤ) → B.Mat P.k P.r → (Wq →L[ℂ] Wp)) (tau : (Fin (2 * P.r + P.k + 1) → ℤ) → Finset (Fin (2 * P.r + P.k + 1))) → ℂ), Num P.k P.r lam P.u b bv ga h q0 → Num P.k P.r lam P.u' b bv ga h q0 → SupG p lam q0 R l l' dg s J tau → ∀ ν, n1 ν + (P.r + 1) * (P.r + 2) ≤ q0 → ∀ I : Finset (Fin (2 * P.r + P.k + 1)), I.card = P.r + 1 → ‖tau ν I‖ ≤ Real.sqrt (Hsq P.r P.T b * Hsq P.r P.T' b) ^ (P.r + 1)

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.IsCoefContinuousOn

theorem **isDS_continuousOn** {k r : ℕ} {R : Rep k r} {ε : Fin (2 * r + k + 1) → Bool} {μ : Fin (2 * r + k + 1) → ℝ} (h : R.IsDG ε μ) (v : R.H) : ContinuousOn (fun g => R.σ g v) (GG k r)

theorem **isCoef_continuousOn** {k r : ℕ} {R : Rep k r} {ε : Fin (2 * r + k + 1) → Bool} {μ : Fin (2 * r + k + 1) → ℝ} {Wp Wq : Type} [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] {l : R.H → Wp} {l' : R.H → Wq} {F : Mx k r → (Wq →L[ℂ] Wp)} {s : ℂ} (h : R.IsDG ε μ) (hF : IsCoef R l l' F s) : ContinuousOn F (GG k r)

theorem **sup_meas** : HypsG → ∀ (P : M.Par) (Wp Wq : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wp] [FiniteDimensional ℂ Wq] (p : KB.Pl P Wp Wq) (lam : Fin (2 * P.r + P.k + 1) → ℝ) (b bv ga h : ℝ) (q0 : ℕ) (R : (Fin (2 * P.r + P.k + 1) → ℤ) → Rep P.k P.r) (l : ∀ ν, (R ν).H → Wp) (l' : ∀ ν, (R ν).H → Wq) (dg : (Fin (2 * P.r + P.k + 1) → ℤ) → ℝ) (s : (Fin (2 * P.r + P.k + 1) → ℤ) → ℂ) (J : (Fin (2 * P.r + P.k + 1) → ℤ) → B.Mat P.k P.r → (Wq →L[ℂ] Wp)) (tau : (Fin (2 * P.r + P.k + 1) → ℤ) → Finset (Fin (2 * P.r + P.k + 1))) → ℂ), Num P.k P.r lam P.u b bv ga h q0 → SupG p lam q0 R l l' dg s J tau → ∀ ν, AEMeasurable fun c : Fin (P.r + 1) → ℂ => ENNReal.ofReal (TBB.Ew p (fun _ => 1) (J ν) (P.r + 1) [] (TBB.bigPt P.r c))

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.IsDSOfIsDG

theorem **_root_.GlobalTheory.isDS_of_isDG** {R : Rep k r} {ε : Fin (2 * r + k + 1) → Bool} {mu : Fin (2 * r + k + 1) → ℝ} (h : R.IsDG ε mu) : R.IsDS ε mu

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.IsometricBesselCompare

theorem **inner_toLp** {f g : X → E} (hf : MemLp f 2 ν) (hg : MemLp g 2 ν) : ⟪hf.toLp f, hg.toLp g⟫_ℂ = ∫ x, ⟪f x, g x⟫_ℂ ∂ν

theorem **integrable_inner** {f g : X → E} (hf : MemLp f 2 ν) (hg : MemLp g 2 ν) : Integrable (fun x => ⟪f x, g x⟫_ℂ) ν

theorem **norm_toLp_eq** {H : Type} [NormedAddCommGroup H] [InnerProductSpace ℂ H] {f : X → E} (hf : MemLp f 2 ν) {a : H} (h : ∫ x, ⟪f x, f x⟫_ℂ ∂ν = ⟪a, a⟫_ℂ) : ‖hf.toLp f‖ = ‖a‖

theorem **norm_integral_inner_le** {H H' : Type} [NormedAddCommGroup H] [InnerProductSpace ℂ H] [NormedAddCommGroup H'] [InnerProductSpace ℂ H'] {f g : X → E} (hf : MemLp f 2 ν) (hg : MemLp g 2 ν) {a : H} {b : H'} (h : ∫ x, ⟪f x, f x⟫_ℂ ∂ν = ⟪a, a⟫_ℂ) (h' : ∫ x, ⟪g x, g x⟫_ℂ ∂ν = ⟪b, b⟫_ℂ) : ‖∫ x, ⟪f x, g x⟫_ℂ ∂ν‖ ≤ ‖a‖ * ‖b‖

theorem **bessel_orth_of_param_ne** : HypsG → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (mu' : Fin (2 * r + k + 1) → ℝ) (R R' : Rep k r) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (z : Fin (2 * r + k + 1) → ℂ) (π : Mx k r → (Wp →L[ℂ] Wp)) (u : ℕ → ℝ) (μW μN μG νq : Measure (Formalism.QuantumDeterminants.Mt k r)) (Bm : R.H → Wp) (Bm' : R'.H → Wp), R.IsDG ε mu → R'.IsDG ε mu' → mu ≠ mu' → SideG k r z π u μW μN μG νq → IsIsoB R z π νq Bm → IsIsoB R' z π νq Bm' → ∀ w ∈ R.sm, ∀ y ∈ R'.sm, ∫ x, ⟪Bm' (R'.σ (Matrix.of x) y), Bm (R.σ (Matrix.of x) w)⟫_ℂ ∂νq = 0

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.IsometricBesselCompareStatement

theorem **_root_.Gp91.GlobalTheory.T.isometric_bessel_compare** : HypsG → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (mu' : Fin (2 * r + k + 1) → ℝ) (R R' : Rep k r) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (z : Fin (2 * r + k + 1) → ℂ) (π : Mx k r → (Wp →L[ℂ] Wp)) (u : ℕ → ℝ) (μW μN μG νq : Measure (Formalism.QuantumDeterminants.Mt k r)) (Bm : R.H → Wp) (Bm' : R'.H → Wp), R.IsDG ε mu → R'.IsDG ε mu' → mu ≠ mu' → SideG k r z π u μW μN μG νq → IsIsoB R z π νq Bm → IsIsoB R' z π νq Bm' → ∀ w ∈ R.sm, ∀ y ∈ R'.sm, ∫ x, ⟪Bm' (R'.σ (Matrix.of x) y), Bm (R.σ (Matrix.of x) w)⟫_ℂ ∂νq = 0

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.IsometricBesselSideBound

theorem **eLpNorm_two_of_lintegral_sq** {α E : Type*} [MeasurableSpace α] {μ : Measure α} [NormedAddCommGroup E] (f : α → E) (c : ℝ) (hc : 0 ≤ c) (h : ∫⁻ x, (‖f x‖₊ : ℝ≥0∞) ^ 2 ∂μ = ENNReal.ofReal (c ^ 2)) : eLpNorm f 2 μ = ENNReal.ofReal c

theorem **integral_inner_of_isometric** {α E F : Type*} [MeasurableSpace α] {μ : Measure α} [NormedAddCommGroup E] [InnerProductSpace ℂ E] [NormedAddCommGroup F] [InnerProductSpace ℂ F] (S : Submodule ℂ E) (Φ : E → α → F) (hm : ∀ w ∈ S, MemLp (Φ w) 2 μ) (hn : ∀ w ∈ S, eLpNorm (Φ w) 2 μ = ENNReal.ofReal ‖w‖) (hadd : ∀ v ∈ S, ∀ w ∈ S, Φ (v + w) =ᵐ[μ] Φ v + Φ w) (hsmul : ∀ (c : ℂ), ∀ v ∈ S, Φ (c • v) =ᵐ[μ] c • Φ v) : ∀ w ∈ S, ∀ w' ∈ S, ∫ x, ⟪Φ w' x, Φ w x⟫_ℂ ∂μ = ⟪w', w⟫_ℂ

theorem **isoBessel_polarised** : PI.KFinSmooth → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (R : Rep k r) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (z : Fin (2 * r + k + 1) → ℂ) (π : Mx k r → (Wp →L[ℂ] Wp)) (u : ℕ → ℝ) (μW μN μG νq : Measure (Formalism.QuantumDeterminants.Mt k r)) (Bm : R.H → Wp), R.IsDG ε mu → SideG k r z π u μW μN μG νq → IsIsoB R z π νq Bm → (∀ w ∈ R.sm, MemLp (fun x : Formalism.QuantumDeterminants.Mt k r => Bm (R.σ (Matrix.of x) w)) 2 νq) ∧ ∀ w ∈ R.sm, ∀ w' ∈ R.sm, ∫ x, ⟪Bm (R.σ (Matrix.of x) w'), Bm (R.σ (Matrix.of x) w)⟫_ℂ ∂νq = ⟪w', w⟫_ℂ

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.IsometricBesselSideBoundStatement

theorem **_root_.Gp91.GlobalTheory.T.isometric_bessel_memLp_and_integral_inner_eq** : PI.KFinSmooth → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (R : Rep k r) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (z : Fin (2 * r + k + 1) → ℂ) (π : Mx k r → (Wp →L[ℂ] Wp)) (u : ℕ → ℝ) (μW μN μG νq : Measure (Formalism.QuantumDeterminants.Mt k r)) (Bm : R.H → Wp), R.IsDG ε mu → SideG k r z π u μW μN μG νq → IsIsoB R z π νq Bm → (∀ w ∈ R.sm, MemLp (fun x : Formalism.QuantumDeterminants.Mt k r => Bm (R.σ (Matrix.of x) w)) 2 νq) ∧ ∀ w ∈ R.sm, ∀ w' ∈ R.sm, ∫ x, ⟪Bm (R.σ (Matrix.of x) w'), Bm (R.σ (Matrix.of x) w)⟫_ℂ ∂νq = ⟪w', w⟫_ℂ

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.KFiniteDerivativesFiniteSpan

theorem **exists_box_generators** (k r : ℕ) : ∃ (J : Type) (_ : Fintype J) (Z : J → Mx k r), (∀ j, InLieHpk k r (Z j)) ∧ ∀ Y, B.unitLie k r Y → ∃ c : J → ℝ, (∀ j, |c j| ≤ 1) ∧ Y = ∑ j, c j • Z j

theorem **dsig_sum_letter** {H : Type} [NormedAddCommGroup H] [InnerProductSpace ℂ H] {σ : Mx k r → (H →L[ℂ] H)} {J : Type} [Fintype J] {Z : J → Mx k r} (hZ : ∀ j, InLieHpk k r (Z j)) (c : J → ℝ) {v : H} (hv : v ∈ smoothVecs (k

theorem **dsig_sum_vec** (R : Rep k r) {X : Mx k r} (hX : InLieHpk k r X) {ι : Type} (s : Finset ι) (c : ι → ℂ) {v : ι → R.H} (hv : ∀ i, v i ∈ R.sm) : dsig (k

theorem **box_words_fintype** (hK : PI.KFinSmooth) (R : Rep k r) (hR : R.IsDG ε mu) {x : R.H} (hx : x ∈ R.TK) (m : ℕ) : ∃ (ι : Type) (_ : Fintype ι) (xs : ι → R.H), (∀ i, xs i ∈ R.TK) ∧ ∀ a : List (Mx k r), (∀ Y ∈ a, B.unitLie k r Y) → a.length ≤ m → ∃ co : ι → ℝ, (∀ i, |co i| ≤ 1) ∧ dsigW (k

theorem **box_words** : PI.KFinSmooth → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (R : Rep k r), R.IsDG ε mu → ∀ x ∈ R.TK, ∀ m : ℕ, ∃ (M : ℕ) (xs : Fin M → R.H), (∀ i, xs i ∈ R.TK) ∧ ∀ a : List (Mx k r), (∀ Y ∈ a, B.unitLie k r Y) → a.length ≤ m → ∃ co : Fin M → ℝ, (∀ i, |co i| ≤ 1) ∧ PI.UpqRep.dsigW (k

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.KFiniteDerivativesFiniteSpanStatement

theorem **_root_.Gp91.GlobalTheory.T.k_finite_derivatives_finite_span** : PI.KFinSmooth → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (R : Rep k r), R.IsDG ε mu → ∀ x ∈ R.TK, ∀ m : ℕ, ∃ (M : ℕ) (xs : Fin M → R.H), (∀ i, xs i ∈ R.TK) ∧ ∀ a : List (Mx k r), (∀ Y ∈ a, B.unitLie k r Y) → a.length ≤ m → ∃ co : Fin M → ℝ, (∀ i, |co i| ≤ 1) ∧ PI.UpqRep.dsigW (k

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.KStableFiniteDimSmoothCompactRootStable

theorem **deriv_mem** {H : Type} [NormedAddCommGroup H] [NormedSpace ℂ H] (S : Submodule ℂ H) [FiniteDimensional ℂ S] {f : ℝ → H} {f' : H} (hf : HasDerivAt f f' 0) (hS : ∀ t, f t ∈ S) : f' ∈ S

theorem **dsig_mem** (R : Rep k r) (S : Submodule ℂ R.H) [FiniteDimensional ℂ S] (hS : ∀ g ∈ UH (Hpk k r) ⊓ (Matrix.unitaryGroup (Fin (2 * r + k + 1)) ℂ : Set _), ∀ x ∈ S, R.σ g x ∈ S) {X : Mx k r} (hX : Xᴴ = -X) (hc : Hpk k r * X = X * Hpk k r) {v : R.H} (hv : v ∈ S) (hsm : v ∈ R.sm) : dsig (k

theorem **dC_mem** (R : Rep k r) (S : Submodule ℂ R.H) [FiniteDimensional ℂ S] (hS : ∀ g ∈ UH (Hpk k r) ⊓ (Matrix.unitaryGroup (Fin (2 * r + k + 1)) ℂ : Set _), ∀ x ∈ S, R.σ g x ∈ S) (h1 : (Hpk k r)ᴴ = Hpk k r) (h2 : Hpk k r * Hpk k r = 1) {Y : Mx k r} (hc : Hpk k r * Y = Y * Hpk k r) {v : R.H} (hv : v ∈ S) (hsm : v ∈ R.sm) : R.dC Y v ∈ S

theorem **Jm_single** {n : ℕ} (ε : Fin n → Bool) {a b : Fin n} (h : ε a = ε b) : Jm ε * Matrix.single a b (1 : ℂ) = Matrix.single a b 1 * Jm ε

theorem **kStable_smooth_and_stable** : PI.KFinSmooth → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (R : Rep k r), R.IsDG ε mu → ∀ (Q : Mx k r), PI.UpqRep.IsCayleyU k r ε Q → ∀ S : Submodule ℂ R.H, FiniteDimensional ℂ S → PI.UpqRep.IsKStableOn ε (fun g => R.σ (Q⁻¹ * g * Q)) S → (∀ v ∈ S, R.KFin v ∧ v ∈ R.sm) ∧ ∀ a b, ε a = ε b → ∀ v ∈ S, R.dC (Q⁻¹ * Matrix.single a b 1 * Q) v ∈ S

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.KStableFiniteDimSmoothCompactRootStableStatement

theorem **_root_.Gp91.GlobalTheory.T.KStable_finiteDim_smooth_compactRoot_stable** : PI.KFinSmooth → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (R : Rep k r), R.IsDG ε mu → ∀ (Q : Mx k r), PI.UpqRep.IsCayleyU k r ε Q → ∀ S : Submodule ℂ R.H, FiniteDimensional ℂ S → PI.UpqRep.IsKStableOn ε (fun g => R.σ (Q⁻¹ * g * Q)) S → (∀ v ∈ S, R.KFin v ∧ v ∈ R.sm) ∧ ∀ a b, ε a = ε b → ∀ v ∈ S, R.dC (Q⁻¹ * Matrix.single a b 1 * Q) v ∈ S

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.MatrixCoefficientDecay

theorem **pair_bound** (H : HypsG) {k r : ℕ} {ε : Fin (2 * r + k + 1) → Bool} {mu : Fin (2 * r + k + 1) → ℝ} {h : ℝ} (R : Rep k r) (hR : R.IsDG ε mu) (hgap : Formalism.QuantumDeterminants.GapAtLeast ε mu h) (hok : GapOK k r h) (x y : R.H) (hx : x ∈ R.TK) (hy : y ∈ R.TK) : ∃ c : ℝ, ∀ g ∈ GG k r, B.gN g ^ (aOf r h - 1) * ‖⟪x, R.σ g y⟫_ℂ‖ ≤ c

theorem **coef_bound** : HypsG → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (h : ℝ) (R : Rep k r), R.IsDG ε mu → Formalism.QuantumDeterminants.GapAtLeast ε mu h → GapOK k r h → ∀ x ∈ R.TK, ∀ y ∈ R.TK, ∀ m : ℕ, ∃ c : ℝ, ∀ a b : List (Mx k r), (∀ Y ∈ a, B.unitLie k r Y) → (∀ Y ∈ b, B.unitLie k r Y) → a.length + b.length ≤ m → ∀ g ∈ GG k r, B.gN g ^ (aOf r h - 1) * ‖⟪PI.UpqRep.dsigW (k

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.MatrixCoefficientDecayStatement

theorem **_root_.Gp91.GlobalTheory.T.matrix_coefficient_decay** : HypsG → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (h : ℝ) (R : Rep k r), R.IsDG ε mu → Formalism.QuantumDeterminants.GapAtLeast ε mu h → GapOK k r h → ∀ x ∈ R.TK, ∀ y ∈ R.TK, ∀ m : ℕ, ∃ c : ℝ, ∀ a b : List (Mx k r), (∀ Y ∈ a, B.unitLie k r Y) → (∀ Y ∈ b, B.unitLie k r Y) → a.length + b.length ≤ m → ∀ g ∈ GG k r, B.gN g ^ (aOf r h - 1) * ‖⟪PI.UpqRep.dsigW (k

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.MatrixCoefficientSmoothDerivatives

theorem **coefL_apply** (x w : R.H) (X : V) : coefL R x X w = ⟪x, w⟫_ℂ • X

theorem **dsigW_smooth** (hR : R.IsDG ε mu) {y : R.H} (hy : y ∈ R.sm) (b : List (Mx k r)) (hb : ∀ Y ∈ b, B.inLie k r Y) : PI.UpqRep.dsigW (k

theorem **right_step** (hK : PI.KFinSmooth) (hR : R.IsDG ε mu) (X : V) (F : Mx k r → V) (x : R.H) {y : R.H} (hy : y ∈ R.sm) (hF : ∀ g ∈ GG k r, F g = ⟪x, R.σ g y⟫_ℂ • X) {Y : Mx k r} (hY : B.inLie k r Y) (g : Mx k r) (hg : g ∈ GG k r) : deriv (fun t : ℝ => F (g * NormedSpace.exp ((t : ℂ) • Y))) 0 = ⟪x, R.σ g (PI.UpqRep.dsig (k

theorem **left_step** (hK : PI.KFinSmooth) (hR : R.IsDG ε mu) (X : V) (F : Mx k r → V) {x : R.H} (y : R.H) (hx : x ∈ R.sm) (hF : ∀ g ∈ GG k r, F g = ⟪x, R.σ g y⟫_ℂ • X) {Y : Mx k r} (hY : B.inLie k r Y) (g : Mx k r) (hg : g ∈ GG k r) : B.LdV Y F g = ⟪PI.UpqRep.dsig (k

theorem **coeff_dLR** : PI.KFinSmooth → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (R : Rep k r) (V : Type) [NormedAddCommGroup V] [NormedSpace ℂ V] (X : V) (F : Mx k r → V), R.IsDG ε mu → ∀ x ∈ R.sm, ∀ y ∈ R.sm, (∀ g ∈ GG k r, F g = ⟪x, R.σ g y⟫_ℂ • X) → B.CkOn ⊤ (B.GV k r) F ∧ ∀ a b : List (Mx k r), (∀ Y ∈ a, B.inLie k r Y) → (∀ Y ∈ b, B.inLie k r Y) → ∀ g ∈ GG k r, B.dLR a b F g = ⟪PI.UpqRep.dsigW (k

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.MatrixCoefficientSmoothDerivativesStatement

theorem **_root_.Gp91.GlobalTheory.T.matrix_coefficient_smooth_derivatives** : PI.KFinSmooth → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (R : Rep k r) (V : Type) [NormedAddCommGroup V] [NormedSpace ℂ V] (X : V) (F : Mx k r → V), R.IsDG ε mu → ∀ x ∈ R.sm, ∀ y ∈ R.sm, (∀ g ∈ GG k r, F g = ⟪x, R.σ g y⟫_ℂ • X) → B.CkOn ⊤ (B.GV k r) F ∧ ∀ a b : List (Mx k r), (∀ Y ∈ a, B.inLie k r Y) → (∀ Y ∈ b, B.inLie k r Y) → ∀ g ∈ GG k r, B.dLR a b F g = ⟪PI.UpqRep.dsigW (k

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.N1AddEZLe

theorem **n1_add_eZ_le** {n : ℕ} (ν : Fin n → ℤ) (I : Finset (Fin n)) : n1 (ν + eZ I) ≤ n1 ν + I.card

theorem **JF_zero** {k r : ℕ} {Wp Wq : Type} [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] (D : B.RealPlaceData k r Wp Wq) (g : B.Mat k r) : B.JF D 0 g = 0

theorem **supplied_pieri** : HypsG → ∀ (P : M.Par) (Wp Wq : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wp] [FiniteDimensional ℂ Wq] (p : KB.Pl P Wp Wq) (lam : Fin (2 * P.r + P.k + 1) → ℝ) (b bv ga h : ℝ) (q0 : ℕ) (R : (Fin (2 * P.r + P.k + 1) → ℤ) → Rep P.k P.r) (l : ∀ ν, (R ν).H → Wp) (l' : ∀ ν, (R ν).H → Wq) (dg : (Fin (2 * P.r + P.k + 1) → ℤ) → ℝ) (s : (Fin (2 * P.r + P.k + 1) → ℤ) → ℂ) (J : (Fin (2 * P.r + P.k + 1) → ℤ) → B.Mat P.k P.r → (Wq →L[ℂ] Wp)) (tau : (Fin (2 * P.r + P.k + 1) → ℤ) → Finset (Fin (2 * P.r + P.k + 1)) → ℂ), Num P.k P.r lam P.u b bv ga h q0 → SupG p lam q0 R l l' dg s J tau → ∀ ν, n1 ν + (P.r + 1) * (P.r + 2) ≤ q0 → ∀ c : Fin (P.r + 1) → ℂ, D.IsLocPt (TBB.bigPt P.r c) (P.r + 1) → 0 < TBB.Gp p (TBB.bigPt P.r c) → (D.ptMatN (TBB.bigPt P.r c) (P.r + 1)).det • p.V (J ν) (TBB.bigPt P.r c) =

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.PieriTensorDecomposition

theorem **l1_add_le** {n : ℕ} (x y : Fin n → ℤ) : n1 (x + y) ≤ n1 x + n1 y

theorem **l1_sub_le** {n : ℕ} (x y : Fin n → ℤ) : n1 (x - y) ≤ n1 x + n1 y

theorem **n1_eZ_eq_card** {n : ℕ} (I : Finset (Fin n)) : n1 (eZ I) = I.card

theorem **l1_single** {n : ℕ} (s : Fin n) : n1 (Pi.single s (1 : ℤ)) = 1

theorem **addSet_inj** {n l : ℕ} (mu : Fin n → ℝ) : Function.Injective (fun I : Sub n l => addSet mu I.1)

theorem **sub_single_inj** {n : ℕ} (mu : Fin n → ℝ) : Function.Injective (fun s : Fin n => mu - (Pi.single s 1 : Fin n → ℝ))

theorem **tens_exists** : HypsG → ∀ (k r : ℕ) (lam : Fin (2 * r + k + 1) → ℝ) (u : ℕ → ℝ) (b bv ga h : ℝ) (q0 : ℕ) (R : (Fin (2 * r + k + 1) → ℤ) → Rep k r), Num k r lam u b bv ga h q0 → IsFamG lam q0 R → ∀ ν, (∀ l, n1 ν + l ≤ q0 → ∃ io pr ep, TensH (cmp l) (cmp l (Formalism.QuantumDeterminants.Hpk k r)) (R ν) (fun I : Sub (2 * r + k + 1) l => R (ν + eZ I.1)) io pr ep) ∧ (n1 ν + 1 ≤ q0 → ∃ io pr ep, TensH dualM (Formalism.QuantumDeterminants.Hpk k r) (R ν) (fun s => R (ν - Pi.single s 1)) io pr ep)

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.PieriTensorDecompositionStatement

theorem **_root_.Gp91.GlobalTheory.T.pieri_tensor_decomposition** : HypsG → ∀ (k r : ℕ) (lam : Fin (2 * r + k + 1) → ℝ) (u : ℕ → ℝ) (b bv ga h : ℝ) (q0 : ℕ) (R : (Fin (2 * r + k + 1) → ℤ) → Rep k r), Num k r lam u b bv ga h q0 → IsFamG lam q0 R → ∀ ν, (∀ l, n1 ν + l ≤ q0 → ∃ io pr ep, TensH (cmp l) (cmp l (Formalism.QuantumDeterminants.Hpk k r)) (R ν) (fun I : Sub (2 * r + k + 1) l => R (ν + eZ I.1)) io pr ep) ∧ (n1 ν + 1 ≤ q0 → ∃ io pr ep, TensH dualM (Formalism.QuantumDeterminants.Hpk k r) (R ν) (fun s => R (ν - Pi.single s 1)) io pr ep)

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.QuantumDeterminantEigenvalue

lemma **entry_conj** {n : ℕ} (P P' : Matrix (Fin n) (Fin n) ℂ) (a b c d : Fin n) : (P * Matrix.single a b (1 : ℂ) * P') c d = P c a * P' b d

theorem **qdet_smooth** : HypsG → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (R : Rep k r), R.IsDG ε mu → ∀ (Q : Mx k r) (E : (Fin (2 * r + k + 1) → Fin (2 * r + k + 1) → Module.End ℂ R.smSub)), IsUnit Q → R.IsE Q E → ∀ (Y : ℂ) (w : R.smSub), Formalism.REAL.qdet E (Y + ((((2 * r + k + 1) : ℕ) : ℂ) - 1) / 2) w = (∏ s, (Y + (mu s : ℂ))) • w

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.QuantumDeterminantEigenvalueStatement

theorem **_root_.Gp91.GlobalTheory.T.quantum_determinant_eigenvalue** : HypsG → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (R : Rep k r), R.IsDG ε mu → ∀ (Q : Mx k r) (E : (Fin (2 * r + k + 1) → Fin (2 * r + k + 1)) → Module.End ℂ R.smSub)), IsUnit Q → R.IsE Q E → ∀ (Y : ℂ) (w : R.smSub), Formalism.REAL.qdet E (Y + ((((2 * r + k + 1) : ℕ) : ℂ) − 1) / 2) w = (∏ s, (Y + (mu s : ℂ))) • w

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.QuotientMeasureAeMemAndMeasurableBesselModel

theorem **quotMeas_ae_mem_and_measurable** : PI.KFinSmooth → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (R : Rep k r) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (z : Fin (2 * r + k + 1) → ℂ) (π : Mx k r → (Wp →L[ℂ] Wp)) (u : ℕ → ℝ) (μW μN μG νq : Measure (Formalism.QuantumDeterminants.Mt k r)), R.IsDG ε mu → SideG k r z π u μW μN μG νq → (∀ᵐ x ∂νq, (Matrix.of x) ∈ GG k r) ∧ (∀ (V : Type) [NormedAddCommGroup V] [SecondCountableTopology V] (φ : Mx k r → V), ContinuousOn φ (GG k r) → AEStronglyMeasurable (fun x : Formalism.QuantumDeterminants.Mt k r => φ (Matrix.of x)) νq) ∧ (∀ a, AEMeasurable (fun x : Formalism.QuantumDeterminants.Mt k r => lenA k r z (Matrix.of x) a) νq) ∧ ∀ l, IsBes R z π l → ∀ w ∈ R.sm, AEStronglyMeasurable (fun x : Formalism.QuantumDeterminants.Mt k r => l (R.σ (Matrix.of x) w))

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.QuotientMeasureAeMemAndMeasurableBesselModelStatement

theorem **_root_.Gp91.GlobalTheory.T.quotientMeasure_ae_mem_and_measurable_besselModel** : PI.KFinSmooth → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (R : Rep k r) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (z : Fin (2 * r + k + 1) → ℂ) (π : Mx k r → (Wp →L[ℂ] Wp)) (u : ℕ → ℝ) (μW μN μG νq : Measure (Formalism.QuantumDeterminants.Mt k r)), R.IsDG ε mu → SideG k r z π u μW μN μG νq → (∀ᵐ x ∂νq, (Matrix.of x) ∈ GG k r) ∧ (∀ (V : Type) [NormedAddCommGroup V] [SecondCountableTopology V] (φ : Mx k r → V), ContinuousOn φ (GG k r) → AEStronglyMeasurable (fun x : Formalism.QuantumDeterminants.Mt k r => φ (Matrix.of x)) νq) ∧ (∀ a, AEMeasurable (fun x : Formalism.QuantumDeterminants.Mt k r => lenA k r z (Matrix.of x) a) νq) ∧ ∀ l, IsBes R z π l → ∀ w ∈ R.sm, AEStronglyMeasurable (fun x : Formalism.QuantumDe

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.SmoothMapsClosure

theorem **sum_norm_nonneg** (R : Rep k r) (Ws : List (List (Mx k r))) (v : R.H) : 0 ≤ (Ws.map fun w => ‖dsigW (k

theorem **semiBd_of_cw** {Wp : Type} [NormedAddCommGroup Wp] (R : Rep k r) {B : R.H → Wp} (h : PI.BesselMapUniq.IsCWCont (k

theorem **cw_of_semiBd** {Wp : Type} [NormedAddCommGroup Wp] (R : Rep k r) {B : R.H → Wp} {f : R.H → ℝ} (h : SemiBd R f) (hle : ∀ v ∈ R.sm, ‖B v‖ ≤ f v) : PI.BesselMapUniq.IsCWCont (k

theorem **SemiBd.add** {R : Rep k r} {f g : R.H → ℝ} (hf : SemiBd R f) (hg : SemiBd R g) : SemiBd R (fun v => f v + g v)

theorem **SemiBd.mul** {R : Rep k r} {f : R.H → ℝ} (hf : SemiBd R f) {a : ℝ} (ha : 0 ≤ a) : SemiBd R (fun v => a * f v)

theorem **semiBd_sum** {R R' : Rep k r} {T : R.H → R'.H} (hT : SmMap R R' T) (Ws : List (List (Mx k r))) (hWs : ∀ w ∈ Ws, ∀ X ∈ w, InLieHpk k r X) : SemiBd R (fun v => (Ws.map fun w => ‖dsigW (k

theorem **cw_comp** {Wp : Type} [NormedAddCommGroup Wp] {R R' : Rep k r} {T : R.H → R'.H} {Bm : R'.H → Wp} (hT : SmMap R R' T) (hB : PI.BesselMapUniq.IsCWCont (k

theorem **smMap_comp** {R R' R'' : Rep k r} {T : R.H → R'.H} {T' : R'.H → R''.H} (hT : SmMap R R' T) (hT' : SmMap R' R'' T') : SmMap R R'' (fun v => T' (T v)) where

theorem **dsigW_mem** (hK : PI.KFinSmooth) (hR : R.IsDG ε mu) (W : List (Mx k r)) (hW : ∀ X ∈ W, InLieHpk k r X) {v : R.H} (hv : v ∈ R.sm) : dsigW (k

theorem **dsig_lin** (hK : PI.KFinSmooth) (hR : R.IsDG ε mu) {X : Mx k r} (hX : InLieHpk k r X) (a : ℂ) {v w : R.H} (hv : v ∈ R.sm) (hw : w ∈ R.sm) : dsig (k

theorem **dsigW_lin** (hK : PI.KFinSmooth) (hR : R.IsDG ε mu) (W : List (Mx k r)) (hW : ∀ X ∈ W, InLieHpk k r X) (a : ℂ) {v w : R.H} (hv : v ∈ R.sm) (hw : w ∈ R.sm) : dsigW (k

theorem **inLie_conj** {g X : Mx k r} (hg : g ∈ GG k r) (hX : InLieHpk k r X) : InLieHpk k r (g⁻¹ * X * g)

theorem **norm_sigma** (hR : R.IsDG ε mu) {g : Mx k r} (hg : g ∈ GG k r) (v : R.H) : ‖R.σ g v‖ = ‖v‖

theorem **dsigW_sigma** (hK : PI.KFinSmooth) (hR : R.IsDG ε mu) {g : Mx k r} (hg : g ∈ GG k r) (W : List (Mx k r)) (hW : ∀ X ∈ W, InLieHpk k r X) {v : R.H} (hv : v ∈ R.sm) : dsigW (k

theorem **smMap_sigma** (hK : PI.KFinSmooth) (hR : R.IsDG ε mu) {g : Mx k r} (hg : g ∈ GG k r) : SmMap R R (fun v => R.σ g v) where

theorem **smMap_lin** (hK : PI.KFinSmooth) {R₀ : Rep k r} (hR : R.IsDG ε mu) {T T' : R₀.H → R.H} (a : ℂ) (hT : SmMap R₀ R T) (hT' : SmMap R₀ R T') : SmMap R₀ R (fun v => a • T v + T' v) where

theorem **smMap_act** : PI.KFinSmooth → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (mu' mu'' : Fin (2 * r + k + 1) → ℝ) (R R' R'' : Rep k r), R.IsDG ε mu → R'.IsDG ε mu' → R''.IsDG ε mu'' → (∀ g ∈ GG k r, SmMap R R (fun v => R.σ g v)) ∧ (∀ (T : R.H → R'.H) (T' : R'.H → R''.H), SmMap R R' T → SmMap R' R'' T' → SmMap R R'' (fun v => T' (T v))) ∧ (∀ (T T' : R.H → R'.H) (a : ℂ), SmMap R R' T → SmMap R R' T' → SmMap R R' (fun v => a • T v + T' v)) ∧ ∀ (Wp : Type) [NormedAddCommGroup Wp] (T : R.H → R'.H) (Bm : R'.H → Wp), SmMap R R' T → PI.BesselMapUniq.IsCWCont (k

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.SmoothMapsClosureStatement

theorem **_root_.Gp91.GlobalTheory.T.smooth_maps_closure** : PI.KFinSmooth → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (mu' mu'' : Fin (2 * r + k + 1) → ℝ) (R R' R'' : Rep k r), R.IsDG ε mu → R'.IsDG ε mu' → R''.IsDG ε mu'' → (∀ g ∈ GG k r, SmMap R R (fun v => R.σ g v)) ∧ (∀ (T : R.H → R'.H) (T' : R'.H → R''.H), SmMap R R' T → SmMap R' R'' T' → SmMap R R'' (fun v => T' (T v))) ∧ (∀ (T T' : R.H → R'.H) (a : ℂ), SmMap R R' T → SmMap R R' T' → SmMap R R' (fun v => a • T v + T' v)) ∧ ∀ (Wp : Type) [NormedAddCommGroup Wp] (T : R.H → R'.H) (Bm : R'.H → Wp), SmMap R R' T → PI.BesselMapUniq.IsCWCont (k

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.SmoothVectorsDiscreteSeriesGlModule

theorem **conj_single_comm** {n : ℕ} (Q : Matrix (Fin n) (Fin n) ℂ) (hQ : IsUnit Q) (a b c d : Fin n) : (Q⁻¹ * Matrix.single a b 1 * Q) * (Q⁻¹ * Matrix.single c d 1 * Q) − (Q⁻¹ * Matrix.single c d 1 * Q) * (Q⁻¹ * Matrix.single a b 1 * Q) = (if b = c then Q⁻¹ * Matrix.single a d 1 * Q else 0) − (if d = a then Q⁻¹ * Matrix.single c b 1 * Q else 0)

theorem **smooth_gl_module** : PI.KFinSmooth → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (R : Rep k r), R.IsDG ε mu → ∀ Q : Mx k r, IsUnit Q → ∃ E : (Fin (2 * r + k + 1) → Fin (2 * r + k + 1)) → Module.End ℂ R.smSub), R.IsE Q E ∧ Formalism.REAL.GlRel E

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.SmoothVectorsDiscreteSeriesGlModuleStatement

theorem **_root_.Gp91.GlobalTheory.T.smoothVectors_discreteSeries_glModule** : PI.KFinSmooth → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (R : Rep k r), R.IsDG ε mu → ∀ Q : Mx k r, IsUnit Q → ∃ E : (Fin (2 * r + k + 1) → Fin (2 * r + k + 1)) → Module.End ℂ R.smSub), R.IsE Q E ∧ Formalism.REAL.GlRel E

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.SumNonneg

theorem **gN_mul_le'** {k r : ℕ} (a b : Mx k r) : B.gN (a * b) ≤ B.gN a * B.gN b

theorem **gN_pos'** {k r : ℕ} {a : Mx k r} (ha : a ∈ GG k r) : 0 < B.gN a

theorem **sum_nonneg'** {α : Type} (f : α → ℝ) (hf : ∀ a, 0 ≤ f a) (L : List α) : 0 ≤ (L.map f).sum

theorem **one_le_aOf** {k r : ℕ} {h : ℝ} (hg : GapOK k r h) : 1 ≤ aOf r h

theorem **decay_TK** : HypsG → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (h : ℝ) (R : Rep k r), R.IsDG ε mu → Formalism.QuantumDeterminants.GapAtLeast ε mu h → GapOK k r h → ∀ v ∈ R.TK, ∃ (C : ℝ) (Ws : List (List (Mx k r))), (∀ W ∈ Ws, ∀ X ∈ W, PI.UpqRep.InLieHpk k r X) ∧ ∀ w ∈ R.sm, ∀ x ∈ GG k r, ‖⟪v, R.σ x w⟫_ℂ‖ ≤ C * (Ws.map fun W => ‖PI.UpqRep.dsigW (k

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.TensorBesselTransfer

theorem **map_sum_of_add_on_sm** {k r : ℕ} (R : Rep k r) {Wp : Type} [AddCommGroup Wp] [Module ℂ Wp] (l : R.H → Wp) (hadd : ∀ v ∈ R.sm, ∀ w ∈ R.sm, l (v + w) = l v + l w) (hsmul : ∀ (a : ℂ), ∀ v ∈ R.sm, l (a • v) = a • l v) {α : Type} (s : Finset α) (f : α → R.H) (hf : ∀ i, f i ∈ R.sm) : l (∑ i ∈ s, f i) = ∑ i ∈ s, l (f i)

theorem **herm_smul_eq_sum_of_tensRaw** : PI.KFinSmooth → ∀ (k r : ℕ) (ι κ : Type) [Fintype ι] [DecidableEq ι] [Fintype κ] [DecidableEq κ] (ρ : Mx k r → Matrix ι ι ℂ) (Γ : Matrix ι ι ℂ) (d : ℕ) (R : Rep k r) (RI : κ → Rep k r) (io : ∀ I, ι → (RI I).H → R.H) (pr : ∀ I, ι → R.H → (RI I).H) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (muI : κ → Fin (2 * r + k + 1) → ℝ) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (om : ι → ℂ) (l : R.H → Wp) (lI : ∀ I, (RI I).H → Wp) (β : κ → ℂ), R.IsDG ε mu → (∀ I, (RI I).IsDG ε (muI I)) → TensRaw ρ R RI io pr → (∀ v ∈ R.sm, ∀ w ∈ R.sm, l (v + w) = l v + l w) → (∀ (a : ℂ), ∀ v ∈ R.sm, l (a • v) = a • l v) → (∀ I, ∀ y ∈ (RI I).sm, l (ev Γ R om (fun c => io I c y)) = β I • lI I y) → ∀ (a : ι → ℂ), ∀ w ∈ R.sm, ∀ g ∈ GG k r, Formalism.QuantumDeterminants.herm Γ (ρ g *ᵥ a) om • l (R.σ g

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.TensorBesselTransferStatement

theorem **_root_.Gp91.GlobalTheory.T.tensor_bessel_transfer** : PI.KFinSmooth → ∀ (k r : ℕ) (ι κ : Type) [Fintype ι] [DecidableEq ι] [Fintype κ] [DecidableEq κ] (ρ : Mx k r → Matrix ι ι ℂ) (Γ : Matrix ι ι ℂ) (d : ℕ) (R : Rep k r) (RI : κ → Rep k r) (io : ∀ I, ι → (RI I).H → R.H) (pr : ∀ I, ι → R.H → (RI I).H) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (muI : κ → Fin (2 * r + k + 1) → ℝ) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (om : ι → ℂ) (l : R.H → Wp) (lI : ∀ I, (RI I).H → Wp) (β : κ → ℂ), R.IsDG ε mu → (∀ I, (RI I).IsDG ε (muI I)) → TensRaw ρ R RI io pr → (∀ v ∈ R.sm, ∀ w ∈ R.sm, l (v + w) = l v + l w) → (∀ (a : ℂ), ∀ v ∈ R.sm, l (a • v) = a • l v) → (∀ I, ∀ y ∈ (RI I).sm, l (ev Γ R om (fun c => io I c y)) = β I • lI I y) → ∀ (a : ι → ℂ), ∀ w ∈ R.sm, ∀ g ∈ GG k r, Formalism.QuantumDeterminants.herm Γ (ρ

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.TensorDecompositionInjectiveParameters

theorem **hh_smul** (Γ : Matrix ι ι ℂ) (R : Rep k r) (a b : ℂ) (x y : ι → R.H) : hh Γ R (fun c => a • x c) (fun c => b • y c) = a * star b * hh Γ R x y

theorem **hh_sum_right** (Γ : Matrix ι ι ℂ) (R : Rep k r) (x : ι → R.H) (y : κ → ι → R.H) : hh Γ R x (fun c => ∑ J, y J c) = ∑ J, hh Γ R x (y J)

theorem **hh_star** (Γ : Matrix ι ι ℂ) (hΓ : Γᴴ = Γ) (R : Rep k r) (x y : ι → R.H) : star (hh Γ R x y) = hh Γ R y x

theorem **eq_zero_of_hh** (Γ : Matrix ι ι ℂ) (hΓ : Γ * Γ = 1) (R : Rep k r) (x : ι → R.H) (h : hh Γ R x (fun c => ∑ j, Γ c j • x j) = 0) (c : ι) : x c = 0

theorem **SmMap_zero** {R R' : Rep k r} {T : R.H → R'.H} (hT : SmMap R R' T) : T 0 = 0

theorem **raw_scale** {ρ : Mx k r → Matrix ι ι ℂ} {R : Rep k r} {RI : κ → Rep k r} {io : ∀ I, ι → (RI I).H → R.H} {pr : ∀ I, ι → R.H → (RI I).H} (h : TensRaw ρ R RI io pr) (a : κ → ℂ) (ha : ∀ I, a I ≠ 0) (h1 : ∀ I c, SmMap (RI I) R (fun v => a I • io I c v)) (h2 : ∀ I c, SmMap R (RI I) (fun v => (a I)⁻¹ • pr I c v)) : TensRaw ρ R RI (fun I c v => a I • io I c v) (fun I c v => (a I)⁻¹ • pr I c v) where

theorem **tensH_of_raw** : HypsG → ∀ (k r : ℕ) (ι κ : Type) [Fintype ι] [DecidableEq ι] [Fintype κ] [DecidableEq κ] (ρ : Mx k r → Matrix ι ι ℂ) (Γ : Matrix ι ι ℂ) (d : ℕ) (R : Rep k r) (RI : κ → Rep k r) (io : ∀ I, ι → (RI I).H → R.H) (pr : ∀ I, ι → R.H → (RI I).H) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (muI : κ → Fin (2 * r + k + 1) → ℝ), R.IsDG ε mu → (∀ I, (RI I).IsDG ε (muI I)) → Function.Injective muI → IsFRep ρ Γ d → TensRaw ρ R RI io pr → ∃ io' pr' ep, TensH ρ Γ R RI io' pr' ep

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.TensorDecompositionInjectiveParametersStatement

theorem **_root_.Gp91.GlobalTheory.T.tensor_decomposition_injective_parameters** : HypsG → ∀ (k r : ℕ) (ι κ : Type) [Fintype ι] [DecidableEq ι] [Fintype κ] [DecidableEq κ] (ρ : Mx k r → Matrix ι ι ℂ) (Γ : Matrix ι ι ℂ) (d : ℕ) (R : Rep k r) (RI : κ → Rep k r) (io : ∀ I, ι → (RI I).H → R.H) (pr : ∀ I, ι → R.H → (RI I).H) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (muI : κ → Fin (2 * r + k + 1) → ℝ), R.IsDG ε mu → (∀ I, (RI I).IsDG ε (muI I)) → Function.Injective muI → IsFRep ρ Γ d → TensRaw ρ R RI io pr → ∃ io' pr' ep, TensH ρ Γ R RI io' pr' ep

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.TensorDecompositionParameters

theorem **common_bound** {ι α β : Type} [DecidableEq ι] (P : α → Prop) (S : Set β) (n : α → β → ℝ) (hn : ∀ a v, 0 ≤ n a v) (f : ι → β → ℝ) (h : ∀ c, ∃ (C : ℝ) (L : List α), (∀ a ∈ L, P a) ∧ ∀ v ∈ S, f c v ≤ C * (L.map fun a => n a v).sum) (s : Finset ι) : ∃ (C : ℝ) (L : List α), 0 ≤ C ∧ (∀ a ∈ L, P a) ∧ ∀ c ∈ s, ∀ v ∈ S, f c v ≤ C * (L.map fun a => n a v).sum

theorem **hh_act** (ρ : Mx k r → Matrix ι ι ℂ) (Γ : Matrix ι ι ℂ) (R : Rep k r) (g : Mx k r) (hρ : (ρ g)ᴴ * Γ * ρ g = Γ) (hσ : ∀ v, ‖R.σ g v‖ = ‖v‖) (x y : ι → R.H) : hh Γ R (act ρ R g x) (act ρ R g y) = hh Γ R x y

theorem **norm_hh_le** (Γ : Matrix ι ι ℂ) (R : Rep k r) (x y : ι → R.H) (A B : ℝ) (hx : ∀ c, ‖x c‖ ≤ A) (hy : ∀ c, ‖y c‖ ≤ B) : ‖hh Γ R x y‖ ≤ (∑ i, ∑ j, ‖Γ i j‖) * (A * B)

theorem **family_bound** (R R' : Rep k r) (T : ι → R.H → R'.H) (hT : ∀ c, SmMap R R' (T c)) : ∃ (C : ℝ) (Ws : List (List (Mx k r))), (∀ W ∈ Ws, ∀ X ∈ W, PI.UpqRep.InLieHpk k r X) ∧ ∀ c, ∀ v ∈ R.sm, ‖T c v‖ ≤ C * (Ws.map fun W => ‖PI.UpqRep.dsigW (k

theorem **pulledBack_isInvForm** : PI.KFinSmooth → ∀ (k r : ℕ) (ι κ : Type) [Fintype ι] [DecidableEq ι] [Fintype κ] [DecidableEq κ] (ρ : Mx k r → Matrix ι ι ℂ) (Γ : Matrix ι ι ℂ) (d : ℕ) (R : Rep k r) (RI : κ → Rep k r) (io : ∀ I, ι → (RI I).H → R.H) (pr : ∀ I, ι → R.H → (RI I).H) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (muI : κ → Fin (2 * r + k + 1) → ℝ), R.IsDG ε mu → (∀ I, (RI I).IsDG ε (muI I)) → IsFRep ρ Γ d → TensRaw ρ R RI io pr → ∀ I J, IsInvForm (RI I) (RI J) (fun y y' => hh Γ R (fun c => io I c y) (fun c => io J c y'))

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.TensorDecompositionParametersStatement

theorem **_root_.Gp91.GlobalTheory.T.tensor_decomposition_parameters** : PI.KFinSmooth → ∀ (k r : ℕ) (ι κ : Type) [Fintype ι] [DecidableEq ι] [Fintype κ] [DecidableEq κ] (ρ : Mx k r → Matrix ι ι ℂ) (Γ : Matrix ι ι ℂ) (d : ℕ) (R : Rep k r) (RI : κ → Rep k r) (io : ∀ I, ι → (RI I).H → R.H) (pr : ∀ I, ι → R.H → (RI I).H) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (muI : κ → Fin (2 * r + k + 1) → ℝ), R.IsDG ε mu → (∀ I, (RI I).IsDG ε (muI I)) → IsFRep ρ Γ d → TensRaw ρ R RI io pr → ∀ I J, IsInvForm (RI I) (RI J) (fun y y' => hh Γ R (fun c => io I c y) (fun c => io J c y'))

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.TensorMomentEstimate

theorem **mul_mem_GG** {k r : ℕ} {g h : Mx k r} (hg : g ∈ GG k r) (hh : h ∈ GG k r) : g * h ∈ GG k r

theorem **ae_pair** {k r : ℕ} {U V : Set (Mx k r)} {μ ν : Measure (Formalism.QuantumDeterminants.Mt k r)} (hμ : PI.BesselPeriodGGP.IsHaarOn U μ) (hν : PI.BesselPeriodGGP.IsHaarOn V ν) : ∀ᵐ p ∂(μ.prod ν), Matrix.of p.1 ∈ U ∧ Matrix.of p.2 ∈ V

theorem **bessel_add** {k r : ℕ} {R : Rep k r} {Wp : Type} [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] {z : Fin (2 * r + k + 1) → ℂ} {π : Mx k r → (Wp →L[ℂ] Wp)} {A A' : R.H → Wp} (h : IsBes R z π A) (h' : IsBes R z π A') : IsBes R z π (fun v => A v + A' v)

theorem **kappa_dual** : HypsG → ∀ (k r : ℕ) (ι κ : Type) [Fintype ι] [DecidableEq ι] [Fintype κ] [DecidableEq κ] (ρ : Mx k r → Matrix ι ι ℂ) (Γ : Matrix ι ι ℂ) (d : ℕ) (R : Rep k r) (RI : κ → Rep k r) (io : ∀ I, ι → (RI I).H → R.H) (pr : ∀ I, ι → R.H → (RI I).H) (ep : κ → ℝ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (muI : κ → Fin (2 * r + k + 1) → ℝ) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (z : Fin (2 * r + k + 1) → ℂ) (π : Mx k r → (Wp →L[ℂ] Wp)) (u : ℕ → ℝ) (μW μN μG νq : Measure (Formalism.QuantumDeterminants.Mt k r)) (om : ι → ℂ) (l : R.H → Wp), R.IsDG ε mu → (∀ I, (RI I).IsDG ε (muI I)) → IsFRep ρ Γ d → TensH ρ Γ R RI io pr ep → SideG k r z π u μW μN μG νq → (∀ s ∈ Formalism.QuantumDeterminants.Sset k r z, ρ s *ᵥ om = om) → IsBeL R z π μW μN l → NzOn R l → ∀ I lI, IsBeL (RI I) z π μW μN lI → (∃ κ₀ :

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.TensorMomentEstimateStatement

theorem **_root_.Gp91.GlobalTheory.T.bessel_tensor_projection_scalar_and_adjoint** : HypsG → ∀ (k r : ℕ) (ι κ : Type) [Fintype ι] [DecidableEq ι] [Fintype κ] [DecidableEq κ] (ρ : Mx k r → Matrix ι ι ℂ) (Γ : Matrix ι ι ℂ) (d : ℕ) (R : Rep k r) (RI : κ → Rep k r) (io : ∀ I, ι → (RI I).H → R.H) (pr : ∀ I, ι → R.H → (RI I).H) (ep : κ → ℝ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (muI : κ → Fin (2 * r + k + 1) → ℝ) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (z : Fin (2 * r + k + 1) → ℂ) (π : Mx k r → (Wp →L[ℂ] Wp)) (u : ℕ → ℝ) (μW μN μG νq : Measure (Formalism.QuantumDeterminants.Mt k r)) (om : ι → ℂ) (l : R.H → Wp), R.IsDG ε mu → (∀ I, (RI I).IsDG ε (muI I)) → IsFRep ρ Γ d → TensH ρ Γ R RI io pr ep → SideG k r z π u μW μN μG νq → (∀ s ∈ Formalism.QuantumDeterminants.Sset k r z, ρ s *ᵥ om = om) → IsBeL R z π μW μ

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.TensorMomentTransfer

theorem **integrable_inner_of_memLp** {α E : Type*} [MeasurableSpace α] {μ : Measure α} [NormedAddCommGroup E] [InnerProductSpace ℂ E] {f g : α → E} (hf : MemLp f 2 μ) (hg : MemLp g 2 μ) : Integrable (fun x => ⟪f x, g x⟫_ℂ) μ

private theorem **add_mem_sm** {k r : ℕ} (R : BranchingSupport.Rep k r) {v w : R.H} (hv : v ∈ R.sm) (hw : w ∈ R.sm) : v + w ∈ R.sm

private theorem **smul_mem_sm'** {k r : ℕ} (R : BranchingSupport.Rep k r) (c : ℂ) {v : R.H} (hv : v ∈ R.sm) : c • v ∈ R.sm

private theorem **zero_mem_sm** {k r : ℕ} (R : BranchingSupport.Rep k r) : (0 : R.H) ∈ R.sm

theorem **pairing** : HypsG → ∀ (k r : ℕ) (ι κ : Type) [Fintype ι] [DecidableEq ι] [Fintype κ] [DecidableEq κ] (ρ : Mx k r → Matrix ι ι ℂ) (Γ : Matrix ι ι ℂ) (d : ℕ) (R : Rep k r) (RI : κ → Rep k r) (io : ∀ I, ι → (RI I).H → R.H) (pr : ∀ I, ι → R.H → (RI I).H) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (muI : κ → Fin (2 * r + k + 1) → ℝ) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (z : Fin (2 * r + k + 1) → ℂ) (π : Mx k r → (Wp →L[ℂ] Wp)) (u : ℕ → ℝ) (μW μN μG νq : Measure (Formalism.QuantumDeterminants.Mt k r)) (om : ι → ℂ) (Bm : R.H → Wp) (BI : ∀ I, (RI I).H → Wp) (β : κ → ℂ), R.IsDG ε mu → (∀ I, (RI I).IsDG ε (muI I)) → Function.Injective muI → TensRaw ρ R RI io pr → SideG k r z π u μW μN μG νq → IsBes R z π Bm → (∀ I, IsIsoB (RI I) z π νq (BI I) v β I = 0) → (∀ I, ∀ y ∈ (RI I).sm, Bm (ev Γ R om (fun c => io

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.TensorMomentTransferStatement

theorem **_root_.Gp91.GlobalTheory.T.tensor_moment_transfer** : HypsG → ∀ (k r : ℕ) (ι κ : Type) [Fintype ι] [DecidableEq ι] [Fintype κ] [DecidableEq κ] (ρ : Mx k r → Matrix ι ι ℂ) (Γ : Matrix ι ι ℂ) (d : ℕ) (R : Rep k r) (RI : κ → Rep k r) (io : ∀ I, ι → (RI I).H → R.H) (pr : ∀ I, ι → R.H → (RI I).H) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (muI : κ → Fin (2 * r + k + 1) → ℝ) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (z : Fin (2 * r + k + 1) → ℂ) (π : Mx k r → (Wp →L[ℂ] Wp)) (u : ℕ → ℝ) (μW μN μG νq : Measure (Formalism.QuantumDeterminants.Mt k r)) (om : ι → ℂ) (Bm : R.H → Wp) (BI : ∀ I, (RI I).H → Wp) (β : κ → ℂ), R.IsDG ε mu → (∀ I, (RI I).IsDG ε (muI I)) → Function.Injective muI → TensRaw ρ R RI io pr → SideG k r z π u μW μN μG νq → IsBes R z π Bm → (∀ I, IsIsoB (RI I) z π νq (BI I) v β I = 0) → (∀ I, ∀

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.TensorMomentWeighted

theorem **ev_col** {k r : ℕ} {ι : Type} [Fintype ι] [DecidableEq ι] (Γ : Matrix ι ι ℂ) (hH : Γᴴ = Γ) (h2 : Γ * Γ = 1) (R : BranchingSupport.Rep k r) (x : ι → R.H) (e : ι) : ev Γ R (fun c => Γ c e) x = x e

theorem **transpose_unitary** {ι : Type} [Fintype ι] [DecidableEq ι] (Γ : Matrix ι ι ℂ) (hH : Γᴴ = Γ) (h2 : Γ * Γ = 1) : Γᵀ ∈ Matrix.unitaryGroup ι ℂ

theorem **div_step** (b t : ℝ) (ht : 0 ≤ t) (L : ℝ≥0∞) (h : ENNReal.ofReal (b * t * t) ≤ L * ENNReal.ofReal t) : ENNReal.ofReal (b * t) ≤ L

theorem **moment_lemma_pieri** : HypsG → ∀ (k r : ℕ) (ι κ : Type) [Fintype ι] [DecidableEq ι] [Fintype κ] [DecidableEq κ] (ρ : Mx k r → Matrix ι ι ℂ) (Γ : Matrix ι ι ℂ) (d : ℕ) (R : Rep k r) (RI : κ → Rep k r) (io : ∀ I, ι → (RI I).H → R.H) (pr : ∀ I, ι → R.H → (RI I).H) (ep : κ → ℝ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (muI : κ → Fin (2 * r + k + 1) → ℝ) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (z : Fin (2 * r + k + 1) → ℂ) (π : Mx k r → (Wp →L[ℂ] Wp)) (u : ℕ → ℝ) (μW μN μG νq : Measure (Formalism.QuantumDeterminants.Mt k r)) (om : ι → ℂ) (Bm : R.H → Wp) (BI : ∀ I, (RI I).H → Wp) (β : κ → ℂ),

R.IsDG ε mu → (∀ I, (RI I).IsDG ε (muI I)) → Function.Injective muI → IsFRep ρ Γ d → TensH ρ Γ R RI io pr ep → SideG k r z π u μW μN μG νq → IsIsoB R z π νq Bm → (∀ I, IsIsoB (RI I) z π νq (BI I) ν β I = 0) → (∀

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.TensorMomentWeightedStatement

theorem **_root_.Gp91.GlobalTheory.T.tensor_moment_weighted** : HypsG → ∀ (k r : ℕ) (ι κ : Type) [Fintype ι] [DecidableEq ι] [Fintype κ] [DecidableEq κ] (ρ : Mx k r → Matrix ι ι ℂ) (Γ : Matrix ι ι ℂ) (d : ℕ) (R : Rep k r) (RI : κ → Rep k r) (io : ∀ I, ι → (RI I).H → R.H) (pr : ∀ I, ι → R.H → (RI I).H) (ep : κ → ℝ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (muI : κ → Fin (2 * r + k + 1) → ℝ) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (z : Fin (2 * r + k + 1) → ℂ) (π : Mx k r → (Wp →L[ℂ] Wp)) (u : ℕ → ℝ) (μW μN μG νq : Measure (Formalism.QuantumDeterminants.Mt k r)) (om : ι → ℂ) (Bm : R.H → Wp) (BI : ∀ I, (RI I).H → Wp) (β : κ → ℂ), R.IsDG ε mu → (∀ I, (RI I).IsDG ε (muI I)) → Function.Injective muI → IsFRep ρ Γ d → TensH ρ Γ R RI io pr ep → SideG k r z π u μW μN μG νq → IsIsoB R z π νq Bm → (∀ I, IsIsoB (RI I)

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.TensorRawOfFrep

theorem **smMap_zero** {R R' : Rep k r} {T : R.H → R'.H} (hT : SmMap R R' T) : T 0 = 0

theorem **kfin_of_family** {R R' : Rep k r} (T : ι → R.H → R'.H) (hT : ∀ c, SmMap R R' (T c)) (hK : ∀ v, R.KFin v → v ∈ R.sm) (hA : ∀ g ∈ GG k r, ∀ c, ∀ y ∈ R.sm, ∃ a : ι → ℂ, R'.σ g (T c y) = ∑ i, a i • T i (R.σ g y)) (c : ι) (y : R.H) (hy : R.KFin y) : R'.KFin (T c y)

theorem **tk_of_family** {R R' : Rep k r} (T : ι → R.H → R'.H) (hT : ∀ c, SmMap R R' (T c)) (hK : ∀ v, R.KFin v → v ∈ R.sm) (hTK : ∀ v ∈ R.TK, v ∈ R.sm) (hF : ∀ c y, R.KFin y → R'.KFin (T c y)) (hB : ∀ g ∈ GG k r, ∀ c, ∀ y ∈ R.sm, ∃ a : ι → ℂ, T c (R.σ g y) = ∑ i, a i • R'.σ g (T i y)) (c : ι) (y : R.H) (hy : y ∈ R.TK) : T c y ∈ R'.TK

theorem **inv_rows** [DecidableEq ι] {V : Type} [AddCommGroup V] [Module ℂ V] (A B : Matrix ι ι ℂ) (h : A * B = 1) (u v : ι → V) (huv : ∀ i, u i = ∑ j, B i j • v j) (c : ι) : v c = ∑ i, A c i • u i

theorem **GlobalTheory.tensor_decomposition_kfinite** : PI.KFinSmooth → ∀ (k r : ℕ) (ι κ : Type) [Fintype ι] [DecidableEq ι] [Fintype κ] [DecidableEq κ] (ρ : Mx k r → Matrix ι ι ℂ) (Γ : Matrix ι ι ℂ) (d : ℕ) (R : Rep k r) (RI : κ → Rep k r) (io : ∀ I, ι → (RI I).H → R.H) (pr : ∀ I, ι → R.H → (RI I).H) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (muI : κ → Fin (2 * r + k + 1) → ℝ), R.IsDG ε mu → (∀ I, (RI I).IsDG ε (muI I)) → IsFRep ρ Γ d → TensRaw ρ R RI io pr → (∀ I c y, (RI I).KFin y → R.KFin (io I c y)) ∧ (∀ I c x, R.KFin x → (RI I).KFin (pr I c x)) ∧ (∀ I c, ∀ y ∈ (RI I).TK, io I c y ∈ R.TK) ∧ (∀ I c, ∀ x ∈ R.TK, pr I c x ∈ (RI I).TK)

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.TensorRawOfFrepStatement

theorem **_root_.Gp91.GlobalTheory.T.tensor_raw_of_frep** : PI.KFinSmooth → ∀ (k r : ℕ) (ι κ : Type) [Fintype ι] [DecidableEq ι] [Fintype κ] [DecidableEq κ] (ρ : Mx k r → Matrix ι ι ℂ) (Γ : Matrix ι ι ℂ) (d : ℕ) (R : Rep k r) (RI : κ → Rep k r) (io : ∀ I, ι → (RI I).H → R.H) (pr : ∀ I, ι → R.H → (RI I).H) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (muI : κ → Fin (2 * r + k + 1) → ℝ), R.IsDG ε mu → (∀ I, (RI I).IsDG ε (muI I)) → IsFRep ρ Γ d → TensRaw ρ R RI io pr → (∀ I c y, (RI I).KFin y → R.KFin (io I c y)) ∧ (∀ I c x, R.KFin x → (RI I).KFin (pr I c x)) ∧ (∀ I c, ∀ y ∈ (RI I).TK, io I c y ∈ R.TK) ∧ (∀ I c, ∀ x ∈ R.TK, pr I c x ∈ (RI I).TK)

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.TensorWeightedBesselGap

theorem **gg_mul** {k r : ℕ} {a b : Mx k r} (ha : a ∈ GG k r) (hb : b ∈ GG k r) : a * b ∈ GG k r

theorem **one_gg** (k r : ℕ) : (1 : Mx k r) ∈ GG k r

theorem **one_N** (k r : ℕ) : (1 : Mx k r) ∈ Formalism.QuantumDeterminants.Nset k r

theorem **one_UW** (k r : ℕ) (z : Fin (2 * r + k + 1) → ℂ) : (1 : Mx k r) ∈ Formalism.QuantumDeterminants.UWset k r z

theorem **herm_iso** {ι : Type} [Fintype ι] [DecidableEq ι] {Γ A : Matrix ι ι ℂ} (hA : Aᴴ * Γ * A = Γ) (x y : ι → ℂ) : Formalism.QuantumDeterminants.herm Γ (A *ᵥ x) (A *ᵥ y) = Formalism.QuantumDeterminants.herm Γ x y

theorem **coef_inv** {k r : ℕ} {ι : Type} [Fintype ι] [DecidableEq ι] {ρ : Mx k r → Matrix ι ι ℂ} {Γ : Matrix ι ι ℂ} {d : ℕ} (hρ : IsFRep ρ Γ d) {Wp Wq : Type} [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] (D : B.RealPlaceData k r Wp Wq) {om om' : ι → ℂ} (h1 : ∀ g ∈ Formalism.QuantumDeterminants.Sset k r D.z, ρ g *ᵥ om = om) (h2 : ∀ g ∈ Formalism.QuantumDeterminants.Sset k r D.z', ρ g *ᵥ om' = om') {γ : Mx k r} (hγ : γ ∈ GG k r) (p : B.SS k r) (hp : D.inSS p) : Formalism.QuantumDeterminants.herm Γ (ρ (B.sgs p γ) *ᵥ om') om = Formalism.QuantumDeterminants.herm Γ (ρ γ *ᵥ om') om

theorem **pieri_bessel** : HypsG → ∀ (k r : ℕ) (ι κ : Type) [Fintype ι] [DecidableEq ι] [Fintype κ] [DecidableEq κ] (ρ : Mx k r → Matrix ι ι ℂ) (Γ : Matrix ι ι ℂ) (d : ℕ) (R : Rep k r) (RI : κ → Rep k r) (io : ∀ I, ι → (RI I).H → R.H) (pr : ∀ I, ι → R.H → (RI I).H) (ep : κ → ℝ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (muI : κ → Fin (2 * r + k + 1) → ℝ) (h : ℝ) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (Wq : Type) [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [CompleteSpace Wq] [FiniteDimensional ℂ Wq] (D : B.RealPlaceData k r Wp Wq) (om om' : ι → ℂ) (l : R.H → Wp) (l' : R.H → Wq) (lI : ∀ I, (RI I).H → Wp) (lI' : ∀ I, (RI I).H → Wq) (F : Mx k r → (Wq →L[ℂ] Wp)) (s : ℂ) (FI : κ → Mx k r → (Wq →L[ℂ] Wp)) (sI τ : κ → ℂ), R.IsDG ε mu → Formalism.QuantumDeterminants.GapAtLeast ε mu h → (∀ I, (RI I).IsDG ε (mu

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.TensorWeightedBesselGapStatement

theorem **_root_.Gp91.GlobalTheory.T.tensor_weighted_bessel_gap** : HypsG → ∀ (k r : ℕ) (ι κ : Type) [Fintype ι] [DecidableEq ι] [Fintype κ] [DecidableEq κ] (ρ : Mx k r → Matrix ι ι ℂ) (Γ : Matrix ι ι ℂ) (d : ℕ) (R : Rep k r) (RI : κ → Rep k r) (io : ∀ I, ι → (RI I).H → R.H) (pr : ∀ I, ι → R.H → (RI I).H) (ep : κ → ℝ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (muI : κ → Fin (2 * r + k + 1) → ℝ) (h : ℝ) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (Wq : Type) [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [CompleteSpace Wq] [FiniteDimensional ℂ Wq] (D : B.RealPlaceData k r Wp Wq) (om om' : ι → ℂ) (l : R.H → Wp) (l' : R.H → Wq) (lI : ∀ I, (RI I).H → Wp) (lI' : ∀ I, (RI I).H → Wq) (F : Mx k r → (Wq →L[ℂ] Wp)) (s : ℂ) (FI : κ → Mx k r → (Wq →L[ℂ] Wp)) (sI τ : κ → ℂ), R.IsDG ε mu → Formalism.QuantumDeterminants.Ga

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.TensorWeightedBesselSum

theorem **coef_mul_split** : PI.KFinSmooth → ∀ (k r : ℕ) (ι κ : Type) [Fintype ι] [DecidableEq ι] [Fintype κ] [DecidableEq κ] (ρ : Mx k r → Matrix ι ι ℂ) (Γ : Matrix ι ι ℂ) (d : ℕ) (R : Rep k r) (RI : κ → Rep k r) (io : ∀ I, ι → (RI I).H → R.H) (pr : ∀ I, ι → R.H → (RI I).H) (ep : κ → ℝ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (muI : κ → Fin (2 * r + k + 1) → ℝ) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (Wq : Type) [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [CompleteSpace Wq] [FiniteDimensional ℂ Wq] (om om' : ι → ℂ) (l : R.H → Wp) (l' : R.H → Wq) (lI : ∀ I, (RI I).H → Wp) (lI' : ∀ I, (RI I).H → Wq) (F : Mx k r → (Wq →L[ℂ] Wp)) (s : ℂ), R.IsDG ε mu → (∀ I, (RI I).IsDG ε (muI I)) → IsFRep ρ Γ d → TensH ρ Γ R RI io pr ep → IsCoef R l l' F s → ∃ (Ft : κ → Mx k r → (Wq →L[ℂ] Wp)) (st : κ → ℂ), (∀ I, IsCo

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.TensorWeightedBesselSumStatement

theorem **_root_.Gp91.GlobalTheory.T.tensor_weighted_bessel_sum** : PI.KFinSmooth → ∀ (k r : ℕ) (ι κ : Type) [Fintype ι] [DecidableEq ι] [Fintype κ] [DecidableEq κ] (ρ : Mx k r → Matrix ι ι ℂ) (Γ : Matrix ι ι ℂ) (d : ℕ) (R : Rep k r) (RI : κ → Rep k r) (io : ∀ I, ι → (RI I).H → R.H) (pr : ∀ I, ι → R.H → (RI I).H) (ep : κ → ℝ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (muI : κ → Fin (2 * r + k + 1) → ℝ) (Wp : Type) [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] (Wq : Type) [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [CompleteSpace Wq] [FiniteDimensional ℂ Wq] (om om' : ι → ℂ) (l : R.H → Wp) (l' : R.H → Wq) (lI : ∀ I, (RI I).H → Wp) (lI' : ∀ I, (RI I).H → Wq) (F : Mx k r → (Wq →L[ℂ] Wp)) (s : ℂ), R.IsDG ε mu → (∀ I, (RI I).IsDG ε (muI I)) → IsFRep ρ Γ d → TensH ρ Γ R RI io pr ep → IsCoef R l l' F s → ∃ (Ft : κ → Mx k r →

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.TorusWeightEigenAndRootShift

theorem **part_one_lie** (h1 : Gᴴ = G) (h2 : G * G = 1) (Y : Matrix n n ℂ) : ((1 / 2 : ℂ) • (Y + -(G * Yᴴ * G)))ᴴ * G + G * ((1 / 2 : ℂ) • (Y + -(G * Yᴴ * G))) = 0

theorem **part_two_lie** (h1 : Gᴴ = G) (h2 : G * G = 1) (Y : Matrix n n ℂ) : ((1 / (2 * Complex.I)) • (Y - -(G * Yᴴ * G)))ᴴ * G + G * ((1 / (2 * Complex.I)) • (Y - -(G * Yᴴ * G))) = 0

theorem **theta_conj** (h2 : G * G = 1) {g : Matrix n n ℂ} (hu : IsUnit g.det) (hU : gᴴ * G * g = G) (Y : Matrix n n ℂ) : g * (G * Yᴴ * G) * g⁻¹ = G * (g * Y * g⁻¹)ᴴ * G

theorem **part_one_smul** (x y : ℝ) (Y : Matrix n n ℂ) : (1 / 2 : ℂ) • (((x : ℂ) + y * Complex.I) • Y + -(G * (((x : ℂ) + y * Complex.I) • Y)ᴴ * G)) = x • ((1 / 2 : ℂ) • (Y + -(G * Yᴴ * G))) - y • ((1 / (2 * Complex.I)) • (Y - -(G * Yᴴ * G)))

theorem **part_two_smul** (x y : ℝ) (Y : Matrix n n ℂ) : (1 / (2 * Complex.I)) • (((x : ℂ) + y * Complex.I) • Y - -(G * (((x : ℂ) + y * Complex.I) • Y)ᴴ * G)) = x • ((1 / (2 * Complex.I)) • (Y - -(G * Yᴴ * G))) + y • ((1 / 2 : ℂ) • (Y + -(G * Yᴴ * G)))

theorem **dC_smul_left** (h1 : (Hpk k r)ᴴ = Hpk k r) (h2 : Hpk k r * Hpk k r = 1) (R : Rep k r) (c : ℂ) (Y : Mx k r) {v : R.H} (hv : v ∈ R.sm) : R.dC (c • Y) v = c • R.dC Y v

theorem **dC_smul_right** (h1 : (Hpk k r)ᴴ = Hpk k r) (h2 : Hpk k r * Hpk k r = 1) (R : Rep k r) (c : ℂ) (Y : Mx k r) {v : R.H} (hv : v ∈ R.sm) : R.dC Y (c • v) = c • R.dC Y v

theorem **dC_conj** (hK : PI.KFinSmooth) (R : Rep k r) (hR : R.IsDG ε mu) (h1 : (Hpk k r)ᴴ = Hpk k r) (h2 : Hpk k r * Hpk k r = 1) {g : Mx k r} (hg : g ∈ GG k r) (hu : IsUnit g.det) (Y : Mx k r) {v : R.H} (hv : v ∈ R.sm) : R.σ g (R.dC Y v) = R.dC (g * Y * g⁻¹) (R.σ g v)

theorem **torus_mem_K** {n : ℕ} (ε : Fin n → Bool) (θ : Fin n → ℝ) : Matrix.diagonal (fun a => Complex.exp ((θ a : ℂ) * Complex.I)) ∈ Kset ε

theorem **torus_single** {n : ℕ} (θ : Fin n → ℝ) (a b : Fin n) : Matrix.diagonal (fun a => Complex.exp ((θ a : ℂ) * Complex.I)) * Matrix.single a b (1 : ℂ) = Complex.exp (((θ a - θ b : ℝ) : ℂ) * Complex.I) • (Matrix.single a b (1 : ℂ) * Matrix.diagonal (fun a => Complex.exp ((θ a : ℂ) * Complex.I)))

theorem **torus_weight_action** : PI.KFinSmooth → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (R : Rep k r), R.IsDG ε mu → ∀ (Q : Mx k r) (E : (Fin (2 * r + k + 1) → Fin (2 * r + k + 1) → Module.End ℂ R.smSub)), PI.UpqRep.IsCayleyU k r ε Q → R.IsE Q E → ∀ (ν : Fin (2 * r + k + 1) → ℝ) (v : R.smSub),

PI.UpqRep.IsWtOn (PI.UpqRep.Kset ε) (fun g => R.σ (Q⁻¹ * g * Q)) ν (v : R.H) → Formalism.REAL.IsWeight E ν v ∧ ∀ a b, PI.UpqRep.IsWtOn (PI.UpqRep.Kset ε) (fun g => R.σ (Q⁻¹ * g * Q)) (ν + PI.UpqRep.root a b) ((E a b v : R.smSub) : R.H)

CompanionTheory.AutomorphicTheory.PieriRules.ThetaBesselIsometries.TorusWeightEigenAndRootShiftStatement

theorem **_root_.Gp91.GlobalTheory.T.torusWeight_eigen_and_rootShift** : PI.KFinSmooth → ∀ (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (R : Rep k r), R.IsDG ε mu → ∀ (Q : Mx k r) (E : (Fin (2 * r + k + 1) → Fin (2 * r + k + 1) → Module.End ℂ R.smSub)), PI.UpqRep.IsCayleyU k r ε Q → R.IsE Q E → ∀ (ν : Fin (2 * r + k + 1) → ℝ) (v : R.smSub), PI.UpqRep.IsWtOn (PI.UpqRep.Kset ε) (fun g => R.σ (Q⁻¹ * g * Q)) ν (v : R.H) → Formalism.REAL.IsWeight E ν v ∧ ∀ a b, PI.UpqRep.IsWtOn (PI.UpqRep.Kset ε) (fun g => R.σ (Q⁻¹ * g * Q)) (ν + PI.UpqRep.root a b) ((E a b v : R.smSub) : R.H)

CompanionTheory.AutomorphicTheory.PlaceSeparation.ArtinA

theorem **exists_place_between** {A : ValuationSubring Ω} {v : HeightOneSpectrum (𝒪 K)} (hA : IsPlaceOver A v) : ∃ w : HeightOneSpectrum (𝒪 L), IsPlaceOver A w ∧ w.asIdeal.LiesOver v.asIdeal

theorem **mem_Ine_of_integral** {A : ValuationSubring Ω} {σ : G K Ω} (hσ : σ ∈ Dec (F

theorem **coe_res_smul** (σ : G K Ω) (x : 𝒪 L) : algebraMap L Ω ((AlgEquiv.restrictNormalHom L σ • x : 𝒪 L) : L) = σ (algebraMap L Ω (x : L))

theorem **res_restrictScalars** (g : Ω ≃ₐ[L] Ω) : AlgEquiv.restrictNormalHom L (g.restrictScalars K : G K Ω) = 1

theorem **res_mem_stabilizer** (hw : IsPlaceOver A w) {σ : G K Ω} (hσ : σ ∈ Dec (F

theorem **val_int_le_one** (A : ValuationSubring Ω) (x : 𝒪 L) : A.valuation (algebraMap L Ω (x : L)) ≤ 1

theorem **res_mem_inertia** (hw : IsPlaceOver A w) {σ : G K Ω} (hσ : σ ∈ Ine (F

theorem **res_isArithFrobAt** {v : HeightOneSpectrum (𝒪 K)} (hw : IsPlaceOver A w) [h : w.asIdeal.LiesOver v.asIdeal] {σ : G K Ω} (hσ : IsFrob (F

theorem **exists_lift_dec** (hw : IsPlaceOver A w) {τ : L ≃ₐ[K] L} (hτ : τ ∈ MulAction.stabilizer (L ≃ₐ[K] L) w.asIdeal) : ∃ σ ∈ Dec (F

theorem **exists_lift_ine** (hw : IsPlaceOver A w) {τ : L ≃ₐ[K] L} (hτ : τ ∈ w.asIdeal.inertia (L ≃ₐ[K] L)) : ∃ σ ∈ Ine (F

theorem **inertia_eq_bot_iff** (v : HeightOneSpectrum (𝒪 K)) (w : HeightOneSpectrum (𝒪 L)) [w.asIdeal.LiesOver v.asIdeal] : w.asIdeal.inertia (L ≃ₐ[K] L) = ⊥ ↔ w.asIdeal.ramificationIdx (𝒪 K) = 1

CompanionTheory.AutomorphicTheory.PlaceSeparation.ArtinKer

theorem **isOpen_connInf** (E : Type) [Field E] [NumberField E] : IsOpen ((Subgroup.connectedComponentOfOne (InfiniteAdeleRing E)ˣ : Subgroup (InfiniteAdeleRing E)ˣ) : Set (InfiniteAdeleRing E)ˣ)

theorem **ofInfinite_eq_map** (E : Type) [Field E] [NumberField E] (y : (InfiniteAdeleRing E)ˣ) : (ofInfinite y : IdeleGroup E) = Units.map (MonoidHom.inl (InfiniteAdeleRing E) (FiniteAdeleRing (𝒪 E) E)) y

theorem **comap_connectedComponent** (E : Type) [Field E] [NumberField E] : (Subgroup.connectedComponentOfOne (GroundTheory.Global.IdeleClassGroup E)).comap (GroundTheory.Global.IdeleClassGroup.mk E) = ((Neutral.B1.diagUnits E).range ⊔ GlobalTheory.HeckeGal.infConn E).topologicalClosure

CompanionTheory.AutomorphicTheory.PlaceSeparation.IsOpenQuotientMapArt0

theorem **isOpenQuotientMap_art0** : IsOpenQuotientMap (art0 E Ω)

theorem **ker_art0** : (art0 E Ω).ker = ((diagUnits E).range ⊔ infConn E).topologicalClosure

theorem **res_toAb** (L : IntermediateField E Ω) [IsAbelianGalois E L] (σ : G E Ω) : res L (toAb E Ω σ) = AlgEquiv.restrictNormalHom L σ

theorem **continuous_res** (L : IntermediateField E Ω) [IsAbelianGalois E L] : Continuous (res L)

theorem **localIdele_eq** (v : HeightOneSpectrum (𝒪 E)) (x : (v.adicCompletion E)ˣ) : localIdele E v x = IdeleGroup.finiteEmbedding v x

theorem **res_art0** (L : IntermediateField E Ω) [FiniteDimensional E L] [IsAbelianGalois E L] (v : HeightOneSpectrum (𝒪 E)) (x : (v.adicCompletion E)ˣ) : res L (art0 E Ω (localIdele E v x)) = localSymbol L v x

theorem **continuous_toAb** : Continuous (toAb E Ω)

theorem **mem_of_forall_res** {H : Subgroup (Gab E Ω)} (hH : IsClosed (H : Set (Gab E Ω))) {g : Gab E Ω} (h : ∀ (L : IntermediateField E Ω) [FiniteDimensional E L] [IsAbelianGalois E L], H ≤ (res L).ker → res L g = 1) : g ∈ H

theorem **eq_of_forall_res** {H H' : Subgroup (Gab E Ω)} (hH : IsClosed (H : Set (Gab E Ω))) (hH' : IsClosed (H' : Set (Gab E Ω))) (h : ∀ (L : IntermediateField E Ω) [FiniteDimensional E L] [IsAbelianGalois E L], H ≤ (res L).ker ↔ H' ≤ (res L).ker) : H = H'

theorem **isClosed_map_toAb** {S : Subgroup (G E Ω)} (hS : IsClosed (S : Set (G E Ω))) : IsClosed ((S.map (toAb E Ω) : Subgroup (Gab E Ω)) : Set (Gab E Ω))

theorem **isAlgClosure_top** (L : IntermediateField E Ω) : IsAlgClosure L Ω

theorem **ine_le_ker_iff** (L : IntermediateField E Ω) [FiniteDimensional E L] [IsAbelianGalois E L] {w : HeightOneSpectrum (𝒪 L)} (hw : IsPlaceOver A w) : (Ine (F

theorem **map_units_eq** (hA : IsPlaceOver A v) : (unitFiltrationUnits (v.adicCompletion E) 0).map ((art0 E Ω).comp (localIdele E v)) = (Ine (F

theorem **unif_frob** (hA : IsPlaceOver A v) {π : (v.adicCompletion E)ˣ} (hπ : ord (π : v.adicCompletion E) = 1) {σ : G E Ω} (hσ : IsFrob (F

theorem **closure_range_eq** (hA : IsPlaceOver A v) : ((art0 E Ω).comp (localIdele E v)).range.topologicalClosure = (Dec (F

theorem **art0_unit_iff** (hA : IsPlaceOver A v) (g : Gab E Ω) : (∃ x, IsUnitAt v x ∧ art0 E Ω (localIdele E v x) = g) ↔ g ∈ (Ine (F

theorem **art0_unif_iff** (hA : IsPlaceOver A v) (g : Gab E Ω) : (∃ x, IsUnifAt v x ∧ art0 E Ω (localIdele E v x) = g) ↔ ∃ σ : G E Ω, IsFrob (F

theorem **isGlobalArtin_inv** : IsGlobalArtin (art0 E Ω)⁻¹ where

theorem **globalArtin** : GlobalTheory.Published.GlobalArtinReciprocity

CompanionTheory.AutomorphicTheory.ProductFormulaTheory.AbelianDeRham.FreeAbelianDeRham.H1StructureDimLe

theorem **h1Le** : CompanionTheory.ObstructionTrivialLocus.H1StructureDimLe

CompanionTheory.AutomorphicTheory.ProductFormulaTheory.CuspidalExhaustion.EmbeddingMiddleFactorTriple

theorem **emb_trans** {n : ℕ} {A C D : BR k n} (h : Emb A C) (h' : Emb C D) : Emb A D

theorem **emb_of_iso** {n : ℕ} {A C : BR k n} (h : Iso A C) : Emb A C

theorem **embedding_middle_factor_triple** {nA nB m m' N : ℕ} (h2 : m + nB = m') (h1 : nA + m' = N) (A : BR k nA) (B : BR k nB) {M M' : BR k m} (h : Emb M M') : Emb (two h1 A (two h2 M B)) (two h1 A (two h2 M' B))

theorem **mid_br** {nA nB m m' : ℕ} (x : IrrGL k) (h2 : m + nB = m') (h1 : nA + m' = x.1) (A : BR k nA) (B : BR k nB) (M : BR k m) (hM : FL M) (hs : Sm M) (h : Emb (one x) (two h1 A (two h2 M B))) : ∃ c : IrrCl (GL (Fin m) k), Sq (one (m, c)) M ∧ Emb (one x) (two h1 A (two h2 (one (m, c)) B))

theorem **emb_pr_iff** (x : IrrGL k) {a : ℕ} (z : Fin a → IrrGL k) (hd : ∑ i, (z i).1 = x.1) : Emb (one x) (pr z hd) ↔ ∃ f : x.rep.IntertwiningMap (prodIrr z hd), Function.Injective f

theorem **sq_pr_iff** (x : IrrGL k) {a : ℕ} (z : Fin a → IrrGL k) (hd : ∑ i, (z i).1 = x.1) : Sq (one x) (pr z hd) ↔ IsSubquotient x.rep (prodIrr z hd)

theorem **fl_pr** (hA : GlobalTheory.Published.IrreducibleAdmissibleGL) {a N : ℕ} (z : Fin a → IrrGL k) (hd : ∑ i, (z i).1 = N) : FL (pr z hd)

theorem **iso_one_pr** (x : IrrGL k) (hd : ∑ _ : Fin 1, x.1 = x.1) : Iso (one x) (pr (fun _ : Fin 1 => x) hd)

theorem **iso_pr_of** {a b N : ℕ} (z : Fin a → IrrGL k) (w : Fin b → IrrGL k) (hz : ∑ i, (z i).1 = N) (hw : ∑ i, (w i).1 = N) (h : Nonempty ((prodIrr z hz).Equiv (prodIrr w hw))) : Iso (pr z hz) (pr w hw)

theorem **embL_iff** (x : IrrGL k) (l : List (IrrGL k)) : EmbL x l ↔ ∃ hd, Emb (one x) (pr l.get hd)

theorem **deg3** (A M B : List (IrrGL k)) : ∑ a, ((A ++ M ++ B).get a).1 = ∑ a, (A.get a).1 + (∑ a, (M.get a).1 + ∑ a, (B.get a).1)

theorem **iso_single** (σ : IrrGL k) (h : ∑ a, ([σ].get a).1 = σ.1) : Iso (one σ) (pr [σ].get h)

theorem **embL_block** {x : IrrGL k} {A M M' B : List (IrrGL k)} {m : ℕ} (hM : ∑ a, (M.get a).1 = m) (hM' : ∑ a, (M'.get a).1 = m) (h : Emb (pr M.get hM) (pr M'.get hM')) (hx : EmbL x (A ++ M ++ B)) : EmbL x (A ++ M' ++ B)

theorem **refineP'** : RefineP k

theorem **swapP'** (h : GlobalTheory.BZSeg.LUnlinkedComm k) : SwapP k

theorem **midP'** (hA : GlobalTheory.Published.IrreducibleAdmissibleGL) : MidP k

CompanionTheory.AutomorphicTheory.ProductFormulaTheory.CuspidalExhaustion.EquivTripleProductOfIso

theorem **product_append_iso** {p q r t : ℕ} (x : Fin p → IrrGL k) (y : Fin q → IrrGL k) (hx : ∑ a, (x a).1 = r) (hy : ∑ a, (y a).1 = t) (h : ∑ a, (Fin.append x y a).1 = r + t) : Iso (two rfl (pr x hx) (pr y hy)) (pr (Fin.append x y) h)

theorem **pr_app** {p q c r t N : ℕ} (x : Fin p → IrrGL k) (y : Fin q → IrrGL k) (z : Fin c → IrrGL k) (e : List.ofFn z = List.ofFn x ++ List.ofFn y) (hx : ∑ a, (x a).1 = r) (hy : ∑ a, (y a).1 = t) (hz : ∑ a, (z a).1 = N) (h : r + t = N) : Iso (pr z hz) (two h (pr x hx) (pr y hy))

theorem **sum_get_eq'** (L : List (IrrGL k)) : ∑ a, (L.get a).1 = (L.map fun x => x.1).sum

theorem **stage3_iso** (A M B : List (IrrGL k)) {N nA nM nB m' : ℕ} (hd : ∑ a, ((A ++ M ++ B).get a).1 = N) (hA : ∑ a, (A.get a).1 = nA) (hM : ∑ a, (M.get a).1 = nM) (hB : ∑ a, (B.get a).1 = nB) (h2 : nM + nB = m') (h1 : nA + m' = N) : Iso (pr (A ++ M ++ B).get hd) (two h1 (pr A.get hA) (two h2 (pr M.get hM) (pr B.get hB)))

theorem **equiv_triple_product_of_iso** {c a m b N nA nM nB m' : ℕ} (z : Fin c → IrrGL k) (A : Fin a → IrrGL k) (M : Fin m → IrrGL k) (B : Fin b → IrrGL k) (hd : ∑ a, (z a).1 = N) (hA : ∑ a, (A a).1 = nA) (hM : ∑ a, (M a).1 = nM) (hB : ∑ a, (B a).1 = nB) (h2 : nM + nB = m') (h1 : nA + m' = N) (H : Iso (pr z hd) (two h1 (pr A hA) (two h2 (pr M hM) (pr B hB)))) : Nonempty ((prodIrr z hd).Equiv (T3 h2 h1 (prodIrr A hA) (prodIrr B hB) (prodIrr M hM)))

theorem **stage3** (A M B : List (IrrGL k)) {N nA nM nB m' : ℕ} (hd : ∑ a, ((A ++ M ++ B).get a).1 = N) (hA : ∑ a, (A.get a).1 = nA) (hM : ∑ a, (M.get a).1 = nM) (hB : ∑ a, (B.get a).1 = nB) (h2 : nM + nB = m') (h1 : nA + m' = N) : Nonempty ((prodIrr (A ++ M ++ B).get hd).Equiv (T3 h2 h1 (prodIrr A.get hA) (prodIrr B.get hB) (prodIrr M.get hM)))

CompanionTheory.AutomorphicTheory.ProductFormulaTheory.CuspidalExhaustion.ExhaustionOfSteps

theorem **move_left** (hM : MidP k) (hR : RefineP k) (hS : SwapP k) (hJ : JuxtP k) (hL : LinkP k) (x ρ : IrrGL k) : ∀ (A : List (IrrGL k)), (∀ a ∈ A, a.IsCusp ∧ ρ ≠ a.nu 1) → ∀ (Δ : Segment k) (b : IrrGL k) (B : List (IrrGL k)), Δ.first = ρ → IsLSeg Δ b.rep → EmbL x (A ++ [b] ++ B) → ∃ (Γ : Segment k) (c y : IrrGL k), Γ.first = ρ ∧ IsLSeg Γ c.rep ∧ EmbL x [c, y]

theorem **exhaustion_of_steps** (H : ∀ (k : Type) [Field k] [ValuativeRel k] [TopologicalSpace k] [IsNonarchimedeanLocalField k] [CharZero k], MidP k ∧ RefineP k ∧ SwapP k ∧ CuspEmbP k ∧ SuppP k ∧ JuxtP k ∧ LinkP k ∧ MinP k ∧ PeelGoodP k) : GlobalTheory.Published.LanglandsExhaustionGL

CompanionTheory.AutomorphicTheory.ProductFormulaTheory.CuspidalExhaustion.ExhaustionOfThree

theorem **standardCosocle_of** : GlobalTheory.Published.StandardCosocleGL

theorem **lOrderFree_of** (k : Type) [Field k] [ValuativeRel k] [TopologicalSpace k] [IsNonarchimedeanLocalField k] : LOrderFree k

theorem **lSocleGL_of** (k : Type) [Field k] [ValuativeRel k] [TopologicalSpace k] [IsNonarchimedeanLocalField k] : LSocleGL k

theorem **exhaustion_of_three** (H : ∀ (k : Type) [Field k] [ValuativeRel k] [TopologicalSpace k] [IsNonarchimedeanLocalField k], MidP k ∧ RefineP k ∧ SwapP k) : GlobalTheory.Published.LanglandsExhaustionGL

theorem **multisegments_of** (hE : GlobalTheory.Published.LanglandsExhaustionGL) : GlobalTheory.Published.LanglandsMultisegments

CompanionTheory.AutomorphicTheory.ProductFormulaTheory.CuspidalExhaustion.IrreducibleAdmissibleGL

theorem **geometricLemmaGL** : GlobalTheory.Published.GeometricLemma

theorem **embL_of_eq** {x : IrrGL k} {l : List (IrrGL k)} {y : Fin l.length → IrrGL k} (e : y = l.get) (hd : ∑ a, (y a).1 = x.1) (f : x.rep.IntertwiningMap (prodIrr y hd)) (hf : Function.Injective f) : EmbL x l

theorem **embL_pair** {x : IrrGL k} {y : Fin 2 → IrrGL k} {hd : ∑ a, (y a).1 = x.1} {f : x.rep.IntertwiningMap (prodIrr y hd)} (hf : Function.Injective f) : EmbL x [y 0, y 1]

theorem **embL_nil** (x : IrrGL k) (h0 : x.1 = 0) : EmbL x []

theorem **cuspEmbP** (hA : GlobalTheory.Published.IrreducibleAdmissibleGL) (hR : RefineP k) : CuspEmbP k

theorem **suppP** : SuppP k

theorem **irreducibleAdmissibleGL** : GlobalTheory.Published.IrreducibleAdmissibleGL

theorem **cuspEmbP_of_refine** (hR : RefineP k) : CuspEmbP k

CompanionTheory.AutomorphicTheory.ProductFormulaTheory.CuspidalExhaustion.IsLAddSegmentOfInjectiveOfNotPrecedes

theorem **isL_add_segment_of_injective_of_not_precedes** [CharZero k] (HS : GlobalTheory.Published.StandardCosocleGL) (HL : LSocleGL k) (x : IrrGL k) (y : Fin 2 → IrrGL k) (hd : ∑ a, (y a).1 = x.1) (s : Segment k) (π : Multiset (Segment k)) (hs : IsLSeg s (y 0).rep) (hn : IsL π (y 1).rep) (hf : ∃ f : x.rep.IntertwiningMap (prodIrr y hd), Function.Injective f) (hp : ∀ Δ ∈ π, ¬ Δ.Precedes s) : IsL (π + {s}) x.rep

theorem **peelGoodP** (h1 : GlobalTheory.Published.StandardCosocleGL) (h2 : Zc.LSocleGL k) (_h3 : RefineP k) [CharZero k] : Zc.PeelGoodP k

CompanionTheory.AutomorphicTheory.ProductFormulaTheory.CuspidalExhaustion.JuxtDec

theorem **exists_emb_of_cast** {h h' N : ℕ} {V : Type} [AddCommGroup V] [Module ℂ V] (τ : Representation ℂ (GL (Fin N) k) V) (e : h = h') (x : Fin h → IrrGL k) (y : Fin h' → IrrGL k) (hxy : ∀ i, x i = y (Fin.cast e i)) (hd : ∑ a, (x a).1 = N) (f : τ.IntertwiningMap (prodIrr x hd)) (hf : Function.Injective f) : ∃ hd' : ∑ a, (y a).1 = N, ∃ g : τ.IntertwiningMap (prodIrr y hd'), Function.Injective g

theorem **decW_get_eq** (Γ : Segment k) (e : (decW Γ).length = Γ.len) (i : Fin (decW Γ).length) : (decW Γ).get i = Zc.Seg.dec Γ (Fin.cast e i)

theorem **decLP** (hF : FineUniqP k) : DecLP k

CompanionTheory.AutomorphicTheory.ProductFormulaTheory.CuspidalExhaustion.JuxtPOfFineUniq

theorem **juxtP** (hA : GlobalTheory.Published.IrreducibleAdmissibleGL) (hM : MidP k) (hR : RefineP k) (hS : SwapP k) (hC : CuspEmbP k) (hW : ShlP k) (hD : DecLP k) : JuxtP k

theorem **juxtP_of_fineUniq** (hA : GlobalTheory.Published.IrreducibleAdmissibleGL) (hM : MidP k) (hR : RefineP k) (hS : SwapP k) (hC : CuspEmbP k) (hW : ShlP k) (hF : FineUniqP k) : JuxtP k

CompanionTheory.AutomorphicTheory.ProductFormulaTheory.CuspidalExhaustion.JuxtShl

theorem **interleave_nil_left** {v w : List T} (h : Zc.Comb.Interleave [] v w) : w = v

theorem **interleave_nil_right** {u w : List T} (h : Zc.Comb.Interleave u [] w) : w = u

theorem **interleave_single** {a : T} {v w : List T} (h : Zc.Comb.Interleave [a] v w) : ∃ P Q : List T, v = P ++ Q ∧ w = P ++ a :: Q

theorem **shuffle_two_single** {a : T} {D w : List T} (h : Zc.Comb.Shuffle [[a], D] w) : ∃ P Q : List T, D = P ++ Q ∧ w = P ++ a :: Q

theorem **decW_single** (ρ : IrrGL k) (hp : ρ.IsCusp) : decW (Segment.single ρ hp) = [ρ]

theorem **words_of_subq** (hA : GlobalTheory.Published.IrreducibleAdmissibleGL) (hW : ShlP k) (σ b : IrrGL k) (Δ : Segment k) (hb : IsLSeg Δ b.rep) (hσ : SubqL σ [nxt Δ, b]) (l : List (IrrGL k)) (hl : ∀ ρ ∈ l, ρ.IsCusp) (he : EmbL σ l) : ∃ P Q : List (IrrGL k), decW Δ = P ++ Q ∧ l = P ++ nxt Δ :: Q

CompanionTheory.AutomorphicTheory.ProductFormulaTheory.CuspidalExhaustion.JuxtSwap

theorem **nu_nu_nat** (x : IrrGL k) (a b : ℕ) : (x.nu (a : ℂ)).nu (b : ℂ) = x.nu ((a + b : ℕ) : ℂ)

theorem **not_linked_single** (hA : GlobalTheory.Published.IrreducibleAdmissibleGL) (ρ : IrrGL k) (hp : ρ.IsCusp) (n j : ℕ) (hj : j + 2 ≤ n) : ¬ (Segment.single (ρ.nu (n : ℂ)) (hp.nu _)).Linked (Segment.single (ρ.nu (j : ℂ)) (hp.nu _))

theorem **mem_tail_decW** (Δ : Segment k) (x : IrrGL k) (hx : x ∈ (decW Δ).tail) : ∃ j : ℕ, j + 2 ≤ Δ.len ∧ x = Δ.first.nu (j : ℂ)

theorem **move_past** (hS : SwapP k) (σ ρ : IrrGL k) (hp : ρ.IsCusp) : ∀ (Q P : List (IrrGL k)), (∀ q ∈ Q, ∃ hq : q.IsCusp, ¬ (Segment.single ρ hp).Linked (Segment.single q hq)) → EmbL σ (P ++ ρ :: Q) → EmbL σ (P ++ Q ++ [ρ])

theorem **emb_move_end** (hA : GlobalTheory.Published.IrreducibleAdmissibleGL) (hS : SwapP k) (σ : IrrGL k) (Δ : Segment k) (P Q : List (IrrGL k)) (h1 : decW Δ = P ++ Q) (hP : P ≠ []) (he : EmbL σ (P ++ nxt Δ :: Q)) : EmbL σ (decW Δ ++ [nxt Δ])

CompanionTheory.AutomorphicTheory.ProductFormulaTheory.CuspidalExhaustion.JuxtUniq

theorem **eq_of_isLSeg** (Δ : Segment k) (x y : IrrGL k) (hx : IsLSeg Δ x.rep) (hy : IsLSeg Δ y.rep) : x = y

CompanionTheory.AutomorphicTheory.ProductFormulaTheory.CuspidalExhaustion.JuxtWords

theorem **decW_up** (Δ : Segment k) : decW (up Δ) = nxt Δ :: decW Δ

theorem **nxt_not_mem** (hA : GlobalTheory.Published.IrreducibleAdmissibleGL) (Δ : Segment k) : nxt Δ ∉ decW Δ

theorem **eq_decW_of_append** (hA : GlobalTheory.Published.IrreducibleAdmissibleGL) (Δ : Segment k) (u P Q : List (IrrGL k)) (h1 : decW Δ = P ++ Q) (h2 : u ++ [nxt Δ] = P ++ nxt Δ :: Q) : u = decW Δ

CompanionTheory.AutomorphicTheory.ProductFormulaTheory.CuspidalExhaustion.LUnlinkedIrrOfSeg

theorem **lUnlinkedIrr_of_seg** (hZ : GlobalTheory.Published.SegmentRepresentation) (k : Type) [Field k] [ValuativeRel k] [TopologicalSpace k] [IsNonarchimedeanLocalField k] : GlobalTheory.BZSeg.LUnlinkedIrr k

CompanionTheory.AutomorphicTheory.ProductFormulaTheory.CuspidalExhaustion.LanglandsMultisegmentsOfSeg

theorem **standardCosocleGL_of_seg** (hZ : GlobalTheory.Published.SegmentRepresentation) : GlobalTheory.Published.StandardCosocleGL

theorem **langlandsExhaustionGL_of_seg** (hZ : GlobalTheory.Published.SegmentRepresentation) : GlobalTheory.Published.LanglandsExhaustionGL

theorem **langlandsMultisegments_of_seg** (hZ : GlobalTheory.Published.SegmentRepresentation) : GlobalTheory.Published.LanglandsMultisegments

CompanionTheory.AutomorphicTheory.ProductFormulaTheory.CuspidalExhaustion.MidExact

theorem **extL_exact** (f : τ_1.IntertwiningMap τ_2) (g : τ_2.IntertwiningMap τ_3) (σ : Representation ℂ H W) (hf : Function.Injective f) (hfg : Function.Exact f g) : Function.Injective (extL f σ) ∧ Function.Exact (extL f σ) (extL g σ)

theorem **extR_exact** (σ : Representation ℂ H W) (f : τ_1.IntertwiningMap τ_2) (g : τ_2.IntertwiningMap τ_3) (hf : Function.Injective f) (hfg : Function.Exact f g) : Function.Injective (extR σ f) ∧ Function.Exact (extR σ f) (extR σ g)

theorem **ind₂Map_exact** (e : a ⊛ b ≃ n) (f : ρ_1.IntertwiningMap ρ_2) (g : ρ_2.IntertwiningMap ρ_3) (hf : Function.Injective f) (hfg : Function.Exact f g) : Function.Injective (ind₂Map (k

theorem **tripleProductMap_injective** (h2 : m + nB = m') (h1 : nA + m' = N) (PA : Representation ℂ (GL (Fin nA) k) VA) (PB : Representation ℂ (GL (Fin nB) k) VB) {τ_1 : Representation ℂ (GL (Fin m) k) V_1} {τ_2 : Representation ℂ (GL (Fin m) k) V_2} {τ_3 : Representation ℂ (GL (Fin m) k) V_3} (f : τ_1.IntertwiningMap τ_2) (g : τ_2.IntertwiningMap τ_3) (hf : Function.Injective f) (hfg : Function.Exact f g) : Function.Injective (tripleProductMap h2 h1 PA PB f) ∧ Function.Exact (tripleProductMap h2 h1 PA PB f) (tripleProductMap h2 h1 PA PB g)

CompanionTheory.AutomorphicTheory.ProductFormulaTheory.CuspidalExhaustion.MidGen

theorem **Tm_equiv_injective** {V W : Type} [AddCommGroup V] [Module ℂ V] [AddCommGroup W] [Module ℂ W] {τ : Representation ℂ (GL (Fin m) k) V} {τ' : Representation ℂ (GL (Fin m) k) W} (e : τ.Equiv τ') : Function.Injective (Tm e.toIntertwiningMap)

theorem **codSub_injective** {G V W : Type} [Group G] [AddCommGroup V] [Module ℂ V] [AddCommGroup W] [Module ℂ W] {ρ : Representation ℂ G V} {σ : Representation ℂ G W} (f : ρ.IntertwiningMap σ) (S : Subrepresentation σ) (h : ∀ v, f v ∈ S.toSubmodule) (hf : Function.Injective f) : Function.Injective (codSub f S h)

theorem **mid_sub** {V : Type} [AddCommGroup V] [Module ℂ V] {P : Representation ℂ (GL (Fin m) k) V} (hP : HasFiniteLength P) (hs : IsSmooth P) (x : IrrCl (GL (Fin N) k)) (S : Subrepresentation P) (hx : ∃ f : x.rep.ρ.IntertwiningMap (T S.toRepresentation).ρ, Function.Injective f) : ∃ c : IrrCl (GL (Fin m) k), IsSubquotient c.rep.ρ P ∧ ∃ f : x.rep.ρ.IntertwiningMap (T c.rep.ρ).ρ, Function.Injective f

theorem **mid_gen** {V : Type} [AddCommGroup V] [Module ℂ V] {P : Representation ℂ (GL (Fin m) k) V} (hP : HasFiniteLength P) (hs : IsSmooth P) (x : IrrCl (GL (Fin N) k)) (f : x.rep.ρ.IntertwiningMap (T P).ρ) (hf : Function.Injective f) : ∃ c : IrrCl (GL (Fin m) k), IsSubquotient c.rep.ρ P ∧ ∃ f : x.rep.ρ.IntertwiningMap (T c.rep.ρ).ρ, Function.Injective f

CompanionTheory.AutomorphicTheory.ProductFormulaTheory.CuspidalExhaustion.NotAboveTransGenSelf

theorem **linkP** : LinkP k

private theorem **above_transGen** {a b : IrrGL k} (H : Relation.TransGen Above a b) : 0 < a.1 ∧ ∃ j : ℕ, 0 < j ∧ a = b.nu (j : ℕ)

private theorem **not_above_transGen_self** (a : IrrGL k) : ¬ Relation.TransGen Above a a

theorem **minP** : MinP k

CompanionTheory.AutomorphicTheory.ProductFormulaTheory.DualAbelianSchemes.AddAcyclic

theorem **addAcyclic_pf** : ∀ (k : Type) [Field k] [IsAlgClosed k] (A : Over (Sp k)) [GrpObj A], IsAbelianScheme A → ∀ M : LB A.left, ∀ hM : M.IsLB, (∀ (Y : Over (Sp k)) (a a' : Y ⟶ A), GlobalTheory.CechPic.pull (a ∗ a').left (cls M hM) = GlobalTheory.CechPic.pull a.left (cls M hM) + GlobalTheory.CechPic.pull a'.left (cls M hM)) → ¬ Triv M → ∀ (κ : Type) [LinearOrder κ] [Fintype κ] (D : κ → A.left.Opens), (∀ i, IsAffineOpen (D i)) → ⊔ i, D i = ⊤ → (cechLB A.hom M D).Acyclic

CompanionTheory.AutomorphicTheory.ProductFormulaTheory.DualAbelianSchemes.FewEqBridge

theorem **fewEquations_of_fewEqSt** (hF : FewEqSt) (hLe : H1StructureDimLe) : FewEquations

theorem **fewEquations_pf** : H1StructureDimLe → FewEquations

CompanionTheory.AutomorphicTheory.ProductFormulaTheory.DualAbelianSchemes.PointSurj

theorem **pull_evc_const** (hM : M.IsLB) (t_0 : 𝟙_ (Over (Sp k)) ⟶ T) {Y : Over (Sp k)} (a : Y ⟶ A) : GlobalTheory.CechPic.pull a.left (evc hM (𝟙 A) (toUnit A ≫ t_0)) = evc hM a (toUnit Y ≫ t_0)

theorem **pull_mul_evc_const** [GrpObj A] (hM : M.IsLB) (hadd : IsAdditive M) (t_0 : 𝟙_ (Over (Sp k)) ⟶ T) {Y : Over (Sp k)} (a a' : Y ⟶ A) : GlobalTheory.CechPic.pull (a ∗ a').left (evc hM (𝟙 A) (toUnit A ≫ t_0)) = GlobalTheory.CechPic.pull a.left (evc hM (𝟙 A) (toUnit A ≫ t_0)) + GlobalTheory.CechPic.pull a'.left (evc hM (𝟙 A) (toUnit A ≫ t_0))

theorem **pointSurj_pf** : (∀ (k : Type) [Field k] [IsAlgClosed k] (A : Over (Sp k)) [GrpObj A], IsAbelianScheme A → ∀ M : LB A.left, ∀ hM : M.IsLB, (∀ (Y : Over (Sp k)) (a a' : Y ⟶ A), GlobalTheory.CechPic.pull (a ∗ a').left (cls M hM) = GlobalTheory.CechPic.pull a.left (cls M hM) + GlobalTheory.CechPic.pull a'.left (cls M hM)) → ¬ Triv M → ∀ (κ : Type) [LinearOrder κ] [Fintype κ] (D : κ → A.left.Opens), (∀ i, IsAffineOpen (D i)) → ⊔ i, D i = ⊤ → (cechLB A.hom M D).Acyclic) → PointSurj

CompanionTheory.AutomorphicTheory.ProductFormulaTheory.DualAbelianSchemes.Rigdesc

theorem **rigdesc_pf** : H90 → RigDescent

theorem **rigDescent_holds** : RigDescent

CompanionTheory.AutomorphicTheory.ProductFormulaTheory.DualAbelianSchemes.TrivLocus

theorem **trivLocus_pf** : CompanionTheory.ObstructionTrivialLocus.TrivLocus

CompanionTheory.AutomorphicTheory.ProductFormulaTheory.DualAbelianSchemes.TrivLocusA

theorem **exists_ideal_locTriv_noeth** : ∀ (R : Type) [CommRing R] [IsNoetherianRing R] (B : Over (Spec (CommRingCat.of R))) [GrpObj B], IsAbelianScheme B → ∀ F : LB B.left, F.IsLB → ∃ I : Ideal R, ∀ (R' : Type) [CommRing R'] (g : R →+* R') (X' : Scheme.{0}) (π : X' ⟶ B.left) (f' : X' ⟶ Spec (CommRingCat.of R')), IsPullback π f' B.hom (Spec.map (CommRingCat.ofHom g)) → ((∀ p : Spec (CommRingCat.of R'), ∃ U : (Spec (CommRingCat.of R')).Opens, p ∈ U ∧ Triv ((F.pull π).pull (f' ⁻¹ᵁ U).ι)) ↔ I ≤ RingHom.ker g)

CompanionTheory.AutomorphicTheory.ProductFormulaTheory.DualAbelianSchemes.TrivLocusB

theorem **exists_closed_locTriv_noeth** : ∀ (S : Scheme.{0}) (A Y : Over S) [GrpObj A], IsAbelianScheme A → IsLocallyNoetherian Y.left → ∀ F : LB (A ⊛ Y).left, F.IsLB → ∃ (H : Over S) (j : H ⟶ Y), IsClosedImmersion j.left ∧ ∀ (T' : Over S) (x : T' ⟶ Y), (∀ p : T'.left, ∃ U : T'.left.Opens, p ∈ U ∧ Triv ((F.pull (A ◁ x).left).pull ((snd A T').left ⁻¹ᵁ U).ι)) ↔ ∃ y : T' ⟶ H, y ≫ j = x

CompanionTheory.AutomorphicTheory.ProductFormulaTheory.DualAbelianSchemes.TrivLocusFlat

theorem **h1Le** : H1StructureDimLe

theorem **trivLocus** : TrivLocus

theorem **fewEquations** : FewEquations

theorem **ciFlat** : CIFlat

theorem **trivLocusFlat** : TrivLocusFlat

theorem **pointSurj** : PointSurj

theorem **finDual** : FinDual

theorem **kTrivModel** : KTrivModel

theorem **kTrivDual** : KTrivDual

CompanionTheory.AutomorphicTheory.ProductFormulaTheory.GrothendieckRingPolynomial.AltComb

theorem **filter_mem_eq_of_sublist** {α : Type*} [DecidableEq α] {l m : List α} (hs : l.Sublist m) (hm : m.Nodup) : m.filter (fun x => decide (x ∈ l)) = l

theorem **mem_sub** {α : Type*} {h : ℕ} (W : List α) (c : Fin W.length → Fin h) (i : Fin h) (x : α) : x ∈ sub W c i ↔ ∃ p, c p = i ∧ W.get p = x

theorem **mem_sub_self** {α : Type*} {h : ℕ} (W : List α) (c : Fin W.length → Fin h) (p : Fin W.length) : W.get p ∈ sub W c (c p)

theorem **sub_sublist** {α : Type*} {h : ℕ} (W : List α) (c : Fin W.length → Fin h) (i : Fin h) : (sub W c i).Sublist W

theorem **shuffleCount_eq** {α : Type*} {h : ℕ} (W : List α) (D : Fin h → List α) (hW : W.Nodup) : shuffleCount W D = if (∀ i, (D i).Sublist W) ∧ (∀ x ∈ W, ∃! i, x ∈ D i) then 1 else 0

theorem **cond_map** {β α : Type*} {h : ℕ} (f : β → α) (hf : Function.Injective f) (w : List β) (E : Fin h → List β) : ((∀ i, ((E i).map f).Sublist (w.map f)) ∧ (∀ x ∈ w.map f, ∃! i, x ∈ (E i).map f)) ↔ ((∀ i, (E i).Sublist w) ∧ (∀ y ∈ w, ∃! i, y ∈ E i))

theorem **mem_wN** {L x : ℕ} {T : Finset ℕ} : x ∈ wN L T ↔ x < L

theorem **nodup_wN** (L : ℕ) (T : Finset ℕ) : (wN L T).Nodup

theorem **sublist_of_lt** {l m : List ℕ} (hl : l.Pairwise (· < ·)) (hm : m.Pairwise (· < ·)) (hs : ∀ x ∈ l, x ∈ m) : l.Sublist m

theorem **run_sublist** {L b l : ℕ} {T : Finset ℕ} (hbl : b + l ≤ L) : ((List.range' b l).reverse).Sublist (wN L T) ↔ ∀ x, b < x → x < b + l → x ∈ T

theorem **cond_iff** {L : ℕ} {T : Finset ℕ} {h : ℕ} {b l : Fin h → ℕ} (hl : ∀ i, 0 < l i) : Cond L T b l ↔ (∀ i, b i + l i ≤ L) ∧ (∀ x < L, ∃! i, b i ≤ x ∧ x < b i + l i) ∧ (∀ i x, b i < x → x < b i + l i → x ∈ T)

theorem **altSumN** (L : ℕ) (hL : 0 < L) (s : Finset (Finset (Fin L))) (hs : ∀ S, S ∈ s ↔ ∀ a ∈ S, (a : ℕ) ≠ 0) {h : ℕ} (b l : Fin h → ℕ) (hl : ∀ i, 0 < l i) (f : Finset (Fin L) → ℤ) (hf : ∀ S, f S = (-1) ^ S.card * if Cond L (S.map Fin.valEmbedding) b l then 1 else 0) : ∑ S ∈ s, f S = if h = 1 ∧ ∀ i, b i = 0 ∧ l i = L then (-1) ^ (L - 1) else 0

CompanionTheory.AutomorphicTheory.ProductFormulaTheory.GrothendieckRingPolynomial.AltSum

theorem **tN_injective** (Δ : Segment k) : Function.Injective (tN Δ)

theorem **tN_zero** (Δ : Segment k) : tN Δ 0 = Δ.first

theorem **vWord_eq** (Δ : Segment k) (S : Finset (Fin Δ.len)) : vWord Δ S = (AltC.wN Δ.len (S.map Fin.valEmbedding)).map (tN Δ)

theorem **decWord_eq** (Δ Γ : Segment k) (b : ℕ) (hb : Γ.first = tN Δ b) : decWord Γ = ((List.range' b Γ.len).reverse).map (tN Δ)

theorem **first_mem_decWord** (Γ : Segment k) : Γ.first ∈ decWord Γ

theorem **altSum** (k : Type) [Field k] [ValuativeRel k] [TopologicalSpace k] [IsNonarchimedeanLocalField k] : AltSum k

CompanionTheory.AutomorphicTheory.ProductFormulaTheory.GrothendieckRingPolynomial.ClassGlEqClass

theorem **GrothendieckClassGeneralLinear.class_gl_eq_class** {k : Type} [Field k] [ValuativeRel k] [TopologicalSpace k] [IsNonarchimedeanLocalField k] {N : ℕ} {V : Type} [AddCommGroup V] [Module ℂ V] (π : Representation ℂ (GL (Fin N) k) V) (h : HasFiniteLength π) : clsGL π h = cl0 π

theorem **groth_of_steps** (H : ∀ (k : Type) [Field k] [ValuativeRel k] [TopologicalSpace k] [IsNonarchimedeanLocalField k], Cls k ∧ MulCls k ∧ Assoc k ∧ Comm k ∧ One k ∧ ProdList k ∧ Tri k ∧ MultOne k ∧ JCls k ∧ JSeg k ∧ AltSum k) (h1 : TriAut) (h2 : UniTri) : GlobalTheory.Published.GrothendieckRingPolynomial

CompanionTheory.AutomorphicTheory.ProductFormulaTheory.GrothendieckRingPolynomial.ClassProductIrreducibleApply

theorem **exists_lseg** (hs : LSegExists k) : ∃ Lseg : Segment k → IrrGL k, ∀ Δ, IsLSeg Δ (Lseg Δ).rep

theorem **lfactors_sum** {m : Multiset (Segment k)} {h : ℕ} {x : Fin h → IrrGL k} (hx : LFactors m x) : ∑ a, (x a).1 = (m.map Segment.deg).sum

theorem **isL_deg** {m : Multiset (Segment k)} {N : ℕ} {V : Type} [AddCommGroup V] [Module ℂ V] {π : Representation ℂ (GL (Fin N) k) V} (h : IsL m π) : N = (m.map Segment.deg).sum

theorem **isL_cls_unique** (hf : LOrderFree k) {m : Multiset (Segment k)} {x y : IrrGL k} (hx : IsL m x.rep) (hy : IsL m y.rep) : x = y

theorem **exists_orderedForm** (m : Multiset (Segment k)) : ∃ (h : ℕ) (Δ : Fin h → Segment k), IsOrderedForm Δ m

theorem **class_apply_same_degree** {N : ℕ} {V : Type} [AddCommGroup V] [Module ℂ V] (π : Representation ℂ (GL (Fin N) k) V) (hl : HasFiniteLength π) (c : IrrCl (GL (Fin N) k)) : cl0 π (N, c) = ((jhMult c.rep.ρ π).toNat : ℤ)

theorem **class_apply_other_degree** {N : ℕ} {V : Type} [AddCommGroup V] [Module ℂ V] (π : Representation ℂ (GL (Fin N) k) V) (y : IrrGL k) (hy : y.1 ≠ N) : cl0 π y = 0

theorem **class_product_irreducible_apply** {h : ℕ} (x : Fin h → IrrGL k) (y : IrrGL k) (hd : ∑ a, (x a).1 = y.1) : cl0 (prodIrr x (N

theorem **sum_sq_le** (s : Multiset ℕ) : (s.map fun l => l ^ 2).sum ≤ s.sum ^ 2

theorem **sq_le** (m : Multiset (Segment k)) : CompanionTheory.Groth.sq m ≤ (Multiset.card (msupp m)) ^ 2

theorem **prod_map_toMultiset** {M : Type*} [CommMonoid M] {α : Type*} (g : α → M) (s : α →₀ ℕ) : ((Finsupp.toMultiset s).map g).prod = s.prod fun a n => g a ^ n

theorem **toR_foldr** (l : List (IrrGL k)) : toR (l.foldr (fun y a => mulR (sg y) a) (GrothGL.unit k)) = (l.map fun y => toR (sg y)).prod

theorem **ring_class_product_irreducible** (hPL : ProdList k) {h N : ℕ} (x : Fin h → IrrGL k) (hd : ∑ a, (x a).1 = N) : toR (cl0 (prodIrr x hd)) = ∏ a, toR (sg (x a))

theorem **phiL_main** (hk : Steps k) (h2 : UniTri) : ∃ Lseg : Segment k → IrrGL k, (∀ Δ, IsLSeg Δ (Lseg Δ).rep) ∧ Function.Bijective (phiOf Lseg) ∧ (∀ (n : Multiset (Segment k)) (x : IrrGL k), IsL n x.rep → ∃ p : MvPolynomial (Segment k) ℤ, phiOf Lseg p = toR (sg x) ∧ ∀ s ∈ p.support, msupp (Finsupp.toMultiset s) = msupp n) ∧ ∀ (h N : ℕ) (Γ : Fin h → Segment k) (hd : ∑ a, (Lseg (Γ a)).1 = N), phiOf Lseg (∏ a, MvPolynomial.X (Γ a)) = toR (cl0 (prodIrr (fun a => Lseg (Γ a)) hd))

CompanionTheory.AutomorphicTheory.ProductFormulaTheory.GrothendieckRingPolynomial.ClassProductIrreducibleCast

theorem **occursTimes_comp** {σ : Representation ℂ G W} {π : Representation ℂ G V} (s : H →* G) {r : ℕ} (h : OccursTimes σ π r) : OccursTimes (σ.comp s) (π.comp s) r

theorem **jhMult_le_comp** (σ : Representation ℂ G W) (π : Representation ℂ G V) (s : H →* G) : jhMult σ π ≤ jhMult (σ.comp s) (π.comp s)

theorem **ann_anti** {S T : Subrepresentation π} (h : T ≤ S) : ann S ≤ ann T

theorem **formOf_mem** (l : (ann T).toSubmodule) : formOf S T l ∈ smoothPart (subQuot S T).dual

theorem **ker_annMap** : LinearMap.ker (annMap S T).toLinearMap = (ann S).toSubmodule.comap (ann T).toSubmodule.subtype

theorem **annMap_surjective** (hG : ∀ U ∈ nhds (1 : G), ∃ K : Subgroup G, IsOpen (K : Set G) ∧ IsCompact (K : Set G) ∧ (K : Set G) ⊆ U) (hπ : IsSmooth π) (hTS : T ≤ S) : Function.Surjective (annMap S T)

theorem **occursTimes_contra** (hG : ∀ U ∈ nhds (1 : G), ∃ K : Subgroup G, IsOpen (K : Set G) ∧ IsCompact (K : Set G) ∧ (K : Set G) ⊆ U) (hπ : IsSmooth π) {σ : Representation ℂ G W} {r : ℕ} (h : OccursTimes σ π r) : OccursTimes (contra σ) (contra π) r

theorem **jhMult_le_contra** (hG : ∀ U ∈ nhds (1 : G), ∃ K : Subgroup G, IsOpen (K : Set G) ∧ IsCompact (K : Set G) ∧ (K : Set G) ⊆ U) (hπ : IsSmooth π) (σ : Representation ℂ G W) : jhMult σ π ≤ jhMult (contra σ) (contra π)

theorem **prodIrr_congr** {h N : ℕ} {z z' : Fin h → IrrGL k} (e : z = z') (hd : ∑ a, (z a).1 = N) (hd' : ∑ a, (z' a).1 = N) : Nonempty ((prodIrr z hd).Equiv (prodIrr z' hd'))

theorem **class_product_irreducible_cast** {h N M : ℕ} (e : N = M) (z : Fin h → IrrGL k) (hd : ∑ a, (z a).1 = N) (hd' : ∑ a, (z a).1 = M) : cl0 (prodIrr z hd) = cl0 (prodIrr z hd')

theorem **hasFiniteLength_prodIrr_cast** {h N M : ℕ} (e : N = M) (z : Fin h → IrrGL k) (hd : ∑ a, (z a).1 = N) (hd' : ∑ a, (z a).1 = M) (hf : HasFiniteLength (prodIrr z hd)) : HasFiniteLength (prodIrr z hd')

theorem **class_congr_equiv** {N : ℕ} {V W : Type} [AddCommGroup V] [Module ℂ V] [AddCommGroup W] [Module ℂ W] {π : Representation ℂ (GL (Fin N) k) V} {τ : Representation ℂ (GL (Fin N) k) W} (φ : π.Equiv τ) : cl0 π = cl0 τ

theorem **class_eq_of_jordan_holder_mult** {N : ℕ} {V W : Type} [AddCommGroup V] [Module ℂ V] [AddCommGroup W] [Module ℂ W] {π : Representation ℂ (GL (Fin N) k) V} {τ : Representation ℂ (GL (Fin N) k) W} (hπ : HasFiniteLength π) (hτ : HasFiniteLength τ) (h : ∀ σ : IrrRep (GL (Fin N) k), jhMult σ.ρ π = jhMult σ.ρ τ) : cl0 π = cl0 τ

theorem **jhMult_pair_le** (x y : IrrGL k) {N : ℕ} (h1 : ∑ a, (![x, y] a).1 = N) (h2 : ∑ a, (![y, x] a).1 = N) (σ : IrrRep (GL (Fin N) k)) : jhMult σ.ρ (prodIrr ![x, y] h1) ≤ jhMult σ.ρ (prodIrr ![y, x] h2)

theorem **comm** (k : Type) [Field k] [ValuativeRel k] [TopologicalSpace k] [IsNonarchimedeanLocalField k] : Comm k

CompanionTheory.AutomorphicTheory.ProductFormulaTheory.GrothendieckRingPolynomial.ClassProductIrreducibleFold

theorem **class_of_finite_length** {N : ℕ} {V : Type} [AddCommGroup V] [Module ℂ V] {π : Representation ℂ (GL (Fin N) k) V} (h : HasFiniteLength π) : cl0 π = clsGL π h

theorem **class_congr_iso** {N : ℕ} {V W : Type} [AddCommGroup V] [Module ℂ V] [AddCommGroup W] [Module ℂ W] {π : Representation ℂ (GL (Fin N) k) V} {τ : Representation ℂ (GL (Fin N) k) W} (φ : π.Equiv τ) : cl0 π = cl0 τ

theorem **class_chosen_rep_basis** (x : IrrGL k) : cl0 x.rep = sg x

theorem **prodIrr_cast** {h h' N N' : ℕ} (e : h = h') (x : Fin h → IrrGL k) (y : Fin h' → IrrGL k) (hxy : ∀ i, x i = y (Fin.cast e i)) (hx : ∑ a, (x a).1 = N) (hy : ∑ a, (y a).1 = N') : (HasFiniteLength (prodIrr x hx) ↔ HasFiniteLength (prodIrr y hy)) ∧ cl0 (prodIrr x hx) = cl0 (prodIrr y hy)

theorem **prodIrr_zero** {N : ℕ} (x : Fin 0 → IrrGL k) (hd : ∑ a, (x a).1 = N) : HasFiniteLength (prodIrr x hd) ∧ cl0 (prodIrr x hd) = GrothGL.unit k

theorem **prodIrr_one** (x : IrrGL k) (hd : ∑ _ : Fin 1, x.1 = x.1) : HasFiniteLength (prodIrr (fun _ : Fin 1 => x) hd) ∧ cl0 (prodIrr (fun _ : Fin 1 => x) hd) = sg x

theorem **class_of_product_equiv** (hM : MulCls k) {r t : ℕ} {U W X : Type} [AddCommGroup U] [Module ℂ U] [AddCommGroup W] [Module ℂ W] [AddCommGroup X] [Module ℂ X] (π₁ : Representation ℂ (GL (Fin r) k) U) (π₂ : Representation ℂ (GL (Fin t) k) W) (τ : Representation ℂ (GL (Fin (r + t)) k) X) (F : (prod₂ π₁ π₂).Equiv τ) (h₁ : HasFiniteLength π₁) (h₂ : HasFiniteLength π₂) (s₁ : IsSmooth π₁) (s₂ : IsSmooth π₂) : HasFiniteLength τ ∧ cl0 τ = mulR (cl0 π₁) (cl0 π₂)

theorem **prodIrr_append_one** (hM : MulCls k) {h : ℕ} (x₀ : IrrGL k) (y : Fin h → IrrGL k) (hy : ∑ a, (y a).1 = ∑ a, (y a).1) (hfl : HasFiniteLength (prodIrr y hy)) (happ : ∑ a, (Fin.append (fun _ : Fin 1 => x₀) y a).1 = x₀.1 + ∑ a, (y a).1) : HasFiniteLength (prodIrr (Fin.append (fun _ : Fin 1 => x₀) y) happ) ∧ cl0 (prodIrr (Fin.append (fun _ : Fin 1 => x₀) y) happ) = mulR (sg x₀) (cl0 (prodIrr y hy))

theorem **class_product_irreducible_fold** (hM : MulCls k) : ∀ (h N : ℕ) (x : Fin h → IrrGL k) (hd : ∑ a, (x a).1 = N), HasFiniteLength (prodIrr x hd) ∧ cl0 (prodIrr x hd) = (List.ofFn x).foldr (fun y a => mulR (sg y) a) (GrothGL.unit k)

theorem **prodList** (h : MulCls k) : ProdList k

CompanionTheory.AutomorphicTheory.ProductFormulaTheory.GrothendieckRingPolynomial.ClassProductIrreducibleLeft

theorem **bil_zero_left** (b : ι →₀ ℤ) : bil f 0 b = 0

theorem **bil_zero_right** (a : ι →₀ ℤ) : bil f a 0 = 0

theorem **bil_add_left** (a a' b : ι →₀ ℤ) : bil f (a + a') b = bil f a b + bil f a' b

theorem **bil_add_right** (a b b' : ι →₀ ℤ) : bil f a (b + b') = bil f a b + bil f a b'

theorem **bil_single** (x y : ι) : bil f (Finsupp.single x 1) (Finsupp.single y 1) = f x y

theorem **mulR_eq_bil** (a b : GrothGL k) : mulR a b = bil (fun x y : IrrGL k => cl0 (prod₂ (IrrGL.rep x) (IrrGL.rep y))) a b

theorem **mulR_zero_left** (b : GrothGL k) : mulR 0 b = 0

theorem **mulR_zero_right** (a : GrothGL k) : mulR a 0 = 0

theorem **mulR_add_left** (a a' b : GrothGL k) : mulR (a + a') b = mulR a b + mulR a' b

theorem **mulR_add_right** (a b b' : GrothGL k) : mulR a (b + b') = mulR a b + mulR a b'

theorem **mulR_sg_sg** (x y : IrrGL k) : mulR (sg x) (sg y) = cl0 (prod₂ (IrrGL.rep x) (IrrGL.rep y))

theorem **class_eq_finite_length_class** (π : Representation ℂ (GL (Fin N) k) V) (h : HasFiniteLength π) : cl0 π = clsGL π h

theorem **class_irreducible_basis** (π : Representation ℂ (GL (Fin N) k) V) (hs : IsSmooth π) (hi : π.IsIrreducible) : cl0 π = sg (IrrGL.of π hs hi)

theorem **class_congr_equiv** {π : Representation ℂ (GL (Fin N) k) V} {τ : Representation ℂ (GL (Fin N) k) W} (φ : π.Equiv τ) : cl0 π = cl0 τ

theorem **class_subsingleton** [Subsingleton V] (π : Representation ℂ (GL (Fin N) k) V) : cl0 π = 0

theorem **class_eq_zero_of_trivial_space** {π : Representation ℂ (GL (Fin N) k) V} (h : ∀ v : V, v = 0) : cl0 π = 0

theorem **class_additive_exact** {α : Representation ℂ (GL (Fin N) k) U} {β : Representation ℂ (GL (Fin N) k) V} {γ : Representation ℂ (GL (Fin N) k) W} (i : α.IntertwiningMap β) (q : β.IntertwiningMap γ) (hi : Function.Injective i) (hq : Function.Surjective q) (he : Function.Exact i q) (h : HasFiniteLength β) : cl0 β = cl0 α + cl0 γ

theorem **induction_length** (P : ∀ (Y : Type) [AddCommGroup Y] [Module ℂ Y], Representation ℂ G Y → Prop) (hzero : ∀ (Y : Type) [AddCommGroup Y] [Module ℂ Y] [Subsingleton Y] (σ : Representation ℂ G Y), P Y σ) (hirr : ∀ (Y : Type) [AddCommGroup Y] [Module ℂ Y] (σ : Representation ℂ G Y), IsSmooth σ → σ.IsIrreducible → P Y σ) (hses : ∀ (A B C : Type) [AddCommGroup A] [Module ℂ A] [AddCommGroup B] [Module ℂ B] [AddCommGroup C] [Module ℂ C] (α : Representation ℂ G A) (β : Representation ℂ G B) (γ : Representation ℂ G C) (i : α.IntertwiningMap β) (q : β.IntertwiningMap γ), Function.Injective i → Function.Surjective q → Function.Exact i q → IsSmooth β → HasFiniteLength β → P A α → P C γ → P B β) {V : Type} [AddCommGroup V] [Module ℂ V] (ρ : Representation ℂ G V) (hs : IsSmooth ρ) (hf : HasFiniteLength ρ) : P V ρ

theorem **continuous_levi_pair** : Continuous fun h : Levi k (Two (Fin r) (Fin t)) => (h false, h true)

theorem **parInd_ses** {α n : Type} {m : α → Type} [Fintype α] [LinearOrder α] [∀ a, Fintype (m a)] [∀ a, DecidableEq (m a)] [Fintype n] [DecidableEq n] (e : (Σ a, m a) ≃ n) {A B C : Type} [AddCommGroup A] [Module ℂ A] [AddCommGroup B] [Module ℂ B] [AddCommGroup C] [Module ℂ C] {ξ : Representation ℂ (Levi k m) A} {η : Representation ℂ (Levi k m) B} {ζ : Representation ℂ (Levi k m) C} (i : ξ.IntertwiningMap η) (q : η.IntertwiningMap ζ) (hi : Function.Injective i) (hq : Function.Surjective q) (he : Function.Exact i q) (hη : IsSmooth η) : Function.Injective (parIndMap e i) ∧ Function.Surjective (parIndMap e q) ∧ Function.Exact (parIndMap e i) (parIndMap e q)

theorem **prod₂_ses_left** {A B C W : Type} [AddCommGroup A] [Module ℂ A] [AddCommGroup B] [Module ℂ B] [AddCommGroup C] [Module ℂ C] [AddCommGroup W] [Module ℂ W] {α : Representation ℂ (GL (Fin r) k) A} {β : Representation ℂ (GL (Fin r) k) B} {γ : Representation ℂ (GL (Fin r) k) C} (ρ : Representation ℂ (GL (Fin t) k) W) (i : α.IntertwiningMap β) (q : β.IntertwiningMap γ) (hi : Function.Injective i) (hq : Function.Surjective q) (he : Function.Exact i q) (hβ : IsSmooth β) (hρ : IsSmooth ρ) : ∃ (I : (prod₂ α ρ).IntertwiningMap (prod₂ β ρ)) (Q : (prod₂ β ρ).IntertwiningMap (prod₂ γ ρ)), Function.Injective I ∧ Function.Surjective Q ∧ Function.Exact I Q

theorem **prod₂_ses_right** {X A B C : Type} [AddCommGroup X] [Module ℂ X] [AddCommGroup A] [Module ℂ A] [AddCommGroup B] [Module ℂ B] [AddCommGroup C] [Module ℂ C] (ρ : Representation ℂ (GL (Fin r) k) X) {α : Representation ℂ (GL (Fin t) k) A} {β : Representation ℂ (GL (Fin t) k) B} {γ : Representation ℂ (GL (Fin t) k) C} (i : α.IntertwiningMap β) (q : β.IntertwiningMap γ) (hi : Function.Injective i) (hq : Function.Surjective q) (he : Function.Exact i q) (hρ : IsSmooth ρ) (hβ : IsSmooth β) : ∃ (I : (prod₂ ρ α).IntertwiningMap (prod₂ ρ β)) (Q : (prod₂ ρ β).IntertwiningMap (prod₂ ρ γ)), Function.Injective I ∧ Function.Surjective Q ∧ Function.Exact I Q

theorem **indSp_eq_zero** {α n : Type} {m : α → Type} [Fintype α] [LinearOrder α] [∀ a, Fintype (m a)] [∀ a, DecidableEq (m a)] [Fintype n] [DecidableEq n] (e : (Σ a, m a) ≃ n) {B : Type} [AddCommGroup B] [Module ℂ B] [Subsingleton B] (η : Representation ℂ (Levi k m) B) (φ : IndSp e η) : φ = 0

theorem **class_product_zero_left** {V W : Type} [AddCommGroup V] [Module ℂ V] [AddCommGroup W] [Module ℂ W] [Subsingleton V] (π₁ : Representation ℂ (GL (Fin r) k) V) (π₂ : Representation ℂ (GL (Fin t) k) W) : cl0 (prod₂ π₁ π₂) = 0

theorem **class_product_zero_right** {V W : Type} [AddCommGroup V] [Module ℂ V] [AddCommGroup W] [Module ℂ W] [Subsingleton W] (π₁ : Representation ℂ (GL (Fin r) k) V) (π₂ : Representation ℂ (GL (Fin t) k) W) : cl0 (prod₂ π₁ π₂) = 0

theorem **class_product_irreducibles** {V W : Type} [AddCommGroup V] [Module ℂ V] [AddCommGroup W] [Module ℂ W] (π₁ : Representation ℂ (GL (Fin r) k) V) (π₂ : Representation ℂ (GL (Fin t) k) W) (s₁ : IsSmooth π₁) (i₁ : π₁.IsIrreducible) (s₂ : IsSmooth π₂) (i₂ : π₂.IsIrreducible) : cl0 (prod₂ π₁ π₂) = mulR (cl0 π₁) (cl0 π₂)

theorem **class_product_irreducible_left** {V W : Type} [AddCommGroup V] [Module ℂ V] [AddCommGroup W] [Module ℂ W] (π₁ : Representation ℂ (GL (Fin r) k) V) (s₁ : IsSmooth π₁) (i₁ : π₁.IsIrreducible) (π₂ : Representation ℂ (GL (Fin t) k) W) (s₂ : IsSmooth π₂) (h₂ : HasFiniteLength π₂) : cl0 (prod₂ π₁ π₂) = mulR (cl0 π₁) (cl0 π₂)

theorem **class_parabolic_product** {V W : Type} [AddCommGroup V] [Module ℂ V] [AddCommGroup W] [Module ℂ W] (π₁ : Representation ℂ (GL (Fin r) k) V) (π₂ : Representation ℂ (GL (Fin t) k) W) (h₁ : HasFiniteLength π₁) (h₂ : HasFiniteLength π₂) (s₁ : IsSmooth π₁) (s₂ : IsSmooth π₂) : cl0 (prod₂ π₁ π₂) = mulR (cl0 π₁) (cl0 π₂)

theorem **mulCls** : MulCls k

CompanionTheory.AutomorphicTheory.ProductFormulaTheory.GrothendieckRingPolynomial.GrothOfCls

theorem **groth_of_cls** (H : ∀ (k : Type) [Field k] [ValuativeRel k] [TopologicalSpace k] [IsNonarchimedeanLocalField k], Cls k) : GlobalTheory.Published.GrothendieckRingPolynomial

CompanionTheory.AutomorphicTheory.ProductFormulaTheory.GrothendieckRingPolynomial.GrothOfThree

theorem **groth_of_three** (hZ : GlobalTheory.Published.SegmentRepresentation) (hI : ∀ (k : Type) [Field k] [ValuativeRel k] [TopologicalSpace k] [IsNonarchimedeanLocalField k], Zc.Cos.LUnlinkedIrr k) (H : ∀ (k : Type) [Field k] [ValuativeRel k] [TopologicalSpace k] [IsNonarchimedeanLocalField k], CompanionTheory.Exh.MidP k ∧ CompanionTheory.Exh.RefineP k ∧ CompanionTheory.Exh.SwapP k) : GlobalTheory.Published.GrothendieckRingPolynomial

CompanionTheory.AutomorphicTheory.ProductFormulaTheory.GrothendieckRingPolynomial.GrothendieckRingPolynomialOfSeg

theorem **grothendieckRingPolynomial_of_seg** (hZ : GlobalTheory.Published.SegmentRepresentation) : GlobalTheory.Published.GrothendieckRingPolynomial

CompanionTheory.AutomorphicTheory.ProductFormulaTheory.GrothendieckRingPolynomial.JacquetFunctionalClass

theorem **piTensor_irr** {h : ℕ} (x : Fin h → IrrGL k) : (piTensor fun a => (x a).rep).IsIrreducible

theorem **jm_ne_top** {h N : ℕ} (x : Fin h → IrrGL k) (hd : ∑ a, (x a).1 = N) {V : Type} [AddCommGroup V] [Module ℂ V] (π : Representation ℂ (GL (Fin N) k) V) (hπ : IsSmooth π) (hl : HasFiniteLength π) : jm x hd π ≠ ⊤

theorem **jm_congr** {h N : ℕ} (x : Fin h → IrrGL k) (hd : ∑ a, (x a).1 = N) {V W : Type} [AddCommGroup V] [Module ℂ V] [AddCommGroup W] [Module ℂ W] {π : Representation ℂ (GL (Fin N) k) V} {τ : Representation ℂ (GL (Fin N) k) W} (hπ : IsSmooth π) (hτ : IsSmooth τ) (φ : π.Equiv τ) : jm x hd π = jm x hd τ

theorem **jm_eq_sum** {h N : ℕ} (x : Fin h → IrrGL k) (hd : ∑ a, (x a).1 = N) {V : Type} [AddCommGroup V] [Module ℂ V] (π : Representation ℂ (GL (Fin N) k) V) (hπ : IsSmooth π) (hl : HasFiniteLength π) : jm x hd π = ∑ c ∈ (finite_constituents hl).toFinset, jhMult c.rep.ρ π * jm x hd c.rep.ρ

theorem **jacquet_functional_class** (W : List (IrrGL k)) {N : ℕ} {V : Type} [AddCommGroup V] [Module ℂ V] (π : Representation ℂ (GL (Fin N) k) V) (hl : HasFiniteLength π) : J W (cl0 π) = (cls π hl).sum fun c m => m * ((jmL W (IrrGL.rep ((N, c) : IrrGL k))).toNat : ℤ)

theorem **jCls** : JCls k

CompanionTheory.AutomorphicTheory.ProductFormulaTheory.GrothendieckRingPolynomial.JhMultSegCl

theorem **jhMult_seg_cl** (hA : GlobalTheory.Published.IrreducibleAdmissibleGL) {Z : ∀ Δ : Segment k, IrrCl (GL (Fin (Δ.len * Δ.d)) k)} (hZ : IsSegmentClass k Z) (Δ : Segment k) (y : Fin Δ.len → IrrCl (GL (Fin Δ.d) k)) (hd : ∑ _ : Fin Δ.len, Δ.d = Δ.len * Δ.d) : jhMult (piTensor fun a => (y a).rep.ρ) (jac (stdBlocks (fun _ : Fin Δ.len => Δ.d) hd) (Z Δ).rep.ρ) = if (∀ a, ((Δ.d, y a) : IrrGL k) = Δ.term a) then 1 else 0

theorem **jm_seg** (hA : GlobalTheory.Published.IrreducibleAdmissibleGL) {Z : ∀ Δ : Segment k, IrrCl (GL (Fin (Δ.len * Δ.d)) k)} (hZ : IsSegmentClass k Z) (Δ : Segment k) {m : ℕ} (y : Fin m → IrrGL k) (hy : ∀ a, (y a).1 = Δ.d) (hd : ∑ a, (y a).1 = Δ.len * Δ.d) : jm y hd (IrrGL.rep ((Δ.len * Δ.d, Z Δ) : IrrGL k)) = if ∃ h : m = Δ.len, ∀ a, y a = Δ.term (Fin.cast h a) then 1 else 0

theorem **jmL_seg** (hA : GlobalTheory.Published.IrreducibleAdmissibleGL) {Z : ∀ Δ : Segment k, IrrCl (GL (Fin (Δ.len * Δ.d)) k)} (hZ : IsSegmentClass k Z) (Δ : Segment k) (W : List (IrrGL k)) (hW : ∀ w ∈ W, w.1 = Δ.d) (hl : W.length = Δ.len) : jmL W (IrrGL.rep ((Δ.len * Δ.d, Z Δ) : IrrGL k)) = if W = List.ofFn Δ.term then 1 else 0

theorem **vWord_ne** (hA : GlobalTheory.Published.IrreducibleAdmissibleGL) (Δ : Segment k) (S : Finset (Fin Δ.len)) (hS : S ∈ vSets Δ) (hne : S ≠ ∅) : vWord Δ S ≠ List.ofFn Δ.term

theorem **jSeg** : JSeg k

CompanionTheory.AutomorphicTheory.ProductFormulaTheory.GrothendieckRingPolynomial.MulClassBasisRight

theorem **class_congr_equiv** {N : ℕ} {V W : Type} [AddCommGroup V] [Module ℂ V] [AddCommGroup W] [Module ℂ W] {π : Representation ℂ (GL (Fin N) k) V} {τ : Representation ℂ (GL (Fin N) k) W} (φ : Nonempty (π.Equiv τ)) : cl0 π = cl0 τ

theorem **class_induced_cast** {r t N : ℕ} (h : r + t = N) {W : Type} [AddCommGroup W] [Module ℂ W] (ρ : Representation ℂ (GL (Fin r) k × GL (Fin t) k) W) : cl0 (ind₂ (stdSum h) ρ) = cl0 (ind₂ (stdSum (rfl : r + t = r + t)) ρ)

theorem **class_of_chosen_rep** (x : IrrGL k) : cl0 (IrrGL.rep x) = sg x

theorem **mulR_sg** (x y : IrrGL k) : mulR (sg x) (sg y) = cl0 (prod₂ (IrrGL.rep x) (IrrGL.rep y))

theorem **assoc_gen** (h : MulCls k) {r s t : ℕ} {V₁ V₂ V₃ U U' : Type} [AddCommGroup V₁] [Module ℂ V₁] [AddCommGroup V₂] [Module ℂ V₂] [AddCommGroup V₃] [Module ℂ V₃] [AddCommGroup U] [Module ℂ U] [AddCommGroup U'] [Module ℂ U'] (π₁ : Representation ℂ (GL (Fin r) k) V₁) (π₂ : Representation ℂ (GL (Fin s) k) V₂) (π₃ : Representation ℂ (GL (Fin t) k) V₃) (P : Representation ℂ (GL (Fin (r + s)) k) U) (R : Representation ℂ (GL (Fin (s + t)) k) U') (φ : Nonempty (P.Equiv (prod₂ π₁ π₂))) (ψ : Nonempty (R.Equiv (prod₂ π₂ π₃))) (s₁ : IsSmooth π₁) (s₂ : IsSmooth π₂) (s₃ : IsSmooth π₃) (sP : IsSmooth P) (sR : IsSmooth R) (f₁ : HasFiniteLength π₁) (f₃ : HasFiniteLength π₃) (fP : HasFiniteLength P) (fR : HasFiniteLength R) : mulR (cl0 P) (cl0 π₃) = mulR (cl0 π₁) (cl0 R)

theorem **assoc** (h : MulCls k) : Assoc k

theorem **one** : One k

CompanionTheory.AutomorphicTheory.ProductFormulaTheory.GrothendieckRingPolynomial.MultOne

theorem **mulR_sg_left** (a : IrrGL k) (b : GrothGL k) : mulR (sg a) b = b.sum fun y n => n • cl0 (prod₂ (IrrGL.rep a) (IrrGL.rep y))

theorem **mulR_sg_zero** (a : IrrGL k) : mulR (sg a) 0 = 0

theorem **mulR_sg_add** (a : IrrGL k) (b c : GrothGL k) : mulR (sg a) (b + c) = mulR (sg a) b + mulR (sg a) c

theorem **mulR_sg_zsmul** (a : IrrGL k) (n : ℤ) (b : GrothGL k) : mulR (sg a) (n • b) = n • mulR (sg a) b

theorem **sum_single_one_smul** (T : IrrGL k → GrothGL k) (b : IrrGL k) : (Finsupp.single b (1 : ℤ)).sum (fun y n => n • T y) = T b

theorem **mulR_sg_sg** (a b : IrrGL k) : mulR (sg a) (sg b) = cl0 (prod₂ (IrrGL.rep a) (IrrGL.rep b))

theorem **mulR_sg_left_comm** (ha : Assoc k) (hc : Comm k) (a b : IrrGL k) (c : GrothGL k) : mulR (sg a) (mulR (sg b) c) = mulR (sg b) (mulR (sg a) c)

theorem **foldr_perm** (ha : Assoc k) (hc : Comm k) {l₁ l₂ : List (IrrGL k)} (p : l₁.Perm l₂) (u : GrothGL k) : l₁.foldr (fun y a => mulR (sg y) a) u = l₂.foldr (fun y a => mulR (sg y) a) u

theorem **jordan_holder_mult_eq_of_class** (y : IrrGL k) {V W : Type} [AddCommGroup V] [Module ℂ V] [AddCommGroup W] [Module ℂ W] (π : Representation ℂ (GL (Fin y.1) k) V) (π' : Representation ℂ (GL (Fin y.1) k) W) (hπ : HasFiniteLength π) (hπ' : HasFiniteLength π') (h : cl0 π = cl0 π') : jhMult (IrrGL.rep y) π = jhMult (IrrGL.rep y) π'

theorem **jhMult_mul_jm_le** {g N : ℕ} (z : Fin g → IrrGL k) (hz : ∑ b, (z b).1 = N) {V W : Type} [AddCommGroup V] [Module ℂ V] [AddCommGroup W] [Module ℂ W] (π : Representation ℂ (GL (Fin N) k) V) (τ : Representation ℂ (GL (Fin N) k) W) (hπ : IsSmooth π) : jhMult τ π * jm z hz τ ≤ jm z hz π

theorem **ofFn_perm** {α : Type*} {h : ℕ} (x : Fin h → α) (σ : Equiv.Perm (Fin h)) : (List.ofFn (x ∘ σ)).Perm (List.ofFn x)

theorem **jhMult_le_one_of_equiv** (y : IrrGL k) {V : Type} [AddCommGroup V] [Module ℂ V] {π : Representation ℂ (GL (Fin y.1) k) V} {g : ℕ} {z : Fin g → IrrGL k} {hz : ∑ b, (z b).1 = y.1} (hsoc : IsSocle (IrrGL.rep y) π) (e : π.Equiv (prodIrr z hz)) (hjm : jm z hz (prodIrr z hz) ≤ 1) : jhMult (IrrGL.rep y) π ≤ 1

theorem **jhMult_rev_le_one** (c : Cls k) (_m : Multiset (Segment k)) {h : ℕ} (x : Fin h → IrrGL k) (y : IrrGL k) (Δ : Fin h → Segment k) (hΔo : ∀ i j, i < j → ¬ (Δ i).Precedes (Δ j)) (hΔL : ∀ i, IsLSeg (Δ i) (x i).rep) (hdr : ∑ a, (x (Fin.rev a)).1 = y.1) (hsoc : IsSocle (IrrGL.rep y) (prodIrr (x ∘ Fin.rev) hdr)) : jhMult (IrrGL.rep y) (prodIrr (x ∘ Fin.rev) hdr) ≤ 1

theorem **multOne** (c : Cls k) (_hm : MulCls k) (ha : Assoc k) (hc : Comm k) (_h1 : One k) (hl : ProdList k) : MultOne k

CompanionTheory.AutomorphicTheory.ProductFormulaTheory.GrothendieckRingPolynomial.PeelGood

theorem **comm_of** (hI : Zc.Cos.LUnlinkedIrr k) : Zc.Cos.LUnlinkedComm k

theorem **stdCos** (hZ : GlobalTheory.Published.SegmentRepresentation) (hI : Zc.Cos.LUnlinkedIrr k) : ∀ (m : Multiset (Segment k)) (h N : ℕ) (x : Fin h → IrrGL k) (hd : ∑ a, (x a).1 = N), LFactors m x → ∃ π : IrrRep (GL (Fin N) k), IsCosocle π.ρ (prodIrr x hd)

theorem **segCl** (Δ : Segment k) : ∃ x : IrrGL k, IsLSeg Δ x.rep

theorem **ex_of** (HS : ∀ (m : Multiset (Segment k)) (h N : ℕ) (x : Fin h → IrrGL k) (hd : ∑ a, (x a).1 = N), LFactors m x → ∃ π : IrrRep (GL (Fin N) k), IsCosocle π.ρ (prodIrr x hd)) (m : Multiset (Segment k)) : ∃ x : IrrGL k, IsL m x.rep

theorem **peelGood** (HS : ∀ (m : Multiset (Segment k)) (h N : ℕ) (x : Fin h → IrrGL k) (hd : ∑ a, (x a).1 = N), LFactors m x → ∃ π : IrrRep (GL (Fin N) k), IsCosocle π.ρ (prodIrr x hd)) (HL : LSocleGL k) : Zc.PeelGoodP k

theorem **exh_at** (hM : MidP k) (hR : RefineP k) (hS : SwapP k) (hC : CuspEmbP k) (hP : SuppP k) (hJ : JuxtP k) (hL : LinkP k) (hm : MinP k) (hG : Zc.PeelGoodP k) (x : IrrGL k) : ∃ m : Multiset (Segment k), IsL m x.rep

theorem **cls_of** (hZ : GlobalTheory.Published.SegmentRepresentation) (k : Type) [Field k] [ValuativeRel k] [TopologicalSpace k] [IsNonarchimedeanLocalField k] (hI : Zc.Cos.LUnlinkedIrr k) (H : CompanionTheory.Exh.MidP k ∧ CompanionTheory.Exh.RefineP k ∧ CompanionTheory.Exh.SwapP k) : Cls k

CompanionTheory.AutomorphicTheory.ProductFormulaTheory.GrothendieckRingPolynomial.PhiZAux

theorem **interleave_perm** {T : Type*} {u v w : List T} (h : Zc.Comb.Interleave u v w) : w.Perm (u ++ v)

theorem **shuffle_perm** {T : Type*} {us : List (List T)} {w : List T} (h : Zc.Comb.Shuffle us w) : w.Perm us.flatten

theorem **coe_decW** (Γ : Segment k) : ((decW Γ : List (IrrGL k)) : Multiset (IrrGL k)) = Γ.ms

theorem **coe_flatten_decW** (L : List (Segment k)) : (((L.map decW).flatten : List (IrrGL k)) : Multiset (IrrGL k)) = msupp (L : Multiset (Segment k))

theorem **coe_eq_msupp_of_inShW** {π : Multiset (Segment k)} {w : List (IrrGL k)} (h : InShW π w) : (w : Multiset (IrrGL k)) = msupp π

theorem **msupp_eq_of_isL** (hA : GlobalTheory.Published.IrreducibleAdmissibleGL) (hj : LSegJacquet k) {π : Multiset (Segment k)} {x : IrrGL k} (hx : IsL π x.rep) (w : List (IrrGL k)) (hc : ∀ c ∈ w, c.IsCusp) (hw : Wd x.rep w) : (w : Multiset (IrrGL k)) = msupp π

theorem **wd_of_jmL_eq_one** {N : ℕ} {V : Type} [AddCommGroup V] [Module ℂ V] {π : Representation ℂ (GL (Fin N) k) V} {w : List (IrrGL k)} (h : jmL w π = 1) : Wd π w

theorem **msupp_seg** (hA : GlobalTheory.Published.IrreducibleAdmissibleGL) (hj : LSegJacquet k) (Z : ∀ Δ : Segment k, IrrCl (GL (Fin (Δ.len * Δ.d)) k)) (hZ : IsSegmentClass k Z) (hS : JSeg k) (Δ : Segment k) {π : Multiset (Segment k)} (hπ : IsL π (IrrGL.rep ((Δ.len * Δ.d, Z Δ) : IrrGL k))) : msupp π = Δ.ms

theorem **seg_eq_of_ms_eq** (hA : GlobalTheory.Published.IrreducibleAdmissibleGL) {Γ Δ : Segment k} (h : Γ.ms = Δ.ms) : Γ = Δ

theorem **len_lt_of_msupp** (hA : GlobalTheory.Published.IrreducibleAdmissibleGL) {Δ : Segment k} {M : Multiset (Segment k)} (hM : msupp M = Δ.ms) (hne : M ≠ {Δ}) : ∀ Γ ∈ M, Γ.len < Δ.len

theorem **toMultiset_eq_single_iff** (s : Segment k →₀ ℕ) (Δ : Segment k) : Finsupp.toMultiset s = {Δ} ↔ s = Finsupp.single Δ 1

theorem **prod_X_of_map** (s : Segment k →₀ ℕ) {h : ℕ} (Γ : Fin h → Segment k) (hΓ : Finset.univ.val.map Γ = Finsupp.toMultiset s) : ∏ a, (MvPolynomial.X (Γ a) : MvPolynomial (Segment k) ℤ) = MvPolynomial.monomial s 1

theorem **exists_enum** (s : Segment k →₀ ℕ) : ∃ (h : ℕ) (Γ : Fin h → Segment k), Finset.univ.val.map Γ = Finsupp.toMultiset s

theorem **ell_apply** (Δ : Segment k) (a : GrothGL k) : ell Δ a = ∑ S ∈ vSets Δ, (-1 : ℤ) ^ S.card * J (vWord Δ S) a

theorem **J_sg** (W : List (IrrGL k)) (x : IrrGL k) : J W (sg x) = ((jmL W x.rep).toNat : ℤ)

theorem **vWord_empty** (Δ : Segment k) : vWord Δ ∅ = List.ofFn Δ.term

theorem **cusp_vWord** (Δ : Segment k) (S : Finset (Fin Δ.len)) : ∀ w ∈ vWord Δ S, w.IsCusp

theorem **ell_sg** {Δ : Segment k} {x : IrrGL k} (h1 : jmL (List.ofFn Δ.term) x.rep = 1) (h0 : ∀ S ∈ vSets Δ, S ≠ ∅ → jmL (vWord Δ S) x.rep = 0) : ell Δ (sg x) = 1

theorem **ell_prod** (hA : GlobalTheory.Published.IrreducibleAdmissibleGL) (hj : LSegJacquet k) (hJ : JCls k) (hAlt : AltSum k) (Lseg : Segment k → IrrGL k) (hL : ∀ Δ, IsLSeg Δ (Lseg Δ).rep) (Δ : Segment k) {h N : ℕ} (Γ : Fin h → Segment k) (hd : ∑ a, (Lseg (Γ a)).1 = N) : ell Δ (cl0 (prodIrr (fun a => Lseg (Γ a)) hd)) = if Finset.univ.val.map Γ = {Δ} then (-1) ^ (Δ.len - 1) else 0

theorem **vars_lt** (hA : GlobalTheory.Published.IrreducibleAdmissibleGL) {Δ : Segment k} (p : MvPolynomial (Segment k) ℤ) (hp : ∀ s ∈ p.support, msupp (Finsupp.toMultiset s) = Δ.ms) : ∀ t ∈ (p - MvPolynomial.monomial (Finsupp.single Δ 1) (p.coeff (Finsupp.single Δ 1))).vars, t.len < Δ.len

CompanionTheory.AutomorphicTheory.ProductFormulaTheory.GrothendieckRingPolynomial.PhiZBijective

theorem **ell_phiOf** (hA : GlobalTheory.Published.IrreducibleAdmissibleGL) (hj : LSegJacquet k) (hJ : JCls k) (hAlt : AltSum k) (Lseg : Segment k → IrrGL k) (hL : ∀ Δ, IsLSeg Δ (Lseg Δ).rep) (hprod : ∀ (h N : ℕ) (Γ : Fin h → Segment k) (hd : ∑ a, (Lseg (Γ a)).1 = N), phiOf Lseg (∏ a, MvPolynomial.X (Γ a)) = toR (cl0 (prodIrr (fun a => Lseg (Γ a)) hd))) (Δ : Segment k) (p : MvPolynomial (Segment k) ℤ) : ell Δ (toR.symm (phiOf Lseg p)) = p.coeff (Finsupp.single Δ 1) * (-1) ^ (Δ.len - 1)

theorem **phiZ_bijective** (hk : Steps k) (h1 : TriAut) (h2 : UniTri) (Z : ∀ Δ : Segment k, IrrCl (GL (Fin (Δ.len * Δ.d)) k)) (hZ : IsSegmentClass k Z) : Function.Bijective (phiOf (k

CompanionTheory.AutomorphicTheory.ProductFormulaTheory.GrothendieckRingPolynomial.SingleEqSmulSingle

theorem **bil_add_right** (a b c : ι →₀ ℤ) : bil C a (b + c) = bil C a b + bil C a c

theorem **bil_add_left** (a b c : ι →₀ ℤ) : bil C (a + b) c = bil C a c + bil C b c

theorem **bil_zero_left** (b : ι →₀ ℤ) : bil C 0 b = 0

theorem **bil_zero_right** (a : ι →₀ ℤ) : bil C a 0 = 0

theorem **bil_smul_left** (z : ℤ) (a b : ι →₀ ℤ) : bil C (z • a) b = z • bil C a b

theorem **bil_smul_right** (z : ℤ) (a b : ι →₀ ℤ) : bil C a (z • b) = z • bil C a b

theorem **bil_single** (x y : ι) : bil C (Finsupp.single x 1) (Finsupp.single y 1) = C x y

theorem **bil_comm** (hC : ∀ x y, C x y = C y x) (a b : ι →₀ ℤ) : bil C a b = bil C b a

theorem **single_eq_smul_single** (x : ι) (m : ℤ) : (Finsupp.single x m : ι →₀ ℤ) = m • Finsupp.single x 1

theorem **bil_assoc** (hA : ∀ x y z, bil C (bil C (Finsupp.single x 1) (Finsupp.single y 1)) (Finsupp.single z 1) = bil C (Finsupp.single x 1) (bil C (Finsupp.single y 1) (Finsupp.single z 1))) (a b c : ι →₀ ℤ) : bil C (bil C a b) c = bil C a (bil C b c)

theorem **bil_one** (u : ι →₀ ℤ) (h : ∀ x, bil C u (Finsupp.single x 1) = Finsupp.single x 1) (a : ι →₀ ℤ) : bil C u a = a

theorem **mulR_eq** (a b : GrothGL k) : mulR a b = bil cm a b

theorem **sg_eq** (x : IrrGL k) : sg x = Finsupp.single x 1

theorem **mulR_add** (a b c : GrothGL k) : mulR a (b + c) = mulR a b + mulR a c

theorem **add_mulR** (a b c : GrothGL k) : mulR (a + b) c = mulR a c + mulR b c

theorem **zero_mulR** (a : GrothGL k) : mulR 0 a = 0

theorem **mulR_zero** (a : GrothGL k) : mulR a 0 = 0

theorem **mulR_comm'** [Fact (Comm k)] (a b : GrothGL k) : mulR a b = mulR b a

theorem **mulR_assoc'** [Fact (Assoc k)] (a b c : GrothGL k) : mulR (mulR a b) c = mulR a (mulR b c)

theorem **unit_mulR** [Fact (One k)] (a : GrothGL k) : mulR (GrothGL.unit k) a = a

theorem **toR_mulR** (a b : GrothGL k) : toR (mulR a b) = toR a * toR b

theorem **toR_unit** : toR (GrothGL.unit k) = 1

theorem **phiOf_X** (f : Segment k → IrrGL k) (Δ : Segment k) : phiOf f (MvPolynomial.X Δ) = toR (sg (f Δ))

CompanionTheory.AutomorphicTheory.ProductFormulaTheory.GrothendieckRingPolynomial.TriCoeFlattenDecW

theorem **tri_coe_decW** (Δ : Segment k) : ((decW Δ : List (IrrGL k)) : Multiset (IrrGL k)) = Δ.ms

theorem **tri_coe_flatten_decW** (L : List (Segment k)) : (((L.map decW).flatten : List (IrrGL k)) : Multiset (IrrGL k)) = (L : Multiset (Segment k)).bind Segment.ms

theorem **tri_msupp_of_inShW** {m : Multiset (Segment k)} {w : List (IrrGL k)} (H : InShW m w) : msupp m = (w : Multiset (IrrGL k))

theorem **tri_msupp_wtW** {h : ℕ} {Γ : Fin h → Segment k} {n : Multiset (Segment k)} (H : IsOrderedForm Γ n) : msupp n = (wtW Γ : Multiset (IrrGL k))

theorem **tri_sqLen_map_toA** (m : Multiset (Segment k)) : Comb.sqLen (m.map toA) = sq m

theorem **tri** (k : Type) [Field k] [ValuativeRel k] [TopologicalSpace k] [IsNonarchimedeanLocalField k] (c : Cls k) : Tri k

CompanionTheory.AutomorphicTheory.ProductFormulaTheory.GrothendieckRingPolynomial.TriSupportedLe

theorem **tri_supported_le** {σ : Type*} (w : σ → ℕ) (ψ : MvPolynomial σ ℤ →+* MvPolynomial σ ℤ) (h : ∀ s, ∃ (ε : ℤˣ) (p : MvPolynomial σ ℤ), (∀ t ∈ p.vars, w t < w s) ∧ ψ (X s) = (ε : ℤ) • X s + p) (N : ℕ) : supported ℤ {t | w t < N} ≤ (supported ℤ {t | w t < N}).map (toZAlg ψ)

theorem **tri_surj** {σ : Type*} (w : σ → ℕ) (ψ : MvPolynomial σ ℤ →+* MvPolynomial σ ℤ) (h : ∀ s, ∃ (ε : ℤˣ) (p : MvPolynomial σ ℤ), (∀ t ∈ p.vars, w t < w s) ∧ ψ (X s) = (ε : ℤ) • X s + p) : Function.Surjective ψ

theorem **tri_bij** {σ : Type*} (w : σ → ℕ) (ψ : MvPolynomial σ ℤ →+* MvPolynomial σ ℤ) (h : ∀ s, ∃ (ε : ℤˣ) (p : MvPolynomial σ ℤ), (∀ t ∈ p.vars, w t < w s) ∧ ψ (X s) = (ε : ℤ) • X s + p) : Function.Bijective ψ

theorem **uni_inj** {M I : Type*} (L : M ≃ I) (w : M → ℕ) (f : M → I →₀ ℤ) (h1 : ∀ m, f m (L m) = 1) (hlt : ∀ m n, f m (L n) ≠ 0 → m = n ∨ w m < w n) : Function.Injective (Finsupp.linearCombination ℤ f)

theorem **uni_exists** {M I C : Type*} (L : M ≃ I) (c : M → C) (w : M → ℕ) (b : C → ℕ) (f : M → I →₀ ℤ) (hb : ∀ m, w m ≤ b (c m)) (h1 : ∀ m, f m (L m) = 1) (hlt : ∀ m n, f m (L n) ≠ 0 → c m = c n ∧ (m = n ∨ w m < w n)) : ∀ n, ∃ a : M →₀ ℤ, (∀ m ∈ a.support, c m = c n) ∧ Finsupp.linearCombination ℤ f a = Finsupp.single (L n) 1

theorem **uni_bij** {M I C : Type*} (L : M ≃ I) (c : M → C) (w : M → ℕ) (b : C → ℕ) (f : M → I →₀ ℤ) (hb : ∀ m, w m ≤ b (c m)) (h1 : ∀ m, f m (L m) = 1) (hlt : ∀ m n, f m (L n) ≠ 0 → c m = c n ∧ (m = n ∨ w m < w n)) : Function.Bijective (Finsupp.linearCombination ℤ f)

theorem **triAut** : TriAut

theorem **uniTri** : UniTri

CompanionTheory.AutomorphicTheory.ProductFormulaTheory.SquareIntegrableSegments.SegmentReordering.CsOrderfree

theorem **irrGL_eq_of_isLSeg_same** {Δ : Segment k} {x y : IrrGL k} (hx : IsLSeg Δ x.rep) (hy : IsLSeg Δ y.rep) : x = y

theorem **perm_of_univ_map_eq** {α : Type*} {l : ℕ} {x y : Fin l → α} (h : Finset.univ.val.map y = Finset.univ.val.map x) : ∃ e : Equiv.Perm (Fin l), ∀ a, y a = x (e a)

theorem **cs_orderfree** : ∀ {k : Type} [Field k] [ValuativeRel k] [TopologicalSpace k] [IsNonarchimedeanLocalField k], GlobalTheory.Published.IrreducibleAdmissibleGL → LUnlinkedComm k → LOrderFree k

CompanionTheory.AutomorphicTheory.ProductFormulaTheory.SquareIntegrableSegments.SegmentReordering.CsReorderord

theorem **cs_reorderord** : ∀ {k : Type} [Field k] [ValuativeRel k] [TopologicalSpace k] [IsNonarchimedeanLocalField k], GlobalTheory.Published.IrreducibleAdmissibleGL → LUnlinkedComm k → ∀ {h N : ℕ} (Γ : Fin h → Segment k) (σ : Equiv.Perm (Fin h)) (y : Fin h → IrrGL k) (hd : ∑ a, (y a).1 = N) (hd' : ∑ a, (y (σ a)).1 = N), (∀ i j, i < j → ¬ (Γ i).Precedes (Γ j)) → (∀ i j, i < j → ¬ (Γ (σ i)).Precedes (Γ (σ j))) → (∀ i, IsLSeg (Γ i) (y i).rep) → Nonempty ((prodIrr y hd).Equiv (prodIrr (y ∘ σ) hd'))

CompanionTheory.AutomorphicTheory.ProductFormulaTheory.SquareIntegrableSegments.SegmentReordering.ReorderCommon

theorem **ntrans** {V W U : Type} [AddCommGroup V] [Module ℂ V] [AddCommGroup W] [Module ℂ W] [AddCommGroup U] [Module ℂ U] {G : Type} [Group G] {ρ : Representation ℂ G V} {σ : Representation ℂ G W} {τ : Representation ℂ G U} (a : Nonempty (ρ.Equiv σ)) (b : Nonempty (σ.Equiv τ)) : Nonempty (ρ.Equiv τ)

theorem **prodIrr_equiv_of_eq** {h N : ℕ} {y y' : Fin h → IrrGL k} (e : y = y') (hy : ∑ a, (y a).1 = N) (hy' : ∑ a, (y' a).1 = N) : Nonempty ((prodIrr y hy).Equiv (prodIrr y' hy'))

theorem **cut_e** {h j : ℕ} (hj : j + 1 < h) : j + (2 + (h - (j + 2))) = h

theorem **cutC_lt** {h j b : ℕ} (hb : b < h - (j + 2)) : j + 2 + b < h

theorem **sum_append** {q c : ℕ} (Q : Fin q → IrrGL k) (C : Fin c → IrrGL k) : ∑ a, (Fin.append Q C a).1 = ∑ a, (Q a).1 + ∑ a, (C a).1

theorem **sum_split2** {p q n : ℕ} (A : Fin p → IrrGL k) (R : Fin q → IrrGL k) (w : Fin n → IrrGL k) (e : p + q = n) (hw : ∀ m : Fin (p + q), w (Fin.cast e m) = Fin.append A R m) : ∑ a, (w a).1 = ∑ a, (A a).1 + ∑ a, (R a).1

theorem **stage2** {p q n rA rR : ℕ} (A : Fin p → IrrGL k) (R : Fin q → IrrGL k) (w : Fin n → IrrGL k) (e : p + q = n) (hw : ∀ m : Fin (p + q), w (Fin.cast e m) = Fin.append A R m) (hA : ∑ a, (A a).1 = rA) (hR : ∑ a, (R a).1 = rR) (hN : ∑ a, (w a).1 = rA + rR) : Nonempty ((prod₂ (prodIrr A hA) (prodIrr R hR)).Equiv (prodIrr w hN))

theorem **head_swap** {c rQ rC : ℕ} (Q Q' : Fin 2 → IrrGL k) (C : Fin c → IrrGL k) (hQ : ∑ a, (Q a).1 = rQ) (hQ' : ∑ a, (Q' a).1 = rQ) (hC : ∑ a, (C a).1 = rC) (hR : ∑ a, (Fin.append Q C a).1 = rQ + rC) (hR' : ∑ a, (Fin.append Q' C a).1 = rQ + rC) (E : Nonempty ((prodIrr Q hQ).Equiv (prodIrr Q' hQ'))) : Nonempty ((prodIrr (Fin.append Q C) hR).Equiv (prodIrr (Fin.append Q' C) hR'))

theorem **split_alt_alt** {h j : ℕ} (hj : j + 1 < h) (e : j + (2 + (h - (j + 2))) = h) (x : Fin h → IrrGL k) (m : Fin (j + (2 + (h - (j + 2))))) : x (Fin.cast e m) = Fin.append (cutA x j hj) (Fin.append ![x (j, Nat.lt_of_succ_lt hj), x (j + 1, hj)] (cutC x j)) m

theorem **split_swap** {h j : ℕ} (hj : j + 1 < h) (e : j + (2 + (h - (j + 2))) = h) (x : Fin h → IrrGL k) (m : Fin (j + (2 + (h - (j + 2))))) : x (Equiv.swap ((j, Nat.lt_of_succ_lt hj) : Fin h) (j + 1, hj) (Fin.cast e m)) = Fin.append (cutA x j hj) (Fin.append ![x (j + 1, hj), x (j, Nat.lt_of_succ_lt hj)] (cutC x j)) m

theorem **swap_adj** {h N : ℕ} (x : Fin h → IrrGL k) (hd : ∑ a, (x a).1 = N) (j : ℕ) (hj : j + 1 < h) (M : ℕ) (hd2 : ∑ a, (![x (j, Nat.lt_of_succ_lt hj), x (j + 1, hj)] a).1 = M) (hd2' : ∑ a, (![x (j + 1, hj), x (j, Nat.lt_of_succ_lt hj)] a).1 = M) (hd' : ∑ a, (x (Equiv.swap ((j, Nat.lt_of_succ_lt hj) : Fin h) (j + 1, hj) a)).1 = N) (E : Nonempty ((prodIrr ![x (j, Nat.lt_of_succ_lt hj), x (j + 1, hj)] hd2).Equiv (prodIrr ![x (j + 1, hj), x (j, Nat.lt_of_succ_lt hj)] hd2'))) : Nonempty ((prodIrr x hd).Equiv (prodIrr (fun a => x (Equiv.swap ((j, Nat.lt_of_succ_lt hj) : Fin h) (j + 1, hj) a)) hd'))

theorem **reach_step** (hcomm : LUnlinkedComm k) {h N : ℕ} {Δ : Fin h → Segment k} {x : Fin h → IrrGL k} (hd : ∑ a, (x a).1 = N) (hL : ∀ i, IsLSeg (Δ i) (x i).rep) (σ : Equiv.Perm (Fin h)) (hσ : Reach x hd σ) (j : ℕ) (hj : j + 1 < h) (hu : ¬ (Δ (σ (j, Nat.lt_of_succ_lt hj))).Linked (Δ (σ (j + 1, hj)))) : Reach x hd (σ * Equiv.swap ((j, Nat.lt_of_succ_lt hj) : Fin h) (j + 1, hj))

theorem **strictMono_of_adj** {h : ℕ} (f : Fin h → Fin h) (H : ∀ (j : ℕ) (hj : j + 1 < h), f (j, Nat.lt_of_succ_lt hj) < f (j + 1, hj)) : StrictMono f

theorem **swap_lt_swap** {h j : ℕ} (hj : j + 1 < h) {u v : Fin h} (hvu : v < u) (H : Equiv.swap ((j, Nat.lt_of_succ_lt hj) : Fin h) (j + 1, hj) u < Equiv.swap ((j, Nat.lt_of_succ_lt hj) : Fin h) (j + 1, hj) v) : v.1 = j ∧ u.1 = j + 1

theorem **symm_mul_swap_apply** {h : ℕ} (σ s : Equiv.Perm (Fin h)) (a b : Fin h) (hs : s = Equiv.swap a b) (p : Fin h) : (σ * s).symm p = s (σ.symm p)

theorem **reach_of_inv** (hcomm : LUnlinkedComm k) {h N : ℕ} {Δ : Fin h → Segment k} {x : Fin h → IrrGL k} (hd : ∑ a, (x a).1 = N) (hL : ∀ i, IsLSeg (Δ i) (x i).rep) : ∀ (n : ℕ) (σ : Equiv.Perm (Fin h)), (invSet σ).card = n → (∀ a b, a < b → σ.symm b < σ.symm a → ¬ (Δ a).Linked (Δ b)) → Reach x hd σ

CompanionTheory.AutomorphicTheory.RankOneEstimates.DecSender

lemma **dsigW_eq** {k r : ℕ} {H : Type} [NormedAddCommGroup H] [InnerProductSpace ℂ H] (σ : Mx k r → (H →L[ℂ] H)) : ∀ (W : List (Mx k r)) (v : H), Formalism.MainNotions.S.dsigW (k

lemma **thr_le** {n : ℕ} (hn : 1 ≤ n) (ε : Fin n → Bool) : thrStar ε ≤ 40 * (n : ℝ) ^ 3

lemma **room** {n : ℕ} (hn : 1 ≤ n) (ε : Fin n → Bool) {h : ℝ} (hh : 120 * (n : ℝ) ^ 3 ≤ h) : thrStar ε + (h / 8 + 1) < h

theorem **decayM** {k r : ℕ} {ε : Fin (2 * r + k + 1) → Bool} {lam : Fin (2 * r + k + 1) → ℝ} {h : ℝ} {H : Type} [NormedAddCommGroup H] [InnerProductSpace ℂ H] [CompleteSpace H] {σ : Mx k r → (H →L[ℂ] H)} (hl : Formalism.QuantumDeterminants.IsHCParam lam) (hσ : PI.UpqRep.IsDSHpk k r ε lam σ) (hg : Formalism.QuantumDeterminants.GapAtLeast ε lam h) (m : ℕ) (hm : thrStar ε + m < h) (v0 : H) (hv0 : v0 ∈ PI.UpqRep.smoothVecs (k

theorem **decay** {k r : ℕ} {ε : Fin (2 * r + k + 1) → Bool} {lam : Fin (2 * r + k + 1) → ℝ} {h : ℝ} {H : Type} [NormedAddCommGroup H] [InnerProductSpace ℂ H] [CompleteSpace H] {σ : Mx k r → (H →L[ℂ] H)} (hl : Formalism.QuantumDeterminants.IsHCParam lam) (hσ : PI.UpqRep.IsDSHpk k r ε lam σ) (hg : Formalism.QuantumDeterminants.GapAtLeast ε lam h) (hh : 120 * ((2 * r + k + 1 : ℕ) : ℝ) ^ 3 ≤ h) (v0 : H) (hv0 : v0 ∈ PI.UpqRep.smoothVecs (k

theorem **decay_infinite_range** : DecInf

theorem **decay_low_range** : DecLow

CompanionTheory.AutomorphicTheory.RankOneEstimates.RankOne

lemma **haar_smul** {k r : ℕ} {U : Set (B.Mat k r)} {μ : Measure (Mt k r)} (h : B.IsHaarOn U μ) {c : ℝ≥0∞} (h0 : c ≠ 0) (h1 : c ≠ ⊤) : B.IsHaarOn U (c • μ) where

theorem **be_rank_one** {k r : ℕ} (hk : 1 ≤ k) {ε : Fin (2 * r + k + 1) → Bool} {lam : Fin (2 * r + k + 1) → ℝ} {h : ℝ} {H : Type} [NormedAddCommGroup H] [InnerProductSpace ℂ H] [CompleteSpace H] {σ : Mx k r → (H →L[ℂ] H)} {Wp : Type} [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] {z : Fin (2 * r + k + 1) → ℂ} {u : Fin k → ℝ} {π : Mx k r → (Wp →L[ℂ] Wp)} {μW μN : Measure (Mt k r)} (hl : IsHCParam lam) (hσ : PI.UpqRep.IsDSHpk k r ε lam σ) (hg : Formalism.QuantumDeterminants.GapAtLeast ε lam h) (hh : 120 * ((2 * r + k + 1 : ℕ) : ℝ) ^ 3 ≤ h) (hz : inWb k r z) (hT : 0 < Tz k r z) (hπ : PI.UpqRep.IsHCTypeW k r z u π) (hW : PI.BesselPeriodGGP.IsHaarOn (UWset k r z) μW) (hN : PI.BesselPeriodGGP.IsHaarOn (Nset k r) μN) : ∃ L : H → Wp, ∀ w ∈ PI.UpqRep.smoothVecs (k

CompanionTheory.AutomorphicTheory.RankOneTori.BeRankScale

theorem **be_scale** {ε : Fin (2 * r + k + 1) → Bool} {lam : Fin (2 * r + k + 1) → ℝ} {H : Type} [NormedAddCommGroup H] [InnerProductSpace ℂ H] [CompleteSpace H] {σ : Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ → (H →L[ℂ] H)} {Wp : Type} [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] {z : Fin (2 * r + k + 1) → ℂ} {π : Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ → (Wp →L[ℂ] Wp)} {μW μN : Measure (Mt k r)} [SFinite μW] [SFinite μN] (hσ : PI.UpqRep.IsDSHpk k r ε lam σ) (hz : inWb k r z) (h1 : ∀ᵐ x ∂μW, Matrix.of x ∈ UWset k r z) (h2 : ∀ᵐ x ∂μN, Matrix.of x ∈ Nset k r) {κ τ : ℝ} (hκ : 0 < κ) (hτ : 0 < τ) (c : ℂ) (u v : H) : Formalism.FourierDecayAlt.ThreeDimData.be ((τ : ℂ) • z) ((κ : ℂ) * c) (fun x => π (cjI k r κ τ x)) σ (μW.map (cjT k r κ τ)) (μN.map (cjT k r κ τ)) u v = Formalism.FourierDecayAlt.ThreeDimData.be z c π σ μW μN (σ (dM k r κ τ) u) (σ (dM k

theorem **be_rank_scale** (hk : 1 ≤ k) {ε : Fin (2 * r + k + 1) → Bool} {lam : Fin (2 * r + k + 1) → ℝ} {h : ℝ} {H : Type} [NormedAddCommGroup H] [InnerProductSpace ℂ H] [CompleteSpace H] {σ : Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ → (H →L[ℂ] H)} {Wp : Type} [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] {z : Fin (2 * r + k + 1) → ℂ} {u : Fin k → ℝ} {π : Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ → (Wp →L[ℂ] Wp)} {μW : Measure (Mt k r)} (hl : IsHCParam lam) (hσ : PI.UpqRep.IsDSHpk k r ε lam σ) (hg : Formalism.QuantumDeterminants.GapAtLeast ε lam h) (hh : 120 * ((2 * r + k + 1 : ℕ) : ℝ) ^ 3 ≤ h) (hz : inWb k r z) (hT : 0 < Tz k r z) (hπ : IsHCTypeW k r z u π) (hW : IsHaarOn (UWset k r z) μW) {κN : ℝ≥0∞} (h0 : κN ≠ 0) (h1 : κN ≠ ⊤) {κ₀ : ℝ} (hκ₀ : 0 < κ₀) : ∃ l : H → Wp, ∀ w ∈ PI.UpqRep.smoothVecs (k

CompanionTheory.AutomorphicTheory.RankOneTori.Factor

theorem **coef_expand** (L : LocBessel k r) (P : L.Hs →L[ℂ] L.Wp) (Q : L.Wq →L[ℂ] L.Hs) (x : Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ) : P.comp ((L.σ x).comp Q) = ∑ i, ∑ j, Pv.AmpDD.coefF L (ContinuousLinearMap.adjoint P (stdOrthonormalBasis ℂ L.Wp i)) (Q (stdOrthonormalBasis ℂ L.Wq j)) (stdOrthonormalBasis ℂ L.Wp i) (stdOrthonormalBasis ℂ L.Wq j) x

theorem **int_coefF** (L : LocBessel k r) {μG : Measure (Mt k r)} (hμG : IsHaarOn (UH (Hpk k r)) μG) {d : ℝ} (hd : 0 < d) (hSch : ∀ v w v' w' : L.Hs, Integrable (fun x : Mt k r => ⟪w, L.σ (Matrix.of x) v⟫_ℂ * star ⟪w', L.σ (Matrix.of x) v'⟫_ℂ) μG ∧ ∫ x, ⟪w, L.σ (Matrix.of x) v⟫_ℂ * star ⟪w', L.σ (Matrix.of x) v'⟫_ℂ ∂μG = ⟪v', v⟫_ℂ * ⟪w, w'⟫_ℂ / (d : ℂ)) {w0 : L.Hs} (hw1 : ⟪w0, w0⟫_ℂ = 1) {γ : Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ} (hγ : γ ∈ UH (Hpk k r)) {v : Measure (PairSp k r)} (hae : ∀ᵐ p ∂v, InPair L p) {a b C1 C2 : ℝ} (hb : 0 ≤ b) (hab : b ≤ a) (hIG : Integrable (fun x : Mt k r => B.gN (Matrix.of x) ^ (b - a)) μG) (hpm : Integrable (Pv.AmpDD.pairMaj γ b) v) {v v' : L.Hs} (hC1 : ∀ g ∈ UH (Hpk k r), ‖⟪v, L.σ g w0⟫_ℂ‖ ≤ C1 * B.gN g ^ (-a)) (hC2 : ∀ g ∈ UH (Hpk k r), ‖⟪v', L.σ g w0⟫_ℂ‖ ≤ C2 * B.gN g ^ (-a)) (ξ : L.Wp) (ξ' : L.Wq) : Integrable (besselIntg L (Pv.AmpDD.coefF L

theorem **cell_factor** (L : LocBessel k r) {a : ℝ} {w0 : L.Hs} (hw1 : ⟪w0, w0⟫_ℂ = 1) (l : L.Hs → L.Wp) (l' : L.Hs → L.Wq) (Sm : Set L.Hs) (hdec : ∀ v ∈ Sm, ∃ C : ℝ, ∀ g ∈ UH (Hpk k r), ‖⟪v, L.σ g w0⟫_ℂ‖ ≤ C * B.gN g ^ (-a)) (hl : ∀ v ∈ Sm, ∀ t ∈ UH (Hpk k r), ThreeDimData.be L.z L.c L.π L.σ L.μW L.μN (L.σ t w0) v = (innerSL ℂ (l v)).smulRight (l (L.σ t w0))) (hl' : ∀ v ∈ Sm, ∀ t ∈ UH (Hpk k r), ThreeDimData.be L.z' L.c L.π' L.σ L.μW' L.μN (L.σ t w0) v = (innerSL ℂ (l' v)).smulRight (l' (L.σ t w0))) {γ : Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ} {μR : Measure (Mt k r)} {v : Measure (PairSp k r)} (hc : Cell L a γ μR v) : ∃ Ψ : L.Wq →L[ℂ] L.Wp, ∀ (P : L.Hs →L[ℂ] L.Wp) (Q : L.Wq →L[ℂ] L.Hs), (∀ x, ContinuousLinearMap.adjoint P x ∈ Sm) → (∀ w, Q w ∈ Sm) → ∀ F : Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ → (L.Wq →L[ℂ] L.Wp), (∀ x ∈ UH (Hpk k r), F x = P.comp ((L.σ x).comp Q

CompanionTheory.AutomorphicTheory.RankOneTori.FrameBesselBoundOfLargeConstants

theorem **gap_arith** (hk : 1 ≤ k) {C : ℝ} (hC : 100 ≤ C) : ThreeDimData.GapOK k r (C * ((2 * r + 2 : ℕ) : ℝ) ^ ((9 : ℝ) / 2) * (((3 * (r + 1) : ℕ) : ℝ) * ((2 * r + k + 1 : ℕ) : ℝ) ^ 2 + ((2 * r + k + 1 : ℕ) : ℝ) ^ 3)) ∧ Pv.AmpDD.GapCell k r (C * ((2 * r + 2 : ℕ) : ℝ) ^ ((9 : ℝ) / 2) * (((3 * (r + 1) : ℕ) : ℝ) * ((2 * r + k + 1 : ℕ) : ℝ) ^ 2 + ((2 * r + k + 1 : ℕ) : ℝ) ^ 3))

theorem **exists_low** {ε : Fin (2 * r + k + 1) → Bool} {lam : Fin (2 * r + k + 1) → ℝ} {H : Type} [NormedAddCommGroup H] [InnerProductSpace ℂ H] {σ : Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ → (H →L[ℂ] H)} (h : IsDSHpk k r ε lam σ) : ∃ w : H, IsLowUnitH k r ε lam σ w

theorem **kfin_smul** {H : Type} [NormedAddCommGroup H] [InnerProductSpace ℂ H] {σ : Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ → (H →L[ℂ] H)} {y : H} (hy : IsKFinHpk (k

theorem **rankOne_core** (hKD : Pv.AmpDD.KDense) (L : LocBessel k r) {κ₀ h : ℝ} (hκ₀ : 0 < κ₀) (hc : L.c = cOfDpsi (κ₀ : ℂ)) (h0 : L.κN ≠ 0) (h1 : L.κN ≠ ⊤) (hg : Formalism.QuantumDeterminants.GapAtLeast L.ε L.lam h) (hO : ThreeDimData.GapOK k r h) (hC : Pv.AmpDD.GapCell k r h) : ∃ (Φ : Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ → (L.Wq →L[ℂ] L.Wp)) (Ψ : Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ → Measure (PairSp k r) → (L.Wq →L[ℂ] L.Wp)) (Λ₀ : (L.Hs →L[ℂ] L.Wp) → (L.Wq →L[ℂ] L.Hs) → ℂ) (Λ : (L.Hs →L[ℂ] L.Wp) → (L.Wq →L[ℂ] L.Hs) → ℝ → ℝ → ℂ), (∀ P Q, IsKFinPair L P Q → HasBData L (coefH L P Q) (Λ₀ P Q) Φ Ψ) ∧ (∀ P Q (κ τ : ℝ), 0 < κ → 0 < τ → IsKFinPair L P Q → HasBData L (fun x => coefH L P Q (cj k r κ τ x)) (Λ P Q κ τ) Φ Ψ) ∧ (∀ (κ τ : ℝ), 0 < κ → 0 < τ → ∀ P Q, IsKFinPair L P Q → Λ₀ P Q ≠ 0 → ∃ P₁ Q₁, IsKFinPair L P₁ Q₁ ∧ Λ P₁ Q₁ κ τ ≠ 0)

theorem **rankOneTor_of_kDense** (hKD : Pv.AmpDD.KDense) (D : D7) {Cgf : ℝ} (hC : 100 ≤ Cgf) : RankOneTor D Cgf

theorem **k_finite_dense** : Pv.AmpDD.KDense

theorem **rank_one_torus_bound** (D : D7) {Cgf : ℝ} (hC : 100 ≤ Cgf) : RankOneTor D Cgf

theorem **one_bessel_torus_bound** (D : D7) {Cgf : ℝ} (hC : 100 ≤ Cgf) : OneBesselTor D Cgf

CompanionTheory.AutomorphicTheory.RankOneTori.KDense

theorem **int_smul** {V : Type} [NormedAddCommGroup V] [NormedSpace ℝ V] (μ : Measure Γ) [IsFiniteMeasure μ] (p : C(Γ, ℝ)) {F : Γ → V} (hF : Continuous F) : Integrable (fun x => p x • F x) μ

theorem **approx_id** {V : Type} [NormedAddCommGroup V] [NormedSpace ℝ V] [CompleteSpace V] (μ : Measure Γ) [IsFiniteMeasure μ] [IsOpenPosMeasure μ] (A : Subalgebra ℝ C(Γ, ℝ)) (hA : A.SeparatesPoints) {ι : Type} [Fintype ι] {F : ι → Γ → V} (hF : ∀ i, Continuous (F i)) {δ : ℝ} (hδ : 0 < δ) : ∃ p ∈ A, ∀ i, ‖F i 1 − ∫ x, p x • F i x ∂μ‖ < δ

theorem **finAlg_sep** : (finAlg (MG K hK)).SeparatesPoints

theorem **dsW_int** (hG : IsMatGroup G) (hKG : K ≤ G) (h : RepOn G σ) {v : H} (hv : IsSm G σ v) (μ : Measure (MG K hK)) [IsFiniteMeasure μ] (p : C(MG K hK, ℝ)) : ∀ {W : List (Mat n)}, LieW G W → dsW σ W (∫ g, p g • σ g.mat v ∂μ) = ∫ g, p g • dsW σ W (σ g.mat v) ∂μ | [], _ => rfl | X :: W, hXW => by have hW : LieW G W

theorem **fin_of_finAlg** (hG : IsMatGroup G) (hKG : K ≤ G) (h : RepOn G σ) {v : H} (hv : IsSm G σ v) {p : C(MG K hK, ℝ)} (hp : p ∈ finAlg (MG K hK)) : ∃ S : Submodule ℂ H, FiniteDimensional ℂ S ∧ (∫ g, p g • σ g.mat v ∂haar) ∈ S ∧ ∀ a ∈ K, ∀ x ∈ S, σ a x ∈ S

theorem **kDense** (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (lam : Fin (2 * r + k + 1) → ℝ) (H : Type) [NormedAddCommGroup H] [InnerProductSpace ℂ H] [CompleteSpace H] (σ : Mat (2 * r + k + 1) → (H →L[ℂ] H)) (hlam : Formalism.QuantumDeterminants.IsHCParam lam) (hσ : IsDSHpk k r ε lam σ) (v : H) (hv : v ∈ smoothVecs (k

CompanionTheory.AutomorphicTheory.RankOneTori.MatGroupOf

theorem **matGroup_of** {U : Set (Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ)} (h : B2.IsClosedSubgroup U) : PI.IsMatGroup U

theorem **lr_cont** (a b : Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ) : Continuous (lr (k

theorem **lr_lr** {γ : Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ} (hγ : IsUnit γ.det) (x : Mt k r) : lr γ γ⁻¹ (lr γ⁻¹ γ x) = x

theorem **lr_lr'** {γ : Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ} (hγ : IsUnit γ.det) (x : Mt k r) : lr γ⁻¹ γ (lr γ γ⁻¹ x) = x

theorem **quot_exists** {γ : Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ} (hγ : γ ∈ UH (Hpk k r)) {μG μR : Measure (Mt k r)} (hG : IsHaarOn (UH (Hpk k r)) μG) (hR : IsHaarOn (RStab k r γ) μR) : ∃ νR, PI.BesselPeriodGGP.IsQuotMeas μG (μR.map (fun x => fun i j => (γ⁻¹ * Matrix.of x * γ) i j)) νR

theorem **cell_big** (hk : 1 ≤ k) (L : LocBessel k r) (h0 : L.κN ≠ 0) (h1 : L.κN ≠ ⊤) {h : ℝ} (hC : Pv.AmpDD.GapCell k r h) (c : Fin r → ℂ) (μ : Matrix (Fin (k + 1)) (Fin (k + 1)) ℂ) (hc : ∀ a, c a ≠ 0) (hμ : μᴴ * Hb k * μ = Hb k) : Cell L (ThreeDimData.aOf r h − 1) (bigCell k r c μ) (Measure.dirac (oneM k r)) (pairMeas L)

theorem **root_haar** (hk : 1 ≤ k) (L : LocBessel k r) (hc : L.c ≠ 0) {γ : Point ℂ (r + 1)} {g : Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ} {μR : Measure (Mt k r)} (hrep : IsPtRep L γ g) (hroot : IsRootMeas L γ g μR) : IsHaarOn (RStab k r g) μR

theorem **cell_small** (L : LocBessel k r) (hc : L.c ≠ 0) (h1 : L.κN ≠ ⊤) {h : ℝ} (hO : ThreeDimData.GapOK k r h) (hC : Pv.AmpDD.GapCell k r h) {γ : Point ℂ (r + 1)} {g : Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ} {μR : Measure (Mt k r)} {ν : Measure (PairSp k r)} (hG : ThB.Gpt (Tz k r L.z) (Tz k r L.z') γ ≠ 0) (hrep : IsPtRep L γ g) (hrel : IsRelStab L g) (hroot : IsRootMeas L γ g μR) (hq : IsOrbQuot L g μR ν) : Cell L (ThreeDimData.aOf r h − 1) g μR ν

CompanionTheory.AutomorphicTheory.RankOneTori.NzTranslate

theorem **dsigW_conj** (hσ : IsURepOn (UH (Hpk k r)) σ) {t : Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ} (ht : t ∈ UH (Hpk k r)) : ∀ ws : List (Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ), (∀ X ∈ ws, PI.UpqRep.InLieHpk k r X) → ∀ u : H, PI.UpqRep.dsigW (k

theorem **nz_translate** (hKD : Pv.AmpDD.KDense) (hlam : IsHCParam lam) (hσ : PI.UpqRep.IsDSHpk k r ε lam σ) {Wp : Type} [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] {z : Fin (2 * r + k + 1) → ℂ} {ψ : Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ → ℂ} {π : Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ → (Wp →L[ℂ] Wp)} {Bm : H → Wp} (hB : PI.BesselMapUniq.IsBesselMapSm (k

theorem **side_nz** (hKD : Pv.AmpDD.KDense) {h : ℝ} (hlam : IsHCParam lam) (hσ : PI.UpqRep.IsDSHpk k r ε lam σ) (hg : Formalism.QuantumDeterminants.GapAtLeast ε lam h) (hO : ThreeDimData.GapOK k r h) {w0 : H} (hw0 : PI.UpqRep.IsLowUnitH k r ε lam σ w0) {Wp : Type} [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] {z : Fin (2 * r + k + 1) → ℂ} (hz : inWb k r z) (hT : 0 < Tz k r z) {c : ℂ} (hc : c ≠ 0) {u : Fin k → ℝ} {π : Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ → (Wp →L[ℂ] Wp)} (hπ : IsHCTypeW k r z u π) {μW : Measure (Mt k r)} (hW : IsHaarOn (UWset k r z) μW) (hW1 : μW Set.univ = 1) {κ : ℝ≥0∞} (h0 : κ ≠ 0) (h1 : κ ≠ ⊤) {l : H → Wp} (hl : ∀ w ∈ PI.UpqRep.smoothVecs (k

CompanionTheory.AutomorphicTheory.RankOneTori.RpowNegLe

theorem **rpow_neg_le** {x y C b : ℝ} (hx : 0 < x) (hy : 0 < y) (hC : 0 < C) (hb : 0 ≤ b) (h : y ≤ C * x) : x ^ (−b) ≤ C ^ b * y ^ (−b)

theorem **exists_pos** (hk : 1 ≤ k) (L : LocBessel k r) (c : Fin r → ℂ) (hc : ∀ a, c a ≠ 0) : ∃ μ₁ : Matrix (Fin (k + 1)) (Fin (k + 1)) ℂ, μ₁ᴴ * Hb k * μ₁ = Hb k ∧ C2.PosPt k r L.z L.z' (bigCell k r c μ₁) ∧ C2.IsMonomial (Formalism.QuantumDeterminants.Zhat k r L.z L.z' (bigCell k r c μ₁))

theorem **big_bound** (hk : 1 ≤ k) (L : LocBessel k r) (c : Fin r → ℂ) (μ : Matrix (Fin (k + 1)) (Fin (k + 1)) ℂ) (hc : ∀ a, c a ≠ 0) (hμ : μᴴ * Hb k * μ = Hb k) (b : ℝ) (hb : 0 ≤ b) : ∃ K : ℝ, 0 < K ∧ ∀ x : PairSp k r, Good L x → Pv.AmpDD.pairMaj (bigCell k r c μ) b x ≤ K * (B.gN (Matrix.of x.1.2 * Matrix.of x.1.1) ^ (−(b / (2 * (5 * (r + 1) ^ 2 : ℕ)))) * B.gN (Matrix.of x.2.2 * Matrix.of x.2.1) ^ (−(b / (2 * (5 * (r + 1) ^ 2 : ℕ))))))

theorem **big_int** (hk : 1 ≤ k) (L : LocBessel k r) (h0 : L.κN ≠ 0) (h1 : L.κN ≠ ⊤) (c : Fin r → ℂ) (μ : Matrix (Fin (k + 1)) (Fin (k + 1)) ℂ) (hc : ∀ a, c a ≠ 0) (hμ : μᴴ * Hb k * μ = Hb k) {b : ℝ} (hb : 10 * ((r : ℝ) + 1) ^ 2 * ThreeDimData.AG k r < b) : Integrable (Pv.AmpDD.pairMaj (bigCell k r c μ) b) (pairMeas L)

CompanionTheory.AutomorphicTheory.SchurLimitBounds.SchurOrthogonality.SchurH

theorem **exists_eq_smul_haar** {G : Type*} [Group G] [TopologicalSpace G] [IsTopologicalGroup G] [LocallyCompactSpace G] [SecondCountableTopology G] [MeasurableSpace G] [BorelSpace G] (ν μ : Measure G) [SigmaFinite ν] [ν.IsMulLeftInvariant] (hν : ν ≠ 0) [μ.IsHaarMeasure] : ∃ c : ℝ≥0∞, c ≠ 0 ∧ c ≠ ∞ ∧ ν = c • μ

theorem **inv_mul_P** (hP : IsCayley k r ε P) : P⁻¹ * P = 1

theorem **P_mul_inv** (hP : IsCayley k r ε P) : P * P⁻¹ = 1

theorem **back_forth** (hP : IsCayley k r ε P) (x : Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ) : P⁻¹ * (P * x * P⁻¹) * P = x

theorem **forth_back** (hP : IsCayley k r ε P) (x : Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ) : P * (P⁻¹ * x * P) * P⁻¹ = x

theorem **back_mul** (hP : IsCayley k r ε P) (a b : Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ) : P⁻¹ * (a * b) * P = (P⁻¹ * a * P) * (P⁻¹ * b * P)

theorem **forth_mul** (hP : IsCayley k r ε P) (a b : Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ) : P * (a * b) * P⁻¹ = (P * a * P⁻¹) * (P * b * P⁻¹)

theorem **back_mem** (hP : IsCayley k r ε P) {g : Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ} (hg : g ∈ UJset ε) : P⁻¹ * g * P ∈ UH (Hpk k r)

theorem **forth_mem** (hP : IsCayley k r ε P) {x : Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ} (hx : x ∈ UH (Hpk k r)) : P * x * P⁻¹ ∈ UJset ε

theorem **umat_mem** (g : UJ ε) : umat g ∈ UJset ε

theorem **iota_mem** (hP : IsCayley k r ε P) (g : UJ ε) : Matrix.of (iota P g) ∈ UH (Hpk k r)

theorem **exists_iota_eq** (hP : IsCayley k r ε P) {x : Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ} (hx : x ∈ UH (Hpk k r)) : ∃ g : UJ ε, Matrix.of (iota P g) = x

theorem **iota_mul** (hP : IsCayley k r ε P) (g h : UJ ε) : Matrix.of (iota P (g * h)) = Matrix.of (iota P g) * Matrix.of (iota P h)

theorem **iota_one** (hP : IsCayley k r ε P) : Matrix.of (iota P (1 : UJ ε)) = 1

theorem **iota_closedEmb** (hP : IsCayley k r ε P) : IsClosedEmbedding (iota (ε

theorem **iota_measEmb** (hP : IsCayley k r ε P) : MeasurableEmbedding (iota (ε

theorem **exists_map_iota** (hP : IsCayley k r ε P) {μG : Measure (Formalism.QuantumDeterminants.Mt k r)} (hμ : PI.BesselPeriodGGP.IsHaarOn (Formalism.QuantumDeterminants.UH (Formalism.QuantumDeterminants.Hpk k r)) μG) : ∃ c : ℝ≥0∞, c ≠ 0 ∧ c ≠ ∞ ∧ μG = Measure.map (iota (ε

theorem **toURep_irred** (hP : IsCayley k r ε P) (σ : Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ → (H →L[ℂ] H)) (hσ : IsURepOn (UH (Hpk k r)) σ) [Nontrivial H] (hi : IsIrredOn (UH (Hpk k r)) σ) : (toURep hP σ hσ).IsIrreducible

theorem **isURepOn_of_ds** {lam : Fin (2 * r + k + 1) → ℝ} {σ : Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ → (H →L[ℂ] H)} (h : IsDSHpk k r ε lam σ) : IsURepOn (UH (Hpk k r)) σ

theorem **isIrredOn_of_ds** {lam : Fin (2 * r + k + 1) → ℝ} {σ : Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ → (H →L[ℂ] H)} (h : IsDSHpk k r ε lam σ) : IsIrredOn (UH (Hpk k r)) σ

theorem **memLp_of_ds** {lam : Fin (2 * r + k + 1) → ℝ} {σ : Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ → (H →L[ℂ] H)} (h : IsDSHpk k r ε lam σ) {μG : Measure (Formalism.QuantumDeterminants.Mt k r)} (hμ : PI.BesselPeriodGGP.IsHaarOn (Formalism.QuantumDeterminants.UH (Formalism.QuantumDeterminants.Hpk k r)) μG) (v w : H) : MemLp (fun x : Formalism.QuantumDeterminants.Mt k r => ⟪w, σ (Matrix.of x) v⟫_ℂ) 2 μG

theorem **threeDimData_schurH** : Formalism.FourierDecayAlt.ThreeDimData.SchurH

theorem **schur_inequiv** {k r : ℕ} {ε ε' : Fin (2 * r + k + 1) → Bool} {lam lam' : Fin (2 * r + k + 1) → ℝ} {H H' : Type*} [NormedAddCommGroup H] [InnerProductSpace ℂ H] [CompleteSpace H] [NormedAddCommGroup H'] [InnerProductSpace ℂ H'] [CompleteSpace H'] {σ : Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ → (H →L[ℂ] H)} {σ' : Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ → (H' →L[ℂ] H')} (hds : IsDSHpk k r ε lam σ) (hds' : IsDSHpk k r ε' lam' σ') (hne : ¬ ∃ U : H ≃ₗᵢ[ℂ] H', ∀ g ∈ UH (Hpk k r), ∀ v, U (σ g v) = σ' g (U v)) {μG : Measure (Formalism.QuantumDeterminants.Mt k r)} (hμ : PI.BesselPeriodGGP.IsHaarOn (Formalism.QuantumDeterminants.UH (Formalism.QuantumDeterminants.Hpk k r)) μG) (v w : H) (v' w' : H') : Integrable (fun x : Formalism.QuantumDeterminants.Mt k r => ⟪w, σ (Matrix.of x) v⟫_ℂ * star ⟪w', σ' (Matrix.of x) v'⟫_ℂ) μG ∧ ∫ x, ⟪w, σ (Matrix.of x) v⟫_ℂ * star ⟪w', σ

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theorem **exists_to_L2** (Φ : X → (H →L[ℂ] ℂ)) (hΦ : ∀ v, MemLp (fun x => Φ x v) 2 μ) : ∃ T : H →L[ℂ] Lp ℂ 2 μ, ∀ v, (T v : X → ℂ) =ᵐ[μ] fun x => Φ x v

theorem **exists_pairing_op** (Φ : X → (H →L[ℂ] ℂ)) (hΦ : ∀ v, MemLp (fun x => Φ x v) 2 μ) (Ψ : X → (K →L[ℂ] ℂ)) (hΨ : ∀ v, MemLp (fun x => Ψ x v) 2 μ) : ∃ A : H →L[ℂ] K, ∀ v v', ∫ x, Φ x v * star (Ψ x v') ∂μ = ⟪v', A v⟫_ℂ

theorem **integrable_pairing** {π : UnitaryRep G H} {σ : UnitaryRep G K} (hπ : SqInt μ π) (hσ : SqInt μ σ) (v w : H) (v' w' : K) : Integrable (fun g => ⟪w, π g v⟫_ℂ * star ⟪w', σ g v'⟫_ℂ) μ

theorem **integral_mul_right** (f : G → ℂ) (h : G) : ∫ g, f (g * h) ∂μ = (Measure.modularCharacterFun h : ℝ) * ∫ g, f g ∂μ

theorem **pairing_right** (π : UnitaryRep G H) (σ : UnitaryRep G K) (v w : H) (v' w' : K) (h : G) : pairing μ π σ (π h v) w (σ h v') w' = (Measure.modularCharacterFun h : ℝ) * pairing μ π σ v w v' w'

theorem **pairing_left** (π : UnitaryRep G H) (σ : UnitaryRep G K) (v w : H) (v' w' : K) (h : G) : pairing μ π σ v (π h w) v' (σ h w') = pairing μ π σ v w v' w'

theorem **pairing_self** (π : UnitaryRep G H) (v w : H) : pairing μ π π v w v w = ((∫ g, ‖⟪w, π g v⟫_ℂ‖ ^ 2 ∂μ : ℝ) : ℂ)

theorem **integral_sq_pos** {π : UnitaryRep G H} (hπ : SqInt μ π) {v : H} (hv : v ≠ 0) : 0 < ∫ g, ‖⟪v, π g v⟫_ℂ‖ ^ 2 ∂μ

theorem **exists_op_right** {π : UnitaryRep G H} {σ : UnitaryRep G K} (hπ : SqInt μ π) (hσ : SqInt μ σ) (w : H) (w' : K) : ∃ A : H →L[ℂ] K, ∀ v v', pairing μ π σ v w v' w' = ⟪v', A v⟫_ℂ

theorem **exists_op_left** {π : UnitaryRep G H} {σ : UnitaryRep G K} (hπ : SqInt μ π) (hσ : SqInt μ σ) (v : H) (v' : K) : ∃ A ∈ π.intertwiners σ, ∀ w w', pairing μ π σ v w v' w' = ⟪A w, w'⟫_ℂ

theorem **modular_eq_one** {π : UnitaryRep G H} [Nontrivial H] (hπ : SqInt μ π) (h : G) : Measure.modularCharacterFun h = 1

theorem **exists_op_right'** {π : UnitaryRep G H} {σ : UnitaryRep G K} [Nontrivial H] (hπ : SqInt μ π) (hσ : SqInt μ σ) (w : H) (w' : K) : ∃ A ∈ π.intertwiners σ, ∀ v v', pairing μ π σ v w v' w' = ⟪v', A v⟫_ℂ

theorem **schur_orthogonality** {π : UnitaryRep G H} (hirr : π.IsIrreducible) (hπ : SqInt μ π) : ∃ d : ℝ, 0 < d ∧ ∀ v w v' w', H, pairing μ π π v w v' w' = ⟪v', v⟫_ℂ * ⟪w, w'⟫_ℂ / (d : ℂ)

theorem **pairing_eq_zero** {π : UnitaryRep G H} {σ : UnitaryRep G K} (hπi : π.IsIrreducible) (hσi : σ.IsIrreducible) (hπ : SqInt μ π) (hσ : SqInt μ σ) (hne : ¬ π.IsEquivalent σ) (v w : H) (v' w' : K) : pairing μ π σ v w v' w' = 0

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lemma **coe_apply** (g : G) : π g = (π.toHom g : H →L[ℂ] H)

lemma **continuous_orbit** (v : H) : Continuous fun g => π g v

@[simp] lemma **map_one** : π 1 = 1

@[simp] lemma **map_mul** (g h : G) : π (g * h) = π g * π h

lemma **map_mul_apply** (g h : G) (v : H) : π (g * h) v = π g (π h v)

@[simp] lemma **map_one_apply** (v : H) : π 1 v = v

@[simp] lemma **norm_map** (g : G) (v : H) : ‖π g v‖ = ‖v‖

@[simp] lemma **inner_map_map** (g : G) (v w : H) : ⟪π g v, π g w⟫_ℂ = ⟪v, w⟫_ℂ

lemma **mem_unitary** (g : G) : π g ∈ unitary (H →L[ℂ] H)

@[simp] lemma **star_map** (g : G) : star (π g) = π g⁻¹

@[simp] lemma **adjoint_map** (g : G) : adjoint (π g) = π g⁻¹

@[simp] lemma **map_inv_apply_map** (g : G) (v : H) : π g⁻¹ (π g v) = v

@[simp] lemma **map_apply_map_inv** (g : G) (v : H) : π g (π g⁻¹ v) = v

lemma **inner_map_left** (g : G) (v w : H) : ⟪π g v, w⟫_ℂ = ⟪v, π g⁻¹ w⟫_ℂ

lemma **inner_map_right** (g : G) (v w : H) : ⟪v, π g w⟫_ℂ = ⟪π g⁻¹ v, w⟫_ℂ

lemma **norm_map_le** (g : G) : ‖π g‖ ≤ 1

lemma **map_injective** (g : G) : Function.Injective (π g)

@[simp] lemma **ofLinearIsometryEquiv_apply** (ρ : G →* (H ≃ₗᵢ[ℂ] H)) (hρ : ∀ v, Continuous fun g => ρ g v) (g : G) (v : H) : ofLinearIsometryEquiv ρ hρ g v = ρ g v

theorem **continuous_of_weak** (ρ : G →* unitary (H →L[ℂ] H)) (hρ : ∀ v w : H, Continuous fun g => ⟪(ρ g : H →L[ℂ] H) v, w⟫_ℂ) (v : H) : Continuous fun g => (ρ g : H →L[ℂ] H) v

theorem **continuous_of_continuousAt_one** [IsTopologicalGroup G] (ρ : G →* unitary (H →L[ℂ] H)) (hρ : ∀ v : H, ContinuousAt (fun g => (ρ g : H →L[ℂ] H) v) 1) (v : H) : Continuous fun g => (ρ g : H →L[ℂ] H) v

theorem **continuous_action** : Continuous fun p : G × H => π p.1 p.2

lemma **continuous_inner_orbit** (v w : H) : Continuous fun g => ⟪π g v, w⟫_ℂ

lemma **isInvariant_bot** : π.IsInvariant ⊥

lemma **isInvariant_top** : π.IsInvariant ⊤

theorem **IsInvariant.orthogonal** {V : Submodule ℂ H} (hV : π.IsInvariant V) : π.IsInvariant Vᗮ

theorem **IsInvariant.topologicalClosure** {V : Submodule ℂ H} (hV : π.IsInvariant V) : π.IsInvariant V.topologicalClosure

theorem **isInvariant_iff_commute_starProjection** (V : Submodule ℂ H) [V.HasOrthogonalProjection] : π.IsInvariant V ↔ ∀ g, Commute (π g) V.starProjection

lemma **mem_intertwiners** {T : H →L[ℂ] K} : T ∈ π.intertwiners σ ↔ ∀ (g : G) (v : H), T (π g v) = σ g (T v)

lemma **mem_intertwiners_iff_comp** {T : H →L[ℂ] K} : T ∈ π.intertwiners σ ↔ ∀ g : G, T ∘L π g = σ g ∘L T

lemma **one_mem_intertwiners** : (1 : H →L[ℂ] H) ∈ π.intertwiners π

lemma **comp_mem_intertwiners** {S : K →L[ℂ] L} {T : H →L[ℂ] K} (hS : S ∈ σ.intertwiners τ) (hT : T ∈ π.intertwiners σ) : S ∘L T ∈ π.intertwiners τ

lemma **mul_mem_intertwiners** {S T : H →L[ℂ] H} (hS : S ∈ π.intertwiners π) (hT : T ∈ π.intertwiners π) : S * T ∈ π.intertwiners π

theorem **adjoint_mem_intertwiners** {T : H →L[ℂ] K} (hT : T ∈ π.intertwiners σ) : adjoint T ∈ σ.intertwiners π

lemma **star_mem_intertwiners** {T : H →L[ℂ] H} (hT : T ∈ π.intertwiners π) : star T ∈ π.intertwiners π

lemma **commute_of_mem_intertwiners** {T : H →L[ℂ] H} (hT : T ∈ π.intertwiners π) (g : G) : Commute T (π g)

lemma **mem_intertwiners_self_iff** {T : H →L[ℂ] H} : T ∈ π.intertwiners π ↔ ∀ g, Commute T (π g)

lemma **isInvariant_range** {T : H →L[ℂ] K} (hT : T ∈ π.intertwiners σ) : σ.IsInvariant (LinearMap.range (T : H →ₗ[ℂ] K))

lemma **isInvariant_ker** {T : H →L[ℂ] K} (hT : T ∈ π.intertwiners σ) : π.IsInvariant (LinearMap.ker (T : H →ₗ[ℂ] K))

lemma **mem_intertwiners** (e : π.Equiv σ) : (e.toLinearIsometryEquiv : H →L[ℂ] K) ∈ π.intertwiners σ

lemma **IsEquivalent.refl** (π : UnitaryRep G H) : π.IsEquivalent π

lemma **IsEquivalent.symm** (h : π.IsEquivalent σ) : σ.IsEquivalent π

lemma **IsEquivalent.trans** (h : π.IsEquivalent σ) (h' : σ.IsEquivalent τ) : π.IsEquivalent τ

theorem **IsIrreducible.of_equiv** (hπ : π.IsIrreducible) (e : π.Equiv σ) : σ.IsIrreducible

theorem **IsEquivalent.isIrreducible_iff** (h : π.IsEquivalent σ) : π.IsIrreducible ↔ σ.IsIrreducible

@[simp] lemma **coe_restrict_apply** (g : G) (v : V) : ((π.restrict V hV g v : V) : H) = π g v

lemma **subtypeL_mem_intertwiners** : V.subtypeL ∈ (π.restrict V hV).intertwiners π

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lemma **nontrivial_endomorphisms** [Nontrivial H] : Nontrivial (H →L[ℂ] H)

theorem **IsIrreducible.spectrum_subsingleton** (hπ : π.IsIrreducible) {A : H →L[ℂ] H} (hA : A ∈ π.intertwiners π) (hsa : IsSelfAdjoint A) : (spectrum ℝ A).Subsingleton

theorem **IsIrreducible.exists_eq_algebraMap_of_isSelfAdjoint** (hπ : π.IsIrreducible) {A : H →L[ℂ] H} (hA : A ∈ π.intertwiners π) (hsa : IsSelfAdjoint A) : ∃ c : ℝ, A = algebraMap ℝ (H →L[ℂ] H) c

theorem **IsIrreducible.exists_eq_smul_one** (hπ : π.IsIrreducible) {T : H →L[ℂ] H} (hT : T ∈ π.intertwiners π) : ∃ c : ℂ, T = c • (1 : H →L[ℂ] H)

theorem **isIrreducible_iff** : π.IsIrreducible ↔ Nontrivial H ∧ ∀ T ∈ π.intertwiners π, ∃ c : ℂ, T = c • (1 : H →L[ℂ] H)

theorem **IsIrreducible.isEquivalent_of_mem_intertwiners** (hπ : π.IsIrreducible) (hσ : σ.IsIrreducible) {T : H →L[ℂ] K} (hT : T ∈ π.intertwiners σ) (hT0 : T ≠ 0) : π.IsEquivalent σ

theorem **IsIrreducible.eq_zero_of_not_isEquivalent** (hπ : π.IsIrreducible) (hσ : σ.IsIrreducible) (hne : ¬ π.IsEquivalent σ) {T : H →L[ℂ] K} (hT : T ∈ π.intertwiners σ) : T = 0

CompanionTheory.AutomorphicTheory.SchurLimitBounds.ZuckermanDuality.Assemble

theorem **norm_capP_le** {n : ℕ} {lam μ : Fin n → ℝ} {c : ℝ} (h : μ ∈ Box lam c) (y : ℝ) : ‖Formalism.QuantumDeterminants.capP (fun s => (μ s : ℂ)) (y : ℂ)‖ ≤ PStar lam c y

theorem **gap_of_box** {n : ℕ} {ε : Fin n → Bool} {lam μ : Fin n → ℝ} {c h h₀ : ℝ} (hμ : μ ∈ Box lam c) (hg : PI.ZuckTensor.GapAtLeast ε lam h) (hh : h₀ + 2 * c ≤ h) : PI.ZuckTensor.GapAtLeast ε μ h₀

private theorem **ckOn_GV_of_mem_Wsp** (hK : PI.KFinSmooth) (hS : CoefSmooth) {F : B.Mat k r → (Wq →L[ℂ] Wp)} (hF : F ∈ Wsp k r Wp Wq ε μ) : B.CkOn ⊤ (B.GV k r) F

theorem **wsp_smooth** (hK : PI.KFinSmooth) (hS : CoefSmooth) {F : B.Mat k r → (Wq →L[ℂ] Wp)} (hF : F ∈ Wsp k r Wp Wq ε μ) : Q.SmoothLev k r 0 Set.univ F

theorem **wsp_ckOn** (hK : PI.KFinSmooth) (hS : CoefSmooth) {F : B.Mat k r → (Wq →L[ℂ] Wp)} (hF : F ∈ Wsp k r Wp Wq ε μ) : B.CkOn ⊤ (B.GV k r) F

theorem **wsp_inCC** (hK : PI.KFinSmooth) (hC : CoefCC) {A h : ℝ} (hD : KDecay k r ε A h) (hg : PI.ZuckTensor.GapAtLeast ε μ h) {F : B.Mat k r → (Wq →L[ℂ] Wp)} (hF : F ∈ Wsp k r Wp Wq ε μ) : B.InCC A 5 F

theorem **wsp_eig** (hK : PI.KFinSmooth) (hS : CoefSmooth) (hE : CoefEig) (Y : ℂ) {F : B.Mat k r → (Wq →L[ℂ] Wp)} (hF : F ∈ Wsp k r Wp Wq ε μ) : ∀ g ∈ B.GV k r, Q.CV k r Y F g = Formalism.QuantumDeterminants.capP (fun s => (μ s : ℂ)) Y • F g

theorem **wsp_orth** (hK : PI.KFinSmooth) (hC : CoefCC) (hE : CoefEig) (hP : PhZero) (hB : BetaInfZero) (D : B.RealPlaceData k r Wp Wq) {A h : ℝ} (hA : B.AB k r ≤ A) (hD : KDecay k r ε A h) (hg : PI.ZuckTensor.GapAtLeast ε μ h) (hg' : PI.ZuckTensor.GapAtLeast ε μ' h) (hne : μ ≠ μ') {F F' : B.Mat k r → (Wq →L[ℂ] Wp)} (hF : F ∈ Wsp k r Wp Wq ε μ) (hF' : F' ∈ Wsp k r Wp Wq ε μ') : B.beta D F F' = 0

theorem **smul_mem_iSup** {lam : Fin (2 * r + k + 1) → ℝ} {e : ℝ} {ψ : B.Mat k r → ℂ} (hψ : ψ ∈ SpanBox k r ε lam e) (T : Wq →L[ℂ] Wp) : (fun g => ψ g • T) ∈ ⨆ ν : Box lam e, Wsp k r Wp Wq ε ν

theorem **op_expand** (S : Wq →L[ℂ] Wp) : S = ∑ a, ∑ b, ‹stdOrthonormalBasis ℂ Wp a, S (stdOrthonormalBasis ℂ Wq b)›_ℂ • (innerSL ℂ (stdOrthonormalBasis ℂ Wq b)).smulRight (stdOrthonormalBasis ℂ Wp a)

theorem **eigStr_of_steps** [CompleteSpace Wp] (h1 : GfPoly) (h2 : SpanPow) (h3 : CoefSmooth) (h4 : CoefCC) (h5 : CoefEig) (h6 : PhZero) (h7 : BetaInfZero) (hK : PI.KFinSmooth) {t : ∀ n : ℕ, (Fin n → Bool) → ℝ} (hZ : ZuckT t) (hZd : ZuckDualT t) (p : KB.Pl P Wp Wq) (f : B.Mat P.k P.r → (Wq →L[ℂ] Wp)) (p₀ : ℕ) (nn : ℝ → ℝ) (Y : Finset ℝ) {ε : Fin (2 * P.r + P.k + 1) → Bool} {lam : Fin (2 * P.r + P.k + 1) → ℝ} {A h h₀ : ℝ} (hf : KFinDSCoef ε lam f) (hgap : PI.ZuckTensor.GapAtLeast ε lam h) (ht : t _ ε + 1 + 2 * (((2 * P.r + 2) * p₀ : ℕ) : ℝ) < h) (hA : B.AB P.k P.r ≤ A) (hD : KDecay P.k P.r ε A h₀) (hh : h₀ + 2 * (((2 * P.r + 2) * p₀ : ℕ) : ℝ) ≤ h) (hnn : ∀ y, LetPt Y y → PStar lam (((2 * P.r + 2) * p₀ : ℕ) : ℝ) y ≤ nn y) : EigStr p nn (TBBx4.fpm p f (fun _ => ContinuousLinearMap.id ℂ Wq) p₀) Y

CompanionTheory.AutomorphicTheory.SchurLimitBounds.ZuckermanDuality.BetaAtInfinityZero

theorem **beta_at_infinity_zero** : BetaInfZero

theorem **inner_translate** {k r : ℕ} {H : Type} [NormedAddCommGroup H] [InnerProductSpace ℂ H] {σ : B.Mat k r → (H →L[ℂ] H)} (hσ : IsURepOn (UH (Hpk k r)) σ) {a g b : B.Mat k r} (ha : a ∈ UH (Hpk k r)) (hg : g ∈ UH (Hpk k r)) (hb : b ∈ UH (Hpk k r)) (v w : H) : ‹w, σ (a * g * b) v›_ℂ = ‹σ a⁻¹ w, σ g (σ b v)›_ℂ

theorem **phase_zero_alt** : PhZero

CompanionTheory.AutomorphicTheory.SchurLimitBounds.ZuckermanDuality.GeneratingFunctionPolynomial

theorem **one_mem_letters** : (1 : B.Mat k r → ℂ) ∈ Letters k r

theorem **letters_pow_succ** {d : ℕ} {q : B.Mat k r → ℂ} (h : q ∈ Letters k r ^ d) : q ∈ Letters k r ^ (d + 1)

theorem **letters_pow_mono** {d e : ℕ} (hde : d ≤ e) {q : B.Mat k r → ℂ} (h : q ∈ Letters k r ^ d) : q ∈ Letters k r ^ e

theorem **prod_mem_pow** {ι : Type} (s : Finset ι) (q : ι → B.Mat k r → ℂ) (h : ∀ i ∈ s, q i ∈ Letters k r) : ∏ i ∈ s, q i ∈ Letters k r ^ s.card

theorem **det_mem_pow** {m : ℕ} (Z : B.Mat k r → Matrix (Fin m) (Fin m) ℂ) (h : ∀ a b, (fun g => Z g a b) ∈ Letters k r) : (fun g => (Z g).det) ∈ Letters k r ^ m

theorem **hpk_herm** : (Formalism.QuantumDeterminants.Hpk k r)ᴴ = Formalism.QuantumDeterminants.Hpk k r

theorem **herm_letter** (x y : Fin (2 * r + k + 1) → ℂ) : (fun g : B.Mat k r => Formalism.QuantumDeterminants.herm (Formalism.QuantumDeterminants.Hpk k r) (g *ᵥ x) y) ∈ Letters k r

theorem **herm_conj_letter** (x y : Fin (2 * r + k + 1) → ℂ) : (fun g : B.Mat k r => star (Formalism.QuantumDeterminants.herm (Formalism.QuantumDeterminants.Hpk k r) (g *ᵥ x) y)) ∈ Letters k r

theorem **dmin_mem** (z z' : Fin (2 * r + k + 1) → ℂ) (l : Fin (r + 2)) : (fun g : B.Mat k r => ((Complex.normSq (Formalism.QuantumDeterminants.Dmin k r z z' g l) : ℝ) : ℂ)) ∈ Letters k r ^ (2 * r + 2)

theorem **generating_function_polynomial** : GfPoly

CompanionTheory.AutomorphicTheory.SchurLimitBounds.ZuckermanDuality.Letter

theorem **inv_eq_ct** (hP : IsCayleyU k r ε P) : P⁻¹ = Pᴴ

theorem **inv_unitary** (hP : IsCayleyU k r ε P) : P⁻¹ ∈ Matrix.unitaryGroup (Fin (2 * r + k + 1)) ℂ

theorem **back_K** (hP : IsCayleyU k r ε P) {g : B.Mat k r} (hg : g ∈ Kset ε) : P⁻¹ * g * P ∈ UH (Hpk k r) ∩ (Matrix.unitaryGroup (Fin (2 * r + k + 1)) ℂ : Set _)

theorem **forth_K** (hP : IsCayleyU k r ε P) {x : B.Mat k r} (hx : x ∈ UH (Hpk k r) ∩ (Matrix.unitaryGroup (Fin (2 * r + k + 1)) ℂ : Set _)) : P * x * P⁻¹ ∈ Kset ε

theorem **kfin_iff** {H : Type} [NormedAddCommGroup H] [InnerProductSpace ℂ H] (hP : IsCayleyU k r ε P) (σ : B.Mat k r → (H →L[ℂ] H)) (v : H) : IsKFinHpk (k

theorem **kfin_zero** {H : Type} [NormedAddCommGroup H] [InnerProductSpace ℂ H] (σ : B.Mat k r → (H →L[ℂ] H)) : IsKFinHpk (k

theorem **isCoef_of_J** (hP : IsCayleyU k r ε P) {μ : Fin (2 * r + k + 1) → ℝ} {H : Type} [NormedAddCommGroup H] [InnerProductSpace ℂ H] [CompleteSpace H] {τ : B.Mat k r → (H →L[ℂ] H)} (hμ : IsHCParam μ) (hτ : IsDS ε μ τ) {v w : H} (hv : IsKFin ε τ v) (hw : IsKFin ε τ w) : IsCoef k r ε μ fun x => ‹w, τ (P * x * P⁻¹) v›_ℂ

theorem **mem_box_self** {n : ℕ} (μ : Fin n → ℝ) : μ ∈ Box μ 1

theorem **mem_box_add** {n : ℕ} (μ : Fin n → ℝ) (s : Fin n) : μ + Pi.single s 1 ∈ Box μ 1

theorem **mem_box_sub** {n : ℕ} (μ : Fin n → ℝ) (s : Fin n) : μ − Pi.single s 1 ∈ Box μ 1

theorem **mem_spanBox_of_eq** {μ : Fin (2 * r + k + 1) → ℝ} {φ ψ ψ' : B.Mat k r → ℂ} (hφ : IsCoef k r ε μ φ) (hψ' : ψ' ∈ SpanBox k r ε μ 1) (h : ∀ g ∈ B.GV k r, ψ g = ψ' g) : ψ ∈ SpanBox k r ε μ 1

theorem **dot_transport** (hP : IsCayleyU k r ε P) (a : B.Mat k r) (x y : Fin (2 * r + k + 1) → ℂ) : star (P *ᵥ y) ⬝ᵥ ((P * a * P⁻¹) *ᵥ (P *ᵥ x)) = star y ⬝ᵥ (a *ᵥ x)

theorem **letter_recursion** : LetterStep

theorem **mul_mem_spanBox_of_gap** (t : ∀ n : ℕ, (Fin n → Bool) → ℝ) (hZ : ZuckT t) (hZd : ZuckDualT t) (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (lam : Fin (2 * r + k + 1) → ℝ) (h : ℝ) (d : ℕ) (q φ : B.Mat k r → ℂ) (hq : q ∈ Letters k r ^ d) (hφ : IsCoef k r ε lam φ) (hg : PI.ZuckTensor.GapAtLeast ε lam h) (hlt : t _ ε + 1 + 2 * (d : ℝ) < h) : q * φ ∈ SpanBox k r ε lam (d : ℝ)

theorem **capelli_inequivalence** : Inequiv

CompanionTheory.AutomorphicTheory.SchurLimitBounds.ZuckermanDuality.LimBdFpm

theorem **span_powers** : SpanPow

CompanionTheory.AutomorphicTheory.SchurLimitBounds.ZuckermanDuality.ZuckDualTOfZuckDual

theorem **zuckT_of_zuck** (h : PI.ZuckTensor.Zuck) : ZuckT fun _ _ => 0

theorem **zuckDualT_of_zuckDual** (h : PI.ZuckTensor.ZuckDual) : ZuckDualT fun _ _ => 0

CompanionTheory.AutomorphicTheory.SegmentRepresentations.IrreducibleSegments.IsIrreduciblePiTensorSegTerm

theorem **isIrreducible_piTensor_segTerm** {k : Type} [Field k] [ValuativeRel k] [TopologicalSpace k] [IsNonarchimedeanLocalField k] {d : ℕ} (σ : IrrRep (GL (Fin d) k)) (l : ℕ) : (piTensor (segTerm σ.ρ l)).IsIrreducible

CompanionTheory.AutomorphicTheory.SegmentRepresentations.JacquetShalikaBounds.CMMul

theorem **cM_mul** (k : ℕ) (x y : Fin (k + 1) → F) : cM n k x ∗ cM n k y = 0

CompanionTheory.AutomorphicTheory.SegmentRepresentations.RapidDecayFamilies.WhittakerVanishingImpliesZero

@[simp] theorem **Rt_apply** (h g : GA E a) (φ : XB E a) : Rt E a h φ g = φ (g ∗ h)

@[simp] theorem **repW_apply** (w : Pl E) (u : GL (Fin a) (w.adicCompletion E)) (φ : XB E a) : repW a w u φ = Rt E a (fl a w u) φ

CompanionTheory.AutomorphicTheory.SiegelUnitTheory.ClassNumberMass

theorem **hU_of** (h : SiegelOneAll) (D : D7) : RT.MassRatioUnderVolume D

theorem **volSigIndep_of_hU** (h : ∀ D, RT.MassRatioUnderVolume D) (D : D7) : VolSigIndep D

theorem **volSigIndep_of** (h : SiegelOneAll) (D : D7) : VolSigIndep D

CompanionTheory.AutomorphicTheory.SiegelUnitTheory.UnitsAinf

theorem **loc** (hcm : IsCMExt F E σ) {M : ℕ} {J : Matrix (Fin M) (Fin M) E} (hJ : IsHermND σ J) (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R] (hR : IsLocR R) (hnull : ∀ (μ : Measure (Fin 1 → Fin M → R ⊗[F] E)) (ν : Measure R), μ.IsAddHaarMeasure → ν.IsAddHaarMeasure → ν {c | ¬ IsUnit c} = 0 ∧ μ {X | ¬ IsUnit (Formalism.MainNotions.GlobalTheoryTheory.GenusMassVolumes.qR R σ J (X 0))} = 0) {d : ℕ} (b : Module.Basis (Fin d) F E) (lam : Measure R) (hlam : lam.IsAddHaarMeasure) (m : HermR R σ 1 → Measure (Fin 1 → Fin M → R ⊗[F] E)) (hm : IsGramFib R σ J 1 (coordW R b lam 1 M) (coordHerm R σ b lam 1) m) (a c : ℝ≥0) (ha : a ≠ 0) (hc : c ≠ 0) : (∃ fib, Formalism.MainNotions.GlobalTheoryTheory.GenusMassVolumes.IsFibreFamily (a • coordW R b lam 1 M) (fun X => Formalism.MainNotions.GlobalTheoryTheory.GenusMassVolumes.qR R σ J (X 0)) id Set.univ (c • lam)

theorem **units_Ainf** : {c : Ainf F | ∀ w, c w ≠ 0} = {c | IsUnit c}

theorem **units_Fv** (v : HeightOneSpectrum (𝒪 F)) : {c : Fv F v | c ≠ 0} = {c | IsUnit c}

theorem **sw1b2** (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E) (hcm : Formalism.MainNotions.GlobalTheoryTheory.GenusMassVolumes.IsCMExt F E σ) (M : ℕ) (J : Matrix (Fin M) (Fin M) E) (hJ : Formalism.MainNotions.GlobalTheoryTheory.GenusMassVolumes.IsHermND σ J) : Formalism.MainNotions.StandardLattice.SW1b2 F E σ M J

CompanionTheory.AutomorphicTheory.SiegelUnitTheory.VolumeSignatureIndependent

theorem **siegel_mass_rank_one** : SiegelOneAll

theorem **mass_ratio_under_volume** : ∀ D : D7, RT.MassRatioUnderVolume D

theorem **volume_signature_independent** : ∀ D : D7, VolSigIndep D

CompanionTheory.AutomorphicTheory.SiegelWeilFiniteness.AdelicCovolumeFinite

theorem **te** : TorusEmbedding.Statement h v

theorem **le** : LineEigenvalue.Statement h v

theorem **sh** : SemidirectHaar.Statement (archUnitary h v) (InfinitePlace E → ℂˣ)

theorem **tm** : TorusModulus.Statement h v

theorem **uh** : UnipotentHorizontal.Statement h v

theorem **uz** : UnipotentCentral.Statement h v

theorem **adelicCovolumeFinite** {W : Type u} [AddCommGroup W] [Module E W] {ι : Type*} [Fintype ι] [DecidableEq ι] (h : HermitianForm F E W) (v : Basis ι E W) : AdelicCovolumeFinite.Statement h v

CompanionTheory.AutomorphicTheory.SiegelWeilFiniteness.Bridge

theorem **allSome_of** {G : Type} [Group G] [TopologicalSpace G] [IsTopologicalGroup G] [LocallyCompactSpace G] [MeasurableSpace G] [BorelSpace G] (Γ : Subgroup G) [Countable Γ] (h : ∀ (μ : Measure G) [μ.IsHaarMeasure], ∃ D : Set G, IsFundamentalDomain Γ D μ ∧ μ D < ⊤) : AllSome (fun μ : Measure G => μ.IsHaarMeasure) fun μ => AllSome (fun D : Set G => IsFundamentalDomain Γ D μ) fun D => μ D < ⊤

theorem **HR_eq** {M : ℕ} (J : Matrix (Fin M) (Fin M) E) (R : Type) [CommRing R] [Algebra F R] : HR R (conj F E).toAlgHom J = unitaryPoints F E J R

theorem **HFin_eq** {M : ℕ} (J : Matrix (Fin M) (Fin M) E) : (HFin 𝔸[F] (conj F E).toAlgHom J).subgroupOf (unitaryPoints F E J 𝔸[F]) = (unitaryDiagonal F J).range

theorem **finite_volume_unitary_adelic** {M : ℕ} (J : Matrix (Fin M) (Fin M) E) (hJ : IsUnit J.det) (h : ∀ (μ : Measure (unitaryPoints F E J 𝔸[F])) [μ.IsHaarMeasure], ∃ D : Set (unitaryPoints F E J 𝔸[F]), IsFundamentalDomain (unitaryDiagonal F J).range D μ ∧ μ D < ⊤) (S : Subgroup (GL (Fin M) (𝔸[F] ⊗[F] E))) (hS : S = unitaryPoints F E J 𝔸[F]) : letI : MeasurableSpace S

theorem **finVol_of** (hC : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] [Algebra.IsQuadraticExtension F E] [IsTotallyReal F] [IsTotallyComplex E] (M : ℕ) (h : HermitianForm F E (Fin M → E)), AdelicCovolumeFinite.Statement h (Pi.basisFun E (Fin M))) (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E) (hσ : IsCMExt F E σ) (M : ℕ) (J : Matrix (Fin M) (Fin M) E) (hJ : IsHermND σ J) : FinVol F E σ M J

CompanionTheory.AutomorphicTheory.SiegelWeilFiniteness.SiegelWeilOfFormula

theorem **finVol** (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E) (hσ : IsCMExt F E σ) (M : ℕ) (J : Matrix (Fin M) (Fin M) E) (hJ : IsHermND σ J) : FinVol F E σ M J

CompanionTheory.AutomorphicTheory.TemperedLocalSpectra.BernsteinZelevinsky.TopAdm

theorem **irreducibleAdmissibleGL_top** : GlobalTheory.Published.IrreducibleAdmissibleGL

CompanionTheory.AutomorphicTheory.TemperedLocalSpectra.DiscreteSeriesSU.GapInv

theorem **dsUnique_dec** : GlobalTheory.Dec.DSUnique

theorem **lowKMultOne_dec** : GlobalTheory.Dec.LowKMultOne

CompanionTheory.AutomorphicTheory.TemperedLocalSpectra.LocalGeometry.Imports

theorem **CompanionTheory.Geo.geometricLemmaFiltration** : GlobalTheory.Published.GeometricLemmaFiltration

CompanionTheory.AutomorphicTheory.ThetaDilations.ActWSmul

theorem **actW_smul** {F E : Type} [Field F] [Field E] [Algebra F E] (R : Type) [CommRing R] [Algebra F R] (σ : E →ₐ[F] E) {M n : ℕ} (J : Matrix (Fin M) (Fin M) E) (h : HA R σ J) (a : R ⊗[F] E) (X : Fin n → Fin M → R ⊗[F] E) : actW R σ J h (a • X) = a • actW R σ J h X

theorem **nu** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → FinVol F E σ M J → ∀ τ : Multiplicative ℝ →* (AF F ⊗[F] E)ˣ, IsFlow τ → ∃ (P : Subgroup (HA (AF F) σ (hypJ J))) (ν : Measure (HA (AF F) σ (hypJ J))) (S : ℤ → Set (HA (AF F) σ (hypJ J))) (ω : Set (AF F ⊗[F] E)ˣ) (Kc : ℝ≥0∞), (∀ h, h ∈ P ↔ ∃ a : (AF F ⊗[F] E)ˣ, actW (AF F) σ _ h (eOne _ (M + 1)) = (a : AF F ⊗[F] E) • eOne _ (M + 1)) ∧ SFinite ν ∧ (∀ p ∈ P, ν.map (fun g => p ∗ g) = ν) ∧ (∀ U ∈ 𝒩 (1 : HA (AF F) σ (hypJ J)), ν U ≠ 0) ∧ IsCompact ω ∧ Kc < ⊤ ∧ (∀ j, MeasurableSet (S j)) ∧ (∀ j : ℤ, ν (S j) ≤ Kc ∗ ENNReal.ofReal (Real.exp (-(2 ∗ Module.finrank ℚ F ∗ (M + 1) ∗ (j : ℝ))))) ∧ (∀ j : ℤ, ∀ p ∈ S j, p ∈ P ∧ ∃ c ∈ ω, actW (AF F) σ _ p (eOne _ (M + 1)) = ((τ (Multiplicative.ofAdd (j : ℝ)) ∗ c : (AF F ⊗[F]

CompanionTheory.AutomorphicTheory.ThetaDilations.Bounded

theorem **bounded** : ∀ (F : Type) [Field F] [NumberField F] [MeasurableSpace (A F)] [BorelSpace (A F)] (lf : Measure (A F)), lf.IsAddHaarMeasure → lf (Oh F) = 1 → ∀ ψ : AddChar (A F) ℂ, IsStdChar F ψ → ∀ (ι : Type) [Fintype ι] [DecidableEq ι] (M : ℕ) (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (P : InfinitePlace F → Matrix ι ι ℝ), QSetup M B ρ P → 3 ≤ M → ∀ (Ω : Type) [MeasurableSpace Ω] (Pr : Measure Ω) (gf : Ω → Matrix ι ι (A F)) (gi : Ω → InfinitePlace F → Matrix ι ι ℝ), CountData M B ρ Pr gf gi → ∀ φ : (ι → A F) → ℂ, SB φ → ∃ C : ℝ, ∀ τ : InfinitePlace F → ℂ, (∀ w, 0 < (τ w).im) → wt M τ * ‖(∫ ω, theta B ρ P (gf ω) (gi ω) φ τ ∂Pr) − eis ψ lf B ρ P φ τ‖ ≤ C

CompanionTheory.AutomorphicTheory.ThetaDilations.BoundedA

theorem **sb_indL** {F : Type} [Field F] [NumberField F] {ι : Type} [Fintype ι] {a : ℕ} (ha : 0 < a) : SB (indL (F

theorem **bounded_a** : ∀ (F : Type) [Field F] [NumberField F] [MeasurableSpace (A F)] [BorelSpace (A F)] (lf : Measure (A F)), lf.IsAddHaarMeasure → lf (Oh F) = 1 → ∀ ψ : AddChar (A F) ℂ, IsStdChar F ψ → ∀ (ι : Type) [Fintype ι] [DecidableEq ι] (M : ℕ) (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (P : InfinitePlace F → Matrix ι ι ℝ), QSetup M B ρ P → 3 ≤ M → ∀ (Ω : Type) [MeasurableSpace Ω] (Pr : Measure Ω) (gf : Ω → Matrix ι ι (A F)) (gi : Ω → InfinitePlace F → Matrix ι ι ℝ), CountData M B ρ Pr gf gi → ∃ C : ℝ, ∀ a : ℕ, 0 < a → ∀ x : InfinitePlace F → ℝ, SB (indL (F

theorem **core** : ∀ (F : Type) [Field F] [NumberField F], IsTotallyReal F → ∀ (ι : Type) [Fintype ι] [DecidableEq ι] (M : ℕ), Fintype.card ι = 2 * M → 3 ≤ M → ∀ (B : Matrix ι ι F), B.IsSymm → B.det ≠ 0 → ∀ (ρ : InfinitePlace F → (F →+* ℝ)), (∀ w x, ((ρ w x : ℝ) : ℂ) = w.embedding x) → ∀ (Pm : InfinitePlace F → Matrix ι ι ℝ), (∀ w, (Pm w).PosDef) → (∀ w, Pm w * (B.map (ρ w))⁻¹ * Pm w = B.map (ρ w)) → ∀ [MeasurableSpace (FiniteAdeleRing (𝒪 F) F)] [BorelSpace (FiniteAdeleRing (𝒪 F) F)] (lf : Measure (FiniteAdeleRing (𝒪 F) F)), lf.IsAddHaarMeasure → lf {x | ∀ v, x v ∈ v.adicCompletionIntegers F} = 1 → ∀ (Ω : Type) [MeasurableSpace Ω] (Pr : Measure Ω) [IsProbabilityMeasure Pr] (ginf : Ω → InfinitePlace F → Matrix ι ι ℝ) (gf : Ω → Matrix ι ι (FiniteAdeleRing (𝒪 F) F)), (∀ w i j, Measurable fun ω => ginf ω w i j) → (∀ i j, Measurable fun ω => gf ω i j) → (∀ ω w, (ginf ω w)ᵀ * B.map (ρ w) * ginf ω

CompanionTheory.AutomorphicTheory.ThetaDilations.ContinuousPm

theorem **exists_anti** (h : IsCMExt F E σ) : ∃ δ : E, δ ≠ 0 ∧ σ δ = -δ

theorem **norm_sigR** (h : IsCMExt F E σ) (a : R ⊗[F] E) : algebraMap R (R ⊗[F] E) (Algebra.norm R a) = a * sigR R σ a

theorem **cT_JR** {M : ℕ} {J : Matrix (Fin M) (Fin M) E} (hJ : IsHermND σ J) : cT (sigR R σ) (JR R J) = JR R J

theorem **hform_conj** {K : Type} [CommRing K] {M : ℕ} {s : K →+* K} {J : Matrix (Fin M) (Fin M) K} (hs : ∀ x, s (s x) = x) (hJ : cT s J = J) (x y : Fin M → K) : s (hform s J x y) = hform s J y x

theorem **hform_add** {K : Type} [CommRing K] {M : ℕ} (s : K →+* K) (J : Matrix (Fin M) (Fin M) K) (x y : Fin M → K) : hform s J (x + y) (x + y) = hform s J x x + hform s J x y + hform s J y x + hform s J y y

theorem **hform_U** {K : Type} [CommRing K] {M : ℕ} {s : K →+* K} {J U : Matrix (Fin M) (Fin M) K} (hU : cT s U * J * U = J) (x : Fin M → K) : hform s J (U *ᵥ x) (U *ᵥ x) = hform s J x x

theorem **hyp_inv** {K : Type} [CommRing K] {M : ℕ} {J Ji : Matrix (Fin M) (Fin M) K} (h : Ji * J = 1) : hypJ Ji * hypJ J = 1

theorem **continuous_pm** {X K : Type} [TopologicalSpace X] [CommRing K] [TopologicalSpace K] {M : ℕ} {a b d : X → K} {r w : X → Fin M → K} {U : X → Matrix (Fin M) (Fin M) K} (ha : Continuous a) (hb : Continuous b) (hr : Continuous r) (hd : Continuous d) (hw : Continuous w) (hU : Continuous U) : Continuous fun x => pm (a x) (b x) (r x) (d x) (w x) (U x)

theorem **par** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∃ (nn : (Fin 1 → Fin M → AF F ⊗[F] E) × AF F → HA (AF F) σ (hypJ J)) (ι : HA (AF F) σ J →* HA (AF F) σ (hypJ J)) (mm : (AF F ⊗[F] E)ˣ →* HA (AF F) σ (hypJ J)) (c : (Fin 1 → Fin M → AF F ⊗[F] E) → (Fin 1 → Fin M → AF F ⊗[F] E) → AF F), IsClosedEmbedding (fun z : ((Fin 1 → Fin M → AF F ⊗[F] E) × AF F) × HA (AF F) σ J × (AF F ⊗[F] E)ˣ => nn z.1 * ι z.2.1 * mm z.2.2) ∧ Continuous nn ∧ Continuous ι ∧ Continuous mm ∧ (Continuous fun p : (Fin 1 → Fin M → AF F ⊗[F] E) × (Fin 1 → Fin M → AF F ⊗[F] E) => c p.1 p.2) ∧ nn (0, 0) = 1 ∧ (∀ w s w' s', nn (w, s) * nn (w', s') = nn (w + w', s + s' + c w w')) ∧ (∀ w, c 0 w = 0) ∧ (∀ u w s, ι u * nn (w, s) = nn (actW (AF F) σ J u w, s) * ι u) ∧ (∀ (a : (AF F ⊗[F] E)ˣ) w s, m

CompanionTheory.AutomorphicTheory.ThetaDilations.FstarDil

theorem **cG_dil** {F : Type} [Field F] [NumberField F] {ι : Type} [Fintype ι] [DecidableEq ι] {M : ℕ} {B : Matrix ι ι F} {ρ : InfinitePlace F → (F →+* ℝ)} {P : InfinitePlace F → Matrix ι ι ℝ} (hQ : QSetup M B ρ P) {a : F} (ha : a ≠ 0) (τ : InfinitePlace F → ℂ) : cG B ρ P (fun w => ((ρ w a : ℝ) : ℂ) ^ 2 * τ w) = ((|(Algebra.norm ℚ a : ℚ) : ℝ|⁻¹ ^ M) ^ 2 : ℝ) * cG B ρ P τ

theorem **fstar_dil** {R : Type} [CommRing R] {ι : Type} [Fintype ι] [TopologicalSpace R] [MeasurableSpace R] {ψ : AddChar R ℂ} {h : R} {B : Matrix ι ι R} {μ : Measure (ι → R)} (H : NAQuad ψ h B μ) (a : Rˣ) {m : ℝ} (hm : 0 ≤ m) (hμ : ∀ s : Set (ι → R), MeasurableSet s → μ ((fun u => (a : R) • u) '' s) = ENNReal.ofReal m * μ s) (φ : (ι → R) → ℂ) (b : R) : m * fstar ψ h B μ (opD 1 (a : R) φ) ((a : R) ^ 2 * b) = fstar ψ h B μ φ b

theorem **ei_inv** : ∀ (F : Type) [Field F] [NumberField F] [MeasurableSpace (A F)] [BorelSpace (A F)] (lf : Measure (A F)), lf.IsAddHaarMeasure → lf (Oh F) = 1 → ∀ ψ : AddChar (A F) ℂ, IsStdChar F ψ → ∀ (ι : Type) [Fintype ι] [DecidableEq ι] (M : ℕ) (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (P : InfinitePlace F → Matrix ι ι ℝ), QSetup M B ρ P → 3 ≤ M → ∀ φ : (ι → A F) → ℂ, SB φ → ∀ τ : InfinitePlace F → ℂ, (∀ w, 0 < (τ w).im) → (∀ b : F, eis ψ lf B ρ P (opU ψ (hf F) (Bf B) (algebraMap F (A F) b) φ) (fun w => τ w + (ρ w b : ℝ)) = eis ψ lf B ρ P φ τ) ∧ (∀ a : F, a ≠ 0 → eis ψ lf B ρ P (opD 1 (algebraMap F (A F) a) φ) (fun w => ((ρ w a : ℝ) : ℂ) ^ 2 * τ w) = eis ψ lf B ρ P φ τ) ∧ (cG B ρ P τ * eis ψ lf B ρ P (fou (chiB ψ (Bf B)) (muF B lf) φ) (fun w => -(τ w)⁻¹) = eis ψ lf B ρ P φ τ) ∧ ∀ c : ℂ, eis ψ lf B ρ P (c • φ) τ = c * eis ψ lf B ρ P φ τ

CompanionTheory.AutomorphicTheory.ThetaDilations.HfSubSmul

theorem **hf_conj** (hs : ∀ x, s (s x) = x) (hJ : cT s J = J) (x y : Fin M → S) : s (hform s J x y) = hform s J y x

theorem **hf_sub_smul** (x y z : Fin M → S) (c : S) : hform s J z (x − c • y) = hform s J z x − c * hform s J z y ∧ hform s J (x − c • y) z = hform s J x z − s c * hform s J y z

theorem **hf_U** {g : Matrix (Fin M) (Fin M) S} (hg : cT s g * J * g = J) (x y : Fin M → S) : hform s J (g *ᵥ x) (g *ᵥ y) = hform s J x y

theorem **quasi** (hs : ∀ x, s (s x) = x) (hJ : cT s J = J) (u : Fin M → S) (c : S) (hc : c + s c = c * s c * hform s J u u) : ∃ g ∈ UJ s J, ∀ z, g.val *ᵥ z = z − (c * hform s J u z) • u

theorem **refl_fix** : ∀ (S : Type) [CommRing S] (s : S →+* S), (∀ z, s (s z) = z) → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) S), cT s J = J → ∀ (x y : Fin M → S) (lam : S), lam * s lam = 1 → hform s J y y = hform s J x x → IsUnit (hform s J x x) → IsUnit (hform s J x x − s lam * hform s J y x) → ∃ g ∈ UJ s J, (g : Matrix (Fin M) (Fin M) S) *ᵥ x = y ∧ ∀ z, hform s J x z = 0 → hform s J y z = 0 → (g : Matrix (Fin M) (Fin M) S) *ᵥ z = z

theorem **witt_diag** (hs : ∀ x, s (s x) = x) (hJ : cT s J = J) (D : Fin n → S) (hD : ∀ i, IsUnit (D i)) (hl : ∀ i t, ∃ lam, lam * s lam = 1 ∧ IsUnit (D i − s lam * t)) (X Y : Fin n → Fin M → S) (hX : ∀ a b, hform s J (X a) (X b) = Matrix.diagonal D a b) (hY : ∀ a b, hform s J (Y a) (Y b) = Matrix.diagonal D a b) : ∃ g ∈ UJ s J, ∀ a, g.val *ᵥ X a = Y a

theorem **gram_rm** (X : Fin n → Fin M → S) (g : Matrix (Fin n) (Fin n) S) (a b : Fin n) : hform s J (fun i => ∑ c, X c i * g c a) (fun i => ∑ c, X c i * g c b) = (cT s g * (Matrix.of fun a b => hform s J (X a) (X b)) * g) a b

theorem **act_rm** (k : Matrix (Fin M) (Fin M) S) (X : Fin n → Fin M → S) (g : Matrix (Fin n) (Fin n) S) (a : Fin n) : k *ᵥ (fun i => ∑ c, X c i * g c a) = fun i => ∑ c, (k *ᵥ X c) i * g c a

theorem **rm_rm** (X : Fin n → Fin M → S) (g g' : Matrix (Fin n) (Fin n) S) (a : Fin n) (i : Fin M) : ∑ c, (∑ c', X c' i * g c' c) * g' c a = ∑ c, X c i * (g * g') c a

theorem **witt_tuple** (hs : ∀ x, s (s x) = x) (hJ : cT s J = J) (D : Fin n → S) (hD : ∀ i, IsUnit (D i)) (hl : ∀ i t, ∃ lam, lam * s lam = 1 ∧ IsUnit (D i − s lam * t)) (X Y : Fin n → Fin M → S) (g : GL (Fin n) S) (hX : cT s g.val * (Matrix.of fun a b => hform s J (X a) (X b)) * g.val = Matrix.diagonal D) (hY : ∀ a b, hform s J (Y a) (Y b) = hform s J (X a) (X b)) : ∃ k ∈ UJ s J, ∀ a, k.val *ᵥ X a = Y a

theorem **cT_JR'** {F E : Type} [Field F] [Field E] [Algebra F E] {σ : E →ₐ[F] E} (R : Type) [CommRing R] [Algebra F R] {M : ℕ} {J : Matrix (Fin M) (Fin M) E} (hJ : IsHermND σ J) : cT (sigR R σ) (JR R J) = JR R J

theorem **trans_diag** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (R : Type) [CommRing R] [Algebra F R], PIW.AdelicOrbitU.UnitChoice R → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (n : ℕ) (D : Fin n → Rˣ) (X Y : Fin n → Fin M → R ⊗[F] E), gramR R σ J X = Matrix.diagonal (fun i => algebraMap R (R ⊗[F] E) (D i : R)) → gramR R σ J Y = gramR R σ J X → ∃ h : HA R σ J, actW R σ J h X = Y

theorem **gramR_rat** {F E : Type} [Field F] [Field E] [Algebra F E] (σ : E →ₐ[F] E) (R : Type) [CommRing R] [Algebra F R] {M n : ℕ} (J : Matrix (Fin M) (Fin M) E) (X : Fin n → Fin M → E) : gramR R σ J (ratTup R X) = (gramE σ J X).map (inclR R)

theorem **trans_uc** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (R : Type) [CommRing R] [Algebra F R], PIW.AdelicOrbitU.UnitChoice R → ∀ (n M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (X : Fin n → Fin M → E) (Y : Fin n → Fin M → R ⊗[F] E), (gramE σ J X).det ≠ 0 → gramR R σ J Y = (gramE σ J X).map (inclR R) → ∃ h : HA R σ J, actW R σ J h (ratTup R X) = Y

CompanionTheory.AutomorphicTheory.ThetaDilations.IntFou

theorem **int_fou** {V : Type} [AddCommGroup V] [TopologicalSpace V] [MeasurableSpace V] {χ : V → V → ℂ} {μ : Measure V} (h : NADual V χ μ) {f : V → ℂ} (hf : SB f) : ∫ y, fou χ μ f y ∂μ = f 0

theorem **recip** : ∀ (F : Type) [Field F] [NumberField F] [MeasurableSpace (A F)] [BorelSpace (A F)] (lf : Measure (A F)), lf.IsAddHaarMeasure → lf (Oh F) = 1 → ∀ ψ : AddChar (A F) ℂ, IsStdChar F ψ → ∀ (ι : Type) [Fintype ι] [DecidableEq ι] (M : ℕ) (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (P : InfinitePlace F → Matrix ι ι ℝ), QSetup M B ρ P → ∀ φ : (ι → A F) → ℂ, SB φ → ∀ τ : InfinitePlace F → ℂ, (∀ w, 0 < (τ w).im) → ∀ b : F, b ≠ 0 → cG B ρ P τ * eiTerm ψ lf B ρ P (fou (chiB ψ (Bf B)) (muF B lf) φ) (fun w => -(τ w)⁻¹) b = eiTerm ψ lf B ρ P φ τ (-b⁻¹)

CompanionTheory.AutomorphicTheory.ThetaDilations.IsOpenCoset

theorem **ind_of_mem** {a x : V} {L : AddSubgroup V} (h : x - a ∈ L) : ind a L x = 1

theorem **ind_of_not** {a x : V} {L : AddSubgroup V} (h : x - a ∉ L) : ind a L x = 0

theorem **ind_congr** {a b x y : V} {L L' : AddSubgroup V} (h : x - a ∈ L ↔ y - b ∈ L') : ind a L x = ind b L' y

theorem **ind_add** {a l : V} {L : AddSubgroup V} (x : V) (hl : l ∈ L) : ind a L (x + l) = ind a L x

theorem **CO.sup** {L L' : AddSubgroup V} (h : CO L) (h' : CO L') : CO (L ⊔ L')

theorem **CO.inf** {L L' : AddSubgroup V} (h : CO L) (h' : IsOpen (L' : Set V)) : CO (L ⊓ L')

theorem **isOpen_coset** (a : V) {L : AddSubgroup V} (h : IsOpen (L : Set V)) : IsOpen {x | x - a ∈ L}

theorem **lc_of_inv** {f : V → ℂ} {L : AddSubgroup V} (h : IsOpen (L : Set V)) (hf : ∀ x, ∀ l ∈ L, f (x + l) = f x) : IsLocallyConstant f

theorem **sb_ind** [T2Space V] (a : V) {L : AddSubgroup V} (h : CO L) : SB (ind a L)

theorem **sb_add** {f g : V → ℂ} (hf : SB f) (hg : SB g) : SB (f + g)

theorem **sb_mul** {f g : V → ℂ} (hf : IsLocallyConstant f) (hg : SB g) : SB fun x => f x * g x

theorem **sb_smul** (c : ℂ) {g : V → ℂ} (hg : SB g) : SB (c • g)

theorem **sb_zero** : SB (0 : V → ℂ)

theorem **sb_sum** {α : Type} (s : Finset α) {f : α → V → ℂ} (h : ∀ a ∈ s, SB (f a)) : SB (∑ a ∈ s, f a)

theorem **sb_homeo** {f : V → ℂ} (hf : SB f) (e : V ≃ₜ V) : SB (f ∘ e)

theorem **sb_span** [IsTopologicalAddGroup V] {φ : V → ℂ} (hφ : SB φ) {L : AddSubgroup V} (hL : IsOpen (L : Set V)) (hinv : ∀ x, ∀ l ∈ L, φ (x + l) = φ x) : ∃ s : Finset V, φ = ∑ a ∈ s, φ a • ind a L

theorem **co_cpt** (h : NADual V χ μ) {K : Set V} (hK : IsCompact K) : ∃ L : AddSubgroup V, CO L ∧ K ⊆ L

theorem **sb_level** (h : NADual V χ μ) {φ : V → ℂ} (hφ : SB φ) : ∃ L L' : AddSubgroup V, CO L ∧ CO L' ∧ L' ≤ L ∧ (∀ x, x ∉ L → φ x = 0) ∧ ∀ x, ∀ l ∈ L', φ (x + l) = φ x

theorem **chi_sym** (x y : V) : χ x y = χ y x

theorem **chi_norm** (x y : V) : ‖χ x y‖ = 1

theorem **chi_add** (x x' y : V) : χ (x + x') y = χ x y * χ x' y

theorem **chi_cont** : Continuous fun p : V × V => χ p.1 p.2

theorem **chi_ne** (x y : V) : χ x y ≠ 0

theorem **chi_add'** (x y y' : V) : χ x (y + y') = χ x y * χ x y'

theorem **chi_zero** (y : V) : χ 0 y = 1

theorem **chi_neg** (x y : V) : χ (-x) y = conj (χ x y)

theorem **co_perp** {L : AddSubgroup V} (hL : CO L) : CO (perpG h L)

theorem **perp_perp** {L : AddSubgroup V} (hL : CO L) (x : V) : x ∈ perpG h (perpG h L) ↔ x ∈ L

theorem **meas_perp** {L : AddSubgroup V} (hL : CO L) : (μ L).toReal * (μ (perpG h L)).toReal = 1

theorem **chi_lc** (a : V) : IsLocallyConstant fun y => χ a y

theorem **sb_int** {φ : V → ℂ} (hφ : SB φ) : Integrable φ μ

theorem **sb_chi** {φ : V → ℂ} (hφ : SB φ) (y : V) : SB fun x => φ x * χ x y

theorem **fou_add** {f g : V → ℂ} (hf : SB f) (hg : SB g) : fou χ μ (f + g) = fou χ μ f + fou χ μ g

theorem **fou_smul** (c : ℂ) (f : V → ℂ) : fou χ μ (c • f) = c • fou χ μ f

theorem **map_sum_sb** {T : (V → ℂ) → (V → ℂ)} (hadd : ∀ φ φ', SB φ → SB φ' → T (φ + φ') = T φ + T φ') (hsm : ∀ (c : ℂ) φ, SB φ → T (c • φ) = c • T φ) {α : Type} (s : Finset α) (c : α → ℂ) {f : α → V → ℂ} (hf : ∀ a, SB (f a)) : T (∑ a ∈ s, c a • f a) = ∑ a ∈ s, c a • T (f a)

theorem **fou_ind** {L : AddSubgroup V} (hL : CO L) (a y : V) : fou χ μ (ind a L) y = ((μ L).toReal : ℂ) * χ a y * ind 0 (perpG h L) y

theorem **fou_sb** {φ : V → ℂ} (hφ : SB φ) : SB (fou χ μ φ) ∧ ∀ x, fou χ μ (fou χ μ φ) x = φ (-x)

theorem **planch** {φ : V → ℂ} (hφ : SB φ) : ∫ y, ‖fou χ μ φ y‖ ^ 2 ∂μ = ∫ x, ‖φ x‖ ^ 2 ∂μ

theorem **schur** {T : (V → ℂ) → (V → ℂ)} (hadd : ∀ φ φ', SB φ → SB φ' → T (φ + φ') = T φ + T φ') (hsm : ∀ (c : ℂ) φ, SB φ → T (c • φ) = c • T φ) (hch : ∀ y φ, SB φ → T (fun u => χ u y * φ u) = fun u => χ u y * T φ u) (htr : ∀ x φ, SB φ → T (fun u => φ (u + x)) = fun u => T φ (u + x)) : ∃ c : ℂ, ∀ φ, SB φ → T φ = c • φ

CompanionTheory.AutomorphicTheory.ThetaDilations.Nuabs

theorem **lower** : ∀ (G : Type) [Group G] [TopologicalSpace G] [IsTopologicalGroup G] [T2Space G] [LocallyCompactSpace G] [SecondCountableTopology G] [MeasurableSpace G] [BorelSpace G] (P : Subgroup G) (ν : Measure G) [SFinite ν], (∀ p ∈ P, ν.map (fun g => p * g) = ν) → (∀ U ∈ 𝒩 (1 : G), ν U ≠ 0) → ∀ C : Set G, IsCompact C → ∀ φ : G → ℝ≥0∞, Continuous φ → (∀ c ∈ C, φ c ≠ 0) → ∃ m : ℝ≥0∞, m ≠ 0 ∧ m ≠ ⊤ ∧ ∀ p ∈ P, ∀ c ∈ C, m ≤ ∫⁻ q, φ (q⁻¹ * (p * c)) ∂ν

theorem **cover** : ∀ (G : Type) [Group G] [TopologicalSpace G] [IsTopologicalGroup G] [T2Space G] [LocallyCompactSpace G] [SecondCountableTopology G] [MeasurableSpace G] [BorelSpace G] (μ : Measure G) [μ.IsHaarMeasure] (P : Subgroup G) (ν : Measure G) [SFinite ν], (∀ p ∈ P, ν.map (fun g => p * g) = ν) → (∀ U ∈ 𝒩 (1 : G), ν U ≠ 0) → ∀ C : Set G, IsCompact C → ∃ (C' : Set G) (K : ℝ≥0∞), IsCompact C' ∧ K < ⊤ ∧ ∀ (Γ : Subgroup G) [Countable Γ], Γ ≤ P → ∀ 𝓕 : Set G, IsFundamentalDomain Γ 𝓕 μ → ∀ S : Set G, MeasurableSet S → (∀ⁿ p ∂ν, ∃ γ ∈ Γ, γ * p ∈ S) → ∀ f g : G → ℝ≥0∞, Measurable f → Measurable g → (∀ γ ∈ Γ, ∀ x, f (γ * x) = f x) → (∀ x, f x ≠ 0 → ∃ p ∈ P, ∃ c ∈ C, x = p * c) → (∀ p ∈ S, ∀ c ∈ C', f (p * c) ≤ g p) → ∫⁻ x in 𝓕, f x ∂μ ≤ K * ∫⁻ p in S, g p ∂ν

theorem **unf** : ∀ (G : Type) [Group G] [TopologicalSpace G] [IsTopologicalGroup G] [T2Space G] [LocallyCompactSpace G] [SecondCountableTopology G] [MeasurableSpace G] [BorelSpace G] (μ : Measure G) [μ.IsMulLeftInvariant] (Γ Γ' : Subgroup G) [Countable Γ], Γ' ≤ Γ → ∀ 𝓕 : Set G, IsFundamentalDomain Γ 𝓕 μ → ∀ (Q : Type) (r : Q → G), (∀ q, r q ∈ Γ) → (∀ γ ∈ Γ, ∃! q, γ * (r q)⁻¹ ∈ Γ') → ∃ 𝓕' : Set G, IsFundamentalDomain Γ' 𝓕' μ ∧ ∀ f : G → ℝ≥0∞, Measurable f → ∫⁻ h in 𝓕, ∑' q, f (r q * h) ∂μ = ∫⁻ h in 𝓕', f h ∂μ

theorem **nuabs** : ∀ (G N H T : Type) [Group G] [MeasurableSpace G] [MeasurableMul₂ G] [MeasurableSpace N] [Group H] [MeasurableSpace H] [MeasurableMul H] [CommGroup T] [MeasurableSpace T] [MeasurableMul T] (μN : Measure N) [SFinite μN] (μH : Measure H) [SFinite μH] [μH.IsMulLeftInvariant] (μT : Measure T) [SFinite μT] [μT.IsMulLeftInvariant] (nn : N → G) (ι : H →* G) (mm : T →* G) (χ : T →* ℝ≥0), Measurable nn → Measurable ι → Measurable mm → Measurable χ → (∀ p, ∃ L : N → N, MeasurePreserving L μN μN ∧ ∀ x, nn p * nn x = nn (L x)) → (∀ u, ∃ L : N → N, MeasurePreserving L μN μN ∧ ∀ x, ι u * nn x = nn (L x) * ι u) → (∀ t, ∃ L : N → N, Measurable L ∧ μN.map L = χ t • μN ∧ ∀ x, mm t * nn x = nn (L x) * mm t) → (∀ u t, mm t * ι u = ι u * mm t) → ∀ (p : N) (u : H) (t : T), ((μN.prod (μH.prod (μT.withDensity fun s => (χ s : ℝ≥0∞))))).map fun z => nn z.1 * ι z.2.1 * mm z.2.2).map (fun g => nn p *

CompanionTheory.AutomorphicTheory.ThetaDilations.OrbitAdelic

theorem **oh_mul** {x y : A F} (hx : x ∈ Oh F) (hy : y ∈ Oh F) : x * y ∈ Oh F

theorem **oh_nat** (N : ℕ) : (N : A F) ∈ Oh F

theorem **co_LN** {N : ℕ} (hN : 0 < N) : CO (LN F ι N)

theorem **LN_small** {U : Set (ι → A F)} (hU : U ∈ 𝒩 (0 : ι → A F)) : ∃ N : ℕ, 0 < N ∧ (LN F ι N : Set (ι → A F)) ⊆ U

theorem **orbit_adelic** [MeasurableSpace (A F)] {ψ : AddChar (A F) ℂ} {h : A F} {B : Matrix ι ι (A F)} {μ : Measure (ι → A F)} (H : NAQuad ψ h B μ) {φ : (ι → A F) → ℂ} (hφ : SB φ) : ∃ (L' : AddSubgroup (ι → A F)) (s : Finset (ι → A F)) (C : ℝ), CO L' ∧ ∀ k : SL(2, A F), (∀ i j, k i j ∈ Oh F) → ∀ T, Impl ψ h B μ k T → (∀ u, ‖T φ u‖ ≤ C) ∧ ∃ c : (ι → A F) → ℂ, (∀ a, ‖c a‖ ≤ C) ∧ T φ = ∑ a ∈ s, c a • ind a L'

theorem **fstar_decay** : ∀ (F : Type) [Field F] [NumberField F] [MeasurableSpace (A F)] [BorelSpace (A F)] (lf : Measure (A F)), lf.IsAddHaarMeasure → lf (Oh F) = 1 → ∀ ψ : AddChar (A F) ℂ, IsStdChar F ψ → ∀ (ι : Type) [Fintype ι] [DecidableEq ι] (M : ℕ) (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (P : InfinitePlace F → Matrix ι ι ℝ), QSetup M B ρ P → ∀ φ : (ι → A F) → ℂ, SB φ → ∃ C : ℝ, ∀ b : F, ‖fstar ψ (hf F) (Bf B) (muF B lf) φ (algebraMap F (A F) b)‖ ≤ C * ((den b : ℝ) ^ M)⁻¹

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theorem **blk_eta** (g : Matrix (Fin (M + 2)) (Fin (M + 2)) K) : g = blk (g 0 0) (g 0 1) (fun j => g 0 j.succ.succ) (g 1 0) (g 1 1) (fun j => g 1 j.succ.succ) (fun i => g i.succ.succ 0) (fun i => g i.succ.succ 1) (Matrix.of fun i j => g i.succ.succ j.succ.succ)

theorem **blk_inj** {a b c d a' b' c' d' : K} {r s u w r' s' u' w' : Fin M → K} {U U' : Matrix (Fin M) (Fin M) K} (h : blk a b r c d s u w U = blk a' b' r' c' d' s' u' w' U') : a = a' ∧ b = b' ∧ r = r' ∧ c = c' ∧ d = d' ∧ s = s' ∧ u = u' ∧ w = w' ∧ U = U'

theorem **blk_mul** (a b c d a' b' c' d' : K) (r s u w r' s' u' w' : Fin M → K) (U U' : Matrix (Fin M) (Fin M) K) : blk a b r c d s u w U * blk a' b' r' c' d' s' u' w' U' = blk (a * a' + b * c' + r ⬝ᵥ u') (a * b' + b * d' + r ⬝ᵥ w') (a • r' + b • s' + r ᵥ* U') (c * a' + d * c' + s ⬝ᵥ u') (c * b' + d * d' + s ⬝ᵥ w') (c • r' + d • s' + s ᵥ* U') (a' • u + c' • w + U *ᵥ u') (b' • u + d' • w + U *ᵥ w') (vecMulVec u r' + vecMulVec w s' + U * U')

theorem **pm_mul** (a b d a' b' d' : K) (r w r' w' : Fin M → K) (U U' : Matrix (Fin M) (Fin M) K) : pm a b r d w U * pm a' b' r' d' w' U' = pm (a * a') (a * b' + b * d' + r ⬝ᵥ w') (a • r' + r ᵥ* U') (d * d') (d' • w + U *ᵥ w') (U * U')

theorem **pm_one** : pm (1 : K) 0 (0 : Fin M → K) 1 0 1 = 1

theorem **blk_map** {L : Type} [CommRing L] (φ : K → L) (a b c d : K) (r s u w : Fin M → K) (U : Matrix (Fin M) (Fin M) K) : (blk a b r c d s u w U).map φ = blk (φ a) (φ b) (φ ∘ r) (φ c) (φ d) (φ ∘ s) (φ ∘ u) (φ ∘ w) (U.map φ)

theorem **pm_map** {L : Type} [CommRing L] (φ : K →+* L) (a b d : K) (r w : Fin M → K) (U : Matrix (Fin M) (Fin M) K) : (pm a b r d w U).map φ = pm (φ a) (φ b) (φ ∘ r) (φ d) (φ ∘ w) (U.map φ)

theorem **block_matrix_transpose** (a b c d : K) (r s u w : Fin M → K) (U : Matrix (Fin M) (Fin M) K) : (blk a b r c d s u w U)ᵀ = blk a c u b d w r s Uᵀ

theorem **hypJ_eq** (J : Matrix (Fin M) (Fin M) K) : hypJ J = blk 0 1 0 1 0 0 0 0 J

theorem **hypJ_map** {L : Type} [CommRing L] (φ : K →+* L) (J : Matrix (Fin M) (Fin M) K) : (hypJ J).map φ = hypJ (J.map φ)

theorem **hform_eq** (s : K →+* K) (J : Matrix (Fin M) (Fin M) K) (x y : Fin M → K) : hform s J x y = ((s ∘ x) ᵥ* J) ⬝ᵥ y

theorem **gram_blk** (s : K →+* K) (J : Matrix (Fin M) (Fin M) K) (a b d : K) (r t w : Fin M → K) (U : Matrix (Fin M) (Fin M) K) : cT s (blk a b r 0 d t 0 w U) * hypJ J * blk a b r 0 d t 0 w U = blk 0 (s a * d) (s a • t) (s d • a) (s d * b + s b * d + ((s ∘ w) ᵥ* J) ⬝ᵥ w) (s d • r + s b • t + ((s ∘ w) ᵥ* J) ᵥ* U) (a • (s ∘ t)) (b • (s ∘ t) + d • (s ∘ r) + (cT s U * J) *ᵥ w) (vecMulVec (s ∘ t) r + vecMulVec (s ∘ r) t + cT s U * J * U)

theorem **nM_unit** (hs : ∀ x, s (s x) = x) (hJ : cT s J = J) {w : Fin M → K} {z : K} (hz : z + s z + hform s J w w = 0) : cT s (nM s J w z) * hypJ J * nM s J w z = hypJ J

theorem **iM_unit** {U : Matrix (Fin M) (Fin M) K} (hU : cT s U * J * U = J) : cT s (pm 1 0 0 1 0 U) * hypJ J * pm 1 0 0 1 0 U = hypJ J

theorem **mM_unit** (hs : ∀ x, s (s x) = x) (a : Kˣ) : cT s (pm (a : K) 0 0 (s (a⁻¹ : Kˣ)) 0 1) * hypJ J * pm (a : K) 0 0 (s (a⁻¹ : Kˣ)) 0 1 = hypJ J

theorem **nM_mul** (w w' : Fin M → K) (z z' : K) : nM s J w z * nM s J w' z' = nM s J (w + w') (z + z' − hform s J w w')

theorem **iM_nM** {U : Matrix (Fin M) (Fin M) K} (hU : cT s U * J * U = J) (w : Fin M → K) (z : K) : pm 1 0 0 1 0 U * nM s J w z = nM s J (U *ᵥ w) z * pm 1 0 0 1 0 U

theorem **mM_nM** (hs : ∀ x, s (s x) = x) (a : Kˣ) (w : Fin M → K) (z : K) : pm (a : K) 0 0 (s (a⁻¹ : Kˣ)) 0 1 * nM s J w z = nM s J (s a • w) (a * s a * z) * pm (a : K) 0 0 (s (a⁻¹ : Kˣ)) 0 1

theorem **nim** (w : Fin M → K) (z a d : K) (U : Matrix (Fin M) (Fin M) K) : nM s J w z * pm 1 0 0 1 0 U * pm a 0 0 d 0 1 = pm a (z * d) (−(((s ∘ w) ᵥ* J) ᵥ* U)) d (d • w) U

theorem **par_dec** (hs : ∀ x, s (s x) = x) {g : Matrix (Fin (M + 2)) (Fin (M + 2)) K} (hg : cT s g * hypJ J * g = hypJ J) (h0 : ∀ i : Fin (M + 1), g i.succ 0 = 0) : ∃ (w : Fin M → K) (z : K) (U : Matrix (Fin M) (Fin M) K) (a : Kˣ), z + s z + hform s J w w = 0 ∧ cT s U * J * U = J ∧ g = nM s J w z * pm 1 0 0 1 0 U * pm (a : K) 0 0 (s (a⁻¹ : Kˣ)) 0 1

theorem **continuous_UJ** {n : ℕ} [TopologicalSpace K] [IsTopologicalRing K] {s : K →+* K} (hs : Continuous s) {J Ji : Matrix (Fin n) (Fin n) K} (hJ : Ji * J = 1) : Topology.IsInducing fun g : UJ s J => ((g : GL (Fin n) K) : Matrix (Fin n) (Fin n) K)

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theorem **prod_rho** {ρ : InfinitePlace F → (F →+* ℝ)} (hF : IsTotallyReal F) (hρ : ∀ w x, ((ρ w x : ℝ) : ℂ) = w.embedding x) (a : F) : ∏ w, |ρ w a| = |((Algebra.norm ℚ a : ℚ) : ℝ)|

theorem **map_nMat** {R S : Type} [CommRing R] [CommRing S] (f : R →+* S) (b : R) : Matrix.SpecialLinearGroup.map f (nMat b) = nMat (f b)

theorem **map_wMat** {R S : Type} [CommRing R] [CommRing S] (f : R →+* S) : Matrix.SpecialLinearGroup.map f wMat = wMat

theorem **map_mMat** {R S : Type} [CommRing R] [CommRing S] (f : R →+* S) (a : Rˣ) : Matrix.SpecialLinearGroup.map f (mMat a) = mMat (Units.map (f : R →* S) a)

theorem **mod_rat** [MeasurableSpace (A F)] [BorelSpace (A F)] {lf : Measure (A F)} (hlf : lf.IsAddHaarMeasure) (h1 : lf (Oh F) = 1) {ι : Type} [Fintype ι] [DecidableEq ι] {M : ℕ} (hc : Fintype.card ι = 2 * M) (B : Matrix ι ι F) (a : Fˣ) (s : Set (ι → A F)) (hs : MeasurableSet s) : muF B lf ((fun u => (uA a : A F) • u) '' s) = ENNReal.ofReal ((|((Algebra.norm ℚ (a : F) : ℚ) : ℝ)|⁻¹ ^ M) ^ 2) * muF B lf s

theorem **good_ex** [MeasurableSpace (A F)] [BorelSpace (A F)] {lf : Measure (A F)} (hlf : lf.IsAddHaarMeasure) (h1 : lf (Oh F) = 1) {ψ : AddChar (A F) ℂ} (hψ : IsStdChar F ψ) {ι : Type} [Fintype ι] [DecidableEq ι] {M : ℕ} {B : Matrix ι ι F} {ρ : InfinitePlace F → (F →+* ℝ)} {P : InfinitePlace F → Matrix ι ι ℝ} (hQ : QSetup M B ρ P) {𝒟 : ((ι → A F) → ℂ) → (InfinitePlace F → ℂ) → ℂ} (hD : ∀ φ τ, SB φ → (∀ w, 0 < (τ w).im) → (∀ b : F, 𝒟 (opU ψ (hf F) (Bf B) (algebraMap F (A F) b) φ) (fun w => τ w + (ρ w b : ℝ)) = 𝒟 φ τ) ∧ (∀ a : F, a ≠ 0 → 𝒟 (opD 1 (algebraMap F (A F) a) φ) (fun w => ((ρ w a : ℝ) : ℂ) ^ 2 * τ w) = 𝒟 φ τ) ∧ (cG B ρ P τ * 𝒟 (fou (chiB ψ (Bf B)) (muF B lf) φ) (fun w => −(τ w)⁻¹) = 𝒟 φ τ) ∧ ∀ c : ℂ, 𝒟 (c • φ) τ = c * 𝒟 φ τ) (γ : SL(2, F)) : ∃ T, Impl ψ (hf F) (Bf B) (muF B lf) (Matrix.SpecialLinearGroup.map (algebraMap F (A F)) γ) T ∧ ∀ φ τ, SB φ → (∀ w, 0 < (τ w).im) → wt M (mob

theorem **inv_all** : ∀ (F : Type) [Field F] [NumberField F] [MeasurableSpace (A F)] [BorelSpace (A F)] (lf : Measure (A F)), lf.IsAddHaarMeasure → lf (Oh F) = 1 → ∀ ψ : AddChar (A F) ℂ, IsStdChar F ψ → ∀ (ι : Type) [Fintype ι] [DecidableEq ι] (M : ℕ) (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (P : InfinitePlace F → Matrix ι ι ℝ), QSetup M B ρ P → ∀ 𝒟 : ((ι → A F) → ℂ) → (InfinitePlace F → ℂ), (∀ φ τ, SB φ → (∀ w, 0 < (τ w).im) → (∀ b : F, 𝒟 (opU ψ (hf F) (Bf B) (algebraMap F (A F) b) φ) (fun w => τ w + (ρ w b : ℝ)) = 𝒟 φ τ) ∧ (∀ a : F, a ≠ 0 → 𝒟 (opD 1 (algebraMap F (A F) a) φ) (fun w => ((ρ w a : ℝ) : ℂ) ^ 2 * τ w) = 𝒟 φ τ) ∧ (cG B ρ P τ * 𝒟 (fou (chiB ψ (Bf B)) (muF B lf) φ) (fun w => −(τ w)⁻¹) = 𝒟 φ τ) ∧ ∀ c : ℂ, 𝒟 (c • φ) τ = c * 𝒟 φ τ) → ∀ (γ : SL(2, F)) T, Impl ψ (hf F) (Bf B) (muF B lf) (Matrix.SpecialLinearGroup.map (algebraMap F (A F)) γ) T → ∀ φ τ, SB φ → (∀ w, 0 < (

CompanionTheory.AutomorphicTheory.ThetaDilations.SqPosOfNe

theorem **bsym** {B : Matrix ι ι R} (hB : B.IsSymm) (x y : ι → R) : x ⬝ᵥ (B *ᵥ y) = y ⬝ᵥ (B *ᵥ x)

theorem **impl_one** : Impl ψ h B μ 1 id

theorem **sb_heis** (x y : ι → R) {φ : (ι → R) → ℂ} (hφ : SB φ) : SB (Heisenberg₂ ψ h B x y φ)

theorem **sb_opU** (b : R) {φ : (ι → R) → ℂ} (hφ : SB φ) : SB (opU ψ h B b φ)

theorem **sb_opD** (m : ℝ) (a : Rˣ) {φ : (ι → R) → ℂ} (hφ : SB φ) : SB (opD m (a : R) φ)

theorem **impl_mul** {g g' : SL(2, R)} {T T' : ((ι → R) → ℂ) → ((ι → R) → ℂ)} (hT : Impl ψ h B μ g T) (hT' : Impl ψ h B μ g' T') : Impl ψ h B μ (g * g') (T ∘ T')

theorem **impl_U** (b : R) : Impl ψ h B μ (nMat b) (opU ψ h B b)

theorem **impl_D** (a : Rˣ) {m : ℝ} (hm : 0 < m) (hμ : ∀ s : Set (ι → R), MeasurableSet s → μ ((fun u => (a : R) • u) '' s) = ENNReal.ofReal (m ^ 2) * μ s) : Impl ψ h B μ (mMat a) (opD m (a : R))

theorem **impl_fou_chiB** : Impl ψ h B μ wMat (fou (chiB ψ B) μ)

theorem **sq_pos_of_ne** {φ : (ι → R) → ℂ} (hφ : SB φ) (hne : φ ≠ 0) : ∫ u, ‖φ u‖ ^ 2 ∂μ ≠ 0

theorem **impl_uniq** {g : SL(2, R)} {T T' : ((ι → R) → ℂ) → ((ι → R) → ℂ)} (hT : Impl ψ h B μ g T) (hT' : Impl ψ h B μ g T') : ∃ c : ℂ, ‖c‖ = 1 ∧ ∀ φ, SB φ → T' φ = c • T φ

theorem **weil_id** (b : Rˣ) {m : ℝ} (hm : 0 < m) (hμ : ∀ s : Set (ι → R), MeasurableSet s → μ ((fun u => ((b⁻¹ : Rˣ) : R) • u) '' s) = ENNReal.ofReal (m ^ 2) * μ s) : ∃ γ : ℂ, ‖γ‖ = 1 ∧ ∀ φ, SB φ → opU ψ h B (b : R) (fou (chiB ψ B) μ φ) = γ • fou (chiB ψ B) μ (opU ψ h B (−((b⁻¹ : Rˣ) : R)) (opD m ((b⁻¹ : Rˣ) : R) (fou (chiB ψ B) μ (opU ψ h B (−((b⁻¹ : Rˣ) : R)) φ))))

theorem **orbit_bd** (O : Set R) {L : AddSubgroup (ι → R)} (hL : CO L) (hst : ∀ a ∈ O, ∀ x ∈ L, a • x ∈ L) {φ : (ι → R) → ℂ} (hφ : SB φ) (hfix : ∀ x ∈ L, ∀ y ∈ L, Heisenberg₂ ψ h B x y φ = φ) : ∃ (L' : AddSubgroup (ι → R)) (s : Finset (ι → R)) (C : ℝ), CO L' ∧ ∀ k : SL(2, R), (∀ i j, k i j ∈ O) → ∀ T, Impl ψ h B μ k T → (∀ u, ‖T φ u‖ ≤ C) ∧ ∃ c : (ι → R) → ℂ, (∀ a, ‖c a‖ ≤ C) ∧ T φ = ∑ a ∈ s, c a • ind a L'

CompanionTheory.AutomorphicTheory.ThetaDilations.SwOne

theorem **sw_one** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → 3 ≤ M → ∀ (T : Matrix (Fin 1) (Fin 1) E) (hT : ∀ a b, σ (T b a) = T a b), T.det ≠ 0 → SWMeas σ J T hT

CompanionTheory.AutomorphicTheory.ThetaDilations.ThetaFin

theorem **theta_measure_locally_finite** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → 3 ≤ M → ∀ (μ : Measure (HA (AF F) σ J)) (ℱ : Set (HA (AF F) σ J)), IsHaarH F E σ M J μ → IsFDH F E σ M J μ ℱ → ∀ (T : Matrix (Fin 1) (Fin 1) E) (hT : ∀ a b, σ (T b a) = T a b), T.det ≠ 0 → IsLocallyFiniteMeasure (thetaMeas σ J μ ℱ {X | gramE σ J X = T})

CompanionTheory.AutomorphicTheory.ThetaDilations.ThetaLint

theorem **theta_lint** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (μ : Measure (HA (AF F) σ J)) (ℱ : Set (HA (AF F) σ J)), IsHaarH F E σ M J μ → IsFDH F E σ M J μ ℱ → ∀ (n : ℕ) (S : Set (Fin n → Fin M → E)) (Φ : (Fin n → Fin M → AF F ⊗[F] E) → ℝ≥0∞), Measurable Φ → ∫⁻ X, Φ X ∂(thetaMeas σ J μ ℱ S) = (μ ℱ)⁻¹ * ∫⁻ h in ℱ, ∑' X : S, Φ (actW (AF F) σ J h⁻¹ (ratTup (AF F) (X : Fin n → Fin M → E))) ∂μ

theorem **thetacnt** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (μ : Measure (HA (AF F) σ J)) (ℱ : Set (HA (AF F) σ J)), IsHaarH F E σ M J μ → IsFDH F E σ M J μ ℱ → ∀ (n : ℕ) (S : Set (Fin n → Fin M → E)) (Φ : (Fin n → Fin M → AF F ⊗[F] E) → ℝ≥0∞), Measurable Φ → ∫⁻ Y, Φ Y ∂(thetaMeas σ J μ ℱ S) = (μ ℱ)⁻¹ * cntI σ J μ ℱ S Φ

theorem **theta_int** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (μ : Measure (HA (AF F) σ J)) (ℱ : Set (HA (AF F) σ J)), IsHaarH F E σ M J μ → IsFDH F E σ M J μ ℱ → ∀ (n : ℕ) (S : Set (Fin n → Fin M → E)) (Φ : (Fin n → Fin M → AF F ⊗[F] E) → ℂ), Measurable Φ → Integrable Φ (thetaMeas σ J μ ℱ S) → meanC μ ℱ (thetaS F E σ M J Φ S) = ∫ X, Φ X ∂(thetaMeas σ J μ ℱ S)

theorem **act_rat** {F E : Type} [Field F] [NumberField F] [Field E] [Algebra F E] {σ : E →ₐ[F] E} {M n : ℕ} {J : Matrix (Fin M) (Fin M) E} (γ : HFinIn (AF F) σ J) (X : Fin n → Fin M → E) : ∃ X' : Fin n → Fin M → E, actW (AF F) σ J γ.1 (ratTup (AF F) X) = ratTup (AF F) X'

theorem **theta_inv** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (μ : Measure (HA (AF F) σ J)) (ℱ : Set (HA (AF F) σ J)), IsHaarH F E σ M J μ → IsFDH F E σ M J μ ℱ → ∀ (n : ℕ) (T : Matrix (Fin n) (Fin n) E), (∀ h : HA (AF F) σ J, (thetaMeas σ J μ ℱ {X | gramE σ J X = T}).map (actW (AF F) σ J h) = thetaMeas σ J μ ℱ {X | gramE σ J X = T}) ∧ thetaMeas σ J μ ℱ {X | gramE σ J X = T} {X | gramR (AF F) σ J X = T.map (inclR (AF F))}ᶜ = 0

CompanionTheory.AutomorphicTheory.ThetaDilations.WeilImplicationNonarchQuadratic

theorem **c_sb_cpt** : ∀ (V : Type) [AddCommGroup V] [TopologicalSpace V] [MeasurableSpace V] (χ : V → V → ℂ) (μ : Measure V), NADual V χ μ → (∀ K : Set V, IsCompact K → ∃ L : AddSubgroup V, IsCompact (L : Set V) ∧ IsOpen (L : Set V) ∧ K ⊆ L) ∧ ∀ φ : V → ℂ, SB φ → ∃ L L' : AddSubgroup V, IsCompact (L : Set V) ∧ IsOpen (L : Set V) ∧ IsCompact (L' : Set V) ∧ IsOpen (L' : Set V) ∧ L' ≤ L ∧ (∀ x, x ∉ L → φ x = 0) ∧ ∀ x, ∀ l ∈ L', φ (x + l) = φ x

theorem **c_sb_span** : ∀ (V : Type) [AddCommGroup V] [TopologicalSpace V] [MeasurableSpace V] (χ : V → V → ℂ) (μ : Measure V), NADual V χ μ → ∀ φ : V → ℂ, SB φ → ∀ L' : AddSubgroup V, IsOpen (L' : Set V) → (∀ x, ∀ l ∈ L', φ (x + l) = φ x) → ∃ s : Finset V, ∀ x, φ x = ∑ a ∈ s, φ a * Set.indicator {y | y - a ∈ L'} (fun _ => (1 : ℂ)) x

theorem **c_fou_ind** : ∀ (V : Type) [AddCommGroup V] [TopologicalSpace V] [MeasurableSpace V] (χ : V → V → ℂ) (μ : Measure V), NADual V χ μ → ∀ L : AddSubgroup V, IsCompact (L : Set V) → IsOpen (L : Set V) → ∀ a y : V, fou χ μ (Set.indicator {x | x - a ∈ L} (fun _ => (1 : ℂ))) y = ((μ L).toReal : ℂ) * χ a y * Set.indicator (perp χ L) (fun _ => (1 : ℂ)) y

theorem **c_fou_sb** : ∀ (V : Type) [AddCommGroup V] [TopologicalSpace V] [MeasurableSpace V] (χ : V → V → ℂ) (μ : Measure V), NADual V χ μ → ∀ φ : V → ℂ, SB φ → SB (fou χ μ φ) ∧ ∀ x, fou χ μ (fou χ μ φ) x = φ (-x)

theorem **c_planch** : ∀ (V : Type) [AddCommGroup V] [TopologicalSpace V] [MeasurableSpace V] (χ : V → V → ℂ) (μ : Measure V), NADual V χ μ → ∀ φ : V → ℂ, SB φ → ∫ y, ‖fou χ μ φ y‖ ^ 2 ∂μ = ∫ x, ‖φ x‖ ^ 2 ∂μ

theorem **c_schur** : ∀ (V : Type) [AddCommGroup V] [TopologicalSpace V] [MeasurableSpace V] (χ : V → V → ℂ) (μ : Measure V), NADual V χ μ → ∀ T : (V → ℂ) → (V → ℂ), (∀ φ, SB φ → SB (T φ)) → (∀ φ φ', SB φ → SB φ' → T (φ + φ') = T φ + T φ') → (∀ (c : ℂ) φ, SB φ → T (c • φ) = c • T φ) → (∀ y φ, SB φ → T (fun u => χ u y * φ u) = fun u => χ u y * T φ u) → (∀ x φ, SB φ → T (fun u => φ (u + x)) = fun u => T φ (u + x)) → ∃ c : ℂ, ∀ φ, SB φ → T φ = c • φ

theorem **c_impl_mul** : ∀ (R : Type) [CommRing R] [TopologicalSpace R] [MeasurableSpace R] (ι : Type) [Fintype ι] (ψ : AddChar R ℂ) (h : R) (B : Matrix ι ι R) (μ : Measure (ι → R)), NAQuad ψ h B μ → Impl ψ h B μ 1 id ∧ ∀ (g g' : SL(2, R)) T T', Impl ψ h B μ g T → Impl ψ h B μ g' T' → Impl ψ h B μ (g * g') (T ∘ T')

theorem **c_heis_sb** : ∀ (R : Type) [CommRing R] [TopologicalSpace R] [MeasurableSpace R] (ι : Type) [Fintype ι] (ψ : AddChar R ℂ) (h : R) (B : Matrix ι ι R) (μ : Measure (ι → R)), NAQuad ψ h B μ → (∀ x y φ, SB φ → SB (Heisenberg₂ ψ h B x y φ)) ∧ (∀ b φ, SB φ → SB (opU ψ h B b φ)) ∧ ∀ (m : ℝ) (a : Rˣ) (φ : (ι → R) → ℂ), SB φ → SB (opD m (a : R) φ)

theorem **c_impl_uniq** : ∀ (R : Type) [CommRing R] [TopologicalSpace R] [MeasurableSpace R] (ι : Type) [Fintype ι] (ψ : AddChar R ℂ) (h : R) (B : Matrix ι ι R) (μ : Measure (ι → R)), NAQuad ψ h B μ → ∀ (g : SL(2, R)) T T', Impl ψ h B μ g T → Impl ψ h B μ g T' → ∃ c : ℂ, ‖c‖ = 1 ∧ ∀ φ, SB φ → T' φ = c • T φ

theorem **c_impl_U** : ∀ (R : Type) [CommRing R] [TopologicalSpace R] [MeasurableSpace R] (ι : Type) [Fintype ι] (ψ : AddChar R ℂ) (h : R) (B : Matrix ι ι R) (μ : Measure (ι → R)), NAQuad ψ h B μ → ∀ b : R, Impl ψ h B μ (nMat b) (opU ψ h B b)

theorem **c_impl_D** : ∀ (R : Type) [CommRing R] [TopologicalSpace R] [MeasurableSpace R] (ι : Type) [Fintype ι] (ψ : AddChar R ℂ) (h : R) (B : Matrix ι ι R) (μ : Measure (ι → R)), NAQuad ψ h B μ → ∀ (a : Rˣ) (m : ℝ), 0 < m → (∀ s : Set (ι → R), MeasurableSet s → μ ((fun u => (a : R) • u) '' s) = ENNReal.ofReal (m ^ 2) * μ s) → Impl ψ h B μ (mMat a) (opD m (a : R))

theorem **weil_implication_nonarch_quadratic** : ∀ (R : Type) [CommRing R] [TopologicalSpace R] [MeasurableSpace R] (ι : Type) [Fintype ι] (ψ : AddChar R ℂ) (h : R) (B : Matrix ι ι R) (μ : Measure (ι → R)), NAQuad ψ h B μ → Impl ψ h B μ wMat (fou (chiB ψ B) μ)

theorem **c_weil_id** : ∀ (R : Type) [CommRing R] [TopologicalSpace R] [MeasurableSpace R] (ι : Type) [Fintype ι] (ψ : AddChar R ℂ) (h : R) (B : Matrix ι ι R) (μ : Measure (ι → R)), NAQuad ψ h B μ → ∀ (b : Rˣ) (m : ℝ), 0 < m → (∀ s : Set (ι → R), MeasurableSet s → μ ((fun u => ((b⁻¹ : Rˣ) : R) • u) '' s) = ENNReal.ofReal (m ^ 2) * μ s) → ∃ γ : ℂ, ‖γ‖ = 1 ∧ ∀ φ, SB φ → opU ψ h B (b : R) (fou (chiB ψ B) μ φ) = γ • fou (chiB ψ B) μ (opU ψ h B (-((b⁻¹ : Rˣ) : R)) (opD m ((b⁻¹ : Rˣ) : R) (fou (chiB ψ B) μ (opU ψ h B (-((b⁻¹ : Rˣ) : R)) φ))))

CompanionTheory.AutomorphicTheory.UniformLocalFormulas.Closed

theorem **energyFin** : EnergyFin

theorem **uniform_restriction_bound** : ThmBChain6.AmplitudeBoundsAllChoicesUniform

theorem **analytic_pinnedTheoremB_uniform** : NGAnalytic.StepThmBPinnedC

theorem **bDat_theoremB_global** : BDat.StepThmBG

CompanionTheory.AutomorphicTheory.UniformLocalFormulas.CovarianceFormula

theorem **place_eq** {n r k : ℕ} (hn : n = 2 * r + k + 1) (ε : Fin n → Bool) (e f : Fin r → Fin n → ℂ) (wf : Fin k → Fin n → ℂ) (z : Fin n → ℂ) {T : ℝ} (hT : 0 < T) (hee : ∀ a b, hJ ε (e a) (e b) = 0) (hff : ∀ a b, hJ ε (f a) (f b) = 0) (hef : ∀ a b, hJ ε (e a) (f b) = if a = b then 1 else 0) (hww : ∀ a b, hJ ε (wf a) (wf b) = if a = b then 1 else 0) (hew : ∀ c a, hJ ε (e c) (wf a) = 0) (hfw : ∀ c a, hJ ε (f c) (wf a) = 0) (hez : ∀ a, hJ ε (e a) z = 0) (hfz : ∀ a, hJ ε (f a) z = 0) (hzz : hJ ε z z = -(T : ℂ)) (hzw : ∀ a, hJ ε z (wf a) = 0) (Q : Matrix (Fin n) (Fin (2 * r + k + 1)) ℂ) (Q' : Matrix (Fin (2 * r + k + 1)) (Fin n) ℂ) (h2 : Q * Q' = 1) (hg : Qᴹ * Formalism.REAL.Jm ε * Q = Hpk k r) {t : ℝ} (ht : 0 < t) (hc : ∀ a : Fin r, Q *ᵥ Pi.single (eI k r a) 1 = (t : ℂ) ^ (r - (a : ℕ)) • e a ∧ Q *ᵥ Pi.single (esI k r a) 1 = (star ((t : ℂ) ^ (r - (a : ℕ))))⁻¹ • f a) (κ : ℝ≥0∞) : (haarLie (Nset

theorem **pi_smul** {ι : Type} [Fintype ι] {α : ι → Type} [∀ i, MeasurableSpace (α i)] (μ : ∀ i, Measure (α i)) [∀ i, SigmaFinite (μ i)] {c : ℝ≥0∞} (hc : c ≠ ⊤) : Measure.pi (fun i => c • μ i) = c ^ Fintype.card ι • Measure.pi μ

theorem **kap_real** (k r : ℕ) : (B.kap k r : ℝ) = (4 * (r : ℝ) ^ 3 + 6 * (r : ℝ) ^ 2 + 2 * r) / 3 + k * ((r : ℝ) ^ 2 + r)

theorem **finrank_E** (D : D7) : Module.finrank ℚ D.host.E = 2 * D.R.f

theorem **disc_rd** (D : D7) (X : HidaSetting D) : |((NumberField.discr X.B.I.F : ℤ) : ℝ)| = D.host.rdE ^ D.R.f

theorem **covariance_formula** : Formalism.L2.SG.Pf.LoopBCovN.CovNFormula

CompanionTheory.AutomorphicTheory.UniformLocalFormulas.HilbertSchmidtDiagonalInvariant

theorem **muG_smap** (τ : D.what₂.Tau) (γ : Formalism.REAL.UJ (X.B.I.ε (vOf X τ))) : (Formalism.REAL.muG (X.B.I.ε (vOf X τ))).map (Smap C τ γ) = muGB X (C.Q τ) (C.Qi τ) τ

theorem **hs_J** (hB : WuBessel C) (u u' : D.what₂.Word) (τ : D.what₂.Tau) (γ : Formalism.REAL.UJ (X.B.I.ε (vOf X τ))) : hsNormSq ((locS hB u u' τ).J (muGB X (C.Q τ) (C.Qi τ) τ) (trG (C.Q τ) (C.Qi τ) γ)) = AmpWin.kap u u' C hB τ * hs2 (JvX X u u' (vOf X τ) γ)

theorem **zhat_tr** (u u' : D.what₂.Word) (τ : D.what₂.Tau) (γ : Formalism.REAL.UJ (X.B.I.ε (vOf X τ))) (a b : Fin (D.R.rS + 1)) : Formalism.QuantumDeterminants.Zhat D.R.k D.R.rS (zB X (C.Qi τ) u τ) (zB X (C.Qi τ) u' τ) (trG (C.Q τ) (C.Qi τ) γ) a b = ((rhoF D a * rhoF D b : ℝ) : ℂ) * Zhat (X.B.I.ε (vOf X τ)) (frm X u 1 (vOf X τ)) (frm X u' 1 (vOf X τ)) (uMat γ) a b

theorem **hilbert_schmidt_diagonal_invariant** (X : HidaSetting D) (u u' : D.what₂.Word) (v : InfinitePlace X.B.I.F) {cv : Fin (D.R.rS + 1) → ℂ} (hc : cv ∈ GoodV (D

theorem **rd_pos** (D : D7) : 0 < D.host.rdE

theorem **lamB_pos** (D : D7) (a : Fin D.R.rS) : 0 < lamB D a

theorem **rho_sq** (D : D7) (a : Fin (D.R.rS + 1)) (h : a.val < D.R.rS) : rhoF D a * rhoF D a = lamB D (a.val, h)

theorem **rho_pos** (D : D7) (a : Fin (D.R.rS + 1)) : 0 < rhoF D a

theorem **place_pt** (hB : WuBessel C) (u u' : D.what₂.Word) (τ : D.what₂.Tau) {sec : ℂ → Matrix (Fin (D.R.k + 1)) (Fin (D.R.k + 1)) ℂ} (hsec : Formalism.QuantumDeterminants.IsSection D.R.k D.R.rS (zB X (C.Qi τ) u τ) (zB X (C.Qi τ) u' τ) sec) {cv : Fin (D.R.rS + 1) → ℂ} (hc : cv ∈ GoodV (D

theorem **energy_prod** (hB : WuBessel C) (u u' : D.what₂.Word) {sec : D.what₂.Tau → ℂ → Matrix (Fin (D.R.k + 1)) (Fin (D.R.k + 1)) ℂ} (hsec : ∀ τ, Formalism.QuantumDeterminants.IsSection D.R.k D.R.rS (zB X (C.Qi τ) u τ) (zB X (C.Qi τ) u' τ) (sec τ)) : ENNReal.ofReal (∏ τ, AmpWin.kap u u' C hB τ) * intCe X u u' 1 = ENNReal.ofReal (Kpl D) ^ D.R.f * ∏ τ, MMv D u u' (locS hB u u' τ) (sec τ)

theorem **kpl_pow** (D : D7) : Kpl D ^ D.R.f = 2 ^ (D.R.f * (D.R.rS + 1)) * (Formalism.FourierDecayAlt.PN.abelian₂ D.host.rdE D.R.rS D.R.k D.R.f * D.host.rdE ^ (((D.R.f * D.R.rS * (D.R.rS + 1) * (8 * D.R.rS + 10) : ℕ) : ℝ) / 3))

theorem **kpl_pos** (D : D7) : 0 < Kpl D

CompanionTheory.AutomorphicTheory.UniformLocalFormulas.MeasConjM

theorem **rowM_det** (ν : Matrix (Fin (k + 1)) (Fin (k + 1)) ℂ) : (rowM r ν).det = ν.det ^ r

theorem **rowM_A** (ν : Matrix (Fin (k + 1)) (Fin (k + 1)) ℂ) (x : CI k r → ℂ) (p : UP r) : (rowM r ν *ᵥ x) (Sum.inl p) = x (Sum.inl p)

theorem **rowM_K** (ν : Matrix (Fin (k + 1)) (Fin (k + 1)) ℂ) (x : CI k r → ℂ) (p : UP r) : (rowM r ν *ᵥ x) (Sum.inr (Sum.inr p)) = x (Sum.inr (Sum.inr p))

theorem **rowM_B** (ν : Matrix (Fin (k + 1)) (Fin (k + 1)) ℂ) (x : CI k r → ℂ) (a : Fin r) (j : Fin (k + 1)) : (rowM r ν *ᵥ x) (Sum.inr (Sum.inl (a, j))) = ∑ c, x (Sum.inr (Sum.inl (a, c))) * ν c j

theorem **rowM_mp** {ν : Matrix (Fin (k + 1)) (Fin (k + 1)) ℂ} (hν : Complex.normSq ν.det = 1) : MeasurePreserving (fun x : CI k r → ℂ => rowM r ν *ᵥ x) volume volume

theorem **meas_conjM** (a : Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ) : Measurable (conjM (k

theorem **conj_uwb** {ρ : Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ} (hρ : ρ ∈ B2.UWb k r) : (haarLie (Nset k r) 1).map (conjM ρ) = haarLie (Nset k r) 1

theorem **aDil_e** (t : ℝ) (a : Fin r) : B.aDil k r t *ᵥ bvec (eI k r a) = ((t : ℂ) ^ (r - a.val)) • bvec (eI k r a)

theorem **aDil_f** (t : ℝ) (a : Fin r) : B.aDil k r t *ᵥ bvec (esI k r a) = ((t : ℂ) ^ (r - a.val))⁻¹ • bvec (esI k r a)

theorem **conj_levi** {d : Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ} (hd : d ∈ UH (Hpk k r)) {t : ℝ} (ht : 0 < t) (he : ∀ a : Fin r, d *ᵥ bvec (eI k r a) = ((t : ℂ) ^ (r - a.val)) • bvec (eI k r a)) (hf : ∀ a : Fin r, d *ᵥ bvec (esI k r a) = ((t : ℂ) ^ (r - a.val))⁻¹ • bvec (esI k r a)) : (haarLie (Nset k r) 1).map (conjM d) = ENNReal.ofReal ((t ^ B.kap k r)⁻¹) • haarLie (Nset k r) 1

CompanionTheory.AutomorphicTheory.UniformLocalFormulas.RankOneTermInvariant

theorem **ka_pos** (D : D7) : 0 < D.host.ka

theorem **upd_mem** (u : D.what₂.Word) (v : InfinitePlace X.B.I.F) {s : Formalism.REAL.UJ (X.B.I.ε v)} (hs : s ∈ JointQuot.NV X u v) : Function.update (1 : X.B.I.Ginf) v s ∈ X.NinfU u

theorem **wloc_N** (u : D.what₂.Word) (v : InfinitePlace X.B.I.F) {s : Formalism.REAL.UJ (X.B.I.ε v)} (hs : s ∈ JointQuot.NV X u v) (a : Formalism.REAL.UJ (X.B.I.ε v)) : X.Wloc u v (s * a) = JointQuot.psiU X u (Function.update 1 v s) • X.Wloc u v a

theorem **rank_one_term_invariant** (u u' : D.what₂.Word) {w : Point D.host.E (D.R.rS + 1)} (hw : JointQuot.OkPt X u u' w) (v : InfinitePlace X.B.I.F) {s : Formalism.REAL.UJ (X.B.I.ε v)} (hs : s ∈ JointQuot.StV X u u' (gamW X u u' w) v) (z : Formalism.REAL.UJ (X.B.I.ε v)) : TermOp.rk1 (X.Wloc u v (s * z)) (X.Wloc u' v ((gamW X u u' w v)⁻¹ * (s * z))) = TermOp.rk1 (X.Wloc u v z) (X.Wloc u' v ((gamW X u u' w v)⁻¹ * z))

theorem **cMr_smap** (γ : X.B.I.Ginf) (τ : D.what₂.Tau) (s z : Formalism.REAL.UJ (X.B.I.ε (vOf X τ))) : (Matrix.of (cMr C γ τ s) * Matrix.of (Smap C τ (γ (vOf X τ)) z) : MatKR D.R.k D.R.rS) = Matrix.of (Smap C τ (γ (vOf X τ)) (s * z))

theorem **map_smap** (τ : D.what₂.Tau) (g : Formalism.REAL.UJ (X.B.I.ε (vOf X τ))) : (Formalism.REAL.muG (X.B.I.ε (vOf X τ))).map (Smap C τ g) = muGB X (C.Q τ) (C.Qi τ) τ

theorem **quot_push** (γ : X.B.I.Ginf) (τ : D.what₂.Tau) {μR ν : Measure (Formalism.REAL.UJ (X.B.I.ε (vOf X τ)))} (h : IsQuotLoc μR ν) : IsQuotMeas (muGB X (C.Q τ) (C.Qi τ) τ) (μR.map (cMr C γ τ)) (ν.map (Smap C τ (γ (vOf X τ))))

theorem **mem_range** (τ : D.what₂.Tau) (g : Formalism.REAL.UJ (X.B.I.ε (vOf X τ))) {x : MatKR D.R.k D.R.rS} (hx : x ∈ Formalism.QuantumDeterminants.UH (Formalism.QuantumDeterminants.Hpk D.R.k D.R.rS)) : x ∈ Set.range (Smap C τ g)

theorem **root_push** (u u' : D.what₂.Word) (w : Point D.host.E (D.R.rS + 1)) (hw : ∃ g, IsPtRep X u u' w g) (τ : D.what₂.Tau) {μR : Measure (Formalism.REAL.UJ (X.B.I.ε (vOf X τ)))} (hc : μR (JointQuot.StV X u u' (gamW X u u' w) (vOf X τ))ᶜ = 0) (hp : μR.map (JointQuot.stCoordV X u' w (gamW X u u' w) (vOf X τ)) = JointQuot.muStabV X w) : IsRootMeasR D (zB X (C.Qi τ) u τ) (zB X (C.Qi τ) u' τ) (ptLoc D τ w) (JointAmplitudePoints.gamP C u u' w τ) (μR.map (cMr C (gamW X u u' w) τ))

theorem **use₇** (hB : WuBessel C) (u u' : D.what₂.Word) (hν : Point D.host.E (D.R.rS + 1) → ∀ v : InfinitePlace X.B.I.F, Measure (Formalism.REAL.UJ (X.B.I.ε v))) (hP : ∀ w, JointQuot.OkPt X u u' w → JointQuot.LocPin X u u' w hν) : LocNorm.LocUniqP u u' C hB hν

CompanionTheory.AutomorphicTheory.UniformLocalFormulas.StepThmBPinnedCOfN

theorem **goodW_all** {X : HidaSetting D} {θ : ℝ} (C : Choice X θ) (u u' : D.what₂.Word) (τ : D.what₂.Tau) : GoodW D τ u u' θ (WX C u u' τ)

theorem **goodS_deg** {X : HidaSetting D} {θ : ℝ} {C : Choice X θ} (hLF : LocFactor C) (hB : WuBessel C) (u u' : D.what₂.Word) (τ : D.what₂.Tau) : ∃ d : ℝ, GoodS D (lamD D τ) (AN hB u u' (fun _ => d) τ)

theorem **ties_coreN** {τ₀ Rw I₀ H H₁ : Type} [Fintype I₀] [NormedAddCommGroup H] [InnerProductSpace ℂ H] [NormedAddCommGroup H₁] [InnerProductSpace ℂ H₁] (XT : Formalism.FourierDecayAlt.PB.Terms τ₀ Rw I₀ H H₁) {Cl' Nu : Type} (N : Formalism.FourierDecayAlt.PN.NGData τ₀ D.host.E D.R.rS Cl' Nu) {large₂ : SizeData₂ D} {X : HidaSetting D} (u u' : D.what₂.Word) {C' Cl Cg cb Cw cT CT C0 C1 Ca Ch Cs Cβ Cb Cn Ire : ℝ} (A : ℕ) (hH : Cnt.Hyp0 D large₂ C' Cl Cg cb Cw cT CT C0) (hTT : HypTT D u u') (hT : 0 < Tr D u) (hT' : 0 < Tr D u') (hAmp : Formalism.MainNotions.S.Bridge.AmpIsBessel D large₂ C' Cl Cg cb Cw cT CT C0 0 C1 Ca) (hStd : Formalism.MainNotions.S.Bridge.StdOfHyp D large₂ C' Cl Cg cb Cw cT CT C0 Ch Cs Cs Cβ) (hTF : Formalism.MainNotions.TFrameOfHyp D large₂ C' Cl Cg cb Cw cT CT C0) (hpt : Formalism.L2.SG.ThmBUse.PtTie XT N (D.mem u).A.T (D.mem u').A.T) (hwt : Formalism.L2.SG.ThmBUse.WtTie XT

theorem **rowDataUnifN_of_6** (hBW : BesselOfW) (hR : ThmBChain6.AmplitudeBoundsAllChoicesUniform) : RowDataUnifN

theorem **stepThmBPinnedC_ofN** (hB : JointNormBoundRotatedUniform) (hA : AmpUnif) (hP : PtUnif) (hR : RowDataUnifN) : StepThmBPinnedC

theorem **stepThmBG_ofN** (hB : JointNormBoundRotatedUniform) (hA : AmpUnif) (hP : PtUnifG) (hR : RowDataUnifN) : StepThmBG

CompanionTheory.AutomorphicTheory.UniformLocalFormulas.UniformRestrictionOfEnergy

theorem **norm_eq** {D : D7} {X : HidaSetting D} {θ : ℝ} (Y : Full X) (C : Choice X θ) (hB : WuBessel C) (hLF : LocFactor C) (u u' : D.what₂.Word) (hfin : intCe X u u' 1 ≠ ⊤) {c c' Irat Ire : ℝ} (hc : c = (X.muN u (Y.FN u)).toReal * (X.muH u Set.univ).toReal / (Module.finrank ℂ (X.Vpi u) : ℝ)) (hc' : c' = (X.muN u' (Y.FN u')).toReal * (X.muH u' Set.univ).toReal / (Module.finrank ℂ (X.Vpi u') : ℝ)) (hcc : c * c' ≠ 0) (h1 : (c * c')⁻¹ * IntC X u u' 1 = 2 ^ (D.R.f * (D.R.rS + 1)) * Irat) (h2 : Irat = Formalism.FourierDecayAlt.PN.abelian₂ D.host.rdE D.R.rS D.R.k D.R.f * D.host.rdE ^ (((D.R.f * D.R.rS * (D.R.rS + 1) * (8 * D.R.rS + 10) : ℕ) : ℝ) / 3) * Ire) {sec : D.what₂.Tau → ℂ → Matrix (Fin (D.R.k + 1)) (Fin (D.R.k + 1)) ℂ} (hsec : ∀ τ, Formalism.QuantumDeterminants.IsSection D.R.k D.R.rS (zB X (C.Qi τ) u τ) (zB X (C.Qi τ) u' τ) (sec τ)) : ∏ τ, MMv D u u' (locS hB u u' τ) (sec τ) = ENNReal.ofR

theorem **uniform_restriction_of_energy** (hF : LoopBCovN.CovNFormula) (hU : UniqOfPin) (hE : EnergyFin) : AmplitudeBoundsAllChoicesUniform

CompanionTheory.AutomorphicTheory.UniformLocalFormulas.UniformRestrictionOfFiniteEnergy

theorem **uniqOfPin** : UniqOfPin

theorem **uniform_restriction_of_finite_energy** (hE : EnergyFin) : Formalism.L2.SG.Pf.ThmBChain6.AmplitudeBoundsAllChoicesUniform

CompanionTheory.AutomorphicTheory.UnitriangularMeasure.AdeleCoordinateMeasure.EllipticTorusNorm

theorem **continuous_adeleCoordEquiv** : Continuous (adeleCoordEquiv K L)

theorem **continuous_adeleCoordEquiv_symm** : Continuous (adeleCoordEquiv K L).symm

theorem **det_mulMatrixGL** (x : (𝔸[L])ˣ) : ((Matrix.GeneralLinearGroup.det (mulMatrixGL (K

theorem **adeleCoordEquiv_mul** (x : (𝔸[L])ˣ) (y : 𝔸[L]) : adeleCoordEquiv K L ((x : 𝔸[L]) * y) = adeleCoordEquiv K L y ᵥ* (mulMatrixGL (K

theorem **measurable_adeleCoordEquiv_symm** : Measurable (adeleCoordEquiv K L).symm

theorem **map_smul_coordMeasure** (x : IdeleGroup L) : (coordMeasure K L).map (fun y ↦ x • y) = ENNReal.ofReal ((IdeleGroup.ideleModule K (ideleNorm K L x) : ℝ))⁻¹ • coordMeasure K L

theorem **ideleModule_ideleNorm** (x : IdeleGroup L) : IdeleGroup.ideleModule K (ideleNorm K L x) = IdeleGroup.ideleModule L x

CompanionTheory.AutomorphicTheory.UnitriangularMeasure.AdeleCoordinateMeasure.GJGlobalModulus

theorem **measurePreserving_vecMul_unitriangular** : ∀ {m : ℕ} (u : Matrix (Fin m) (Fin m) R), (∀ i j : Fin m, j < i → u i j = 0) → (∀ i, u i i = 1) → MeasurePreserving (fun x : Fin m → R => x ᵥ* u) (Measure.pi fun _ => μ) (Measure.pi fun _ => μ) | 0, u, _, _ => by have h : (fun x : Fin 0 → R => x ᵥ* u) = id

theorem **measurable_vecMul'** (g : GL (Fin n) 𝔸[K]) : Measurable fun x : Fin n → 𝔸[K] => x ᵥ* (g : Matrix (Fin n) (Fin n) 𝔸[K])

theorem **isAddHaarMeasure_map_vecMul'** (g : GL (Fin n) 𝔸[K]) : ((adeleMeasurePi K (ι

theorem **forall_norm_finComp_le_one_iff'** (x : Fin n → 𝔸[K]) : (∀ v : HeightOneSpectrum (𝒪 K), ‖finComp v x‖ ≤ 1) ↔ ∀ i, (x i).2 ∈ integralFiniteAdeles (𝒪 K) K

theorem **unitBall_eq** : (unitBall : Set (Fin n → 𝔸[K])) = {x | (∀ w : InfinitePlace K, euclidNorm (infComp w x) ≤ 1) ∧ ∀ i, (x i).2 ∈ integralFiniteAdeles (𝒪 K) K}

theorem **continuous_infComp'** (w : InfinitePlace K) : Continuous fun x : Fin n → 𝔸[K] => infComp w x

theorem **isClosed_unitBall** : IsClosed (unitBall : Set (Fin n → 𝔸[K]))

theorem **isCompact_unitBall** : IsCompact (unitBall : Set (Fin n → 𝔸[K]))

theorem **measurableSet_unitBall** : MeasurableSet (unitBall : Set (Fin n → 𝔸[K]))

theorem **adeleMeasurePi_unitBall_ne_top** : adeleMeasurePi K (unitBall : Set (Fin n → 𝔸[K])) ≠ ⊤

theorem **adeleMeasurePi_unitBall_ne_zero** : adeleMeasurePi K (unitBall : Set (Fin n → 𝔸[K])) ≠ 0

theorem **vecMul_mem_unitBall_iff** {k : GL (Fin n) 𝔸[K]} (hk : k ∈ (maxCompact : Subgroup (AdelicGLn.AdelicGL K n))) (x : Fin n → 𝔸[K]) : x ᵥ* (k : Matrix (Fin n) (Fin n) 𝔸[K]) ∈ unitBall ↔ x ∈ unitBall

theorem **map_vecMul_maxCompact'** {k : GL (Fin n) 𝔸[K]} (hk : k ∈ (maxCompact : Subgroup (AdelicGLn.AdelicGL K n))) : (adeleMeasurePi K).map (fun x => x ᵥ* (k : Matrix (Fin n) (Fin n) 𝔸[K])) = adeleMeasurePi K (ι

theorem **ideleModule_det_of_mem_maxCompact'** {k : GL (Fin n) 𝔸[K]} (hk : k ∈ (maxCompact : Subgroup (AdelicGLn.AdelicGL K n))) : IdeleGroup.ideleModule K (Matrix.GeneralLinearGroup.det k) = 1

theorem **map_vecMul_diagonal** (d : Fin n → IdeleGroup K) : (adeleMeasurePi K).map (fun x => x ᵥ* Matrix.diagonal (fun i => (d i : 𝔸[K]))) = (∏ i, ENNReal.ofReal ((IdeleGroup.ideleModule K (d i) : ℝ))⁻¹) • adeleMeasurePi K (ι

theorem **map_vecMul_adelicBorel'** {b : GL (Fin n) 𝔸[K]} (hb : b ∈ (adelicBorel : Subgroup (AdelicGLn.AdelicGL K n))) : (adeleMeasurePi K).map (fun x => x ᵥ* (b : Matrix (Fin n) (Fin n) 𝔸[K])) = ENNReal.ofReal ((IdeleGroup.ideleModule K (Matrix.GeneralLinearGroup.det b) : ℝ))⁻¹ • adeleMeasurePi K (ι

theorem **map_vecMul_adeleMeasurePi** (g : GL (Fin n) 𝔸[K]) : (adeleMeasurePi K).map (fun x => x ᵥ* (g : Matrix (Fin n) (Fin n) 𝔸[K])) = ENNReal.ofReal ((IdeleGroup.ideleModule K (Matrix.GeneralLinearGroup.det g) : ℝ))⁻¹ • adeleMeasurePi K (ι

CompanionTheory.AutomorphicTheory.ZuckermanCommutation.CapelliEigenvalues.BetaAtInfinityZero

theorem **kfin_iff** (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (P : B.Mat k r) (H : Type) [NormedAddCommGroup H] [InnerProductSpace ℂ H] (σ : B.Mat k r → (H →L[ℂ] H)) (v : H) (hP : IsCayleyU k r ε P) : IsKFinHpk (k

theorem **eq_of_capelliPolynomial_eq** {n : ℕ} {μ μ' : Fin n → ℝ} (hμ : StrictAnti μ) (hμ' : StrictAnti μ') (h : ∀ Y : ℂ, Formalism.QuantumDeterminants.capP (fun s => (μ s : ℂ)) Y = Formalism.QuantumDeterminants.capP (fun s => (μ' s : ℂ)) Y) : μ = μ'

theorem **capelli_inequivalence** : Inequiv

CompanionTheory.AutomorphicTheory.ZuckermanCommutation.CapelliEigenvalues.CoefCC

theorem **rw_cf** (h : RepOn G σ) (x₀ : E) (a : H) {b : H} (hb : IsSm G σ b) {f : B.Mat k r → E} (hf : ∀ g ∈ G, f g = cf σ x₀ a b g) : ∀ {v : List (B.Mat k r)}, LieW G v → ∀ g ∈ G, B.Rw v f g = cf σ x₀ a (dsW σ v b) g | [], _, g, hg => hf g hg | X :: v, hv, g, hg => by have hv' : LieW G v

theorem **lw_cf** (h : RepOn G σ) (hiso : ∀ g ∈ G, ∀ v, ‖σ g v‖ = ‖v‖) (x₀ : E) {a : H} (ha : IsSm G σ a) (b : H) {f : B.Mat k r → E} (hf : ∀ g ∈ G, f g = cf σ x₀ a b g) : ∀ {u : List (B.Mat k r)}, LieW G u → ∀ g ∈ G, u.foldr B.LdV f g = cf σ x₀ (dsW σ u a) b g | [], _, g, hg => hf g hg | X :: u, hu, g, hg => by have hu' : LieW G u

theorem **capelli_coefficient_smooth** : CoefSmooth

theorem **capelli_coefficient_continuity** : CoefCC

CompanionTheory.AutomorphicTheory.ZuckermanCommutation.CapelliEigenvalues.CoefEig

theorem **capC_vec** (hI : PI.DSInfCharCap.DSInfCharCap) {k r : ℕ} {ε : Fin (2 * r + k + 1) → Bool} {lam : Fin (2 * r + k + 1) → ℝ} {Hs : Type} [NormedAddCommGroup Hs] [InnerProductSpace ℂ Hs] {σ : Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ → (Hs →L[ℂ] Hs)} (hlam : IsHCParam lam) (hσ : IsDSHpk k r ε lam σ) {E : Type} [NormedAddCommGroup E] [NormedSpace ℂ E] (L : Hs →L[ℂ] E) {v : Hs} (hv : v ∈ smoothVecs (k

theorem **qminorOp_cast** {n m : ℕ} {E : Type*} [NormedAddCommGroup E] [NormedSpace ℂ E] (Eop : Fin n → Fin n → (Matrix (Fin n) (Fin n) ℂ → E) → (Matrix (Fin n) (Fin n) ℂ → E)) (h : m = n) (a : Fin m → Fin n) (ha : ∀ j, (a j).val = j.val) (u : ℂ) (f : Matrix (Fin n) (Fin n) ℂ → E) : Formalism.QuantumDeterminants.qminorOp Eop a a u f = Formalism.QuantumDeterminants.qminorOp Eop id id u f

theorem **capelli_eigenvalue_transfer** : CompanionTheory.LimBd.Main.CVBridge

theorem **coefEig_of** (hI : PI.DSInfCharCap.DSInfCharCap) (hK : PI.KFinSmooth) : CompanionTheory.LimBd.Main.CoefEig

CompanionTheory.AutomorphicTheory.ZuckermanCommutation.CapelliEigenvalues.CoefEigOfInfChar

theorem **coefEig_of_infChar** (hI : PI.DSInfCharCap.DSInfCharCap) : CompanionTheory.LimBd.Main.CoefEig

CompanionTheory.AutomorphicTheory.ZuckermanCommutation.CapelliEigenvalues.FunAct

theorem **algHom_qdet** {A B : Type} [Ring A] [Algebra ℂ A] [Ring B] [Algebra ℂ B] (φ : A →ₐ[ℂ] B) {n : ℕ} (E : Fin n → Fin n → A) (u : ℂ) : φ (qdet E u) = qdet (fun a b => φ (E a b)) u

theorem **lift_eq_sum** {X A : Type} [Ring A] [Algebra ℂ A] (f : X → A) (p : FreeAlgebra ℂ X) : FreeAlgebra.lift ℂ f p = (FreeAlgebra.equivMonoidAlgebraFreeMonoid p).coeff.sum fun w c => c • (w.toList.map f).prod

theorem **inUJ_one** : inUJ ε (1 : Matrix (Fin n) (Fin n) ℂ)

theorem **rd_vec** (hm : ∀ a b, inUJ ε a → inUJ ε b → σ (a * b) = (σ a).comp (σ b)) {v : Hs} (hv : SmoothOnGV ε fun x => σ x v) {X g : Matrix (Fin n) (Fin n) ℂ} (hX : inG ε X) (hg : inUJ ε g) : RdV X (fun x => σ x v) g = σ g (PI.PhMeasCore.dsig σ X v)

theorem **eab_vec** (hm : ∀ a b, inUJ ε a → inUJ ε b → σ (a * b) = (σ a).comp (σ b)) {v : Hs} (hv : SmoothOnGV ε fun x => σ x v) (a b : Fin n) {g : Matrix (Fin n) (Fin n) ℂ} (hg : inUJ ε g) : EabV ε a b (fun x => σ x v) g = σ g (dsigC ε σ (unitM (a, b)) v)

theorem **word_rep** (hm : ∀ a b, inUJ ε a → inUJ ε b → σ (a * b) = (σ a).comp (σ b)) {v : Hs} {x : ioS ε Hs} (hx : IORep (fun y => σ y v) x.1) : ∀ w : List (Fin n × Fin n), IORep (fun y => σ y (dsigCW ε σ (w.map unitM) v)) ((w.map fun ab => ioE ε Hs ab.1 ab.2).prod x).1 | [] => by simpa [dsigCW] using hx | ab :: w => by have ih

theorem **actU_rep** (hm : ∀ a b, inUJ ε a → inUJ ε b → σ (a * b) = (σ a).comp (σ b)) {v : Hs} {x : ioS ε Hs} (hx : IORep (fun y => σ y v) x.1) (p : FreeAlgebra ℂ (Fin n × Fin n)) : IORep (fun y => σ y (actU ε σ p v)) (FreeAlgebra.lift ℂ (fun ab : Fin n × Fin n => ioE ε Hs ab.1 ab.2) p x).1

theorem **capelli_vec** (hm : ∀ a b, inUJ ε a → inUJ ε b → σ (a * b) = (σ a).comp (σ b)) {v : Hs} (hv : SmoothOnGV ε fun x => σ x v) (u : ℂ) {g : Matrix (Fin n) (Fin n) ℂ} (hg : inUJ ε g) : capelli ε u (fun x => σ x v) g = σ g (actU ε σ (qdet (Egl n) u) v)

CompanionTheory.AutomorphicTheory.ZuckermanCommutation.CapelliEigenvalues.Transport

theorem **foldr_rel** {Eop : Fin n → Fin n → (Matrix (Fin n) (Fin n) ℂ → E) → (Matrix (Fin n) (Fin n) ℂ → E)} {Eop' : Fin n → Fin n → (Matrix (Fin n) (Fin n) ℂ → E') → (Matrix (Fin n) (Fin n) ℂ → E')} (Rel : (Matrix (Fin n) (Fin n) ℂ → E) → (Matrix (Fin n) (Fin n) ℂ → E') → Prop) (hT : ∀ u a b f f', Rel f f' → Rel (Top Eop u a b f) (Top Eop' u a b f')) {f : Matrix (Fin n) (Fin n) ℂ → E} {f' : Matrix (Fin n) (Fin n) ℂ → E'} (h : Rel f f') : ∀ (t : ℕ) (u : Fin t → ℂ) (a b : Fin t → Fin n), Rel ((List.ofFn fun k => Top Eop (u k) (a k) (b k)).foldr (fun T h => T h) f) ((List.ofFn fun k => Top Eop' (u k) (a k) (b k)).foldr (fun T h => T h) f') | 0, _, _, _ => by simpa using h | t + 1, u, a, b => by rw [List.ofFn_succ, List.ofFn_succ, List.foldr_cons, List.foldr_cons] exact hT _ _ _ _ _ (foldr_rel Rel hT h t (fun k => u k.succ) (fun k => a k.succ) (fun k => b k.succ)) theorem qminorOp_rel {Eop :

theorem **ec_congr** {f f' : Matrix (Fin n) (Fin n) ℂ → E} (h : ∀ g, inUJ ε g → f g = f' g) (Z : Matrix (Fin n) (Fin n) ℂ) {g : Matrix (Fin n) (Fin n) ℂ} (hg : inUJ ε g) : EcV ε Z f g = EcV ε Z f' g

theorem **top_smooth** {f : Matrix (Fin n) (Fin n) ℂ → E} (hf : SmoothOnGV ε f) (u : ℂ) (a b : Fin n) : SmoothOnGV ε (Top (EabV ε) u a b f)

theorem **eab_clm** (φ : E →L[ℂ] E') {f : Matrix (Fin n) (Fin n) ℂ → E} (hf : SmoothOnGV ε f) (a b : Fin n) {g : Matrix (Fin n) (Fin n) ℂ} (hg : inUJ ε g) : φ (EabV ε a b f g) = EabV ε a b (fun x => φ (f x)) g

theorem **capelli_clm** (φ : E →L[ℂ] E') {f : Matrix (Fin n) (Fin n) ℂ → E} (hf : SmoothOnGV ε f) (u : ℂ) {g : Matrix (Fin n) (Fin n) ℂ} (hg : inUJ ε g) : φ (capelli ε u f g) = capelli ε u (fun x => φ (f x)) g

theorem **capelli_localization** : PI.UnitaryGroupMassFormula.CapelliEig.Loc

theorem **capelli_transport** : PI.UnitaryGroupMassFormula.CapelliEig.CapTransport

CompanionTheory.AutomorphicTheory.ZuckermanCommutation.ZuckermanFunctors.AdjFromApply

theorem **cMT_mul** {n : ℕ} {A B : Matrix (Fin n) (Fin n) ℂ} (h : A * Bᵀ = 1) : (cM n)ᵀ * (1 : Matrix Unit Unit ℂ) = (A ⊗ₖ B) * (cM n)ᵀ

private theorem **hpk_apply_lemma** (k r : ℕ) (i l : Fin (2 * r + k + 1)) : Formalism.QuantumDeterminants.Hpk k r i l = if (l : ℕ) = (if (i : ℕ) < r ∨ r + k < i then 2 * r + k - i else i) then (if (i : ℕ) = r + k then -1 else 1) else 0

private theorem **hpk_mul_self_lemma_2** (k r : ℕ) : Formalism.QuantumDeterminants.Hpk k r * Formalism.QuantumDeterminants.Hpk k r = 1

private theorem **isUnit_det_lemma_3** {k r : ℕ} {g : Formalism.FourierDecayAlt.ThreeDimData.Mx k r} (hg : g ∈ BranchingSupport.GG k r) : IsUnit g.det

theorem **cMT_hom** (hM : MatHom) (b : Bool) (B : URep k r) : IsHom (one B) ((vr k r b).tens (vr k r (!b)), B) (mAct (cM _)ᵀ)

theorem **uR_int** (F : FRep k r) (g : Mx k r) : uR F.ι * F.ρ g = (F.tens (triv k r)).ρ g * uR F.ι

theorem **uMT_int** (F : FRep k r) (g : Mx k r) : (uM F.ι)ᵀ * ((triv k r).tens F).ρ g = F.ρ g * (uM F.ι)ᵀ

theorem **adjTo_apply** (p : (Fin n → V) →ₗ[ℂ] (Unit → V')) (x : Unit → V) (d : Fin n) : adjTo p x d = p (Pi.single d (x ())) ()

theorem **adjFrom_apply** (i : (Unit → V) →ₗ[ℂ] (Fin n → V')) (y : Fin n → V) (u : Unit) : adjFrom i y u = ∑ c, i (fun _ => y c) c

theorem **adj_of** (hC : HomCat) (hM : MatHom) (hT : TensHom) : Adj

CompanionTheory.AutomorphicTheory.ZuckermanCommutation.ZuckermanFunctors.Bridge

theorem **n1_add_le'** {n : ℕ} (x y : Fin n → ℤ) : n1 (x + y) ≤ n1 x + n1 y

theorem **n1_single_eq_natAbs** {n : ℕ} (s : Fin n) (m : ℤ) : n1 (Pi.single s m) = m.natAbs

theorem **n1_eZ_eq_card_finset** {n : ℕ} (I : Finset (Fin n)) : n1 (eZ I) = I.card

theorem **zuckG_of_sm** {t : ∀ n : ℕ, (Fin n → Bool) → ℝ} (ht : ∀ n, 1 ≤ n → ∀ ε, t n ε ≤ 43 * (n : ℝ) ^ 3) (hZ : ZuckSm t) : BranchingSupport.ZuckG

CompanionTheory.AutomorphicTheory.ZuckermanCommutation.ZuckermanFunctors.DecompositionOfInjective

theorem **decomposition_of_injective** : DecOfInj

theorem **trivial_rep_admissible** : OKTriv

theorem **vector_rep_admissible** : OKVr

CompanionTheory.AutomorphicTheory.ZuckermanCommutation.ZuckermanFunctors.IsSimpleFam

theorem **URep.repOn** (U : URep k r) : RepOn (GG k r) U.R.σ where

theorem **URep.sig_coe** (U : URep k r) {g : Mx k r} (hg : g ∈ GG k r) (v : U.R.smSub) : (U.sig g v : U.R.H) = U.R.σ g v

theorem **URep.dsig_coe** (U : URep k r) {X : Mx k r} (hX : InLieHpk k r X) (v : U.R.smSub) : (U.dsig X v : U.R.H) = PI.UpqRep.dsig (k

CompanionTheory.AutomorphicTheory.ZuckermanCommutation.ZuckermanFunctors.MultOne

theorem **mAct_apply** {ι ι' : Type} [Fintype ι] {V : Type} [AddCommGroup V] [Module ℂ V] (A : Matrix ι' ι ℂ) (x : ι → V) (c : ι') : mAct A x c = ∑ d, A c d • x d

theorem **mAct_mul** {ι ι' ι'' : Type} [Fintype ι] [Fintype ι'] {V : Type} [AddCommGroup V] [Module ℂ V] (A : Matrix ι'' ι' ℂ) (B : Matrix ι' ι ℂ) : (mAct (A * B) : (ι → V) →ₗ[ℂ] _) = mAct A ∘ₗ mAct B

theorem **mAct_smul** {ι ι' : Type} [Fintype ι] {V : Type} [AddCommGroup V] [Module ℂ V] (a : ℂ) (A : Matrix ι' ι ℂ) : (mAct (a • A) : (ι → V) →ₗ[ℂ] _) = a • mAct A

theorem **mAct_sub** {ι ι' : Type} [Fintype ι] {V : Type} [AddCommGroup V] [Module ℂ V] (A B : Matrix ι' ι ℂ) : (mAct (A - B) : (ι → V) →ₗ[ℂ] _) = mAct A - mAct B

theorem **mAct_one** {ι : Type} [Fintype ι] [DecidableEq ι] {V : Type} [AddCommGroup V] [Module ℂ V] : (mAct (1 : Matrix ι ι ℂ) : (ι → V) →ₗ[ℂ] _) = LinearMap.id

theorem **hom_cas** (hL : HomLie) {X Y : Obj k r} {f : M X →ₗ[ℂ] M Y} (hX : X.1.OK) (hY : Y.1.OK) (hf : IsHom X Y f) (x : M X) : f (cas X x) = cas Y (f x)

theorem **cM_mul** {n : ℕ} {A B : Matrix (Fin n) (Fin n) ℂ} (h : Aᵀ * B = 1) : cM n * (A ⊗ₖ B) = (1 : Matrix Unit Unit ℂ) * cM n

theorem **cM_hom** (hM : MatHom) (b : Bool) (B : URep k r) : IsHom ((vr k r b).tens (vr k r (!b)), B) (one B) (mAct (cM _))

theorem **casimir_kronecker_sum** (n : ℕ) : (∑ a : Fin n, ∑ b : Fin n, Matrix.single a b (1 : ℂ) ⊗ₖ (-(Matrix.single b a (1 : ℂ))ᵀ) = (-1 : ℂ) • ((cM n)ᵀ * cM n)) ∧ (∑ a : Fin n, ∑ b : Fin n, (-(Matrix.single a b (1 : ℂ))ᵀ) ⊗ₖ Matrix.single b a (1 : ℂ) = (-1 : ℂ) • ((cM n)ᵀ * cM n))

theorem **qMat_vr** (hS : StdD) (b : Bool) : qMat (vr k r b) (vr k r (!b)) = (-1 : ℂ) • ((cM _)ᵀ * cM _ : Matrix _ _ ℂ)

theorem **uM_int** (F : FRep k r) (g : Mx k r) : uM F.ι * F.ρ g = ((triv k r).tens F).ρ g * uM F.ι

theorem **multOne_of** (hC : HomCat) (hM : MatHom) (hT : TensHom) (hL : HomLie) (hS : CasSplit) (hD : StdD) (hK : KerZero) (o1 : OKTriv) (o2 : OKVr) (o3 : OKTens) : MultOne

CompanionTheory.AutomorphicTheory.ZuckermanCommutation.ZuckermanFunctors.ShiftApply

theorem **dec_iso** (C : IsLinCat N Hom) {κ : Type} [Fintype κ] {X : O} {T T' : κ → O} (h : ∃ io pr, IsDec Hom X T io pr) (e : ∀ s, IsIso Hom (T s) (T' s)) : ∃ io pr, IsDec Hom X T' io pr

theorem **shift_shift** (b : Bool) (μ : Fin n → ℝ) (s : Fin n) : shift (!b) (shift b μ s) s = μ

theorem **shift_apply** (b : Bool) (μ : Fin n → ℝ) (s a : Fin n) : shift b μ s a = μ a + if a = s then (if b then -1 else 1) else 0

theorem **gap_shift** {ε : Fin n → Bool} {μ : Fin n → ℝ} {h : ℝ} (hg : GapAtLeast ε μ h) (b : Bool) (s : Fin n) : GapAtLeast ε (shift b μ s) (h - 1)

theorem **casG_shift** (b : Bool) (μ : Fin n → ℝ) (s : Fin n) : casG (shift b μ s) = casG μ + 2 * (if b then -1 else 1) * μ s + 1

theorem **cas_ne** {μ : Fin n → ℝ} (hμ : StrictAnti μ) (b : Bool) (s t : Fin n) : (casG (shift b (shift b μ s) t) : ℂ) ≠ 2 * (casG (shift b μ s) : ℂ) - (casG μ : ℂ)

theorem **shift_inj** (b : Bool) (μ : Fin n → ℝ) : Function.Injective (shift b μ)

theorem **near** {ε : Fin n → Bool} {μ : Fin n → ℝ} {h D : ℝ} (hμ : IsHCParam μ) (hg : GapAtLeast ε μ h) (hD : ∀ a c, a < c → D ≤ μ a - μ c) {ν : Fin n → ℝ} (j : ℕ) (hj : (j : ℝ) < D) (hx : ∀ a, ∃ x : ℕ, x ≤ j ∧ ν a = μ a + x) : IsHCParam ν ∧ GapAtLeast ε ν (h - j) ∧ ∀ a c, a < c → D - j ≤ ν a - ν c

theorem **ind_le** {m : ℕ} (J : Sub n m) (a : Fin n) : (if a ∈ J.1 then 1 else 0 : ℕ) ≤ m

theorem **addSet_inj** {l : ℕ} (μ : Fin n → ℝ) : Function.Injective fun I : Sub n l => addSet μ I.1

theorem **iso_of** (S : Steps) (Lb : Lib t) {μ : Fin (2 * r + k + 1) → ℝ} {U U' : URep k r} {h : ℝ} (hU : U.R.IsDS ε μ) (hU' : U'.R.IsDS ε μ) (hg : GapAtLeast ε μ h) (ht : t _ ε < h) : IsIso (N

theorem **injV** (S : Steps) (Lb : Lib t) (b : Bool) {μ : Fin (2 * r + k + 1) → ℝ} {U : URep k r} {h : ℝ} (hU : U.R.IsDS ε μ) (hg : GapAtLeast ε μ h) (ht : t _ ε + (((2 * r + k + 1) ^ 2 + 2 : ℕ) : ℝ) < h) : ∃ T : {s // IsHCParam (shift b μ s)} → URep k r, (∀ s, (T s).R.IsDS ε (shift b μ s)) ∧ JInj (N

theorem **decV** (S : Steps) (Lb : Lib t) (hk : 1 ≤ k) (b : Bool) {μ : Fin (2 * r + k + 1) → ℝ} {U : URep k r} {h : ℝ} (hU : U.R.IsDS ε μ) (hg : GapAtLeast ε μ h) (ht : t _ ε + (((2 * r + k + 1) ^ 2 + 2 : ℕ) : ℝ) + 1 < h) (T : Fin (2 * r + k + 1) → URep k r) (hT : ∀ s, (T s).R.IsDS ε (shift b μ s)) : ∃ io pr, IsDec (N

theorem **decW** (S : Steps) (Lb : Lib t) (hk : 1 ≤ k) : ∀ (l : ℕ) {μ : Fin (2 * r + k + 1) → ℝ} {U : URep k r} {h : ℝ}, U.R.IsDS ε μ → GapAtLeast ε μ h → t _ ε + (((2 * r + k + 1) ^ 2 + 2 : ℕ) : ℝ) + 1 + l < h → (∀ a c, a < c → (l : ℝ) + 1 ≤ μ a - μ c) → ∀ T : Sub (2 * r + k + 1) l → URep k r, (∀ I, (T I).R.IsDS ε (addSet μ I.1)) → ∃ io pr, IsDec (N

theorem **zuckSm_of** (S : Steps) (Lb : Lib t) : ZuckSm t

CompanionTheory.AutomorphicTheory.ZuckermanCommutation.ZuckermanFunctors.ZuckDualTThrS

theorem **zuckT_thrS** : CompanionTheory.LimBd.Main.ZuckT fun _ ε => ReciprocityTheory.ChartRestriction.DS.G2.thrS ε

theorem **zuckDualT_thrS** : CompanionTheory.LimBd.Main.ZuckDualT fun _ ε => ReciprocityTheory.ChartRestriction.DS.G2.thrS ε

CompanionTheory.AutomorphicTheory.ZuckermanCommutation.ZuckermanFunctors.ZuckermanGapSmall

theorem **zuckerman_gap_small** : ZuckSm thr

theorem **zuckerman_gap** : BranchingSupport.ZuckG

CompanionTheory.AutomorphicTheory.ZuckermanCommutation.ZuckermanFunctors.ZuckermanIngredients

theorem **zuckerman_ingredients** : Steps where

CompanionTheory.AutomorphicTheory.ZuckermanCommutation.ZuckermanFunctors.ZuckermanLibraryBounds

theorem **thr_le** {n : ℕ} (hn : 1 ≤ n) (ε : Fin n → Bool) : thr n ε ≤ 43 * (n : ℝ) ^ 3

theorem **dsigW_SK** {k r : ℕ} {H : Type} [NormedAddCommGroup H] [InnerProductSpace ℂ H] (σ : Mx k r → (H →L[ℂ] H)) : ∀ (W : List (Mx k r)) (v : H), Formalism.MainNotions.S.dsigW (k

theorem **zuckerman_library_bounds** : Lib thr where

CompanionTheory.HarishChandraModules.BesselParameterIntegrals.BesselOrbitConvolutions.AtForm

theorem **smooth_K_finite_form_one_side** : CompanionTheory.D.Upq.BesselFunctionalConstruction.AtFormOneOfSteps

CompanionTheory.HarishChandraModules.BesselParameterIntegrals.BesselOrbitConvolutions.AtFormAdm

theorem **smooth_admissible_form_one_side** : CompanionTheory.D.Upq.BesselFunctionalConstruction.AtFormAdmOneOfSteps

CompanionTheory.HarishChandraModules.BesselParameterIntegrals.BesselOrbitConvolutions.AtFormAdmC

theorem **smooth_admissible_form_one_side** : CompanionTheory.D.Upq.BesselFunctionalConstruction.C.AtFormAdmOneOfSteps

CompanionTheory.HarishChandraModules.BesselParameterIntegrals.BesselOrbitConvolutions.AtFormC

theorem **smooth_K_finite_form_one_side** : CompanionTheory.D.Upq.BesselFunctionalConstruction.C.AtFormOneOfSteps

CompanionTheory.HarishChandraModules.BesselParameterIntegrals.BesselOrbitConvolutions.BeBessel

theorem **beBessel_of** (hGS : Formalism.FourierDecayAlt.ThreeDimData.GrowthS) : CompanionTheory.D.Upq.BesselFunctionalConstruction.BeBessel

CompanionTheory.HarishChandraModules.BesselParameterIntegrals.BesselOrbitConvolutions.BeConv

theorem **bessel_integral_convergence_of_growth** (hGS : Formalism.FourierDecayAlt.ThreeDimData.GrowthS) : CompanionTheory.D.Upq.BesselFunctionalConstruction.BeConv

CompanionTheory.HarishChandraModules.BesselParameterIntegrals.BesselOrbitConvolutions.BesselNonvanishingFromL2

theorem **bessel_nonvanishing_from_L2** : CompanionTheory.D.Upq.BesselFunctionalConstruction.BesselNonvanishingReduction

CompanionTheory.HarishChandraModules.BesselParameterIntegrals.BesselOrbitConvolutions.BesselNonvanishingL2

theorem **decDD** : DecDD

theorem **beConv** : BeConv

theorem **beBessel** : BeBessel

theorem **normIdSm** : NormIdSm

theorem **kFinForm** : KFinForm

theorem **zeroCase** : ZeroCase

theorem **zeroCaseDD** : ZeroCaseDD

theorem **ooConv** : OOConv

theorem **quadConv** : QuadConv

theorem **bessel_nonvanishing_L2** : BesselNonvanishingL2

CompanionTheory.HarishChandraModules.BesselParameterIntegrals.BesselOrbitConvolutions.DDForm

theorem **discrete_form_one_side** : CompanionTheory.D.Upq.BesselFunctionalConstruction.DDFormOneOfSteps

CompanionTheory.HarishChandraModules.BesselParameterIntegrals.BesselOrbitConvolutions.DDFormC

theorem **discrete_form_one_side** : CompanionTheory.D.Upq.BesselFunctionalConstruction.C.DDFormOneOfSteps

CompanionTheory.HarishChandraModules.BesselParameterIntegrals.BesselOrbitConvolutions.DoubleCosetDecomposition

theorem **double_coset_decomposition** : CompanionTheory.D.Upq.BesselFunctionalConstruction.DecDD

CompanionTheory.HarishChandraModules.BesselParameterIntegrals.BesselOrbitConvolutions.FinalC

theorem **exists_move** {z : Fin (2 * r + k + 1) → ℂ} (hz : inWb k r z) {c : ℂ} (hc : c ≠ 0) : ∃ b ∈ UH (Hpk k r), (∀ h ∈ UWset k r z, b * h = h * b) ∧ ∀ m ∈ Nset k r, b * m * b⁻¹ ∈ Nset k r ∧ b⁻¹ * m * b ∈ Nset k r ∧ psi k r z cStd (b * m * b⁻¹) = psi k r z c m

theorem **isHaarOn_map_cjE** {b : Mx k r} (hb : b ∈ UH (Hpk k r)) {N : Set (Mx k r)} (hN : ∀ m ∈ N, b * m * b⁻¹ ∈ N ∧ b⁻¹ * m * b ∈ N) {μ : Measure (Mt k r)} (hμ : PI.BesselPeriodGGP.IsHaarOn N μ) : PI.BesselPeriodGGP.IsHaarOn N (μ.map (cjE hb))

theorem **be_rank_one_scale** (hS : Side k r ε lam h σ w0 z c u π μW μN μG νq) : ∃ L : H → Wp, ∀ w ∈ PI.UpqRep.smoothVecs (k

theorem **shapeC** : C.Shape

theorem **dDForm_scale** (hS : Side k r ε lam h σ w0 z c u π μW μN μG νq) : DDForm z c π μW μN μG νq σ w0

theorem **atForm_scale** (hS : Side k r ε lam h σ w0 z c u π μW μN μG νq) : AtForm z c π μW μN μG νq σ

theorem **atFormAdm_scale** (hS : Side k r ε lam h σ w0 z c u π μW μN μG νq) : AtFormAdm z c π μW μN μG νq σ w0

CompanionTheory.HarishChandraModules.BesselParameterIntegrals.BesselOrbitConvolutions.JacquetFunctionalNormalization

theorem **inputs_holds** : Inputs

theorem **rankOne_holds** : RankOne

CompanionTheory.HarishChandraModules.BesselParameterIntegrals.BesselOrbitConvolutions.KFinFormOf

theorem **shape_of** (hR : RankOne) (hBB : BeBessel) : Shape

theorem **bessel_add** {k r : ℕ} {H Wp : Type} [NormedAddCommGroup H] [InnerProductSpace ℂ H] [NormedAddCommGroup Wp] [NormedSpace ℂ Wp] {z : Fin (2 * r + k + 1) → ℂ} {ψ : Mx k r → ℂ} {σ : Mx k r → (H →L[ℂ] H)} {π : Mx k r → (Wp →L[ℂ] Wp)} {B B' : H → Wp} (hB : PI.BesselMapUniq.IsBesselMapSm (k

theorem **kFinForm_of** (hBC : BeConv) (hBB : BeBessel) : KFinForm

CompanionTheory.HarishChandraModules.BesselParameterIntegrals.BesselOrbitConvolutions.NormId

theorem **normIdSm_of** (hGS : Formalism.FourierDecayAlt.ThreeDimData.GrowthS) (hGG : Formalism.FourierDecayAlt.ThreeDimData.GrowthG) (hMS : MeasSideP) : CompanionTheory.D.Upq.BesselFunctionalConstruction.NormIdSm

CompanionTheory.HarishChandraModules.BesselParameterIntegrals.BesselOrbitConvolutions.OOConv

theorem **ooConv_of** (hSch : SchurH) (hNIs : CompanionTheory.D.Upq.BesselFunctionalConstruction.NormIdSm) (hNI : NormInv) (hGG : GrowthG) (hDD : CompanionTheory.D.Upq.BesselFunctionalConstruction.DecDD) (hBC : CompanionTheory.D.Upq.BesselFunctionalConstruction.BeConv) : CompanionTheory.D.Upq.BesselFunctionalConstruction.OOConv

CompanionTheory.HarishChandraModules.BesselParameterIntegrals.BesselOrbitConvolutions.PhiReg

theorem **phiReg_of** (hSch : SchurH) (hNI : CompanionTheory.D.Upq.BesselFunctionalConstruction.NormIdSm) (hGG : GrowthG) (hDD : CompanionTheory.D.Upq.BesselFunctionalConstruction.DecDD) (hBC : CompanionTheory.D.Upq.BesselFunctionalConstruction.BeConv) : CompanionTheory.D.Upq.BesselFunctionalConstruction.PhiReg

CompanionTheory.HarishChandraModules.BesselParameterIntegrals.BesselOrbitConvolutions.QuadAOf

theorem **quadA_of** (hSch : SchurH) (hNIs : CompanionTheory.D.Upq.BesselFunctionalConstruction.NormIdSm) (hNI : NormInv) (hGG : GrowthG) (hDD : CompanionTheory.D.Upq.BesselFunctionalConstruction.DecDD) (hBC : CompanionTheory.D.Upq.BesselFunctionalConstruction.BeConv) (hIn : CompanionTheory.D.Upq.BesselFunctionalConstruction.Inputs) {k r : ℕ} {Wp Wq : Type} [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [CompleteSpace Wq] [FiniteDimensional ℂ Wq] (D : B.RealPlaceData k r Wp Wq) {ε : Fin (2 * r + k + 1) → Bool} {lam : Fin (2 * r + k + 1) → ℝ} {h : ℝ} {H : Type} [NormedAddCommGroup H] [InnerProductSpace ℂ H] [CompleteSpace H] {σ : Mx k r → (H →L[ℂ] H)} {w0 : H} {u u' : Fin k → ℝ} {νq νq' : Measure (Formalism.QuantumDeterminants.Mt k r)} (hS1 : Side k r ε lam h σ w0 D.z Formalism.QuantumDeterminants.c

CompanionTheory.HarishChandraModules.BesselParameterIntegrals.BesselOrbitConvolutions.QuadBOf

theorem **quadB_of** (hNI : NormInv) (hGS : GrowthS) (hGG : GrowthG) (hDD : CompanionTheory.D.Upq.BesselFunctionalConstruction.DecDD) (hIn : CompanionTheory.D.Upq.BesselFunctionalConstruction.Inputs) {k r : ℕ} {Wp Wq : Type} [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] (D : B.RealPlaceData k r Wp Wq) {ε : Fin (2 * r + k + 1) → Bool} {lam : Fin (2 * r + k + 1) → ℝ} {h : ℝ} {H : Type}

[NormedAddCommGroup H] [InnerProductSpace ℂ H] [CompleteSpace H] {σ : Mx k r → (H →L[ℂ] H)} {w0 : H} {u : Fin k → ℝ} {νq : Measure (Formalism.QuantumDeterminants.Mt k r)} (hS1 : Side k r ε lam h σ w0 D.z Formalism.QuantumDeterminants.cStd u D.π D.μW D.μN D.μG νq) {l : H → Wp} (hbe : ∀ x ∈ DDsp σ w0, ∀ y ∈ DDsp σ w0, be D.z Formalism.QuantumDeterminants.cStd D.π σ D.μW D.μN x y = (innerSL ℂ (l y)).smulRight (l x)) : ∫⁻

CompanionTheory.HarishChandraModules.BesselParameterIntegrals.BesselOrbitConvolutions.VanishingOfL2Equivariant

theorem **vanishing_of_L2_equivariant** {k r : ℕ} {ε : Fin (2 * r + k + 1) → Bool} {lam : Fin (2 * r + k + 1) → ℝ} {h : ℝ} {H : Type} [NormedAddCommGroup H] [InnerProductSpace ℂ H] [CompleteSpace H] {σ : Mx k r → (H →L[ℂ] H)} {w0 : H} {Wp : Type} [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [CompleteSpace Wp] [FiniteDimensional ℂ Wp] {z : Fin (2 * r + k + 1) → ℂ} {c : ℂ} {u : Fin k → ℝ} {π : Mx k r → (Wp →L[ℂ] Wp)} {μW μN μG νq : Measure (Formalism.QuantumDeterminants.Mt k r)} (hBC : CompanionTheory.D.Upq.BesselFunctionalConstruction.BeConv) (hIn : CompanionTheory.D.Upq.BesselFunctionalConstruction.Inputs) (hS : Side k r ε lam h σ w0 z c u π μW μN μG νq) {x : H} (hx : x ∈ PI.UpqRep.smoothVecs (k

theorem **zeroCaseDD_of** (hBC : CompanionTheory.D.Upq.BesselFunctionalConstruction.BeConv) (hNI : CompanionTheory.D.Upq.BesselFunctionalConstruction.NormIdSm) (hSch : SchurH) : CompanionTheory.D.Upq.BesselFunctionalConstruction.ZeroCaseDD

theorem **zeroCase_of** (hBC : CompanionTheory.D.Upq.BesselFunctionalConstruction.BeConv) (hNI : CompanionTheory.D.Upq.BesselFunctionalConstruction.NormIdSm) (hSch : SchurH) : CompanionTheory.D.Upq.BesselFunctionalConstruction.ZeroCase

CompanionTheory.HarishChandraModules.BesselParameterIntegrals.CoePolarExpHomeomorphComplexApply

theorem **continuousOn_matrix_inv** : ContinuousOn (fun A : Matrix n n 𝕜 => A⁻¹) {A | IsUnit A}

theorem **posDef_conjTranspose_mul_self_GL** (g : GL n 𝕜) : ((g : Matrix n n 𝕜)ᴴ * g).PosDef

theorem **polarPos_posDef** (g : GL n 𝕜) : (polarPos g).PosDef

theorem **polarPos_mul_self** (g : GL n 𝕜) : polarPos g * polarPos g = (g : Matrix n n 𝕜)ᴴ * g

theorem **isUnit_det_of_posDef** {p : Matrix n n 𝕜} (hp : p.PosDef) : IsUnit p.det

theorem **polarUnitary_mem** (g : GL n 𝕜) : polarUnitary g ∈ unitaryGroup n 𝕜

theorem **polarUnitary_mul_polarPos** (g : GL n 𝕜) : polarUnitary g * polarPos g = g

theorem **polar_unique** {g : GL n 𝕜} {k p : Matrix n n 𝕜} (hk : k ∈ unitaryGroup n 𝕜) (hp : p.PosDef) (h : k * p = g) : polarPos g = p ∧ polarUnitary g = k

@[simp] theorem **coe_polarEquiv_apply** (kp : unitaryGroup n 𝕜 × {p : Matrix n n 𝕜 // p.PosDef}) : ((polarEquiv kp : GL n 𝕜) : Matrix n n 𝕜) = kp.1 * kp.2

theorem **continuous_polarPos** (hsqrt : ContinuousOn (CFC.sqrt : Matrix n n 𝕜 → Matrix n n 𝕜) {a | 0 ≤ a}) : Continuous (polarPos : GL n 𝕜 → Matrix n n 𝕜)

theorem **posDef_exp_of_isHermitian_complex** {X : Matrix n n ℂ} (hX : X.IsHermitian) : (NormedSpace.exp X).PosDef

theorem **posDef_exp_of_isHermitian_real** {X : Matrix n n ℝ} (hX : X.IsHermitian) : (NormedSpace.exp X).PosDef

CompanionTheory.HarishChandraModules.BesselParameterIntegrals.ContDiffOnExpPartialHomeomorphSymm

theorem **contDiff_exp** {n : WithTop ℕ∞} : ContDiff 𝕂 n (exp : 𝔸 → 𝔸)

theorem **exp_add_of_commute'** {x y : 𝔸} (h : Commute x y) : exp (x + y) = exp x * exp y

theorem **isOpen_expRegularSet** : IsOpen (expRegularSet 𝕂 𝔸)

theorem **zero_mem_expRegularSet** : (0 : 𝔸) ∈ expRegularSet 𝕂 𝔸

theorem **hasStrictFDerivAt_exp_zero_equiv** : HasStrictFDerivAt (exp : 𝔸 → 𝔸) ((ContinuousLinearEquiv.refl 𝕂 𝔸 : 𝔸 →L[𝕂] 𝔸)) 0

theorem **coe_expPartialHomeomorph** : ⇑(expPartialHomeomorph 𝕂 𝔸) = exp

theorem **zero_mem_expPartialHomeomorph_source** : (0 : 𝔸) ∈ (expPartialHomeomorph 𝕂 𝔸).source

theorem **one_mem_expPartialHomeomorph_target** : (1 : 𝔸) ∈ (expPartialHomeomorph 𝕂 𝔸).target

theorem **expPartialHomeomorph_symm_one** : (expPartialHomeomorph 𝕂 𝔸).symm 1 = 0

theorem **expPartialHomeomorph_source_subset** : (expPartialHomeomorph 𝕂 𝔸).source ⊆ expRegularSet 𝕂 𝔸

theorem **contDiffOn_expPartialHomeomorph_symm** {n : WithTop ℕ∞} : ContDiffOn 𝕂 n (expPartialHomeomorph 𝕂 𝔸).symm (expPartialHomeomorph 𝕂 𝔸).target

@[simp] theorem **coe_expUnit** (X : 𝔸) : (expUnit 𝕂 𝔸 X : 𝔸) = exp X

@[simp] theorem **coe_expUnit_inv** (X : 𝔸) : ((expUnit 𝕂 𝔸 X)⁻¹ : 𝔸ˣ) = exp (-X)

theorem **continuous_expUnit** : Continuous (expUnit 𝕂 𝔸)

theorem **mem_expChartUnits_source** (g : 𝔸ˣ) : g ∈ (expChartUnits 𝕂 g).source

theorem **expChartUnits_self** (g : 𝔸ˣ) : expChartUnits 𝕂 g g = 0

theorem **expChartUnits_symm_apply** (g : 𝔸ˣ) (X : 𝔸) : (((expChartUnits 𝕂 g).symm X : 𝔸ˣ) : 𝔸) = g * exp X

theorem **expChartUnits_mem_maximalAtlas** (g : 𝔸ˣ) (n : WithTop ℕ∞) : expChartUnits 𝕂 g ∈ IsManifold.maximalAtlas 𝓘(𝕂, 𝔸) n 𝔸ˣ

theorem **lieGroup_GL** (𝕂 : Type*) [RCLike 𝕂] (n : ℕ) (k : WithTop ℕ∞) : LieGroup 𝓘(𝕂, Matrix (Fin n) (Fin n) 𝕂) k (GL (Fin n) 𝕂)

theorem **lieGroup_GL_complex_real** (n : ℕ) (k : WithTop ℕ∞) : LieGroup 𝓘(ℝ, Matrix (Fin n) (Fin n) ℂ) k (GL (Fin n) ℂ)

theorem **expChart_GL_mem_maximalAtlas** (𝕂 : Type*) [RCLike 𝕂] (n : ℕ) (g : GL (Fin n) 𝕂) (k : WithTop ℕ∞) : expChartUnits 𝕂 g ∈ IsManifold.maximalAtlas 𝓘(𝕂, Matrix (Fin n) (Fin n) 𝕂) k (GL (Fin n) 𝕂)

@[simp] theorem **subspaceProj_coe** (𝔤 : Submodule ℝ 𝔸) (X : 𝔤) : subspaceProj 𝔤 (X : 𝔸) = X

theorem **subspaceProj_of_mem** {𝔤 : Submodule ℝ 𝔸} {X : 𝔸} (hX : X ∈ 𝔤) : (subspaceProj 𝔤 X : 𝔸) = X

theorem **continuous_val_val** (G : Subgroup 𝔸ˣ) : Continuous (fun x : G => ((x : 𝔸ˣ) : 𝔸))

theorem **isOpen_nhd** : IsOpen h.nhd

theorem **zero_mem_nhd** : (0 : 𝔸) ∈ h.nhd

theorem **nhd_subset** : h.nhd ⊆ (ℰ).source

theorem **mem_of_mem_nhd** {X : 𝔸} (hX : X ∈ h.nhd) (hG : expUnit ℝ 𝔸 X ∈ G) : X ∈ 𝔤

theorem **isOpen_nhdOne** : IsOpen h.nhdOne

theorem **nhdOne_subset_target** : h.nhdOne ⊆ (ℰ).target

theorem **symm_mem_nhd** {a : 𝔸} (ha : a ∈ h.nhdOne) : (ℰ).symm a ∈ h.nhd

theorem **exp_symm** {a : 𝔸} (ha : a ∈ h.nhdOne) : exp ((ℰ).symm a) = a

theorem **exp_mem_nhdOne** {X : 𝔸} (hX : X ∈ h.nhd) : exp X ∈ h.nhdOne

theorem **symm_exp** {X : 𝔸} (hX : X ∈ h.nhd) : (ℰ).symm (exp X) = X

theorem **one_mem_nhdOne** : (1 : 𝔸) ∈ h.nhdOne

@[simp] theorem **coe_expG** (X : 𝔤) : ((h.expG X : 𝔸ˣ) : 𝔸) = exp (X : 𝔸)

theorem **continuous_expG** : Continuous h.expG

theorem **symm_mem_lie** {x : G} (hx : ((x : 𝔸ˣ) : 𝔸) ∈ h.nhdOne) : (ℰ).symm ((x : 𝔸ˣ) : 𝔸) ∈ 𝔤

theorem **contDiffOn_proj_symm_mul_exp** {n : WithTop ℕ∞} (c : 𝔸) (s : Set 𝔤) (hs : ∀ X ∈ s, c * exp (X : 𝔸) ∈ (ℰ).target) : ContDiffOn ℝ n (fun X : 𝔤 => subspaceProj 𝔤 ((ℰ).symm (c * exp (X : 𝔸)))) s

theorem **isManifold** (n : WithTop ℕ∞) : letI

theorem **contMDiff_val** (n : WithTop ℕ∞) : letI

theorem **contMDiffAt_of_val** {E : Type*} [NormedAddCommGroup E] [NormedSpace ℝ E] {H : Type*} [TopologicalSpace H] {I : ModelWithCorners ℝ E H} {M : Type*} [TopologicalSpace M] [ChartedSpace H M] {n : WithTop ℕ∞} {f : M → G} {x : M} (hf : ContMDiffAt I 𝓘(ℝ, 𝔸) n (fun y => ((f y : 𝔸ˣ) : 𝔸)) x) : letI

theorem **lieGroup** (n : WithTop ℕ∞) : letI

theorem **indefJ_mul_self** : indefJ p q * indefJ p q = 1

theorem **indefJ_conjTranspose** : (indefJ p q)ᴴ = indefJ p q

theorem **isUnit_indefJ** : IsUnit (indefJ p q)

theorem **inv_indefJ** : (indefJ p q)⁻¹ = indefJ p q

theorem **norm_indefJ_le** : ‖indefJ p q‖ ≤ 1

theorem **mem_indefiniteUnitaryLieAlgebra_iff** {X : 𝔐} : X ∈ indefiniteUnitaryLieAlgebra p q ↔ Xᴴ = -(indefJ p q * X * indefJ p q)

theorem **mem_indefiniteUnitaryGroup_iff** {g : GL (Fin p ⊕ Fin q) ℂ} : g ∈ indefiniteUnitaryGroup p q ↔ g.valᴴ * indefJ p q * g.val = indefJ p q

theorem **exp_neg_conj_indefJ** (X : 𝔐) : exp (-(indefJ p q * X * indefJ p q)) = indefJ p q * exp (-X) * indefJ p q

theorem **exp_mul_exp_neg** (X : 𝔐) : exp X * exp (-X) = 1

theorem **exp_neg_mul_exp** (X : 𝔐) : exp (-X) * exp X = 1

theorem **isExpLieAlgebra_indefiniteUnitary** : IsExpLieAlgebra (indefiniteUnitaryGroup p q) (indefiniteUnitaryLieAlgebra p q) where

CompanionTheory.HarishChandraModules.BesselParameterIntegrals.ExistsUniqueComplexification

theorem **mem_lieJ** {X : Matrix n n ℂ} : X ∈ lieJ J ↔ Xᴴ * J + J * X = 0

theorem **conjTranspose_mul_eq_of_mem** {X : Matrix n n ℂ} (hX : X ∈ lieJ J) : Xᴴ * J = -(J * X)

theorem **commutator_mem_lieJ** {X Y : Matrix n n ℂ} (hX : X ∈ lieJ J) (hY : Y ∈ lieJ J) : X * Y - Y * X ∈ lieJ J

@[simp] theorem **mem_lieSubJ** {X : Matrix n n ℂ} : X ∈ lieSubJ J ↔ X ∈ lieJ J

theorem **lie_mem_lieSubJ** {X Y : Matrix n n ℂ} (hX : X ∈ lieSubJ J) (hY : Y ∈ lieSubJ J) : ⁅X, Y⁆ ∈ lieSubJ J

theorem **mul_J_J** (Z : Matrix n n ℂ) : Z * J * J = Z

theorem **J_J_mul** (Z : Matrix n n ℂ) : J * (J * Z) = Z

theorem **mem_lieJ_iff** {X : Matrix n n ℂ} : X ∈ lieJ J ↔ Xᴴ = -(J * X * J)

theorem **conjTranspose_mem_lieJ** {X : Matrix n n ℂ} (hX : X ∈ lieJ J) : Xᴴ ∈ lieJ J

theorem **J_mul_mul_J_of_mem** {X : Matrix n n ℂ} (hX : X ∈ lieJ J) : J * X * J = -Xᴴ

theorem **J_mul_mul_J_mem** {X : Matrix n n ℂ} (hX : X ∈ lieJ J) : J * X * J ∈ lieJ J

theorem **mem_kJ** {X : Matrix n n ℂ} : X ∈ kJ J ↔ X ∈ lieJ J ∧ X * J = J * X

theorem **mem_pJ** {X : Matrix n n ℂ} : X ∈ pJ J ↔ X ∈ lieJ J ∧ X * J = -(J * X)

theorem **kJ_le** : kJ J ≤ lieJ J

theorem **pJ_le** : pJ J ≤ lieJ J

theorem **mem_kJ_iff** {X : Matrix n n ℂ} : X ∈ kJ J ↔ X ∈ lieJ J ∧ Xᴴ = -X

theorem **mem_pJ_iff** {X : Matrix n n ℂ} : X ∈ pJ J ↔ X ∈ lieJ J ∧ Xᴴ = X

theorem **kJ_inf_pJ** : kJ J ⊓ pJ J = ⊥

theorem **kJ_sup_pJ** : kJ J ⊔ pJ J = lieJ J

theorem **commutator_mem_kJ** {X Y : Matrix n n ℂ} (hX : X ∈ kJ J) (hY : Y ∈ kJ J) : X * Y - Y * X ∈ kJ J

theorem **commutator_mem_pJ** {X Y : Matrix n n ℂ} (hX : X ∈ kJ J) (hY : Y ∈ pJ J) : X * Y - Y * X ∈ pJ J

theorem **commutator_mem_kJ_of_pJ** {X Y : Matrix n n ℂ} (hX : X ∈ pJ J) (hY : Y ∈ pJ J) : X * Y - Y * X ∈ kJ J

theorem **thetaG_add** (M N : Matrix n n ℂ) : thetaG J (M + N) = thetaG J M + thetaG J N

theorem **thetaG_sub** (M N : Matrix n n ℂ) : thetaG J (M - N) = thetaG J M - thetaG J N

theorem **thetaG_smul** (c : ℂ) (M : Matrix n n ℂ) : thetaG J (c • M) = star c • thetaG J M

theorem **thetaG_thetaG** (M : Matrix n n ℂ) : thetaG J (thetaG J M) = M

theorem **mem_lieJ_iff_thetaG** {X : Matrix n n ℂ} : X ∈ lieJ J ↔ thetaG J X = X

theorem **reG_mem** (M : Matrix n n ℂ) : reG J M ∈ lieJ J

theorem **imG_mem** (M : Matrix n n ℂ) : imG J M ∈ lieJ J

theorem **reG_add_I_smul_imG** (M : Matrix n n ℂ) : M = reG J M + Complex.I • imG J M

theorem **eq_reG_imG_of_eq** {M X Y : Matrix n n ℂ} (hX : X ∈ lieJ J) (hY : Y ∈ lieJ J) (hM : M = X + Complex.I • Y) : X = reG J M ∧ Y = imG J M

theorem **existsUnique_complexification** (M : Matrix n n ℂ) : ∃! XY : Matrix n n ℂ × Matrix n n ℂ, XY.1 ∈ lieJ J ∧ XY.2 ∈ lieJ J ∧ M = XY.1 + Complex.I • XY.2

CompanionTheory.HarishChandraModules.BesselParameterIntegrals.InPCommutatorOfInK

theorem **exp_conj_J** (hJJ : J * J = 1) (X : Matrix m m ℂ) : exp (J * X * J) = J * exp X * J

theorem **exp_mul_exp_neg'** (X : Matrix m m ℂ) : exp X * exp (-X) = 1

theorem **exp_neg_mul_exp'** (X : Matrix m m ℂ) : exp (-X) * exp X = 1

theorem **isExpLieAlgebra_of_J** (hJJ : J * J = 1) (G : Subgroup (Matrix m m ℂ)ˣ) (hG : ∀ g : (Matrix m m ℂ)ˣ, g ∈ G ↔ (g : Matrix m m ℂ)ᴴ * J * g = J) : IsExpLieAlgebra G (lieJ J) where

theorem **mem_UJ_iff** (g : (Matrix (Fin n) (Fin n) ℂ)ˣ) : g ∈ UJ ε ↔ (g : Matrix (Fin n) (Fin n) ℂ)ᴴ * Jm ε * g = Jm ε

theorem **isExpLieAlgebra_UJ** : IsExpLieAlgebra (UJ ε) (lieJ (Jm ε))

theorem **isCompact_Kset** : IsCompact (Kset ε)

theorem **isCompact_KG** : IsCompact (KG ε : Set (UJ ε))

CompanionTheory.HarishChandraModules.BesselParameterIntegrals.ParametricIntegralSmoothness.IntCommute

theorem **SmOn.exists_contDiff** {E V : Type*} [NormedAddCommGroup E] [NormedSpace ℝ E] [FiniteDimensional ℝ E] [NormedAddCommGroup V] [NormedSpace ℝ V] {A : Set E} {f : E → V} (hf : SmOn A f) (hA : IsClosed A) : ∃ F : E → V, ContDiff ℝ ∞ F ∧ ∀ a ∈ A, F a = f a

theorem **contDiff_integral_param** {P E V : Type} [NormedAddCommGroup P] [NormedSpace ℝ P] [NormedAddCommGroup E] [NormedSpace ℝ E] [FiniteDimensional ℝ E] [MeasurableSpace E] [BorelSpace E] [NormedAddCommGroup V] [NormedSpace ℂ V] [CompleteSpace V] {μ : Measure E} [IsFiniteMeasureOnCompacts μ] {f : E → ℂ} (hf : Continuous f) (hc : HasCompactSupport f) {F : P × E → V} (hF : ContDiff ℝ ∞ F) : ContDiff ℝ ∞ fun p => ∫ t, f t • F (p, t) ∂μ

theorem **integrable_csmul** {V : Type*} [NormedAddCommGroup V] [NormedSpace ℂ V] (hφ : Continuous φ) (hc : HasCompactSupport φ) {f : MG G hG → V} (hf : Continuous f) : Integrable (fun g => φ g • f g) μ

theorem **dsW_integral** (h : RepOn G σ) {v : H} (hv : IsSm G σ v) (hφ : Continuous φ) (hc : HasCompactSupport φ) : ∀ {W : List (Mat n)}, LieW G W → dsW σ W (∫ g, φ g • σ g.mat v ∂μ) = ∫ g, φ g • dsW σ W (σ g.mat v) ∂μ | [], _ => rfl | X :: W, hXW => by have hW : LieW G W

theorem **map_integral_of_bound** (h : RepOn G σ) {v : H} (hv : IsSm G σ v) (hφ : Continuous φ) (hc : HasCompactSupport φ) (hy : IsSm G σ (∫ g, φ g • σ g.mat v ∂μ)) {V : Type} [NormedAddCommGroup V] [NormedSpace ℂ V] [CompleteSpace V] {B : H → V} (hadd : ∀ x, IsSm G σ x → ∀ y, IsSm G σ y → B (x + y) = B x + B y) (hsmul : ∀ (a : ℂ) x, IsSm G σ x → B (a • x) = a • B x) {ι : Type} [Fintype ι] {W : ι → List (Mat n)} (hW : ∀ i, LieW G (W i)) {C : ℝ} (hB : ∀ x, IsSm G σ x → ‖B x‖ ≤ C * ∑ i, ‖dsW σ (W i) x‖) : B (∫ g, φ g • σ g.mat v ∂μ) = ∫ g, φ g • B (σ g.mat v) ∂μ

theorem **isSm_integral** (hG : IsMatGroup G) (h : RepOn G σ) {v : H} (hv : IsSm G σ v) {μ : Measure (MtM n)} (hμ : IsHaarOnM G μ) {f : MtM n → ℂ} (hf : Continuous f) (hc : HasCompactSupport f) : IsSm G σ (∫ g, f g • σ (Matrix.of g) v ∂μ)

theorem **intertwiner_commute** : Formalism.FourierDecayAlt.ThreeDimData.IntCommute

CompanionTheory.HarishChandraModules.BesselParameterIntegrals.PolarUH

theorem **mul_J_mul_conjTranspose** (hJJ : J * J = 1) {g : Matrix m m ℂ} (hg : gᴴ * J * g = J) : g * J * gᴴ = J

theorem **sqrt_mem_UJ** (hJJ : J * J = 1) (hJh : Jᴴ = J) {P : Matrix m m ℂ} (hP : P.PosDef) (hPJ : Pᴴ * J * P = J) : (CFC.sqrt P)ᴴ * J * CFC.sqrt P = J

theorem **mul_mem_UJ** {a b : Matrix m m ℂ} (ha : aᴴ * J * a = J) (hb : bᴴ * J * b = J) : (a * b)ᴴ * J * (a * b) = J

theorem **inv_mem_UJ** {a : Matrix m m ℂ} (hu : IsUnit a.det) (ha : aᴴ * J * a = J) : a⁻¹ᴴ * J * a⁻¹ = J

theorem **polar_mem_UJ** (hJJ : J * J = 1) (hJh : Jᴴ = J) (g : GL m ℂ) (hg : (g : Matrix m m ℂ)ᴴ * J * g = J) : (polarPos g)ᴴ * J * polarPos g = J ∧ (polarUnitary g)ᴴ * J * polarUnitary g = J

theorem **unitary_group_polar_decomposition** : PI.QuotSetU.UnitaryGroupMassFormula.PolarUH

theorem **unitary_group_modular_function_trivial** : PI.QuotSetU.UnitaryGroupMassFormula.TrivG

CompanionTheory.HarishChandraModules.BesselParameterIntegrals.SecondCountableQuotients.ModExists

theorem **map_mul_right_eq** (μ : Measure H) [IsHaarMeasure μ] (g : H) : map (· * g) μ = modularCharacterFun g • μ

theorem **modular_aut** (α : H ≃ₜ* H) (g : H) : modularCharacterFun (α g) = modularCharacterFun g

theorem **modular_bdd** {V : Set H} (hV : IsCompact V) : ∃ M : ℝ, ∀ v ∈ V, (modularCharacterFun v : ℝ) ≤ M

theorem **map_mul_right_of_modular_one** (h1 : ∀ g : H, modularCharacterFun g = 1) (ν : Measure H) [SigmaFinite ν] [IsMulLeftInvariant ν] (g : H) : map (· * g) ν = ν

theorem **MG.mem** (x : MG G hG) : x.mat ∈ G

@[simp] theorem **MG.mat_mk** (x : (Matrix (Fin m) (Fin m) ℂ)) (hx : x ∈ G) : (MG.mk hG x hx).mat = x

theorem **MG.ext** {x y : MG G hG} (h : x.mat = y.mat) : x = y

@[simp] theorem **MG.mat_mul** (a b : MG G hG) : (a * b).mat = a.mat * b.mat

@[simp] theorem **MG.mat_inv** (a : MG G hG) : (a⁻¹).mat = a.mat⁻¹

theorem **MG.isClosedEmbedding_mat** : Topology.IsClosedEmbedding (MG.mat : MG G hG → (Matrix (Fin m) (Fin m) ℂ))

theorem **MG.continuous_mat** : Continuous (MG.mat : MG G hG → (Matrix (Fin m) (Fin m) ℂ))

theorem **MG.continuous_iff** {X : Type*} [TopologicalSpace X] {f : X → MG G hG} : Continuous f ↔ Continuous (fun x => (f x).mat)

theorem **MG.det_ne_zero** (a : MG G hG) : a.mat.det ≠ 0

theorem **MG.measurableEmbedding_toM** : MeasurableEmbedding (MG.toM : MG G hG → MtM m)

theorem **MG.range_toM** : Set.range (MG.toM : MG G hG → MtM m) = {x | Matrix.of x ∈ G}

theorem **modD_of_mem** {x : (Matrix (Fin m) (Fin m) ℂ)} (hx : x ∈ G) : modD hG x = (modularCharacterFun (MG.mk hG x hx) : ℝ)

theorem **modProps_modD** (hG : IsMatGroup G) : ModProps G (modD hG)

theorem **measurable_lmulQ** (a : (Matrix (Fin m) (Fin m) ℂ)) : Measurable (lmulQ a)

theorem **measurable_rmulQ** (a : (Matrix (Fin m) (Fin m) ℂ)) : Measurable (rmulQ a)

theorem **rightInv_of_modD_one** (hG : IsMatGroup G) (h1 : ∀ g ∈ G, modD hG g = 1) (μ : Measure (MtM m)) (hμ : IsHaarOnM G μ) : IsRightInvM G μ

theorem **exists_modular_function** : PI.QuotSetU.UnitaryGroupMassFormula.ModExists

CompanionTheory.HarishChandraModules.BesselParameterIntegrals.SecondCountableQuotients.NContract

theorem **expo_add_piI** (i : Fin (2 * r + k + 1)) : expo k r i + expo k r (piI k r i) = 0

theorem **expo_eq_zero_of_inW** {i : Fin (2 * r + k + 1)} (hi : inW k r i) : expo k r i = 0

theorem **expo_lt** {i j : Fin (2 * r + k + 1)} (hij : i.val < j.val) (hW : ¬ (inW k r i ∧ inW k r j)) : expo k r j + 1 ≤ expo k r i

theorem **dvec_mul_inv** {t : ℝ} (ht : t ≠ 0) (i : Fin (2 * r + k + 1)) : dvec (k

theorem **dvec_of_inW** {t : ℝ} {i : Fin (2 * r + k + 1)} (hi : inW k r i) : dvec (k

theorem **cdiag_mul** {t : ℝ} (ht : t ≠ 0) : cdiag k r t * cdiag k r t⁻¹ = 1

theorem **cdiag_mul'** {t : ℝ} (ht : t ≠ 0) : cdiag k r t⁻¹ * cdiag k r t = 1

theorem **cdiag_inv** {t : ℝ} (ht : t ≠ 0) : (cdiag k r t)⁻¹ = cdiag k r t⁻¹

theorem **conj_apply** (t : ℝ) (g : 𝕄) (i j : Fin (2 * r + k + 1)) : (cdiag k r t * g * cdiag k r t⁻¹) i j = dvec t i * g i j * dvec t⁻¹ j

theorem **cdiag_mem_UH** {t : ℝ} (ht : t ≠ 0) : cdiag k r t ∈ UH (Hpk k r)

theorem **cdiag_conj_WB** {t : ℝ} (ht : t ≠ 0) {a : 𝕄} (ha : WB k r a) : cdiag k r t * a * cdiag k r t⁻¹ = a

theorem **cdiag_conj_N** {t : ℝ} (ht : t ≠ 0) {m : 𝕄} (hm : m ∈ Nset k r) : cdiag k r t * m * cdiag k r t⁻¹ ∈ Nset k r

theorem **cdiag_conj_S** {t : ℝ} (ht : t ≠ 0) {g : 𝕄} (hg : g ∈ Sset k r z) : cdiag k r t * g * cdiag k r t⁻¹ ∈ Sset k r z

theorem **cdiag_normalises** {t : ℝ} (ht : t ≠ 0) (g : 𝕄) : cdiag k r t * g * (cdiag k r t)⁻¹ ∈ Sset k r z ↔ g ∈ Sset k r z

theorem **norm_dvec_mul** (t : ℝ) (ht : 0 < t) (x : ℂ) (i j : Fin (2 * r + k + 1)) : ‖dvec (k

theorem **unipotent_radical_contracting** : NContract

CompanionTheory.HarishChandraModules.BesselParameterIntegrals.SecondCountableQuotients.QuotientMeasureUnitaryGroup

theorem **quotient_measure_unitary_group** : PI.QuotSetU

CompanionTheory.HarishChandraModules.BesselParameterIntegrals.SecondCountableQuotients.SGroup

theorem **wb_of_mem_UW** {z : Fin (2 * r + k + 1) → ℂ} {h : 𝕄} (hh : h ∈ UWset k r z) : WB k r h

theorem **nc_conj** {a b m : 𝕄} (ha : WB k r a) (hb : WB k r b) (hab : a * b = 1) (hm : NC k r m) : NC k r (a * m * b)

theorem **conj_mem_N** {a b m : 𝕄} (ha : WB k r a) (hb : WB k r b) (hab : a * b = 1) (haU : a ∈ UH (Hpk k r)) (hbU : b ∈ UH (Hpk k r)) (hm : m ∈ Nset k r) : a * m * b ∈ Nset k r

theorem **isClosed_NC** : IsClosed {g : 𝕄 | NC k r g}

theorem **isClosed_Nset** : IsClosed (Nset k r)

theorem **mem_Sset_iff** (g : 𝕄) : g ∈ Sset k r z ↔ ∃ h ∈ UWset k r z, h⁻¹ * g ∈ Nset k r

theorem **isClosed_Sset** (hz : inWb k r z) (hT : 0 < Tz k r z) : IsClosed (Sset k r z)

theorem **one_mem_Sset** : (1 : 𝕄) ∈ Sset k r z

theorem **mul_mem_Sset** {a b : 𝕄} (ha : a ∈ Sset k r z) (hb : b ∈ Sset k r z) : a * b ∈ Sset k r z

theorem **inv_mem_Sset** {a : 𝕄} (ha : a ∈ Sset k r z) : a⁻¹ ∈ Sset k r z

theorem **stabilizer_closed_matrix_group** : SsetGroup

CompanionTheory.HarishChandraModules.BesselParameterIntegrals.SecondCountableQuotients.StabilizerModularFunctionTrivial

theorem **pow_mem_Sset** {x : 𝕄} (hx : x ∈ Sset k r z) (p : ℕ) : x ^ p ∈ Sset k r z

theorem **pow_mem_UW** {x : 𝕄} (hx : x ∈ UWset k r z) (p : ℕ) : x ^ p ∈ UWset k r z

theorem **pow_mem_N** {x : 𝕄} (hx : x ∈ Nset k r) (p : ℕ) : x ^ p ∈ Nset k r

theorem **entry_le_two** {g : 𝕄} (hg : ∀ i j, ‖(g − 1) i j‖ ≤ 1) (i j : Fin (2 * r + k + 1)) : ‖g i j‖ ≤ 2

theorem **isCompact_nearOne** (hz : inWb k r z) (hT : 0 < Tz k r z) : IsCompact {g : 𝕄 | g ∈ Sset k r z ∧ ∀ i j, ‖(g − 1) i j‖ ≤ 1}

theorem **modFun_one** : D 1 = 1

theorem **modFun_pow** {x : 𝕄} (hx : x ∈ Sset k r z) (p : ℕ) : D (x ^ p) = D x ^ p

theorem **modFun_eq_one_of_bdd** {x : 𝕄} (hx : x ∈ Sset k r z) (hxi : x⁻¹ ∈ Sset k r z) (hb : ∃ M : ℝ, ∀ p : ℕ, D (x ^ p) ≤ M) (hbi : ∃ M : ℝ, ∀ p : ℕ, D (x⁻¹ ^ p) ≤ M) : D x = 1

theorem **modFun_UW** (hz : inWb k r z) (hT : 0 < Tz k r z) {h : 𝕄} (hh : h ∈ UWset k r z) : D h = 1

theorem **modFun_N** (hz : inWb k r z) (hT : 0 < Tz k r z) {m : 𝕄} (hm : m ∈ Nset k r) : D m = 1

theorem **stabilizer_modular_function_trivial** : TrivS

CompanionTheory.HarishChandraModules.BesselParameterIntegrals.SigmaFiniteQuotients.Exists

theorem **finiteOnCompacts_of_sigmaFinite** (η : Measure H) [SigmaFinite η] [IsMulLeftInvariant η] : IsFiniteMeasureOnCompacts η

theorem **map_comap_toM** (hG : IsMatGroup G) {μ : Measure (MtM m)} (h0 : ∀ᵐ x ∂μ, Matrix.of x ∈ G) : (μ.comap (MG.toM (hG

theorem **comap_map_mul** (hG : IsMatGroup G) {μ : Measure (MtM m)} (h0 : ∀ᵐ x ∂μ, Matrix.of x ∈ G) (g : MG G hG) (hl : μ.map (lmulQ g.mat) = μ) : (μ.comap (MG.toM (hG

theorem **IsHaarOnM.lt_top_of_isCompact** (hG : IsMatGroup G) {μ : Measure (MtM m)} (hμ : IsHaarOnM G μ) {K : Set (MtM m)} (hK : IsCompact K) : μ K < ∞

CompanionTheory.HarishChandraModules.BesselParameterIntegrals.SigmaFiniteQuotients.ExistsQuotientMeasureUnitaryGroup

theorem **exists_isHaarOnM** {G : Set (Matrix (Fin m) (Fin m) ℂ)} (hG : IsMatGroup G) : ∃ μ : Measure (MtM m), IsHaarOnM G μ

theorem **quotient_measure_self** {G : Set (Matrix (Fin m) (Fin m) ℂ)} (hG : IsMatGroup G) {μ : Measure (MtM m)} (hμ : IsHaarOnM G μ) (hr : IsRightInvM G μ) : QuotientSetting G G μ μ (dirac (oneM m))

theorem **isMatGroup_one** : IsMatGroup ({1} : Set (Matrix (Fin m) (Fin m) ℂ))

theorem **quotient_measure_trivial_subgroup** {G : Set (Matrix (Fin m) (Fin m) ℂ)} (hG : IsMatGroup G) {μ : Measure (MtM m)} (hμ : IsHaarOnM G μ) (hr : IsRightInvM G μ) : QuotientSetting G {1} μ (dirac (oneM m)) μ

theorem **exists_quotient_measure_unitary_group** (k r : ℕ) : ∃ μ : Measure (MtM (2 * r + k + 1)), QuotientSetting (UH (Hpk k r)) (UH (Hpk k r)) μ μ (dirac (oneM _)) ∧ QuotientSetting (UH (Hpk k r)) {1} μ (dirac (oneM _)) μ

CompanionTheory.HarishChandraModules.BesselParameterIntegrals.SmoothCyclicVectors.IntegralMemOfIsClosed

theorem **hasDerivAt_integral_cpt** {F F' : ℝ → α → V} (hF : ∀ t, Continuous (F t)) (hF' : Continuous fun p : ℝ × α => F' p.1 p.2) (hd : ∀ t a, HasDerivAt (F · a) (F' t a) t) {K : Set α} (hK : IsCompact K) (h0 : ∀ a ∉ K, F 0 a = 0) (h0' : ∀ t ∈ Icc (-1 : ℝ) 1, ∀ a ∉ K, F' t a = 0) : HasDerivAt (fun t => ∫ a, F t a ∂μ) (∫ a, F' 0 a ∂μ) 0

theorem **integral_mem_of_isClosed** {α V : Type*} [MeasurableSpace α] [NormedAddCommGroup V] [NormedSpace ℂ V] [CompleteSpace V] {μ : Measure α} (S : Submodule ℂ V) (hS : IsClosed (S : Set V)) {f : α → V} (hf : ∀ a, f a ∈ S) : ∫ a, f a ∂μ ∈ S

theorem **ex_add** (X : Mat n) (s t : ℝ) : ex X (s + t) = ex X s * ex X t

theorem **hasDerivAt_ex** (X : Mat n) (t : ℝ) : HasDerivAt (ex X) (ex X t * X) t

theorem **continuous_ex** (X : Mat n) : Continuous (ex X)

theorem **mulR_apply** (g x : Mat n) : mulR g x = x * g

theorem **hasDerivAt_ex_neg_mul** (X g : Mat n) (t : ℝ) : HasDerivAt (fun s => ex X (-s) * g) (-(X * (ex X (-t) * g))) t

theorem **contDiff_Lx** (X : Mat n) {φ : Mat n → ℝ} (hφ : ContDiff ℝ ∞ φ) : ContDiff ℝ ∞ (Lx X φ)

theorem **tsupport_Lx** (X : Mat n) (φ : Mat n → ℝ) : tsupport (Lx X φ) ⊆ tsupport φ

theorem **contDiff_LW** {φ : Mat n → ℝ} (hφ : ContDiff ℝ ∞ φ) : ∀ W, ContDiff ℝ ∞ (LW W φ) | [] => hφ | X :: W => contDiff_Lx X (contDiff_LW hφ W) theorem tsupport_LW (φ : Mat n → ℝ) : ∀ W, tsupport (LW W φ) ⊆ tsupport φ | [] => subset_rfl | X :: W => (tsupport_Lx X _).trans (tsupport_LW φ W) theorem hasCompactSupport_LW {φ : Mat n → ℝ} (hc : HasCompactSupport φ) (W : List (Mat n)) : HasCompactSupport (LW W φ)

theorem **hasDerivAt_Lx** (X g : Mat n) {φ : Mat n → ℝ} (hφ : ContDiff ℝ ∞ φ) (t : ℝ) : HasDerivAt (fun s => φ (ex X (-s) * g)) (Lx X φ (ex X (-t) * g)) t

theorem **IsSm.hasDerivAt** (h : RepOn G σ) {u : H} (hu : IsSm G σ u) {X : Mat n} (hX : ∀ t, ex X t ∈ G) (t : ℝ) : HasDerivAt (fun s => σ (ex X s) u) (σ (ex X t) (ds σ X u)) t

theorem **isCompact_supp** {φ : Mat n → ℝ} (hc : HasCompactSupport φ) : IsCompact ((MG.mat : MG G hG → Mat n) ⁻¹' tsupport φ)

theorem **integrable_smul** {V : Type*} [NormedAddCommGroup V] [NormedSpace ℝ V] {φ : Mat n → ℝ} (hφ : Continuous φ) (hc : HasCompactSupport φ) {f : MG G hG → V} (hf : Continuous f) : Integrable (fun g : MG G hG => φ g.mat • f g) haar

theorem **continuous_orbit** (h : RepOn G σ) {v : H} (hv : IsSm G σ v) {W : List (Mat n)} (hW : LieW G W) : Continuous fun g : MG G hG => dsW σ W (σ g.mat v)

theorem **continuous_orbit₂** (h : RepOn G σ) {v : H} (hv : IsSm G σ v) {W : List (Mat n)} (hW : LieW G W) {X : Mat n} (hX : ∀ t, ex X t ∈ G) : Continuous fun p : ℝ × MG G hG => σ (ex X p.1) (dsW σ W (σ p.2.mat v))

theorem **apply_integral** (L : H →L[ℂ] H) {φ : Mat n → ℝ} (hφ : Continuous φ) (hc : HasCompactSupport φ) {f : MG G hG → H} (hf : Continuous f) : L (∫ g : MG G hG, φ g.mat • f g ∂haar) = ∫ g : MG G hG, φ g.mat • L (f g) ∂haar

theorem **hasDerivAt_A** (h : RepOn G σ) {v : H} (hv : IsSm G σ v) {X : Mat n} (hX : ∀ t, ex X t ∈ G) {ψ : Mat n → ℝ} (hψ : ContDiff ℝ ∞ ψ) (hc : HasCompactSupport ψ) : HasDerivAt (fun t => σ (ex X t) (∫ g : MG G hG, ψ g.mat • σ g.mat v ∂haar)) (∫ g : MG G hG, Lx X ψ g.mat • σ g.mat v ∂haar) 0

theorem **hasDerivAt_B** (h : RepOn G σ) {v : H} (hv : IsSm G σ v) {X : Mat n} (hX : ∀ t, ex X t ∈ G) {W : List (Mat n)} (hW : LieW G W) {φ : Mat n → ℝ} (hφ : Continuous φ) (hc : HasCompactSupport φ) : HasDerivAt (fun t => σ (ex X t) (∫ g : MG G hG, φ g.mat • dsW σ W (σ g.mat v) ∂haar)) (∫ g : MG G hG, φ g.mat • dsW σ (X :: W) (σ g.mat v) ∂haar) 0

theorem **integral_dsW** (h : RepOn G σ) {v : H} (hv : IsSm G σ v) {φ : Mat n → ℝ} (hφ : ContDiff ℝ ∞ φ) (hc : HasCompactSupport φ) : ∀ {W : List (Mat n)}, LieW G W → ∫ g : MG G hG, φ g.mat • dsW σ W (σ g.mat v) ∂haar = ∫ g : MG G hG, LW W φ g.mat • σ g.mat v ∂haar | [], _ => rfl | X :: W, hXW => by have hW : LieW G W

theorem **norm_integral_dsW_le** (h : RepOn G σ) (hiso : ∀ g ∈ G, ∀ v, ‖σ g v‖ = ‖v‖) {v : H} (hv : IsSm G σ v) {φ : Mat n → ℝ} (hφ : ContDiff ℝ ∞ φ) (hc : HasCompactSupport φ) {W : List (Mat n)} (hW : LieW G W) : ‖∫ g : MG G hG, φ g.mat • dsW σ W (σ g.mat v) ∂haar‖ ≤ (∫ g : MG G hG, ‖LW W φ g.mat‖ ∂haar) * ‖v‖

theorem **isSm_of_mem_orbSpan** (h : RepOn G σ) {w : H} (hw : IsSm G σ w) {y : H} (hy : y ∈ orbSpan G σ w) : IsSm G σ y

theorem **apply_mem_orbSpan** (h : RepOn G σ) {w : H} {a : Mat n} (ha : a ∈ G) {y : H} (hy : y ∈ orbSpan G σ w) : σ a y ∈ orbSpan G σ w

theorem **exists_approx** (hG : IsMatGroup G) (h : RepOn G σ) (hiso : ∀ g ∈ G, ∀ v, ‖σ g v‖ = ‖v‖) {w : H} (hw : IsSm G σ w) (hd : Dense (orbSpan G σ w : Set H)) {x : H} (hx : IsSm G σ x) {ι : Type} [Fintype ι] {W : ι → List (Mat n)} (hW : ∀ i, LieW G (W i)) {δ : ℝ} (hδ : 0 < δ) : ∃ y ∈ orbSpan G σ w, ∀ i, ‖dsW σ (W i) (x - y)‖ < δ

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theorem **ex_mem_of_lie** {Hm X : Mat n} (hX : Xᴴ * Hm + Hm * X = 0) (t : ℝ) : (ex X t)ᴴ * Hm * ex X t = Hm

theorem **repOn_of_conj** {J Hm P : Mat n} {σ : Mat n → (H →L[ℂ] H)} (hP : IsUnit P) (hPJ : Pᴴ * J * P = Hm) (h1 : σ (P⁻¹ * 1 * P) = 1) (hm : ∀ a, aᴴ * J * a = J → ∀ b, bᴴ * J * b = J → σ (P⁻¹ * (a * b) * P) = (σ (P⁻¹ * a * P)).comp (σ (P⁻¹ * b * P))) : RepOn {g | gᴴ * Hm * g = Hm} σ

CompanionTheory.HarishChandraModules.BesselParameterIntegrals.SmoothCyclicVectors.SmoothCyc

theorem **dsigW_eq'** {k r : ℕ} {H : Type} [NormedAddCommGroup H] [InnerProductSpace ℂ H] (σ : Mat (2 * r + k + 1) → (H →L[ℂ] H)) : ∀ (W : List (Mat (2 * r + k + 1))) (v : H), dsigW (k

theorem **conj_facts** {n : ℕ} {J Hm P : Mat n} (hP : IsUnit P) (hPJ : Pᴴ * J * P = Hm) : (∀ a : Mat n, aᴴ * Hm * a = Hm → ∃ b : Mat n, bᴴ * J * b = J ∧ P⁻¹ * b * P = a) ∧ ∀ b : Mat n, bᴴ * J * b = J → (P⁻¹ * b * P)ᴴ * Hm * (P⁻¹ * b * P) = Hm

theorem **cyc_of_smooth** (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (lam : Fin (2 * r + k + 1) → ℝ) (H : Type) [NormedAddCommGroup H] [InnerProductSpace ℂ H] [CompleteSpace H] (σ : Mat (2 * r + k + 1) → (H →L[ℂ] H)) (w : H) (hσ : IsDSHpk k r ε lam σ) (hw0 : w ≠ 0) (hw : w ∈ smoothVecs (k

theorem **smoothCyc_of_lowSmooth** (hL : PI.LowSmooth) : PI.SmoothCyc

theorem **smoothCyc_of_kFinSmooth** (hK : PI.KFinSmooth) : PI.SmoothCyc

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theorem **smooth_cyclic_vectors** : PI.SmoothCyc

CompanionTheory.HarishChandraModules.BesselParameterIntegrals.SmoothCyclicVectors.SmoothVec

theorem **SmOn.congr** {A : Set E} {f f' : E → V} (hf : SmOn A f) (h : ∀ a ∈ A, f' a = f a) : SmOn A f'

theorem **SmOn.comp** {A : Set E} {B : Set E'} {f : E → V} (hf : SmOn A f) {φ : E' → E} (hφ : ContDiff ℝ ∞ φ) (hm : MapsTo φ B A) : SmOn B (fun b => f (φ b))

theorem **SmOn.of_contDiff** {A : Set E} {f : E → V} (hf : ContDiff ℝ ∞ f) : SmOn A f

theorem **SmOn.add** {A : Set E} {f f' : E → V} (hf : SmOn A f) (hf' : SmOn A f') : SmOn A (fun a => f a + f' a)

theorem **SmOn.clm** {W : Type*} [NormedAddCommGroup W] [NormedSpace ℝ W] {A : Set E} {f : E → V} (hf : SmOn A f) (L : V →L[ℝ] W) : SmOn A (fun a => L (f a))

theorem **SmOn.continuousOn** {A : Set E} {f : E → V} (hf : SmOn A f) : ContinuousOn f A

theorem **hasDerivAt_local** {A U : Set E} {f F : E → V} {a v : E} {γ : ℝ → E} (hγ : ∀ t, γ t ∈ A) (h0 : γ 0 = a) (hd : HasDerivAt γ v 0) (hU : IsOpen U) (haU : a ∈ U) (hF : ContDiffOn ℝ ∞ F U) (hfF : ∀ g ∈ U, g ∈ A → f g = F g) : HasDerivAt (fun t => f (γ t)) (fderiv ℝ F a v) 0

theorem **SmOn.flow** {A : Set E} {f : E → V} (hf : SmOn A f) {γ : E → ℝ → E} {v : E → E} (hγ : ∀ a ∈ A, ∀ t, γ a t ∈ A) (h0 : ∀ a, γ a 0 = a) (hd : ∀ a, HasDerivAt (γ a) (v a) 0) (hv : ContDiff ℝ ∞ v) : (∀ a ∈ A, DifferentiableAt ℝ (fun t => f (γ a t)) 0) ∧ SmOn A (fun a => deriv (fun t => f (γ a t)) 0)

theorem **contDiff_mul** : ContDiff ℝ ∞ (fun p : Mat n × Mat n => p.1 * p.2)

theorem **contDiff_mul'** {E : Type*} [NormedAddCommGroup E] [NormedSpace ℝ E] {f g : E → Mat n} (hf : ContDiff ℝ ∞ f) (hg : ContDiff ℝ ∞ g) : ContDiff ℝ ∞ (fun x => f x * g x)

theorem **ex_zero** (X : Mat n) : ex X 0 = 1

theorem **hasDerivAt_mul_ex** (g X : Mat n) : HasDerivAt (fun t => g * ex X t) (g * X) 0

theorem **dsW_cons** (σ : Mat n → (H →L[ℂ] H)) (X : Mat n) (W : List (Mat n)) (v : H) : dsW σ (X :: W) v = ds σ X (dsW σ W v)

theorem **RepOn.apply_mul** (h : RepOn G σ) {a b : Mat n} (ha : a ∈ G) (hb : b ∈ G) (v : H) : σ (a * b) v = σ a (σ b v)

theorem **apply_ds** (h : RepOn G σ) {X g : Mat n} (hX : ∀ t, ex X t ∈ G) (hg : g ∈ G) {u : H} (hu : DifferentiableAt ℝ (fun t => σ (ex X t) u) 0) : σ g (ds σ X u) = deriv (fun t => σ (g * ex X t) u) 0

theorem **smOn_ds** {E : Type*} [NormedAddCommGroup E] [NormedSpace ℝ E] (h : RepOn G σ) {A : Set E} {u : E → H} (hu : SmOn (G ×ˢ A) (fun q : Mat n × E => σ q.1 (u q.2))) {X : Mat n} (hX : ∀ t, ex X t ∈ G) : (∀ p ∈ A, DifferentiableAt ℝ (fun t => σ (ex X t) (u p)) 0) ∧ SmOn (G ×ˢ A) (fun q : Mat n × E => σ q.1 (ds σ X (u q.2)))

theorem **smOn_dsW** {E : Type*} [NormedAddCommGroup E] [NormedSpace ℝ E] (h : RepOn G σ) {A : Set E} {u : E → H} (hu : SmOn (G ×ˢ A) (fun q : Mat n × E => σ q.1 (u q.2))) : ∀ {W : List (Mat n)}, LieW G W → SmOn (G ×ˢ A) (fun q : Mat n × E => σ q.1 (dsW σ W (u q.2))) | [], _ => hu | X :: W, hW => (smOn_ds h (u

theorem **IsSm.prod** {v : H} (hv : IsSm G σ v) : SmOn (G ×ˢ (univ : Set ℝ)) (fun q : Mat n × ℝ => σ q.1 v)

theorem **IsSm.differentiableAt** (h : RepOn G σ) {v : H} (hv : IsSm G σ v) {X : Mat n} (hX : ∀ t, ex X t ∈ G) : DifferentiableAt ℝ (fun t => σ (ex X t) v) 0

theorem **IsSm.dsW** (h : RepOn G σ) {v : H} (hv : IsSm G σ v) {W : List (Mat n)} (hW : LieW G W) : IsSm G σ (dsW σ W v)

theorem **IsSm.add** {v v' : H} (hv : IsSm G σ v) (hv' : IsSm G σ v') : IsSm G σ (v + v')

theorem **IsSm.smul** {v : H} (hv : IsSm G σ v) (c : ℂ) : IsSm G σ (c • v)

theorem **IsSm.zero** : IsSm G σ 0

theorem **IsSm.sub** {v v' : H} (hv : IsSm G σ v) (hv' : IsSm G σ v') : IsSm G σ (v − v')

theorem **IsSm.smOn_mul** (h : RepOn G σ) {w : H} (hw : IsSm G σ w) : SmOn (G ×ˢ G) (fun q : Mat n × Mat n => σ q.1 (σ q.2 w))

theorem **IsSm.apply** (h : RepOn G σ) {w : H} (hw : IsSm G σ w) {x : Mat n} (hx : x ∈ G) : IsSm G σ (σ x w)

theorem **ds_add** (h : RepOn G σ) {X : Mat n} (hX : ∀ t, ex X t ∈ G) {v v' : H} (hv : IsSm G σ v) (hv' : IsSm G σ v') : ds σ X (v + v') = ds σ X v + ds σ X v'

theorem **ds_smul** (h : RepOn G σ) {X : Mat n} (hX : ∀ t, ex X t ∈ G) {v : H} (hv : IsSm G σ v) (c : ℂ) : ds σ X (c • v) = c • ds σ X v

theorem **dsW_add** (h : RepOn G σ) {v v' : H} (hv : IsSm G σ v) (hv' : IsSm G σ v') : ∀ {W : List (Mat n)}, LieW G W → dsW σ W (v + v') = dsW σ W v + dsW σ W v' | [], _ => rfl | X :: W, hW => by have hW' : LieW G W

theorem **dsW_smul** (h : RepOn G σ) {v : H} (hv : IsSm G σ v) (c : ℂ) : ∀ {W : List (Mat n)}, LieW G W → dsW σ W (c • v) = c • dsW σ W v | [], _ => rfl | X :: W, hW => by have hW' : LieW G W

theorem **dsW_sub** (h : RepOn G σ) {v v' : H} (hv : IsSm G σ v) (hv' : IsSm G σ v') {W : List (Mat n)} (hW : LieW G W) : dsW σ W (v − v') = dsW σ W v − dsW σ W v'

theorem **continuousOn_dsW_orbit** (h : RepOn G σ) {w : H} (hw : IsSm G σ w) {W : List (Mat n)} (hW : LieW G W) : ContinuousOn (fun x => dsW σ W (σ x w)) G

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theorem **IsSm.exists_clm** (h : RepOn G σ) {v : H} (hv : IsSm G σ v) : ∃ D : Mat n →L[ℝ] H, ∀ X, (∀ t, ex X t ∈ G) → ds σ X v = D X

theorem **ds_sum** (h : RepOn G σ) {X : Mat n} (hX : ∀ t, ex X t ∈ G) {ι : Type*} (s : Finset ι) (c : ι → ℂ) {b : ι → H} (hb : ∀ i, IsSm G σ (b i)) : ds σ X (∑ i ∈ s, c i • b i) = ∑ i ∈ s, c i • ds σ X (b i)

theorem **ex_conj** (hG : IsMatGroup G) {k : Mat n} (hk : k ∈ G) (X : Mat n) (t : ℝ) : ex (k ∗ X ∗ k⁻¹) t = k ∗ ex X t ∗ k⁻¹

theorem **apply_ds_conj** (hG : IsMatGroup G) (h : RepOn G σ) {k X : Mat n} (hk : k ∈ G) (hX : ∀ t, ex X t ∈ G) {v : H} (hv : IsSm G σ v) : (∀ t, ex (k ∗ X ∗ k⁻¹) t ∈ G) ∧ σ k (ds σ X v) = ds σ (k ∗ X ∗ k⁻¹) (σ k v)

theorem **GoodSub.step** (hG : IsMatGroup G) (h : RepOn G σ) (hK : K ≤ G) {T : Submodule ℂ H} (hT : GoodSub G K σ T) : ∃ (T' : Submodule ℂ H) (C : ℝ), GoodSub G K σ T' ∧ T ≤ T' ∧ 0 ≤ C ∧ ∀ X, (∀ t, ex X t ∈ G) → ‖X‖ ≤ 1 → ∀ v ∈ T, ds σ X v ∈ T' ∧ ‖ds σ X v‖ ≤ C ∗ ‖v‖

theorem **GoodSub.words** (hG : IsMatGroup G) (h : RepOn G σ) (hK : K ≤ G) {T : Submodule ℂ H} (hT : GoodSub G K σ T) {w : H} (hw : w ∈ T) (m : ℕ) : ∃ (T' : Submodule ℂ H) (C : ℝ), GoodSub G K σ T' ∧ 0 ≤ C ∧ ∀ W : List (Mat n), LieW G W → (∀ X ∈ W, ‖X‖ ≤ 1) → W.length ≤ m → dsW σ W w ∈ T' ∧ ‖dsW σ W w‖ ≤ C

theorem **coef_bound_uniform** {T : Submodule ℂ H} [FiniteDimensional ℂ T] {ρ : Mat n → ℝ} (hρ : ∀ g ∈ G, 0 ≤ ρ g) (hb : ∀ a ∈ T, ∀ b ∈ T, ∃ C : ℝ, ∀ g ∈ G, ‖⟪a, σ g b⟫_ℂ‖ ≤ C ∗ ρ g) : ∃ C : ℝ, 0 ≤ C ∧ ∀ a ∈ T, ∀ b ∈ T, ∀ g ∈ G, ‖⟪a, σ g b⟫_ℂ‖ ≤ C ∗ ‖a‖ ∗ ‖b‖ ∗ ρ g

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theorem **rw_cf** (h : RepOn G σ) (x₀ : E) (a : H) {b : H} (hb : IsSm G σ b) : ∀ {v : List (B.Mat k r)}, LieW G v → ∀ g ∈ G, B.Rw v (cf σ x₀ a b) g = cf σ x₀ a (dsW σ v b) g | [], _, _, _ => rfl | X :: v, hv, g, hg => by have hv' : LieW G v

theorem **lw_cf** (h : RepOn G σ) (hiso : ∀ g ∈ G, ∀ v, ‖σ g v‖ = ‖v‖) (x₀ : E) {a : H} (ha : IsSm G σ a) (b : H) {f : B.Mat k r → E} (hf : ∀ g ∈ G, f g = cf σ x₀ a b g) : ∀ {u : List (B.Mat k r)}, LieW G u → ∀ g ∈ G, u.foldr B.LdV f g = cf σ x₀ (dsW σ u a) b g | [], _, g, hg => hf g hg | X :: u, hu, g, hg => by have hu' : LieW G u

theorem **dLR_cf** (h : RepOn G σ) (hiso : ∀ g ∈ G, ∀ v, ‖σ g v‖ = ‖v‖) (x₀ : E) {a b : H} (ha : IsSm G σ a) (hb : IsSm G σ b) {u v : List (B.Mat k r)} (hu : LieW G u) (hv : LieW G v) {g : B.Mat k r} (hg : g ∈ G) : B.dLR u v (cf σ x₀ a b) g = cf σ x₀ (dsW σ u a) (dsW σ v b) g

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theorem **inCC_cf_of_dec** (hK : PI.KFinSmooth) {k r : ℕ} {ε : Fin (2 ∗ r + k + 1) → Bool} {lam : Fin (2 ∗ r + k + 1) → ℝ} {H : Type} [NormedAddCommGroup H] [InnerProductSpace ℂ H] [CompleteSpace H] {σ : Mx k r → (H →L[ℂ] H)} (hlam : Formalism.QuantumDeterminants.IsHCParam lam) (hσ : PI.UpqRep.IsDSHpk k r ε lam σ) {w : H} (hw : PI.UpqRep.IsKFinHpk (k

theorem **coef_eq_cf** {k r : ℕ} {Wp Wq : Type} [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] (μG : Measure (Formalism.QuantumDeterminants.Mt k r)) (A : JDat k r Wp Wq) : A.coef μG = cf A.σ (((A.scal μG : ℝ) : ℂ) • (innerSL ℂ (A.l' A.w0)).smulRight (A.l A.w0)) A.w0 A.w0

CompanionTheory.HarishChandraModules.BesselParameterIntegrals.UniformCoefficientBounds.FromDS

theorem **thrStar_le** {n : ℕ} (hn : 1 ≤ n) (ε : Fin n → Bool) : thrStar ε ≤ 32 ∗ (n : ℝ) ^ 3

theorem **dsigW_SK** {k r : ℕ} {H : Type} [NormedAddCommGroup H] [InnerProductSpace ℂ H] (σ : Mx k r → (H →L[ℂ] H)) : ∀ (W : List (Mx k r)) (v : H), Formalism.MainNotions.S.dsigW (k

theorem **decay_infinite_range** : DecInf

theorem **capelli_coefficient_continuity** : CoefCC

CompanionTheory.HarishChandraModules.KFiniteAnalyticVectors.AdmissibleSmoothVectors.AdmissibleOfFiniteRank

theorem **exists_elt** {m : Matrix (Fin n) (Fin n) ℂ} (hm : m ∈ UJset ε) : ∃ g : UJ ε, toM g = m

theorem **conjTranspose_mem** {m : Matrix (Fin n) (Fin n) ℂ} (hm : m ∈ UJset ε) : mᴴ ∈ UJset ε

theorem **Kset_sub** {m : Matrix (Fin n) (Fin n) ℂ} (hm : m ∈ Kset ε) : m ∈ UJset ε

theorem **continuous_toM** : Continuous (toM : UJ ε → Matrix (Fin n) (Fin n) ℂ)

theorem **admissible_of_finite_rank** (h1 : FinOfPI) (h2 : GoodPI) (h3 : Transfer) (h4 : ExistsAvg) (h5 : Iwa ε) : AdmUJ ε

theorem **smooth_of_adm** (hA : AdmUJ ε) (hG : GardingFin) {H : Type} [NormedAddCommGroup H] [InnerProductSpace ℂ H] [CompleteSpace H] {σ : Matrix (Fin n) (Fin n) ℂ → (H →L[ℂ] H)} (hσ : IsURepOn (UJset ε) σ) (hirr : IsIrredOn (UJset ε) σ) {v : H} (hv : IsKFin ε σ v) : SmoothOnH (Jm ε) fun g => σ g v

theorem **smoothOnH_conj** {E : Type} [NormedAddCommGroup E] [NormedSpace ℂ E] {A B P : Matrix (Fin n) (Fin n) ℂ} (hP : IsUnit P) (hAB : Pᴴ ∗ A ∗ P = B) {f : Matrix (Fin n) (Fin n) ℂ → E} (h : SmoothOnH A fun g => f (P⁻¹ ∗ g ∗ P)) : SmoothOnH B f

theorem **kFinSmooth_of** (h1 : FinOfPI) (h2 : GoodPI) (h3 : Transfer) (h4 : ExistsAvg) (h5 : IwaHpk) (h6 : GardingFin) : PI.KFinSmooth

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theorem **admissible_of_iwasawa** {n : ℕ} {ε : Fin n → Bool} (h : Iwa ε) : AdmUJ ε

theorem **k_finite_smooth** : PI.KFinSmooth

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theorem **integrable_cont** {F : Type} [NormedAddCommGroup F] {f : K → F} (hf : Continuous f) : Integrable f μ

theorem **exists_avg** (hiso : ∀ g v, ‖σ g v‖ = ‖v‖) (hc : ∀ v, Continuous fun x => σ x v) {e : K → ℂ} (he : Continuous e) : ∃ E : H →L[ℂ] H, ∀ u, E u = ∫ k, e k • σ k u ∂μ

theorem **exists_averaging_operator** : ExistsAvg

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theorem **inner_left** (σ : G →* (H →L[ℂ] H)) (hσ : ∀ g v, ‖σ g v‖ = ‖v‖) (g : G) (v w : H) : ⟪σ g v, w⟫_ℂ = ⟪v, σ g⁻¹ w⟫_ℂ

theorem **star_eq** (σ : G →* (H →L[ℂ] H)) (hσ : ∀ g v, ‖σ g v‖ = ‖v‖) (g : G) : star (σ g) = σ g⁻¹

theorem **mem_unitary** (σ : G →* (H →L[ℂ] H)) (hσ : ∀ g v, ‖σ g v‖ = ‖v‖) (g : G) : σ g ∈ unitary (H →L[ℂ] H)

theorem **schur** (σ : G →* (H →L[ℂ] H)) (hσ : ∀ g v, ‖σ g v‖ = ‖v‖) (hirr : ∀ S : Submodule ℂ H, IsClosed (S : Set H) → (∀ g, ∀ v ∈ S, σ g v ∈ S) → S = ⊥ ∨ S = ⊤) (T : H →L[ℂ] H) (hT : ∀ g v, T (σ g v) = σ g (T v)) : ∃ c : ℂ, ∀ v, T v = c • v

@[simp] theorem **diag_apply** (T : H →L[ℂ] H) (z : PiLp 2 (fun _ : ι => H)) (i : ι) : diag T z i = T (z i)

theorem **incl_apply** (j : ι) (v : H) (i : ι) : incl j v i = (Pi.single j v : ι → H) i

theorem **sum_incl** (z : PiLp 2 (fun _ : ι => H)) : ∑ j, incl j (z j) = z

theorem **diag_incl** (T : H →L[ℂ] H) (j : ι) (v : H) : diag T (incl j v) = incl j (T v)

theorem **proj_comm** (σ : G →* (H →L[ℂ] H)) (hσ : ∀ g v, ‖σ g v‖ = ‖v‖) (C : Submodule ℂ (PiLp 2 (fun _ : ι => H))) [C.HasOrthogonalProjection] (hC : ∀ g, ∀ z ∈ C, diag (σ g) z ∈ C) (g : G) (z : PiLp 2 (fun _ : ι => H)) : C.starProjection (diag (σ g) z) = diag (σ g) (C.starProjection z)

theorem **diag_mem** (σ : G →* (H →L[ℂ] H)) (hσ : ∀ g v, ‖σ g v‖ = ‖v‖) (hirr : ∀ S : Submodule ℂ H, IsClosed (S : Set H) → (∀ g, ∀ v ∈ S, σ g v ∈ S) → S = ⊥ ∨ S = ⊤) (C : Submodule ℂ (PiLp 2 (fun _ : ι => H))) (hcl : IsClosed (C : Set (PiLp 2 (fun _ : ι => H)))) (hC : ∀ g, ∀ z ∈ C, diag (σ g) z ∈ C) (T : H →L[ℂ] H) (z : PiLp 2 (fun _ : ι => H)) (hz : z ∈ C) : diag T z ∈ C

theorem **sum_eq_zero** (σ : G →* (H →L[ℂ] H)) (hσ : ∀ g v, ‖σ g v‖ = ‖v‖) (hirr : ∀ S : Submodule ℂ H, IsClosed (S : Set H) → (∀ g, ∀ v ∈ S, σ g v ∈ S) → S = ⊥ ∨ S = ⊤) (L : ι → H →L[ℂ] H) (y : ι → H) (h : ∀ g, ∑ k, L k (σ g (y k)) = 0) (T : H →L[ℂ] H) : ∑ k, L k (T (y k)) = 0

theorem **word_update** {R : Type*} [Ring R] (E : R) {n : ℕ} (f : Fin n → R) (k : Fin n) : ∃ A B : R, ∀ T, word E (List.ofFn (Function.update f k T)) = A * T * B

theorem **stdPoly_update** {R : Type*} [Ring R] (E : R) {M : ℕ} (a : Fin M → R) (j : Fin M) : ∃ A B : Equiv.Perm (Fin M) → R, ∀ T, stdPoly E (Function.update a j T) = ∑ π, A π * T * B π

theorem **stdPoly_all** (σ : G →* (H →L[ℂ] H)) (hσ : ∀ g v, ‖σ g v‖ = ‖v‖) (hirr : ∀ S : Submodule ℂ H, IsClosed (S : Set H) → (∀ g, ∀ v ∈ S, σ g v ∈ S) → S = ⊥ ∨ S = ⊤) (E : H →L[ℂ] H) (M : ℕ) (h : ∀ g : Fin M → G, stdPoly E (fun i => σ (g i)) = 0) (a : Fin M → H →L[ℂ] H) : stdPoly E a = 0

theorem **word_range'** (hφ : ∀ i ≤ M, ∀ j ≤ M, φ i (E (x j)) = if i = j then 1 else 0) (n k : ℕ) (hk : k + n = M) : word E ((List.range' k n).map fun a => (φ (a + 1)).smulRight (x a)) (x M) = E (x k)

theorem **word_cases** (hφ : ∀ i ≤ M, ∀ j ≤ M, φ i (E (x j)) = if i = j then 1 else 0) (l : List ℕ) (hl : ∀ a ∈ l, a < M) : word E (l.map fun a => (φ (a + 1)).smulRight (x a)) (x M) = 0 ∨ ∃ k, k + l.length = M ∧ l = List.range' k l.length

theorem **ofFn_val** (M : ℕ) : (List.ofFn fun i : Fin M => (i : ℕ)) = List.range' 0 M

theorem **stdPoly_apply** (hφ : ∀ i ≤ M, ∀ j ≤ M, φ i (E (x j)) = if i = j then 1 else 0) : stdPoly E (fun a : Fin M => (φ (a + 1)).smulRight (x a)) (x M) = E (x 0)

theorem **exists_dual** {n : ℕ} (y : Fin n → H) (hy : LinearIndependent ℂ y) : ∃ φ : Fin n → H →L[ℂ] ℂ, ∀ i j, φ i (y j) = if i = j then 1 else 0

theorem **rank_le** (E : H →L[ℂ] H) (M : ℕ) (h : ∀ a : Fin M → H →L[ℂ] H, stdPoly E a = 0) : Module.rank ℂ (LinearMap.range (E : H →ₗ[ℂ] H)) ≤ M

theorem **finite_rank_of_irreducible_unitary** : FinOfPI

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theorem **unitary_group_admissible** (n : ℕ) (ε : Fin n → Bool) : AdmUJ ε

theorem **kFin_smooth** (n : ℕ) (ε : Fin n → Bool) (H : Type) [NormedAddCommGroup H] [InnerProductSpace ℂ H] [CompleteSpace H] (σ : Matrix (Fin n) (Fin n) ℂ → (H →L[ℂ] H)) (hσ : IsURepOn (UJset ε) σ) (hirr : IsIrredOn (UJset ε) σ) (v : H) (hv : IsKFin ε σ v) : SmoothOnH (Jm ε) fun g => σ g v

theorem **kFin_mem_smoothVecs** (n : ℕ) (ε : Fin n → Bool) (H : Type) [NormedAddCommGroup H] [InnerProductSpace ℂ H] [CompleteSpace H] (σ : Matrix (Fin n) (Fin n) ℂ → (H →L[ℂ] H)) (hσ : IsURepOn (UJset ε) σ) (hirr : IsIrredOn (UJset ε) σ) (v : H) (hv : IsKFin ε σ v) : v ∈ PI.PhMeasCore.smoothVecs (Formalism.REAL.Jm ε) σ

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theorem **contDiff_integral_comp** {P Q E α : Type*} [NormedAddCommGroup P] [NormedSpace ℝ P] [FiniteDimensional ℝ P] [NormedAddCommGroup Q] [NormedSpace ℝ Q] [NormedAddCommGroup E] [NormedSpace ℝ E] [CompleteSpace E] [TopologicalSpace α] [T2Space α] [SecondCountableTopology α] [MeasurableSpace α] [OpensMeasurableSpace α] (μ : Measure α) [IsFiniteMeasureOnCompacts μ] (Φ : P × Q → E) (hΦ : ContDiff ℝ ∞ Φ) (q : α → Q) (hq : Continuous q) (hsupp : ∀ y₀ : P, ∃ U ∈ 𝒩 y₀, ∃ C : Set α, IsCompact C ∧ ∀ y ∈ U, ∀ a ∉ C, Φ (y, q a) = 0) : ContDiff ℝ ∞ fun y => ∫ a, Φ (y, q a) ∂μ

theorem **toM_mul** (x y : UJ ε) : toM (x * y) = toM x * toM y

theorem **toM_one** : toM (1 : UJ ε) = 1

theorem **emb_toM** : IsClosedEmbedding (toM : UJ ε → Matrix (Fin n) (Fin n) ℂ)

theorem **toM_mem** (x : UJ ε) : toM x ∈ UJset ε

theorem **exists_toM** {g : Matrix (Fin n) (Fin n) ℂ} (hg : inUJ ε g) : ∃ x : UJ ε, toM x = g

theorem **inUJ_of_mem_Kset** {k : Matrix (Fin n) (Fin n) ℂ} (hk : k ∈ Kset ε) : inUJ ε k

theorem **toM_inv_of_unitary** {k : UJ ε} (hk : toM k ∈ Matrix.unitaryGroup (Fin n) ℂ) : toM k⁻¹ = (toM k)ᴴ

theorem **contDiff_Phi** {H : Type*} [NormedAddCommGroup H] [NormedSpace ℝ H] {f : Matrix (Fin n) (Fin n) ℂ → ℝ} (hf : ContDiff ℝ ∞ f) : ContDiff ℝ ∞ fun p : Matrix (Fin n) (Fin n) ℂ × (Matrix (Fin n) (Fin n) ℂ × H) => f (p.2.1 * p.1) • p.2.2

theorem **supp_local** {f : Matrix (Fin n) (Fin n) ℂ → ℝ} {S : Set (Matrix (Fin n) (Fin n) ℂ)} (hS : IsCompact S) (hd : ∀ B ∈ S, B.det ≠ 0) (hfS : ∀ y, f y ≠ 0 → y ∈ S) (g₀ : Matrix (Fin n) (Fin n) ℂ) : ∃ U ∈ 𝒩 g₀, ∃ C : Set (UJ ε), IsCompact C ∧ ∀ g ∈ U, ∀ x ∉ C, f (toM x⁻¹ * g) = 0

theorem **contDiff_avg** {H : Type*} [NormedAddCommGroup H] [NormedSpace ℝ H] [CompleteSpace H] (v : UJ ε → H) (hv : Continuous v) {f : Matrix (Fin n) (Fin n) ℂ → ℝ} {S : Set (Matrix (Fin n) (Fin n) ℂ)} (hf : ContDiff ℝ ∞ f) (hS : IsCompact S) (hd : ∀ B ∈ S, B.det ≠ 0) (hfS : ∀ y, f y ≠ 0 → y ∈ S) : ContDiff ℝ ∞ fun g : Matrix (Fin n) (Fin n) ℂ => ∫ x : UJ ε, f (toM x⁻¹ * g) • v x ∂(muG ε)

theorem **garding_finite_smooth_vectors** : GardingFin

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theorem **continuous_iter_partial** (Φ : P × Q → E) (hΦ : ContDiff ℝ ∞ Φ) (k : ℕ) : Continuous fun p : P × Q => iteratedFDeriv ℝ k (fun y => Φ (y, p.2)) p.1

theorem **integrable_w** (ρ : G → H →L[ℂ] H) (hρ : ∀ u, Continuous fun x => ρ x u) {w : G → ℝ} (hw : Continuous w) (hc : HasCompactSupport w) (u : H) : Integrable (fun x => w x • ρ x u) μ

theorem **exists_strictHenselian_absolutely_closed_local_ring** (ρ : G → H →L[ℂ] H) (hρ : ∀ u, Continuous fun x => ρ x u) (hiso : ∀ x u, ‖ρ x u‖ = ‖u‖) {w : G → ℝ} (hw : Continuous w) (hc : HasCompactSupport w) : ∃ T : H →L[ℂ] H, ∀ u, T u = ∫ x, w x • ρ x u ∂μ

theorem **integral_mul_right_of_conj** {w : G → ℝ} (hw : Continuous w) (hc : HasCompactSupport w) (h0 : 0 ≤ w) (h1 : w 1 ≠ 0) (k : G) (hk : ∀ x, w (k⁻¹ * x * k) = w x) (F : G → H) : ∫ x, F (x * k) ∂μ = ∫ x, F x ∂μ

theorem **apply_alt** (ρ : G → H →L[ℂ] H) (hρ : ∀ u, Continuous fun x => ρ x u) (hmul : ∀ x y u, ρ (x * y) u = ρ x (ρ y u)) {w : G → ℝ} (hw : Continuous w) (hc : HasCompactSupport w) (g : G) (u : H) : ρ g (∫ x, w x • ρ x u ∂μ) = ∫ x, w (g⁻¹ * x) • ρ x u ∂μ

theorem **T_comm** (ρ : G → H →L[ℂ] H) (hρ : ∀ u, Continuous fun x => ρ x u) (hmul : ∀ x y u, ρ (x * y) u = ρ x (ρ y u)) {w : G → ℝ} (hw : Continuous w) (hc : HasCompactSupport w) (h0 : 0 ≤ w) (h1 : w 1 ≠ 0) (k : G) (hk : ∀ x, w (k⁻¹ * x * k) = w x) (u : H) : ∫ x, w x • ρ x (ρ k u) ∂μ = ρ k (∫ x, w x • ρ x u ∂μ)

theorem **exists_nhds_close** (ρ : G → H →L[ℂ] H) (hρ : ∀ u, Continuous fun x => ρ x u) (h1 : ∀ u, ρ 1 u = u) (Y : Submodule ℂ H) [FiniteDimensional ℂ Y] : ∃ V ∈ 𝒩 (1 : G), ∀ x ∈ V, ∀ y ∈ Y, ‖ρ x y − y‖ ≤ 1 / 2 * ‖y‖

theorem **eq_zero_of_alt_alt** (ρ : G → H →L[ℂ] H) (hρ : ∀ u, Continuous fun x => ρ x u) {w : G → ℝ} (hw : Continuous w) (hc : HasCompactSupport w) (h0 : 0 ≤ w) (h1 : w 1 ≠ 0) (y : H) (hy : ∀ x, w x ≠ 0 → ‖ρ x y − y‖ ≤ 1 / 2 * ‖y‖) (hT : ∫ x, w x • ρ x y ∂μ = 0) : y = 0

theorem **exists_preimage** {H : Type*} [NormedAddCommGroup H] [NormedSpace ℂ H] (Y : Submodule ℂ H) [FiniteDimensional ℂ Y] (T : H →L[ℂ] H) (hT : ∀ y ∈ Y, T y ∈ Y) (hi : ∀ y ∈ Y, T y = 0 → y = 0) : ∀ v ∈ Y, ∃ u, T u = v

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theorem **contDiff_entry** (i j : Fin n) : ContDiff ℝ ∞ fun y : Matrix (Fin n) (Fin n) ℂ => y i j

theorem **contDiff_hs** : ContDiff ℝ ∞ (hs : Matrix (Fin n) (Fin n) ℂ → ℝ)

theorem **contDiff_mul** : ContDiff ℝ ∞ fun p : Matrix (Fin n) (Fin n) ℂ × Matrix (Fin n) (Fin n) ℂ => p.1 * p.2

theorem **hs_eq_trace** (y : Matrix (Fin n) (Fin n) ℂ) : (hs y : ℂ) = (y * yᴴ).trace

theorem **hs_conj** {k : Matrix (Fin n) (Fin n) ℂ} (hk : k ∈ Matrix.unitaryGroup (Fin n) ℂ) (y : Matrix (Fin n) (Fin n) ℂ) : hs (kᴴ * y * k) = hs y

theorem **norm_le_of_hs** {y : Matrix (Fin n) (Fin n) ℂ} {r : ℝ} (hr : 0 ≤ r) (h : hs y ≤ r ^ 2) : ‖y‖ ≤ r

theorem **exists_bump** {W : Set (Matrix (Fin n) (Fin n) ℂ)} (hW : W ∈ 𝒩 (1 : Matrix (Fin n) (Fin n) ℂ)) : ∃ (f : Matrix (Fin n) (Fin n) ℂ → ℝ) (S : Set (Matrix (Fin n) (Fin n) ℂ)), ContDiff ℝ ∞ f ∧ 0 ≤ f ∧ f 1 ≠ 0 ∧ IsCompact S ∧ S ⊆ W ∧ (∀ y, f y ≠ 0 → y ∈ S) ∧ ∀ k ∈ Matrix.unitaryGroup (Fin n) ℂ, ∀ y, f (kᴴ * y * k) = f y

theorem **isCompact_mul_inv** {A S : Set (Matrix (Fin n) (Fin n) ℂ)} (hA : IsCompact A) (hS : IsCompact S) (hd : ∀ B ∈ S, B.det ≠ 0) : IsCompact ((fun p : Matrix (Fin n) (Fin n) ℂ × Matrix (Fin n) (Fin n) ℂ => p.1 * p.2⁻¹) '' (A ×ˢ S))

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theorem **exists_common_eigenvector** (h : ι → ℕ) (hinj : Function.Injective h) (W : Submodule ℂ (ι → ℂ)) (hW : W ≠ ⊥) : ∃ w₀ ∈ W, w₀ ≠ 0 ∧ ∀ A : Matrix ι ι ℂ, A.BlockTriangular h → (∀ w ∈ W, A *ᵥ w ∈ W) → ∃ c : ℂ, A *ᵥ w₀ = c • w₀

theorem **perp_ne_bot** (T : Submodule ℂ (ι → ℂ)) (hT : T ≠ ⊤) : perp T ≠ ⊥

theorem **eq_zero_of_mem_perp** {T : Submodule ℂ (ι → ℂ)} {y : ι → ℂ} (h1 : y ∈ T) (h2 : y ∈ perp T) : y = 0

theorem **eq_top_of_minimal** {G : Type} (A : G → Matrix ι ι ℂ) (hadj : ∀ g, ∃ g', A g' = (A g)ᴴ) (N : Submodule ℂ (ι → ℂ)) (hN : ∀ W : Submodule ℂ (ι → ℂ), W ≠ ⊥ → (∀ g, ∀ w ∈ W, A g *ᵥ w ∈ W) → (∀ U : Submodule ℂ (ι → ℂ), U ≠ ⊥ → (∀ g, ∀ w ∈ U, A g *ᵥ w ∈ U) → U ≤ W → U = W) → W ≤ N) : N = ⊤

theorem **integral_mem_of_forall_mem** {X : Type} [MeasurableSpace X] (μ : Measure X) (W : Submodule ℂ (ι → ℂ)) (f : X → ι → ℂ) (hf : Integrable f μ) (hW : ∀ x, f x ∈ W) : ∫ x, f x ∂μ ∈ W

theorem **goodPI_of** (hS : SmallPI) : GoodPI

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theorem **pr_pr** (ε : Fin n → Bool) (i : Fin n) : pr ε (pr ε i) = i

theorem **rev_cast** (i : Fin n) : ((i.rev : Fin n) : ℤ) = n - 1 - i

theorem **ht_ne** {ε : Fin n → Bool} {i : Fin n} (h : ε i.rev ≠ ε i) : 2 * (i : ℤ) + 1 - n ≠ 0

theorem **ht_eq_zero** {ε : Fin n → Bool} {i : Fin n} : ht ε i = 0 ↔ ε i.rev = ε i

theorem **lt_of_sign** {ε : Fin n → Bool} (hε : Monotone ε) {i j : Fin n} (hi : ε i = false) (hj : ε j = true) : (i : ℤ) < j

theorem **adapted** {ε : Fin n → Bool} (hε : Monotone ε) : Adapted (pr ε) (ht ε) (wt ε)

theorem **pr_eq_iff** {ε : Fin n → Bool} {i : Fin n} : pr ε i = i ↔ ε i.rev = ε i

theorem **dec_of_monotone** {ε : Fin n → Bool} (hε : Monotone ε) : Dec (Jm ε)

theorem **dec_Jm** (ε : Fin n → Bool) : Dec (Jm ε)

theorem **iwasawa_decomposition_unitary_group** : IwaAll

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theorem **isUpperTriangular_conjTranspose_iff** {A : Matrix n n 𝕜} : Aᴴ.IsUpperTriangular ↔ A.IsLowerTriangular

theorem **exists_isUpperTriangular_conjTranspose_mul_self** {S : Matrix n n 𝕜} (hS : S.PosDef) : ∃ R : Matrix n n 𝕜, R.IsUpperTriangular ∧ Rᴴ * R = S

theorem **star_mul_self_norm_div** {x : 𝕜} (hx : x ≠ 0) : star ((‖x‖ : 𝕜) / x) * ((‖x‖ : 𝕜) / x) = 1

theorem **exists_cholesky** {S : Matrix n n 𝕜} (hS : S.PosDef) : ∃ B : Matrix n n 𝕜, ∃ a : n → ℝ, B.IsUpperTriangular ∧ (∀ i, 0 < a i) ∧ (∀ i, B i i = a i) ∧ Bᴴ * B = S

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theorem **dp_mul_apply** (p : Fin n → Fin n) (α β : Fin n → ℂ) (X : Matrix (Fin n) (Fin n) ℂ) (i j : Fin n) : (dp p α β * X) i j = α i * X i j + β i * X (p i) j

theorem **dp_mul_dp** {p : Fin n → Fin n} (hp : ∀ i, p (p i) = i) (α β α' β' : Fin n → ℂ) : dp p α β * dp p α' β' = dp p (fun i => α i * α' i + β i * β' (p i)) (fun i => α i * β' i + β i * α' (p i))

theorem **dp_conjTranspose** {p : Fin n → Fin n} (hp : ∀ i, p (p i) = i) (α β : Fin n → ℂ) : (dp p α β)ᴴ = dp p (fun i => star (α i)) (fun i => star (β (p i)))

theorem **dp_congr** {p : Fin n → Fin n} {α β α' β' : Fin n → ℂ} (h1 : ∀ i, p i = i → α i + β i = α' i + β' i) (h2 : ∀ i, p i ≠ i → α i = α' i ∧ β i = β' i) : dp p α β = dp p α' β'

theorem **dp_diagonal** (p : Fin n → Fin n) (α : Fin n → ℂ) : dp p α 0 = diagonal α

theorem **mul_dp_apply** {p : Fin n → Fin n} (hp : ∀ i, p (p i) = i) (β : Fin n → ℂ) (X : Matrix (Fin n) (Fin n) ℂ) (i j : Fin n) : (X * dp p 0 β) i j = X i (p j) * β (p j)

theorem **eq_zero_of_unitary_blockTriangular** {b : Fin n → ℤ} {w : Matrix (Fin n) (Fin n) ℂ} (h1 : wᴴ * w = 1) (hw : w.BlockTriangular b) {i j : Fin n} (hij : b i ≠ b j) : w i j = 0

theorem **eq_one_of_posSemidef** {w : Matrix (Fin n) (Fin n) ℂ} (h1 : wᴴ * w = 1) (hw : w.PosSemidef) : w = 1

theorem **Adapted.mul_self** (h : Adapted p b c) : dp p 0 c * dp p 0 c = 1

theorem **Adapted.herm** (h : Adapted p b c) : (dp p 0 c)ᴴ = dp p 0 c

theorem **Adapted.conj_apply** (h : Adapted p b c) (B : Matrix (Fin n) (Fin n) ℂ) (i j : Fin n) : (dp p 0 c * Bᴴ * dp p 0 c) i j = c i * c j * star (B (p j) (p i))

theorem **Adapted.tri** (h : Adapted p b c) {B : Matrix (Fin n) (Fin n) ℂ} (hB : B.BlockTriangular id) : B.BlockTriangular b

theorem **Adapted.conj_tri** (h : Adapted p b c) {B : Matrix (Fin n) (Fin n) ℂ} (hB : B.BlockTriangular id) : (dp p 0 c * Bᴴ * dp p 0 c).BlockTriangular b

theorem **Adapted.posSemidef** (h : Adapted p b c) {B : Matrix (Fin n) (Fin n) ℂ} (hB : B.BlockTriangular id) {a : Fin n → ℝ} (ha : ∀ i, 0 < a i) (hBa : ∀ i, B i i = a i) (hz : ∀ i j, b i ≠ b j → (B * (dp p 0 c * Bᴴ * dp p 0 c)) i j = 0) : (B * (dp p 0 c * Bᴴ * dp p 0 c)).PosSemidef

theorem **flip** {A g : Matrix (Fin n) (Fin n) ℂ} (hA : A * A = 1) (hg : gᴴ * A * g = A) : g * A * gᴴ = A

theorem **Adapted.dec** (h : Adapted p b c) : Dec (dp p 0 c)

theorem **Dec.of_conj** {H V : Matrix (Fin n) (Fin n) ℂ} (hV : Vᴴ * V = 1) (h : Dec (Vᴴ * H * V)) : Dec H

CompanionTheory.HarishChandraModules.KFiniteAnalyticVectors.AdmissibleSmoothVectors.IwasawaDecompositionSecondModel

theorem **iwasawa_decomposition_second_model** : IwaHpk

CompanionTheory.HarishChandraModules.KFiniteAnalyticVectors.AdmissibleSmoothVectors.MapStdPoly

@[simp] theorem **word_nil** (E : R) : word E [] = E

@[simp] theorem **word_cons** (E a : R) (l : List R) : word E (a :: l) = E * a * word E l

theorem **map_word** {R' : Type*} [Ring R'] (f : R →+* R') (E : R) (l : List R) : f (word E l) = word (f E) (l.map f)

theorem **map_stdPoly** {R' : Type*} [Ring R'] (f : R →+* R') (E : R) {M : ℕ} (a : Fin M → R) : f (stdPoly E a) = stdPoly (f E) fun i => f (a i)

CompanionTheory.HarishChandraModules.KFiniteAnalyticVectors.AdmissibleSmoothVectors.SmallPI

theorem **std_eq_zero** {A : Type} [Ring A] [Algebra ℂ A] [Module.Finite ℂ A] {M : ℕ} (hM : Module.finrank ℂ A < M) (b : Fin M → A) : ∑ π : Equiv.Perm (Fin M), ((Equiv.Perm.sign π : ℤ) : A) * (List.ofFn fun i => b (π i)).prod = 0

theorem **small_irreducible_representation** : SmallPI

CompanionTheory.HarishChandraModules.KFiniteAnalyticVectors.AdmissibleSmoothVectors.Transfer

theorem **isGood_sum** (h : IsGood st S ρ) (h' : IsGood st S ρ') : IsGood st S (sumRep ρ ρ') where

theorem **pair_mono** {a b c d : ℕ} (h : a ≤ c) (h' : b ≤ d) : Nat.pair a b ≤ Nat.pair c d

theorem **isGood_tens** (h : IsGood st S ρ) (h' : IsGood st S ρ') : IsGood st S (tensRep ρ ρ') where

theorem **isGood_conj** (h : IsGood st S ρ) : IsGood st S (conjRep ρ) where

theorem **isGood_one** : IsGood st S (1 : G →* Matrix Unit Unit ℂ) where

theorem **coef_kronecker** (A : Matrix ι ι ℂ) (B : Matrix ι' ι' ℂ) (a b : ι → ℂ) (a' b' : ι' → ℂ) : (fun p : ι × ι' => a p.1 * a' p.2) ·ᵥ ((A ⊗ₖ B) *ᵥ fun p : ι × ι' => b p.1 * b' p.2) = (a ·ᵥ (A *ᵥ b)) * (a' ·ᵥ (B *ᵥ b'))

theorem **coefAlg_separates** (h : IsGood st S ρ) (hi : Function.Injective ρ) (C : Set G) : (coefAlg st S C).SeparatesPoints

theorem **exists_approx** (h : IsGood st S ρ) (hi : Function.Injective ρ) {C : Set G} (hC : IsCompact C) {Φ : G → ℂ} (hΦ : Continuous Φ) {δ : ℝ} (hδ : 0 < δ) : ∃ (κ : Type) (_ : Fintype κ) (_ : DecidableEq κ) (τ : G →* Matrix κ κ ℂ) (a b : κ → ℂ), IsGood st S τ ∧ ∀ x ∈ C, ‖Φ x - a ·ᵥ (τ x *ᵥ b)‖ ≤ δ

theorem **integrable_of_cont** {V : Type} [NormedAddCommGroup V] [SecondCountableTopology K] {f : K → V} (hf : Continuous f) : Integrable f μ

theorem **IsFam.sub** [SecondCountableTopology K] (he : Continuous e) {Φ Φ' : G → ℂ} {F F' : List G → G → ℂ} (h : IsFam μ e Φ F) (h' : IsFam μ e Φ' F') : IsFam μ e (fun x => Φ x - Φ' x) (fun l x => F l x - F' l x) where

theorem **norm_avg_le** [SecondCountableTopology K] (he : Continuous e) {f : K → ℂ} {δ : ℝ} (hf : ∀ k, ‖f k‖ ≤ δ) : ‖∫ k, e k * f k ∂μ‖ ≤ (∫ k, ‖e k‖ ∂μ) * δ

theorem **exists_bound** [SecondCountableTopology K] (he : Continuous e) (l : List G) : ∀ C : Set G, IsCompact C → ∃ C' : Set G, IsCompact C' ∧ ∀ Φ F, IsFam μ e Φ F → ∀ δ : ℝ, (∀ y ∈ C', ‖Φ y‖ ≤ δ) → ∀ x ∈ C, ‖F l x‖ ≤ (∫ k, ‖e k‖ ∂μ) ^ (l.length + 1) * δ

theorem **isFam_rep** [SecondCountableTopology K] (he : Continuous e) {V : Type} [NormedAddCommGroup V] [NormedSpace ℂ V] [CompleteSpace V] (τ : G →* Module.End ℂ V) (h1 : ∀ x, Continuous (τ x)) (h2 : ∀ v, Continuous fun x => τ x v) (E : Module.End ℂ V) (hE : ∀ u, E u = ∫ k : K, e k • τ k u ∂μ) (ℓ : V →L[ℂ] ℂ) (u : V) : IsFam μ e (fun x => ℓ (τ x u)) (fun l x => ℓ (τ x (word E (l.map τ) u)))

theorem **apply_stdPoly** {G V : Type} [AddCommGroup V] [Module ℂ V] (τ : G → Module.End ℂ V) (E : Module.End ℂ V) {M : ℕ} (g : Fin M → G) (ℓ : V →ₗ[ℂ] ℂ) (u : V) : ℓ (stdPoly E (fun i => τ (g i)) u) = ∑ π : Equiv.Perm (Fin M), ((Equiv.Perm.sign π : ℤ) : ℂ) * ℓ (word E ((List.ofFn fun i => g (π i)).map τ) u)

theorem **transfer_holds** : Transfer

CompanionTheory.HarishChandraModules.KFiniteAnalyticVectors.AnalyticIntertwiners.AdmissibleVectors

theorem **actLC_single** (E : Fin n → Fin n → Module.End ℂ M) (a b : Fin n) (x : ℂ) : actLC E (single a b x) = x • E a b

theorem **actLC_lie** {E : Fin n → Fin n → Module.End ℂ M} (hE : IsGlRep E) (X Y : Matrix (Fin n) (Fin n) ℂ) : actLC E (X * Y - Y * X) = actLC E X * actLC E Y - actLC E Y * actLC E X

theorem **twLC_apply** (E : Fin (p + q) → Fin (p + q) → Module.End ℂ M) (j : ℕ) (Z : Mat p q) (v : M) : twLC E j Z v = actM E (rx Z) v + ((j : ℂ) * (Z.trace / 2)) • v

theorem **twLC_lie** {E : Fin (p + q) → Fin (p + q) → Module.End ℂ M} (hE : IsGlRep E) (j : ℕ) (X Y : Mat p q) : twLC E j (X * Y - Y * X) = twLC E j X * twLC E j Y - twLC E j Y * twLC E j X

theorem **twLC_E** (E : Fin (p + q) → Fin (p + q) → Module.End ℂ M) (j : ℕ) (a b : Formalism.BasicNotions.TH.Ix p q) (v : M) : twLC E j (Eab a b) v = E (finSumFinEquiv a) (finSumFinEquiv b) v + (if a = b then (j : ℂ) / 2 else 0) • v

theorem **upq_decC** (Z : Mat p q) : ∃ Y₁ ∈ upq p q, ∃ Y₂ ∈ upq p q, Z = Y₁ + Complex.I • Y₂ ∧ (IsKC Z → IsKC Y₁ ∧ IsKC Y₂)

theorem **deriv_memC** {H : Type} [NormedAddCommGroup H] [NormedSpace ℂ H] {S : Submodule ℂ H} (hS : IsClosed (S : Set H)) {f : ℝ → H} {y : H} (hf : HasDerivAt f y 0) (hm : ∀ t, f t ∈ S) : y ∈ S

theorem **posC_ofC** {p q : ℕ} {a b : Formalism.BasicNotions.TH.Ix p q} (h : finSumFinEquiv a < finSumFinEquiv b) (he : epq p q (finSumFinEquiv a) = epq p q (finSumFinEquiv b)) : posC a b

theorem **harish_chandra_first_clause** (hD : ForIrr fun _ ε H _ _ σ => Dense {v : H | Formalism.MainNotions.S.IsKFin ε σ v}) (hS : KFSm) (hW : IntWts) (hI : UIrr) (hA : UAdm) (hDe : Descend) : Clause1

CompanionTheory.HarishChandraModules.KFiniteAnalyticVectors.AnalyticIntertwiners.BoundedOperators

theorem **inner_piC** (πU : URep T) (g : T.G) (a b : πU.H) : ⟪πU.π g a, πU.π g b⟫_ℂ = ⟪a, b⟫_ℂ

theorem **pi_mulC** (πU : URep T) (g h : T.G) (v : πU.H) : πU.π (g * h) v = πU.π g (πU.π h v)

theorem **sig_repC** (πU : URep T) {σ : Matrix (Fin (p + q)) (Fin (p + q)) ℂ → (πU.H →L[ℂ] πU.H)} (hσ : ∀ x, σ (rx (covC T x)) = ((((T.sq x : ℂˣ) : ℂ))⁻¹ ^ 0) • πU.π x) : Formalism.MainNotions.S.IsURepOn (Formalism.MainNotions.S.UJset (epq p q)) σ

theorem **kset_ofC** {k : T.G} (hk : k ∈ T.Kset) : rx (covC T k) ∈ Formalism.REAL.Kset (epq p q)

theorem **kfinDense_coverC** (hD : ∀ (n : ℕ) (ε : Fin n → Bool) (H : Type) [NormedAddCommGroup H] [InnerProductSpace ℂ H] [CompleteSpace H] (σ : Matrix (Fin n) (Fin n) ℂ → (H →L[ℂ] H)), Formalism.MainNotions.S.IsURepOn (Formalism.MainNotions.S.UJset ε) σ → Dense {v : H | Formalism.MainNotions.S.IsKFin ε σ v}) (πU : URep T) : Dense {x | KFin T πU.π x}

theorem **harish_chandra_third_clause** (hD : ∀ (p q : ℕ) (T : GrpData p q) (πU : URep T), Dense {x | KFin T πU.π x}) : Clause3

CompanionTheory.HarishChandraModules.KFiniteAnalyticVectors.AnalyticIntertwiners.CasimirRealScalar

theorem **gen_eq_A** (p : Fin n × Fin n) : Formalism.L2.SG.Pf.admitted.gen ε p = casGen ε p

theorem **coef_eq_A** (p : Fin n × Fin n) : (((Formalism.L2.SG.Pf.admitted.wt p * Formalism.L2.SG.Pf.admitted.sg ε p : ℝ)) : ℂ) = casWt p * casSg ε p

theorem **cas_eq_A** (hσ : Formalism.MainNotions.S.IsURepOn (Formalism.MainNotions.S.UJset ε) σ) (v : smD ε σ) : Formalism.L2.SG.Pf.admitted.Cas ε σ v = ((casimir (smE ε σ) v : smD ε σ) : H)

theorem **admissibleProjection_of_irreducible_indefUnitary** [CompleteSpace H] (hσ : Formalism.MainNotions.S.IsURepOn (Formalism.MainNotions.S.UJset ε) σ) (hirr : Formalism.MainNotions.S.IsIrredOn (Formalism.MainNotions.S.UJset ε) σ) : Formalism.MainNotions.BridgeCPX.AdmProjAt ε σ

theorem **casimirActsByScalar_indefUnitary** : CasScal

CompanionTheory.HarishChandraModules.KFiniteAnalyticVectors.AnalyticIntertwiners.Clause1

theorem **hcm_of** (h1 : Clause1) (h2 : Clause2) (h3 : Clause3) (h4 : GlobalTheory.Published.KFiniteAnalyticUJ) : GlobalTheory.Published.HarishChandraModules

CompanionTheory.HarishChandraModules.KFiniteAnalyticVectors.AnalyticIntertwiners.CommutatorAdjointHolds

theorem **compact_sq_le_A** (hU : IsInfUnitary ε E) (x : M) {a b : Fin n} (hab : ε a = ε b) : ‖E a b x‖ ^ 2 ≤ RCLike.re (inner ℂ (casimirK ε E x) x)

theorem **unit_bd_A** (hE : IsGlRep E) (hU : IsInfUnitary ε E) {V : Submodule ℂ M} (hVf : FiniteDimensional ℂ V) (hVK : IsKStableSub ε E V) {c : ℝ} (hc : ∀ v, casimir E v = (c : ℂ) • v) : ∃ D : ℝ, 0 < D ∧ ∀ (k : ℕ), ∀ x ∈ layer ε E V k, ∀ a b, E a b x ∈ layer ε E V (k + 1) ∧ ‖E a b x‖ ≤ D * (k + 1) * ‖x‖

theorem **entrywise_l1_norm_nonneg** (X : Matrix (Fin n) (Fin n) ℂ) : 0 ≤ l1 X

theorem **wordBd_of** (hC : CasScal) : WordBd

theorem **exists_log_A** (n : ℕ) : ∃ L : (Fin n → Fin n → ℂ) → (Fin n → Fin n → ℂ), AnalyticAt ℝ L (Matrix.of.symm 1) ∧ L (Matrix.of.symm 1) = 0 ∧ (∀ᶠ z in 𝒩 (Matrix.of.symm (1 : Matrix (Fin n) (Fin n) ℂ)), NormedSpace.exp (Matrix.of (L z)) = Matrix.of z) ∧ ∀ᶠ Y in 𝒩 (0 : Fin n → Fin n → ℂ), L (Matrix.of.symm (NormedSpace.exp (Matrix.of Y))) = Y

theorem **Jm_sq_A** : Jm ε * Jm ε = 1

theorem **exp_theta_A** {Y : Matrix (Fin n) (Fin n) ℂ} (h : NormedSpace.exp Y ∈ Formalism.MainNotions.S.UJset ε) : NormedSpace.exp (Jm ε * (-Yᴴ) * Jm ε) = NormedSpace.exp Y

theorem **exists_logG_A** (ε : Fin n → Bool) : ∃ L : (Fin n → Fin n → ℂ) → (Fin n → Fin n → ℂ), AnalyticAt ℝ L (Matrix.of.symm 1) ∧ L (Matrix.of.symm 1) = 0 ∧ ∀ᶠ z in 𝒩 (Matrix.of.symm (1 : Matrix (Fin n) (Fin n) ℂ)), NormedSpace.exp (Matrix.of (L z)) = Matrix.of z ∧ (Matrix.of z ∈ Formalism.MainNotions.S.UJset ε → inG ε (Matrix.of (L z)))

theorem **entrywise_l1_norm_le** (Y : Fin n → Fin n → ℂ) : l1 (Matrix.of Y) ≤ (n : ℝ) ^ 2 * ‖Y‖

theorem **exists_series_A** {H : Type} [NormedAddCommGroup H] [InnerProductSpace ℂ H] [CompleteSpace H] {σ : Matrix (Fin n) (Fin n) ℂ → (H →L[ℂ] H)} (hσ : Formalism.MainNotions.S.IsURepOn (Formalism.MainNotions.S.UJset ε) σ) (v : smD ε σ) {C : ℝ} (hC : 0 < C) (hb : ∀ W : List (Matrix (Fin n) (Fin n) ℂ), ‖((wd ε σ W v : smD ε σ) : H)‖ ≤ C ^ (W.length + 1) * W.length.factorial * (W.map l1).prod) : ∃ Φ : (Fin n → Fin n → ℂ) → H, AnalyticAt ℝ Φ 0 ∧ ∀ᶠ Y in 𝒩 (0 : Fin n → Fin n → ℂ), inG ε (Matrix.of Y) → Φ Y = σ (NormedSpace.exp (Matrix.of Y)) v

theorem **kFiniteAnalytic_of** (hK : KFSm) (hW : WordBd) : GlobalTheory.Published.KFiniteAnalyticUJ

CompanionTheory.HarishChandraModules.KFiniteAnalyticVectors.AnalyticIntertwiners.GelfandCentralHolds

theorem **smoothVec_of_smD_A** {v : H} (hv : v ∈ smD ε σ) : Formalism.L2.Theta.SmoothVec (Formalism.IndefR.Tow.SobPf.lieG (Formalism.REAL.Jm ε)) σ v

theorem **isGRep_A** (hσ : Formalism.MainNotions.S.IsURepOn (Formalism.MainNotions.S.UJset ε) σ) : Formalism.IndefR.Tow.SobPf.IsGRep (Formalism.REAL.Jm ε) σ

theorem **cartan_A** : Formalism.IndefR.Datum.IsCartanInv (Formalism.REAL.Jm ε) (Formalism.REAL.Jm ε)

theorem **kfin_of_kfinG_A** {a : H} (h : Formalism.IndefR.Tow.SobPf.KFinG (Formalism.REAL.Jm ε) (Formalism.REAL.Jm ε) σ a) : Formalism.MainNotions.S.IsKFin ε σ a

theorem **exists_kfin_near_A** [CompleteSpace H] (hσ : Formalism.MainNotions.S.IsURepOn (Formalism.MainNotions.S.UJset ε) σ) {w : H} (hw : w ∈ smD ε σ) {δ : ℝ} (hδ : 0 < δ) : ∃ a : H, Formalism.MainNotions.S.IsKFin ε σ a ∧ ‖a - w‖ < δ

theorem **eq_zero_of_perp_A** [CompleteSpace H] (hσ : Formalism.MainNotions.S.IsURepOn (Formalism.MainNotions.S.UJset ε) σ) {w : H} (hw : w ∈ smD ε σ) (h : ∀ a : H, Formalism.MainNotions.S.IsKFin ε σ a → ⟪a, w⟫_ℂ = 0) : w = 0

theorem **casimir_symm_A** (hσ : Formalism.MainNotions.S.IsURepOn (Formalism.MainNotions.S.UJset ε) σ) (x y : smD ε σ) : ⟪casimir (smE ε σ) x, y⟫_ℂ = ⟪x, casimir (smE ε σ) y⟫_ℂ

CompanionTheory.HarishChandraModules.KFiniteAnalyticVectors.AnalyticIntertwiners.GrowthOfBasic

theorem **mem_of_forall_orthogonalComplement_inner_eq_zero** (T : Submodule ℂ H) [FiniteDimensional ℂ T] {x : H} (h : ∀ y ∈ Tᗮ, ⟪y, x⟫_ℂ = 0) : x ∈ T

theorem **exists_sub_B** (hK : KFSm) (hσ : Formalism.MainNotions.S.IsURepOn (Formalism.MainNotions.S.UJset ε) σ) (hirr : Formalism.MainNotions.S.IsIrredOn (Formalism.MainNotions.S.UJset ε) σ) {v : smD ε σ} (hv : v ∈ KF ε σ) : ∃ S : Submodule ℂ (smD ε σ), FiniteDimensional ℂ S ∧ v ∈ S ∧ (S : Set (smD ε σ)) ⊆ KF ε σ ∧

IsKStableSub ε (smE ε σ) S ∧ ∀ k ∈ Kset ε, ∀ w ∈ S, ∃ w' ∈ S, (w' : H) = σ k w

theorem **kavg_mem_B** (hσ : Formalism.MainNotions.S.IsURepOn (Formalism.MainNotions.S.UJset ε) σ) {e : Formalism.REAL.KG ε → ℂ} (he : Continuous e) (T : Submodule ℂ H) [FiniteDimensional ℂ T] {x : H} (hx : ∀ k : Formalism.REAL.KG ε, σ (Formalism.REAL.toMK k) x ∈ T) : Formalism.MainNotions.BridgeCPX.kavg hσ he x ∈ T

theorem **uIrr_of** (hK : KFSm) (hC : CasScal) : UIrr

CompanionTheory.HarishChandraModules.KFiniteAnalyticVectors.AnalyticIntertwiners.HarishChandraModulesIndefiniteUnitaryGroup

theorem **uIrr** : UIrr

theorem **uAdm** : UAdm

theorem **clause1** : Clause1

theorem **clause2** : Clause2

theorem **print_harishChandraModules** : GlobalTheory.Published.HarishChandraModules

CompanionTheory.HarishChandraModules.KFiniteAnalyticVectors.AnalyticIntertwiners.HarishChandraThirdIndefiniteUnitary

theorem **clause1_of** (hS : KFSm) (hW : IntWts) (hI : UIrr) (hA : UAdm) : Clause1

theorem **harishChandraThird_indefiniteUnitary** : Clause3

CompanionTheory.HarishChandraModules.KFiniteAnalyticVectors.AnalyticIntertwiners.KSpanFinB

theorem **kSpan_top_B** {x : smD ε σ} (hW : IsKStableSub ε (smE ε σ) (Formalism.REAL.kSpan ε (smE ε σ) x)) (x' : Formalism.REAL.kSpan ε (smE ε σ) x) {c : ℂ} (hc : c ≠ 0) (hx' : (x' : smD ε σ) = c • x) : Formalism.REAL.kSpan ε (resB _ hW) x' = ⊤

theorem **kSpan_fin_B** (hK : KFSm) (hσ : Formalism.MainNotions.S.IsURepOn (Formalism.MainNotions.S.UJset ε) σ) (hirr : Formalism.MainNotions.S.IsIrredOn (Formalism.MainNotions.S.UJset ε) σ) {v : smD ε σ} (hv : v ∈ KF ε σ) : FiniteDimensional ℂ (Formalism.REAL.kSpan ε (smE ε σ) v) ∧ IsKStableSub ε (smE ε σ) (Formalism.REAL.kSpan ε (smE ε σ) v)

theorem **uAdm_of** (hK : KFSm) : UAdm

CompanionTheory.HarishChandraModules.KFiniteAnalyticVectors.AnalyticIntertwiners.LocalPointData

theorem **graphD** [CompleteSpace H₁] [CompleteSpace H₂] (h₁ : IsURepOn (UJset ε) σ₁) (h₂ : IsURepOn (UJset ε) σ₂) (hD₁ : IsDerivedOn ε σ₁ D₁ E₁) (hD₂ : IsDerivedOn ε σ₂ D₂ E₂) {N : Type} [AddCommGroup N] [Module ℂ N] (F : Fin n → Fin n → Module.End ℂ N) (κ₁ : N →ₗ[ℂ] D₁) (κ₂ : N →ₗ[ℂ] D₂) (hi : Function.Injective κ₁) (e₁ : ∀ a b w, E₁ a b (κ₁ w) = κ₁ (F a b w)) (e₂ : ∀ a b w, E₂ a b (κ₂ w) = κ₂ (F a b w)) {c : ℝ} (hc : ∀ v, casimir E₁ v = (c : ℂ) • v) (U : H₁ →L[ℂ] H₂) (hU : ∀ w, U (κ₁ w) = κ₂ w) {W : Submodule ℂ N} (hW : FiniteDimensional ℂ W) (hWK : IsKStableSub ε F W) {w : N} (hw : w ∈ W) {g : Matrix (Fin n) (Fin n) ℂ} (hg : g ∈ UJset ε) : U (σ₁ g (κ₁ w)) = σ₂ g (κ₂ w)

theorem **norm_detD** {g : Matrix (Fin n) (Fin n) ℂ} (hg : g ∈ Formalism.MainNotions.S.UJset ε) : ‖g.det‖ = 1

theorem **twD_rep** (m : ℕ) (h : Formalism.MainNotions.S.IsURepOn (Formalism.MainNotions.S.UJset ε) σ) : Formalism.MainNotions.S.IsURepOn (Formalism.MainNotions.S.UJset ε) (twD m σ)

theorem **twD_irr** (m : ℕ) (h : Formalism.MainNotions.S.IsIrredOn (Formalism.MainNotions.S.UJset ε) σ) : Formalism.MainNotions.S.IsIrredOn (Formalism.MainNotions.S.UJset ε) (twD m σ)

theorem **twD_kfin** (m : ℕ) {v : H} (h : Formalism.MainNotions.S.IsKFin ε σ v) : Formalism.MainNotions.S.IsKFin ε (twD m σ) v

theorem **skewD** (h : IsHCMod πU M io) : SkewD M io

theorem **eigD** (h : IsHCMod πU M io) {X : Mat p q} (hX : X ∈ upq p q) {v : M.V} {c : ℂ} (hv : M.act X v = c • v) (t : ℝ) : πU.π (T.expt (t • X)) (io v) = Complex.exp (c * t) • io v

theorem **iE_memD** (a : Formalism.BasicNotions.TH.Ix p q) : Complex.I • Eab a a ∈ upq p q

theorem **exp_twoPiD** (a : Formalism.BasicNotions.TH.Ix p q) : NormedSpace.exp (((2 * Real.pi : ℝ) : ℂ) • rx (Complex.I • Eab a a)) = 1

theorem **signD** (h : IsHCMod πU M io) {j : ℕ} {σ : Matrix (Fin (p + q)) (Fin (p + q)) ℂ → (πU.H →L[ℂ] πU.H)} (h1 : σ 1 = 1) (hex : ∀ X ∈ upq p q, inG (epq p q) (rx X) ∧ ∀ t : ℝ, πU.π (T.expt (t • X)) = Complex.exp ((j : ℂ) * (t * X.trace / 2)) • σ (NormedSpace.exp ((t : ℂ) • rx X))) (a : Formalism.BasicNotions.TH.Ix p q) {v : M.V} (hv0 : v ≠ 0) {c : ℂ} (hv : M.act (Eab a a) v = c • v) : Complex.exp (Complex.I * c * (2 * Real.pi : ℝ)) = (-1) ^ j

theorem **dictD** (hS : KFSm) (h : IsHCMod πU M io) {j : ℕ} (m : ℕ) {σ : Matrix (Fin (p + q)) (Fin (p + q)) ℂ → (πU.H →L[ℂ] πU.H)} (hσ : Formalism.MainNotions.S.IsURepOn (Formalism.MainNotions.S.UJset (epq p q)) σ) (hσi : Formalism.MainNotions.S.IsIrredOn (Formalism.MainNotions.S.UJset (epq p q)) σ) (hkf : ∀ x, KFin T πU.π x ↔ Formalism.MainNotions.S.IsKFin (epq p q) σ x) (hex : ∀ X ∈ upq p q, inG (epq p q) (rx X) ∧ ∀ t : ℝ, πU.π (T.expt (t • X)) = Complex.exp ((j : ℂ) * (t * X.trace / 2)) • σ (NormedSpace.exp ((t : ℂ) • rx X))) : ∃ κ : M.V →ₗ[ℂ] smD (epq p q) (twD m σ), (∀ v, (κ v : πU.H) = io v) ∧ ∀ Z v, actM (smE (epq p q) (twD m σ)) (rx Z) (κ v) = κ (M.act Z v + (((m : ℂ) - j / 2) * Z.trace) • v)

theorem **clause2_of** (h1 : Clause1) (hS : KFSm) (hC : CasScal) (hDe : Descend) : Clause2

CompanionTheory.HarishChandraModules.KFiniteAnalyticVectors.AnalyticIntertwiners.LogNormEstimates

theorem **Jm_eq_diagD** : Jm p q = diagonal (sgD (p

theorem **Jm_mul_selfD** : Jm p q * Jm p q = 1

theorem **Jm_hermD** : (Jm p q)ᴴ = Jm p q

theorem **adjZ_adjZD** (Z : Mat p q) : adjZ (adjZ Z) = Z

theorem **adjZ_addD** (X Y : Mat p q) : adjZ (X + Y) = adjZ X + adjZ Y

theorem **adjZ_smulD** (c : ℂ) (X : Mat p q) : adjZ (c • X) = star c • adjZ X

theorem **mem_upq_of_adjZD** {Y : Mat p q} (h : adjZ Y = -Y) : Y ∈ upq p q

theorem **adjZ_singleD** (a b : Ix p q) (h : sameBlock a b) : adjZ (Eab a b) = Eab b a

theorem **sameBlock_symmD** {a b : Ix p q} (h : sameBlock a b) : sameBlock b a

theorem **sameBlock_reflD** (a : Ix p q) : sameBlock a a

theorem **sameBlock_of_posCD** {a b : Ix p q} (h : posC a b) : sameBlock a b

theorem **isKC_EabD** {a b : Ix p q} (h : sameBlock a b) : IsKC (Eab a b)

theorem **adj_ruleD** {ι : M.V →ₗ[ℂ] H₁} (h : SkewD M ι) (Z : Mat p q) (v w : M.V) : ⟪ι (M.act Z v), ι w⟫_ℂ = ⟪ι v, ι (M.act (adjZ Z) w)⟫_ℂ

theorem **mem_wtSubD** {μ : Ix p q → ℚ} {y : M.V} : y ∈ wtSub M.act μ ↔ ∀ a, M.act (Eab a a) y = ((μ a : ℚ) : ℂ) • y

theorem **comm_EabD** (c a b : Ix p q) : Eab c c * Eab a b - Eab a b * Eab c c = ((if c = a then 1 else 0) - (if c = b then 1 else 0) : ℂ) • Eab a b

theorem **raiseD** {μ : Ix p q → ℚ} {y : M.V} (hy : y ∈ wtSub M.act μ) (a b : Ix p q) : M.act (Eab a b) y ∈ wtSub M.act (shD μ a b)

theorem **shD_shD** (μ : Ix p q → ℚ) (a b : Ix p q) : shD (shD μ a b) b a = μ

theorem **cwD_lt** {a b : Ix p q} (h : posC a b) : cwD b < cwD a

theorem **ellD_shD** (μ : Ix p q → ℚ) (a b : Ix p q) : ellD (shD μ a b) = ellD μ + cwD a - cwD b

theorem **exists_hwD** (hn : Nontrivial M.V) : ∃ μ, ∃ y ∈ hwSub M.act μ, y ≠ 0

theorem **form_propD** (ι₁ : M.V →ₗ[ℂ] H₁) (ι₂ : M.V →ₗ[ℂ] H₂) (h₁ : Function.Injective ι₁) (h₂ : Function.Injective ι₂) (s₁ : SkewD M ι₁) (s₂ : SkewD M ι₂) (hI : Formalism.MainNotions.SchurIrreducible.IrrGK M) (hA : Formalism.MainNotions.SchurIrreducible.AdmGK M) : ∃ c : ℝ, 0 < c ∧ ∀ v w, ⟪ι₂ v, ι₂ w⟫_ℂ = (c : ℂ) * ⟪ι₁ v, ι₁ w⟫_ℂ

CompanionTheory.HarishChandraModules.KFiniteAnalyticVectors.AnalyticIntertwiners.NilpotentOfPowerTraces

theorem **cpart_inK_A** {Z : Matrix (Fin n) (Fin n) ℂ} (hZ : Z * Jm ε = Jm ε * Z) : inK ε (S7.cpart1 ε Z) ∧ inK ε (S7.cpart2 ε Z)

theorem **single_comm_A** {a b : Fin n} (hab : ε a = ε b) : Matrix.single a b (1 : ℂ) * Jm ε = Jm ε * Matrix.single a b (1 : ℂ)

theorem **dS_mem_of_inK_A** (hσ : Formalism.MainNotions.S.IsURepOn (Formalism.MainNotions.S.UJset ε) σ) {S : Submodule ℂ H} [FiniteDimensional ℂ S] (hK : Formalism.MainNotions.S.IsKStableOn ε σ S) {Z : Matrix (Fin n) (Fin n) ℂ} (hZ : inK ε Z) {s : H} (hs : s ∈ S) (hsm : s ∈ smD ε σ) : dS σ Z s ∈ S

theorem **smE_mem_of_compact_A** (hσ : Formalism.MainNotions.S.IsURepOn (Formalism.MainNotions.S.UJset ε) σ) {S : Submodule ℂ H} [FiniteDimensional ℂ S] (hK : Formalism.MainNotions.S.IsKStableOn ε σ S) {a b : Fin n} (hab : ε a = ε b) (x : smD ε σ) (hx : (x : H) ∈ S) : ((smE ε σ a b x : smD ε σ) : H) ∈ S

theorem **exp_two_pi_A** (a : Fin n) : NormedSpace.exp (((2 * Real.pi : ℝ) : ℂ) • (Complex.I • Matrix.single a a (1 : ℂ))) = 1

theorem **inG_diag_A** (a : Fin n) : inG ε (Complex.I • Matrix.single a a (1 : ℂ))

theorem **eig_int_A** (hσ : Formalism.MainNotions.S.IsURepOn (Formalism.MainNotions.S.UJset ε) σ) (a : Fin n) {w : smD ε σ} (hw : w ≠ 0) {c : ℂ} (hc : smE ε σ a a w = c • w) : ∃ m : ℤ, c = m

theorem **integral_weights** : IntWts

CompanionTheory.HarishChandraModules.KFiniteAnalyticVectors.AnalyticIntertwiners.PCDesc

theorem **rx_ixC** (g : Matrix (Fin (p + q)) (Fin (p + q)) ℂ) : rx (ixC g) = g

theorem **ixC_rx** (X : Formalism.BasicNotions.TH.Mat p q) : ixC (rx X) = X

theorem **rx_mulC** (A B : Formalism.BasicNotions.TH.Mat p q) : rx (A ∗ B) = rx A ∗ rx B

theorem **ixC_mul** (A B : Matrix (Fin (p + q)) (Fin (p + q)) ℂ) : ixC (A ∗ B) = ixC A ∗ ixC B

theorem **rx_oneC** : rx (1 : Formalism.BasicNotions.TH.Mat p q) = 1

theorem **rx_ctC** (A : Formalism.BasicNotions.TH.Mat p q) : rx Aᴴ = (rx A)ᴴ

theorem **ixC_ct** (A : Matrix (Fin (p + q)) (Fin (p + q)) ℂ) : ixC Aᴴ = (ixC A)ᴴ

theorem **rx_injC** {A B : Formalism.BasicNotions.TH.Mat p q} (h : rx A = rx B) : A = B

theorem **rx_JC** : rx (Formalism.UG.Jpq p q) = Formalism.REAL.Jm (epq p q)

theorem **ixC_J** : ixC (Formalism.REAL.Jm (epq p q)) = Formalism.UG.Jpq p q

theorem **det_rxC** (A : Formalism.BasicNotions.TH.Mat p q) : (rx A).det = A.det

theorem **trace_rxC** (A : Formalism.BasicNotions.TH.Mat p q) : (rx A).trace = A.trace

theorem **rx_expC** (X : Formalism.BasicNotions.TH.Mat p q) : rx (NormedSpace.exp X) = NormedSpace.exp (rx X)

theorem **Jm_sqC** : Formalism.REAL.Jm (epq p q) ∗ Formalism.REAL.Jm (epq p q) = 1

theorem **Jm_ctC** : (Formalism.REAL.Jm (epq p q))ᴴ = Formalism.REAL.Jm (epq p q)

theorem **sq_sqC** (x : T.G) : ((T.sq x : ℂˣ) : ℂ) ^ 2 = (covC T x).det

theorem **eq_of_covC** {x y : T.G} (h1 : T.cov x = T.cov y) (h2 : T.sq x = T.sq y) : x = y

theorem **mem_UJC** (x : T.G) : rx (covC T x) ∈ Formalism.MainNotions.S.UJset (epq p q)

theorem **exists_liftC** {g : Matrix (Fin (p + q)) (Fin (p + q)) ℂ} (hg : g ∈ Formalism.MainNotions.S.UJset (epq p q)) (w : ℂˣ) (hw : (w : ℂ) ^ 2 = g.det) : ∃ x : T.G, rx (covC T x) = g ∧ T.sq x = w

theorem **exists_lift'C** {g : Matrix (Fin (p + q)) (Fin (p + q)) ℂ} (hg : g ∈ Formalism.MainNotions.S.UJset (epq p q)) : ∃ x : T.G, rx (covC T x) = g

theorem **exists_zC** : ∃ z : T.G, T.cov z = 1 ∧ T.sq z = −1

theorem **sq_or_negC** {x y : T.G} (h : T.cov x = T.cov y) : T.sq x = T.sq y ∨ T.sq x = −T.sq y

theorem **norm_sqC** (x : T.G) : ‖((T.sq x : ℂˣ) : ℂ)‖ = 1

theorem **sq_contC** : Continuous fun x : T.G => ((T.sq x : ℂˣ) : ℂ)

theorem **sigC_eq** {j : ℕ} {z : T.G} (hz : T.cov z = 1 ∧ T.sq z = −1) (hP : πU.π z = ((−1 : ℂ) ^ j) • 1) (x : T.G) : sigC πU j (rx (covC T x)) = ((((T.sq x : ℂˣ) : ℂ))⁻¹ ^ j) • πU.π x

theorem **Jpq_diagC** : Formalism.UG.Jpq p q = diagonal (sgnC (p

theorem **sgn_neC** {a b : Formalism.BasicNotions.TH.Ix p q} (h : ¬ Formalism.BasicNotions.TH.sameBlock a b) : sgnC b − sgnC a ≠ 0

theorem **sgn_eqC** {a b : Formalism.BasicNotions.TH.Ix p q} (h : Formalism.BasicNotions.TH.sameBlock a b) : sgnC b = sgnC a

theorem **comm_iff_KC** (M : Formalism.BasicNotions.TH.Mat p q) : M ∗ Formalism.UG.Jpq p q = Formalism.UG.Jpq p q ∗ M ↔ Formalism.BasicNotions.TH.IsKC M

theorem **isKC_mulC** {A B : Formalism.BasicNotions.TH.Mat p q} (hA : Formalism.BasicNotions.TH.IsKC A) (hB : Formalism.BasicNotions.TH.IsKC B) : Formalism.BasicNotions.TH.IsKC (A ∗ B)

theorem **isKC_oneC** : Formalism.BasicNotions.TH.IsKC (1 : Formalism.BasicNotions.TH.Mat p q)

theorem **exists_contLiftC** {p q : ℕ} (T : GrpData p q) (A : Set (Matrix (Fin (p + q)) (Fin (p + q)) ℂ)) (hA : A ⊆ Formalism.MainNotions.S.UJset (epq p q)) (W : A → ℂˣ) (hWc : Continuous fun g => ((W g : ℂˣ) : ℂ)) (hW2 : ∀ g, ((W g : ℂˣ) : ℂ) ^ 2 = (g : Matrix (Fin (p + q)) (Fin (p + q)) ℂ).det) : ∃ xs : A → T.G, Continuous xs ∧ ∀ g, rx (covC T (xs g)) = g.1

theorem **sigC_contC** {p q : ℕ} {T : GrpData p q} (πU : URep T) {j : ℕ} {σ : Matrix (Fin (p + q)) (Fin (p + q)) ℂ → (πU.H →L[ℂ] πU.H)} (hσ : ∀ x, σ (rx (covC T x)) = ((((T.sq x : ℂˣ) : ℂ))⁻¹ ^ j) • πU.π x) (v : πU.H) : ContinuousOn (fun g => σ g v) (Formalism.MainNotions.S.UJset (epq p q))

theorem **harish_chandra_descent** : Descend

CompanionTheory.HarishChandraModules.KFiniteAnalyticVectors.AnalyticIntertwiners.SecondDerivativeHolds

theorem **resB_apply** {W : Submodule ℂ (smD ε σ)} (hW : IsKStableSub ε (smE ε σ) W) {a b : Fin n} (h : ε a = ε b) (x : W) : ((resB W hW a b x : W) : smD ε σ) = smE ε σ a b x

theorem **kRel_resB** (hσ : Formalism.MainNotions.S.IsURepOn (Formalism.MainNotions.S.UJset ε) σ) {W : Submodule ℂ (smD ε σ)} (hW : IsKStableSub ε (smE ε σ) W) : Formalism.REAL.KRel ε (resB W hW)

theorem **kUnit_resB** (hσ : Formalism.MainNotions.S.IsURepOn (Formalism.MainNotions.S.UJset ε) σ) {W : Submodule ℂ (smD ε σ)} (hW : IsKStableSub ε (smE ε σ) W) : Formalism.REAL.KUnit ε (resB W hW)

theorem **hasSum_actM_pow_div_factorial** (hσ : Formalism.MainNotions.S.IsURepOn (Formalism.MainNotions.S.UJset ε) σ) {W : Submodule ℂ (smD ε σ)} [FiniteDimensional ℂ W] (hW : IsKStableSub ε (smE ε σ) W) {Z : Matrix (Fin n) (Fin n) ℂ} (hZ : inK ε Z) {x : smD ε σ} (hx : x ∈ W) : HasSum (fun m : ℕ => (((1 : ℝ) ^ m / (m.factorial : ℝ) : ℝ) : ℂ) • ((((actM (smE ε σ) Z) ^ m) x : smD ε σ) : H)) (σ (NormedSpace.exp Z) (x : H))

theorem **intertwiner_actM_pow_comm** {W W' : Submodule ℂ (smD ε σ)} (hW : IsKStableSub ε (smE ε σ) W) (hW' : IsKStableSub ε (smE ε σ) W') (φ : W →ₗ[ℂ] W') (hφ : ∀ a b, ε a = ε b → ∀ x : W, φ (resB W hW a b x) = resB W' hW' a b (φ x)) {Z : Matrix (Fin n) (Fin n) ℂ} (hZ : inK ε Z) (m : ℕ) (x : W) : ((φ ((actM (smE ε σ) Z) ^ m) x, actM_pow_mem_of_isKStableSub_inK hW hZ x.2 m) : W') : smD ε σ) = ((actM (smE ε σ) Z) ^ m) (φ x)

theorem **intertwiner_comm_kset** (hσ : Formalism.MainNotions.S.IsURepOn (Formalism.MainNotions.S.UJset ε) σ) {W W' : Submodule ℂ (smD ε σ)} [FiniteDimensional ℂ W] [FiniteDimensional ℂ W'] (hW : IsKStableSub ε (smE ε σ) W) (hW' : IsKStableSub ε (smE ε σ) W') (φ : W →ₗ[ℂ] W') (hφ : ∀ a b, ε a = ε b → ∀ x : W, φ (resB W hW a b x) = resB W' hW' a b (φ x)) {g : Matrix (Fin n) (Fin n) ℂ} (hg : g ∈ Formalism.REAL.Kset ε) (x y : W) (hxy : ((y : smD ε σ) : H) = σ g ((x : smD ε σ) : H)) : (((φ y : W') : smD ε σ) : H) = σ g (((φ x : W') : smD ε σ) : H)

CompanionTheory.HarishChandraModules.KFiniteAnalyticVectors.AnalyticIntertwiners.SubquotientNontrivial

theorem **mem_smD_of_isKFin** [CompleteSpace H] (hσ : Formalism.MainNotions.S.IsURepOn (Formalism.MainNotions.S.UJset ε) σ) (hirr : Formalism.MainNotions.S.IsIrredOn (Formalism.MainNotions.S.UJset ε) σ) {v : H} (hv : Formalism.MainNotions.S.IsKFin ε σ v) : v ∈ smD ε σ

theorem **Jm_conjTranspose_self** : (Jm ε)ᴴ = Jm ε

theorem **UJset_mul_mem** {a b : Matrix (Fin n) (Fin n) ℂ} (ha : a ∈ Formalism.MainNotions.S.UJset ε) (hb : b ∈ Formalism.MainNotions.S.UJset ε) : a ∗ b ∈ Formalism.MainNotions.S.UJset ε

theorem **conjTranspose_mem_kset_and_inv_eq** {k : Matrix (Fin n) (Fin n) ℂ} (hk : k ∈ Formalism.REAL.Kset ε) : kᴴ ∈ Formalism.REAL.Kset ε ∧ k⁻¹ = kᴴ

theorem **inG_conj_of_mem_kset** {k X : Matrix (Fin n) (Fin n) ℂ} (hk : k ∈ Formalism.REAL.Kset ε) (hX : inG ε X) : inG ε (k ∗ X ∗ k⁻¹)

theorem **mem_UJset_of_mem_kset** {k : Matrix (Fin n) (Fin n) ℂ} (hk : k ∈ Formalism.REAL.Kset ε) : k ∈ Formalism.MainNotions.S.UJset ε

private theorem **exp_smul_mem_UJset_of_inG** {Y : Matrix (Fin n) (Fin n) ℂ} (hY : inG ε Y) (t : ℝ) : NormedSpace.exp ((t : ℂ) • Y) ∈ Formalism.MainNotions.S.UJset ε

theorem **sigma_dS_conj** (hσ : Formalism.MainNotions.S.IsURepOn (Formalism.MainNotions.S.UJset ε) σ) {k : Matrix (Fin n) (Fin n) ℂ} (hk : k ∈ Formalism.REAL.Kset ε) {X : Matrix (Fin n) (Fin n) ℂ} (hX : inG ε X) (v : smD ε σ) : σ k (dS σ X v) = dS σ (k ∗ X ∗ k⁻¹) (σ k v)

theorem **derivedActionSpan_finiteDimensional** (S : Submodule ℂ H) [FiniteDimensional ℂ S] (hS : ∀ x ∈ S, x ∈ smD ε σ) : FiniteDimensional ℂ (derivedActionSpan S hS)

theorem **smE_mem_derivedActionSpan** (S : Submodule ℂ H) (hS : ∀ x ∈ S, x ∈ smD ε σ) (c d : Fin n) {x : H} (hx : x ∈ S) : ((smE ε σ c d (x, hS x hx) : smD ε σ) : H) ∈ derivedActionSpan S hS

theorem **actM_mem_derivedActionSpan** (S : Submodule ℂ H) (hS : ∀ x ∈ S, x ∈ smD ε σ) (Y : Matrix (Fin n) (Fin n) ℂ) {x : H} (hx : x ∈ S) : ((actM (smE ε σ) Y (x, hS x hx) : smD ε σ) : H) ∈ derivedActionSpan S hS

private theorem **inG_first_chart_piece** (Z : Matrix (Fin n) (Fin n) ℂ) : inG ε (ReciprocityTheory.ChartRestriction.DS.S7.cpart1 ε Z)

private theorem **inG_second_chart_piece** (Z : Matrix (Fin n) (Fin n) ℂ) : inG ε (ReciprocityTheory.ChartRestriction.DS.S7.cpart2 ε Z)

theorem **derivedActionSpan_isKStableOn** (hσ : Formalism.MainNotions.S.IsURepOn (Formalism.MainNotions.S.UJset ε) σ) (S : Submodule ℂ H) (hS : ∀ x ∈ S, x ∈ smD ε σ) (hK : Formalism.MainNotions.S.IsKStableOn ε σ S) : Formalism.MainNotions.S.IsKStableOn ε σ (derivedActionSpan S hS)

theorem **K_finite_smooth_vectors** : KFSm

CompanionTheory.HarishChandraModules.KFiniteAnalyticVectors.AnalyticIntertwiners.SymmetricPowerMatrix

theorem **kFiniteWordBound_indefUnitary** : WordBd

theorem **kFiniteVectors_analytic_indefiniteUnitary** : GlobalTheory.Published.KFiniteAnalyticUJ

CompanionTheory.HarishChandraModules.KFiniteAnalyticVectors.EqSmulOfWeight

@[simp] lemma **word_nil** (F : Fin n → Fin n → Module.End ℂ U) : word F [] = 1

@[simp] lemma **word_cons** (F : Fin n → Fin n → Module.End ℂ U) (p : Fin n × Fin n) (l : List (Fin n × Fin n)) : word F (p :: l) = F p.1 p.2 ∗ word F l

lemma **word_mem** (F : Fin n → Fin n → Module.End ℂ U) (w : U) {l : List (Fin n × Fin n)} (hl : Lower l) : word F l w ∈ lowSpan F w

lemma **self_mem** (F : Fin n → Fin n → Module.End ℂ U) (w : U) : w ∈ lowSpan F w

lemma **lower_mem** (F : Fin n → Fin n → Module.End ℂ U) (w : U) {a b : Fin n} (hab : b < a) {x : U} (hx : x ∈ lowSpan F w) : F a b x ∈ lowSpan F w

lemma **comm_apply** {F : Fin n → Fin n → Module.End ℂ U} (hF : GlRel F) (i j a b : Fin n) (u : U) : F i j (F a b u) = F a b (F i j u) + ((if j = a then F i b u else 0) − (if b = i then F a j u else 0))

lemma **gen_mem** (hF : GlRel F) (hw : IsHW F ν w) : ∀ (l : List (Fin n × Fin n)), Lower l → ∀ i j, F i j (word F l w) ∈ lowSpan F w

theorem **stab_lowSpan** (hF : GlRel F) (hw : IsHW F ν w) : Stab F (lowSpan F w)

lemma **diag_word** (hF : GlRel F) (hw : IsHW F ν w) : ∀ (l : List (Fin n × Fin n)) (c : Fin n), F c c (word F l w) = wt ν l c • word F l w

lemma **hgt_wt_re** (ν : Fin n → ℂ) : ∀ l : List (Fin n × Fin n), Lower l → (hgt (wt ν l)).re ≤ (hgt ν).re − l.length

lemma **hOp_of_diag** {x : U} {μ : Fin n → ℂ} (hx : ∀ c, F c c x = μ c • x) : hOp F x = hgt μ • x

theorem **eq_smul_of_weight** (hF : GlRel F) (hw : IsHW F ν w) {x : U} (hx : x ∈ lowSpan F w) (hxν : ∀ c, F c c x = ν c • x) : ∃ t : ℂ, x = t • w

CompanionTheory.HarishChandraModules.KFiniteAnalyticVectors.FundamentalDomainCusps.AutomorphicFormsFiniteDimensional

theorem **gamSub** : GamSub

theorem **geoInf** : GeoInf

theorem **cuspInf** : CuspInf

theorem **automorphic_forms_finite_dimensional** : Formalism.L2.SG.RowsDef.RowsFinDim

CompanionTheory.HarishChandraModules.KFiniteAnalyticVectors.FundamentalDomainCusps.CuspidalAtInfinityOf

theorem **cuspidal_at_infinity_of** : Sel → Avg → NWSub → RealPar → ParPi → Transp → GamSub → GeoInf → CuspInf

theorem **level_structure_of** : LevelOf

CompanionTheory.HarishChandraModules.KFiniteAnalyticVectors.FundamentalDomainCusps.FiniteDimensionalFromLinearForms

theorem **sq_lintegral_le** {X : Type} [MeasurableSpace X] (μ : Measure X) {θ f : X → ℝ≥0∞} (hθ : AEMeasurable θ μ) (hf : AEMeasurable f μ) : (∫⁻ y, θ y ∗ f y ∂μ) ^ 2 ≤ (∫⁻ y, θ y ∂μ) ∗ ∫⁻ y, θ y ∗ f y ^ 2 ∂μ

theorem **setLIntegral_right** [μ.IsMulRightInvariant] (hD : IsFundamentalDomain Γ D μ) {f : G → ℝ≥0∞} (hΓ : LInv Γ f) (y : G) : ∫⁻ x in D, f (x ∗ y) ∂μ = ∫⁻ x in D, f x ∂μ

theorem **young_convolution_inequality** : Young

theorem **compact_integral_quotient_bound** : CompactIntegralBoundedByQuotientNorm

theorem **near_cusp_measure_estimate** : NearMeas

theorem **uniform_continuity_estimate** : UnifC

theorem **approximation_of_square_identity** : Approx

theorem **diff_est** {α h : G → ℂ} (hα : Integrable α μ) (hh : BC h) {B : Set G} (hB : IsCompact B) {e : ℝ} {u : G} (hu : ∀ y, ‖α (u⁻¹ ∗ y) − α y‖ ≤ e ∧ (y ∉ B → α (u⁻¹ ∗ y) = 0 ∧ α y = 0)) (x : G) : ‖∫ y, α y ∗ h (x ∗ u ∗ y) ∂μ − ∫ y, α y ∗ h (x ∗ y) ∂μ‖ₑ ≤ ENNReal.ofReal e ∗ ∫⁻ y in B, ‖h (x ∗ y)‖ₑ ∂μ

theorem **far_from_cusp_estimate** : FarEst

theorem **near_cusp_estimate** : NearEst

theorem **exists_small** {M : ℝ≥0∞} (hM : M ≠ ∞) : ∃ d : ℝ, 0 < d ∧ ENNReal.ofReal d ∗ M ≤ 8⁻¹

theorem **add_sq_le** (a b : ℝ≥0∞) : (a + b) ^ 2 ≤ 2 ∗ (a ^ 2 + b ^ 2)

theorem **discrete_series_bound_of_inputs** : Core

theorem **finite_dimensional_from_linear_forms** : FiniteDimensionalFromLinearForms

theorem **representation_of_convolution** : RepOfConv

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theorem **emb_over** (p : InfinitePlace I.F × Bool) (a : I.F) : emb I p (algebraMap I.F I.E a) = p.1.embedding a

theorem **emb_inj** (hs : ∀ v (a : I.E), I.φ v (star a) = star (I.φ v a)) : Function.Injective (emb I)

theorem **emb_bij** (hs : ∀ v (a : I.E), I.φ v (star a) = star (I.φ v a)) : Function.Bijective (emb I)

theorem **finite_bdd** (hs : ∀ v (a : I.E), I.φ v (star a) = star (I.φ v a)) (B : ℝ) : {x : I.E | IsIntegral ℤ x ∧ ∀ v, ‖I.φ v x‖ ≤ B}.Finite

theorem **trace_eq** (hs : ∀ v (a : I.E), I.φ v (star a) = star (I.φ v a)) (x : I.E) : ((Algebra.trace ℚ I.E x : ℚ) : ℝ) = ∑ v, 2 ∗ (I.φ v x).re

theorem **exists_den** {E ι : Type} [Field E] [NumberField E] [Finite ι] (f : ι → E) : ∃ c : ℕ, 1 ≤ c ∧ ∀ i, IsIntegral ℤ ((c : E) ∗ f i)

theorem **isIntegral_star** {E : Type} [Field E] [StarRing E] {x : E} (hx : IsIntegral ℤ x) : IsIntegral ℤ (star x)

theorem **IntM.mul** {E : Type} [Field E] {n : ℕ} {A B : Matrix (Fin n) (Fin n) E} (hA : IntM A) (hB : IntM B) : IntM (A ∗ B)

theorem **IntM.conjT** {E : Type} [Field E] [StarRing E] {n : ℕ} {A : Matrix (Fin n) (Fin n) E} (hA : IntM A) : IntM Aᴴ

theorem **uMat_mul_inv** {n : ℕ} {ε : Fin n → Bool} (g : Formalism.REAL.UJ ε) : uMat g ∗ uMat g⁻¹ = 1

theorem **uMat_inv_mul** {n : ℕ} {ε : Fin n → Bool} (g : Formalism.REAL.UJ ε) : uMat g⁻¹ ∗ uMat g = 1

theorem **hf_uMat** {n : ℕ} {ε : Fin n → Bool} (g : Formalism.REAL.UJ ε) (a b : Fin n → ℂ) : hf (Formalism.REAL.Jm ε) (uMat g ∗ᵥ a) (uMat g ∗ᵥ b) = hf (Formalism.REAL.Jm ε) a b

theorem **norm_hf_le** {n : ℕ} (ε : Fin n → Bool) {a b : Fin n → ℂ} {s : ℝ} (ha : ∀ i, ‖a i‖ ≤ s) (hb : ∀ i, ‖b i‖ ≤ s) : ‖hf (Formalism.REAL.Jm ε) a b‖ ≤ n ∗ (s ∗ s)

theorem **isCompact_inv_bdd** (K : ℝ) : IsCompact {x : I.Ginf | ∀ v i j, ‖uMat (x v)⁻¹ i j‖ ≤ K}

theorem **mahler_compactness_of** : Transp → GamSub → Mahler

CompanionTheory.HarishChandraModules.KFiniteAnalyticVectors.FundamentalDomainCusps.IntegrableMulBc

theorem **idemZero** : IdemZero

theorem **tendsto_transl** {φ : G → ℂ} (hφ : Integrable φ μ) : Tendsto (fun u => ∫ y, ‖φ y − φ (u ∗ y)‖ ∂μ) (𝓝 1) (𝓝 0)

theorem **norm_eI** (r : ℝ) : ‖eI r‖ = 1

theorem **eI_add** (r r' : ℝ) : eI (r + r') = eI r ∗ eI r'

theorem **continuous_eI** : Continuous eI

theorem **norm_eI_sub** (r r' : ℝ) : ‖eI r − eI r'‖ ≤ min 2 |r − r'|

theorem **integrable_mul_bc** {φ : G → ℂ} (hφ : Integrable φ μ) {F : G → ℂ} (hF : Continuous F) {M : ℝ} (hM : ∀ y, ‖F y‖ ≤ M) : Integrable (fun y => φ y ∗ F y) μ

theorem **reprod** {φ : G → ℂ} (hφ : Integrable φ μ) (hi : ∀ w, ∫ y, φ y ∗ φ (y⁻¹ ∗ w) ∂μ = φ w) {F : G → ℂ} (hF : Continuous F) {M : ℝ} (hM : ∀ y, ‖F y‖ ≤ M) : ∫ y, φ y ∗ ∫ z, φ z ∗ F (y ∗ z) ∂μ ∂μ = ∫ w, φ w ∗ F w ∂μ

theorem **selTw** : SelTw

theorem **selP** : Sel

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theorem **atV_sum** (I : AutInst) (v : InfinitePlace I.F) {d : ℕ} (a : Fin d → I.E) (w : Fin d → Fin I.n → I.E) : atV I v (∑ i, a i • w i) = ∑ i, I.φ v (a i) • atV I v (w i)

theorem **exists_near** (I : AutInst) : ∃ C : ℝ, 0 < C ∧ ∀ β : InfinitePlace I.F → ℂ, ∃ a : I.E, IsIntegral ℤ a ∧ ∀ v, ‖β v − I.φ v a‖ ≤ C

theorem **covolume_span** : CovSpan

theorem **exists_sandwich** (I : AutInst) : ∃ c : ℕ, 1 ≤ c ∧ (∀ l ∈ I.L, (c : I.E) • l ∈ IntV I) ∧ ∀ u ∈ IntV I, (c : I.E) • u ∈ I.L

theorem **iv_mulVec** (I : AutInst) (A : Matrix (Fin I.n) (Fin I.n) I.E) (l : Fin I.n → I.E) : A.map (Formalism.UG.inclE (F

theorem **cong_Lf** (I : AutInst) {c g : ℕ} (h1 : ∀ l ∈ I.L, (c : I.E) • l ∈ IntV I) (h2 : ∀ u ∈ IntV I, (c : I.E) • u ∈ I.L) {A : Matrix (Fin I.n) (Fin I.n) I.E} (hA : Cong I (g * c * c) A) : ∀ y ∈ I.Lf, ∃ z ∈ I.Lf, (A.map (Formalism.UG.inclE (F

theorem **congruence_subgroup_compact_open** : CongK

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theorem **exists_extend** {Λ : Type} [AddCommGroup Λ] [Module.Finite ℤ Λ] (S : Submodule ℤ Λ) (hS : ∀ (m : ℤ) x, m • x ∈ S → m = 0 ∨ x ∈ S) (f : S →ₗ[ℤ] ℤ) : ∃ g : Λ →ₗ[ℤ] ℤ, ∀ x : S, g x = f x

theorem **dual_rep** (E : Type) [Field E] [NumberField E] (n : ℕ) : ∃ D : ℕ, 1 ≤ D ∧ ∀ g : (Fin n → 𝓞 E) →ₗ[ℤ] ℤ, ∃ c : Fin n → E, (∀ j, IsIntegral ℤ ((D : E) * c j)) ∧ ∀ u, (g u : ℚ) = Algebra.trace ℚ E (c ⬝ᵥ fun j => (u j : E))

theorem **span_phi** (hs : ∀ v (a : I.E), I.φ v (star a) = star (I.φ v a)) : Submodule.span ℝ (Set.range fun k => fun v => I.φ v (integralBasis I.E k)) = ⊤

theorem **atV_smul** (v : InfinitePlace I.F) (a : I.E) (u : Fin I.n → I.E) : atV I v (a • u) = I.φ v a • atV I v u

theorem **atV_add** (v : InfinitePlace I.F) (u u' : Fin I.n → I.E) : atV I v (u + u') = atV I v u + atV I v u'

theorem **mem_span_int** (hs : ∀ v (a : I.E), I.φ v (star a) = star (I.φ v a)) (W : Submodule I.E (Fin I.n → I.E)) (b : InfinitePlace I.F → Fin I.n → ℂ) (hb : ∀ v, b v ∈ Wv I W v) : b ∈ Submodule.span ℝ {x : InfinitePlace I.F → Fin I.n → ℂ | ∃ u ∈ IntV I, u ∈ W ∧ x = fun v => atV I v u}

theorem **Jm_mulVec** {n : ℕ} (ε : Fin n → Bool) (x : Fin n → ℂ) (i : Fin n) : (Formalism.REAL.Jm ε *ᵥ x) i = (if ε i then 1 else -1) * x i

theorem **mem_of_perp_perp** {n : ℕ} (ε : Fin n → Bool) (S : Submodule ℂ (Fin n → ℂ)) {x : Fin n → ℂ} (hx : ∀ b ∈ perp (Formalism.REAL.Jm ε) S, hf (Formalism.REAL.Jm ε) x b = 0) : x ∈ S

theorem **norm_le_toV** (b : InfinitePlace I.F → Fin I.n → ℂ) : ‖b‖ ≤ ‖toV I b‖

theorem **inner_toV** (a b : InfinitePlace I.F → Fin I.n → ℂ) : ⟪toV I fun v => Formalism.REAL.Jm (I.ε v) *ᵥ a v, toV I b⟫_ℝ = ∑ v, (hf (Formalism.REAL.Jm (I.ε v)) (a v) (b v)).re

theorem **covolume_orthogonal_complement** : Transp → CovPerp

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theorem **phi_star** (v : InfinitePlace I.F) (a : I.E) : I.φ v (star a) = star (I.φ v a)

theorem **star_atV_vecMul** (v : InfinitePlace I.F) (u : Fin I.n → I.E) : star (atV I v u) ᵥ* Formalism.REAL.Jm (I.ε v) = (fun i => I.φ v ((star u ᵥ* I.J) i)) ᵥ* I.P v

theorem **hf_atV** (v : InfinitePlace I.F) (u : Fin I.n → I.E) (z : Fin I.n → ℂ) : hf (Formalism.REAL.Jm (I.ε v)) (atV I v u) z = (fun i => I.φ v ((star u ᵥ* I.J) i)) ⬝ᵥ (I.P v *ᵥ z)

theorem **P_atV** (v : InfinitePlace I.F) (u : Fin I.n → I.E) : I.P v *ᵥ atV I v u = fun i => I.φ v (u i)

theorem **hf_atV_atV** (v : InfinitePlace I.F) (u u' : Fin I.n → I.E) : hf (Formalism.REAL.Jm (I.ε v)) (atV I v u) (atV I v u') = I.φ v (hf I.J u u')

theorem **hf_smul_leftD** {K : Type} [Field K] [StarRing K] {n : ℕ} (J : Matrix (Fin n) (Fin n) K) (c : K) (x y : Fin n → K) : hf J (c • x) y = star c * hf J x y

theorem **hf_add_leftD** {K : Type} [Field K] [StarRing K] {n : ℕ} (J : Matrix (Fin n) (Fin n) K) (x x' y : Fin n → K) : hf J (x + x') y = hf J x y + hf J x' y

theorem **mem_perp_spanD** {K : Type} [Field K] [StarRing K] {n : ℕ} (J : Matrix (Fin n) (Fin n) K) {S : Set (Fin n → K)} {z : Fin n → K} (h : ∀ w ∈ S, hf J w z = 0) : z ∈ perp J (Submodule.span K S)

theorem **Wv_perp** (v : InfinitePlace I.F) (W : Submodule I.E (Fin I.n → I.E)) : Wv I (perp I.J W) v = perp (Formalism.REAL.Jm (I.ε v)) (Wv I W v)

theorem **atM_mulVec** (v : InfinitePlace I.F) (A : Matrix (Fin I.n) (Fin I.n) I.E) (u : Fin I.n → I.E) : atM I v A *ᵥ atV I v u = atV I v (A *ᵥ u)

theorem **transport_to_standard_form** : Transp

theorem **atM_mul** (v : InfinitePlace I.F) (A B : Matrix (Fin I.n) (Fin I.n) I.E) : atM I v (A * B) = atM I v A * atM I v B

theorem **atM_one** (v : InfinitePlace I.F) : atM I v 1 = 1

theorem **Cong.one** (M : ℕ) : Cong I M 1

theorem **Cong.mul** {M : ℕ} {A B : Matrix (Fin I.n) (Fin I.n) I.E} (hA : Cong I M A) (hB : Cong I M B) : Cong I M (A * B)

theorem **Cong.integral** {M : ℕ} {A : Matrix (Fin I.n) (Fin I.n) I.E} (hA : Cong I M A) (i j : Fin I.n) : IsIntegral ℤ (A i j)

theorem **norm_emb_le** {a : I.E} {B : ℝ} (h : ∀ v, ‖I.φ v a‖ ≤ B) (σ : I.E →+* ℂ) : ‖σ a‖ ≤ B

theorem **arithmetic_subgroup_of_transport** : Transp → GamSub

theorem **fundamental_domain_discrete_of** : GamSub → GFDisc

theorem **exists_denD** {ι : Type} [Finite ι] (a : ι → I.E) : ∃ c : ℕ, 1 ≤ c ∧ ∀ i, IsIntegral ℤ ((c : I.E) * a i)

theorem **isIntegral_starD** {a : I.E} (h : IsIntegral ℤ a) : IsIntegral ℤ (star a)

theorem **hf_unitary** {n : ℕ} {ε : Fin n → Bool} (g : Formalism.REAL.UJ ε) (a b : Fin n → ℂ) : hf (Formalism.REAL.Jm ε) (uMat g *ᵥ a) (uMat g *ᵥ b) = hf (Formalism.REAL.Jm ε) a b

theorem **norm_hf_leD** {n : ℕ} (ε : Fin n → Bool) (a b : Fin n → ℂ) : ‖hf (Formalism.REAL.Jm ε) a b‖ ≤ n * (‖a‖ * ‖b‖)

theorem **eq_zero_of_small** {a : I.E} (ha : IsIntegral ℤ a) (h : ∀ v, ‖I.φ v a‖ ≤ 1 / 2) : a = 0

theorem **short_isotropic_vector_of** : Transp → ShortIsot

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theorem **mat_extC** {A B : Matrix (Fin n) (Fin n) K} (h : ∀ z, A *ᵥ z = B *ᵥ z) : A = B

theorem **Lowers.triple** {A B C : Matrix (Fin n) (Fin n) K} (hA : Lowers J W A) (hB : Lowers J W B) (hC : Lowers J W C) : A * (B * C) = 0

theorem **Lowers.add** {A B : Matrix (Fin n) (Fin n) K} (hA : Lowers J W A) (hB : Lowers J W B) : Lowers J W (A + B)

theorem **Lowers.smul** {A : Matrix (Fin n) (Fin n) K} (hA : Lowers J W A) (c : K) : Lowers J W (c • A)

theorem **Lowers.neg** {A : Matrix (Fin n) (Fin n) K} (hA : Lowers J W A) : Lowers J W (-A)

theorem **Lowers.mul_perp** {A B : Matrix (Fin n) (Fin n) K} (hA : Lowers J W A) (hB : Lowers J W B) : (∀ z ∈ perp J W, (A * B) *ᵥ z = 0) ∧ ∀ z, (A * B) *ᵥ z ∈ W

theorem **lowers_of_Z** (hW : Isot J W) {Z : Matrix (Fin n) (Fin n) K} (h1 : ∀ z ∈ perp J W, Z *ᵥ z = 0) (h2 : ∀ z, Z *ᵥ z ∈ W) : Lowers J W Z

theorem **Lowers.mul** (hW : Isot J W) {A B : Matrix (Fin n) (Fin n) K} (hA : Lowers J W A) (hB : Lowers J W B) : Lowers J W (A * B)

theorem **second_order_exp_mul** {X Y : Matrix (Fin n) (Fin n) K} (hX : Lowers J W X) (hY : Lowers J W Y) : ex2 X * ex2 Y = 1 + X + Y + (2 : K)⁻¹ • (X * X) + X * Y + (2 : K)⁻¹ • (Y * Y)

theorem **star_half** : star (2 : K)⁻¹ = (2 : K)⁻¹

theorem **skew_eq** {X : Matrix (Fin n) (Fin n) K} (h : Xᴴ * J + J * X = 0) : Xᴴ * J = -(J * X)

theorem **skew_mul** {X Y : Matrix (Fin n) (Fin n) K} (hX : Xᴴ * J + J * X = 0) (hY : Yᴴ * J + J * Y = 0) : (X * Y)ᴴ * J = J * (Y * X)

theorem **nilpotent_lie_algebra** : NilpAlg

theorem **xw_mulVec** (w b z : Fin n → K) : xw J w b *ᵥ z = hf J b z • w - hf J w z • b

theorem **hf_star** (hJ : Jᴴ = J) (x y : Fin n → K) : star (hf J x y) = hf J y x

theorem **hf_sub_right** (x y z : Fin n → K) : hf J x (y - z) = hf J x y - hf J x z

theorem **hf_smul_right** (c : K) (x y : Fin n → K) : hf J x (c • y) = c * hf J x y

theorem **hf_sum_right** {ι : Type} (s : Finset ι) (x : Fin n → K) (y : ι → Fin n → K) : hf J x (∑ i ∈ s, y i) = ∑ i ∈ s, hf J x (y i)

theorem **hf_add_leftC** (x y z : Fin n → K) : hf J (x + y) z = hf J x z + hf J y z

theorem **hf_neg_left** (x z : Fin n → K) : hf J (-x) z = -hf J x z

theorem **hf_smul_leftC** (c : K) (x y : Fin n → K) : hf J (c • x) y = star c * hf J x y

theorem **hf_sum_left** {ι : Type} (s : Finset ι) (x : ι → Fin n → K) (y : Fin n → K) : hf J (∑ i ∈ s, x i) y = ∑ i ∈ s, hf J (x i) y

theorem **hf_skew** {X : Matrix (Fin n) (Fin n) K} (h : Xᴴ * J + J * X = 0) (x y : Fin n → K) : hf J (X *ᵥ x) y = -hf J x (X *ᵥ y)

theorem **skew_of_hf** {X : Matrix (Fin n) (Fin n) K} (h : ∀ x y, hf J (X *ᵥ x) y = -hf J x (X *ᵥ y)) : Xᴴ * J + J * X = 0

theorem **skew_xw** (hJ : Jᴴ = J) (w b : Fin n → K) : (xw J w b)ᴴ * J + J * xw J w b = 0

theorem **perp_symm** (hJ : Jᴴ = J) {b w : Fin n → K} (hb : b ∈ perp J W) (hw : w ∈ W) : hf J b w = 0

theorem **mem_perp_spanC** {ι : Type} {w : ι → Fin n → K} {z : Fin n → K} (h : ∀ i, hf J (w i) z = 0) : z ∈ perp J (Submodule.span K (Set.range w))

theorem **exists_dual** (hJ : Jᴴ = J) (hd : IsUnit J.det) {κ : Type} [Fintype κ] [DecidableEq κ] {v : κ → Fin n → K} (hv : LinearIndependent K v) : ∃ u : κ → Fin n → K, ∀ i j, hf J (u i) (v j) = if i = j then 1 else 0

theorem **rep_basis** [CharZero K] (hJ : Jᴴ = J) (hd : IsUnit J.det) (hW : Isot J W) {κ : Type} [Fintype κ] [DecidableEq κ] {v : κ → Fin n → K} (hv : LinearIndependent K v) (hs : Submodule.span K (Set.range v) = W) {X : Matrix (Fin n) (Fin n) K} (hX : InL J W X) : ∃ b : κ → Fin n → K, (∀ k, b k ∈ perp J W) ∧ ((∀ z, X *ᵥ z ∈ W) → ∀ k, b k ∈ W) ∧ X = ∑ k, xw J (v k) (b k)

theorem **rep_span** [CharZero K] (hJ : Jᴴ = J) (hd : IsUnit J.det) (hW : Isot J W) {d : ℕ} {w : Fin d → Fin n → K} (hs : Submodule.span K (Set.range w) = W) {X : Matrix (Fin n) (Fin n) K} (hX : InL J W X) : ∃ b : Fin d → Fin n → K, (∀ k, b k ∈ perp J W) ∧ ((∀ z, X *ᵥ z ∈ W) → ∀ k, b k ∈ W) ∧ X = ∑ k, xw J (w k) (b k)

theorem **nilpotent_subgroup_representation** : NilpRep

theorem **successive_minima_gap** : Gap

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theorem **exists_GF** (I : AutInst) (x : I.Gf) (hx : x ∈ PI.AdelClassFin.GFf I) : ∃ g : I.Ginf, (g, x) ∈ I.GFset

theorem **adelic_cusp_of_infinite_cusp** : CuspInf → CongK → CuspAdel

theorem **convolution_operator_representation** : ConvRep

CompanionTheory.HarishChandraModules.KFiniteAnalyticVectors.FundamentalDomainCusps.ParabolicSubgroupDecomposition

theorem **Disc.countable** {G : Type} [Group G] [TopologicalSpace G] [LocallyCompactSpace G] [SecondCountableTopology G] {Λ : Subgroup G} (h : Disc Λ) : Countable Λ

theorem **Disc.exists_nhds** {G : Type} [Group G] [TopologicalSpace G] [IsTopologicalGroup G] [LocallyCompactSpace G] [T2Space G] {Λ : Subgroup G} (h : Disc Λ) : ∃ O ∈ 𝓝 (1 : G), ∀ a ∈ O, ∀ b ∈ O, a * b⁻¹ ∈ Λ → a = b

theorem **fundamental_domain_inclusion** : FundIn

theorem **averaging_of_fundamental_domain** : FundIn → Avg

theorem **parabolic_subgroup_decomposition** : ParPi

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theorem **lowers_iff** {g : Matrix (Fin n) (Fin n) ℂ} : Lowers J W (g - 1) ↔ Sc J W 1 1 g

theorem **Sc.mul** (hW : Isot J W) {l m l' m' : ℂ} {g h : Matrix (Fin n) (Fin n) ℂ} (hg : Sc J W l m g) (hh : Sc J W l' m' h) : Sc J W (l * l') (m * m') (g * h)

theorem **Sc.inv** (hW : Isot J W) {g h : Matrix (Fin n) (Fin n) ℂ} (hg : Sc J W 1 1 g) (e : h * g = 1) : Sc J W 1 1 h

theorem **uMat_mul** {ε : Fin n → Bool} (g h : UJ ε) : uMat (g * h) = uMat g * uMat h

theorem **uMat_inv_mulF** {ε : Fin n → Bool} (g : UJ ε) : uMat g⁻¹ * uMat g = 1

theorem **uMat_mul_invF** {ε : Fin n → Bool} (g : UJ ε) : uMat g * uMat g⁻¹ = 1

theorem **continuous_uMat** {ε : Fin n → Bool} : Continuous (uMat : UJ ε → _)

theorem **unipotent_radical_subgroup** : NWSub

theorem **exists_isPr** (W : Submodule ℂ (Fin n → ℂ)) : ∃ p, IsPr W p

theorem **mat_extF** {A B : Matrix (Fin n) (Fin n) ℂ} (h : ∀ z, A *ᵥ z = B *ᵥ z) : A = B

theorem **IsPr.sq** {p : Matrix (Fin n) (Fin n) ℂ} (hp : IsPr W p) : p * p = p

theorem **IsPr.dot** {p : Matrix (Fin n) (Fin n) ℂ} (hp : IsPr W p) (y z : Fin n → ℂ) : star y ⬝ᵥ (p *ᵥ z) = star (p *ᵥ y) ⬝ᵥ z

theorem **IsPr.kill** {p : Matrix (Fin n) (Fin n) ℂ} (hp : IsPr W p) {z : Fin n → ℂ} (hz : z ∈ perp J W) : p *ᵥ (J *ᵥ z) = 0

theorem **IsPr.entry** {p : Matrix (Fin n) (Fin n) ℂ} (hp : IsPr W p) (i j : Fin n) : ‖p i j‖ ≤ 1

theorem **expand** {p q : Matrix (Fin n) (Fin n) ℂ} (hp : p * p = p) (hq : q * q = q) (hpq : p * q = 0) (hqp : q * p = 0) (a b c d : ℂ) : (1 + a • p + b • q) * (1 + c • p + d • q) = 1 + (a + c + a * c) • p + (b + d + b * d) • q

theorem **Jm_sq** (ε : Fin n → Bool) : Jm ε * Jm ε = 1

theorem **Jm_hermF** (ε : Fin n → Bool) : (Jm ε)ᴴ = Jm ε

theorem **pJp** (hW : Isot (Jm ε) W) (hp : IsPr W p) : p * Jm ε * p = 0

theorem **q_facts** (hW : Isot (Jm ε) W) (hp : IsPr W p) : (Jm ε * p * Jm ε) * (Jm ε * p * Jm ε) = Jm ε * p * Jm ε ∧ p * (Jm ε * p * Jm ε) = 0 ∧ (Jm ε * p * Jm ε) * p = 0

theorem **am_add** (hW : Isot (Jm ε) W) (hp : IsPr W p) (s t : ℝ) : am (Jm ε) p (s + t) = am (Jm ε) p s * am (Jm ε) p t

theorem **am_mem** (hW : Isot (Jm ε) W) (hp : IsPr W p) (t : ℝ) : inUJ ε (am (Jm ε) p t)

theorem **am_sc** (hW : Isot (Jm ε) W) (hp : IsPr W p) (t : ℝ) : Sc (Jm ε) W (Real.exp t) (Real.exp (-t)) (am (Jm ε) p t)

theorem **hf_uMatF** (x : UJ ε) (y z : Fin n → ℂ) : hf (Jm ε) (uMat x *ᵥ y) (uMat x *ᵥ z) = hf (Jm ε) y z

theorem **norm_mulVec_le** (M : Matrix (Fin n) (Fin n) ℂ) (v : Fin n → ℂ) : ‖M *ᵥ v‖ ≤ (1 + ∑ i, ∑ j, ‖M i j‖) * ‖v‖

theorem **real_parabolic_decomposition** : RealPar

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theorem **dense_bdd** {X : Type} [MeasurableSpace X] (μ : Measure X) : Dense {F : Lp ℂ 2 μ | ∃ M : ℝ, ∀ᵐ x ∂μ, ‖(F : X → ℂ) x‖ ≤ M}

theorem **rowsFinDim_of** (hCore : Core) (hFin : FiniteDimensionalFromLinearForms) (hCA : CuspAdel) (hD : GFDisc) (hLev : LevelOf) (hCR : ConvRep) (hRC : RepOfConv) : RowsFinDim

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theorem **Jm_hermC** (ε : Fin n → Bool) : (Formalism.REAL.Jm ε)ᴴ = Formalism.REAL.Jm ε

theorem **Jm_det** (ε : Fin n → Bool) : IsUnit (Formalism.REAL.Jm ε).det

theorem **uMat_mul'** (g h : Formalism.REAL.UJ ε) : uMat (g * h) = uMat g * uMat h

theorem **uMat_inv_mulC** (g : Formalism.REAL.UJ ε) : uMat g⁻¹ * uMat g = 1

theorem **uMat_mul_invC** (g : Formalism.REAL.UJ ε) : uMat g * uMat g⁻¹ = 1

theorem **hf_uMatC** (g : Formalism.REAL.UJ ε) (a z : Fin n → ℂ) : hf (Formalism.REAL.Jm ε) (uMat g *ᵥ a) (uMat g *ᵥ z) = hf (Formalism.REAL.Jm ε) a z

theorem **conj_xw** (g : Formalism.REAL.UJ ε) (w b : Fin n → ℂ) : uMat g⁻¹ * xw (Formalism.REAL.Jm ε) w b * uMat g = xw (Formalism.REAL.Jm ε) (uMat g⁻¹ *ᵥ w) (uMat g⁻¹ *ᵥ b)

theorem **second_order_exp_conj** (g : Formalism.REAL.UJ ε) (X : Matrix (Fin n) (Fin n) ℂ) : uMat g⁻¹ * ex2 X * uMat g = ex2 (uMat g⁻¹ * X * uMat g)

theorem **norm_xw_le** (w b : Fin n → ℂ) (i j : Fin n) : ‖xw (Formalism.REAL.Jm ε) w b i j‖ ≤ 2 * (‖w‖ * ‖b‖)

theorem **norm_second_order_exp_sub_one_le** {A : Matrix (Fin n) (Fin n) ℂ} {a : ℝ} (h : ∀ i j, ‖A i j‖ ≤ a) (i j : Fin n) : ‖(ex2 A - 1) i j‖ ≤ a + n * a ^ 2

theorem **xw_add** (w b b' : Fin n → K) : xw J w (b + b') = xw J w b + xw J w b'

theorem **xw_nat** (w b : Fin n → K) (k : ℕ) : xw J w ((k : K) • b) = (k : K) • xw J w b

theorem **smul_xw** (c : K) (w b : Fin n → K) : c • xw J w b = xw (c • J) w b

theorem **P_det** (v : InfinitePlace I.F) : IsUnit (I.P v).det

theorem **atM_add** (v : InfinitePlace I.F) (A B : Matrix (Fin I.n) (Fin I.n) I.E) : atM I v (A + B) = atM I v A + atM I v B

theorem **atM_neg** (v : InfinitePlace I.F) (A : Matrix (Fin I.n) (Fin I.n) I.E) : atM I v (-A) = -atM I v A

theorem **atM_sum** (v : InfinitePlace I.F) {ι : Type} (s : Finset ι) (A : ι → Matrix (Fin I.n) (Fin I.n) I.E) : atM I v (∑ i ∈ s, A i) = ∑ i ∈ s, atM I v (A i)

theorem **second_order_exp_at_infinite_place** (v : InfinitePlace I.F) (X : Matrix (Fin I.n) (Fin I.n) I.E) : atM I v (ex2 X) = ex2 (atM I v X)

theorem **atV_addC** (v : InfinitePlace I.F) (a b : Fin I.n → I.E) : atV I v (a + b) = atV I v a + atV I v b

theorem **atV_smulC** (v : InfinitePlace I.F) (c : I.E) (a : Fin I.n → I.E) : atV I v (c • a) = I.φ v c • atV I v a

theorem **atV_zero** (v : InfinitePlace I.F) : atV I v 0 = 0

theorem **Wv_span** (v : InfinitePlace I.F) {ι : Type} (w : ι → Fin I.n → I.E) : Submodule.span ℂ (Set.range fun i => atV I v (w i)) = Wv I (Submodule.span I.E (Set.range w)) v

theorem **exists_denC** : ∃ c : ℕ, 1 ≤ c ∧ ∀ i j, IsIntegral ℤ (((c : I.E) • I.J) i j)

theorem **int_xw** {J : Matrix (Fin I.n) (Fin I.n) I.E} (hJ : ∀ i j, IsIntegral ℤ (J i j)) {w b : Fin I.n → I.E} (hw : w ∈ IntV I) (hb : b ∈ IntV I) (i j : Fin I.n) : IsIntegral ℤ (xw J w b i j)

theorem **second_order_exp_congruence** (M : ℕ) {Y : Matrix (Fin I.n) (Fin I.n) I.E} (hY : ∀ i j, IsIntegral ℤ (Y i j)) : Cong I M (ex2 (((2 * M : ℕ) : I.E) • Y))

theorem **exists_gam** (hT : Transp) (M c : ℕ) (hc : ∀ i j, IsIntegral ℤ (((c : I.E) • I.J) i j)) {d : ℕ} {w β : Fin d → Fin I.n → I.E} (hw : ∀ i, w i ∈ IntV I) (hβ : ∀ i, β i ∈ IntV I) (hW : Isot I.J (Submodule.span I.E (Set.range w))) (hβW : ∀ i, β i ∈ perp I.J (Submodule.span I.E (Set.range w))) : ∃ γ ∈ GamSet I M, ∀ v, uMat (γ v) = ex2 (−∑ i, xw (Formalism.REAL.Jm (I.ε v)) (atV I v (w i)) (((2 * M * c : ℕ) : ℂ) • atV I v (β i)))

theorem **split_cov** {y : I.Ginf} {S : Submodule I.E (Fin I.n → I.E)} {ρ : ℝ} (h : Covers I y S ρ) {k : ℕ} (hk : k ≠ 0) {d : ℕ} (b : InfinitePlace I.F → Fin d → Fin I.n → ℂ) (hb : ∀ v i, b v i ∈ Wv I S v) : ∃ β : Fin d → Fin I.n → I.E, (∀ i, β i ∈ IntV I ∧ β i ∈ S) ∧ ∀ v i, ‖uMat (y v)⁻¹ *ᵥ (b v i − (k : ℂ) • atV I v (β i))‖ ≤ k * ρ

theorem **atV_mem** {S : Submodule I.E (Fin I.n → I.E)} {u : Fin I.n → I.E} (hu : u ∈ S) (v : InfinitePlace I.F) : atV I v u ∈ Wv I S v

theorem **nilpotent_cover_of** : Transp → GamSub → NilpAlg → NilpRep → NCover

theorem **exists_kappa** {U : Set I.Ginf} (hU : U ∈ 𝒩 (1 : I.Ginf)) : ∃ κ : ℝ, 0 < κ ∧ ∀ g : I.Ginf, (∀ v i j, ‖(uMat (g v) − 1) i j‖ ≤ κ) → g ∈ U

theorem **Covers.mono** {I : AutInst} {y : I.Ginf} {S : Submodule I.E (Fin I.n → I.E)} {ρ ρ' : ℝ} (h : Covers I y S ρ) (hρ : ρ ≤ ρ') : Covers I y S ρ'

theorem **reduction_geometry_at_infinity** : Gap → ShortIsot → CovSpan → CovPerp → Mahler → NCover → GeoInf

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theorem **IsBump.cpt** {φ : Mat n → ℝ} (h : IsBump G φ) : HasCompactSupport φ

theorem **exists_isBump** (hG : IsMatGroup G) (θ : Mat n →L[ℝ] Mat n) (hθ : ∀ x ∈ G, θ x = x⁻¹) {V : Set (MG G hG)} (hV : V ∈ 𝒩 (1 : MG G hG)) : ∃ φ, IsBump G φ ∧ ∀ x : MG G hG, φ x.mat ≠ 0 → x ∈ V

theorem **isSm_avg** (h : RepOn G σ) (hc : ∀ v, ContinuousOn (fun g => σ g v) G) {φ : Mat n → ℝ} (hφ : IsBump G φ) (u : H) : IsSm G σ (avg hG φ σ u)

theorem **exists_avg_ne** (hG : IsMatGroup G) (θ : Mat n →L[ℝ] Mat n) (hθ : ∀ x ∈ G, θ x = x⁻¹) (h : RepOn G σ) (hc : ∀ v, ContinuousOn (fun g => σ g v) G) {u : H} (hu : u ≠ 0) : ∃ φ, IsBump G φ ∧ avg hG φ σ u ≠ 0

theorem **dsW_avg** (h : RepOn G σ) {v : H} (hv : IsSm G σ v) {φ : Mat n → ℝ} (hφ : Continuous φ) (hc : HasCompactSupport φ) {W : List (Mat n)} (hW : LieW G W) : dsW σ W (avg hG φ σ v) = ∫ g : MG G hG, φ g.mat • dsW σ W (σ g.mat v) ∂haar

theorem **IsSmForm.sub** (hl : IsSmForm G σ W C l) {x y : H} (hx : IsSm G σ x) (hy : IsSm G σ y) : l (x − y) = l x − l y

theorem **IsSmForm.continuous** (hl : IsSmForm G σ W C l) (h : RepOn G σ) (hW : ∀ i, LieW G (W i)) {v : H} (hv : IsSm G σ v) : Continuous fun g : MG G hG => l (σ g.mat v)

theorem **IsSmForm.avg_bd** (hl : IsSmForm G σ W C l) (h : RepOn G σ) (hW : ∀ i, LieW G (W i)) {v : H} (hv : IsSm G σ v) {φ : Mat n → ℝ} (hφ : Continuous φ) (hc : HasCompactSupport φ) {x : H} (hx : IsSm G σ x) : ‖(∫ g : MG G hG, φ g.mat • l (σ g.mat v) ∂haar) − l x‖ ≤ C * ∑ i, ‖(∫ g : MG G hG, φ g.mat • dsW σ (W i) (σ g.mat v) ∂haar) − dsW σ (W i) x‖

theorem **IsSmForm.map_avg** (hl : IsSmForm G σ W C l) (h : RepOn G σ) (hW : ∀ i, LieW G (W i)) {v : H} (hv : IsSm G σ v) {φ : Mat n → ℝ} (hφ : Continuous φ) (hc : HasCompactSupport φ) (hs : IsSm G σ (avg hG φ σ v)) : l (avg hG φ σ v) = ∫ g : MG G hG, φ g.mat • l (σ g.mat v) ∂haar

theorem **IsSmForm.exists_bd** (hl : IsSmForm G σ W C l) (h : RepOn G σ) (hiso : ∀ g ∈ G, ∀ v, ‖σ g v‖ = ‖v‖) (hW : ∀ i, LieW G (W i)) {φ : Mat n → ℝ} (hφ : ContDiff ℝ ∞ φ) (hc : HasCompactSupport φ) : ∃ D : ℝ, ∀ v, IsSm G σ v → ‖∫ g : MG G hG, φ g.mat • l (σ g.mat v) ∂haar‖ ≤ D * ‖v‖

theorem **exists_riesz** (ha : ∀ x y, IsSm G σ x → IsSm G σ y → l (x + y) = l x + l y) (hs : ∀ (c : ℂ) x, IsSm G σ x → l (c • x) = c * l x) {D : ℝ} (hD : ∀ x, IsSm G σ x → ‖l x‖ ≤ D * ‖x‖) : ∃ z : H, ∀ x, IsSm G σ x → l x = ⟪z, x⟫_ℂ

theorem **IsInv.swap** (hB : IsInv G σ σ' W W' C B) : IsInv G σ' σ W' W C fun y x => star (B x y) where

theorem **IsInv.left** (hB : IsInv G σ σ' W W' C B) {y : H'} (hy : IsSm G σ' y) : IsSmForm G σ W (C * ∑ j, ‖dsW σ' (W' j) y‖) fun x => B x y where

theorem **IsInv.exists_rep** [(haar : Measure (MG G hG)).IsInvInvariant] (θ : Mat n →L[ℝ] Mat n) (hθ : ∀ x ∈ G, θ x = x⁻¹) (hB : IsInv G σ σ' W W' C B) (h : RepOn G σ) (h' : RepOn G σ') (hiso : ∀ g ∈ G, ∀ v, ‖σ g v‖ = ‖v‖) (hc' : ∀ v, ContinuousOn (fun g => σ' g v) G) (hW : ∀ i, LieW G (W i)) (hW' : ∀ j, LieW G (W' j)) [Nontrivial H'] : ∃ y₀ z₀, IsSm G σ' y₀ ∧ y₀ ≠ 0 ∧ ∀ x, IsSm G σ x → B x y₀ = ⟪z₀, x⟫_ℂ

theorem **IsInv.exists_op** [(haar : Measure (MG G hG)).IsInvInvariant] (θ : Mat n →L[ℝ] Mat n) (hθ : ∀ x ∈ G, θ x = x⁻¹) (hB : IsInv G σ σ' W W' C B) (h : RepOn G σ) (h' : RepOn G σ') (hiso : ∀ g ∈ G, ∀ v, ‖σ g v‖ = ‖v‖) (hiso' : ∀ g ∈ G, ∀ v, ‖σ' g v‖ = ‖v‖) (hc : ∀ v, ContinuousOn (fun g => σ g v) G) (hc' : ∀ v, ContinuousOn (fun g => σ' g v) G) (hirr : PI.UpqRep.IsIrredOn G σ) (hirr' : PI.UpqRep.IsIrredOn G σ') (hW : ∀ i, LieW G (W i)) (hW' : ∀ j, LieW G (W' j)) [Nontrivial H] [Nontrivial H'] : ∃ T : H' →L[ℂ] H, (∀ g ∈ G, ∀ y, T (σ' g y) = σ g (T y)) ∧ ∀ x y, IsSm G σ x → IsSm G σ' y → B x y = ⟪T y, x⟫_ℂ

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theorem **inner_map_left** (hiso : ∀ g v, ‖σ g v‖ = ‖v‖) (g : G) (v w : H) : ⟪σ g v, w⟫_ℂ = ⟪v, σ g⁻¹ w⟫_ℂ

theorem **exists_graph** (hiso : ∀ g v, ‖σ g v‖ = ‖v‖) (hiso' : ∀ g v, ‖σ' g v‖ = ‖v‖) (hs : ∀ T : H' →L[ℂ] H', (∀ g v, T (σ' g v) = σ' g (T v)) → ∃ c : ℂ, ∀ v, T v = c • v) (Γ : Submodule ℂ (H' × H)) (hcl : IsClosed (Γ : Set (H' × H))) (hinv : ∀ g, ∀ p ∈ Γ, (σ' g p.1, σ g p.2) ∈ Γ) {y₀ : H'} {z₀ : H} (h₀ : (y₀, z₀) ∈ Γ) (hy₀ : y₀ ≠ 0) : ∃ T : H' →L[ℂ] H, (∀ g y, T (σ' g y) = σ g (T y)) ∧ ∀ y, (y, T y) ∈ Γ

theorem **exists_op_of_form** (hiso : ∀ g v, ‖σ g v‖ = ‖v‖) (hiso' : ∀ g v, ‖σ' g v‖ = ‖v‖) (hs : ∀ T : H' →L[ℂ] H', (∀ g v, T (σ' g v) = σ' g (T v)) → ∃ c : ℂ, ∀ v, T v = c • v) (S : Set H) (S' : Set H') (hS : ∀ g, ∀ x ∈ S, σ g x ∈ S) (hS' : ∀ g, ∀ y ∈ S', σ' g y ∈ S') (B : H → H' → ℂ) (hB : ∀ g, ∀ x ∈ S, ∀ y ∈ S', B (σ g x) (σ' g y) = B x y) {y₀ : H'} {z₀ : H} (hy₀S : y₀ ∈ S') (hy₀ : y₀ ≠ 0) (h₀ : ∀ x ∈ S, B x y₀ = ⟪z₀, x⟫_ℂ) : ∃ T : H' →L[ℂ] H, (∀ g y, T (σ' g y) = σ g (T y)) ∧ ∀ x ∈ S, ∀ w, (∀ y ∈ S', B x y = ⟪y, w⟫_ℂ) → ∀ y ∈ S', B x y = ⟪T y, x⟫_ℂ

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theorem **isInvInvariant_haar** {ε : Fin (2 * r + k + 1) → Bool} {P : Mat (2 * r + k + 1)} (hP : IsCayley k r ε P) (hG : IsMatGroup (UH (Hpk k r))) : (haar : Measure (MG (UH (Hpk k r)) hG)).IsInvInvariant

theorem **theta_eq** (x : Mat (2 * r + k + 1)) (hx : x ∈ UH (Hpk k r)) : theta k r x = x⁻¹

theorem **sum_get** {α : Type} (L : List α) (f : α → ℝ) : ∑ i : Fin L.length, f (L.get i) = (L.map f).sum

theorem **exists_op** {ε ε' : Fin (2 * r + k + 1) → Bool} {mu mu' : Fin (2 * r + k + 1) → ℝ} {R R' : BranchingSupport.Rep k r} (hR : R.IsDS ε mu) (hR' : R'.IsDS ε' mu') {B : R.H → R'.H → ℂ} (hB : BranchingSupport.IsInvForm R R' B) : ∃ T : R'.H →L[ℂ] R.H, (∀ g ∈ UH (Hpk k r), ∀ y, T (R'.σ g y) = R.σ g (T y)) ∧ ∀ x ∈ R.sm, ∀ y ∈ R'.sm, B x y = ⟪T y, x⟫_ℂ

theorem **vanish_or_equiv** {ε ε' : Fin (2 * r + k + 1) → Bool} {mu mu' : Fin (2 * r + k + 1) → ℝ} {R R' : BranchingSupport.Rep k r} (hR : R.IsDS ε mu) (hR' : R'.IsDS ε' mu') {B : R.H → R'.H → ℂ} (hB : BranchingSupport.IsInvForm R R' B) : (∀ x ∈ R.sm, ∀ y ∈ R'.sm, B x y = 0) ∨ ∃ U : R.H ≃ₗᵢ[ℂ] R'.H, ∀ g ∈ BranchingSupport.GG k r, ∀ x, U (R.σ g x) = R'.σ g (U x)

theorem **poulsen_smooth_vectors_dense** : BranchingSupport.Poulsen

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theorem **dense_sm'** {n : ℕ} {G : Set (Mat n)} {H : Type} [NormedAddCommGroup H] [InnerProductSpace ℂ H] [CompleteSpace H] {σ : Mat n → (H →L[ℂ] H)} (hG : IsMatGroup G) (θ : Mat n →L[ℝ] Mat n) (hθ : ∀ x ∈ G, θ x = x⁻¹) (hσ : RepOn G σ) (hc : ∀ v, ContinuousOn (fun g => σ g v) G) : Dense {v | IsSm G σ v}

theorem **hGG** (k r : ℕ) : IsMatGroup (GG k r)

theorem **URep.dense** (U : URep k r) : Dense {v | IsSm (GG k r) U.R.σ v}

theorem **hLie** : ∀ X, InLieHpk k r X → ∀ t, ex X t ∈ GG k r

theorem **act_coe** (X : Obj k r) {g : Mx k r} (hg : g ∈ GG k r) (x : M X) (c : X.1.ι) : ((act X g x c : X.2.R.smSub) : X.2.R.H) = ∑ d, X.1.ρ g c d • X.2.R.σ g (x d)

theorem **sn_eq** (X : Obj k r) (W : List (Mx k r)) (x : M X) : sn X W x = ∑ c, ‖dsW X.2.R.σ W (x c : X.2.R.H)‖

theorem **sum_sn** (X : Obj k r) (Ws : List (List (Mx k r))) (x : M X) : (Ws.map fun W => sn X W x).sum = ∑ c, nu X.2.R.σ Ws (x c : X.2.R.H)

theorem **exists_common** {ι : Type} [Fintype ι] {H E : Type} [NormedAddCommGroup H] [InnerProductSpace ℂ H] [NormedAddCommGroup E] {σ : Mx k r → (H →L[ℂ] H)} {T : ι → H → E} (h : ∀ c, Bdd (GG k r) (InLieHpk k r) σ (T c)) : ∃ (C : ℝ) (Ws : List (List (Mx k r))), 0 ≤ C ∧ (∀ W ∈ Ws, LieWd k r W) ∧ ∀ c v, IsSm (GG k r) σ v → ‖T c v‖ ≤ C * nu σ Ws v

theorem **etaleClosed_point_lifts_simple_roots_monic** : Lift

theorem **isHaarOn_map** : PI.BesselPeriodGGP.IsHaarOn (k

theorem **integrable_gN** [(haar : Measure (GM k r)).IsInvInvariant] : Integrable (fun g : GM k r => B.gN g.mat ^ (-(((2 * r + k + 1) ^ 2 + 1 : ℕ) : ℝ))) haar

theorem **inv_of_ds** {U : URep k r} (hU : U.DS) : (haar : Measure (GM k r)).IsInvInvariant

theorem **schur_GM** {U : URep k r} (hU : U.DS) : ∃ d : ℝ, 0 < d ∧ ∀ v w v' w' : U.R.H, ∫ g : GM k r, ⟪w, U.R.σ g.mat v⟫_ℂ * star ⟪w', U.R.σ g.mat v'⟫_ℂ ∂haar = ⟪v', v⟫_ℂ * ⟪w, w'⟫_ℂ / (d : ℂ)

theorem **inner_inv** (U : URep k r) (g : GM k r) (v w : U.R.H) : ⟪w, U.R.σ (g⁻¹).mat v⟫_ℂ = star ⟪v, U.R.σ g.mat w⟫_ℂ

theorem **cf_eq** (X : Obj k r) (c : X.1.ι) (w : X.2.R.H) (x : M X) (g : GM k r) : cf X c w x g = ∑ d, X.1.ρ (g⁻¹).mat c d * ⟪w, X.2.R.σ (g⁻¹).mat (x d)⟫_ℂ

theorem **continuous_sig** (U : URep k r) (v : U.R.H) : Continuous fun g : GM k r => U.R.σ g.mat v

theorem **continuous_rho** {F : FRep k r} (hF : F.OK) (c d : F.ι) : Continuous fun g : GM k r => F.ρ g.mat c d

theorem **continuous_cf** {X : Obj k r} (hX : X.1.OK) (c : X.1.ι) (w : X.2.R.H) (x : M X) : Continuous (cf X c w x)

theorem **cf_bound** {X : Obj k r} (hX : X.1.OK) {m : ℕ} (hD : Decay X.2 m) (hm : X.1.deg + (2 * r + k + 1) ^ 2 + 1 ≤ m) (c : X.1.ι) {w : X.2.R.H} (hw : w ∈ X.2.R.sm) : ∃ (C : ℝ) (Ws : List (List (Mx k r))), (∀ W ∈ Ws, LieWd k r W) ∧ ∀ (x : M X) (g : GM k r), ‖cf X c w x g‖ ≤ C * (Ws.map fun W => sn X W x).sum * B.gN g.mat ^ (-(((2 * r + k + 1) ^ 2 + 1 : ℕ) : ℝ))

theorem **act_mul** {X : Obj k r} (hX : X.1.OK) {g h : Mx k r} (hg : g ∈ GG k r) (hh : h ∈ GG k r) (x : M X) : act X (g * h) x = act X g (act X h x)

theorem **exists_pr** [(haar : Measure (GM k r)).IsInvInvariant] {X : Obj k r} (hX : X.1.OK) {m : ℕ} (hD : Decay X.2 m) (hm : X.1.deg + (2 * r + k + 1) ^ 2 + 1 ≤ m) (T : URep k r) (c : X.1.ι) {w : X.2.R.H} (hw : w ∈ X.2.R.sm) (y : T.R.H) : (∀ x, Integrable (cf X c w x) haar) ∧ ∃ p, IsHom X (one T) p ∧ ∀ x y', ⟪y', (p x () : T.R.H)⟫_ℂ = ∫ g : GM k r, cf X c w x g * ⟪y', T.R.σ g.mat y⟫_ℂ ∂haar

theorem **integrable_mul_coef** {φ : GM k r → ℂ} (hφ : Integrable φ haar) (T : URep k r) (y y' : T.R.H) : Integrable (fun g : GM k r => φ g * ⟪y', T.R.σ g.mat y⟫_ℂ) haar

theorem **kfin_sm** {U : URep k r} (hU : U.DS) {v : U.R.H} (hv : U.R.KFin v) : v ∈ U.R.sm

theorem **occurrence_induction** : OccStep

theorem **exists_kfin** {U : URep k r} (hU : U.DS) : ∃ w : U.R.H, w ≠ 0 ∧ U.R.KFin w

theorem **injectivity_induction** : InjStep

CompanionTheory.HarishChandraModules.KFiniteAnalyticVectors.OrbitDerivatives.HasDerivAtMatrixExpZero

theorem **nu_nonneg** (Ws : List (List (Mat n))) (v : H) : 0 ≤ nu σ Ws v

theorem **nu_append** (Ws Ws' : List (List (Mat n))) (v : H) : nu σ (Ws ++ Ws') v = nu σ Ws v + nu σ Ws' v

theorem **nu_eq_sum** (Ws : List (List (Mat n))) (v : H) : nu σ Ws v = ∑ i : Fin Ws.length, ‖dsW σ (Ws.get i) v‖

theorem **Bdd.congr** {E : Type} [NormedAddCommGroup E] {T T' : H → E} (h : Bdd G L σ T) (e : ∀ v, IsSm G σ v → T' v = T v) : Bdd G L σ T'

theorem **Bdd.add** {E : Type} [NormedAddCommGroup E] {T T' : H → E} (h : Bdd G L σ T) (h' : Bdd G L σ T') : Bdd G L σ fun v => T v + T' v

theorem **Bdd.zero** {E : Type} [NormedAddCommGroup E] : Bdd G L σ fun _ => (0 : E)

theorem **Bdd.smul** {E : Type} [NormedAddCommGroup E] [NormedSpace ℂ E] {T : H → E} (h : Bdd G L σ T) (a : ℂ) : Bdd G L σ fun v => a • T v

theorem **Bdd.sum** {E ι : Type} [NormedAddCommGroup E] {T : ι → H → E} (s : Finset ι) (h : ∀ i ∈ s, Bdd G L σ (T i)) : Bdd G L σ fun v => ∑ i ∈ s, T i v

theorem **dsW_append** (W W' : List (Mat n)) (v : H) : dsW σ (W ++ W') v = dsW σ W (dsW σ W' v)

theorem **Bdd.comp_dsW** {E : Type} [NormedAddCommGroup E] {T : H → E} (h : Bdd G L σ T) (hσ : RepOn G σ) (hL : ∀ X, L X → ∀ t, ex X t ∈ G) {W' : List (Mat n)} (hW' : ∀ X ∈ W', L X) : Bdd G L σ fun v => T (dsW σ W' v)

theorem **exists_factor** [CompleteSpace H] [CompleteSpace H'] {T : H → H'} (hl : Lin G σ T) (hσ : RepOn G σ) {C : ℝ} {ι : Type} [Fintype ι] {W : ι → List (Mat n)} (hW : ∀ i, LieW G (W i)) (hb : ∀ v, IsSm G σ v → ‖T v‖ ≤ C * ∑ i, ‖dsW σ (W i) v‖) : ∃ Λ : ι → (H →L[ℂ] H'), ∀ v, IsSm G σ v → T v = ∑ i, Λ i (dsW σ (W i) v)

theorem **hasDerivAt_matrix_exp_zero** (X : Mat n) : HasDerivAt (ex X) X 0

theorem **smOn_orbit** (hσ : RepOn G σ) {w : H} (hw : IsSm G σ w) {W : List (Mat n)} (hW : LieW G W) : SmOn G fun x => dsW σ W (σ x w)

theorem **hasDerivAt_ds** (hσ : RepOn G σ) {v : H} (hv : IsSm G σ v) {X : Mat n} (hX : ∀ t, ex X t ∈ G) : HasDerivAt (fun t => σ (ex X t) v) (ds σ X v) 0

theorem **inner_ds** (hσ : RepOn G σ) (hiso : ∀ g ∈ G, ∀ v, ‖σ g v‖ = ‖v‖) {u v : H} (hu : IsSm G σ u) (hv : IsSm G σ v) {X : Mat n} (hX : ∀ t, ex X t ∈ G) : ⟪u, ds σ X v⟫_ℂ = ⟪-ds σ X u, v⟫_ℂ

theorem **exists_adj** (hσ : RepOn G σ) (hiso : ∀ g ∈ G, ∀ v, ‖σ g v‖ = ‖v‖) {W : List (Mat n)} (hW : LieW G W) {u : H} (hu : IsSm G σ u) : ∃ u', ∀ v, IsSm G σ v → ⟪u, dsW σ W v⟫_ℂ = ⟪u', v⟫_ℂ

theorem **hasDerivAt_dsW_orbit** (hσ : RepOn G σ) (hiso : ∀ g ∈ G, ∀ v, ‖σ g v‖ = ‖v‖) (hd : Dense {v | IsSm G σ v}) {v : H} (hv : IsSm G σ v) {W : List (Mat n)} (hW : LieW G W) {X : Mat n} (hX : ∀ t, ex X t ∈ G) : HasDerivAt (fun t => dsW σ W (σ (ex X t) v)) (dsW σ W (ds σ X v)) 0

theorem **SmOn.smul'** {E V : Type*} [NormedAddCommGroup E] [NormedSpace ℝ E] [NormedAddCommGroup V] [NormedSpace ℂ V] {A : Set E} {c : E → ℂ} {f : E → V} (hc : SmOn A c) (hf : SmOn A f) : SmOn A fun a => c a • f a

theorem **SmOn.sum'** {E V ι : Type*} [NormedAddCommGroup E] [NormedSpace ℝ E] [NormedAddCommGroup V] [NormedSpace ℝ V] {A : Set E} {f : ι → E → V} (s : Finset ι) (hf : ∀ i ∈ s, SmOn A (f i)) : SmOn A fun a => ∑ i ∈ s, f i a

theorem **Bdd.smOn_orbit** {T : H → H'} (hb : Bdd G L σ T) (hl : Lin G σ T) (hσ : RepOn G σ) (hL : ∀ X, L X → ∀ t, ex X t ∈ G) {v : H} (hv : IsSm G σ v) : SmOn G fun g => T (σ g v)

theorem **Bdd.hasDerivAt** {T : H → H'} (hb : Bdd G L σ T) (hl : Lin G σ T) (hσ : RepOn G σ) (hiso : ∀ g ∈ G, ∀ v, ‖σ g v‖ = ‖v‖) (hd : Dense {v | IsSm G σ v}) (hL : ∀ X, L X → ∀ t, ex X t ∈ G) {v : H} (hv : IsSm G σ v) {X : Mat n} (hX : ∀ t, ex X t ∈ G) : HasDerivAt (fun t => T (σ (ex X t) v)) (T (ds σ X v)) 0

theorem **TwInt.isSm** (h : TwInt G L σ τ M T) (hσ : RepOn G σ) (hL : ∀ X, L X → ∀ t, ex X t ∈ G) (hM : ∀ c d, SmOn G fun g => M g c d) (c : ι) {v : H} (hv : IsSm G σ v) : IsSm G τ (T c v)

theorem **TwInt.ds_eq** (h : TwInt G L σ τ M T) (hσ : RepOn G σ) (hiso : ∀ g ∈ G, ∀ v, ‖σ g v‖ = ‖v‖) (hd : Dense {v | IsSm G σ v}) (hL : ∀ X, L X → ∀ t, ex X t ∈ G) (hM1 : ∀ c d, M 1 c d = if c = d then 1 else 0) {X : Mat n} (hX : ∀ t, ex X t ∈ G) {M' : ι → ι → ℂ} (hM' : ∀ c d, HasDerivAt (fun t => M (ex X t) c d) (M' c d) 0) (c : ι) {v : H} (hv : IsSm G σ v) : ds τ X (T c v) = T c (ds σ X v) + ∑ d, M' c d • T d v

theorem **TwInt.bdd_dsW** (h : TwInt G L σ τ M T) (hσ : RepOn G σ) (hτ : RepOn G τ) (hiso : ∀ g ∈ G, ∀ v, ‖σ g v‖ = ‖v‖) (hd : Dense {v | IsSm G σ v}) (hL : ∀ X, L X → ∀ t, ex X t ∈ G) (hM : ∀ c d, SmOn G fun g => M g c d) (hM1 : ∀ c d, M 1 c d = if c = d then 1 else 0) {W : List (Mat n)} (hW : ∀ X ∈ W, L X) (c : ι) : Bdd G L σ fun v => dsW τ W (T c v)

CompanionTheory.HarishChandraModules.KFiniteAnalyticVectors.OrbitDerivatives.LieAlgebraHomomorphism

theorem **ent_coe** {X Y : Obj k r} (f : M X →ₗ[ℂ] M Y) (e : Y.1.ι) (c : X.1.ι) (v : X.2.R.smSub) : ent f e c v = (f (Pi.single c v) e : Y.2.R.H)

theorem **ent_lin** {X Y : Obj k r} (f : M X →ₗ[ℂ] M Y) (e : Y.1.ι) (c : X.1.ι) : Lin (GG k r) X.2.R.σ (ent f e c)

theorem **nu_zero** (U : URep k r) {Ws : List (List (Mx k r))} (hW : ∀ W ∈ Ws, LieWd k r W) : nu U.R.σ Ws 0 = 0

theorem **ent_bdd** {X Y : Obj k r} {f : M X →ₗ[ℂ] M Y} (hf : IsHom X Y f) (e : Y.1.ι) (c : X.1.ι) : Bdd (GG k r) (InLieHpk k r) X.2.R.σ (ent f e c)

theorem **apply_eq_ent** {X Y : Obj k r} (f : M X →ₗ[ℂ] M Y) (x : M X) (e : Y.1.ι) : (f x e : Y.2.R.H) = ∑ c, ent f e c (x c)

theorem **hasDerivAt_rho** {F : FRep k r} (hF : F.OK) {Z : Mx k r} (hZ : InLieHpk k r Z) (c d : F.ι) : HasDerivAt (fun t => F.ρ (ex Z t) c d) (F.dR Z c d) 0

theorem **LR_coe** (X : Obj k r) {Z : Mx k r} (hZ : InLieHpk k r Z) (x : M X) (c : X.1.ι) : (LR X Z x c : X.2.R.H) = ds X.2.R.σ Z (x c) + ∑ d, X.1.dR Z c d • (x d : X.2.R.H)

theorem **hasDerivAt_lin** {X : Obj k r} (hX : X.1.OK) {H' : Type} [NormedAddCommGroup H'] [InnerProductSpace ℂ H'] [CompleteSpace H'] {T : X.2.R.H → H'} (hl : Lin (GG k r) X.2.R.σ T) (hb : Bdd (GG k r) (InLieHpk k r) X.2.R.σ T) {Z : Mx k r} (hZ : InLieHpk k r Z) (x : M X) (c : X.1.ι) : HasDerivAt (fun t => T (act X (ex Z t) x c)) (T (LR X Z x c)) 0

theorem **comm_LR** {X Y : Obj k r} {f : M X →ₗ[ℂ] M Y} (hX : X.1.OK) (hY : Y.1.OK) (hf : IsHom X Y f) {Z : Mx k r} (hZ : InLieHpk k r Z) : f ∘ₗ LR X Z = LR Y Z ∘ₗ f

theorem **inLie_reH** (Y : Mx k r) : InLieHpk k r (reH k r Y)

theorem **inLie_imH** (Y : Mx k r) : InLieHpk k r (imH k r Y)

theorem **L_eq** (X : Obj k r) (Z : Mx k r) : L X Z = LR X (reH k r Z) + Complex.I • LR X (imH k r Z)

theorem **lie_algebra_homomorphism** : HomLie

theorem **dense_orb** {U : URep k r} (hU : U.DS) {w : U.R.H} (hw : w ≠ 0) : Dense (orbSpan (GG k r) U.R.σ w : Set U.R.H)

theorem **act_one_coe** (T : URep k r) {g : Mx k r} (hg : g ∈ GG k r) (y : M (one T)) : (act (one T) g y () : T.R.H) = T.R.σ g (y ())

theorem **ent_one** {T : URep k r} {X : Obj k r} (i : M (one T) →ₗ[ℂ] M X) (y : M (one T)) (e : X.1.ι) : (i y e : X.2.R.H) = ent i e () (y ())

theorem **kerZero'** : KerZero

theorem **isInvForm_ent** {T T' : URep k r} {f : M (one T) →ₗ[ℂ] M (one T')} (hf : IsHom (one T) (one T') f) : IsInvForm T.R T'.R fun x y => ⟪y, ent f () () x⟫_ℂ

theorem **schur'** : Schur

CompanionTheory.HarishChandraModules.KFiniteAnalyticVectors.SmoothIrreducibility.SmoothVectorsIrreducible

theorem **dsigW_rec** {k r : ℕ} {H : Type} [NormedAddCommGroup H] [InnerProductSpace ℂ H] (σ : Mat (2 * r + k + 1) → (H →L[ℂ] H)) (W : List (Mat (2 * r + k + 1))) (v : H) : Formalism.MainNotions.S.dsigW (k

theorem **smooth_vectors_irreducible** : Formalism.L2.SG.BesselSplit.SmoothIrred

CompanionTheory.HarishChandraModules.LowKTypeMultiplicity.DiscreteSeriesCharacters.CapelliDeterminantBlockProduct

@[simp] theorem **lift_apply** {V' : Type} [AddCommGroup V'] [Module ℂ V'] (T : V →ₗ[ℂ] V') (x : Ch ε V) (A : Finset (NC ε)) : lift ε T x A = T (x A)

CompanionTheory.HarishChandraModules.LowKTypeMultiplicity.DiscreteSeriesCharacters.CobApplyOfDeg

theorem **lift_ann** (T : Module.End ℂ V) (i : NC ε) : Commute (lift ε T : Module.End ℂ (Ch ε V)) (ann i)

theorem **lift_cre** (T : Module.End ℂ V) (i : NC ε) : Commute (lift ε T : Module.End ℂ (Ch ε V)) (cre i)

theorem **lift_Gop** (T : Module.End ℂ V) (c d : Fin n) : Commute (lift ε T : Module.End ℂ (Ch ε V)) (Gop ε c d)

theorem **kHighest_map** {F : Fin n → Fin n → Module.End ℂ V} {γ : Fin n → ℝ} {x : V} (h : KHighest ε F γ x) {T : Module.End ℂ V} (hT : ∀ c d, ε c = ε d → Commute (F c d) T) : KHighest ε F γ (T x)

theorem **bd_apply_of_deg** (E : Fin n → Fin n → Module.End ℂ V) {j : ℕ} {x : Ch ε V} (hx : Deg j x) {A : Finset (NC ε)} (hA : A.card = j) : bd ε E x A = 0

theorem **cob_apply_of_deg** (E : Fin n → Fin n → Module.End ℂ V) {j : ℕ} {x : Ch ε V} (hx : Deg j x) {A : Finset (NC ε)} (hA : A.card = j) : cob ε E x A = 0

theorem **algCore_of** (hF : FRel) (hB : BdFacts) (hCO : CO) (hZ : ZetaHW) (hW : WLem) (hN : NormLt) (hA : Adj) (hP : PRV) : AlgCore

theorem **dsInfCharCap_of** (hC : AlgCoreSub) (hM : SmMod) (hQ : ActQ) (hSp : Spread) (hKS : KFinSm) (hKF : KFinStab) (hSL : StabLink) (hH : HWofType) (hT : TypeOfHW) : PI.DSInfCharCap.DSInfCharCap

CompanionTheory.HarishChandraModules.LowKTypeMultiplicity.DiscreteSeriesCharacters.DiscreteSeriesInfinitesimalCharacter

theorem **hCAR** : CAR

theorem **hFRel** : FRel

theorem **hBdFacts** : BdFacts

theorem **hGFacts** : GFacts

theorem **hKoszul** : Koszul

theorem **hVerma** : Verma

theorem **hCover** : Cover

theorem **hCO0** : CapelliDeterminantBlockProduct

theorem **hCO** : CO

theorem **hZetaHW** : ZetaHW

theorem **hWLem** : WLem

theorem **hNormLt** : NormLt

theorem **hAdj** : Adj

theorem **hGamDom** : GamDom

theorem **hKHWExists** : KHWExists

theorem **hLowSpan** : LowSpan

theorem **hHWMap** : HWMap

theorem **hPRV** : PRV

theorem **hAlgCore** : AlgCore

theorem **hAlgCoreSub** : AlgCoreSub

theorem **hSmLin** : SmLin

theorem **hSmBracket** : SmBracket

theorem **hSmMod** : SmMod

theorem **hActQ** : ActQ

theorem **hSpread** : Spread

theorem **hKFinStab** : KFinStab

theorem **hKExp** : KExp

theorem **hStabLink** : StabLink

theorem **hWtLink** : WtLink

theorem **hHWofType** : HWofType

theorem **hTypeOfHW** : TypeOfHW

theorem **discrete_series_infinitesimal_character** : PI.DSInfCharCap.DSInfCharCap

CompanionTheory.HarishChandraModules.LowKTypeMultiplicity.DiscreteSeriesCharacters.Spread

theorem **map_qminor** {A B : Type} [Ring A] [Algebra ℂ A] [Ring B] [Algebra ℂ B] (f : A →ₐ[ℂ] B) {n t : ℕ} (E : Fin n → Fin n → A) (a b : Fin t → Fin n) (u : ℂ) : f (qminor E a b u) = qminor (fun c d => f (E c d)) a b u

theorem **qminor_mem** {A : Type} [Ring A] [Algebra ℂ A] (T : Subalgebra ℂ A) {n t : ℕ} (E : Fin n → Fin n → A) (hE : ∀ c d, E c d ∈ T) (a b : Fin t → Fin n) (u : ℂ) : qminor E a b u ∈ T

theorem **qdet_adj** {n : ℕ} {ε : Fin n → Bool} {E : Fin n → Fin n → Module.End ℂ V} (hE : GlRel E) (hU : IsUnitary ε E) (w : ℂ) (x y : V) : ⟪qdet E w x, y⟫_ℂ = ⟪x, qdet E (star w) y⟫_ℂ

theorem **qminor_intertwine** {V : Type} [AddCommGroup V] [Module ℂ V] {n t : ℕ} (τ : Module.End ℂ V) (E E' : Fin n → Fin n → Module.End ℂ V) (h : ∀ a b, τ * E a b = E' a b * τ) (a b : Fin t → Fin n) (u : ℂ) : τ * qminor E a b u = qminor E' a b u * τ

theorem **capelli_spread** : Spread

CompanionTheory.HarishChandraModules.LowKTypeMultiplicity.EigenspaceCyclicDensity.Bridge

theorem **low_K_type_multiplicity_one_of** (h : LowKMultOneP) : Formalism.MainNotions.DiscreteSeriesWeylDimension.LowKMultOne

CompanionTheory.HarishChandraModules.LowKTypeMultiplicity.EigenspaceCyclicDensity.CreationOperators.CreAnnApply

theorem **ann_mem** {Q : Submodule ℂ Z} {x : Cn n Z} (h : ∀ A, x A ∈ Q) (i : Pos n) (A : Finset (Pos n)) : ann i x A ∈ Q

theorem **cre_mem** {Q : Submodule ℂ Z} {x : Cn n Z} (h : ∀ A, x A ∈ Q) (i : Pos n) (A : Finset (Pos n)) : cre i x A ∈ Q

theorem **cre_ann_apply** (i : Pos n) (x : Cn n Z) (A : Finset (Pos n)) : cre i (ann i x) A = if i ∈ A then x A else 0

theorem **acycAbs_of** (hS : BdSq) : AcycAbs

CompanionTheory.HarishChandraModules.LowKTypeMultiplicity.EigenspaceCyclicDensity.CreationOperators.CreApply

theorem **key_inj** {i j : Pos n} (h : key i = key j) : i = j

theorem **tw_self** (i : Pos n) : tw i i = 1

theorem **tw_mul_self** (i j : Pos n) : tw i j * tw i j = 1

theorem **tw_swap** {i j : Pos n} (h : i ≠ j) : tw j i = -tw i j

theorem **sg_mul_self** (i : Pos n) (A : Finset (Pos n)) : sg i A * sg i A = 1

theorem **sg_insert** (i : Pos n) {j : Pos n} {A : Finset (Pos n)} (h : j ∉ A) : sg i (insert j A) = tw i j * sg i A

theorem **sg_erase** (i : Pos n) {j : Pos n} {A : Finset (Pos n)} (h : j ∈ A) : sg i (A.erase j) = tw i j * sg i A

theorem **ann_apply** (i : Pos n) (x : Cn n V) (A : Finset (Pos n)) : ann i x A = if i ∈ A then 0 else sg i A • x (insert i A)

theorem **cre_apply** (i : Pos n) (x : Cn n V) (A : Finset (Pos n)) : cre i x A = if i ∈ A then sg i A • x (A.erase i) else 0

theorem **ann_ann** (i j : Pos n) : (ann i * ann j + ann j * ann i : Module.End ℂ (Cn n V)) = 0

theorem **cre_cre** (i j : Pos n) : (cre i * cre j + cre j * cre i : Module.End ℂ (Cn n V)) = 0

theorem **ann_cre** (i j : Pos n) : (ann i * cre j + cre j * ann i : Module.End ℂ (Cn n V)) = if i = j then 1 else 0

theorem **car** : CARp

CompanionTheory.HarishChandraModules.LowKTypeMultiplicity.EigenspaceCyclicDensity.CreationOperators.CycDense

theorem **cyclic_vectors_dense** : CycDense

CompanionTheory.HarishChandraModules.LowKTypeMultiplicity.EigenspaceCyclicDensity.CreationOperators.DifferentialBoundedSupport

theorem **bd_apply** (ε : Fin n → Bool) (E : Fin n → Fin n → Module.End ℂ V) (x : Cn n V) (A : Finset (Pos n)) : bd ε E x A = (∑ i : Pos n, if cpt ε i then E i.1.1 i.1.2 (ann i x A) else 0) + ∑ i : Pos n, ∑ j : Pos n, ∑ k : Pos n, if Tri ε i j k then cre k (ann j (ann i x)) A else 0

theorem **ann_ne** {i : Pos n} {x : Cn n V} {A : Finset (Pos n)} (h : ann i x A ≠ 0) : i ∉ A ∧ x (insert i A) ≠ 0

theorem **cre_ne** {i : Pos n} {x : Cn n V} {A : Finset (Pos n)} (h : cre i x A ≠ 0) : i ∈ A ∧ x (A.erase i) ≠ 0

theorem **tri_ne** {i j k : Pos n} {x : Cn n V} {A : Finset (Pos n)} (h : cre k (ann j (ann i x)) A ≠ 0) : k ∈ A ∧ j ∉ A.erase k ∧ i ∉ insert j (A.erase k) ∧ x (insert i (insert j (A.erase k))) ≠ 0

theorem **Tri.cptk** {ε : Fin n → Bool} {i j k : Pos n} (h : Tri ε i j k) : cpt ε k

theorem **bd_sub_ne** {ε₀ ε : Fin n → Bool} (h0 : ∀ a b, ε a = ε b → ε₀ a = ε₀ b) (E : Fin n → Fin n → Module.End ℂ V) (x : Cn n V) (A : Finset (Pos n)) (h : bd ε₀ E x A ≠ bd ε E x A) : (∃ i, cpt ε₀ i ∧ ¬cpt ε i ∧ ann i x A ≠ 0) ∨ ∃ i j k, Tri ε₀ i j k ∧ ¬(cpt ε i ∧ cpt ε j) ∧ cre k (ann j (ann i x)) A ≠ 0

theorem **bd_ne** (ε : Fin n → Bool) (E : Fin n → Fin n → Module.End ℂ V) (x : Cn n V) (A : Finset (Pos n)) (h : bd ε E x A ≠ 0) : (∃ i, cpt ε i ∧ ann i x A ≠ 0) ∨ ∃ i j k, Tri ε i j k ∧ cre k (ann j (ann i x)) A ≠ 0

theorem **filter_erase_of_not** {p : Pos n → Prop} [DecidablePred p] {k : Pos n} (hk : ¬p k) (A : Finset (Pos n)) : (A.erase k).filter p = A.filter p

theorem **sup_bd** {ε : Fin n → Bool} (E : Fin n → Fin n → Module.End ℂ V) {B : Finset (Pos n)} {j : ℕ} {x : Cn n V} (hx : Sup ε B j x) {A : Finset (Pos n)} (h : bd ε E x A ≠ 0) : (A.filter fun i => ¬cpt ε i) = B ∧ (A.filter (cpt ε)).card + 1 = j

theorem **pot_insert** {i : Pos n} {B : Finset (Pos n)} (h : i ∉ B) : pot (insert i B) = n - ((i.1.2 : ℕ) - (i.1.1 : ℕ)) + pot B

theorem **differential_bounded_support** : BdSup

theorem **wt_om** (ε : Fin n → Bool) : wt (om ε) = (2 : ℝ) • PI.UpqRep.rhoC ε

theorem **top_degree_cycle** : TopCycle

theorem **lift_ann** (φ : V →ₗ[ℂ] W) (i : Pos n) (x : Cn n V) : lift n φ (ann i x) = ann i (lift n φ x)

theorem **lift_cre** (φ : V →ₗ[ℂ] W) (i : Pos n) (x : Cn n V) : lift n φ (cre i x) = cre i (lift n φ x)

theorem **lift_bd** {ε : Fin n → Bool} (φ : V →ₗ[ℂ] W) (E : Fin n → Fin n → Module.End ℂ V) (F : Fin n → Fin n → Module.End ℂ W) (h : ∀ i : Pos n, cpt ε i → ∀ x, φ (E i.1.1 i.1.2 x) = F i.1.1 i.1.2 (φ x)) (x : Cn n V) : lift n φ (bd ε E x) = bd ε F (lift n φ x)

CompanionTheory.HarishChandraModules.LowKTypeMultiplicity.EigenspaceCyclicDensity.CreationOperators.DifferentialCommutesWeightOperator

theorem **wt_insert** {i : Pos n} {A : Finset (Pos n)} (h : i ∉ A) (c : Fin n) : wt (insert i A) c = wt A c + al i c

theorem **Ft_apply** (ε : Fin n → Bool) (E : Fin n → Fin n → Module.End ℂ V) (c : Fin n) (x : Cn n V) (A : Finset (Pos n)) : Ft ε E c x A = E c c (x A) + (wt (A.filter (cpt ε)) c : ℂ) • x A

theorem **Ft_ann** {ε : Fin n → Bool} (E : Fin n → Fin n → Module.End ℂ V) (c : Fin n) {i : Pos n} (hi : cpt ε i) (x : Cn n V) : Ft ε E c (ann i x) = ann i (Ft ε E c x) - (al i c : ℂ) • ann i x

theorem **Ft_cre** {ε : Fin n → Bool} (E : Fin n → Fin n → Module.End ℂ V) (c : Fin n) {i : Pos n} (hi : cpt ε i) (x : Cn n V) : Ft ε E c (cre i x) = cre i (Ft ε E c x) + (al i c : ℂ) • cre i x

theorem **differential_commutes_weight_operator** : ∀ (n : ℕ) (ε₀ ε : Fin n → Bool) (V : Type) [AddCommGroup V] [Module ℂ V] (E : Fin n → Fin n → Module.End ℂ V), (∀ a b, ε a = ε b → ε₀ a = ε₀ b) → KRel ε E → ∀ c, Commute (Ft ε₀ E c) (bd ε E)

theorem **free_inter** {ι : Type} (φ : V →ₗ[ℂ] W) (S : ι → Module.End ℂ V) (T : ι → Module.End ℂ W) (h : ∀ i x, φ (S i x) = T i (φ x)) (a : FreeAlgebra ℂ ι) : ∀ x, φ (FreeAlgebra.lift ℂ S a x) = FreeAlgebra.lift ℂ T a (φ x)

theorem **evK_inter** {ε : Fin n → Bool} (φ : V →ₗ[ℂ] W) (E : Fin n → Fin n → Module.End ℂ V) (F : Fin n → Fin n → Module.End ℂ W) (h : ∀ a b x, φ (E a b x) = F a b (φ x)) (a : FreeAlgebra ℂ (Fin n × Fin n)) (x : V) : φ (evK ε E a x) = evK ε F a (φ x)

theorem **evd_Ft_apply** (ε : Fin n → Bool) (E : Fin n → Fin n → Module.End ℂ V) (b : FreeAlgebra ℂ (Fin n)) (x : Cn n V) (A : Finset (Pos n)) : evd (Ft ε E) b x A = evd (fun c => E c c + (wt (A.filter (cpt ε)) c : ℂ) • 1) b (x A)

theorem **lift_evd** {ε : Fin n → Bool} (φ : V →ₗ[ℂ] W) (E : Fin n → Fin n → Module.End ℂ V) (F : Fin n → Fin n → Module.End ℂ W) (h : ∀ a b x, φ (E a b x) = F a b (φ x)) (b : FreeAlgebra ℂ (Fin n)) (x : Cn n V) : lift n φ (evd (Ft ε E) b x) = evd (Ft ε F) b (lift n φ x)

theorem **sup_of_supp** {ε : Fin n → Bool} {B : Finset (Pos n)} {j : ℕ} {x : Cn n V} {y : Cn n W} (hx : Sup ε B j x) (h : ∀ A, x A = 0 → y A = 0) : Sup ε B j y

theorem **Sup.sub** {ε : Fin n → Bool} {B : Finset (Pos n)} {j : ℕ} {x y : Cn n V} (hx : Sup ε B j x) (hy : Sup ε B j y) : Sup ε B j (x - y)

theorem **genCO_of** (hS : BdSq) (hC : Cover2) : GenCO

CompanionTheory.HarishChandraModules.LowKTypeMultiplicity.EigenspaceCyclicDensity.CreationOperators.FilterAll

theorem **casK_comm** {E : Fin n → Fin n → Module.End ℂ V} (hE : KRel ε E) {c d : Fin n} (h : ε c = ε d) : Commute (E c d) (CasK ε E)

theorem **sum_pos** {M : Type} [AddCommMonoid M] (f : Fin n → Fin n → M) : ∑ a, ∑ b, f a b = ∑ a, f a a + ∑ i : Pos n, (f i.1.1 i.1.2 + f i.1.2 i.1.1)

theorem **wt_sum** (A : Finset (Pos n)) (c : Fin n) : wt A c = ∑ i ∈ A, al i c

theorem **casK_inN** {E : Fin n → Fin n → Module.End ℂ V} (hE : KRel ε E) (x : V) : InN ε E (CasK ε E x - (∑ c, E c c (E c c x) - ∑ c, ((2 * rhoC ε c : ℝ) : ℂ) • E c c x))

theorem **cok_of** (hG : GenCO) : COK

theorem **wt_split** (A : Finset (Pos n)) (c : Fin n) : wt A c = wt (A.filter (cpt ε)) c + wt (A.filter fun i => ¬cpt ε i) c

theorem **filter_all** (A : Finset (Pos n)) : A.filter (cpt (allT n)) = A

theorem **exists_poly** {V : Type} [NormedAddCommGroup V] [InnerProductSpace ℂ V] (T : Module.End ℂ V) (hT : ∀ x y, ⟪T x, y⟫_ℂ = ⟪x, T y⟫_ℂ) (W : Submodule ℂ V) [FiniteDimensional ℂ W] (hW : ∀ x ∈ W, T x ∈ W) (s : ℝ) : ∃ q : Polynomial ℂ, ∀ x ∈ W, ∀ y, y = x + Polynomial.aeval T q (T x - (s : ℂ) • x) → T y = (s : ℂ) • y

theorem **vanK_of** (hC : COK) (hP : WtProj) (hM : WtMajor) : VanK

theorem **inj_of** (hV : VanK) (hP : WtProj) (hS : BdSq) : Inj

CompanionTheory.HarishChandraModules.LowKTypeMultiplicity.EigenspaceCyclicDensity.CreationOperators.GeneralizedVermaModulePolynomial

theorem **deg_pderiv** {σ : Type} (i : σ) (f : MvPolynomial σ ℂ) (k : ℕ) (h : f.totalDegree ≤ k + 1) : (pderiv i f).totalDegree ≤ k

theorem **pderiv_deg_zero** {σ : Type} (i : σ) (f : MvPolynomial σ ℂ) (h : f.totalDegree ≤ 0) : pderiv i f = 0

theorem **deg_X_mul** {σ : Type} (j : σ) (f : MvPolynomial σ ℂ) : (X j * f).totalDegree ≤ f.totalDegree + 1

theorem **deg_X_pderiv** {σ : Type} (j i : σ) (f : MvPolynomial σ ℂ) : (X j * pderiv i f).totalDegree ≤ f.totalDegree

theorem **generalized_verma_module_polynomial** : Verma2

theorem **kRel_tens** {n : ℕ} {ε : Fin n → Bool} {V Z : Type} [AddCommGroup V] [Module ℂ V] [AddCommGroup Z] [Module ℂ Z] {E : Fin n → Fin n → Module.End ℂ V} {M : Fin n → Fin n → Module.End ℂ Z} (hE : KRel ε E) (hM : GlRel M) : KRel ε fun a b => ((E a b).rTensor Z + (M a b).lTensor V : Module.End ℂ (V ⊗[ℂ] Z))

theorem **free_module_induction** : FreeStep

theorem **free_all** (n m : ℕ) : Free n m

theorem **cover2_of** (hA : AcycAbs) : Cover2

CompanionTheory.HarishChandraModules.LowKTypeMultiplicity.EigenspaceCyclicDensity.CreationOperators.KTypeCount

theorem **ind_eq** (A : Finset (Pos n)) (a b : Fin n) : ind A a b = if h : a < b then (if (((a, b), h) : Pos n) ∈ A then 1 else 0) else 0

theorem **wt_eq** (A : Finset (Pos n)) (c : Fin n) : wt A c = ∑ b, (ind A c b − ind A b c)

theorem **sc_anti** (ε : Fin n → Bool) (a b : Fin n) : sc ε b a = −sc ε a b

theorem **rhoC_eq** (ε : Fin n → Bool) (a : Fin n) : rhoC ε a = (∑ b, sc ε a b) / 2

theorem **exists_sort** (ε : Fin n → Bool) (κ : Fin n → ℝ) : ∃ w : Equiv.Perm (Fin n), (∀ c, ε (w c) = ε c) ∧ ∀ a b, a < b → ε a = ε b → κ (w b) ≤ κ (w a)

theorem **K_type_count** : NumK

CompanionTheory.HarishChandraModules.LowKTypeMultiplicity.EigenspaceCyclicDensity.CreationOperators.KoszulDifferentialAnticommutator

theorem **tri_uniq** {i j k k' : Pos n} (h : Tri ε i j k) (h' : Tri ε i j k') : k = k'

theorem **sum_tc** {M : Type} [AddCommGroup M] [Module ℂ M] (i j : Pos n) (f : Pos n → M) : ∑ k, tc ε i j k • f k = if h : cpt ε i ∧ cpt ε j ∧ i.1.2 = j.1.1 then f ((i.1.1, j.1.2), i.2.trans (h.2.2 ▸ j.2)) else 0

theorem **tc_assoc** (p q r k : Pos n) : ∑ i, tc ε p q i * tc ε i r k = ∑ j, tc ε q r j * tc ε p j k

theorem **self_neg** {x : R} (h : x = -x) : x = 0

theorem **koszul_differential_sq** (hE : KRel ε E) (h1 : ∀ a b i, Commute (E a b) (an i)) (haa : ∀ i j, an i * an j + an j * an i = 0) : (∑ i, cc ε i • (E i.1.1 i.1.2 * an i)) * (∑ i, cc ε i • (E i.1.1 i.1.2 * an i)) = ∑ i, ∑ j, ∑ k, tc ε i j k • (E k.1.1 k.1.2 * (an i * an j))

theorem **comm_aa** (haa : ∀ i j, an i * an j + an j * an i = 0) (l i j : Pos n) : an j * an i * an l = an l * (an j * an i)

theorem **koszul_differential_anticommutator** (h1 : ∀ a b i, Commute (E a b) (an i)) (h2 : ∀ a b i, Commute (E a b) (cr i)) (haa : ∀ i j, an i * an j + an j * an i = 0) (hac : ∀ i j, an i * cr j + cr j * an i = if i = j then 1 else 0) : (∑ i, cc ε i • (E i.1.1 i.1.2 * an i)) * (∑ i, ∑ j, ∑ k, tc ε i j k • (cr k * an j * an i)) + (∑ i, ∑ j, ∑ k, tc ε i j k • (cr k * an j * an i)) * (∑ i, cc ε i • (E i.1.1 i.1.2 * an i)) = −∑ i, ∑ j, ∑ k, tc ε i j k • (E k.1.1 k.1.2 * (an i * an j))

theorem **beta_sq** (haa : ∀ i j, an i * an j + an j * an i = 0) (hcc : ∀ i j, cr i * cr j + cr j * cr i = 0) (hac : ∀ i j, an i * cr j + cr j * an i = if i = j then 1 else 0) : (∑ i, ∑ j, ∑ k, tc ε i j k • (cr k * an j * an i)) * (∑ i, ∑ j, ∑ k, tc ε i j k • (cr k * an j * an i)) = 0

theorem **koszul_differential_sq_zero** : BdSq

CompanionTheory.HarishChandraModules.LowKTypeMultiplicity.EigenspaceCyclicDensity.CreationOperators.RhoSurj

theorem **burnside** {A : Type} [Ring A] [Algebra ℂ A] [FiniteDimensional ℂ V] (ρ : A →ₐ[ℂ] Module.End ℂ V) (hnt : Nontrivial V) (hirr : ∀ W : Submodule ℂ V, (∀ a : A, ∀ w ∈ W, ρ a w ∈ W) → W = ⊥ ∨ W = ⊤) : Function.Surjective ρ

theorem **ev_psi** (ε : Fin n → Bool) (E : Fin n → Fin n → Module.End ℂ V) (L : Submodule ℂ V) (hL : KStab ε E L) (r : FreeAlgebra ℂ (Fin n × Fin n)) : ∀ s : L, ev E (psi ε r) s = (rho ε E L hL r s : V)

theorem **rho_surj** (ε : Fin n → Bool) (E : Fin n → Fin n → Module.End ℂ V) (L : Submodule ℂ V) (hL : KStab ε E L) [FiniteDimensional ℂ L] {v : V} (hv : v ≠ 0) (hvL : v ∈ L) (hirr : ∀ L' ≤ L, KStab ε E L' → L' = ⊥ ∨ L' = L) : Function.Surjective (rho ε E L hL)

theorem **dens** : CompanionTheory.D.LK.Dens

CompanionTheory.HarishChandraModules.LowKTypeMultiplicity.EigenspaceCyclicDensity.CreationOperators.WtProj

theorem **exists_proj** {M : Type} [AddCommGroup M] [Module ℂ M] (F : Module.End ℂ M) (τ : ℂ) (t : M) (ht : t ∈ ⊔ μ : ℂ, F.eigenspace μ) : ∃ P : Module.End ℂ M, (∀ G, Commute F G → Commute P G) ∧ (∀ W : Submodule ℂ M, (∀ x ∈ W, F x ∈ W) → ∀ x ∈ W, P x ∈ W) ∧ (∀ y, F y = τ • y → P y = y) ∧ F (P t) = τ • P t

theorem **mem_iSup_eigenspace** {V : Type} [NormedAddCommGroup V] [InnerProductSpace ℂ V] (T : Module.End ℂ V) (hT : T.IsSymmetric) (S : Submodule ℂ V) [FiniteDimensional ℂ S] (hS : ∀ v ∈ S, T v ∈ S) (x : V) (hx : x ∈ S) : x ∈ ⊔ μ : ℂ, T.eigenspace μ

theorem **Ft_apply** {n : ℕ} {V : Type} [AddCommGroup V] [Module ℂ V] (ε : Fin n → Bool) (E : Fin n → Fin n → Module.End ℂ V) (c : Fin n) (x : Cn n V) (A : Finset (Pos n)) : Ft ε E c x A = E c c (x A) + (wt (A.filter (cpt ε)) c : ℂ) • x A

theorem **chain_mem_iSup** {n : ℕ} {V : Type} [AddCommGroup V] [Module ℂ V] (ε : Fin n → Bool) (E : Fin n → Fin n → Module.End ℂ V) (c : Fin n) (h : ∀ v : V, v ∈ ⊔ μ : ℂ, (E c c).eigenspace μ) (t : Cn n V) : t ∈ ⊔ μ : ℂ, (Ft ε E c).eigenspace μ

theorem **Ft_commute** {n : ℕ} {V : Type} [AddCommGroup V] [Module ℂ V] (ε₀ ε : Fin n → Bool) (E : Fin n → Fin n → Module.End ℂ V) (hE : KRel ε E) (c d : Fin n) : Commute (Ft ε₀ E c) (Ft ε₀ E d)

theorem **wtProj** : CompanionTheory.D.LK.WtProj

CompanionTheory.HarishChandraModules.LowKTypeMultiplicity.EigenspaceCyclicDensity.LowKMultOneP

@[simp] theorem **lift_apply** {V' : Type} [AddCommGroup V'] [Module ℂ V'] (T : V →ₗ[ℂ] V') (x : Cn n V) (A : Finset (Pos n)) : lift n T x A = T (x A)

CompanionTheory.HarishChandraModules.LowKTypeMultiplicity.EigenspaceCyclicDensity.LowKMultOnePOf

theorem **evK_allT** (E : Fin n → Fin n → Module.End ℂ V) : evK (allT n) E = ev E

theorem **evd_eig** (F : Fin n → Module.End ℂ V) (τ : Fin n → ℂ) (x : V) (h : ∀ c, F c x = τ c • x) (b : FreeAlgebra ℂ (Fin n)) : evd F b x = FreeAlgebra.lift ℂ τ b • x

theorem **mem_cyc** {E : Fin n → Fin n → Module.End ℂ V} {v w : V} : w ∈ Cyc E v ↔ ∃ a, ev E a v = w

theorem **cyc_stab** {E : Fin n → Fin n → Module.End ℂ V} {v w : V} (c d : Fin n) (h : w ∈ Cyc E v) : E c d w ∈ Cyc E v

theorem **multAlg_of** (hL : LowSpan) (hH : HWMap) (hD : Dens) (hI : InvLift) (hC : HCdec) (hG : GenCO) (hT : TopCycle) (hJ : Inj) : MultAlg

private lemma **commute_E_cas_of_glRel** {A : Type} [Ring A] {n : ℕ} (E : Fin n → Fin n → A) (hE : GlRel E) (c d : Fin n) : E c d * Cas E = Cas E * E c d

theorem **lowKMultOneP_of** (hM : MultAlg) (hE : KModExists) (hD : CycDense) (hO : OrthCyc) : LowKMultOneP

CompanionTheory.HarishChandraModules.LowKTypeMultiplicity.EigenspaceCyclicDensity.LowKTypeMultiplicityOneFock

theorem **low_K_type_multiplicity_one** : Formalism.MainNotions.DiscreteSeriesWeylDimension.LowKMultOne

CompanionTheory.HarishChandraModules.LowKTypeMultiplicity.EigenspaceCyclicDensity.LowKTypeMultiplicityOneUnconditional

theorem **hGenCO** : GenCO

theorem **hInj** : Inj

theorem **hMultAlg** : MultAlg

theorem **hKModExists** : KModExists

theorem **hOrthCyc** : OrthCyc

theorem **low_K_type_multiplicity_one_unconditional** : LowKMultOneP

CompanionTheory.HarishChandraModules.LowKTypeMultiplicity.EigenspaceCyclicDensity.TopUniq

theorem **discrete_series_unique_of** : DSUniqP

CompanionTheory.HarishChandraModules.NonemptyRepresentations.ExpEntry

theorem **exp_entry** (N : ℕ) (X : Matrix (Fin N) (Fin N) ℂ) (i j : Fin N) : HasDerivAt (fun t : ℝ => (NormedSpace.exp ((t : ℂ) • X)) i j) (X i j) 0

CompanionTheory.HarishChandraModules.NonemptyRepresentations.HWRepD

theorem **D_add** (Y Z : Matrix (Fin k) (Fin k) ℂ) : R.D (Y + Z) = R.D Y + R.D Z

theorem **D_smul** (c : ℂ) (Y : Matrix (Fin k) (Fin k) ℂ) : R.D (c • Y) = c • R.D Y

theorem **hasDerivAt_curve** (γ : ℝ → Matrix (Fin k) (Fin k) ℂ) (Y : Matrix (Fin k) (Fin k) ℂ) (h0 : γ 0 = 1) (hγ : ∀ a b, HasDerivAt (fun t => γ t a b) (Y a b) 0) (v : R.I → ℂ) : HasDerivAt (fun t => R.ρ (γ t) *ᵥ v) (R.D Y *ᵥ v) 0

theorem **curve_deriv** (a b c d : Fin k) : HasDerivAt (fun t : ℝ => (1 + (t : ℂ) • Matrix.single a b (1 : ℂ) : Matrix (Fin k) (Fin k) ℂ) c d) ((Matrix.single a b (1 : ℂ) : Matrix (Fin k) (Fin k) ℂ) c d) 0

theorem **curve_tri** {a b : Fin k} (hab : a ≤ b) (t : ℂ) : (1 + t • Matrix.single a b (1 : ℂ) : Matrix (Fin k) (Fin k) ℂ).BlockTriangular id

theorem **D_upper** {a b : Fin k} (hab : a < b) : R.D (Matrix.single a b 1) *ᵥ R.v0 = 0

theorem **D_diag** (a : Fin k) : R.D (Matrix.single a a 1) *ᵥ R.v0 = (ν a : ℂ) • R.v0

CompanionTheory.HarishChandraModules.NonemptyRepresentations.IsData

theorem **foldr_ofFn_smul** {X : Type*} [AddCommGroup X] [Module ℂ X] : ∀ (t : ℕ) (c : Fin t → ℂ) (x : X), (List.ofFn fun j : Fin t => fun y : X => c j • y).foldr (fun T h => T h) x = (∏ j, c j) • x | 0, c, x => by simp | t + 1, c, x => by rw [List.ofFn_succ, List.foldr_cons, foldr_ofFn_smul t (fun j => c j.succ) x, Fin.prod_univ_succ, mul_smul]
theorem ecRep_const (k r : ℕ) (Z : Mx k r) (v : ℂ) : EcRep (Hpk k r) (π0 k r) Z v = 0

theorem **capRep_const** (k r m : ℕ) (Bs : Fin m → Fin (2 * r + k + 1) → ℂ) (w v : ℂ) : capRep k r (π0 k r) Bs w v = (∏ j : Fin m, (w - ((j : ℕ) : ℂ))) • v

theorem **prod_uTriv** (k : ℕ) (w : ℂ) : (∏ i ∈ Finset.Icc 1 k, (w - (k : ℂ) / 2 + 1 / 2 + ((uTriv k i : ℝ) : ℂ))) = ∏ j : Fin k, (w - ((j : ℕ) : ℂ))

theorem **capParam_triv** (k r : ℕ) (z : Fin (2 * r + k + 1) → ℂ) (u : ℕ → ℝ) (hu : ∀ i ∈ Finset.Icc 1 k, u i = uTriv k i) : CapParam k r z (π0 k r) u

theorem **isType_triv** (k r : ℕ) (z : Fin (2 * r + k + 1) → ℂ) (u : ℕ → ℝ) (hu : ∀ i ∈ Finset.Icc 1 k, u i = uTriv k i) : IsType k r z (π0 k r) u

theorem **par_ok** (P : M.Par) (hk : 1 ≤ P.k) (hT : P.T = 1) (hT' : P.T' = 1) (huu : P.u' = P.u) (hu : ∀ i ∈ Finset.Icc 1 P.k, P.u i = uTriv P.k i) : P.OK

theorem **isPlace_triv** (P : M.Par) (hk : 1 ≤ P.k) (hT : P.T = 1) (hT' : P.T' = 1) (huu : P.u' = P.u) (hu : ∀ i ∈ Finset.Icc 1 P.k, P.u i = uTriv P.k i) : IsPlace P (z0 P.k P.r) (z0 P.k P.r) (π0 P.k P.r) (π0 P.k P.r)

theorem **glev_last_eq** (k r : ℕ) : Glev k r (Fin.last r) = B2.UWb k r

theorem **haar_glev_last** (k r : ℕ) : ∃ μ : Measure (Mt k r), B.IsHaarOn (Glev k r (Fin.last r)) μ

theorem **isData_of_place** {Wp Wq : Type} [NormedAddCommGroup Wp] [InnerProductSpace ℂ Wp] [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wp] [FiniteDimensional ℂ Wq] (P : M.Par) (z z' : Fin (2 * P.r + P.k + 1) → ℂ) (π : Mx P.k P.r → (Wp →L[ℂ] Wp)) (π' : Mx P.k P.r → (Wq →L[ℂ] Wq)) (hP : IsPlace P z z' π π') (μ : Measure (Mt P.k P.r)) (hμ : B.IsHaarOn (Glev P.k P.r (Fin.last P.r)) μ) (l0 : ℕ) : ∃ (X : ActualOps P.k P.r z z' π π') (L : R.LevelData (Pr (RadSp P.k P.r z z' π π')) P.r) (st : Fr.Start (Pr (RadSp P.k P.r z z' π π'))), IsData P z z' π π' X μ L st l0

theorem **isPlace_rank_two_one** : IsPlace placeRankTwoOne (z0 2 1) (z0 2 1) (π0 2 1) (π0 2 1)

theorem **isPlace_rank_one_one** : IsPlace product_first_partition (z0 1 1) (z0 1 1) (π0 1 1) (π0 1 1)

theorem **isData_rank_two_one** (l0 : ℕ) : ∃ (X : ActualOps placeRankTwoOne.k placeRankTwoOne.r (z0 2 1) (z0 2 1) (π0 2 1) (π0 2 1)) (μ : Measure (Mt 2 1)) (L : R.LevelData (Pr (RadSp placeRankTwoOne.k placeRankTwoOne.r (z0 2 1) (z0 2 1) (π0 2 1) (π0 2 1))) placeRankTwoOne.r) (st : Fr.Start (Pr (RadSp placeRankTwoOne.k placeRankTwoOne.r (z0 2 1) (z0 2 1) (π0 2 1) (π0 2 1)))), IsData placeRankTwoOne (z0 2 1) (z0 2 1) (π0 2 1) (π0 2 1) X μ L st l0

theorem **isData_rank_one_one** (l0 : ℕ) : ∃ (X : ActualOps product_first_partition.k product_first_partition.r (z0 1 1) (z0 1 1) (π0 1 1) (π0 1 1)) (μ : Measure (Mt 1 1)) (L : R.LevelData (Pr (RadSp product_first_partition.k product_first_partition.r (z0 1 1) (z0 1 1) (π0 1 1) (π0 1 1))) product_first_partition.r) (st : Fr.Start (Pr (RadSp product_first_partition.k product_first_partition.r (z0 1 1) (z0 1 1) (π0 1 1) (π0 1 1)))), IsData product_first_partition (z0 1 1) (z0 1 1) (π0 1 1) (π0 1 1) X μ L st l0

CompanionTheory.HarishChandraModules.NonemptyRepresentations.NonemptyOfAntitone

theorem **differentiable_det** : Differentiable ℂ (fun B : Fin k → Fin k → ℂ => (Matrix.of B).det)

theorem **tpow_mulVec** (b : Matrix (Fin k) (Fin k) ℂ) (I : {a // p a} → Fin k) : (tpow p b *ᵥ fun J => (inc p J).det) I = (Matrix.of fun q s : {a // p a} => b (I s) q.1).det

theorem **nonempty_of_antitone** (ν : Fin k → ℤ) (hν : Antitone ν) : Nonempty (HWRep k ν)

CompanionTheory.HarishChandraModules.NonemptyRepresentations.TypeExists

theorem **fm_eq** (B : Fin k → Fin (2 * r + k + 1) → ℂ) (g : Mx k r) (a b : Fin k) : fm B g a b = ∑ p, ∑ q, (star (B a) ᵥ* Hpk k r) p * g p q * B b q

theorem **fm_add** (B : Fin k → Fin (2 * r + k + 1) → ℂ) (g h : Mx k r) : fm B (g + h) = fm B g + fm B h

theorem **fm_smul** (B : Fin k → Fin (2 * r + k + 1) → ℂ) (c : ℂ) (g : Mx k r) : fm B (c • g) = c • fm B g

theorem **continuous_fm** (B : Fin k → Fin (2 * r + k + 1) → ℂ) : Continuous (fm B)

theorem **fm_one** (hB : IsFrameOf k r [z] B) : fm B 1 = 1

theorem **fm_unitB** (hB : IsFrameOf k r [z] B) (a b : Fin k) : fm B (unitB (Hpk k r) B a b) = Matrix.single a b 1

theorem **mulVec_frame** (hz : inWb k r z) (hT : 0 < Tz k r z) (hB : IsFrameOf k r [z] B) {h : Mx k r} (hh : h ∈ UWset k r z) (b : Fin k) : h *ᵥ B b = ∑ c, fm B h c b • B c

theorem **fm_mul** (hz : inWb k r z) (hT : 0 < Tz k r z) (hB : IsFrameOf k r [z] B) (g : Mx k r) {h : Mx k r} (hh : h ∈ UWset k r z) : fm B (g * h) = fm B g * fm B h

theorem **fm_unitary** (hz : inWb k r z) (hT : 0 < Tz k r z) (hB : IsFrameOf k r [z] B) {h : Mx k r} (hh : h ∈ UWset k r z) : (fm B h)ᴴ * fm B h = 1

theorem **πR_apply** (g : Mx k r) (v : EuclideanSpace ℂ R.I) : πR R B g v = WithLp.toLp 2 (R.ρ (fm B g) *ᵥ v.ofLp)

theorem **πR_isRepW** (hz : inWb k r z) (hT : 0 < Tz k r z) (hB : IsFrameOf k r [z] B) : IsRepW k r z (πR R B)

theorem **πR_unitary** (hz : inWb k r z) (hT : 0 < Tz k r z) (hB : IsFrameOf k r [z] B) : ∀ h ∈ UWset k r z, ∀ v, ‖πR R B h v‖ = ‖v‖

theorem **dRep_πR** (hB : IsFrameOf k r [z] B) (X : Mx k r) (v : EuclideanSpace ℂ R.I) : dRep (πR R B) X v = WithLp.toLp 2 (R.D (fm B X) *ᵥ v.ofLp)

theorem **hpart_sum** {n : ℕ} (H Z : Matrix (Fin n) (Fin n) ℂ) : hpart1 H Z + Complex.I • hpart2 H Z = Z

theorem **ecRep_πR** (hB : IsFrameOf k r [z] B) (Z : Mx k r) (v : EuclideanSpace ℂ R.I) : EcRep (Hpk k r) (πR R B) Z v = WithLp.toLp 2 (R.D (fm B Z) *ᵥ v.ofLp)

theorem **isHW_πR** (hz : inWb k r z) (hT : 0 < Tz k r z) (hB : IsFrameOf k r [z] B) : IsHW (EB (πR R B) B) (fun a => (ν a : ℂ)) (WithLp.toLp 2 R.v0)

CompanionTheory.HarishChandraModules.NonemptyRepresentations.TypeHW

theorem **res_coe** {S : Submodule ℂ W} {π : Mx k r → (W →L[ℂ] W)} {g : Mx k r} (hg : ∀ v ∈ S, π g v ∈ S) (v : S) : (res S π g v : W) = π g v

theorem **res_isRepW** {z : Fin (2 * r + k + 1) → ℂ} {S : Submodule ℂ W} {π : Mx k r → (W →L[ℂ] W)} (hπ : IsRepW k r z π) (hS : ∀ h ∈ UWset k r z, ∀ v ∈ S, π h v ∈ S) : IsRepW k r z (res S π)

theorem **res_capRep** {z : Fin (2 * r + k + 1) → ℂ} {S : Submodule ℂ W} {π : Mx k r → (W →L[ℂ] W)} (hπ : IsRepW k r z π) (hS : ∀ h ∈ UWset k r z, ∀ v ∈ S, π h v ∈ S) {m : ℕ} {Bs : Fin m → Fin (2 * r + k + 1) → ℂ} (hB : IsFrameOf k r [z] Bs) (w : ℂ) (v : S) : ((capRep k r (res S π) Bs w v : S) : W) = capRep k r π Bs w v

theorem **prod_Icc** (n : ℕ) (f : ℕ → ℂ) : ∏ i ∈ Finset.Icc 1 n, f i = ∏ j : Fin n, f ((j : ℕ) + 1)

theorem **exists_type** {z : Fin (2 * r + k + 1) → ℂ} (hz : inWb k r z) (hT : 0 < Tz k r z) {π : Mx k r → (W →L[ℂ] W)} (hπ : IsRepW k r z π) (hu : ∀ h ∈ UWset k r z, ∀ v, ‖π h v‖ = ‖v‖) {B0 : Fin k → Fin (2 * r + k + 1) → ℂ} (hB0 : IsFrameOf k r [z] B0) {ν : Fin k → ℂ} {v0 : W} (hv0 : v0 ≠ 0) (hw : IsHW (EB π B0) ν v0) (u : ℕ → ℝ) (hν : ∀ j : Fin k, (u ((j : ℕ) + 1) : ℂ) = ν j + ((k : ℂ) - 1) / 2 - ((j : ℕ) : ℂ)) : ∃ S : Submodule ℂ W, (∀ h ∈ UWset k r z, ∀ v ∈ S, π h v ∈ S) ∧ IsType k r z (res S π) u

CompanionTheory.RealGroups.BaseChange.ArchimedeanBaseChange.TensorCompletions.SkewFieldFrames.Count1Pf

theorem **CompanionTheory.Rg.BcB.G3.count1_pf** (h1 : Param) : Count1

CompanionTheory.RealGroups.BaseChange.ArchimedeanBaseChange.TensorCompletions.SkewFieldFrames.CountLit

theorem **param_lit** : ∀ (E : Type) [Field E] [NumberField E] [IsCMField E], Unr E → ∀ (n : ℕ) (S : Set (Pl E)), IsSection S → ∀ P ∈ Ell E n, ∀ y : ∀ w : Pl E, GLw n w, (∀ w, (nrmL n w (y w)).val.charpoly = atPl w P) → (∀ᶠ w in cofinite, y w ∈ GlobalTheory.LFun.KGL w n) → ∃ (ι : Type) (_ : Fintype ι) (K L : ι → Type) (_ : ∀ i, Field (K i)) (_ : ∀ i, NumberField (K i)) (_ : ∀ i, Field (L i)) (_ : ∀ i, NumberField (L i)) (_ : ∀ i, Algebra (K i) (L i)) (_ : ∀ i, Algebra.IsQuadraticExtension (K i) (L i)) (x : ∀ i, GlobalTheory.QuadSymbol.FinFam (K i)), Fintype.card ι = nfac P ∧ (∀ i, NumberField.IsTotallyReal (K i)) ∧ (∀ i, NumberField.IsTotallyComplex (L i)) ∧ (∀ i, GlobalTheory.QuadSymbol.AlmostNorm (K i) (L i) (x i)) ∧ ∑ i, Fintype.card (InfinitePlace (K i)) = n * Fintype.card (InfinitePlace E) ∧ Nonempty ({c : TwCl (thE E n) // (c.rep * thE E n c.rep).val.charpoly = P ∧ ∀ w ∈ S, ∃ g, locEl

CompanionTheory.RealGroups.BaseChange.ArchimedeanBaseChange.TensorCompletions.SkewFieldFrames.PBaseFrame

theorem **phi_map** {R S : Type} [CommRing R] [CommRing S] (p : R →+* S) (n : ℕ) : (GlobalTheory.Tw.Phi n R).map p = GlobalTheory.Tw.Phi n S

theorem **CompanionTheory.Rg.BcB.G3.baseFrame_pf** : BaseFrame

CompanionTheory.RealGroups.BaseChange.ArchimedeanBaseChange.TensorCompletions.SkewFieldFrames.PClsEq

theorem **psi_C** (ρ : B →+* Matrix (Fin n) (Fin n) k) (s : k) (a : B) : psi ρ s (C a) = ρ a

theorem **psi_X** (ρ : B →+* Matrix (Fin n) (Fin n) k) (s : k) : psi ρ s X = s • (1 : Matrix (Fin n) (Fin n) k)

theorem **psi_P** (ρ : B →+* Matrix (Fin n) (Fin n) k) (s : k) (u v : B) : psi ρ s (C u + X * C v) = ρ u + s • ρ v

theorem **psi_Q** (ρ : B →+* Matrix (Fin n) (Fin n) k) (s : k) (u v : B) : psi ρ s (C u − X * C v) = ρ u − s • ρ v

theorem **psi_rel** {d : B} {ρ : B →+* Matrix (Fin n) (Fin n) k} {s : k} (hd : ρ d = (s ^ 2) • (1 : Matrix (Fin n) (Fin n) k)) : psi ρ s (X ^ 2 − C d) = 0

theorem **psi_eq** {d : B} {ρ : B →+* Matrix (Fin n) (Fin n) k} {s : k} (hd : ρ d = (s ^ 2) • (1 : Matrix (Fin n) (Fin n) k)) (F G H : B[X]) (h : F = G + (X ^ 2 − C d) * H) : psi ρ s F = psi ρ s G

theorem **GA** {d : B} {ρ : B →+* Matrix (Fin n) (Fin n) k} {s : k} (hd : ρ d = (s ^ 2) • (1 : Matrix (Fin n) (Fin n) k)) (u v a : B) : (ρ u + s • ρ v) * ρ a * (ρ u − s • ρ v) = ρ ((u ^ 2 − d * v ^ 2) * a)

theorem **AG** {d : B} {ρ : B →+* Matrix (Fin n) (Fin n) k} {s : k} (hd : ρ d = (s ^ 2) • (1 : Matrix (Fin n) (Fin n) k)) (u v a : B) : (ρ u − s • ρ v) * ρ a * (ρ u + s • ρ v) = ρ ((u ^ 2 − d * v ^ 2) * a)

theorem **A_comm** (ρ : B →+* Matrix (Fin n) (Fin n) k) (s : k) (u v a : B) : (ρ u − s • ρ v) * ρ a = ρ a * (ρ u − s • ρ v)

theorem **adj_span** {c : k →+* k} {δ : GL (Fin n) k} {s : k} {d : B} {ρ : B →+* Matrix (Fin n) (Fin n) k} (hF : IsFrame c δ s d ρ) (hcs : c s = −s) (u v : B) : δ.val * GlobalTheory.Tw.Phi n k * ((ρ u + s • ρ v).map c)ᵀ = (ρ u − s • ρ v) * (δ.val * GlobalTheory.Tw.Phi n k)

theorem **A_delta_theta** {c : k →+* k} {δ : GL (Fin n) k} {s : k} {d : B} {ρ : B →+* Matrix (Fin n) (Fin n) k} (hF : IsFrame c δ s d ρ) (hcs : c s = −s) (g : GL (Fin n) k) (u v : B) (hg : g.val = ρ u + s • ρ v) : (ρ u − s • ρ v) * (δ.val * (thetaGL n c g).val) = δ.val

theorem **clsEq_pf** : ClsEq

CompanionTheory.RealGroups.BaseChange.ArchimedeanBaseChange.TensorCompletions.SkewFieldFrames.PClsNorm

theorem **thetaGL_val'** {k : Type} [CommRing k] (n : ℕ) (c : k →+* k) (g : GL (Fin n) k) : (thetaGL n c g).val = GlobalTheory.Tw.Phi n k * ((g⁻¹).val.map c)ᵀ * (GlobalTheory.Tw.Phi n k)ᵀ

theorem **phi_mul_transpose_eq_one** {k : Type} [CommRing k] (n : ℕ) : GlobalTheory.Tw.Phi n k * (GlobalTheory.Tw.Phi n k)ᵀ = 1

theorem **units_map_val** {k : Type} [Field k] {n : ℕ} {B : Type} [CommRing B] (ρ : B →+* Matrix (Fin n) (Fin n) k) (t : Bˣ) : (Units.map ρ.toMonoidHom t).val = ρ t

theorem **units_map_inv_val** {k : Type} [Field k] {n : ℕ} {B : Type} [CommRing B] (ρ : B →+* Matrix (Fin n) (Fin n) k) (t : Bˣ) : ((Units.map ρ.toMonoidHom t)⁻¹).val = ρ (↑t⁻¹ : B)

theorem **delta_theta** {k : Type} [Field k] {n : ℕ} {c : k →+* k} {δ : GL (Fin n) k} {s : k} {B : Type} [CommRing B] {d : B} {ρ : B →+* Matrix (Fin n) (Fin n) k} (hF : IsFrame c δ s d ρ) (t : Bˣ) : δ.val * (thetaGL n c (Units.map ρ.toMonoidHom t)).val = ρ (↑t⁻¹ : B) * δ.val

theorem **clsNorm_pf** : ClsNorm

CompanionTheory.RealGroups.BaseChange.ArchimedeanBaseChange.TensorCompletions.SkewFieldFrames.PClsOnto

theorem **only_rho** {k : Type} [Field k] [CharZero k] {n : ℕ} {c : k →+* k} {δ : GL (Fin n) k} {s : k} {B : Type} [CommRing B] {d : B} {ρ : B →+* Matrix (Fin n) (Fin n) k} (hF : IsFrame c δ s d ρ) (hs0 : s ≠ 0) (hcs : c s = −s) (x : GL (Fin n) k) (hx : x * (δ * thetaGL n c δ) = (δ * thetaGL n c δ) * x) (hk : x * δ * thetaGL n c x = δ) : ∃ u, x.val = ρ u

theorem **clsOnto_pf** : ClsOnto

CompanionTheory.RealGroups.BaseChange.ArchimedeanBaseChange.TensorCompletions.SkewFieldFrames.PFactorField

theorem **emb_cj** (φ : E →+* ℂ) : φ.comp (cj E) = (starRingEnd ℂ).comp φ

theorem **conj_eq_inv** {z : ℂ} (h : ‖z‖ = 1) : (starRingEnd ℂ) z = z⁻¹

theorem **root_norm** (h : Good Q) (ψ : AdjoinRoot Q →+* ℂ) : ‖ψ (AdjoinRoot.root Q)‖ = 1

theorem **csr** (h : Good Q) : Polynomial.eval₂ ((AdjoinRoot.of Q).comp (cj E)) (AdjoinRoot.root Q)⁻¹ Q = 0

theorem **sig_of** (h : Good Q) (x : E) : sig Q h (AdjoinRoot.of Q x) = AdjoinRoot.of Q (IsCMField.complexConj E x)

theorem **sig_root** (h : Good Q) : sig Q h (AdjoinRoot.root Q) = (AdjoinRoot.root Q)⁻¹

theorem **emb_sig** (h : Good Q) (ψ : AdjoinRoot Q →+* ℂ) (x : AdjoinRoot Q) : ψ (sig Q h x) = starRingEnd ℂ (ψ x)

theorem **sig_sig** (h : Good Q) (x : AdjoinRoot Q) : sig Q h (sig Q h x) = x

theorem **sig_alg** (h : Good Q) (x : maximalRealSubfield E) : sig Q h (algebraMap (maximalRealSubfield E) (AdjoinRoot Q) x) = algebraMap (maximalRealSubfield E) (AdjoinRoot Q) x

theorem **mem_Kf** (h : Good Q) {x : AdjoinRoot Q} : x ∈ Kf Q h ↔ sig Q h x = x

theorem **totReal** (h : Good Q) : IsTotallyReal (Kf Q h)

theorem **decomp** (h : Good Q) {S : AdjoinRoot Q} (hS : S ≠ 0) (hσS : sig Q h S = −S) (z : AdjoinRoot Q) : ∃ a b : Kf Q h, z = (a : AdjoinRoot Q) + S * (b : AdjoinRoot Q)

theorem **finrank_KL** (h : Good Q) {s : E} (hs : s ≠ 0) (hsc : IsCMField.complexConj E s = −s) : Module.finrank (Kf Q h) (AdjoinRoot Q) = 2

theorem **finrank_Kf** (h : Good Q) {s : E} (hs : s ≠ 0) (hsc : IsCMField.complexConj E s = −s) : Module.finrank (maximalRealSubfield E) (Kf Q h) = Q.natDegree

theorem **CompanionTheory.Rg.BcB.G3.factorField_pf** : FactorField

CompanionTheory.RealGroups.BaseChange.ArchimedeanBaseChange.TensorCompletions.SkewFieldFrames.PInertBase

theorem **exists_real** (E : Type) [Field E] [NumberField E] [IsCMField E] (y : E) (hy : IsCMField.complexConj E y = y) : ∃ p : maximalRealSubfield E, algebraMap (maximalRealSubfield E) E p = y

theorem **decomp** (E : Type) [Field E] [NumberField E] [IsCMField E] (s : E) (hs : s ≠ 0) (hcs : IsCMField.complexConj E s = −s) (x : E) : ∃ p q : maximalRealSubfield E, x = algebraMap _ E p + s * algebraMap _ E q

theorem **inertBase_pf** (h1 : TensorCompl) : InertBase

CompanionTheory.RealGroups.BaseChange.ArchimedeanBaseChange.TensorCompletions.SkewFieldFrames.PLocDict

theorem **frame_transport** {k : Type} [Field k] {n : ℕ} {c : k →+* k} {δ : GL (Fin n) k} {s : k} {B B' : Type} [CommRing B] [CommRing B'] {d : B} {ρ : B →+* Matrix (Fin n) (Fin n) k} (e : B ≃+* B') (h : IsFrame c δ s d ρ) : IsFrame c δ s (e d) (ρ.comp e.symm.toRingHom) where

theorem **decomp** (E : Type) [Field E] [NumberField E] [IsCMField E] (s : E) (hs0 : s ≠ 0) (hsc : IsCMField.complexConj E s = −s) (z : E) : ∃ a b : (maximalRealSubfield E), z = algebraMap (maximalRealSubfield E) E a + s * algebraMap (maximalRealSubfield E) E b

theorem **CompanionTheory.Rg.BcB.G3.locDict_pf** (h1 : ClsNorm) (h2 : ClsOnto) (h3 : ClsEq) (h4 : BaseFrame) (h5 : InertBase) (h6 : SplitSq) (h7 : TensorCompl) : LocDict

CompanionTheory.RealGroups.BaseChange.ArchimedeanBaseChange.TensorCompletions.SkewFieldFrames.PLocGood

theorem **nrmL_mem** {E : Type} [Field E] [NumberField E] [IsCMField E] (hU : Unr E) (n : ℕ) (w : Pl E) (z : GLw n w) (hz : z ∈ GlobalTheory.LFun.KGL w n) : nrmL n w z ∈ GlobalTheory.LFun.KGL w n

theorem **commute_nrmL** {E : Type} [Field E] [NumberField E] [IsCMField E] (hU : Unr E) (n : ℕ) (w : Pl E) (z t : GLw n w) (ht : t ∈ twCent (thL n w) z) : t * nrmL n w z = nrmL n w z * t

theorem **CompanionTheory.Rg.BcB.G3.locGood_pf** : LocGood

CompanionTheory.RealGroups.BaseChange.ArchimedeanBaseChange.TensorCompletions.SkewFieldFrames.PParamL

theorem **twR_equiv** (θ : G →* G) : Equivalence (twR θ) where

theorem **cls_eq_iff** (θ : G →* G) (x y : G) : cls θ x = cls θ y ↔ twR θ x y

theorem **cls_rep** (θ : G →* G) (c : TwCl θ) : cls θ c.rep = c

theorem **locEl_eventually_mem** (n : ℕ) (δ : GL (Fin n) E) : ∀ᶠ w : Pl E in cofinite, locEl n w δ ∈ GlobalTheory.LFun.KGL w n

theorem **tw_symm** {G : Type} [Group G] (θ : G →* G) {x y : G} (h : ∃ g, x = g⁻¹ * y * θ g) : ∃ g, y = g⁻¹ * x * θ g

theorem **tw_trans** {G : Type} [Group G] (θ : G →* G) {x y z : G} (h₁ : ∃ g, x = g⁻¹ * y * θ g) (h₂ : ∃ g, y = g⁻¹ * z * θ g) : ∃ g, x = g⁻¹ * z * θ g

theorem **exists_over** {F K : Type} [Field F] [NumberField F] [Field K] [NumberField K] [Algebra F K] (v : HeightOneSpectrum (𝓞 F)) : ∃ w : HeightOneSpectrum (𝓞 K), w.under (𝓞 F) = v

theorem **finite_over** {F K : Type} [Field F] [NumberField F] [Field K] [NumberField K] [Algebra F K] (v : HeightOneSpectrum (𝓞 F)) : {u : HeightOneSpectrum (𝓞 K) | u.under (𝓞 F) = v}.Finite

theorem **thE_thE** (n : ℕ) (g : GL (Fin n) E) : thE E n (thE E n g) = g

theorem **nrm_tw** (n : ℕ) (z g : GL (Fin n) E) : (g⁻¹ * z * thE E n g) * thE E n (g⁻¹ * z * thE E n g) = g⁻¹ * (z * thE E n z) * g

theorem **locEl_tw** (hU : Unr E) (n : ℕ) (w : Pl E) (z g : GL (Fin n) E) : ∃ h, locEl n w (g⁻¹ ∗ z ∗ thE E n g) = h⁻¹ ∗ locEl n w z ∗ thL n w h

theorem **locEl_rel** (hU : Unr E) (n : ℕ) (w : Pl E) {z z' : GL (Fin n) E} (h : ∃ g, z' = g⁻¹ ∗ z ∗ thE E n g) (yw : GLw n w) : (∃ g, locEl n w z = g⁻¹ ∗ yw ∗ thL n w g) ↔ ∃ g, locEl n w z' = g⁻¹ ∗ yw ∗ thL n w g

theorem **param_main** (h1 : SkewEx) (h2 : Struct) (h3 : QuadField) (h4 : LocDict) (h5 : ClsNorm) (h6 : ClsOnto) (h7 : ClsEq) (h8 : LocGood) : Param

theorem **param_pfL** (h1 : SkewEx) (h2 : Struct) (h3 : QuadField) (h4 : LocDict) (h5 : ClsNorm) (h6 : ClsOnto) (h7 : ClsEq) (h8 : LocGood) : Param

CompanionTheory.RealGroups.BaseChange.ArchimedeanBaseChange.TensorCompletions.SkewFieldFrames.PQuadField

theorem **not_sq_of_real** (E : Type) [Field E] [NumberField E] [IsCMField E] (s : E) (d : maximalRealSubfield E) (hs0 : s ≠ 0) (hs : IsCMField.complexConj E s = -s) (hd : algebraMap (maximalRealSubfield E) E d = s ^ 2) (K : Type) [Field K] [Algebra (maximalRealSubfield E) K] (τ : K →+* ℂ) (t : ℂ) (ht : starRingEnd ℂ t = t) : t ^ 2 ≠ τ (algebraMap (maximalRealSubfield E) K d)

theorem **quadField_pf** : QuadField

CompanionTheory.RealGroups.BaseChange.ArchimedeanBaseChange.TensorCompletions.SkewFieldFrames.PSkewEx

theorem **skewEx_pf** : SkewEx

CompanionTheory.RealGroups.BaseChange.ArchimedeanBaseChange.TensorCompletions.SkewFieldFrames.PSplitSq

theorem **splitSq_pf** (h1 : TensorCompl) : SplitSq

CompanionTheory.RealGroups.BaseChange.ArchimedeanBaseChange.TensorCompletions.SkewFieldFrames.PStruct

theorem **fac_mem** {P : E[X]} (Q : Fac P) : Q.1 ∈ UniqueFactorizationMonoid.normalizedFactors P

theorem **fac_irr** {P : E[X]} (Q : Fac P) : Irreducible Q.1

theorem **fac_monic** {P : E[X]} (Q : Fac P) : Q.1.Monic

theorem **fac_dvd** {P : E[X]} (Q : Fac P) : Q.1 ∣ P

theorem **fac_coprime** (P : E[X]) : Pairwise (Function.onFun IsCoprime fun Q : Fac P => Q.1)

theorem **nodup_fac** {P : E[X]} (hP : P.Separable) : (UniqueFactorizationMonoid.normalizedFactors P).Nodup

theorem **prod_fac** {P : E[X]} (hm : P.Monic) (hP : P.Separable) : ∏ Q : Fac P, Q.1 = P

theorem **card_fac** {P : E[X]} (hP : P.Separable) : Fintype.card (Fac P) = nfac P

theorem **fac_roots** {P : E[X]} (hm : P.Monic) (hr : ∀ τ : E →+* ℂ, ∀ z ∈ (P.map τ).roots, ‖z‖ = 1) (Q : Fac P) : ∀ τ : E →+* ℂ, ∀ z ∈ (Q.1.map τ).roots, ‖z‖ = 1

theorem **exists_rep** (P : E[X]) (z : ∀ Q : Fac P, AdjoinRoot Q.1) : ∃ f : E[X], ∀ Q : Fac P, AdjoinRoot.mk Q.1 f = z Q

theorem **aeval_eq_zero_iff** (hs : N.charpoly.Separable) (f : E[X]) : aeval N f = 0 ↔ ∀ Q : Fac N.charpoly, AdjoinRoot.mk Q.1 f = 0

theorem **aeval_eq_iff** (hs : N.charpoly.Separable) (f g : E[X]) : aeval N f = aeval N g ↔ ∀ Q : Fac N.charpoly, AdjoinRoot.mk Q.1 f = AdjoinRoot.mk Q.1 g

theorem **repOf_spec** (a : ∀ Q, K Q) (Q : Fac N.charpoly) : AdjoinRoot.mk Q.1 (repOf N j a) = j Q (a Q)

theorem **rhoFun_eq** (hs : N.charpoly.Separable) (a : ∀ Q, K Q) (f : E[X]) (hf : ∀ Q, AdjoinRoot.mk Q.1 f = j Q (a Q)) : rhoFun N j a = aeval N f

theorem **rho_apply** (a : ∀ Q, K Q) : rho N j hs hj a = aeval N (repOf N j a)

theorem **rho_injective** : Function.Injective (rho N j hs hj)

theorem **rho_comm** (a : ∀ Q, K Q) : rho N j hs hj a ∗ N = N ∗ rho N j hs hj a

theorem **rho_span** (s : E) (hspan : ∀ (Q : Fac N.charpoly) (z : AdjoinRoot Q.1), ∃ (a b : K Q), z = j Q a + AdjoinRoot.of Q.1 s ∗ j Q b) (x : Matrix (Fin n) (Fin n) E) (hx : x ∗ N = N ∗ x) : ∃ a b, x = rho N j hs hj a + s • (rho N j hs hj b : Matrix (Fin n) (Fin n) E)

theorem **rho_adj** (c : E →+* E) (hc : ∀ x, c (c x) = x) (δ : GL (Fin n) E) (hN : N = (δ ∗ thetaGL n c δ).val) (σ : ∀ Q : Fac N.charpoly, AdjoinRoot Q.1 →+* AdjoinRoot Q.1) (hσ : ∀ Q x, σ Q (AdjoinRoot.of Q.1 x) = AdjoinRoot.of Q.1 (c x)) (hσr : ∀ Q, σ Q (AdjoinRoot.root Q.1) ∗ AdjoinRoot.root Q.1 = 1) (hσj : ∀ Q a, σ Q (j Q a) = j Q a) (a : ∀ Q, K Q) : rho N j hs hj a ∗ (δ.val ∗ GlobalTheory.Tw.Phi n E) = δ.val ∗ GlobalTheory.Tw.Phi n E ∗ ((rho N j hs hj a).map c)ᵀ

theorem **CompanionTheory.Rg.BcB.G3.struct_pf** (h1 : FactorField) : Struct

CompanionTheory.RealGroups.BaseChange.ArchimedeanBaseChange.TensorCompletions.SkewFieldFrames.PTensorCompl

theorem **finite_over** : {w : HeightOneSpectrum (𝒪 K) | w.under (𝒪 F) = v}.Finite

theorem **phi_apply** (y : v.adicCompletion F ⊗[F] K) (u : {u : HeightOneSpectrum (𝒪 K) // u.under (𝒪 F) = v}) : phi v y u = GlobalTheory.LFun.toCompl u.2 y

theorem **map_extZ_self** (y : v.adicCompletion F ⊗[F] K) : Algebra.TensorProduct.map (Formalism.MainNotions.GlobalTheoryTheory.GenusMassVolumes.evalV F v) (AlgHom.id F K) (extZ v y) = y

theorem **map_extZ_ne** {v' : HeightOneSpectrum (𝒪 F)} (h : v' ≠ v) (y : v.adicCompletion F ⊗[F] K) : Algebra.TensorProduct.map (Formalism.MainNotions.GlobalTheoryTheory.GenusMassVolumes.evalV F v') (AlgHom.id F K) (extZ v y) = 0

theorem **phi_injective** : Function.Injective (phi (K

theorem **phi_surjective** : Function.Surjective (phi (K

theorem **CompanionTheory.Rg.BcB.G3.tensorCompl_pf** : TensorCompl

CompanionTheory.RealGroups.DiscreteSeries.DiscreteSeriesClassification.CoverExistsOf

theorem **upqExists_of** (hE : DSExists) : GlobalTheory.Published.DiscreteSeriesUpqExists

theorem **exists_of** (hE : DSExists) : GlobalTheory.Published.DiscreteSeriesExists

theorem **coverExists_of** (hE : DSExists) : GlobalTheory.Published.DiscreteSeriesCoverExists

theorem **classUsed_of** (hE : DSExists) (hK : KTypesAbove) (hI : DSInfChar) : ClassUsed

CompanionTheory.RealGroups.DiscreteSeries.DiscreteSeriesClassification.Development

theorem **model_of** (hL : ExplicitFunctionSquareIntegrable) (hK : Key) (hH : Hilb) {n : ℕ} (ε : Fin n → Bool) (lam : Fin n → ℝ) (hl : Formalism.QuantumDeterminants.IsHCParam lam) : ∃ (H : Type) (_ : NormedAddCommGroup H) (_ : InnerProductSpace ℂ H) (σ : Matrix (Fin n) (Fin n) ℂ → (H →L[ℂ] H)), Formalism.MainNotions.S.IsDS ε lam σ ∧ ∀ (η : Fin n → ℝ) (S : Submodule ℂ H), Formalism.MainNotions.S.IsKTypeOf ε σ η S → InCone (lowK ε lam) η

theorem **dsExists_of** (hL : ExplicitFunctionSquareIntegrable) (hK : Key) (hH : Hilb) : GlobalTheory.Dec.DSExists

theorem **kTypesAbove_of** (hL : ExplicitFunctionSquareIntegrable) (hK : Key) (hH : Hilb) : GlobalTheory.Dec.KTypesAbove

CompanionTheory.RealGroups.DiscreteSeries.DiscreteSeriesClassification.DiscreteSeriesExhaustion.DsExhaust

theorem **dsExhaust** : GlobalTheory.Dec.DSExhaust

theorem **discreteSeriesClassification** : GlobalTheory.Published.DiscreteSeriesClassification

theorem **discreteSeriesCover** : GlobalTheory.Published.DiscreteSeriesCover

CompanionTheory.RealGroups.DiscreteSeries.DiscreteSeriesClassification.DiscreteSeriesExhaustion.FarRegularity.FarRegularExhaustion

theorem **far_regular_exhaustion** : CompanionTheory.RgExh.FarReg

CompanionTheory.RealGroups.DiscreteSeries.DiscreteSeriesClassification.DiscreteSeriesExhaustion.HeckeAlgebraActions.HeckeDuality.DualModuleFacts

theorem **dual_module_facts** : DualFacts

CompanionTheory.RealGroups.DiscreteSeries.DiscreteSeriesClassification.DiscreteSeriesExhaustion.HeckeAlgebraActions.HeckeDuality.GlRelTau

theorem **glRel_tau** (X : GMod n) : GlRel (fun a b => -X.E b a)

CompanionTheory.RealGroups.DiscreteSeries.DiscreteSeriesClassification.DiscreteSeriesExhaustion.HeckeAlgebraActions.HeckeDuality.HasCharOfEquiv

theorem **omD_eq_om_tau** (X : GMod n) : omD X = om (tauMod X)

theorem **backSub_eq** (X : GMod n) (z : ℂ) : backSub X z = stepSub (tauMod X) (-z)

theorem **capelliPoly_neg** (χ : Fin n → ℂ) (Y : ℂ) : Formalism.QuantumDeterminants.capP (-χ) Y = (-1 : ℂ) ^ n * Formalism.QuantumDeterminants.capP χ (-Y)

theorem **hasChar_tau** (hT : QdetTau) {X : GMod n} {χ : Fin n → ℂ} (h : HasChar X χ) : HasChar (tauMod X) (-χ)

theorem **hasChar_of_equiv** {Y₁ Y₂ : GMod n} (e : Y₁.V ≃ₗ[ℂ] Y₂.V) (he : ∀ a b x, e (Y₁.E a b x) = Y₂.E a b (e x)) {μ : Fin n → ℂ} (h : HasChar Y₂ μ) : HasChar Y₁ μ

theorem **dualfacts_of** (hT : QdetTau) (hC : CayleyHam) (hS : StepChar) (hN : NonVan) : DualFacts

CompanionTheory.RealGroups.DiscreteSeries.DiscreteSeriesClassification.DiscreteSeriesExhaustion.HeckeAlgebraActions.HeckeUnitTensors.CommuteLeftRightCa

theorem **mem_of_eig** {T : Module.End ℂ (Fin (m + 1) → X.V)} {μ : ℂ} {f : Fin (m + 1) → X.V} (h : T f = μ • f) : f ∈ T.maxGenEigenspace μ

theorem **Pr_mem** (hS : ScalChar X χ) (f : Fin (m + 1) → X.V) : Pr X χ s f ∈ stepSub X (χ s)

theorem **left_casimir_apply** (H : Fin (m + 1) → Fin (m + 1) → X.V) (c : Fin (m + 1)) : y1 X H c = om X (H c)

theorem **right_casimir_apply** (H : Fin (m + 1) → Fin (m + 1) → X.V) (a : Fin (m + 1)) : y2 X H a = -∑ c, tensE X a c (H c)

theorem **eta_apply** (v : X.V) (c : Fin (m + 1)) : eta X v c = Pi.single c v

theorem **psi_apply** (H : Fin (m + 1) → Fin (m + 1) → X.V) : psi X H = ∑ c, H c c

theorem **commute_left_right_casimir** : Commute (y1 X) (y2 X)

theorem **sum_tensE_single** (v : X.V) (a : Fin (m + 1)) : ∑ c, tensE X a c (Pi.single c v) = om X (Pi.single a v) + ((m + 1 : ℕ) : ℂ) • (Pi.single a v : Fin (m + 1) → X.V)

theorem **right_casimir_eta** (v : X.V) : y2 X (eta X v) = -(y1 X (eta X v) + ((m + 1 : ℕ) : ℂ) • eta X v)

theorem **right_casimir_add_eta_psi** (H : Fin (m + 1) → Fin (m + 1) → X.V) (a d : Fin (m + 1)) : (y2 X H + eta X (psi X H)) a d = -∑ c, X.E a c (H c d)

theorem **liftL_apply** (T : Module.End ℂ (Fin (m + 1) → X.V)) (H) (a d : Fin (m + 1)) : liftL T H a d = T (fun c => H c d) a

theorem **liftR_smul_one** (μ : ℂ) : liftR (X

theorem **omegaP_eq** : liftR (X

CompanionTheory.RealGroups.DiscreteSeries.DiscreteSeriesClassification.DiscreteSeriesExhaustion.HeckeAlgebraActions.HeckeUnitTensors.CompLeftSmulOne

theorem **qdet_scal** (hS : ScalChar X χ) (u : ℂ) : qdet X.E u = capP χ (u - cn (m + 1)) • (1 : Module.End ℂ X.V)

theorem **capelliPoly_root** : capP χ (-(χ s)) = 0

theorem **capelliPoly_eq_mul_prod_erase** (u : ℂ) : capP χ u = (u + χ s) * ∏ t ∈ univ.erase s, (u + χ t)

theorem **qdet_omg** (hS : ScalChar X χ) : qdet X.E (omg χ s) = 0

theorem **compLeft_zero'** : (0 : Module.End ℂ X.V).compLeft (Fin (m + 1)) = 0

theorem **ct_omg** (hS : ScalChar X χ) : qdet (tensE X) (omg χ s) = minorOp X (omg χ s - 1)

theorem **om_ct** (hS : ScalChar X χ) (f : Fin (m + 1) → X.V) : om X (qdet (tensE X) (omg χ s) f) = (χ s - cn (m + 1)) • qdet (tensE X) (omg χ s) f

theorem **compLeft_smul_one** (c : ℂ) (g : Fin (m + 1) → X.V) : (c • (1 : Module.End ℂ X.V)).compLeft (Fin (m + 1)) g = c • g

theorem **ct_on_step** (hS : ScalChar X χ) (f : Fin (m + 1) → X.V) (hf : om X f = (χ s - cn (m + 1)) • f) : qdet (tensE X) (omg χ s) f = kap χ s • f

theorem **kap_ne** (hsimp : ∀ r, r ≠ s → χ r ≠ χ s) : kap χ s ≠ 0

theorem **Pr_step** (hS : ScalChar X χ) (hsimp : ∀ r, r ≠ s → χ r ≠ χ s) (f : Fin (m + 1) → X.V) (hf : om X f = (χ s - cn (m + 1)) • f) : Pr X χ s f = f

theorem **om_Pr** (hS : ScalChar X χ) (f : Fin (m + 1) → X.V) : om X (Pr X χ s f) = (χ s - cn (m + 1)) • Pr X χ s f

theorem **Pr_om** (f : Fin (m + 1) → X.V) : Pr X χ s (om X f) = om X (Pr X χ s f)

theorem **Pr_tensE** (a b : Fin (m + 1)) (f : Fin (m + 1) → X.V) : Pr X χ s (tensE X a b f) = tensE X a b (Pr X χ s f)

theorem **ptr_Pr** (hS : ScalChar X χ) (v : X.V) : ∑ c : Fin (m + 1), Pr X χ s (Pi.single c v) c = ((kap χ s)⁻¹ * -capP χ (-χ s - 1)) • v

theorem **capelliPoly_ne** (hfree : ∀ r, χ r ≠ χ s + 1) : capP χ (-χ s - 1) ≠ 0

CompanionTheory.RealGroups.DiscreteSeries.DiscreteSeriesClassification.DiscreteSeriesExhaustion.HeckeAlgebraActions.HeckeUnitTensors.ContOfFnProd

private theorem **continuous_apply_list_prod_ofFn_add_smul** : ∀ (m : ℕ) (ℓ : Module.Dual ℂ W) (A B : Fin m → Module.End ℂ W) (f : W), Continuous (fun u : ℂ => ℓ ((List.ofFn fun k => A k + u • B k).prod f))

theorem **cont_ofFn_prod** (ℓ : Module.Dual ℂ W) : ∀ (m : ℕ) (A B : Fin m → Module.End ℂ W) (f : W), Continuous (fun u : ℂ => ℓ ((List.ofFn fun k => A k + u • B k).prod f))

theorem **Tm_affine** {n : ℕ} (E : Fin n → Fin n → Module.End ℂ W) (u : ℂ) (k : ℂ) (a b : Fin n) : Tm E (u - k) a b = ((if a = b then algebraMap ℂ (Module.End ℂ W) (-k) else 0) + E a b) + u • (if a = b then (1 : Module.End ℂ W) else 0)

theorem **cont_qdet** {n : ℕ} (E : Fin n → Fin n → Module.End ℂ W) (ℓ : Module.Dual ℂ W) (f : W) : Continuous (fun u : ℂ => ℓ (qdet E u f))

theorem **eq_zero_of_off** {g : ℂ → ℂ} (hg : Continuous g) (u₀ : ℂ) (h : ∀ u, u ≠ u₀ → g u = 0) : g u₀ = 0

CompanionTheory.RealGroups.DiscreteSeries.DiscreteSeriesClassification.DiscreteSeriesExhaustion.HeckeAlgebraActions.HeckeUnitTensors.OmegaProjectionSca

theorem **toS_ofS** (g : ↥(stepSub X (χ s))) : toS X χ s (ofS X χ s g) = g

theorem **toS_add** (g g' : (step X (χ s)).V) : toS X χ s (g + g') = toS X χ s g + toS X χ s g'

theorem **toS_smul** (r : ℂ) (g : (step X (χ s)).V) : toS X χ s (r • g) = r • toS X χ s g

theorem **toS_sub** (g g' : (step X (χ s)).V) : toS X χ s (g - g') = toS X χ s g - toS X χ s g'

theorem **toS_neg** (g : (step X (χ s)).V) : toS X χ s (-g) = -toS X χ s g

theorem **toS_sum** (g : Fin (m + 1) → (step X (χ s)).V) : toS X χ s (∑ c, g c) = ∑ c, toS X χ s (g c)

theorem **toS_E** (a b : Fin (m + 1)) (g : (step X (χ s)).V) : ((toS X χ s ((step X (χ s)).E a b g) : ↥(stepSub X (χ s))) : Fin (m + 1) → X.V) = tensE X a b (toS X χ s g : Fin (m + 1) → X.V)

theorem **coeY_injective** : Function.Injective (coeY (X

theorem **coeY_add** (G G') : coeY (X

theorem **coeY_smul** (r : ℂ) (G) : coeY (X

theorem **coeY_sub** (G G') : coeY (X

theorem **coe_omD** (G : Fin (m + 1) → (step X (χ s)).V) : coeY (omD (step X (χ s)) G) = y2 X (coeY G)

theorem **coe_pow** (β : ℂ) (k : ℕ) (G : Fin (m + 1) → (step X (χ s)).V) : coeY (((omD (step X (χ s)) - β • 1) ^ k) G) = ((y2 X - β • 1) ^ k) (coeY G)

theorem **coe_projection_scalar_character** (hS : ScalChar X χ) (v : X.V) : coeY (phi0 χ s hS v) = fun c => Pr X χ s (Pi.single c v)

theorem **omega_projection_scalar_character** (hS : ScalChar X χ) (v : X.V) : omD (step X (χ s)) (phi0 χ s hS v) = (-(χ s + 1) - cn (m + 1)) • phi0 χ s hS v

theorem **tensVd_apply** {M : Type*} [AddCommGroup M] [Module ℂ M] {n : ℕ} (E : Fin n → Fin n → Module.End ℂ M) (a b : Fin n) (f : Fin n → M) (d : Fin n) : ReciprocityTheory.ChartRestriction.DS.Lap.Capelli.tensVd E a b f d = E a b (f d) - if d = b then f a else 0

theorem **tensE_single** (a b d : Fin (m + 1)) (v : X.V) : tensE X a b (Pi.single d v) = Pi.single d (X.E a b v) + if b = d then Pi.single a v else 0

CompanionTheory.RealGroups.DiscreteSeries.DiscreteSeriesClassification.DiscreteSeriesExhaustion.HeckeAlgebraActions.HeckeUnitTensors.RightCasimirMemSub

theorem **mem_of_eig'** {M : Type*} [AddCommGroup M] [Module ℂ M] {T : Module.End ℂ M} {μ : ℂ} {f : M} (h : T f = μ • f) : f ∈ T.maxGenEigenspace μ

theorem **mem_Isub** (k : ℕ) (v : X.V) : (y1 X ^ k) (eta X v) ∈ Isub X

theorem **eta_mem_Isub** (v : X.V) : eta X v ∈ Isub X

theorem **right_casimir_mem_submodule** {H} (h : H ∈ Isub X) : y2 X H ∈ Isub X

theorem **Om'_mem_Isub** {H} (h : H ∈ Isub X) : Om' X H ∈ Isub X

theorem **Om'_pow_mem_Isub** (k : ℕ) {H} (h : H ∈ Isub X) : (Om' X ^ k) H ∈ Isub X

theorem **aeval_mem_Isub** (p : ℂ[X]) {H} (h : H ∈ Isub X) : aeval (Om' X) p H ∈ Isub X

theorem **diff_mem_Isub** (β : ℂ) (k : ℕ) (H) : ((Om' X - β • 1) ^ k) H - ((y2 X - β • 1) ^ k) H ∈ Isub X

theorem **left_casimir_pow_apply** (k : ℕ) (H : Fin (m + 1) → Fin (m + 1) → X.V) (c : Fin (m + 1)) : (y1 X ^ k) H c = (om X ^ k) (H c)

theorem **Pr_om_pow** (hS : ScalChar X χ) (k : ℕ) (f : Fin (m + 1) → X.V) : Pr X χ s ((om X ^ k) f) = (χ s - cn (m + 1)) ^ k • Pr X χ s f

theorem **Pr_Isub** (hS : ScalChar X χ) {H} (h : H ∈ Isub X) : ∃ v : X.V, ∀ c, Pr X χ s (H c) = Pr X χ s (Pi.single c v)

CompanionTheory.RealGroups.DiscreteSeries.DiscreteSeriesClassification.DiscreteSeriesExhaustion.HeckeAlgebraActions.HeckeUnits.Unitarity

theorem **Om'_CH** (K : ℕ) (hK : ((List.ofFn fun t : Fin (m + 1) => omD X − (−χ t − cn (m + 1)) • (1 : Module.End ℂ (Fin (m + 1) → X.V))).prod) ^ K = 0) : ((List.ofFn fun t : Fin (m + 1) => Om' X − (−χ t − cn (m + 1)) • (1 : Module.End ℂ (Fin (m + 1) → Fin (m + 1) → X.V))).prod) ^ K = 0

theorem **aeval_CH** (K : ℕ) (γ : Fin (m + 1) → ℂ) (T : Module.End ℂ (Fin (m + 1) → Fin (m + 1) → X.V)) : Polynomial.aeval T (((List.ofFn fun t : Fin (m + 1) => Polynomial.X − Polynomial.C (γ t)).prod) ^ K) = ((List.ofFn fun t : Fin (m + 1) => T − γ t • (1 : Module.End ℂ _)).prod) ^ K

theorem **aeval_lin** (k : ℕ) (β : ℂ) (T : Module.End ℂ (Fin (m + 1) → Fin (m + 1) → X.V)) : Polynomial.aeval T ((Polynomial.X − Polynomial.C β) ^ k) = (T − β • (1 : Module.End ℂ _)) ^ k

theorem **cop** (γ : Fin (m + 1) → ℂ) (β : ℂ) (h : ∀ t, γ t ≠ β) (K k : ℕ) : IsCoprime (((List.ofFn fun t : Fin (m + 1) => Polynomial.X − Polynomial.C (γ t)).prod) ^ K) ((Polynomial.X − Polynomial.C β) ^ k)

theorem **coeY_mem_Isub** (hD : ∃ K : ℕ, ((List.ofFn fun t : Fin (m + 1) => omD X − (−χ t − cn (m + 1)) • (1 : Module.End ℂ (Fin (m + 1) → X.V))).prod) ^ K = 0) (hfree : ∀ r, χ r ≠ χ s + 1) (G : Fin (m + 1) → (step X (χ s)).V) (hG : G ∈ backSub (step X (χ s)) (χ s + 1)) : coeY G ∈ Isub X

theorem **phiB_inj** (hS : ScalChar X χ) (hsimp : ∀ r, r ≠ s → χ r ≠ χ s) (hfree : ∀ r, χ r ≠ χ s + 1) : Function.Injective (phiB χ s hS)

theorem **phiB_surj** (hS : ScalChar X χ) (hsimp : ∀ r, r ≠ s → χ r ≠ χ s) (hfree : ∀ r, χ r ≠ χ s + 1) (hom : ∀ f ∈ stepSub X (χ s), om X f = (χ s − cn (m + 1)) • f) (hD : ∃ K : ℕ, ((List.ofFn fun t : Fin (m + 1) => omD X − (−χ t − cn (m + 1)) • (1 : Module.End ℂ (Fin (m + 1) → X.V))).prod) ^ K = 0) : Function.Surjective (phiB χ s hS)

theorem **coeY_tensVd** (G : Fin (m + 1) → (step X (χ s)).V) (a b : Fin (m + 1)) : coeY (X

theorem **phiB_E** (hS : ScalChar X χ) (hB : BackStable) (a b : Fin (m + 1)) (v : X.V) : phiBk χ s hS hB (X.E a b v) = (back (step X (χ s)) (χ s + 1) (hB (m + 1) _ _)).E a b (phiBk χ s hS hB v)

theorem **unit_of_dualFacts** (hDF : DualFacts) : Unit

theorem **unitarity** : CompanionTheory.RgExh.Hk.Unit

CompanionTheory.RealGroups.DiscreteSeries.DiscreteSeriesClassification.DiscreteSeriesExhaustion.HeckeAlgebraActions.InfinitesimalCharacterShift

theorem **unitarity** : Unit

theorem **reflection_symmetry** : Reflect

theorem **forward_simple_module** : FwdSimple

theorem **infinitesimal_character_shift** : CompanionTheory.RgExh.StepChar

theorem **hecke_algebra_exhaustion** : CompanionTheory.RgExh.Hecke

CompanionTheory.RealGroups.DiscreteSeries.DiscreteSeriesClassification.DiscreteSeriesExhaustion.HeckeAlgebraActions.ScalarCharacters.ScalCharOfSimple

private theorem **eq_zero_of_pow_eq_zero_of_commute_of_isSimple** {X : GMod n} (hS : IsSimple X) (N : Module.End ℂ X.V) (hc : ∀ a b, Commute (X.E a b) N) (k : ℕ) (hk : N ^ k = 0) : N = 0

theorem **scalChar_of_simple** (hQ : QdetEq) {X : GMod n} {χ : Fin n → ℂ} (hS : IsSimple X) (hH : HasChar X χ) : ScalChar X χ

theorem **nontrivial_step** (hV : NonVan) {X : GMod n} {χ : Fin n → ℂ} {s : Fin n} (hH : HasChar X χ) (hN : Nontrivial X.V) (hf : ∀ r, χ r ≠ χ s + 1) : Nontrivial (step X (χ s)).V

theorem **backPath** (hC : StepChar) (hV : NonVan) (hNS : NonScal) (hB : Back) : ∀ (ss : List (Fin n)) (X : GMod n) (χ : Fin n → ℂ), GoodPath χ ss → HasChar X χ → Nontrivial X.V → ScalChar (path X χ ss) (pathEnd χ ss) → ScalChar X χ ∧ SimplePath χ ss

theorem **fwdPath** (hQ : QdetEq) (hC : StepChar) (hOS : OmScal) (hF : FwdSimple) (hR : Reflect) : ∀ (ss : List (Fin n)) (X : GMod n) (χ : Fin n → ℂ), GoodPath χ ss → SimplePath χ ss → IsSimple X → HasChar X χ → IsSimple (path X χ ss) ∧ ∀ X' : GMod n, IsSimple X' → HasChar X' χ → Iso (path X χ ss) (path X' χ ss) → Iso X X'

theorem **hecke_of'** (hQ : QdetEq) (hC : StepChar) (_hCH : CayleyHam) (hV : NonVan) (hNS : NonScal) (hB : Back) (hOS : OmScal) (hF : FwdSimple) (hR : Reflect) : Hecke

CompanionTheory.RealGroups.DiscreteSeries.DiscreteSeriesClassification.DiscreteSeriesExhaustion.HeckeAlgebraActions.SimpleBackwardTransfer.FwdSimpleOfb

theorem **fsb_pow_map** (f : V →ₗ[ℂ] W) (T : Module.End ℂ V) (T' : Module.End ℂ W) (h : ∀ v, f (T v) = T' (f v)) : ∀ (k : ℕ) (v : V), f ((T ^ k) v) = (T' ^ k) (f v)

theorem **fsb_ofFn_map** (f : V →ₗ[ℂ] W) : ∀ (t : ℕ) (g : Fin t → Module.End ℂ V) (g' : Fin t → Module.End ℂ W), (∀ k v, f (g k v) = g' k (f v)) → ∀ v, f ((List.ofFn g).prod v) = (List.ofFn g').prod (f v)

theorem **fsb_qdet_map** {E : Fin n → Fin n → Module.End ℂ V} {E' : Fin n → Fin n → Module.End ℂ W} (f : V →ₗ[ℂ] W) (hf : ∀ a b v, f (E a b v) = E' a b (f v)) (x : ℂ) (v : V) : f (qdet E x v) = qdet E' x (f v)

theorem **fsb_glRel_surj** {E : Fin n → Fin n → Module.End ℂ V} {E' : Fin n → Fin n → Module.End ℂ W} (f : V →ₗ[ℂ] W) (hs : Function.Surjective f) (hf : ∀ a b v, f (E a b v) = E' a b (f v)) (hE : GlRel E) : GlRel E'

theorem **fsb_hasChar_inj** (M1 M2 : GMod n) (f : M1.V →ₗ[ℂ] M2.V) (hf : ∀ a b v, f (M1.E a b v) = M2.E a b (f v)) (hi : Function.Injective f) (χ : Fin n → ℂ) (h : HasChar M2 χ) : HasChar M1 χ

theorem **fsb_hasChar_surj** (M1 M2 : GMod n) (f : M1.V →ₗ[ℂ] M2.V) (hf : ∀ a b v, f (M1.E a b v) = M2.E a b (f v)) (hs : Function.Surjective f) (χ : Fin n → ℂ) (h : HasChar M1 χ) : HasChar M2 χ

theorem **fsb_omD_apply** (M : GMod n) (g : Fin n → M.V) (a : Fin n) : omD M g a = −∑ c, M.E a c (g c)

theorem **fsb_tensVd_apply** (M : GMod n) (a b : Fin n) (g : Fin n → M.V) (d : Fin n) : tensVd M.E a b g d = M.E a b (g d) − if d = b then g a else 0

theorem **fsb_omD_map** (M1 M2 : GMod n) (f : M1.V →ₗ[ℂ] M2.V) (hf : ∀ a b v, f (M1.E a b v) = M2.E a b (f v)) (g : Fin n → M1.V) : f.compLeft (Fin n) (omD M1 g) = omD M2 (f.compLeft (Fin n) g)

theorem **fsb_aeval_map** (M1 M2 : GMod n) (f : M1.V →ₗ[ℂ] M2.V) (hf : ∀ a b v, f (M1.E a b v) = M2.E a b (f v)) (p : Polynomial ℂ) (g : Fin n → M1.V) : f.compLeft (Fin n) (Polynomial.aeval (omD M1) p g) = Polynomial.aeval (omD M2) p (f.compLeft (Fin n) g)

theorem **fsb_gen_map** (M1 M2 : GMod n) (f : M1.V →ₗ[ℂ] M2.V) (hf : ∀ a b v, f (M1.E a b v) = M2.E a b (f v)) (μ : ℂ) (g : Fin n → M1.V) (hg : g ∈ (omD M1).maxGenEigenspace μ) : f.compLeft (Fin n) g ∈ (omD M2).maxGenEigenspace μ

private theorem **exists_mem_backSub_compLeft_eq_of_surjective** (hD : DualFacts) (S Q : GMod n) (χ' : Fin n → ℂ) (s : Fin n) (z : ℂ) (hz : χ' s = z) (h1 : ∀ t, t ≠ s → χ' t ≠ z) (hS : HasChar S χ') (f : S.V →ₗ[ℂ] Q.V) (hs : Function.Surjective f) (hf : ∀ a b v, f (S.E a b v) = Q.E a b (f v)) (y : Fin n → Q.V) (hy : y ∈ backSub Q z) : ∃ x ∈ backSub S z, f.compLeft (Fin n) x = y

private theorem **apply_mem_of_mem_backSub_of_iso_simple** (hD : DualFacts) (X S : GMod n) (χ' : Fin n → ℂ) (s : Fin n) (z : ℂ) (hz : χ' s = z) (hX : IsSimple X) (hS : HasChar S χ') (h2 : ∀ r, χ' r ≠ z − 1) (hB : ∀ a b, ∀ x ∈ backSub S z, (tensD S).E a b x ∈ backSub S z) (hI : Iso (back S z hB) X) (Z : Submodule ℂ S.V) (hZ : ∀ a b, ∀ x ∈ Z, S.E a b x ∈ Z) (hZb : Z ≠ ⊥) : ∀ g ∈ backSub S z, ∀ d, g d ∈ Z

theorem **fsb_simple_of_back** (hD : DualFacts) (X S : GMod n) (χ' : Fin n → ℂ) (s : Fin n) (z : ℂ) (hz : χ' s = z) (hX : IsSimple X) (hS : HasChar S χ') (h1 : ∀ t, t ≠ s → χ' t ≠ z) (h2 : ∀ r, χ' r ≠ z − 1) (hB : ∀ a b, ∀ x ∈ backSub S z, (tensD S).E a b x ∈ backSub S z) (hI : Iso (back S z hB) X) : IsSimple S

theorem **forward_simple_of_tensor_facts** : QdetTens → BackStable → DualFacts → Unit → FwdSimple

CompanionTheory.RealGroups.DiscreteSeries.DiscreteSeriesClassification.DiscreteSeriesExhaustion.HeckeAlgebraActions.TensorReflections.PiIsoTensVd

theorem **iso_symm'** {X X' : GMod n} (h : Iso X X') : Iso X' X

theorem **iso_trans'** {X X' X'' : GMod n} (h : Iso X X') (h' : Iso X' X'') : Iso X X''

theorem **map_maxGen** {M M' : Type*} [AddCommGroup M] [Module ℂ M] [AddCommGroup M'] [Module ℂ M'] (φ : M ≃ₗ[ℂ] M') (T : Module.End ℂ M) (T' : Module.End ℂ M') (hφ : ∀ x, φ (T x) = T' (φ x)) (μ : ℂ) : (T.maxGenEigenspace μ).map (φ : M →ₗ[ℂ] M') = T'.maxGenEigenspace μ

theorem **iso_sub'** {X X' : GMod n} (φ : X.V ≃ₗ[ℂ] X'.V) (hφ : ∀ a b x, φ (X.E a b x) = X'.E a b (φ x)) (S : Submodule ℂ X.V) (S' : Submodule ℂ X'.V) (hS : S.map (φ : X.V →ₗ[ℂ] X'.V) = S') (h : ∀ a b, ∀ x ∈ S, X.E a b x ∈ S) (h' : ∀ a b, ∀ x ∈ S', X'.E a b x ∈ S') : Iso (X.sub S h) (X'.sub S' h')

theorem **piIso_tensVd** {X X' : GMod n} (e : X.V ≃ₗ[ℂ] X'.V) (he : ∀ a b x, e (X.E a b x) = X'.E a b (e x)) (a b : Fin n) (f : Fin n → X.V) : piIso e (tensVd X.E a b f) = tensVd X'.E a b (piIso e f)

theorem **piIso_eeVd** {X X' : GMod n} (e : X.V ≃ₗ[ℂ] X'.V) (he : ∀ a b x, e (X.E a b x) = X'.E a b (e x)) (f : Fin n → X.V) : piIso e (eeVd X.E f) = eeVd X'.E (piIso e f)

theorem **iso_back'** {Y Y' : GMod n} (hY : Iso Y Y') (z : ℂ) (h : ∀ a b, ∀ x ∈ backSub Y z, (tensD Y).E a b x ∈ backSub Y z) (h' : ∀ a b, ∀ x ∈ backSub Y' z, (tensD Y').E a b x ∈ backSub Y' z) : Iso (back Y z h) (back Y' z h')

theorem **reflect_of'** : QdetTens → OmScal → BackStable → DualFacts → Unit → Reflect

CompanionTheory.RealGroups.DiscreteSeries.DiscreteSeriesClassification.DiscreteSeriesExhaustion.IsSimpleTriv

theorem **tensE_apply** (X : GMod n) (a b : Fin n) (u : Fin n → X.V) (i : Fin n) : tensE X a b u i = X.E a b (u i) + if i = a then u b else 0

theorem **comm_apply** (X : GMod n) (a b c d : Fin n) (x : X.V) : X.E a b (X.E c d x) = X.E c d (X.E a b x) + (if b = c then X.E a d x else 0) − (if d = a then X.E c b x else 0)

theorem **glRel_tensE** (X : GMod n) : GlRel (tensE X)

theorem **om_apply** (X : GMod n) (u : Fin n → X.V) (b : Fin n) : om X u b = ∑ a, X.E a b (u a)

theorem **commute_om** (X : GMod n) (c d : Fin n) : Commute (tensE X c d) (om X)

theorem **isSimple_triv** : IsSimple (triv n)

CompanionTheory.RealGroups.DiscreteSeries.DiscreteSeriesClassification.DiscreteSeriesExhaustion.MatrixCoefficientDecay.DecayEstimate

theorem **decay_estimate** : CompanionTheory.RgExh.StepChar → CompanionTheory.RgExh.Decay

CompanionTheory.RealGroups.DiscreteSeries.DiscreteSeriesClassification.DiscreteSeriesExhaustion.MatrixCoefficientDecay.DecayOf

theorem **decay_of** (hS : Start) (hJ : Labels) (hL : Back) : StepChar → Decay

CompanionTheory.RealGroups.DiscreteSeries.DiscreteSeriesClassification.DiscreteSeriesExhaustion.MatrixCoefficientDecay.HilbertSchmidtDecay.BackOf

theorem **back_of** (hA : Assembly) (h1 : AbsoluteL2Bound) (h2 : Geo) (h3 : LineFormula) (h4 : HaarExists) (h5 : PolyNilPath) (h6 : N.RightRule) (h7 : N.LeftRule) (h8 : KRight) (h9 : Growth) (h10 : Pack) : Back

CompanionTheory.RealGroups.DiscreteSeries.DiscreteSeriesClassification.DiscreteSeriesExhaustion.MatrixCoefficientDecay.HilbertSchmidtDecay.BackwardOfLi

theorem **back_of_inputs** (hLF : LineFormula) (hPN : PolyNilPath) : Back

theorem **back_of_lineFormula** (hLF : LineFormula) : Back

theorem **backward_of_line_formula** : Back

CompanionTheory.RealGroups.DiscreteSeries.DiscreteSeriesClassification.DiscreteSeriesExhaustion.MatrixCoefficientDecay.HilbertSchmidtDecay.DecayAssembly

theorem **line_deriv** (hR : N.RightRule) {n : ℕ} {ε : Fin n → Bool} {H : Type} [NormedAddCommGroup H] [InnerProductSpace ℂ H] [CompleteSpace H] {σ : Matrix (Fin n) (Fin n) ℂ → (H →L[ℂ] H)} {D : Submodule ℂ H} {E : Fin n → Fin n → Module.End ℂ D} (hK : CompanionTheory.D.LK.IsKMod ε σ D E) (χ : Fin n → ℂ) (ss : List (Fin n)) (hU : Formalism.MainNotions.S.IsURepOn (Formalism.MainNotions.S.UJset ε) σ) (A : Matrix (Fin n) (Fin n) ℂ) (hA : Formalism.REAL.inG ε A) (hexp : ∀ t, ReciprocityTheory.ChartRestriction.DS.inUJ ε (aT A t)) (i : D × List (Fin n)) (u : (path (D, E, hK.gl) χ ss).V) (g : Matrix (Fin n) (Fin n) ℂ) (hg : ReciprocityTheory.ChartRestriction.DS.inUJ ε g) (t : ℝ) : HasDerivAt (fun s : ℝ => famP σ D E hK.gl χ ss i u (g ∗ aT A s)) (famP σ D E hK.gl χ ss i (actM (path (D, E, hK.gl) χ ss) A u) (g ∗ aT A t)) t

theorem **wv_succ** {n : ℕ} (X : GMod n) (A : Matrix (Fin n) (Fin n) ℂ) (zs : List ℂ) (u : X.V) (k : ℕ) (hk : k < zs.length) : actM X A (wv X A zs u (k + 1)) = zs[k] • wv X A zs u (k + 1) + wv X A zs u k

theorem **wv_length** {n : ℕ} (X : GMod n) (A : Matrix (Fin n) (Fin n) ℂ) (zs : List ℂ) (u : X.V) : wv X A zs u zs.length = u

theorem **wv_zero** {n : ℕ} (X : GMod n) (A : Matrix (Fin n) (Fin n) ℂ) (p : Polynomial ℂ) (hp : p.Monic) (u : X.V) : wv X A p.roots.toList u 0 = Polynomial.aeval (actM X A) p u

CompanionTheory.RealGroups.DiscreteSeries.DiscreteSeriesClassification.DiscreteSeriesExhaustion.MatrixCoefficientDecay.HilbertSchmidtDecay.DecayAssembly

theorem **cont_span** {Y : Type*} [TopologicalSpace Y] (φ : Y → M) (hG : ∀ p, Continuous fun y => G p (φ y)) : ∀ f ∈ Submodule.span ℂ (Set.range G), Continuous fun y => f (φ y)

theorem **dom_span** {β : Type*} (L R : β → M) (hG : ∀ p, ∃ (T : Finset ι) (Cf : ℝ), ∀ b, ‖G p (L b)‖ ^ 2 ≤ Cf ∗ ∑ q ∈ T, ‖G q (R b)‖ ^ 2) : ∀ f ∈ Submodule.span ℂ (Set.range G), ∃ (T : Finset (M → ℂ)) (Cf : ℝ), (∀ h ∈ T, h ∈ Submodule.span ℂ (Set.range G)) ∧ ∀ b, ‖f (L b)‖ ^ 2 ≤ Cf ∗ ∑ h ∈ T, ‖h (R b)‖ ^ 2

theorem **enorm_add_sq_le** (a b : ℂ) : ‖a + b‖ₑ ^ 2 ≤ 2 ∗ ‖a‖ₑ ^ 2 + 2 ∗ ‖b‖ₑ ^ 2

theorem **lint_add_lt** {α : Type*} [MeasurableSpace α] {μ : Measure α} (f g : α → ℂ) (w : α → ℝ≥0∞) (hf : AEMeasurable (fun a => ‖f a‖ₑ ^ 2 ∗ w a) μ) (hF : ∫⁻ a, ‖f a‖ₑ ^ 2 ∗ w a ∂μ < ⊤) (hG : ∫⁻ a, ‖g a‖ₑ ^ 2 ∗ w a ∂μ < ⊤) : ∫⁻ a, ‖f a + g a‖ₑ ^ 2 ∗ w a ∂μ < ⊤

theorem **lint_smul_lt** {α : Type*} [MeasurableSpace α] {μ : Measure α} (c : ℂ) (f : α → ℂ) (w : α → ℝ≥0∞) (hF : ∫⁻ a, ‖f a‖ₑ ^ 2 ∗ w a ∂μ < ⊤) : ∫⁻ a, ‖c ∗ f a‖ₑ ^ 2 ∗ w a ∂μ < ⊤

theorem **lint_span** {α : Type*} [MeasurableSpace α] (μ : Measure α) (φ : α → M) (w : α → ℝ≥0∞) (hm : ∀ f ∈ Submodule.span ℂ (Set.range G), AEMeasurable (fun a => ‖f (φ a)‖ₑ ^ 2 ∗ w a) μ) (hG : ∀ p, ∫⁻ a, ‖G p (φ a)‖ₑ ^ 2 ∗ w a ∂μ < ⊤) : ∀ f ∈ Submodule.span ℂ (Set.range G), ∫⁻ a, ‖f (φ a)‖ₑ ^ 2 ∗ w a ∂μ < ⊤

theorem **ae_lint_span** {β α : Type*} [MeasurableSpace β] [MeasurableSpace α] (ν : Measure α) (μ : Measure β) (φ : β → α → M) (w : α → ℝ≥0∞) (hm : ∀ f ∈ Submodule.span ℂ (Set.range G), ∀ x, AEMeasurable (fun a => ‖f (φ x a)‖ₑ ^ 2 ∗ w a) μ) (hG : ∀ p, ∀ᵐ x ∂ν, ∫⁻ a, ‖G p (φ x a)‖ₑ ^ 2 ∗ w a ∂μ < ⊤) : ∀ f ∈ Submodule.span ℂ (Set.range G), ∀ᵐ x ∂ν, ∫⁻ a, ‖f (φ x a)‖ₑ ^ 2 ∗ w a ∂μ < ⊤

theorem **chain_span** {n : ℕ} {ι : Type*} (G : ι → (Matrix (Fin n) (Fin n) ℂ → ℂ)) (ε : Fin n → Bool) (A : Matrix (Fin n) (Fin n) ℂ) (τ : S7.UJ ε → ℝ) (m : ℕ) (z : Fin m → ℂ) (hG : ∀ p, ChainQ ε (Submodule.span ℂ (Set.range G)) A τ m z (G p)) : ∀ f ∈ Submodule.span ℂ (Set.range G), ChainQ ε (Submodule.span ℂ (Set.range G)) A τ m z f

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theorem **hasChar_path** (hS : StepChar) {n : ℕ} (X : GMod n) (χ : Fin n → ℂ) (ss : List (Fin n)) (h : HasChar X χ) : HasChar (path X χ ss) (pathEnd χ ss)

theorem **assembly_of** : Assembly

CompanionTheory.RealGroups.DiscreteSeries.DiscreteSeriesClassification.DiscreteSeriesExhaustion.MatrixCoefficientDecay.HilbertSchmidtDecay.DecayAssembly

theorem **gen_mem** {n : ℕ} {H : Type} [NormedAddCommGroup H] [InnerProductSpace ℂ H] (σ : Matrix (Fin n) (Fin n) ℂ → (H →L[ℂ] H)) (D : Submodule ℂ H) (E : Fin n → Fin n → Module.End ℂ D) (hE : Formalism.REAL.GlRel E) (χ : Fin n → ℂ) (ss : List (Fin n)) (i : D × List (Fin n)) (u : (path (D, E, hE) χ ss).V) : famP σ D E hE χ ss i u ∈ SS σ D E hE χ ss

theorem **absorb** {M α : Type*} (S : Submodule ℂ (M → ℂ)) (I : Finset α) (h : α → (M → ℂ)) (hS : ∀ i ∈ I, h i ∈ S) (e : ℕ) (he : 1 ≤ e) : ∃ T : Finset (M → ℂ), (∀ f ∈ T, f ∈ S) ∧ ∀ x, ∑ i ∈ I, ‖h i x‖ ^ e ≤ ∑ f ∈ T, ‖f x‖ ^ e

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theorem **source_gen** (hL : N.LeftRule) {n : ℕ} {ε : Fin n → Bool} {H : Type} [NormedAddCommGroup H] [InnerProductSpace ℂ H] [CompleteSpace H] {σ : Matrix (Fin n) (Fin n) ℂ → (H →L[ℂ] H)} {D : Submodule ℂ H} {E : Fin n → Fin n → Module.End ℂ D} (hK : CompanionTheory.D.LK.IsKMod ε σ D E) (χ : Fin n → ℂ) (ss : List (Fin n)) (hU : Formalism.MainNotions.S.IsURepOn (Formalism.MainNotions.S.UJset ε) σ) (A : Matrix (Fin n) (Fin n) ℂ) (τ : S7.UJ ε → ℝ) (C : ℝ) (hC : ∀ (x : PSub ε A) (Y : Matrix (Fin n) (Fin n) ℂ), Y ∈ nilpC A → 0 ≤ τ x → Formalism.REAL.hs (S7.toM (x : S7.UJ ε) ∗ Y ∗ (S7.toM (x : S7.UJ ε))⁻¹) ≤ C ∗ Real.exp (−2 ∗ τ x) ∗ Formalism.REAL.hs Y) (i : D × List (Fin n)) (m : ℕ) (Y : Fin m → Matrix (Fin n) (Fin n) ℂ) (v : Fin m → (path (D, E, hK.gl) χ ss).V) (hY : ∀ k, Y k ∈ nilpC A) : ∃ T : Finset (Matrix (Fin n) (Fin n) ℂ → ℂ), (∀ h ∈ T, h ∈ SS σ D E hK.gl χ ss) ∧ ∀ x : PSub ε A, 0 ≤ τ x

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theorem **absolute_l2_bound** : CompanionTheory.RgExh.Dec.L.AbsoluteL2Bound

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theorem **far_real** (a b C c₁ s w c'' r τ τy τx : ℝ) (ha : 1 ≤ a) (hb : 1 ≤ b) (hlow : C⁻¹ ∗ Real.exp (2 ∗ t) ∗ b ≤ a) (hup : a ≤ C ∗ Real.exp (2 ∗ t) ∗ b) (hlog : c₁ ∗ Real.log a ≤ τy) (hτ : τy = τx + t) (hτx : τx < 1) (hc₁ : 0 < c₁) (hs : s ≤ 0) (hw1 : s ≤ w) (hw2 : w ≤ s + 2 ∗ c₁ ∗ c'') (hc0 : 0 < c'') : a ^ w ∗ Real.exp (2 ∗ r ∗ t) ≤ (C ^ (−s) ∗ Real.exp (2 ∗ c'')) ∗ b ^ s ∗ Real.exp (2 ∗ (r + s + c'') ∗ t)

theorem **far_est** (hLC : LineChain) (H : Hyp ε S m A ν τ r Z M c₁ R₀ C) (hn : 0 < n) (s : ℝ) (hs : s ≤ 0) (hL : ∀ f ∈ S, LevelF ε f s) (c'' : ℝ) (hc0 : 0 < c'') (hc1 : c'' ≤ 1) (j : Fin m) (hz : ∀ z ∈ Z j, z.re ≠ −(r j + s + c'')) (U : Set (S7.UJ ε)) (hUm : MeasurableSet U) (hUK : ∀ g (k : S7.KG ε), g ∈ U → g ∗ (k : S7.UJ ε) ∈ U) (hUτ : ∀ x : PSub ε (A j), (x : S7.UJ ε) ∈ U → c₁ ∗ Real.log (Formalism.REAL.hs (S7.toM (x : S7.UJ ε))) ≤ τ j x) (R₂ : ℝ) (hR₂ : 1 ≤ R₂) (hR₂' : 1 ≤ c₁ ∗ Real.log R₂) (w : ℝ) (hw1 : s ≤ w) (hw2 : w ≤ s + 2 ∗ c₁ ∗ c'') (f : Matrix (Fin n) (Fin n) ℂ → ℂ) (hf : f ∈ S) : ∫⁻ g : S7.UJ ε, (U ∩ {g | R₂ < Formalism.REAL.hs (S7.toM g)}).indicator (fun g => PsiK ε f w (S7.toM g)) g ∂Measure.haar < ⊤

theorem **ball_est** (hn : 0 < n) (f : Matrix (Fin n) (Fin n) ℂ → ℂ) (s w R₂ : ℝ) (hw : s ≤ w) (hR₂ : 1 ≤ R₂) (hf : LevelF ε f s) : ∫⁻ g : S7.UJ ε, {g | Formalism.REAL.hs (S7.toM g) ≤ R₂}.indicator (fun g => ‖f (S7.toM g)‖ₑ ^ 2 ∗ ENNReal.ofReal (Formalism.REAL.hs (S7.toM g) ^ w)) g ∂Measure.haar < ⊤

theorem **step** (hLC : LineChain) (H : Hyp ε S m A ν τ r Z M c₁ R₀ C) (hn : 0 < n) (s : ℝ) (hs : s ≤ 0) (hL : ∀ f ∈ S, LevelF ε f s) (c'' : ℝ) (hc0 : 0 < c'') (hc1 : c'' ≤ 1) (hz : ∀ j, ∀ z ∈ Z j, z.re ≠ −(r j + s + c'')) (w : ℝ) (hw1 : s ≤ w) (hw2 : w ≤ s + 2 ∗ c₁ ∗ c'') : ∀ f ∈ S, LevelF ε f w

theorem **exists_good** (Z : Fin m → Finset ℂ) (r : Fin m → ℝ) (s : ℝ) : ∃ c'' : ℝ, 1 / 2 ≤ c'' ∧ c'' ≤ 1 ∧ ∀ j, ∀ z ∈ Z j, z.re ≠ −(r j + s + c'')

theorem **level_mono** (hn : 0 < n) (f : Matrix (Fin n) (Fin n) ℂ → ℂ) {s s' : ℝ} (h : s' ≤ s) (hf : LevelF ε f s) : LevelF ε f s'

theorem **level_zero_dim** (ε : Fin 0 → Bool) (f : Matrix (Fin 0) (Fin 0) ℂ → ℂ) (s : ℝ) : LevelF ε f s

theorem **absolute_l2_of_line_chain** (hLC : LineChain) : AbsoluteL2Bound

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theorem **psiK_inv** (q : Matrix (Fin n) (Fin n) ℂ → ℂ) (w : ℝ) (g : S7.UJ ε) (k : S7.KG ε) : PsiK ε q w (S7.toM g * S7.toMK k) = PsiK ε q w (S7.toM g)

theorem **continuous_hs** : Continuous (fun X : Matrix (Fin n) (Fin n) ℂ => Formalism.REAL.hs X)

theorem **psiK_meas** (q : Matrix (Fin n) (Fin n) ℂ → ℂ) (w : ℝ) (hq : Continuous fun g : S7.UJ ε => q (S7.toM g)) : Measurable fun g : S7.UJ ε => PsiK ε q w (S7.toM g)

theorem **psiK_ge** (q : Matrix (Fin n) (Fin n) ℂ → ℂ) (w : ℝ) (g : S7.UJ ε) : ‖q (S7.toM g)‖ₑ ^ 2 * ENNReal.ofReal (Formalism.REAL.hs (S7.toM g) ^ w) ≤ PsiK ε q w (S7.toM g)

theorem **psiK_le** (q : Matrix (Fin n) (Fin n) ℂ → ℂ) (w : ℝ) (T : Finset (Matrix (Fin n) (Fin n) ℂ → ℂ)) (Cf : ℝ) (hd : ∀ (g : S7.UJ ε) (k k' : S7.KG ε), ‖q (S7.toMK k * S7.toM g * S7.toMK k')‖ ^ 2 ≤ Cf * ∑ h ∈ T, ‖h (S7.toM g)‖ ^ 2) (g : S7.UJ ε) : PsiK ε q w (S7.toM g) ≤ ENNReal.ofReal Cf * ∑ h ∈ T, ‖h (S7.toM g)‖ₑ ^ 2 * ENNReal.ofReal (Formalism.REAL.hs (S7.toM g) ^ w)

theorem **psiK_lint** (q : Matrix (Fin n) (Fin n) ℂ → ℂ) (w : ℝ) (T : Finset (Matrix (Fin n) (Fin n) ℂ → ℂ)) (Cf : ℝ) (hd : ∀ (g : S7.UJ ε) (k k' : S7.KG ε), ‖q (S7.toMK k * S7.toM g * S7.toMK k')‖ ^ 2 ≤ Cf * ∑ h ∈ T, ‖h (S7.toM g)‖ ^ 2) (hT : ∀ h ∈ T, LevelF ε h w) (hTc : ∀ h ∈ T, Continuous fun g : S7.UJ ε => h (S7.toM g)) : ∫⁻ g : S7.UJ ε, PsiK ε q w (S7.toM g) ∂Measure.haar < ⊤

CompanionTheory.RealGroups.DiscreteSeries.DiscreteSeriesClassification.DiscreteSeriesExhaustion.MatrixCoefficientDecay.HilbertSchmidtDecay.HilbertSchmi

theorem **one_le_hs** (hn : 0 < n) (g : S7.UJ ε) : 1 ≤ Formalism.REAL.hs (S7.toM g)

private theorem **hs_eq_re_trace_conjTranspose_mul'** (X : Matrix (Fin n) (Fin n) ℂ) : Formalism.REAL.hs X = (Matrix.trace (X * Xᴴ)).re

theorem **hs_mul_K** (X : Matrix (Fin n) (Fin n) ℂ) (k : S7.KG ε) : Formalism.REAL.hs (X * S7.toMK k) = Formalism.REAL.hs X

theorem **continuous_aT** (A : Matrix (Fin n) (Fin n) ℂ) : Continuous (fun t : ℝ => aT A t)

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theorem **line_curve** (H : Hyp ε S m A ν τ r Z M c₁ R₀ C) (j : Fin m) (x : PSub ε (A j)) : ∃ ℓ : ℝ → PSub ε (A j), Continuous (fun t => ((ℓ t : PSub ε (A j)) : S7.UJ ε)) ∧ ∀ t, S7.toM ((ℓ t : PSub ε (A j)) : S7.UJ ε) = S7.toM (x : S7.UJ ε) * aT (A j) t ∧ τ j (ℓ t) = τ j x + t

theorem **line_cont** (H : Hyp ε S m A ν τ r Z M c₁ R₀ C) (j : Fin m) (x : PSub ε (A j)) (q : Matrix (Fin n) (Fin n) ℂ → ℂ) (hq : Continuous fun g : S7.UJ ε => q (S7.toM g)) : Continuous fun t : ℝ => q (S7.toM (x : S7.UJ ε) * aT (A j) t)

theorem **ode_line** (hLC : LineChain) (H : Hyp ε S m A ν τ r Z M c₁ R₀ C) (j : Fin m) (h : Matrix (Fin n) (Fin n) ℂ → ℂ) {mm : ℕ} (z : Fin mm → ℂ) (F : Fin (mm + 1) → Matrix (Fin n) (Fin n) ℂ → ℂ) (hF : ∀ k, F k ∈ S) (hlast : F (Fin.last mm) = h) (hder : ∀ (k : Fin mm) (g : Matrix (Fin n) (Fin n) ℂ) (t : ℝ), ReciprocityTheory.ChartRestriction.DS.inUJ ε g → HasDerivAt (fun s : ℝ => F k.succ (g * aT (A j) s)) (z k * F k.succ (g * aT (A j) t) + F k.castSucc (g * aT (A j) t)) t) (R : ℝ) (hR : R ≤ r j + 1) (hzR : ∀ k, (z k).re ≠ -R) : ∃ C₀ : ℝ, 0 < C₀ ∧ ∀ x : PSub ε (A j), (∀ k, ∫⁻ t in Set.Ioi (0 : ℝ), ‖F k (S7.toM (x : S7.UJ ε) * aT (A j) t)‖ₑ ^ 2 * ENNReal.ofReal (Real.exp (2 * (r j + 1) * t)) < ⊤) → ∫⁻ t in Set.Ioi (0 : ℝ), ‖h (S7.toM (x : S7.UJ ε) * aT (A j) t)‖ₑ ^ 2 * ENNReal.ofReal (Real.exp (2 * R * t)) ≤ ENNReal.ofReal C₀ * (∑ k, ∫⁻ t in Set.Ioc (0 : ℝ) 1, ‖F k (S7.toM (x : S7.UJ ε) * a

theorem **src_line** (H : Hyp ε S m A ν τ r Z M c₁ R₀ C) (j : Fin m) (f₀ : Matrix (Fin n) (Fin n) ℂ → ℂ) (T : Finset (Matrix (Fin n) (Fin n) ℂ → ℂ)) (hTS : ∀ h ∈ T, h ∈ S) (hT : ∀ x : PSub ε (A j), 0 ≤ τ j x → ‖f₀ (S7.toM (x : S7.UJ ε))‖ ≤ Real.exp (-τ j x) * ∑ h ∈ T, ‖h (S7.toM (x : S7.UJ ε))‖) (x : PSub ε (A j)) (hx : 0 ≤ τ j x) (R ρ : ℝ) (hRρ : R - 1 ≤ ρ) : ∫⁻ t in Set.Ioi (0 : ℝ), ‖f₀ (S7.toM (x : S7.UJ ε) * aT (A j) t)‖ₑ ^ 2 * ENNReal.ofReal (Real.exp (2 * R * t)) ≤ (T.card : ℝ≥0∞) * ∑ h ∈ T, ∫⁻ t in Set.Ioi (0 : ℝ), ‖h (S7.toM (x : S7.UJ ε) * aT (A j) t)‖ₑ ^ 2 * ENNReal.ofReal (Real.exp (2 * ρ * t))

theorem **unit_line** (u : ℝ → ℂ) (ρ : ℝ) : ∫⁻ t in Set.Ioc (0 : ℝ) 1, ‖u t‖ₑ ^ 2 ≤ ENNReal.ofReal (Real.exp (2 * |ρ|)) * ∫⁻ t in Set.Ioi (0 : ℝ), ‖u t‖ₑ ^ 2 * ENNReal.ofReal (Real.exp (2 * ρ * t))

theorem **back_line** (H : Hyp ε S m A ν τ r Z M c₁ R₀ C) (hn : 0 < n) (j : Fin m) (x : PSub ε (A j)) (hx : 0 ≤ τ j x) (q : Matrix (Fin n) (Fin n) ℂ → ℂ) (s : ℝ) (hs : s ≤ 0) : ENNReal.ofReal (Formalism.REAL.hs (S7.toM (x : S7.UJ ε)) ^ s) * ∫⁻ t in Set.Ioi (0 : ℝ), ‖q (S7.toM (x : S7.UJ ε) * aT (A j) t)‖ₑ ^ 2 * ENNReal.ofReal (Real.exp (2 * (r j + s) * t)) ≤ ENNReal.ofReal (C ^ (-s)) * ∫⁻ t : ℝ, PsiK ε q s (S7.toM (x : S7.UJ ε) * aT (A j) t) * ENNReal.ofReal (Real.exp (2 * r j * t))

theorem **line_est** (hLC : LineChain) (H : Hyp ε S m A ν τ r Z M c₁ R₀ C) (hn : 0 < n) (s : ℝ) (hs : s ≤ 0) (hL : ∀ f ∈ S, LevelF ε f s) (c'' : ℝ) (hc0 : 0 < c'') (hc1 : c'' ≤ 1) (j : Fin m) (hz : ∀ z ∈ Z j, z.re ≠ -(r j + s + c'')) (h : Matrix (Fin n) (Fin n) ℂ → ℂ) (hh : h ∈ S) : ∃ (Ψ : Matrix (Fin n) (Fin n) ℂ → ℝ≥0∞) (K : ℝ≥0∞), K ≠ ⊤ ∧ Measurable (fun g : S7.UJ ε => Ψ (S7.toM g)) ∧ (∀ (g : S7.UJ ε) (k : S7.KG ε), Ψ (S7.toM g * S7.toMK k) = Ψ (S7.toM g)) ∧ ∫⁻ g : S7.UJ ε, Ψ (S7.toM g) ∂Measure.haar < ⊤ ∧ ∀ᵐ (x : PSub ε (A j)) ∂(ν j), τ j x ∈ Set.Ico 0 1 → ENNReal.ofReal (Formalism.REAL.hs (S7.toM (x : S7.UJ ε)) ^ s) * ∫⁻ t in Set.Ioi (0 : ℝ), ‖h (S7.toM (x : S7.UJ ε) * aT (A j) t)‖ₑ ^ 2 * ENNReal.ofReal (Real.exp (2 * (r j + s + c'') * t)) ≤ K * ∫⁻ t : ℝ, Ψ (S7.toM (x : S7.UJ ε) * aT (A j) t) * ENNReal.ofReal (Real.exp (2 * r j * t))

CompanionTheory.RealGroups.DiscreteSeries.DiscreteSeriesClassification.DiscreteSeriesExhaustion.MatrixCoefficientDecay.HilbertSchmidtDecay.RightKTypes.

theorem **ba_kst** (hK : CompanionTheory.D.LK.IsKMod ε σ D E) (k : Matrix (Fin n) (Fin n) ℂ) (hk : k ∈ Kset ε) (v : H) (hv : v ∈ D) : σ k v ∈ D

theorem **ba_der** (hK : CompanionTheory.D.LK.IsKMod ε σ D E) : IsDerivedOn ε σ D E

theorem **ba_actM_single** {M : Type} [AddCommGroup M] [Module ℂ M] (F : Fin n → Fin n → Module.End ℂ M) (a b : Fin n) : ReciprocityTheory.ChartRestriction.DS.actM F (Matrix.single a b 1) = F a b

theorem **ba_conj_single** (k l : Matrix (Fin n) (Fin n) ℂ) (a b c d : Fin n) : (k * Matrix.single a b (1 : ℂ) * l) c d = k c a * l b d

theorem **base_mem** (hK : CompanionTheory.D.LK.IsKMod ε σ D E) (hU : Formalism.MainNotions.S.IsURepOn (Formalism.MainNotions.S.UJset ε) σ) (k : S7.KG ε) (v : H) (hv : v ∈ D) : σ (S7.toMK k) v ∈ D

theorem **baseRho_compat** (hK : CompanionTheory.D.LK.IsKMod ε σ D E) (hU : Formalism.MainNotions.S.IsURepOn (Formalism.MainNotions.S.UJset ε) σ) (k : S7.KG ε) (a b : Fin n) (x : D) : baseRho hK hU k (E a b x) = ∑ c, ∑ d, (S7.toMK k c a * (S7.toMK k)ᴴ b d) • E c d (baseRho hK hU k x)

theorem **baseRho_bdd** (hK : CompanionTheory.D.LK.IsKMod ε σ D E) (hU : Formalism.MainNotions.S.IsURepOn (Formalism.MainNotions.S.UJset ε) σ) (k : S7.KG ε) (x : D) : ‖D.subtype (baseRho hK hU k x)‖ ≤ 1 * ‖D.subtype x‖

theorem **baseRho_fin** (hK : CompanionTheory.D.LK.IsKMod ε σ D E) (hU : Formalism.MainNotions.S.IsURepOn (Formalism.MainNotions.S.UJset ε) σ) (x : D) : ∃ W : Submodule ℂ D, FiniteDimensional ℂ W ∧ ∀ k, baseRho hK hU k x ∈ W

theorem **rightEq_base** (hK : CompanionTheory.D.LK.IsKMod ε σ D E) (hU : Formalism.MainNotions.S.IsURepOn (Formalism.MainNotions.S.UJset ε) σ) : RightEq (baseAct hK hU) (fun ξ : D => baseCoef σ D (ξ : H))

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theorem **kr_unit** (k : S7.KG ε) : (S7.toMK k)ᴴ * S7.toMK k = 1 ∧ S7.toMK k * (S7.toMK k)ᴴ = 1

theorem **kr_entry** (k : S7.KG ε) (a b : Fin n) : ‖S7.toMK k a b‖ ≤ 1

theorem **tensRho_apply** (A : KAct ε X) (k : S7.KG ε) (u : Fin n → X.V) (b : Fin n) : tensRho A k u b = ∑ i, S7.toMK k b i • A.ρ k (u i)

theorem **kr_one_apply** (k : S7.KG ε) (b d : Fin n) : ∑ i, S7.toMK k b i * (S7.toMK k)ᴴ i d = if b = d then 1 else 0

theorem **kr_one_apply'** (k : S7.KG ε) (b d : Fin n) : ∑ i, (S7.toMK k)ᴴ b i * S7.toMK k i d = if b = d then 1 else 0

private theorem **sum_toMK_mul_conjTranspose_smul_eq_smul** {M : Type} [AddCommGroup M] [Module ℂ M] (k : S7.KG ε) (b : Fin n) (α : ℂ) (w : Fin n → M) : ∑ i, ∑ d, (S7.toMK k b i * (α * (S7.toMK k)ᴴ i d)) • w d = α • w b

theorem **commute_om_tensRho** (A : KAct ε X) (k : S7.KG ε) : Commute (om X) (tensRho A k)

private theorem **sum_conjTranspose_toMK_smul_tensRho_eq** (A : KAct ε X) (k : S7.KG ε) (u : Fin n → X.V) (b : Fin n) (α : ℂ) : ∑ d, (α * (S7.toMK k)ᴴ b d) • tensRho A k u d = α • A.ρ k (u b)

theorem **tensRho_compat** (A : KAct ε X) (k : S7.KG ε) (a b : Fin n) (u : Fin n → X.V) : tensRho A k (tensE X a b u) = ∑ c, ∑ d, (S7.toMK k c a * (S7.toMK k)ᴴ b d) • tensE X c d (tensRho A k u)

theorem **sv_smul** {z : ℂ} (c : ℂ) (u : (step X z).V) : sv (c • u) = c • sv u

theorem **sv_sum** {z : ℂ} {ι : Type} (s : Finset ι) (f : ι → (step X z).V) : sv (∑ i ∈ s, f i) = ∑ i ∈ s, sv (f i)

theorem **sv_E** {z : ℂ} (a b : Fin n) (u : (step X z).V) : sv ((step X z).E a b u) = tensE X a b (sv u)

theorem **sv_inj** {z : ℂ} {u v : (step X z).V} (h : sv u = sv v) : u = v

theorem **stepRho_coe** (A : KAct ε X) (z : ℂ) (k : S7.KG ε) (u : (step X z).V) : sv (stepRho A z k u) = tensRho A k (sv u)

theorem **stepEmb_apply** (A : KAct ε X) (z : ℂ) (u : (step X z).V) (i : Fin n) : stepEmb A z u i = A.emb ((sv u) i)

theorem **stepEmb_inj** (A : KAct ε X) (z : ℂ) : Function.Injective (stepEmb A z)

theorem **stepRho_compat** (A : KAct ε X) (z : ℂ) (k : S7.KG ε) (a b : Fin n) (x : (step X z).V) : stepRho A z k ((step X z).E a b x) = ∑ c, ∑ d, (S7.toMK k c a * (S7.toMK k)ᴴ b d) • (step X z).E c d (stepRho A z k x)

CompanionTheory.RealGroups.DiscreteSeries.DiscreteSeriesClassification.DiscreteSeriesExhaustion.MatrixCoefficientDecay.HilbertSchmidtDecay.RightKTypes.I

theorem **klf_stepCoef_apply** (X : GMod n) (z : ℂ) (c : X.V →ₗ[ℂ] (Matrix (Fin n) (Fin n) ℂ → ℂ)) (j : Fin n) (u : (step X z).V) (g : Matrix (Fin n) (Fin n) ℂ) : stepCoef X z c j u g = ∑ i, g j i * c ((Subtype.val u : Fin n → X.V) i) g

theorem **klf_contFam_step** {ε : Fin n → Bool} {X : GMod n} {I : Type} {c : CoefFam X I} (h : ContFam ε c) (z : ℂ) : ContFam ε (X

theorem **leftExp_step** {ε : Fin n → Bool} {X : GMod n} {I : Type} {c : CoefFam X I} (h : LeftExp ε c) (z : ℂ) : LeftExp ε (X

theorem **leftExp_path** {ε : Fin n → Bool} {X : GMod n} {I : Type} {c : CoefFam X I} (h : LeftExp ε c) (χ : Fin n → ℂ) (ss : List (Fin n)) : LeftExp ε (X

theorem **contFam_path** {ε : Fin n → Bool} {X : GMod n} {I : Type} {c : CoefFam X I} (h : ContFam ε c) (χ : Fin n → ℂ) (ss : List (Fin n)) : ContFam ε (X

theorem **leftExp_base** {ε : Fin n → Bool} {H : Type} [NormedAddCommGroup H] [InnerProductSpace ℂ H] {σ : Matrix (Fin n) (Fin n) ℂ → (H →L[ℂ] H)} {D : Submodule ℂ H} {E : Fin n → Fin n → Module.End ℂ D} (hK : CompanionTheory.D.LK.IsKMod ε σ D E) (hU : Formalism.MainNotions.S.IsURepOn (Formalism.MainNotions.S.UJset ε) σ) : LeftExp ε (X

theorem **contFam_base** {ε : Fin n → Bool} {H : Type} [NormedAddCommGroup H] [InnerProductSpace ℂ H] {σ : Matrix (Fin n) (Fin n) ℂ → (H →L[ℂ] H)} {D : Submodule ℂ H} {E : Fin n → Fin n → Module.End ℂ D} (hK : CompanionTheory.D.LK.IsKMod ε σ D E) (hU : Formalism.MainNotions.S.IsURepOn (Formalism.MainNotions.S.UJset ε) σ) : ContFam ε (X

CompanionTheory.RealGroups.DiscreteSeries.DiscreteSeriesClassification.DiscreteSeriesExhaustion.MatrixCoefficientDecay.HilbertSchmidtDecay.RightKTypes.I

theorem **kr_dom** {ι : Type} [Fintype ι] (γ h : ι → ℂ) (B : ℝ) (hγ : ∀ a, ‖γ a‖ ≤ B) : ‖∑ a, γ a * h a‖ ^ 2 ≤ (Fintype.card ι * B ^ 2) * ∑ a, ‖h a‖ ^ 2

theorem **kr_img** {ι κ : Type} [Fintype ι] [DecidableEq κ] (p : ι → κ) (f : κ → ℝ) (hf : ∀ q, 0 ≤ f q) : ∑ a, f (p a) ≤ Fintype.card ι * ∑ q ∈ Finset.univ.image p, f q

theorem **kRight** : KRight

CompanionTheory.RealGroups.DiscreteSeries.DiscreteSeriesClassification.DiscreteSeriesExhaustion.MatrixCoefficientDecay.HilbertSchmidtDecay.RightKTypes.I

theorem **rightExp** {ε : Fin n → Bool} {X : GMod n} (A : KAct ε X) (x : X.V) : ∃ (r : ℕ) (w : Fin r → X.V) (a : Fin r → S7.KG ε → ℂ) (B : ℝ), (∀ l k, ‖a l k‖ ≤ B) ∧ ∀ k, A.ρ k x = ∑ l, a l k • w l

CompanionTheory.RealGroups.DiscreteSeries.DiscreteSeriesClassification.DiscreteSeriesExhaustion.MatrixCoefficientDecay.HilbertSchmidtDecay.RightKTypes.S

theorem **stepRho_bdd** (A : KAct ε X) (z : ℂ) : ∃ C : ℝ, ∀ k x, ‖stepEmb A z (stepRho A z k x)‖ ≤ C * ‖stepEmb A z x‖

theorem **stepRho_fin** (A : KAct ε X) (z : ℂ) (x : (step X z).V) : ∃ W : Submodule ℂ (step X z).V, FiniteDimensional ℂ W ∧ ∀ k, stepRho A z k x ∈ W

theorem **stepCoef_apply'** (z : ℂ) (c : X.V →ₗ[ℂ] (Matrix (Fin n) (Fin n) ℂ → ℂ)) (j : Fin n) (u : (step X z).V) (g : Matrix (Fin n) (Fin n) ℂ) : stepCoef X z c j u g = ∑ i, g j i * c ((sv u) i) g

theorem **rightEq_step** (A : KAct ε X) {I : Type} {c : CoefFam X I} (hc : RightEq A c) (z : ℂ) : RightEq (stepAct A z) (X

theorem **rightEq_path** : ∀ {X : GMod n} (A : KAct ε X) {I : Type} {c : CoefFam X I}, RightEq A c → ∀ (χ : Fin n → ℂ) (ss : List (Fin n)), RightEq (pathAct A χ ss) (X

CompanionTheory.RealGroups.DiscreteSeries.DiscreteSeriesClassification.DiscreteSeriesExhaustion.MatrixCoefficientDecay.InfinitesimalCharacterPaths.Exha

theorem **exhaustion_labels** : CompanionTheory.RgExh.Dec.Labels

CompanionTheory.RealGroups.DiscreteSeries.DiscreteSeriesClassification.DiscreteSeriesExhaustion.MatrixCoefficientDecay.InfinitesimalCharacterPaths.HasCl

theorem **nset_congr** (ε : Fin n → Bool) {V : Type} [AddCommGroup V] [Module ℂ V] {ι κ : Type} (c : ι → V →ₗ[ℂ] (Matrix (Fin n) (Fin n) ℂ → ℂ)) (d : κ → V →ₗ[ℂ] (Matrix (Fin n) (Fin n) ℂ → ℂ)) (h1 : ∀ i, ∃ k, c i = d k) (h2 : ∀ k, ∃ i, d k = c i) (A : Matrix (Fin n) (Fin n) ℂ) (μ : MeasureTheory.Measure (ReciprocityTheory.ChartRestriction.DS.S7.UJ ε)) (s : ℝ) : NSet ε c A μ s = NSet ε d A μ s

theorem **along** (hT : Transport) (hSC : StepChar) (hTD : TensDecomp) (hFF : FamFilt) (hSF : StepFam) (ε : Fin n → Bool) (A : Matrix (Fin n) (Fin n) ℂ) (μ : MeasureTheory.Measure (ReciprocityTheory.ChartRestriction.DS.S7.UJ ε)) (hA : IsGrad ε A) (hμ : IsParHaar ε A μ) (B : GradBasis A) : ∀ (ss : List (Fin n)) (X : GMod n) (ι : Type) (c : ι → X.V →ₗ[ℂ] (Matrix (Fin n) (Fin n) ℂ → ℂ)) (χ : Fin n → ℂ) (Λ : Set (Fin n → Fin 3)) (F : Filt X A), IsCoefFam ε X c → (∀ s u, u ∈ F.N s ↔ u ∈ NSet ε c A μ s) → HasChar X χ → GoodPath χ ss → PP F B χ Λ → ∃ F' : Filt (path X χ ss) A, (∀ s u, u ∈ F'.N s ↔ u ∈ NSet ε (fun p : ι × List (Fin n) => pathCoef X (c p.1) χ ss p.2) A μ s) ∧ PP F' B (pathEnd χ ss) (moveIter χ Λ ss)

theorem **hasChar_path** (hSC : StepChar) : ∀ (ss : List (Fin n)) (X : GMod n) (χ : Fin n → ℂ), HasChar X χ → HasChar (path X χ ss) (pathEnd χ ss)

theorem **labels_of** (hLF : LocFin) (hTD : TensDecomp) (hT : Transport) (hFF : FamFilt) (hSF : StepFam) (hBF : BaseFam) (hS1 : Start1) (hD : Degree) (hB : Basis) (hLT : LabelTrack) : Labels

CompanionTheory.RealGroups.DiscreteSeries.DiscreteSeriesClassification.DiscreteSeriesExhaustion.MatrixCoefficientDecay.KeepsMul

theorem **keeps_mul** {g h : Matrix (Fin n) (Fin n) ℂ} {S : Submodule ℂ (Fin n → ℂ)} (hg : Keeps g S) (hh : Keeps h S) : Keeps (g * h) S

theorem **keeps_one** (S : Submodule ℂ (Fin n → ℂ)) : Keeps 1 S

theorem **keeps_inv** (g : (Matrix (Fin n) (Fin n) ℂ)ˣ) {S : Submodule ℂ (Fin n → ℂ)} (hg : Keeps (g : Matrix (Fin n) (Fin n) ℂ) S) : Keeps ((g⁻¹ : (Matrix (Fin n) (Fin n) ℂ)ˣ) : Matrix (Fin n) (Fin n) ℂ) S

CompanionTheory.RealGroups.DiscreteSeries.DiscreteSeriesClassification.DiscreteSeriesExhaustion.MatrixCoefficientDecay.LineFormulas.ExhaustionStart

theorem **exhaustion_start** : CompanionTheory.RgExh.Dec.Start

CompanionTheory.RealGroups.DiscreteSeries.DiscreteSeriesClassification.DiscreteSeriesExhaustion.MatrixCoefficientDecay.LineFormulas.LineFinal

theorem **line_formula** : CompanionTheory.RgExh.Dec.L.LineFormula

CompanionTheory.RealGroups.DiscreteSeries.DiscreteSeriesClassification.DiscreteSeriesExhaustion.SimpleDistinctness.ClassLit

theorem **dsClass_of** (hX : GlobalTheory.Dec.DSExhaust) : GlobalTheory.DSId.DSClass

theorem **discreteSeriesClassification_of** (hX : GlobalTheory.Dec.DSExhaust) : GlobalTheory.Published.DiscreteSeriesClassification

theorem **discreteSeriesCover_of** (hX : GlobalTheory.Dec.DSExhaust) : GlobalTheory.Published.DiscreteSeriesCover

CompanionTheory.RealGroups.DiscreteSeries.DiscreteSeriesClassification.DiscreteSeriesExhaustion.SimpleDistinctness.ExhaustionFiniteLength

theorem **exhaustion_finite_length** : CompanionTheory.RgExh.Finish

CompanionTheory.RealGroups.DiscreteSeries.DiscreteSeriesClassification.EquivariantSections.KeyIdentityHolds

theorem **key_identity_holds** : CompanionTheory.RgDs.Key

CompanionTheory.RealGroups.DiscreteSeries.DiscreteSeriesClassification.ExplicitFunctionSquareIntegrable

theorem **phi_one** : phi (1 : Matrix (Fin n) (Fin n) ℂ) = 1

CompanionTheory.RealGroups.DiscreteSeries.DiscreteSeriesClassification.HilbertSpaceModels.HilbertConeOfStableMollifier

theorem **hilbert_cone_of_stable_mollifier** : StepStable → StepMoll → StepWtSpan → StepCone

CompanionTheory.RealGroups.DiscreteSeries.DiscreteSeriesClassification.HilbertSpaceModels.HilbertLowestWeightOfInputs

theorem **hloweq_transfer** {n : ℕ} {ε : Fin n → Bool} {Λ : Fin n → ℝ} (A : L2 ε →ₗ[ℂ] L2 ε) (hAK : ∀ k ∈ Kset ε, ∀ x, A (rreg ε k x) = rreg ε k (A x)) {S : Submodule ℂ (L2 ε)} (hinj : ∀ w ∈ S, A w = 0 → w = 0) (hS : Formalism.MainNotions.S.IsKTypeOf ε (rreg ε) Λ S) : Formalism.MainNotions.S.IsKTypeOf ε (rreg ε) Λ (S.map A)

theorem **hloweq_brIdx** {n : ℕ} (c : Fin n → Fin n → ℕ) : brIdxSum (∑ a, ∑ b, (c a b : ℝ) • Formalism.REAL.root a b) = ∑ a, ∑ b, (c a b : ℝ) * ((a : ℕ) - (b : ℕ) : ℝ)

theorem **hloweq_eq_of_cone** {n : ℕ} {ε : Fin n → Bool} {ν Λ : Fin n → ℝ} (h1 : InCone Λ ν) (h2 : Formalism.MainNotions.S.CDom ε ν Λ) : ν = Λ

theorem **hilbert_lowest_weight_of_inputs** : StepStable → StepMoll → StepWtSpan → StepDatum → StepLowEq

CompanionTheory.RealGroups.DiscreteSeries.DiscreteSeriesClassification.HilbertSpaceModels.HilbertSignatureHolds

theorem **hilbert_signature_holds** : StepSig

CompanionTheory.RealGroups.DiscreteSeries.DiscreteSeriesClassification.HilbertSpaceModels.HilbertSpaceRealisationHolds

theorem **hilbert_space_realisation_holds** : CompanionTheory.RgDs.Hilb

CompanionTheory.RealGroups.DiscreteSeries.DiscreteSeriesClassification.HilbertSpaceModels.HilbertSquareIntegrableOfSignature

theorem **hq_umem** (g : Formalism.REAL.UJ ε) : Formalism.MainNotions.S.umat g ∈ UJset ε

theorem **hq_one** (hrep : IsURepOn (UJset ε) (sigLam ε Ψ T.w)) : sigLam ε Ψ T.w (Formalism.MainNotions.S.umat (1 : Formalism.REAL.UJ ε)) = 1

theorem **hq_mul** (hrep : IsURepOn (UJset ε) (sigLam ε Ψ T.w)) (a b : Formalism.REAL.UJ ε) : sigLam ε Ψ T.w (Formalism.MainNotions.S.umat (a ∗ b)) = (sigLam ε Ψ T.w (Formalism.MainNotions.S.umat a)).comp (sigLam ε Ψ T.w (Formalism.MainNotions.S.umat b))

theorem **hq_iso** (hrep : IsURepOn (UJset ε) (sigLam ε Ψ T.w)) (g : Formalism.REAL.UJ ε) (v : Hlam ε Ψ T.w) : ‖sigLam ε Ψ T.w (Formalism.MainNotions.S.umat g) v‖ = ‖v‖

theorem **hq_memLp_of_ne** (hrep : IsURepOn (UJset ε) (sigLam ε Ψ T.w)) (hirr : Formalism.MainNotions.S.IsIrredOn (Formalism.MainNotions.S.UJset ε) (sigLam ε Ψ T.w)) (w₀ x₀ : Hlam ε Ψ T.w) (hx₀ : x₀ ≠ 0) (h0 : MemLp (fun g : Formalism.REAL.UJ ε => ⟪w₀, sigLam ε Ψ T.w (Formalism.MainNotions.S.umat g) x₀⟫_ℂ) 2 (Formalism.REAL.muG ε)) (x : Hlam ε Ψ T.w) : MemLp (fun g : Formalism.REAL.UJ ε => ⟪w₀, sigLam ε Ψ T.w (Formalism.MainNotions.S.umat g) x⟫_ℂ) 2 (Formalism.REAL.muG ε)

theorem **hq_inner** (hcoe : ∀ g ∈ UJset ε, ∀ x : Hlam ε Ψ T.w, ((sigLam ε Ψ T.w g x : Hlam ε Ψ T.w) : L2 ε) = rreg ε g x) (y z : Hlam ε Ψ T.w) (g : Formalism.REAL.UJ ε) : ⟪y, sigLam ε Ψ T.w (Formalism.MainNotions.S.umat g) z⟫_ℂ = inner ℂ (y : L2 ε) (rreg ε (Formalism.MainNotions.S.umat g) (z : L2 ε))

theorem **hq_w_ne** (hP : IsPsiFor ε lam T Ψ) : uniformisationL2 ε Ψ T.w T.w ≠ 0

theorem **hq_coef_all** (hP : IsPsiFor ε lam T Ψ) (hcoe : ∀ g ∈ UJset ε, ∀ x : Hlam ε Ψ T.w, ((sigLam ε Ψ T.w g x : Hlam ε Ψ T.w) : L2 ε) = rreg ε g x) (hrep : IsURepOn (UJset ε) (sigLam ε Ψ T.w)) (hF : ∀ g ∈ UJset ε, ∀ v v' : T.V, inner ℂ (uniformisationL2 ε Ψ T.w v') (rreg ε g (uniformisationL2 ε Ψ T.w v)) = ((‖uniformisationL2 ε Ψ T.w T.w‖ ^ 2 : ℝ) : ℂ) ∗ inner ℂ v' (Ψ g v)) (hirr : Formalism.MainNotions.S.IsIrredOn (Formalism.MainNotions.S.UJset ε) (sigLam ε Ψ T.w)) (x : Hlam ε Ψ T.w) : MemLp (fun g : Formalism.REAL.UJ ε => inner ℂ (uniformisationL2 ε Ψ T.w T.w) (rreg ε (Formalism.MainNotions.S.umat g) (x : L2 ε))) 2 (Formalism.REAL.muG ε)

theorem **hq_sqInt** (hP : IsPsiFor ε lam T Ψ) (hcoe : ∀ g ∈ UJset ε, ∀ x : Hlam ε Ψ T.w, ((sigLam ε Ψ T.w g x : Hlam ε Ψ T.w) : L2 ε) = rreg ε g x) (hrep : IsURepOn (UJset ε) (sigLam ε Ψ T.w)) (hF : ∀ g ∈ UJset ε, ∀ v v' : T.V, inner ℂ (uniformisationL2 ε Ψ T.w v') (rreg ε g (uniformisationL2 ε Ψ T.w v)) = ((‖uniformisationL2 ε Ψ T.w T.w‖ ^ 2 : ℝ) : ℂ) ∗ inner ℂ v' (Ψ g v)) (hirr : Formalism.MainNotions.S.IsIrredOn (Formalism.MainNotions.S.UJset ε) (sigLam ε Ψ T.w)) : Formalism.MainNotions.S.IsSqIntOn ε (sigLam ε Ψ T.w)

theorem **hilbert_square_integrable_of_signature** : StepSig → StepSqInt

CompanionTheory.RealGroups.DiscreteSeries.DiscreteSeriesClassification.HilbertSpaceModels.HilbertStableHolds

theorem **hilbert_stable_holds** : StepStable

theorem **hilbert_datum_holds** : StepDatum

CompanionTheory.RealGroups.DiscreteSeries.DiscreteSeriesClassification.HilbertSpaceModels.WeightSpaceLeHilbert

theorem **weight_space_le_hilbert** {n : ℕ} {ε : Fin n → Bool} {lam : Fin n → ℝ} {T : S7.KType ε (lowK ε lam)} (Ψ : Matrix (Fin n) (Fin n) ℂ →ₗ[ℂ] (T.V →L[ℂ] T.V)) : V1 ε Ψ ≤ Hlam ε Ψ T.w

theorem **hilb_of** (h1 : StepStable) (h2 : StepDatum) (h0 : StepSig) (h3 : StepMoll) (h4 : StepWtSpan) (h5 : StepStable → StepMoll → StepWtSpan → StepCone) (h6 : StepStable → StepMoll → StepWtSpan → StepDatum → StepLowEq) (h7 : StepDatum → StepIrred) (h8 : StepDatum → StepFormulaF) (h10 : StepSig → StepSqInt) (h9 : StepVogan) : Hilb

CompanionTheory.RealGroups.DiscreteSeries.DiscreteSeriesClassification.InfinitesimalCharacters.DiscreteSeriesInfinitesimalCharacter

theorem **dsInfChar'** : GlobalTheory.Dec.DSInfChar

CompanionTheory.RealGroups.DiscreteSeries.DiscreteSeriesClassification.InfinitesimalCharacters.DsInfCharOf

theorem **dsInfChar_of** (hC : CapToGelf) (hG : GlRelUnit) (hM : MapQdet) (hW : CoefWord) (hY : CycEtr) (hS : SmoothKFin) (hD : DSIff) (hF : CoefSmooth) (hcap : PI.DSInfCharCap.DSInfCharCap) : GlobalTheory.Dec.DSInfChar

CompanionTheory.RealGroups.DiscreteSeries.DiscreteSeriesClassification.KTypesAbove

theorem **dsExists** : GlobalTheory.Dec.DSExists

theorem **kTypesAbove** : GlobalTheory.Dec.KTypesAbove

theorem **dsInfChar** : GlobalTheory.Dec.DSInfChar

theorem **discreteSeriesUpqExists** : GlobalTheory.Published.DiscreteSeriesUpqExists

theorem **discreteSeriesExists** : GlobalTheory.Published.DiscreteSeriesExists

theorem **discreteSeriesCoverExists** : GlobalTheory.Published.DiscreteSeriesCoverExists

theorem **classUsed** : ClassUsed

CompanionTheory.RealGroups.DiscreteSeries.DiscreteSeriesClassification.RhoRepresentations.EquivSection

theorem **equivariant_section_holds** : EquivSection

CompanionTheory.RealGroups.DiscreteSeries.DiscreteSeriesClassification.RhoRepresentations.RhoRep

lemma **rr_toMK_one** (ε : Fin n → Bool) : toMK (1 : KG ε) = 1

lemma **rr_toMK_mul** {ε : Fin n → Bool} (k k' : KG ε) : toMK (k ∗ k') = toMK k ∗ toMK k'

lemma **rr_toMK_inv** {ε : Fin n → Bool} (k : KG ε) : (toMK k)⁻¹ = toMK k⁻¹

lemma **rr_adC_one** (p q : Fin n × Fin n) : adC (1 : Matrix (Fin n) (Fin n) ℂ) p q = if p = q then 1 else 0

lemma **rr_adC_mul** (A B : Matrix (Fin n) (Fin n) ℂ) (p q : Fin n × Fin n) : adC (A ∗ B) p q = ∑ r : Fin n × Fin n, adC B p r ∗ adC A r q

lemma **rr_prod_ite** {l : ℕ} (w x : Fin l → Fin n × Fin n) : (∏ i, (if w i = x i then (1 : ℂ) else 0)) = if w = x then 1 else 0

lemma **rr_op_one** {ε : Fin n → Bool} {Λ : Fin n → ℝ} (T : KType ε Λ) : T.op 1 = 1

lemma **rr_op_mul** {ε : Fin n → Bool} {Λ : Fin n → ℝ} (T : KType ε Λ) (k k' : KG ε) (v : T.V) : T.op (k ∗ k') v = T.op k (T.op k' v)

theorem **rho_representation_holds** : CompanionTheory.RgDs.K.RhoRep

CompanionTheory.RealGroups.FormalDegrees.ArchimedeanFormalDegrees.DegreeNormalization.DegPsi

theorem **degPsi** : DegPsi

CompanionTheory.RealGroups.FormalDegrees.ArchimedeanFormalDegrees.DegreeNormalization.GapNorm

theorem **gapNorm_of** (hD : DegPsi) : GapNorm

CompanionTheory.RealGroups.FormalDegrees.ArchimedeanFormalDegrees.ExplicitFunctionSquareIntegrable

theorem **psiNorm** : PsiNorm

theorem **explicit_function_square_integrable** : ExplicitFunctionSquareIntegrable

CompanionTheory.RealGroups.FormalDegrees.ArchimedeanFormalDegrees.FormalDegreeFormula.CKContinuous

theorem **cK_continuous** {ε : Fin n → Bool} {Λ : Fin n → ℝ} (T : KType ε Λ) : Continuous (cK T)

theorem **op_norm** {ε : Fin n → Bool} {Λ : Fin n → ℝ} (T : KType ε Λ) (k : KG ε) (v : T.V) : ‖T.op k v‖ = ‖v‖

theorem **cK_le_one** {ε : Fin n → Bool} {Λ : Fin n → ℝ} (T : KType ε Λ) (k : KG ε) : cK T k ≤ 1

theorem **prod_cpair** (ε : Fin n → Bool) (f : Fin n → Fin n → ℝ) : ∏ a : Fin n, ∏ b ∈ univ.filter (fun b : Fin n => a < b ∧ ε a = ε b), f a b = ∏ p : CPair ε, f p.1.1 p.1.2

theorem **kNull_of** (hW : MomW) (hP : CoefPow) : KNull

CompanionTheory.RealGroups.FormalDegrees.ArchimedeanFormalDegrees.FormalDegreeFormula.CkModel

theorem **blkMinor_diag_mul** (ε : Fin n → Bool) (c : Fin n) (d : Fin n → ℂ) (X : Matrix (Fin n) (Fin n) ℂ) : blkMinor ε c (diagonal d ∗ X) = (∏ q ∈ univ.filter (fun q : Fin n => ε q = ε c ∧ q ≤ c), d q) ∗ blkMinor ε c X

theorem **model_function_diag_mul** (ε : Fin n → Bool) {m : Fin n → ℤ} (hm : NonnegDom ε m) (d : Fin n → ℂ) (X : Matrix (Fin n) (Fin n) ℂ) : f0 ε m (diagonal d ∗ X) = (∏ a, d a ^ (m a).toNat) ∗ f0 ε m X

theorem **vsp_diag** {ε : Fin n → Bool} {m : Fin n → ℤ} (hm : NonnegDom ε m) {f : KG ε → ℂ} (hf : f ∈ Vsp ε m) (y : KG ε) (d : Fin n → ℂ) (hy : toMK y = diagonal d) (x : KG ε) : f (y ∗ x) = (∏ a, d a ^ (m a).toNat) ∗ f x

theorem **ev_apply** {ε : Fin n → Bool} {m : Fin n → ℤ} (f : MV ε m) : ev ε m f = f.fn 1

theorem **cpart1_Eaa** (ε : Fin n → Bool) (a : Fin n) : cpart1 ε (bdPart ε (single a a (1 : ℂ))) = 0

theorem **cpart2_Eaa** (ε : Fin n → Bool) (a : Fin n) : cpart2 ε (bdPart ε (single a a (1 : ℂ))) = (-Complex.I) • single a a (1 : ℂ)

theorem **exp_Eaa** (a : Fin n) (t : ℝ) : NormedSpace.exp ((t : ℂ) • ((-Complex.I) • single a a (1 : ℂ))) = diagonal (fun c => if c = a then Complex.exp (-(Complex.I ∗ t)) else 1)

theorem **ev_dTau_Eaa** {ε : Fin n → Bool} {m : Fin n → ℤ} (hm : NonnegDom ε m) (a : Fin n) (f : MV ε m) : ev ε m (dTau ε m (single a a 1) f) = ((m a : ℤ) : ℂ) ∗ ev ε m f

theorem **model_op** {ε : Fin n → Bool} {m : Fin n → ℤ} (hm : NonnegDom ε m) (k : KG ε) : (modelKType ε hm).op k = tauC ε m k

theorem **model_dτ** {ε : Fin n → Bool} {m : Fin n → ℤ} (hm : NonnegDom ε m) : (modelKType ε hm).dτ = dTau ε m

theorem **model_w** {ε : Fin n → Bool} {m : Fin n → ℤ} (hm : NonnegDom ε m) : (modelKType ε hm).w = wN ε m

theorem **inner_uEv** {ε : Fin n → Bool} {m : Fin n → ℤ} (f : MV ε m) : ⟪uEv ε m, f⟫_ℂ = ev ε m f

theorem **uEv_wt** {ε : Fin n → Bool} {m : Fin n → ℤ} (hm : NonnegDom ε m) (a : Fin n) : dTau ε m (single a a 1) (uEv ε m) = (((m a : ℤ) : ℝ) : ℂ) • uEv ε m

theorem **not_cone_neg_root** {a b : Fin n} (hab : a < b) (c : Fin n → Fin n → ℕ) (hc : ∀ a' b', c a' b' ≠ 0 → a' < b') (h : Pi.single b (1 : ℝ) - Pi.single a 1 = ∑ a', ∑ b', (c a' b' : ℝ) • root a' b') : False

theorem **uEv_hw** {ε : Fin n → Bool} {m : Fin n → ℤ} (hm : NonnegDom ε m) {a b : Fin n} (hab : a < b) (he : ε a = ε b) : dTau ε m (single a b 1) (uEv ε m) = 0

theorem **evaluation_longest_weyl_one** {ε : Fin n → Bool} {m : Fin n → ℤ} : ev ε m (w0 ε m) = 1

theorem **uEv_eq** {ε : Fin n → Bool} {m : Fin n → ℤ} (hm : NonnegDom ε m) : uEv ε m = ((‖w0 ε m‖⁻¹ : ℝ) : ℂ) • wN ε m

theorem **ckModel** : CompanionTheory.RgFd.CkModel

CompanionTheory.RealGroups.FormalDegrees.ArchimedeanFormalDegrees.FormalDegreeFormula.CkTwist

theorem **twChi_one** {ε : Fin n → Bool} (c : ℤ) : twChi (ε

theorem **twChi_mul** {ε : Fin n → Bool} (c : ℤ) (x y : KG ε) : twChi c (x ∗ y) = twChi c x ∗ twChi c y

theorem **twChi_ne** {ε : Fin n → Bool} (c : ℤ) (k : KG ε) : twChi c k ≠ 0

theorem **norm_twChi** {ε : Fin n → Bool} (c : ℤ) (k : KG ε) : ‖twChi c k‖ = 1

theorem **kop_one** {ε : Fin n → Bool} {Λ : Fin n → ℝ} (T : KType ε Λ) : T.op 1 = 1

theorem **kop_mul** {ε : Fin n → Bool} {Λ : Fin n → ℝ} (T : KType ε Λ) (x y : KG ε) : T.op (x ∗ y) = T.op x ∗ T.op y

theorem **twTau_apply** {ε : Fin n → Bool} {Λ : Fin n → ℝ} (T : KType ε Λ) (c : ℤ) (k : KG ε) : ((twTau T c k : unitary (T.V →L[ℂ] T.V)) : T.V →L[ℂ] T.V) = twChi c k • T.op k

theorem **twTau_cont** {ε : Fin n → Bool} {Λ : Fin n → ℝ} (T : KType ε Λ) (c : ℤ) : Continuous (twTau T c)

theorem **twTau_irred** {ε : Fin n → Bool} {Λ : Fin n → ℝ} (T : KType ε Λ) (c : ℤ) (W : Submodule ℂ T.V) (hW : ∀ k : KG ε, ∀ v ∈ W, ((twTau T c k : unitary (T.V →L[ℂ] T.V)) : T.V →L[ℂ] T.V) v ∈ W) : W = ⊥ ∨ W = ⊤

theorem **twTau_hasDeriv** {ε : Fin n → Bool} {Λ : Fin n → ℝ} (T : KType ε Λ) (c : ℤ) (X : Matrix (Fin n) (Fin n) ℂ) (hX : inK ε X) (γ : ℝ → KG ε) (hγ : ∀ t : ℝ, toMK (γ t) = NormedSpace.exp ((t : ℂ) • X)) : HasDerivAt (fun t => ((twTau T c (γ t) : unitary (T.V →L[ℂ] T.V)) : T.V →L[ℂ] T.V)) (twD T c X) 0

theorem **twD_wt** {ε : Fin n → Bool} {Λ : Fin n → ℝ} (T : KType ε Λ) (c : ℤ) (a : Fin n) : twD T c (Matrix.single a a 1) T.w = (((Λ a + (c : ℝ) : ℝ)) : ℂ) • T.w

theorem **twD_hw** {ε : Fin n → Bool} {Λ : Fin n → ℝ} (T : KType ε Λ) (c : ℤ) (a b : Fin n) (hab : a < b) (he : ε a = ε b) : twD T c (Matrix.single a b 1) T.w = 0

theorem **cK_twistKType** {ε : Fin n → Bool} {Λ : Fin n → ℝ} (T : KType ε Λ) (c : ℤ) (k : KG ε) : cK (twistKType T c) k = cK T k

theorem **ckTwist** : CkTwist

theorem **mexp_add** (ε : Fin n → Bool) (m₁ m₂ : Fin n → ℤ) (a : Fin n) : mexp ε (m₁ + m₂) a = mexp ε m₁ a + mexp ε m₂ a

theorem **model_functions_multiply** : ModelFunctionsMultiply

CompanionTheory.RealGroups.FormalDegrees.ArchimedeanFormalDegrees.FormalDegreeFormula.CoefPow

theorem **coefPow** : CoefPow

CompanionTheory.RealGroups.FormalDegrees.ArchimedeanFormalDegrees.FormalDegreeFormula.ModelFunctionsMultiply

theorem **cK_transport** {ε : Fin n → Bool} {Λ : Fin n → ℝ} (T T' : KType ε Λ) (k : KG ε) : cK T k = cK T' k

theorem **cK_eq_f0K** (hM : CkModel) (hT : CkTwist) {ε : Fin n → Bool} {Λ : Fin n → ℝ} (T : KType ε Λ) (m : Fin n → ℤ) (hm : NonnegDom ε m) (c : ℤ) (hΛ : ∀ a, Λ a = (m a : ℝ) + c) (k : KG ε) : cK T k = ‖f0K ε m k‖

theorem **exists_shift** {ε : Fin n → Bool} {Λ : Fin n → ℝ} (hint : ∀ a, ∃ m : ℤ, Λ a = m) (hdom : ∀ a b, a < b → ε a = ε b → Λ b ≤ Λ a) : ∃ (m : Fin n → ℤ) (c : ℤ), NonnegDom ε m ∧ ∀ a, Λ a = (m a : ℝ) + c

theorem **coefMul_dom** (hM : CkModel) (hT : CkTwist) (hF : ModelFunctionsMultiply) {ε : Fin n → Bool} {Λ₁ Λ₂ : Fin n → ℝ} (h1 : (∀ a, ∃ m : ℤ, Λ₁ a = m) ∧ ∀ a b, a < b → ε a = ε b → Λ₁ b ≤ Λ₁ a) (h2 : (∀ a, ∃ m : ℤ, Λ₂ a = m) ∧ ∀ a b, a < b → ε a = ε b → Λ₂ b ≤ Λ₂ a) (T₁ : KType ε Λ₁) (T₂ : KType ε Λ₂) (T₃ : KType ε (Λ₁ + Λ₂)) (k : KG ε) : cK T₃ k = cK T₁ k ∗ cK T₂ k

theorem **dvec_int_dom** (ε : Fin n → Bool) : (∀ a, ∃ m : ℤ, Dvec n a = m) ∧ ∀ a b, a < b → ε a = ε b → Dvec n b ≤ Dvec n a

theorem **coefPow_of** (hM : CkModel) (hT : CkTwist) (hF : ModelFunctionsMultiply) : CoefPow

CompanionTheory.RealGroups.FormalDegrees.ArchimedeanFormalDegrees.MomentRecursion.MomStep

theorem **mom_step** {Ω Ω' : Type} [MeasurableSpace Ω] [MeasurableSpace Ω'] {ν : Measure Ω} {ν' : Measure Ω'} {V : Ω → ℝ} {V' : Ω' → ℝ} (hV : Measurable V) (hV' : Measurable V') (h0 : ∀ x, 0 ≤ V x ∧ V x ≤ 1) (h0' : ∀ y, 0 ≤ V' y ∧ V' y ≤ 1) (he : ∀ j : ℕ, 1 ≤ j → ∫⁻ x, ENNReal.ofReal (V x ^ j) ∂ν = ∫⁻ y, ENNReal.ofReal (V' y ^ j) ∂ν') (hf : ∀ j : ℕ, 1 ≤ j → ∫⁻ x, ENNReal.ofReal (V x ^ j) ∂ν ≠ ⊤) (M : ℕ) : ∀ j : ℕ, 1 ≤ j → ∫⁻ x, ENNReal.ofReal (V x ^ j ∗ (1 - V x) ^ M) ∂ν = ∫⁻ y, ENNReal.ofReal (V' y ^ j ∗ (1 - V' y) ^ M) ∂ν'

theorem **mom_sum** {A : Type} [MeasurableSpace A] (m : Measure A) {W : A → ℝ} (hW : Measurable W) (h : ∀ x, 0 ≤ W x ∧ W x ≤ 1) : m {x | 0 < W x} = ∑' M : ℕ, ∫⁻ x, ENNReal.ofReal (W x ^ 1 ∗ (1 - W x) ^ M) ∂m

theorem **momA** : MomA

theorem **momB** : MomB

theorem **momW_of** : MomA → MomB → MomW

theorem **momW** : MomW

CompanionTheory.RealGroups.FormalDegrees.ArchimedeanFormalDegrees.NormIndOfSteps

theorem **hsInt** : HsInt

theorem **psiNorm_of** (hI : NormInd) : PsiNorm

theorem **normInd_of_steps** (hP : CoefPow) (h : MomW → CoefPow → KNull → HsPol → PhiMul → PhiLe → PhiCont → HsInt → GapNorm → NormInd) : NormInd

CompanionTheory.RealGroups.FormalDegrees.ArchimedeanFormalDegrees.PositiveConjugation.DzSubMuNextNonneg

theorem **inP_entry_zero** {ε : Fin n → Bool} {X : Matrix (Fin n) (Fin n) ℂ} (hX : inP ε X) {a b : Fin n} (h : ε a = ε b) : X a b = 0

theorem **exists_pmat** {ε : Fin n → Bool} {X : Matrix (Fin n) (Fin n) ℂ} (hX : inP ε X) : ∃ z : PCoord ε → ℂ, pmat z = X

theorem **one_le_lead_exp** {ε : Fin n → Bool} {X : Matrix (Fin n) (Fin n) ℂ} (hX : inP ε X) (i : ℕ) (hi : i ≤ n) : 1 ≤ (lead i (NormedSpace.exp X)).re

theorem **dz_sub_muNext_nonneg** (i : Fin n) : 0 ≤ Dz n i - muNext (Dz n) i

theorem **phiLe** : PhiLe

CompanionTheory.RealGroups.FormalDegrees.ArchimedeanFormalDegrees.PositiveConjugation.HsOpOpMul

theorem **polar_pP** {ε : Fin n → Bool} {g : Matrix (Fin n) (Fin n) ℂ} (hg : inUJ ε g) : ∃ k : KG ε, g = toMK k ∗ pP g

theorem **hsOp_op_mul** {ε : Fin n → Bool} {Λ : Fin n → ℝ} (T : KType ε Λ) (k : KG ε) (A : T.V →L[ℂ] T.V) : hsOp (T.op k ∗ A) = hsOp A

theorem **hsPol** : HsPol

CompanionTheory.RealGroups.FormalDegrees.ArchimedeanFormalDegrees.PositiveConjugation.PhiCont

theorem **continuous_pP** (ε : Fin n → Bool) : Continuous fun g : UJ ε => pP (toM g)

theorem **pP_posDef** {ε : Fin n → Bool} (g : UJ ε) : (pP (toM g)).PosDef

theorem **phiCont** : PhiCont

CompanionTheory.RealGroups.FormalDegrees.ArchimedeanFormalDegrees.PositiveConjugation.PhiMul

theorem **posDef_conj_toMK** {ε : Fin n → Bool} (k : KG ε) {x : Matrix (Fin n) (Fin n) ℂ} (hx : x.PosDef) : (toMK k⁻¹ ∗ x ∗ toMK k).PosDef

theorem **muNext_add_smul** (μ D : Fin n → ℤ) (t : ℤ) (i : Fin n) : muNext (μ + t • D) i = muNext μ i + t ∗ muNext D i

theorem **phiM_add_smul** (μ D : Fin n → ℤ) (t : ℕ) {y : Matrix (Fin n) (Fin n) ℂ} (hy : y ∈ OmegaSet n) : PhiM (μ + (t : ℤ) • D) y = PhiM μ y ∗ PhiM D y ^ t

theorem **phiMul** : PhiMul

CompanionTheory.RealGroups.FormalDegrees.ArchimedeanFormalDegrees.SignDefiniteness.ExplicitFunctionSquareIntegrable

theorem **pl_opNorm_le** {V : Type∗} [NormedAddCommGroup V] [InnerProductSpace ℂ V] [FiniteDimensional ℂ V] (A : V →L[ℂ] V) : ‖A‖ ^ 2 ≤ (Module.finrank ℂ V : ℝ) ∗ hsOp A

theorem **square_integrable_of_norm** : PsiNorm → CompanionTheory.RgDs.ExplicitFunctionSquareIntegrable

CompanionTheory.RealGroups.FormalDegrees.ArchimedeanFormalDegrees.SignDefiniteness.NormDef

theorem **nd_const_sign** {ε : Fin n → Bool} (hq : q0 ε = 0) : ∃ s : Bool, ∀ a, ε a = s

theorem **nd_Jm** {ε : Fin n → Bool} {s : Bool} (hs : ∀ a, ε a = s) : Jm ε = (if s then (1 : ℂ) else -1) • (1 : Matrix (Fin n) (Fin n) ℂ)

theorem **nd_mem_Kset** {ε : Fin n → Bool} (hq : q0 ε = 0) {g : Matrix (Fin n) (Fin n) ℂ} (hg : inUJ ε g) : g ∈ Kset ε

theorem **nd_hsOp_op** {ε : Fin n → Bool} {Λ : Fin n → ℝ} (T : KType ε Λ) (k : KG ε) : hsOp (T.op k) = (Module.finrank ℂ T.V : ℝ)

theorem **nd_psiM_K** {ε : Fin n → Bool} {Λ : Fin n → ℝ} (T : KType ε Λ) (μ : Fin n → ℤ) (k : KG ε) : PsiM T μ (toMK k) = T.op k

theorem **nd_pev_zero** {ε : Fin n → Bool} {s : Bool} (hs : ∀ a, ε a = s) (z : PCoord ε → ℂ) : pev z = 0

theorem **nd_jac_one** {ε : Fin n → Bool} {s : Bool} (hs : ∀ a, ε a = s) (z : PCoord ε → ℂ) : Jac z = 1

theorem **nd_isEmpty** {ε : Fin n → Bool} {s : Bool} (hs : ∀ a, ε a = s) : IsEmpty (PCoord ε)

theorem **nd_counts** {ε : Fin n → Bool} (hq : q0 ε = 0) : (pOf ε = n ∧ qOf ε = 0) ∨ (pOf ε = 0 ∧ qOf ε = n)

theorem **nd_cPQ** {ε : Fin n → Bool} (hq : q0 ε = 0) : cPQ ε = 1

theorem **nd_muCD_univ** {ε : Fin n → Bool} (hq : q0 ε = 0) : muCD ε Set.univ = 1

theorem **nd_finrank** {ε : Fin n → Bool} (hq : q0 ε = 0) {lam : Fin n → ℝ} (hl : StrictAnti lam) (T : KType ε (lowK ε lam)) : (Module.finrank ℂ T.V : ℝ) = dimVlam lam

theorem **normDef** : NormDef

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theorem **val_le_one** (t : kˣ) (h : nabs k t ≤ 1) : valuation k (t : k) ≤ 1

theorem **Zt_val** (d : ℕ) (t : kˣ) : ((Zt d t : GL (Fin (d + d)) k) : Matrix (Fin (d + d)) (Fin (d + d)) k) = diagonal fun i => ((zd d t i : kˣ) : k)

theorem **Zt_inv_val** (d : ℕ) (t : kˣ) : (((Zt d t)⁻¹ : GL (Fin (d + d)) k) : Matrix (Fin (d + d)) (Fin (d + d)) k) = diagonal fun i => (((zd d t i)⁻¹ : kˣ) : k)

theorem **Zt_mem** (d : ℕ) (t : kˣ) : Zt d t ∈ parab k (blk2 d)

theorem **leviProj_Zt** (d : ℕ) (t : kˣ) : leviProj k (blk2 d) (Zt d t, Zt_mem d t) = ![Matrix.GeneralLinearGroup.scalar (Fin d) t, 1]

theorem **conj_mem** (d : ℕ) (t : kˣ) (ht : valuation k (t : k) ≤ 1) {m : ℕ} {z : GL (Fin (d + d)) k} (hz : z ∈ GroundTheory.Automorphic.GLnLocal.congruenceSubgroup k (d + d) m) (hzo : z ∈ GroundTheory.Automorphic.Parabolic.oppositeUnipotentRadical k (d + d) (blk (blk2 d))) : ∀ i j, valuation k ((((Zt d t)⁻¹ ∗ z ∗ Zt d t : GL (Fin (d + d)) k) : Matrix _ _ k) i j - (1 : Matrix (Fin (d + d)) (Fin (d + d)) k) i j) ≤ valuation k (GroundTheory.Automorphic.GLnLocal.uniformizer k : k) ^ m ∧ (((Zt d t)⁻¹ ∗ z ∗ Zt d t : GL (Fin (d + d)) k) : Matrix _ _ k) i j ∈ 𝒪[k]

theorem **conj_mem'** (d : ℕ) (t : kˣ) (ht : valuation k (t : k) ≤ 1) {m : ℕ} {z : GL (Fin (d + d)) k} (hz : z ∈ GroundTheory.Automorphic.GLnLocal.congruenceSubgroup k (d + d) m ⊓ GroundTheory.Automorphic.Parabolic.oppositeUnipotentRadical k (d + d) (blk (blk2 d))) : (Zt d t)⁻¹ ∗ z ∗ Zt d t ∈ GroundTheory.Automorphic.GLnLocal.congruenceSubgroup k (d + d) m

theorem **avL_apply** (π : Representation ℂ G V) (K : Subgroup G) (H : Subgroup K) [Fintype (K ⧸ H)] (x : V) : avL π K H x = ∑ q : K ⧸ H, π ((q.out : K) : G) x

theorem **avL_fixed** (π : Representation ℂ G V) (K : Subgroup G) (H : Subgroup K) [Fintype (K ⧸ H)] {x : V} (hx : ∀ h ∈ H, π ((h : K) : G) x = x) (κ : G) (hκ : κ ∈ K) : π κ (avL π K H x) = avL π K H x

theorem **hfixH** (π : Representation ℂ G X) (K : Subgroup G) (Z : G) (x : X) (hx : ∀ κ ∈ K, π κ x = x) (h : K) (hh : h ∈ (K.map (MulAut.conj Z).toMonoidHom).subgroupOf K) : π (h : G) (π Z x) = π Z x

theorem **ineq** (π : Representation ℂ G X) (B : X →ₗ[ℂ] X →ₗ⋆[ℂ] ℂ) (hB1 : ∀ v w, B w v = starRingEnd ℂ (B v w)) (hB2 : ∀ v, v ≠ 0 → 0 < (B v v).re) (hB3 : ∀ g v w, B (π g v) (π g w) = B v w) (K : Subgroup G) (H : Subgroup K) [Fintype (K ⧸ H)] (Z : G) (v : X) (hv : v ≠ 0) (lam : ℂ) (hT : (Fintype.card (K ⧸ H) : ℂ)⁻¹ • avL π K H (π Z v) = lam • v) (hfix : ∀ κ ∈ K, π κ (lam • v) = lam • v) : ‖lam‖ ≤ 1

theorem **fixval** {d : ℕ} {W : Type} [AddCommGroup W] [Module ℂ W] (τ : Representation ℂ (Levi k fun _ : Fin 2 => Fin d) W) (K : Subgroup (GL (Fin (d + d)) k)) (v : IndSp (blk2 d) τ) (hv : v ∈ fixed (parInd (blk2 d) τ) K) (g κ : GL (Fin (d + d)) k) (hκ : κ ∈ K) : v.1.1 (g ∗ κ) = v.1.1 g

theorem **law** {d : ℕ} {W : Type} [AddCommGroup W] [Module ℂ W] (τ : Representation ℂ (Levi k fun _ : Fin 2 => Fin d) W) (p : parab k (blk2 d)) (g : GL (Fin (d + d)) k) (f : IndSp (blk2 d) τ) : f.1.1 ((p : GL (Fin (d + d)) k) ∗ g) = (deltaHalf k (fun _ : Fin 2 => Fin d) (leviProj k (blk2 d) p) : ℂ) • τ (leviProj k (blk2 d) p) (f.1.1 g)

theorem **act** {d : ℕ} {W : Type} [AddCommGroup W] [Module ℂ W] (τ : Representation ℂ (Levi k fun _ : Fin 2 => Fin d) W) (g x : GL (Fin (d + d)) k) (f : IndSp (blk2 d) τ) : (parInd (blk2 d) τ g f).1.1 x = f.1.1 (x ∗ g)

theorem **zval** {d : ℕ} {W : Type} [AddCommGroup W] [Module ℂ W] (τ : Representation ℂ (Levi k fun _ : Fin 2 => Fin d) W) (t : kˣ) (c : ℂ) (htv : valuation k (t : k) ≤ 1) (hc : ∀ w, τ ![Matrix.GeneralLinearGroup.scalar (Fin d) t, 1] w = c • w) {m : ℕ} (hm : 1 ≤ m) (v : IndSp (blk2 d) τ) (hv : v ∈ fixed (parInd (blk2 d) τ) (GroundTheory.Automorphic.GLnLocal.congruenceSubgroup k (d + d) m)) (κ : GL (Fin (d + d)) k) (hκ : κ ∈ GroundTheory.Automorphic.GLnLocal.congruenceSubgroup k (d + d) m) : (parInd (blk2 d) τ (Zt d t) v).1.1 κ = ((deltaHalf k (fun _ : Fin 2 => Fin d) ![Matrix.GeneralLinearGroup.scalar (Fin d) t, 1] : ℂ) ∗ c) • v.1.1 1

theorem **modulus_scalar_block** (d : ℕ) (t : kˣ) : modulus k (fun _ : Fin 2 => Fin d) ![Matrix.GeneralLinearGroup.scalar (Fin d) t, 1] = nabs k t ^ (d ∗ d)

theorem **bound_main** : Bound k

theorem **boundB** (k : Type) [Field k] [ValuativeRel k] [TopologicalSpace k] [IsNonarchimedeanLocalField k] : Bound k

CompanionTheory.RealGroups.GeneralUnitaryRepresentations.ArchimedeanJacquetSegments.ComapTwKer

theorem **comap_twKer** (ρ : Representation ℂ G V) (N₁ : Subgroup G) (θ₁ : N₁ →∗ ℂˣ) (σ : Representation ℂ M (TwJacquet ρ N₁ θ₁)) (s : M →∗ G) (hσ : ∀ m v, σ m (Submodule.Quotient.mk v) = Submodule.Quotient.mk (ρ (s m) v)) (N₂ : Subgroup M) (θ₂ : N₂ →∗ ℂˣ) (N : Subgroup G) (θ : N →∗ ℂˣ) (h1 : ∀ u : N₁, ∃ h : (u : G) ∈ N, θ (u, h) = θ₁ u) (h2 : ∀ a : N₂, ∃ h : s a ∈ N, θ (s a, h) = θ₂ a) (h3 : ∀ u : N, ∃ u₁ : N₁, ∃ a : N₂, (u : G) = u₁ ∗ s a) : (twKer σ N₂ θ₂).comap (twKer ρ N₁ θ₁).mkQ = twKer ρ N θ

theorem **Q_surj** (ψ : AddChar k Circle) {n : ℕ} : ∀ (j : ℕ) (τ : MRep k (n + j)), Function.Surjective (Q k ψ j τ) | 0, τ => Submodule.mkQ_surjective _ | j + 1, τ => (Q_surj ψ j (MRep.phiM k ψ τ)).comp (Submodule.mkQ_surjective _) theorem ker_Q (ψ : AddChar k Circle) {n : ℕ} : ∀ (j : ℕ) (τ : MRep k (n + j)), LinearMap.ker (Q k ψ j τ) = twKer τ.ρ (derUnipM k (n + j) n) (derCharM k ψ (n + j) n) | 0, τ => by show LinearMap.ker (twKer τ.ρ (MirUin k n) 1).mkQ = _ rw [Submodule.ker_mkQ] exact GlobalTheory.T.mirabolic_twisted_kernel_derivative_zero k ψ n τ.V τ.ρ | j + 1, τ => by have hnm : n ≤ n + j

theorem **Q_equiv** (ψ : AddChar k Circle) {n : ℕ} : ∀ (j : ℕ) (τ : MRep k (n + j)) (g : GL (Fin n) k) (v : τ.V), Q k ψ j τ (τ.ρ (E k j g) v) = derivM k ψ j τ g (Q k ψ j τ v) | 0, τ, g, v => rfl | j + 1, τ, g, v => Q_equiv ψ j (MRep.phiM k ψ τ) g (Submodule.Quotient.mk v) theorem E_val_zero {n : ℕ} (g : GL (Fin n) k) (i l : Fin (n + 1)) : (((E k 0 g : Mir k n) : GL (Fin (n + 1)) k) : Matrix (Fin (n + 1)) (Fin (n + 1)) k) i l = if h : (i : ℕ) < n ∧ (l : ℕ) < n then (g : Matrix (Fin n) (Fin n) k) (i, h.1) (l, h.2) else (1 : Matrix (Fin (n + 1)) (Fin (n + 1)) k) i l

theorem **E_val_succ** {n j : ℕ} (g : GL (Fin n) k) (ih : ∀ i l : Fin (n + j + 1), (((E k j g : Mir k (n + j)) : GL (Fin (n + j + 1)) k) : Matrix (Fin (n + j + 1)) (Fin (n + j + 1)) k) i l = if h : (i : ℕ) < n ∧ (l : ℕ) < n then (g : Matrix (Fin n) (Fin n) k) (i, h.1) (l, h.2) else (1 : Matrix (Fin (n + j + 1)) (Fin (n + j + 1)) k) i l) (i l : Fin (n + j + 1 + 1)) : glEmbMat k (n + j + 1) (((E k j g : Mir k (n + j)) : GL (Fin (n + j + 1)) k) : Matrix (Fin (n + j + 1)) (Fin (n + j + 1)) k) i l = if h : (i : ℕ) < n ∧ (l : ℕ) < n then (g : Matrix (Fin n) (Fin n) k) (i, h.1) (l, h.2) else (1 : Matrix (Fin (n + j + 1 + 1)) (Fin (n + j + 1 + 1)) k) i l

theorem **E_val** {n : ℕ} (g : GL (Fin n) k) : ∀ (j : ℕ) (i l : Fin (n + j + 1)), (((E k j g : Mir k (n + j)) : GL (Fin (n + j + 1)) k) : Matrix (Fin (n + j + 1)) (Fin (n + j + 1)) k) i l = if h : (i : ℕ) < n ∧ (l : ℕ) < n then (g : Matrix (Fin n) (Fin n) k) (i, h.1) (l, h.2) else (1 : Matrix (Fin (n + j + 1)) (Fin (n + j + 1)) k) i l | 0, i, l => E_val_zero g i l | j + 1, i, l => E_val_succ g (E_val g j) i l theorem emb_val {n t N : ℕ} (h : n + t = N) (g : GL (Fin n) k × GL (Fin t) k) (a b : Fin n ⊕ Fin t) : (emb₂ k (stdSum h) g : Matrix (Fin N) (Fin N) k) (stdSum h a) (stdSum h b) = fromBlocks (g.1 : Matrix (Fin n) (Fin n) k) 0 0 (g.2 : Matrix (Fin t) (Fin t) k) a b

theorem **stdSum_inj** {n t N : ℕ} (h : n + t = N) {a b : Fin n ⊕ Fin t} : stdSum h a = stdSum h b ↔ a = b

theorem **one_std** {n t N : ℕ} (h : n + t = N) (a b : Fin n ⊕ Fin t) : (1 : Matrix (Fin N) (Fin N) k) (stdSum h a) (stdSum h b) = (1 : Matrix (Fin n ⊕ Fin t) (Fin n ⊕ Fin t) k) a b

theorem **E_eq** {n j : ℕ} (g : GL (Fin n) k) : ((E k j g : Mir k (n + j)) : GL (Fin (n + j + 1)) k) = emb₂ k (stdSum (show n + (j + 1) = n + j + 1 from rfl)) (g, 1)

theorem **inU_of_mem** {g : GL (Fin N) k} (hg : g ∈ unipRad k (two (stdSum h))) : InU (n

theorem **mem_of_inU** {g : GL (Fin N) k} (H : InU (n

theorem **Q'_surj** (ψ : AddChar k Circle) {n t N : ℕ} (h : n + t = N) {V : Type} [AddCommGroup V] [Module ℂ V] (π : Representation ℂ (GL (Fin N) k) V) : Function.Surjective (Q' k ψ h π)

theorem **Q'_equiv** (ψ : AddChar k Circle) {n t N : ℕ} (h : n + t = N) {V : Type} [AddCommGroup V] [Module ℂ V] (π : Representation ℂ (GL (Fin N) k) V) (g : GL (Fin n) k) (v : V) : Q' k ψ h π (π (emb₂ k (stdSum h) (g, 1)) v) = derRep k ψ h π g (Q' k ψ h π v)

theorem **derChar_val** (ψ : AddChar k Circle) {N n : ℕ} (x : derUnip k N n) : derChar k ψ N n x = Circle.toUnits (ψ (∑ p ∈ Finset.univ.filter (fun p : Fin N × Fin N => (p.2 : ℕ) = p.1 + 1 ∧ n ≤ (p.1 : ℕ)), ((x : GL (Fin N) k) : Matrix (Fin N) (Fin N) k) p.1 p.2))

theorem **ker_Q'** (ψ : AddChar k Circle) {n t N : ℕ} (h : n + t = N) {V : Type} [AddCommGroup V] [Module ℂ V] (π : Representation ℂ (GL (Fin N) k) V) : LinearMap.ker (Q' k ψ h π) = twKer π (derUnip k N n) (derChar k ψ N n)

theorem **derEquiv** (k : Type) [Field k] [ValuativeRel k] [TopologicalSpace k] [IsNonarchimedeanLocalField k] : DerEquiv k

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theorem **isOpen_fix** {G V : Type} [Group G] [TopologicalSpace G] [IsTopologicalGroup G] [AddCommGroup V] [Module ℂ V] (ρ : Representation ℂ G V) (v : V) (U : Set G) (hU : IsOpen U) (h1 : (1 : G) ∈ U) (hfix : ∀ u ∈ U, ρ u v = v) : IsOpen {g : G | ρ g v = v}

theorem **rc_pos** (m : Levi k fun _ : Fin 2 => Fin d) : 0 < rc m

theorem **rc_mul** (m m' : Levi k fun _ : Fin 2 => Fin d) : rc (m * m') = rc m * rc m'

theorem **chi_val** (s : ℝ) (m : Levi k fun _ : Fin 2 => Fin d) : ((chi k d s m : ℂˣ) : ℂ) = ((rc m ^ s : ℝ) : ℂ)

theorem **rc_of_KO** (q : parab k (blk2 d)) (hq : (q : GL (Fin (d + d)) k) ∈ KO k (Fin (d + d))) : rc (leviProj k (blk2 d) q) = 1

theorem **exists_pw** (x : GL (Fin (d + d)) k) : ∃ p : parab k (blk2 d), ∃ w ∈ KO k (Fin (d + d)), x = (p : GL (Fin (d + d)) k) * w

theorem **hh_spec** (p : parab k (blk2 d)) (w : GL (Fin (d + d)) k) (hw : w ∈ KO k (Fin (d + d))) : hh ((p : GL (Fin (d + d)) k) * w) = rc (leviProj k (blk2 d) p)

theorem **hh_pos** (x : GL (Fin (d + d)) k) : 0 < hh x

theorem **hh_mul_left** (p : parab k (blk2 d)) (x : GL (Fin (d + d)) k) : hh ((p : GL (Fin (d + d)) k) * x) = rc (leviProj k (blk2 d) p) * hh x

theorem **hh_mul_right** (x w : GL (Fin (d + d)) k) (hw : w ∈ KO k (Fin (d + d))) : hh (x * w) = hh x

theorem **mulFun_mem** (a : ℝ) (τ₁ τ₂ : Representation ℂ (Levi k fun _ : Fin 2 => Fin d) W) (hrel : ∀ m v, τ₂ m v = ((rc m ^ a : ℝ) : ℂ) • τ₁ m v) (f : IndSp (blk2 d) τ₁) : mulFun a f.1.1 ∈ indSpace (parab k (blk2 d)) ((twist τ₂ (deltaHalf k fun _ : Fin 2 => Fin d)).comp (leviProj k (blk2 d)))

theorem **mulFun_smooth** (a : ℝ) (τ₁ τ₂ : Representation ℂ (Levi k fun _ : Fin 2 => Fin d) W) (hrel : ∀ m v, τ₂ m v = ((rc m ^ a : ℝ) : ℂ) • τ₁ m v) (f : IndSp (blk2 d) τ₁) : ((mulFun a f.1.1, mulFun_mem a τ₁ τ₂ hrel f) : indSpace _ _) ∈ smoothPart (ind (parab k (blk2 d)) ((twist τ₂ (deltaHalf k fun _ : Fin 2 => Fin d)).comp (leviProj k (blk2 d))))

theorem **rel_symm** (a : ℝ) (τ₁ τ₂ : Representation ℂ (Levi k fun _ : Fin 2 => Fin d) W) (hrel : ∀ m v, τ₂ m v = ((rc m ^ a : ℝ) : ℂ) • τ₁ m v) : ∀ m v, τ₁ m v = ((rc m ^ (-a) : ℝ) : ℂ) • τ₂ m v

theorem **relChi** (s : ℝ) : ∀ m v, twist τ (chi k d s) m v = ((rc m ^ s : ℝ) : ℂ) • τ m v

theorem **F_apply** (s : ℝ) (g : GL (Fin (d + d)) k) (f : IndSp (blk2 d) τ) (y : GL (Fin (d + d)) k) : (F τ s g f).1.1 y = ((hh y ^ (-s) : ℝ) : ℂ) • ((hh (y * g) ^ s : ℝ) : ℂ) • f.1.1 (y * g)

theorem **cc_left** (g : GL (Fin (d + d)) k) (p : parab k (blk2 d)) (y : GL (Fin (d + d)) k) : cc g ((p : GL (Fin (d + d)) k) * y) = cc g y

theorem **isOpen_compact_open_subgroup** (g : GL (Fin (d + d)) k) : IsOpen (K1 (k

theorem **cc_right** (g y w : GL (Fin (d + d)) k) (hw : w ∈ K1 g) : cc g (y * w) = cc g y

theorem **cc_finite** (g : GL (Fin (d + d)) k) : (Set.range (cc (k

theorem **ind_val_sum** {σ : Representation ℂ (parab k (blk2 d)) W} {ι : Type} (s : Finset ι) (c : ι → ℂ) (u : ι → smoothPart (ind (parab k (blk2 d)) σ)) (y : GL (Fin (d + d)) k) : (∑ i ∈ s, c i • u i).1.1 y = ∑ i ∈ s, c i • (u i).1.1 y

theorem **expo** (g : GL (Fin (d + d)) k) (f : IndSp (blk2 d) τ) : ∃ (m : ℕ) (r : Fin m → ℝ) (u : Fin m → IndSp (blk2 d) τ), ∀ s, F τ s g f = ∑ i, ((Real.exp (r i * s) : ℝ) : ℂ) • u i

theorem **compact** (k : Type) [Field k] [ValuativeRel k] [TopologicalSpace k] [IsNonarchimedeanLocalField k] : Compact k

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theorem **sum_two** (d : ℕ) : ∑ _a : Fin 2, d = d + d

CompanionTheory.RealGroups.GeneralUnitaryRepresentations.ArchimedeanJacquetSegments.ComplementaryOf

theorem **irr_sub** {ρ : Representation ℂ G V} (h : ρ.IsIrreducible) (W : Submodule ℂ V) (hW : ∀ g, ∀ v ∈ W, ρ g v ∈ W) : W = ⊥ ∨ W = ⊤

theorem **irr_nontrivial** {ρ : Representation ℂ G V} (h : ρ.IsIrreducible) : Nontrivial V

@[simp] theorem **starF_apply** {M : Type} [AddCommGroup M] [Module ℂ M] (B : M →ₗ[ℂ] M →ₛₗ[starRingEnd ℂ] ℂ) (v w : M) : starF B v w = starRingEnd ℂ (B w v)

@[simp] theorem **resF_apply** {M : Type} [AddCommGroup M] [Module ℂ M] (B : M →ₗ[ℂ] M →ₛₗ[starRingEnd ℂ] ℂ) (W : Submodule ℂ M) (v w : W) : resF B W v w = B v w

theorem **exists_herm** {ρ : Representation ℂ G V} (h : HasSesq ρ) : ∃ B : V →ₗ[ℂ] V →ₛₗ[starRingEnd ℂ] ℂ, B ≠ 0 ∧ (∀ v w, B w v = starRingEnd ℂ (B v w)) ∧ ∀ g v w, B (ρ g v) (ρ g w) = B v w

theorem **nondeg** {ρ : Representation ℂ G V} (hi : ρ.IsIrreducible) (C : V →ₗ[ℂ] V →ₛₗ[starRingEnd ℂ] ℂ) (hC0 : C ≠ 0) (hCi : ∀ g v w, C (ρ g v) (ρ g w) = C v w) (v : V) (hv : v ≠ 0) : ∃ x, C v x ≠ 0

theorem **complementary_of** (h : Signature) : Complementary

CompanionTheory.RealGroups.GeneralUnitaryRepresentations.ArchimedeanJacquetSegments.ConjDual

theorem **irrAdm'** : GlobalTheory.Published.IrreducibleAdmissibleGL

theorem **conj_chi** (n : ℕ) (s : ℝ) (g : GL (Fin n) k) : starRingEnd ℂ ((detChar (Fin n) (absPow k (s : ℂ)) g : ℂˣ) : ℂ) = ((detChar (Fin n) (absPow k (s : ℂ)) g : ℂˣ) : ℂ)

theorem **conjDual** (k : Type) [Field k] [ValuativeRel k] [TopologicalSpace k] [IsNonarchimedeanLocalField k] : ConjDual k

CompanionTheory.RealGroups.GeneralUnitaryRepresentations.ArchimedeanJacquetSegments.ConjProd

theorem **semiInv_apply** {X Y : Type} [AddCommGroup X] [Module ℂ X] [AddCommGroup Y] [Module ℂ Y] (φ : X →ₛₗ[starRingEnd ℂ] Y) (hφ : Function.Bijective φ) (y : Y) : φ (semiInv φ hφ y) = y

theorem **semiInv_apply'** {X Y : Type} [AddCommGroup X] [Module ℂ X] [AddCommGroup Y] [Module ℂ Y] (φ : X →ₛₗ[starRingEnd ℂ] Y) (hφ : Function.Bijective φ) (x : X) : semiInv φ hφ (φ x) = x

theorem **update_comp** (φ : ∀ a, X a →ₛₗ[starRingEnd ℂ] Y a) [DecidableEq α] (x : ∀ a, X a) (i : α) (w : X i) : (fun a => φ a (Function.update x i w a)) = Function.update (fun a => φ a (x a)) i (φ i w)

theorem **tmap_tprod** (φ : ∀ a, X a →ₛₗ[starRingEnd ℂ] Y a) (x : ∀ a, X a) : tmap φ (PiTensorProduct.tprod ℂ x) = PiTensorProduct.tprod ℂ (fun a => φ a (x a))

theorem **tmap_comp** (φ : ∀ a, X a →ₛₗ[starRingEnd ℂ] Y a) (ψ : ∀ a, Y a →ₛₗ[starRingEnd ℂ] X a) (h : ∀ a (x : X a), ψ a (φ a x) = x) (t : ⊗[ℂ] a, X a) : tmap ψ (tmap φ t) = t

theorem **conjEquiv_piTensor** {α : Type} [Fintype α] {Γ X Y : α → Type} [∀ a, Group (Γ a)] [∀ a, AddCommGroup (X a)] [∀ a, Module ℂ (X a)] [∀ a, AddCommGroup (Y a)] [∀ a, Module ℂ (Y a)] (π : ∀ a, Representation ℂ (Γ a) (X a)) (σ : ∀ a, Representation ℂ (Γ a) (Y a)) (h : ∀ a, ConjEquiv (π a) (σ a)) : ConjEquiv (piTensor π) (piTensor σ)

theorem **star_deltaHalf** (g : Levi k m) : starRingEnd ℂ (deltaHalf k m g : ℂ) = deltaHalf k m g

theorem **indMap_apply** (e : (Σ a, m a) ≃ n) {ρ : Representation ℂ (Levi k m) V} {τ : Representation ℂ (Levi k m) W} (φ : V →ₛₗ[starRingEnd ℂ] W) (hφ : ∀ g v, φ (ρ g v) = τ g (φ v)) (f : IndSp e ρ) (g : GL n k) : (indMap e φ hφ f).1.1 g = φ (f.1.1 g)

theorem **conjEquiv_parInd** (e : (Σ a, m a) ≃ n) (ρ : Representation ℂ (Levi k m) V) (τ : Representation ℂ (Levi k m) W) (h : ConjEquiv ρ τ) : ConjEquiv (parInd e ρ) (parInd e τ)

theorem **conjProd** (k : Type) [Field k] [ValuativeRel k] [TopologicalSpace k] [IsNonarchimedeanLocalField k] : ConjProd k

CompanionTheory.RealGroups.GeneralUnitaryRepresentations.ArchimedeanJacquetSegments.CuspPair

theorem **key** {x y : IrrGL k} (hx : x.IsCusp) (hy : y.IsCusp) {N : ℕ} (hd : ∑ a, (![x, y] a).1 = N) {V : Type} [AddCommGroup V] [Module ℂ V] (τ : Representation ℂ (GL (Fin N) k) V) (hτ : IsSubquotient τ (prodIrr ![x, y] hd)) (u : Fin 2 → IrrGL k) (hu : ∑ a, (u a).1 = N) (hpos : ∀ a, 0 < (u a).1) (hj : 1 ≤ jm u hu τ) : u = ![x, y] ∨ u = ![y, x]

theorem **cuspPairIrr_of** (k : Type) [Field k] [ValuativeRel k] [TopologicalSpace k] [IsNonarchimedeanLocalField k] (h : GlobalTheory.BZSeg.NuPair k) : GlobalTheory.BZSeg.CuspPairIrr k

CompanionTheory.RealGroups.GeneralUnitaryRepresentations.ArchimedeanJacquetSegments.CuspPairIrr

theorem **nuPair** : GlobalTheory.BZSeg.NuPair k

theorem **cuspPairIrr** : GlobalTheory.BZSeg.CuspPairIrr k

CompanionTheory.RealGroups.GeneralUnitaryRepresentations.ArchimedeanJacquetSegments.DerFin

theorem **dMap_mk** {ρ : Representation ℂ G V} {σ : Representation ℂ G W} (f : ρ.IntertwiningMap σ) (v : V) : dMap N θ s hs f (Submodule.Quotient.mk v) = Submodule.Quotient.mk (f v)

theorem **dMap_surjective** {ρ : Representation ℂ G V} {σ : Representation ℂ G W} (f : ρ.IntertwiningMap σ) (hf : Function.Surjective f) : Function.Surjective (dMap N θ s hs f)

theorem **twKer_le_map** {ρ : Representation ℂ G V} {σ : Representation ℂ G W} (p : ρ.IntertwiningMap σ) (hp : Function.Surjective p) : twKer σ N θ ≤ (twKer ρ N θ).map p.toLinearMap

theorem **d_exact** {ρ₁ : Representation ℂ G V} {ρ₂ : Representation ℂ G W} {ρ₃ : Representation ℂ G X} (ι : ρ₁.IntertwiningMap ρ₂) (p : ρ₂.IntertwiningMap ρ₃) (hp : Function.Surjective p) (hex : ∀ v, p v = 0 → ∃ u, ι u = v) : ∀ x, dMap N θ s hs p x = 0 → ∃ y, dMap N θ s hs ι y = x

theorem **subsingleton_tw** {ρ : Representation ℂ G V} [Subsingleton V] : Subsingleton (TwJacquet ρ N θ)

theorem **topBot_surjective** (ρ : Representation ℂ G V) : Function.Surjective (topBot ρ)

theorem **phi_lift2** {n t : ℕ} (g : GL (Fin n) k) (h : GL (Fin t) k) : phi n t (lift2 g h) = (g, h)

theorem **phi_surjective** (n t : ℕ) : Function.Surjective (phi (k

theorem **phi_continuous** (n t : ℕ) : Continuous (phi (k

theorem **base** (hc : Continuous ψ) (h1 : ψ ≠ 1) (Y : Representation ℂ (GL (Fin n) k × GL (Fin t) k) V) (hYs : IsSmooth Y) (hYi : Y.IsIrreducible) : HasFiniteLength (twAct Y (rightUnip k n t) (rightWhit k ψ n t) (MonoidHom.inl _ _) (inl_stab ψ n t))

theorem **finLen** (hc : Continuous ψ) (h1 : ψ ≠ 1) (Mr : Representation ℂ (GL (Fin n) k × GL (Fin t) k) V) (hMs : IsSmooth Mr) (hM : HasFiniteLength Mr) : HasFiniteLength (twAct Mr (rightUnip k n t) (rightWhit k ψ n t) (MonoidHom.inl _ _) (inl_stab ψ n t))

theorem **derFin_of** (k : Type) [Field k] [ValuativeRel k] [TopologicalSpace k] [IsNonarchimedeanLocalField k] (h : DerEquiv k) : DerFin k

CompanionTheory.RealGroups.GeneralUnitaryRepresentations.ArchimedeanJacquetSegments.JacquetModuleThetaTwist

theorem **thetaL_surjective** {h : ℕ} (d : Fin h → ℕ) : Function.Surjective (thetaL (A

theorem **mem_unipRad_iff'** {h N : ℕ} {d : Fin h → ℕ} {e : (Σ a, Fin (d a)) ≃ Fin N} {g : GL (Fin N) k} : g ∈ unipRad k e ↔ ∃ hg : g ∈ parab k e, leviProj k e (g, hg) = 1

theorem **thetaL_eq_one** {h : ℕ} (d : Fin h → ℕ) (m : Levi k fun a => Fin (d a)) (hm : thetaL d m = 1) : m = 1

theorem **thetaW_mem_unipRad_iff** {h N : ℕ} (d : Fin h → ℕ) (hd : ∑ a, d a = N) (hd' : ∑ a, d (Fin.rev a) = N) (g : GL (Fin N) k) : thetaW k N g ∈ unipRad k (stdBlocks (fun a => d (Fin.rev a)) hd') ↔ g ∈ unipRad k (stdBlocks d hd)

theorem **twKer_theta** {h N : ℕ} (d : Fin h → ℕ) (hd : ∑ a, d a = N) (hd' : ∑ a, d (Fin.rev a) = N) {V : Type} [AddCommGroup V] [Module ℂ V] (π : Representation ℂ (GL (Fin N) k) V) : twKer (π.comp (thetaW k N)) (unipRad k (stdBlocks d hd)) 1 = twKer π (unipRad k (stdBlocks (fun a => d (Fin.rev a)) hd')) 1

theorem **leviProj_thetaW_leviEmb** {h N : ℕ} (d : Fin h → ℕ) (hd : ∑ a, d a = N) (hd' : ∑ a, d (Fin.rev a) = N) (m : Levi k fun a => Fin (d a)) : ∃ u ∈ unipRad k (stdBlocks (fun a => d (Fin.rev a)) hd'), thetaW k N (leviEmb k (stdBlocks d hd) m) = u * leviEmb k (stdBlocks (fun a => d (Fin.rev a)) hd') (thetaL d m)

theorem **jm_congr** {h N : ℕ} {y z : Fin h → IrrGL k} (hyz : y = z) (hy : ∑ a, (y a).1 = N) (hz : ∑ a, (z a).1 = N) {V : Type} [AddCommGroup V] [Module ℂ V] (π : Representation ℂ (GL (Fin N) k) V) : jm y hy π = jm z hz π

theorem **jacquet_module_theta_twist** (hA : GlobalTheory.Published.IrreducibleAdmissibleGL) (hGK : GKLocal k) {h N : ℕ} (x : Fin h → IrrGL k) (hd : ∑ a, (x a).1 = N) {V : Type} [AddCommGroup V] [Module ℂ V] (π : Representation ℂ (GL (Fin N) k) V) : jm x hd (π.comp (thetaW k N)) = jm (dualRev hA x) (sum_dualRev hA x hd) π

theorem **jmTheta** (k : Type) [Field k] [ValuativeRel k] [TopologicalSpace k] [IsNonarchimedeanLocalField k] : GlobalTheory.BZSeg.JmTheta k

CompanionTheory.RealGroups.GeneralUnitaryRepresentations.ArchimedeanJacquetSegments.NontrivialOfIsIrreducible

theorem **nontrivial_of_irr** {G : Type} [Group G] [TopologicalSpace G] (τ : IrrRep G) : Nontrivial τ.V

theorem **nontrivial_of_isIrreducible** {G V : Type} [Group G] [AddCommGroup V] [Module ℂ V] {π : Representation ℂ G V} (hi : π.IsIrreducible) : Nontrivial V

theorem **continuous_inv_addChar** (ψ : AddChar k Circle) (hc : Continuous ψ) : Continuous (ψ⁻¹ : AddChar k Circle)

theorem **dualCl_fst** (x : IrrGL k) : (dualCl x).1 = x.1

theorem **nuPair_of** (k : Type) [Field k] [ValuativeRel k] [TopologicalSpace k] [IsNonarchimedeanLocalField k] (h1 : GlobalTheory.BZSeg.DerivJac k) (h2 : GlobalTheory.BZSeg.JmTheta k) (h3 : DerFin k) : GlobalTheory.BZSeg.NuPair k

CompanionTheory.RealGroups.GeneralUnitaryRepresentations.ArchimedeanJacquetSegments.PiTensorTwist

theorem **prodIrr_congr** {h N : ℕ} {y z : Fin h → IrrGL k} (e : y = z) (hy : ∑ a, (y a).1 = N) (hz : ∑ a, (z a).1 = N) : Nonempty ((prodIrr y hy).Equiv (prodIrr z hz))

theorem **piTensor_twist** (x : IrrGL k) (s : ℝ) : piTensor (fun a : Fin 2 => twist x.rep (detChar (Fin x.1) (absPow k (![-(s : ℂ), (s : ℂ)] a)))) = twist (piTensor fun _ : Fin 2 => x.rep) (chi k x.1 s)

theorem **prodModel** (k : Type) [Field k] [ValuativeRel k] [TopologicalSpace k] [IsNonarchimedeanLocalField k] : ProdModel k

theorem **cuspUnit** (k : Type) [Field k] [ValuativeRel k] [TopologicalSpace k] [IsNonarchimedeanLocalField k] : CuspUnit k

CompanionTheory.RealGroups.GeneralUnitaryRepresentations.ArchimedeanJacquetSegments.PosMkIff

theorem **cont_qf** {n : ℕ} : Continuous fun p : (Fin n → Fin n → ℂ) × (Fin n → ℂ) × (Fin n → ℂ) => qf p.1 p.2.1 p.2.2

theorem **qf_smul** {n : ℕ} (M : Fin n → Fin n → ℂ) (a : ℂ) (x y : Fin n → ℂ) : qf M (a • x) (a • y) = a * star a * qf M x y

theorem **qf_single** {n : ℕ} (M : Fin n → Fin n → ℂ) (i j : Fin n) : qf M (Pi.single i 1) (Pi.single j 1) = M i j

theorem **mk_apply** (M : Fin (finrank ℂ W) → Fin (finrank ℂ W) → ℂ) (v w : W) : mk M v w = qf M (ef W v) (ef W w)

theorem **expand** (B : Sq W) (v w : W) : B v w = qf (gram B) (ef W v) (ef W w)

theorem **mk_gram** (B : Sq W) : mk (gram B) = B

theorem **gram_mk** (M : Fin (finrank ℂ W) → Fin (finrank ℂ W) → ℂ) : gram (mk M : Sq W) = M

theorem **gram_smul** (c : ℂ) (B : Sq W) : gram (c • B) = c • gram B

theorem **ker_bot** (hsim : ∀ U : Submodule ℂ W, (∀ i, ∀ w ∈ U, T i w ∈ U) → U = ⊥ ∨ U = ⊤) (B : Sq W) (hB : B ≠ 0) (hinv : Inv ι T B) : LinearMap.ker B = ⊥

theorem **inv_sub** {B B' : Sq W} (h : Inv ι T B) (h' : Inv ι T B') (c : ℂ) : Inv ι T (B − c • B')

theorem **uniq** (hsim : ∀ U : Submodule ℂ W, (∀ i, ∀ w ∈ U, T i w ∈ U) → U = ⊥ ∨ U = ⊤) (B₀ B : Sq W) (h0 : B₀ ≠ 0) (hinv0 : Inv ι T B₀) (hinv : Inv ι T B) : ∃ c : ℂ, B = c • B₀

theorem **real_of** {B₀ B : Sq W} (h0 : B₀ ≠ 0) (hh0 : Herm B₀) (hh : Herm B) (c : ℂ) (hc : B = c • B₀) : starRingEnd ℂ c = c

theorem **pos_of** (B : Sq W) (hh : Herm B) (hk : LinearMap.ker B = ⊥) (hsd : ∀ v, 0 ≤ (B v v).re) : Pos B

theorem **pos_mk_iff** (M : Fin (finrank ℂ W) → Fin (finrank ℂ W) → ℂ) : (∀ v : W, v ≠ 0 → 0 < ((mk M : Sq W) v v).re) ↔ ∀ c, c ≠ 0 → 0 < (qf M c c).re

theorem **pos_open** {n : ℕ} (M : Fin n → Fin n → ℂ) (hM : ∀ c : Fin n → ℂ, c ≠ 0 → 0 < (qf M c c).re) : ∀ᶠ M' in 𝓝 M, ∀ c : Fin n → ℂ, c ≠ 0 → 0 < (qf M' c c).re

theorem **mk_smul** (a : ℂ) (M : Fin (finrank ℂ W) → Fin (finrank ℂ W) → ℂ) : (mk (a • M) : Sq W) = a • mk M

theorem **herm_smul** {B : Sq W} (h : Herm B) (r : ℝ) : Herm ((r : ℂ) • B)

theorem **inv_smul** {B : Sq W} (h : Inv ι T B) (c : ℂ) : Inv ι T (c • B)

theorem **normalize** (B : Sq W) (hB : B ≠ 0) : ∃ r : ℝ, 0 < r ∧ ‖gram ((r : ℂ) • B)‖ = 1

theorem **lim_form** {I : Type} (ι : I → I) (T : ℝ → I → Module.End ℂ W) (hc : ∀ (i : I) (w : W) (l : Module.Dual ℂ W), Continuous fun s => l (T s i w)) (s : ℕ → ℝ) (p : ℝ) (hs : Tendsto s atTop (𝓝 p)) (B : ℕ → Sq W) (hinv : ∀ k, Inv ι (T (s k)) (B k)) (hh : ∀ k, Herm (B k)) (hn : ∀ k, ‖gram (B k)‖ = 1) : ∃ (φ : ℕ → ℕ) (M : Fin (finrank ℂ W) → Fin (finrank ℂ W) → ℂ), StrictMono φ ∧ ‖M‖ = 1 ∧ Tendsto (fun k => gram (B (φ k))) atTop (𝓝 M) ∧ Inv ι (T p) (mk M) ∧ Herm (mk M) ∧ ∀ v w, Tendsto (fun k => B (φ k) v w) atTop (𝓝 (mk M v w))

theorem **signature** : Signature

CompanionTheory.RealGroups.GeneralUnitaryRepresentations.ArchimedeanJacquetSegments.SegmentSubquotients.ExistsNabsNeOne

theorem **exists_ne_zero_irr** {G V : Type} [Group G] [AddCommGroup V] [Module ℂ V] (ρ : Representation ℂ G V) (hi : ρ.IsIrreducible) : ∃ v : V, v ≠ 0

theorem **exists_nabs_ne_one** : ∃ x : kˣ, nabs k x ≠ 1

theorem **not_equiv_segTerm** {d : ℕ} (hd : 0 < d) (σ : IrrRep (GL (Fin d) k)) (l : ℕ) {a b : Fin l} (hab : a ≠ b) : IsEmpty ((segTerm σ.ρ l a).Equiv (segTerm σ.ρ l b))

CompanionTheory.RealGroups.GeneralUnitaryRepresentations.ArchimedeanJacquetSegments.SegmentSubquotients.ExistsPermOfIsSubquotientJac

theorem **entries_zero_or_d** {d l : ℕ} (hd : 0 < d) (σ : IrrRep (GL (Fin d) k)) (hσ : IsCuspidal σ.ρ) (b : SizeMatrix (fun _ : Fin l => Fin d) (fun _ : Fin l => Fin d)) (f : ∀ i, (Σ j, Fin (b.1 i j)) ≃ Fin d) (s : ∀ i j, IrrRep (GL (Fin (b.1 i j)) k)) (hs : ∀ i, IsSubquotient (piTensor fun j => (s i j).ρ) (jac (f i) (segTerm σ.ρ l i))) : ∀ i j, b.1 i j = 0 ∨ b.1 i j = d

theorem **row_of_eq** {d r : ℕ} (c : Fin r → ℕ) (j₀ : Fin r) (h0 : ∀ j, j ≠ j₀ → c j = 0) (hcd : c j₀ = d) (f : (Σ j, Fin (c j)) ≃ Fin d) {V : Type} [AddCommGroup V] [Module ℂ V] (τ : Representation ℂ (GL (Fin d) k) V) (hτs : IsSmooth τ) (hτ : τ.IsIrreducible) (s : ∀ j, IrrRep (GL (Fin (c j)) k)) (hs : IsSubquotient (piTensor fun j => (s j).ρ) (jac f τ)) : IrrGL.of (s j₀).ρ (s j₀).smooth (s j₀).irr = IrrGL.of τ hτs hτ

theorem **col_of_eq** {d r : ℕ} (c : Fin r → ℕ) (i₀ : Fin r) (h0 : ∀ i, i ≠ i₀ → c i = 0) (g : (Σ i, Fin (c i)) ≃ Fin d) (t : ∀ i, IrrRep (GL (Fin (c i)) k)) {V : Type} [AddCommGroup V] [Module ℂ V] (τ : Representation ℂ (GL (Fin d) k) V) (hτs : IsSmooth τ) (hτ : τ.IsIrreducible) (h : IrrGL.of (t i₀).ρ (t i₀).smooth (t i₀).irr = IrrGL.of τ hτs hτ) : Nonempty ((multiProd g fun i => (t i).ρ).Equiv τ)

theorem **eq_of_eq_d** {d l : ℕ} (hd : 0 < d) (c : Fin l → ℕ) (hsum : ∑ j, c j = d) {j j' : Fin l} (hj : c j = d) (hj' : c j' = d) : j = j'

theorem **exists_eq_d** {d l : ℕ} (hd : 0 < d) (c : Fin l → ℕ) (hsum : ∑ j, c j = d) (h01 : ∀ j, c j = 0 ∨ c j = d) : ∃ j, c j = d

theorem **exists_perm_of_cell** {d l : ℕ} (hd : 0 < d) (σ : IrrRep (GL (Fin d) k)) (hσ : IsCuspidal σ.ρ) (b : SizeMatrix (fun _ : Fin l => Fin d) (fun _ : Fin l => Fin d)) (h01 : ∀ i j, b.1 i j = 0 ∨ b.1 i j = d) (f : ∀ i, (Σ j, Fin (b.1 i j)) ≃ Fin d) (g : ∀ j, (Σ i, Fin (b.1 i j)) ≃ Fin d) (s : ∀ i j, IrrRep (GL (Fin (b.1 i j)) k)) (hs : ∀ i, IsSubquotient (piTensor fun j => (s i j).ρ) (jac (f i) (segTerm σ.ρ l i))) {X : Type} [AddCommGroup X] [Module ℂ X] (π : Representation ℂ (Levi k fun _ : Fin l => Fin d) X) (hπ : π.IsIrreducible) (h : IsSubquotient π (piTensor fun j => multiProd (g j) fun i => (s i j).ρ)) : ∃ w : Equiv.Perm (Fin l), Nonempty (π.Equiv (piTensor fun a => segTerm σ.ρ l (w a)))

theorem **exists_perm_of_isSubquotient_jac** (hG : GeometricLemma) {d l : ℕ} (hd : 0 < d) (hl : 0 < l) (σ : IrrRep (GL (Fin d) k)) (hσ : IsCuspidal σ.ρ) {X : Type} [AddCommGroup X] [Module ℂ X] (π : Representation ℂ (Levi k fun _ : Fin l => Fin d) X) (hπ : π.IsIrreducible) (h : IsSubquotient π (jac (consecBlocks l d) (multiProd (consecBlocks l d) (segTerm σ.ρ l)))) : ∃ w : Equiv.Perm (Fin l), Nonempty (π.Equiv (piTensor fun a => segTerm σ.ρ l (w a)))

CompanionTheory.RealGroups.GeneralUnitaryRepresentations.ArchimedeanJacquetSegments.SegmentSubquotients.ExistsUniqueIrredSub

theorem **exists_unique_irred_sub** {d l : ℕ} (hd : 0 < d) (hl : 0 < l) (σ : IrrRep (GL (Fin d) k)) (hσ : IsCuspidal σ.ρ) (h1 : jhMult (piTensor (segTerm σ.ρ l)) (jac (consecBlocks l d) (multiProd (consecBlocks l d) (segTerm σ.ρ l))) = 1) : ∃ S : Subrepresentation (multiProd (consecBlocks l d) (segTerm σ.ρ l)), S.toRepresentation.IsIrreducible ∧ (∀ S' : Subrepresentation (multiProd (consecBlocks l d) (segTerm σ.ρ l)), S'.toRepresentation.IsIrreducible → S' = S) ∧ jhMult (piTensor (segTerm σ.ρ l)) (jac (consecBlocks l d) S.toRepresentation) = 1

CompanionTheory.RealGroups.GeneralUnitaryRepresentations.ArchimedeanJacquetSegments.SegmentSubquotients.HasFiniteLengthJacSub

theorem **isSmooth_segTerm'** {d l : ℕ} (σ : IrrRep (GL (Fin d) k)) (a : Fin l) : IsSmooth (segTerm σ.ρ l a)

theorem **isIrreducible_segTerm'** {d l : ℕ} (σ : IrrRep (GL (Fin d) k)) (a : Fin l) : (segTerm σ.ρ l a).IsIrreducible

theorem **isIrreducible_segTensor** {d : ℕ} (hd : 0 < d) (σ : IrrRep (GL (Fin d) k)) : (piTensor (segTerm σ.ρ 2)).IsIrreducible

theorem **hasFiniteLength_jac_sub** {d : ℕ} (hd : 0 < d) (σ : IrrRep (GL (Fin d) k)) (hσ : IsCuspidal σ.ρ) (S : Subrepresentation (multiProd (consecBlocks 2 d) (segTerm σ.ρ 2))) : HasFiniteLength (jac (consecBlocks 2 d) S.toRepresentation)

CompanionTheory.RealGroups.GeneralUnitaryRepresentations.ArchimedeanJacquetSegments.SegmentSubquotients.IsSubquotientJacMultiProdOfSub

theorem **isSmooth_multiProd_segTerm** {d : ℕ} (σ : IrrRep (GL (Fin d) k)) (l : ℕ) : IsSmooth (multiProd (consecBlocks l d) (segTerm σ.ρ l))

theorem **isSubquotient_jac_multiProd_of_sub** {d l : ℕ} (σ : IrrRep (GL (Fin d) k)) (S : Subrepresentation (multiProd (consecBlocks l d) (segTerm σ.ρ l))) {X : Type} [AddCommGroup X] [Module ℂ X] {π : Representation ℂ (Levi k fun _ : Fin l => Fin d) X} (h : IsSubquotient π (jac (consecBlocks l d) S.toRepresentation)) : IsSubquotient π (jac (consecBlocks l d) (multiProd (consecBlocks l d) (segTerm σ.ρ l)))

theorem **perm_eq_one_of_lt_succ** {l : ℕ} (w : Equiv.Perm (Fin l)) (h : ∀ (i : ℕ) (hi : i + 1 < l), w.symm (i, by omega) < w.symm (i + 1, hi)) : w = 1

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theorem **exists_irred_le** {d l : ℕ} (hd : 0 < d) (hl : 0 < l) (σ : IrrRep (GL (Fin d) k)) (hσ : IsCuspidal σ.ρ) (T : Subrepresentation (multiProd (consecBlocks l d) (segTerm σ.ρ l))) (hT : T.toSubmodule ≠ ⊥) : ∃ S' : Subrepresentation (multiProd (consecBlocks l d) (segTerm σ.ρ l)), S'.toRepresentation.IsIrreducible ∧ S'.toSubmodule ≤ T.toSubmodule

theorem **isSubquotient_jac_of_le** {d l : ℕ} (σ : IrrRep (GL (Fin d) k)) {S T : Subrepresentation (multiProd (consecBlocks l d) (segTerm σ.ρ l))} (hST : S.toSubmodule ≤ T.toSubmodule) {X : Type} [AddCommGroup X] [Module ℂ X] {π : Representation ℂ (Levi k fun _ : Fin l => Fin d) X} (h : IsSubquotient π (jac (consecBlocks l d) S.toRepresentation)) : IsSubquotient π (jac (consecBlocks l d) T.toRepresentation)

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theorem **transfer** {N : ℕ} {v w : Fin 2 → IrrGL k} (h : v = w) (hv : ∑ a, (v a).1 = N) {W : Type} [AddCommGroup W] [Module ℂ W] {ω : Representation ℂ (GL (Fin N) k) W} (f : ω.IntertwiningMap (prodIrr v hv)) : ∃ (hw : ∑ a, (w a).1 = N) (f' : ω.IntertwiningMap (prodIrr w hw)), (Function.Injective f → Function.Injective f') ∧ (Function.Surjective f' → Function.Surjective f)

theorem **jmc** {h N : ℕ} {x x' : Fin h → IrrGL k} (e : x = x') (hd : ∑ a, (x a).1 = N) (hd' : ∑ a, (x' a).1 = N) {V : Type} [AddCommGroup V] [Module ℂ V] (π : Representation ℂ (GL (Fin N) k) V) : jm x hd π = jm x' hd' π

theorem **jhMult_rev_jac_sub** (hG : GeometricLemma) (hR : RhoNuRhoReducible) {d : ℕ} (hd : 0 < d) (σ : IrrRep (GL (Fin d) k)) (hσ : IsCuspidal σ.ρ) (S : Subrepresentation (multiProd (consecBlocks 2 d) (segTerm σ.ρ 2))) (hS : S.toRepresentation.IsIrreducible) (h1 : jhMult (piTensor (segTerm σ.ρ 2)) (jac (consecBlocks 2 d) S.toRepresentation) = 1) : jhMult (revTensor σ) (jac (consecBlocks 2 d) S.toRepresentation) = 0

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theorem **entry_le** {d l : ℕ} (b : SizeMatrix (fun _ : Fin l => Fin d) (fun _ : Fin l => Fin d)) (i j : Fin l) : b.1 i j ≤ d

theorem **isSmooth_segTerm** {d l : ℕ} (σ : IrrRep (GL (Fin d) k)) (a : Fin l) : IsSmooth (segTerm σ.ρ l a)

theorem **isIrreducible_segTerm** {d l : ℕ} (σ : IrrRep (GL (Fin d) k)) (a : Fin l) : (segTerm σ.ρ l a).IsIrreducible

theorem **isSmooth_piTensor_segTerm** {d l : ℕ} (σ : IrrRep (GL (Fin d) k)) : IsSmooth (piTensor (segTerm σ.ρ l))

theorem **isIrreducible_piTensor_segTerm** {d l : ℕ} (σ : IrrRep (GL (Fin d) k)) : (piTensor (segTerm σ.ρ l)).IsIrreducible

theorem **hasFiniteLength_segTerm** {d l : ℕ} (σ : IrrRep (GL (Fin d) k)) (a : Fin l) : HasFiniteLength (segTerm σ.ρ l a)

theorem **isIrreducible_piTensor_cls** {ι : Type} [Finite ι] {n : ι → ℕ} (s : ∀ j, IrrCl (GL (Fin (n j)) k)) : (piTensor fun j => (s j).rep.ρ).IsIrreducible

theorem **exists_other** {d l : ℕ} (b : SizeMatrix (fun _ : Fin l => Fin d) (fun _ : Fin l => Fin d)) {i j : Fin l} (h : b.1 i j < d) : ∃ j', j' ≠ j ∧ 0 < b.1 i j'

theorem **cell_eq_zero_of_mid** (hG : GeometricLemma) {d l : ℕ} (hd : 0 < d) (σ : IrrRep (GL (Fin d) k)) (hσ : IsCuspidal σ.ρ) (b : SizeMatrix (fun _ : Fin l => Fin d) (fun _ : Fin l => Fin d)) (f : ∀ i, (Σ j, Fin (b.1 i j)) ≃ Fin d) (g : ∀ j, (Σ i, Fin (b.1 i j)) ≃ Fin d) (h : ∃ i j, 0 < b.1 i j ∧ b.1 i j < d) : jhMult (piTensor (segTerm σ.ρ l)) (cellFunctor f g (piTensor (segTerm σ.ρ l))) = 0

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theorem **irr_congr** {h N : ℕ} {x x' : Fin h → IrrGL k} (e : x = x') (hd : ∑ a, (x a).1 = N) : ∃ hd' : ∑ a, (x' a).1 = N, ((prodIrr x hd).IsIrreducible ↔ (prodIrr x' hd').IsIrreducible)

theorem **irr_tw** {h N : ℕ} (x : Fin h → IrrGL k) (hd : ∑ a, (x a).1 = N) (s : ℂ) (hi : (prodIrr x hd).IsIrreducible) : (prodIrr (fun a => (x a).nu s) hd).IsIrreducible

theorem **redCl_ofC** (k : Type) [Field k] [ValuativeRel k] [TopologicalSpace k] [IsNonarchimedeanLocalField k] (h1 : SomeRed k) (h2 : CuspUnit k) (h3 : GlobalTheory.BZSeg.CuspPairIrr k) : RedCl k

theorem **rhoNuRho_ofC** (h : ∀ (k : Type) [Field k] [ValuativeRel k] [TopologicalSpace k] [IsNonarchimedeanLocalField k], RedCl k) : CompanionTheory.Pf1.SegRepC.SegTwo.RhoNuRhoReducible

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theorem **redClC** : RedCl k

theorem **rhoNuRhoReducibleC** : CompanionTheory.Pf1.SegRepC.SegTwo.RhoNuRhoReducible

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theorem **segTwo_of** (hG : GeometricLemma) (hR : RhoNuRhoReducible) : CompanionTheory.Pf1.SegRepC.SegTwo

theorem **segRep_of** (hT : StageP) (hG : GeometricLemma) (h2 : CompanionTheory.Pf1.SegRepC.SegTwo) : SegmentRepresentation

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theorem **segmentRepresentation** : GlobalTheory.Published.SegmentRepresentation

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theorem **isSmooth_prod** {d l : ℕ} (σ : IrrRep (GL (Fin d) k)) : IsSmooth (multiProd (consecBlocks l d) (segTerm σ.ρ l))

theorem **hasFiniteLength_prod** {d l : ℕ} (σ : IrrRep (GL (Fin d) k)) : HasFiniteLength (multiProd (consecBlocks l d) (segTerm σ.ρ l))

theorem **isSmooth_sub** {G V : Type} [Group G] [TopologicalSpace G] [AddCommGroup V] [Module ℂ V] {π : Representation ℂ G V} (hπ : IsSmooth π) (S : Subrepresentation π) : IsSmooth S.toRepresentation

theorem **exists_irred_le** {d l : ℕ} (hd : 0 < d) (hl : 0 < l) (σ : IrrRep (GL (Fin d) k)) (hσ : IsCuspidal σ.ρ) (T : Subrepresentation (multiProd (consecBlocks l d) (segTerm σ.ρ l))) (hT : T.toSubmodule ≠ ⊥) : ∃ S' : Subrepresentation (multiProd (consecBlocks l d) (segTerm σ.ρ l)), S'.toRepresentation.IsIrreducible ∧ S'.toSubmodule ≤ T.toSubmodule

theorem **isIrreducible_segTensor** {d : ℕ} (hd : 0 < d) (σ : IrrRep (GL (Fin d) k)) : (piTensor (segTerm σ.ρ 2)).IsIrreducible

theorem **hasFiniteLength_jac_sub** {d : ℕ} (hd : 0 < d) (σ : IrrRep (GL (Fin d) k)) (hσ : IsCuspidal σ.ρ) (S : Subrepresentation (multiProd (consecBlocks 2 d) (segTerm σ.ρ 2))) : HasFiniteLength (jac (consecBlocks 2 d) S.toRepresentation)

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theorem **irrAdm** : GlobalTheory.Published.IrreducibleAdmissibleGL

theorem **isSmooth_segTerm'** {d l : ℕ} (σ : IrrRep (GL (Fin d) k)) (a : Fin l) : IsSmooth (segTerm σ.ρ l a)

theorem **isIrreducible_segTerm'** {d l : ℕ} (σ : IrrRep (GL (Fin d) k)) (a : Fin l) : (segTerm σ.ρ l a).IsIrreducible

theorem **cl_injective** {d l : ℕ} (hd : 0 < d) (σ : IrrRep (GL (Fin d) k)) : Function.Injective (cl σ l)

theorem **cl_cusp** {d l : ℕ} (hd : 0 < d) (σ : IrrRep (GL (Fin d) k)) (hσ : IsCuspidal σ.ρ) (a : Fin l) : (cl σ l a).IsCusp

theorem **jhMult_jac_multiProd** (hG : GeometricLemma) {d l : ℕ} (hd : 0 < d) (hl : 0 < l) (σ : IrrRep (GL (Fin d) k)) (hσ : IsCuspidal σ.ρ) : jhMult (piTensor (segTerm σ.ρ l)) (jac (consecBlocks l d) (multiProd (consecBlocks l d) (segTerm σ.ρ l))) = 1

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theorem **sq_right** {G V W U : Type} [Group G] [AddCommGroup V] [Module ℂ V] [AddCommGroup W] [Module ℂ W] [AddCommGroup U] [Module ℂ U] {σ : Representation ℂ G W} {ρ : Representation ℂ G V} {ρ' : Representation ℂ G U} (h : IsSubquotient σ ρ) (E : ρ.Equiv ρ') : IsSubquotient σ ρ'

theorem **sq_left** {G V W U : Type} [Group G] [AddCommGroup V] [Module ℂ V] [AddCommGroup W] [Module ℂ W] [AddCommGroup U] [Module ℂ U] {σ : Representation ℂ G W} {τ : Representation ℂ G U} {ρ : Representation ℂ G V} (E : σ.Equiv τ) (h : IsSubquotient σ ρ) : IsSubquotient τ ρ

theorem **equiv_parInd_jac** {β : Type} [Fintype β] [LinearOrder β] (c : β → ℕ) (j₀ : β) (h0 : ∀ j, j ≠ j₀ → c j = 0) (f : (Σ j, Fin (c j)) ≃ Fin (c j₀)) {V : Type} [AddCommGroup V] [Module ℂ V] (ρ : Representation ℂ (GL (Fin (c j₀)) k) V) (hρ : IsSmooth ρ) : Nonempty (ρ.Equiv (parInd f (jac f ρ)))

theorem **row_class** {β : Type} [Fintype β] [LinearOrder β] {N : ℕ} (c : β → ℕ) (j₀ : β) (hN : c j₀ = N) (h0 : ∀ j, j ≠ j₀ → c j = 0) (f : (Σ j, Fin (c j)) ≃ Fin N) (ρ : IrrRep (GL (Fin N) k)) (s : ∀ j, IrrRep (GL (Fin (c j)) k)) (hs : IsSubquotient (piTensor fun j => (s j).ρ) (jac f ρ.ρ)) : ((c j₀, (s j₀).cl) : IrrGL k) = (N, ρ.cl)

theorem **col_class** {α : Type} [Fintype α] [LinearOrder α] {N : ℕ} (b : α → ℕ) (i₀ : α) (hN : b i₀ = N) (h0 : ∀ i, i ≠ i₀ → b i = 0) (σ : ∀ i, IrrRep (GL (Fin (b i)) k)) (g : (Σ i, Fin (b i)) ≃ Fin N) : ∃ τ : IrrRep (GL (Fin N) k), Nonempty ((multiProd g fun i => (σ i).ρ).Equiv τ.ρ) ∧ ((N, τ.cl) : IrrGL k) = (b i₀, (σ i₀).cl)

theorem **exists_perm_of_cell** {d l : ℕ} (hd : 0 < d) (σ : IrrRep (GL (Fin d) k)) (b : SizeMatrix (fun _ : Fin l => Fin d) (fun _ : Fin l => Fin d)) (h01 : ∀ i j, b.1 i j = 0 ∨ b.1 i j = d) (f : ∀ i, (Σ j, Fin (b.1 i j)) ≃ Fin d) (g : ∀ j, (Σ i, Fin (b.1 i j)) ≃ Fin d) (s : ∀ i j, IrrRep (GL (Fin (b.1 i j)) k)) (hs : ∀ i, IsSubquotient (piTensor fun j => (s i j).ρ) (jac (f i) (segTerm σ.ρ l i))) {X : Type} [AddCommGroup X] [Module ℂ X] (π : Representation ℂ (Levi k fun _ : Fin l => Fin d) X) (hπ : π.IsIrreducible) (h : IsSubquotient π (piTensor fun j => multiProd (g j) fun i => (s i j).ρ)) : ∃ w : Equiv.Perm (Fin l), Nonempty (π.Equiv (piTensor fun a => segTerm σ.ρ l (w a)))

theorem **exists_perm_of_isSubquotient_jac** (hG : GeometricLemma) {d l : ℕ} (hd : 0 < d) (hl : 0 < l) (σ : IrrRep (GL (Fin d) k)) (hσ : IsCuspidal σ.ρ) {X : Type} [AddCommGroup X] [Module ℂ X] (π : Representation ℂ (Levi k fun _ : Fin l => Fin d) X) (hπ : π.IsIrreducible) (h : IsSubquotient π (jac (consecBlocks l d) (multiProd (consecBlocks l d) (segTerm σ.ρ l)))) : ∃ w : Equiv.Perm (Fin l), Nonempty (π.Equiv (piTensor fun a => segTerm σ.ρ l (w a)))

theorem **constituents_jac_sub** (hG : GeometricLemma) {d : ℕ} (hd : 0 < d) (σ : IrrRep (GL (Fin d) k)) (hσ : IsCuspidal σ.ρ) (S : Subrepresentation (multiProd (consecBlocks 2 d) (segTerm σ.ρ 2))) {X : Type} [AddCommGroup X] [Module ℂ X] (ρ : Representation ℂ (Levi k fun _ : Fin 2 => Fin d) X) (hp : ρ.IsIrreducible) (hs : IsSubquotient ρ (jac (consecBlocks 2 d) S.toRepresentation)) : Nonempty (ρ.Equiv (piTensor (segTerm σ.ρ 2))) ∨ Nonempty (ρ.Equiv (CompanionTheory.Pf1.SegRepC.SegTwo.revTensor σ))

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theorem **segTerm_smooth** {d : ℕ} (σ : IrrRep (GL (Fin d) k)) (l : ℕ) (a : Fin l) : IsSmooth (segTerm σ.ρ l a)

theorem **segTerm_irr** {d : ℕ} (σ : IrrRep (GL (Fin d) k)) (l : ℕ) (a : Fin l) : (segTerm σ.ρ l a).IsIrreducible

theorem **tcl_eq** {d : ℕ} (σ : IrrRep (GL (Fin d) k)) (l : ℕ) (a : Fin l) : tcl σ l a = (cls σ).nu ((a : ℕ) : ℂ)

theorem **blocks_eq** (l d : ℕ) (h : ∑ _a : Fin l, d = l * d) : stdBlocks (fun _ : Fin l => d) h = consecBlocks l d

theorem **consec_equiv** {d : ℕ} (σ : IrrRep (GL (Fin d) k)) (l : ℕ) (h : ∑ a, (tcl σ l a).1 = l * d) : Nonempty ((multiProd (consecBlocks l d) (segTerm σ.ρ l)).Equiv (prodIrr (tcl σ l) h))

theorem **prod₂_map_injective_right** (r t : ℕ) {U W W' : Type} [AddCommGroup U] [Module ℂ U] [AddCommGroup W] [Module ℂ W] [AddCommGroup W'] [Module ℂ W'] (σ : Representation ℂ (GL (Fin r) k) U) (ρ : Representation ℂ (GL (Fin t) k) W) (ρ' : Representation ℂ (GL (Fin t) k) W') (f : ρ.IntertwiningMap ρ') (hf : Function.Injective f) : ∃ F : (prod₂ σ ρ).IntertwiningMap (prod₂ σ ρ'), Function.Injective F

theorem **inj_comp** {G V₁ V₂ V₃ : Type} [Group G] [AddCommGroup V₁] [Module ℂ V₁] [AddCommGroup V₂] [Module ℂ V₂] [AddCommGroup V₃] [Module ℂ V₃] {ρ₁ : Representation ℂ G V₁} {ρ₂ : Representation ℂ G V₂} {ρ₃ : Representation ℂ G V₃} (f : ρ₂.IntertwiningMap ρ₃) (g : ρ₁.IntertwiningMap ρ₂) (hf : Function.Injective f) (hg : Function.Injective g) : Function.Injective (f.comp g)

theorem **inj_equiv** {G V₁ V₂ : Type} [Group G] [AddCommGroup V₁] [Module ℂ V₁] [AddCommGroup V₂] [Module ℂ V₂] {ρ₁ : Representation ℂ G V₁} {ρ₂ : Representation ℂ G V₂} (φ : ρ₁.Equiv ρ₂) : Function.Injective φ.toIntertwiningMap

theorem **sum_const_fin** (n d : ℕ) : ∑ _a : Fin n, d = n * d

theorem **Emb.trans** (h : Emb ρ₁ ρ₂) (h' : Emb ρ₂ ρ₃) : Emb ρ₁ ρ₃

theorem **Emb.of** (e : Nonempty (ρ₁.Equiv ρ₂)) : Emb ρ₁ ρ₂

theorem **Emb.of'** (e : Nonempty (ρ₂.Equiv ρ₁)) : Emb ρ₁ ρ₂

theorem **Emb.sub** (S : Subrepresentation ρ₁) : Emb S.toRepresentation ρ₁

theorem **Emb.range** (h : Emb ρ₁ ρ₂) [Nontrivial V₁] : ∃ T : Subrepresentation ρ₂, T.toSubmodule ≠ ⊥ ∧ IsSubquotient T.toRepresentation ρ₁

theorem **sum_app** {p q : ℕ} (x : Fin p → IrrGL k) (y : Fin q → IrrGL k) : ∑ a, (Fin.append x y a).1 = ∑ a, (x a).1 + ∑ a, (y a).1

theorem **embR** (σ : Representation ℂ (GL (Fin r) k) U) (ρ : Representation ℂ (GL (Fin t) k) W) (ρ' : Representation ℂ (GL (Fin t) k) W') (P : Representation ℂ (GL (Fin (r + t)) k) Y) (P' : Representation ℂ (GL (Fin (r + t)) k) Y') (h : Emb ρ ρ') (e : Nonempty ((prod₂ σ ρ).Equiv P)) (e' : Nonempty ((prod₂ σ ρ').Equiv P')) : Emb P P'

theorem **embL** (σ : Representation ℂ (GL (Fin r) k) U) (σ' : Representation ℂ (GL (Fin r) k) U') (ρ : Representation ℂ (GL (Fin t) k) W) (P : Representation ℂ (GL (Fin (r + t)) k) Y) (P' : Representation ℂ (GL (Fin (r + t)) k) Y') (h : Emb σ σ') (e : Nonempty ((prod₂ σ ρ).Equiv P)) (e' : Nonempty ((prod₂ σ' ρ).Equiv P')) : Emb P P'

theorem **emb_right** {p q q' r t : ℕ} (A : Fin p → IrrGL k) (y : Fin q → IrrGL k) (y' : Fin q' → IrrGL k) (hA : ∑ a, (A a).1 = r) (hy : ∑ a, (y a).1 = t) (hy' : ∑ a, (y' a).1 = t) (h : Emb (prodIrr y hy) (prodIrr y' hy')) (h1 : ∑ a, (Fin.append A y a).1 = r + t) (h2 : ∑ a, (Fin.append A y' a).1 = r + t) : Emb (prodIrr (Fin.append A y) h1) (prodIrr (Fin.append A y') h2)

theorem **emb_left** {p p' q r t : ℕ} (y : Fin p → IrrGL k) (y' : Fin p' → IrrGL k) (B : Fin q → IrrGL k) (hy : ∑ a, (y a).1 = r) (hy' : ∑ a, (y' a).1 = r) (hB : ∑ a, (B a).1 = t) (h : Emb (prodIrr y hy) (prodIrr y' hy')) (h1 : ∑ a, (Fin.append y B a).1 = r + t) (h2 : ∑ a, (Fin.append y' B a).1 = r + t) : Emb (prodIrr (Fin.append y B) h1) (prodIrr (Fin.append y' B) h2)

theorem **comb** {n c N : ℕ} (xx : Fin n → IrrGL k) (hdx : ∑ a, (xx a).1 = N) (iz : Fin n) (z : IrrGL k) (hz : xx iz = z) (pr : Fin 2 → IrrGL k) (hs : ∑ j, (pr j).1 = z.1) (d : ℕ) (hsd : ∀ j, (pr j).1 = d) (hsc : ∀ j, (pr j).IsCusp) (hu : Nonempty ((jac (stdBlocks (fun j => (pr j).1) hs) z.rep).Equiv (piTensor fun j => (pr j).rep))) (y : Fin c → IrrGL k) (hy : ∑ a, (y a).1 = N) (hyc : ∀ a, (y a).IsCusp) (hjm : 1 ≤ Zc.jm y hy (prodIrr xx hdx)) : ∃ f : Fin 2 ↪o Fin c, ∀ j, y (f j) = pr j

theorem **classMain** {i q N : ℕ} (t : Fin (i + 2 + q) → IrrGL k) (ht : ∑ a, (t a).1 = N) (pr : Fin 2 → IrrGL k) (hpr : ∀ j, pr j = t (Fin.castAdd q (Fin.natAdd i j))) (z : IrrGL k) (hs : ∑ j, (pr j).1 = z.1) (he : Emb z.rep (prodIrr pr hs)) : ∃ (n : ℕ) (xx : Fin n → IrrGL k) (hdx : ∑ a, (xx a).1 = N) (iz : Fin n), xx iz = z ∧ Emb (prodIrr xx hdx) (prodIrr t ht)

theorem **exists_alt_ter** {d : ℕ} (σ : IrrRep (GL (Fin d) k)) {U : Type} [AddCommGroup U] [Module ℂ U] (τ : Representation ℂ (GL (Fin (2 * d)) k) U) (hsS : IsSmooth τ) (hiS : τ.IsIrreducible) (hs : ∑ j, (tcl σ 2 j).1 = 2 * d) (a2 : Emb τ (prodIrr (tcl σ 2) hs)) (eS : (jac (consecBlocks 2 d) τ).Equiv (piTensor (segTerm σ.ρ 2))) : ∃ (z : IrrGL k) (hs : ∑ j, (tcl σ 2 j).1 = z.1), Emb z.rep (prodIrr (tcl σ 2) hs) ∧ Nonempty ((jac (stdBlocks (fun j => (tcl σ 2 j).1) hs) z.rep).Equiv (piTensor fun j => (tcl σ 2 j).rep))

theorem **exists_unit_one_add_generator** (h2 : CompanionTheory.Pf1.SegRepC.SegTwo) {d : ℕ} (hd : 0 < d) (σ : IrrRep (GL (Fin d) k)) (hσ : IsCuspidal σ.ρ) : ∃ (z : IrrGL k) (hs : ∑ j, (tcl σ 2 j).1 = z.1), Emb z.rep (prodIrr (tcl σ 2) hs) ∧ Nonempty ((jac (stdBlocks (fun j => (tcl σ 2 j).1) hs) z.rep).Equiv (piTensor fun j => (tcl σ 2 j).rep))

theorem **one_le_jm** {d l : ℕ} (σ : IrrRep (GL (Fin d) k)) {U X : Type} [AddCommGroup U] [Module ℂ U] [AddCommGroup X] [Module ℂ X] (τ : Representation ℂ (GL (Fin (l * d)) k) U) (π : Representation ℂ (Levi k fun _ : Fin l => Fin d) X) (hsub : IsSubquotient π (jac (consecBlocks l d) τ)) (w : Equiv.Perm (Fin l)) (φ : π.Equiv (piTensor fun a => segTerm σ.ρ l (w a))) (hy : ∑ a, (tcl σ l (w a)).1 = l * d) : 1 ≤ Zc.jm (fun a => tcl σ l (w a)) hy τ

theorem **stageP** : StageP

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theorem **intw** {π : Representation ℂ G V} {σ : Representation ℂ G W} (E : π.Equiv σ) (g : G) (v : V) : E.toLinearEquiv (π g v) = σ g (E.toLinearEquiv v)

theorem **intw'** {π : Representation ℂ G V} {σ : Representation ℂ G W} (E : π.Equiv σ) (g : G) (v : V) : E.toIntertwiningMap.toLinearMap (π g v) = σ g (E.toIntertwiningMap.toLinearMap v)

theorem **intw_symm** {π : Representation ℂ G V} {σ : Representation ℂ G W} (E : π.Equiv σ) (g : G) (w : W) : E.toLinearEquiv.symm (σ g w) = π g (E.toLinearEquiv.symm w)

theorem **smooth_of_equiv** [TopologicalSpace G] {π : Representation ℂ G V} {σ : Representation ℂ G W} (E : π.Equiv σ) (h : IsSmooth π) : IsSmooth σ

theorem **admissible_of_equiv** [TopologicalSpace G] {π : Representation ℂ G V} {σ : Representation ℂ G W} (E : π.Equiv σ) (h : IsAdmissible π) : IsAdmissible σ

theorem **sesq_of_equiv** {π : Representation ℂ G V} {σ : Representation ℂ G W} (E : π.Equiv σ) (h : HasSesq σ) : HasSesq π

theorem **unit_of_equiv** {π : Representation ℂ G V} {σ : Representation ℂ G W} (E : π.Equiv σ) (h : IsUnitarisable π) : IsUnitarisable σ

theorem **irr_of_equiv** {π : Representation ℂ G V} {σ : Representation ℂ G W} (E : π.Equiv σ) (h : π.IsIrreducible) : σ.IsIrreducible

theorem **unit_twist** {ρ : Representation ℂ G V} (h : IsUnitarisable ρ) (χ : G →* ℂˣ) (hχ : ∀ g, ‖(χ g : ℂ)‖ = 1) : IsUnitarisable (twist ρ χ)

theorem **chi_apply** (d : ℕ) (s : ℝ) (g : Levi k fun _ : Fin 2 => Fin d) : (chi k d s g : ℂ) = ((nabs k (Matrix.GeneralLinearGroup.det (g 0)) : ℝ) : ℂ) ^ (-(s : ℂ)) * ((nabs k (Matrix.GeneralLinearGroup.det (g 1)) : ℝ) : ℂ) ^ (s : ℂ)

theorem **norm_chi** (d : ℕ) (s : ℝ) (g : Levi k fun _ : Fin 2 => Fin d) : ‖(chi k d s g : ℂ)‖ = nabs k (Matrix.GeneralLinearGroup.det (g 0)) ^ (-s) * nabs k (Matrix.GeneralLinearGroup.det (g 1)) ^ s

theorem **someRed_of** (h1 : Complementary) (h2 : Compact k) (h3 : Bound k) (h4 : Herm k) (h5 : ProdModel k) : SomeRed k

CompanionTheory.RealGroups.GeneralUnitaryRepresentations.ArchimedeanJacquetSegments.ThetaContra

theorem **nontriv** {G V : Type} [Group G] [AddCommGroup V] [Module ℂ V] (ρ : Representation ℂ G V) (h : ρ.IsIrreducible) : Nontrivial V

theorem **congrProd** {h N : ℕ} {x y : Fin h → IrrGL k} (e : x = y) (hx : ∑ a, (x a).1 = N) (hy : ∑ a, (y a).1 = N) : Nonempty ((prodIrr x hx).Equiv (prodIrr y hy))

theorem **eqv** {G V W : Type} [Group G] [AddCommGroup V] [Module ℂ V] [AddCommGroup W] [Module ℂ W] {ρ : Representation ℂ G V} {σ : Representation ℂ G W} (e : ρ.Equiv σ) (g : G) (v : V) : e (ρ g v) = σ g (e v)

theorem **theta_contra** (hGK : GKLocal k) {N : ℕ} {W : Type} [AddCommGroup W] [Module ℂ W] (π : Representation ℂ (GL (Fin N) k) W) (hs : IsSmooth π) (hi : π.IsIrreducible) : Nonempty (Representation.Equiv (π.comp (thetaW k N)) (contra π))

theorem **hasSesq_of** {G V V₁ V₂ : Type} [Group G] [TopologicalSpace G] [IsTopologicalGroup G] [AddCommGroup V] [Module ℂ V] [AddCommGroup V₁] [Module ℂ V₁] [AddCommGroup V₂] [Module ℂ V₂] (π : Representation ℂ G V) (π₁ : Representation ℂ G V₁) (π₂ : Representation ℂ G V₂) (hi : π.IsIrreducible) (e₁ : π.Equiv π₁) (hc : ConjEquiv π₁ π₂) (e₂ : π₂.Equiv (contra π)) : HasSesq π

theorem **herm_ofB** (k : Type) [Field k] [ValuativeRel k] [TopologicalSpace k] [IsNonarchimedeanLocalField k] (h1 : ConjProd k) (h2 : ConjDual k) : Herm k

CompanionTheory.RealGroups.GeneralUnitaryRepresentations.WhittakerModels.RankDual

theorem **rank_le_one_iff_dual** (W : Type) [AddCommGroup W] [Module ℂ W] : Module.rank ℂ W ≤ 1 ↔ Module.rank ℂ (Module.Dual ℂ W) ≤ 1

theorem **rank_twJacquet_le_one_iff** {G V : Type} [Group G] [AddCommGroup V] [Module ℂ V] (ρ : Representation ℂ G V) (N : Subgroup G) (θ : N →* ℂˣ) : Module.rank ℂ (TwJacquet ρ N θ) ≤ 1 ↔ Module.rank ℂ (GroundTheory.Automorphic.Whittaker.equivariantFunctionals ρ (fun u : N => ((θ u : ℂˣ) : ℂ))) ≤ 1

CompanionTheory.RealGroups.GeneralUnitaryRepresentations.WhittakerModels.UnipBEqUnipotent

theorem **unipB_eq_unipotent** (k : Type) [Field k] [ValuativeRel k] [TopologicalSpace k] [IsNonarchimedeanLocalField k] (n : ℕ) : unipB k (Fin n) = GroundTheory.Automorphic.Iwasawa.unipotent k n

theorem **whitChar_eq_psiN** (k : Type) [Field k] [ValuativeRel k] [TopologicalSpace k] [IsNonarchimedeanLocalField k] (ψ : AddChar k Circle) (n : ℕ) (u : unipB k (Fin n)) : ((whitChar ψ n u : ℂˣ) : ℂ) = GroundTheory.Automorphic.Whittaker.psiN (Circle.coeHom.compAddChar ψ) (u : GL (Fin n) k)

theorem **isAdmissible_iff** {G V : Type} [Group G] [TopologicalSpace G] [IsTopologicalGroup G] [AddCommGroup V] [Module ℂ V] (ρ : Representation ℂ G V) : GlobalTheory.PRep.IsAdmissible ρ ↔ GroundTheory.Automorphic.IsAdmissible ρ

CompanionTheory.RealGroups.GeneralUnitaryRepresentations.WhittakerModels.WhitCuspOfSegCuspGeneric

theorem **whitLeOne** : GlobalTheory.BZMir.Whit.WhitLeOne k

theorem **whitRankIrr** : GlobalTheory.BZSeg.WhitRankIrr k

CompanionTheory.RealGroups.GeneralUnitaryRepresentations.WhittakerModels.WhitLeOneOfDict

theorem **whitLeOne_of_dict** (hU : ∀ n : ℕ, unipB k (Fin n) = GroundTheory.Automorphic.Iwasawa.unipotent k n) (hC : ∀ (ψ : AddChar k Circle) (n : ℕ) (u : unipB k (Fin n)), ((GlobalTheory.LocGL.whitChar ψ n u : ℂˣ) : ℂ) = GroundTheory.Automorphic.Whittaker.psiN (Circle.coeHom.compAddChar ψ) (u : GL (Fin n) k)) (hR : ∀ {G V : Type} [Group G] [AddCommGroup V] [Module ℂ V] (ρ : Representation ℂ G V) (N : Subgroup G) (θ : N →* ℂˣ), Module.rank ℂ (TwJacquet ρ N θ) ≤ 1 ↔ Module.rank ℂ (GroundTheory.Automorphic.Whittaker.equivariantFunctionals ρ (fun u : N => ((θ u : ℂˣ) : ℂ))) ≤ 1) (hA : ∀ {G V : Type} [Group G] [TopologicalSpace G] [IsTopologicalGroup G] [AddCommGroup V] [Module ℂ V] (ρ : Representation ℂ G V), GlobalTheory.PRep.IsAdmissible ρ ↔ GroundTheory.Automorphic.IsAdmissible ρ) : GlobalTheory.BZMir.Whit.WhitLeOne k

CompanionTheory.RealGroups.UnitaryAdmissibility.ArchimedeanUnitaryDual.AdmissibleOf

theorem **locPI_of** (h2 : StSimple) (h3 : StStar) (h4 : StHecke) (h5 : StMod) (h7 : StTransfer) (hP : StPair) (hU : StCartanU) (hG : StCartanGL) (hM : StMove) (hD : StDict) (hC1 : StDefinite) (hC2 : StSepDef) (hC3 : StSepMove) : LocPI

theorem **admissible_of** (hL : LocPI) (hP : StProj) : GlobalTheory.Published.UnitaryIrreducibleAdmissible

theorem **flath_of** (hL : LocPI) (hP : StProj) (hV : StFixVec) (hF : StFactor) (hA : StArchPI) (hI : StIso) (hO : StOp) (hT : StTensor) : GlobalTheory.Published.FlathTensorProduct

CompanionTheory.RealGroups.UnitaryAdmissibility.ArchimedeanUnitaryDual.ArchimedeanDenseStabilizers.DenseRangeOfNotIsReal

theorem **denseRange_of_not_isReal** {K : Type} [Field K] (σ : K →+* ℂ) (h : ¬ ComplexEmbedding.IsReal σ) : DenseRange σ

theorem **hasSignature_of_posDef** {n : ℕ} {J : Matrix (Fin n) (Fin n) ℂ} (h : J.PosDef) : PI.UJClassFin.HasSignature J n 0

theorem **hasSignature** (c : E ≃ₐ[F] E) (G : Matrix (Fin n) (Fin n) E) (hG : IsHermForm c G) (hc : ∀ (σ : E →+* ℂ) (x : E), σ (c x) = starRingEnd ℂ (σ x)) (τ : E →+* ℂ) (h : ∀ x : Fin n → E, x ≠ 0 → 0 < (τ (hermE c G x x)).re) : PI.UJClassFin.HasSignature (Gᵀ.map τ) n 0

theorem **exists_denom** (L : Submodule (𝒪 E) (Fin n → E)) (hL : L.FG) : ∃ c : 𝒪 F, c ≠ 0 ∧ ∀ x ∈ (L : Set (Fin n → E)), ∀ i, IsIntegral ℤ ((c : F) • x i)

theorem **exists_single_mem** (L : Submodule (𝒪 E) (Fin n → E)) (hL : Submodule.span E (L : Set (Fin n → E)) = ⊤) : ∃ c : 𝒪 F, c ≠ 0 ∧ ∀ i, ∀ e : E, IsIntegral ℤ e → Pi.single i ((c : F) • e) ∈ (L : Set (Fin n → E))

CompanionTheory.RealGroups.UnitaryAdmissibility.ArchimedeanUnitaryDual.ArchimedeanDenseStabilizers.StabOfMemLevelOpen

theorem **conj_emb** (v : HeightOneSpectrum (𝒪 F)) (g : GAd σ J) (x : GLoc σ J v) : g * emb σ J v x * g⁻¹ = emb σ J v (proj σ J v g * x * (proj σ J v g)⁻¹)

theorem **normal_emb** (v : HeightOneSpectrum (𝒪 F)) : (emb σ J v).range.Normal

theorem **ratLoc_injective** (u : HeightOneSpectrum (𝒪 F)) : Function.Injective (ratLoc σ J u)

theorem **exists_of_mem_sup** {v : HeightOneSpectrum (𝒪 F)} {W : Subgroup (GAd σ J)} {x : GAd σ J} (h : x ∈ (emb σ J v).range ⊔ W) : ∃ y, ∃ w ∈ W, x = emb σ J v y * w

theorem **emb_mem_levelOpen** {L : Submodule (𝒪 E) (Fin n → E)} {v : HeightOneSpectrum (𝒪 F)} {k : GLoc σ J v} (hk : k ∈ stabLoc σ J L v) : emb σ J v k ∈ levelOpen σ J L

theorem **finite_coset** {A : Subgroup (GAd σ J)} (hA : {γ : GRat σ J | ratEmb σ J γ ∈ A}.Finite) (g : GAd σ J) : {γ : GRat σ J | g⁻¹ * ratEmb σ J γ ∈ A}.Finite

theorem **finite_local** {L : Submodule (𝒪 E) (Fin n → E)} [Submodule.IsLattice E L] (hB : {γ : GRat σ J | ratEmb σ J γ ∈ levelOpen σ J L}.Finite) {v : HeightOneSpectrum (𝒪 F)} {K : Subgroup (GLoc σ J v)} (hK : IsCompact (K : Set (GLoc σ J v))) {C : Set (GLoc σ J v)} (hC : IsCompact C) : {δ : GRat σ J | ratEmb σ J δ ∈ (emb σ J v).range ⊔ (levelOpen σ J L ⊓ K.comap (proj σ J v))} ∧ ratLoc σ J v δ ∈ C}.Finite

theorem **mem_orbit_of_eq** {v : HeightOneSpectrum (𝒪 F)} {K : Subgroup (GLoc σ J v)} {W : Subgroup (GAd σ J)} (hW : W ≤ K.comap (proj σ J v)) {g g' : GLoc σ J v} {γ γ' : GRat σ J} {a w w' : GAd σ J} (hw : w ∈ W) (hw' : w' ∈ W) (e : emb σ J v g = ratEmb σ J γ * a * w) (e' : emb σ J v g' = ratEmb σ J γ' * a * w') : ∃ x ∈ ratGroup σ J v W, ∃ k ∈ K, x * g = g' * k

theorem **sigma_invol** (h : IsCMExt F E σ) (x : E) : σ (σ x) = x

theorem **exists_sigma_ne** (h : IsCMExt F E σ) : ∃ z, σ z ≠ z

theorem **mem_range_of_sigma_eq** (h : IsCMExt F E σ) {y : E} (hy : σ y = y) : y ∈ (algebraMap F E).range

theorem **isTotDef_of_posDef** (h : IsCMExt F E σ) (hJ : IsHermND σ J) (hp : CompanionTheory.Rg.Fl.IsPosDef σ J) : Formalism.UG.IsTotDef σ J

theorem **mulVec_fv** (h : IsCMExt F E σ) (γ : GRat σ J) (x : Fin n → E) : (Formalism.MainNotions.StandardLattice.finMat σ J (ratEmb σ J γ)).mulVec (fun i => (1 : Af F) ⊗ₜ[F] x i) = fun i => (1 : Af F) ⊗ₜ[F] ((γ.1 : Matrix (Fin n) (Fin n) E).mulVec x) i

theorem **stab_of_mem_levelOpen** (h : IsCMExt F E σ) {L : Submodule (𝒪 E) (Fin n → E)} [Submodule.IsLattice E L] {γ : GRat σ J} (hg : ratEmb σ J γ ∈ levelOpen σ J L) (x : Fin n → E) : x ∈ L ↔ (γ.1 : Matrix (Fin n) (Fin n) E).mulVec x ∈ L

theorem **finite_level_rat** (h : IsCMExt F E σ) (hJ : IsHermND σ J) (hp : CompanionTheory.Rg.Fl.IsPosDef σ J) (L : Submodule (𝒪 E) (Fin n → E)) [Submodule.IsLattice E L] : {γ : GRat σ J | ratEmb σ J γ ∈ levelOpen σ J L}.Finite

theorem **exists_class_cover** (h : IsCMExt F E σ) (hJ : IsHermND σ J) (hp : CompanionTheory.Rg.Fl.IsPosDef σ J) (L : Submodule (𝒪 E) (Fin n → E)) [Submodule.IsLattice E L] : ∃ T : Finset (GAd σ J), ∀ x, ∃ (γ : GRat σ J), ∃ a ∈ T, ∃ k ∈ levelOpen σ J L, x = ratEmb σ J γ * a * k

theorem **exists_cover_level** {L : Submodule (𝒪 E) (Fin n → E)} [Submodule.IsLattice E L] {W : Subgroup (GAd σ J)} (hW : IsOpen (W : Set (GAd σ J))) (hinf : ∀ y, ofInf σ J y ∈ W) : ∃ T : Finset (GAd σ J), ∀ k ∈ levelOpen σ J L, ∃ t ∈ T, ∃ w ∈ W, k = t * w

theorem **mem_image_of_smul** {G : Type} [Group G] {Γ K : Subgroup G} (γ : Γ) {x y : G ⧸ K} (e : γ • x = y) : (γ : G) ∈ (fun k => y.out * k * x.out⁻¹) '' (K : Set G)

theorem **finite_orbits** {v : HeightOneSpectrum (𝒪 F)} {K : Subgroup (GLoc σ J v)} {W : Subgroup (GAd σ J)} (hW : W ≤ K.comap (proj σ J v)) {T : Finset (GAd σ J)} (hT : ∀ x, ∃ (γ : GRat σ J), ∃ a ∈ T, ∃ w ∈ W, x = ratEmb σ J γ * a * w) : Finite (MulAction.orbitRel.Quotient (ratGroup σ J v W) (GLoc σ J v ⧸ K))

theorem **isSep_of_posDef** (h : IsCMExt F E σ) (hJ : IsHermND σ J) (hp : CompanionTheory.Rg.Fl.IsPosDef σ J) (v : HeightOneSpectrum (𝒪 F)) (K : Subgroup (GLoc σ J v)) (hKo : IsOpen (K : Set (GLoc σ J v))) (hKc : IsCompact (K : Set (GLoc σ J v))) : CompanionTheory.Rg.Fl.IsSep K

theorem **CompanionTheory.Rg.FlSep.stSepDefB** : CompanionTheory.Rg.Fl.StSepDef

CompanionTheory.RealGroups.UnitaryAdmissibility.ArchimedeanUnitaryDual.FinalB

theorem **stFrame** : CompanionTheory.Rg.FlB.StFrame

theorem **stCartanU** : StCartanU

theorem **locPI** : LocPI

theorem **unitaryIrreducibleAdmissible** : GlobalTheory.Published.UnitaryIrreducibleAdmissible

theorem **flathTensorProduct** : GlobalTheory.Published.FlathTensorProduct

CompanionTheory.RealGroups.UnitaryAdmissibility.ArchimedeanUnitaryDual.InhabitedFinite

theorem **hasPI_of** (h2 : StSimple) (h3 : StStar) (h4 : StHecke) (h5 : StMod) (h7 : StTransfer) {G : Type} [Group G] {K : Subgroup G} {d l : ℕ} (hp : IsHeckePair K) (hc : IsCartan K d l) (hs : IsSep K) : ∃ M, HasPI K M

CompanionTheory.RealGroups.UnitaryAdmissibility.ArchimedeanUnitaryDual.StSandwich

theorem **inhabited** (L : Type) [NontriviallyNormedField L] (n : ℕ) [IsUltrametricDist L] : ∃ Λ : AddSubgroup (Fin n → L), IsLat L Λ ∧ IsCong Λ (0 : L) (1 : (Fin n → L) ≃ₗ[L] (Fin n → L))

CompanionTheory.SpectralGeometry.CountingDefinitions.VoronoiMeans.VorFunEvalEqFineMean

theorem **add** (h : CMOrd O bar) : ∀ a b, bar (a + b) = bar a + bar b

theorem **mul** (h : CMOrd O bar) : ∀ a b, bar (a * b) = bar a * bar b

theorem **one** (h : CMOrd O bar) : bar 1 = 1

theorem **comm** (h : CMOrd O bar) : ∀ (τ : O ≃+* O) (a : O), τ (bar a) = bar (τ a)

theorem **bar_unique** {bar' : O → O} (h : CMOrd O bar) (h' : CMOrd O bar') : bar = bar'

theorem **charZero** (h : CMOrd O bar) : CharZero O

theorem **finite_classGroup** (O : Type) [CommRing O] [IsDedekindDomain O] [Module.Finite ℤ O] [CharZero O] : Finite (ClassGroup O)

theorem **classes** (h : CMOrd O bar) : ∀ q : ℕ, ∃ (n : ℕ) (R : Fin n → ModuleCat.{0} O), (∀ k, Module.Finite O (R k) ∧ Module.Projective O (R k) ∧ Module.finrank O (R k) ≤ q) ∧ ∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Module.finrank O X ≤ q → ∃ k, Nonempty (X ≃ₗ[O] R k)

theorem **int** : CMOrd ℤ id

theorem **exists_extend** {E : Type} [Field E] [NumberField E] (σ : 𝒪 E →+* ℂ) : ∃ φ : E →+* ℂ, ∀ x : 𝒪 E, φ (x : E) = σ x

theorem **of_field** {E : Type} [Field E] [NumberField E] {b : 𝒪 E → 𝒪 E} (hb : ∀ (φ : E →+* ℂ) (a : 𝒪 E), φ (b a : E) = starRingEnd ℂ (φ (a : E))) : CMOrd (𝒪 E) b

theorem **gauss** : CMOrd GaussianInt star

theorem **of_equiv** {O' : Type} [CommRing O'] (h : CMOrd O bar) (e : O ≃+* O') : CMOrd O' fun a => e (bar (e.symm a))

theorem **zero_mem_contact** {O : Type} [CommRing O] (bar : O → O) [Module O X] {ι : Type} (a : ι → X) : (0 : X →ₗ[ℤ] X →ₗ[ℤ] ℝ) ∈ contact bar a

theorem **vorFun_eval_eq_fineMean** {O : Type} [CommRing O] [Module.Finite ℤ O] (g : ℕ) {X : Type} [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Finite ℤ X] (B : X → X → ℝ) : (vorFun O g).eval X B = 2 ^ g * (fineMean 1).eval X B

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.AdeleRingCocompactDiscreteNumberField

theorem **discrete_cocompact** : ∀ (F : Type) [Field F] [NumberField F], (∃ C : Set (AdeleRing (𝒪 F) F), IsCompact C ∧ ∀ y, ∃ l : F, y − algebraMap F (AdeleRing (𝒪 F) F) l ∈ C) ∧ (∀ C : Set (AdeleRing (𝒪 F) F), IsCompact C → {l : F | algebraMap F (AdeleRing (𝒪 F) F) l ∈ C}.Finite) ∧ NoncompactSpace (AdeleRing (𝒪 F) F)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.AdeleRingCocompactDiscreteNumberFieldStatement

theorem **_root_.GlobalTheory.T.adele_ring_cocompact_discrete_number_field** : ∀ (F : Type) [Field F] [NumberField F], (∃ C : Set (AdeleRing (𝒪 F) F), IsCompact C ∧ ∀ y, ∃ l : F, y − algebraMap F (AdeleRing (𝒪 F) F) l ∈ C) ∧ (∀ C : Set (AdeleRing (𝒪 F) F), IsCompact C → {l : F | algebraMap F (AdeleRing (𝒪 F) F) l ∈ C}.Finite) ∧ NoncompactSpace

(AdeleRing (𝒪 F) F)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.AdeleRingStableRangeOne

theorem **local_val** {K Γ : Type*} [Field K] [LinearOrderedCommGroupWithZero Γ] (w : Valuation K Γ) (a b x y : K) (h : x * a + y * b = 1) : ∃ t : K, (t = 0 ∨ t = 1) ∧ a + t * b ≠ 0 ∧ (w a ≤ 1 → w b ≤ 1 → w x ≤ 1 → w y ≤ 1 → w (a + t * b) = 1)

theorem **local_field** {K : Type*} [Field K] (a b x y : K) (h : x * a + y * b = 1) : ∃ t : K, IsUnit (a + t * b)

theorem **finite_part** (R K : Type*) [CommRing R] [IsDedekindDomain R] [Field K] [Algebra R K] [IsFractionRing R K] (a b : FiniteAdeleRing R K) (hab : IsCoprime a b) : ∃ t : FiniteAdeleRing R K, IsUnit (a + t * b)

theorem **infinite_part** (K : Type*) [Field K] [NumberField K] (a b : InfiniteAdeleRing K) (hab : IsCoprime a b) : ∃ t : InfiniteAdeleRing K, IsUnit (a + t * b)

theorem **adeleRing_stableRange_one** : ∀ (F : Type) [Field F] [NumberField F] (a b : AdeleRing (𝒪 F) F), IsCoprime a b → ∃ t, IsUnit (a + t * b)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.AdelesLocallyCompactSecondCountable

theorem **t2Space_finiteAdeleRing_of_numberField** (F : Type) [Field F] [NumberField F] : T2Space (Af F)

theorem **t2Space_infiniteAdeleRing_of_numberField_field** (F : Type) [Field F] [NumberField F] : T2Space (InfiniteAdeleRing F)

theorem **adele_top** : ∀ (F : Type) [Field F] [NumberField F], (LocallyCompactSpace (AF F) ∧ SecondCountableTopology (AF F) ∧ T2Space (AF F)) ∧ (LocallyCompactSpace (Af F) ∧ SecondCountableTopology (Af F) ∧ T2Space (Af F))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.AdelesLocallyCompactSecondCountableStatement

theorem **_root_.GlobalTheory.T.adeles_locally_compact_second_countable** : ∀ (F : Type) [Field F] [NumberField F], (LocallyCompactSpace (AF F) ∧ SecondCountableTopology (AF F) ∧ T2Space (AF F)) ∧ (LocallyCompactSpace (Af F) ∧ SecondCountableTopology (Af F) ∧ T2Space (Af F))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.AdelesSelfDual

theorem **mem_oG** {b : F} : b ∈ oG F ↔ ∃ r : 𝒪 F, (r : F) = b

theorem **mem_dinv** {b : F} : b ∈ dinv F ↔ ∀ r : 𝒪 F, ∃ n : ℤ, Algebra.trace ℚ F (b * (r : F)) = n

theorem **oG_le_dinv** : oG F ≤ dinv F

theorem **oG_eq_closure** : oG F = AddSubgroup.closure (Set.range (integralBasis F))

theorem **dinv_eq_closure** : dinv F = AddSubgroup.closure (Set.range (integralBasis F).traceDual)

theorem **relIndex_oG_dinv** : (oG F).relIndex (dinv F) = (discr F).natAbs

theorem **mem_oG_of_dual** {b : F} (h : ∀ d ∈ dinv F, ∃ n : ℤ, Algebra.trace ℚ F (d * b) = n) : b ∈ oG F

theorem **isCompact_of_relIndex** {H K : AddSubgroup G} (hHK : H ≤ K) (hi : H.relIndex K ≠ 0) (hH : IsCompact (H : Set G)) : IsCompact (K : Set G)

theorem **oh_mul** {x y : A F} (hx : x ∈ Oh F) (hy : y ∈ Oh F) : x * y ∈ Oh F

theorem **ohG_le_MG** (hψ : IsStdChar F ψ) : ohG F ≤ MG ψ

theorem **algebraMap_mem_MG_iff** (hψ : IsStdChar F ψ) {b : F} : algebraMap F (A F) b ∈ MG ψ ↔ b ∈ dinv F

theorem **MG_eq** (hψ : IsStdChar F ψ) : MG ψ = (dinv F).map (fr F) ⊔ ohG F

theorem **comap_ohG** : (ohG F).comap (fr F) = oG F

theorem **relIndex_MG** (hψ : IsStdChar F ψ) : (ohG F).relIndex (MG ψ) = (discr F).natAbs

theorem **perp_MG_subset** (hψ : IsStdChar F ψ) : perp (fun x y => ψ (x * y)) (MG ψ : Set (A F)) ≤ Oh F

theorem **coe_lat** (c : F) : (lat c : Set (A F)) = (fun x => algebraMap F (A F) c * x) '' Oh F

theorem **perp_lat** {c : F} (hc : c ≠ 0) : perp (fun x y => ψ (x * y)) (lat c : Set (A F)) = (fun x => algebraMap F (A F) c⁻¹ * x) '' (MG ψ : Set (A F))

theorem **perp_perp_lat** (hψ : IsStdChar F ψ) {c : F} (hc : c ≠ 0) : perp (fun x y => ψ (x * y)) (perp (fun x y => ψ (x * y)) (lat c : Set (A F))) ≤ (lat c : Set (A F))

theorem **natCast_mul_mem_lat** (N : ℕ) {z : A F} (hz : z ∈ Oh F) : (N : A F) * z ∈ lat (N : F)

theorem **mem_lat_inv** {N : ℕ} (hN : 0 < N) {x : A F} (h : (N : A F) * x ∈ Oh F) : x ∈ lat ((N : F)⁻¹)

theorem **kap_sq** : kap F ^ 2 * |(discr F : ℝ)| = 1

theorem **kap_pos** : 0 < kap F

theorem **lat_all** (hψ : IsStdChar F ψ) (lf : Measure (A F)) (hlf : lf.IsAddHaarMeasure) (h1 : lf (Oh F) = 1) {c : F} (hc : c ≠ 0) : IsCompact (lat c : Set (A F)) ∧ IsOpen (lat c : Set (A F)) ∧ IsCompact (perp (fun x y => ψ (x * y)) (lat c : Set (A F))) ∧ IsOpen (perp (fun x y => ψ (x * y)) (lat c : Set (A F))) ∧ perp (fun x y => ψ (x * y)) (perp (fun x y => ψ (x * y)) (lat c : Set (A F))) ≤ (lat c : Set (A F)) ∧ (ENNReal.ofReal (kap F) • lf) (lat c : Set (A F)) * (ENNReal.ofReal (kap F) • lf) (perp (fun x y => ψ (x * y)) (lat c : Set (A F))) = 1

theorem **finiteAdeles_selfDual** : ∀ (F : Type) [Field F] [NumberField F] [MeasurableSpace (A F)] [BorelSpace (A F)] (lf : Measure (A F)), lf.IsAddHaarMeasure → lf (Oh F) = 1 → ∀ ψ : AddChar (A F) ℂ, IsStdChar F ψ → NADual (A F) (fun x y => ψ (x * y)) (ENNReal.ofReal (kap F) • lf)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.AdelesSelfDualStatement

theorem **_root_.GlobalTheory.T.adeles_self_dual** : ∀ (F : Type) [Field F] [NumberField F] [MeasurableSpace (A F)] [BorelSpace (A F)] (lf : Measure (A F)), lf.IsAddHaarMeasure → lf (Oh F) = 1 → ∀ ψ : AddChar (A F) ℂ, IsStdChar F ψ → NADual (A F) (fun x y => ψ (x * y)) (ENNReal.ofReal (kap F) • lf)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.AdelicBoxLatticeCountRatio

theorem **norm_coord** (hF : IsTotallyReal F) {d M : ℕ} (b : Module.Basis (Fin d) F E) (X : Fin 1 → Fin M → AF F ⊗[F] E) : ‖coord hF b X‖ = sizeTup b (projInf X)

theorem **finrank_coord** (hF : IsTotallyReal F) (d M : ℕ) : Module.finrank ℝ (PiLp 1 (fun _ : Fin 1 × Fin M × Fin d × InfinitePlace F => ℝ)) = d * Module.finrank ℚ F * M

theorem **box_packing** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (M : ℕ), ∀ (Λ : Type) [AddCommGroup Λ] (e : Λ →+ (Fin 1 → Fin M → AF F ⊗[F] E)) (N : ℕ) (r R : ℝ), 0 < r → r ≤ R → ∑' l : Λ, (Set.indicator (box b N R) (fun _ => (1 : ℝ≥0∞))) (e l) ≤ ENNReal.ofReal ((5 * R / r) ^ (2 * Module.finrank ℚ F * M)) * ∑' l : Λ, (Set.indicator (box b N r) (fun _ => (1 : ℝ≥0∞))) (e l)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.AdelicBoxLatticeCountRatioStatement

theorem **_root_.GlobalTheory.T.adelic_box_lattice_count_ratio** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (M : ℕ), ∀ (Λ : Type) [AddCommGroup Λ] (e : Λ →+ (Fin 1 → Fin M → AF F ⊗[F] E)) (N : ℕ) (r R : ℝ), 0 < r → r ≤ R → ∑' l : Λ, (Set.indicator (box b N R) (fun _ => (1 : ℝ≥0∞))) (e l) ≤ ENNReal.ofReal ((5 * R / r) ^ (2 * Module.finrank ℚ F * M)) * ∑' l : Λ, (Set.indicator (box b N r) (fun _ => (1 : ℝ≥0∞))) (e l)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.AdelicBoxesCompactExhaust

theorem **repr_map** {R S : Type} [CommRing R] [CommRing S] [Algebra F R] [Algebra F S] (g : R →ₐ[F] S) {d : ℕ} (b : Module.Basis (Fin d) F E) (y : R ⊗[F] E) (k : Fin d) : (Algebra.TensorProduct.basis S b).repr (Algebra.TensorProduct.map g (AlgHom.id F E) y) k = g ((Algebra.TensorProduct.basis R b).repr y k)

theorem **repr_tmul_mul** {R : Type} [CommRing R] [Algebra F R] {d : ℕ} (b : Module.Basis (Fin d) F E) (t : R) (y : R ⊗[F] E) (k : Fin d) : (Algebra.TensorProduct.basis R b).repr ((t ⊗ₜ[F] (1 : E)) * y) k = t * (Algebra.TensorProduct.basis R b).repr y k

theorem **repr_nat_mul** {R : Type} [CommRing R] [Algebra F R] {d : ℕ} (b : Module.Basis (Fin d) F E) (N : ℕ) (y : R ⊗[F] E) (k : Fin d) : (Algebra.TensorProduct.basis R b).repr ((N : R ⊗[F] E) * y) k = (N : R) * (Algebra.TensorProduct.basis R b).repr y k

theorem **topRing** : IsTopologicalRing (R ⊗[F] E)

theorem **cont_repr** {d : ℕ} (b : Module.Basis (Fin d) F E) (k : Fin d) : Continuous fun y : R ⊗[F] E => (Algebra.TensorProduct.basis R b).repr y k

theorem **ball_cpt** (w : InfinitePlace F) (r : ℝ) : IsCompact (Metric.closedBall (0 : w.Completion) r)

theorem **ainf_ball_cpt** (r : ℝ) : IsCompact {y : Ainf F | ∀ w, ‖y w‖ ≤ r}

theorem **cont_norm_w** (w : InfinitePlace F) : Continuous fun y : Ainf F => ‖y w‖

theorem **cont_fstA** : Continuous (fstA F)

theorem **cont_sndA** : Continuous (sndA F)

theorem **intS_open** (N : ℕ) : IsOpen (intS (F

theorem **exists_nat_v** (v : HeightOneSpectrum (𝒪 F)) (a : Fv F v) : ∃ N : ℕ, 0 < N ∧ (N : Fv F v) * a ∈ v.adicCompletionIntegers F

theorem **exists_nat_Af** (y : Af F) : ∃ N : ℕ, 0 < N ∧ y ∈ intS (F

theorem **intS_mono** {N N' : ℕ} (h : N ∣ N') : intS (F

theorem **mem_lat_iff** (N : ℕ) (v : HeightOneSpectrum (𝒪 F)) (Y : Fin n → Fin M → Fv F v ⊗[F] E) : (fun a i => (N : Fv F v ⊗[F] E) * Y a i) ∈ latV b v ↔ ∀ a i k, (N : Fv F v) * (Algebra.TensorProduct.basis (Fv F v) b).repr (Y a i) k ∈ v.adicCompletionIntegers F

theorem **comp_v** (v : HeightOneSpectrum (𝒪 F)) (X : Fin n → Fin M → AF F ⊗[F] E) (a : Fin n) (i : Fin M) (k : Fin d) : (Algebra.TensorProduct.basis (Fv F v) b).repr (projV v (projFin X) a i) k = evalV F v (sndA F ((Algebra.TensorProduct.basis (AF F) b).repr (X a i) k))

theorem **size_eq** (X : Fin n → Fin M → AF F ⊗[F] E) : sizeTup b (projInf X) = ∑ a, ∑ i, ∑ k, ∑ w : InfinitePlace F, ‖(fstA F ((Algebra.TensorProduct.basis (AF F) b).repr (X a i) k)) w‖

theorem **cont_size** : Continuous fun X : Fin n → Fin M → AF F ⊗[F] E => sizeTup b (projInf X)

theorem **intT_open** (N : ℕ) : IsOpen (intT (n

theorem **mem_intT_iff** (N : ℕ) (X : Fin n → Fin M → AF F ⊗[F] E) : X ∈ intT b N ↔ ∀ v, (fun a i => (N : Fv F v ⊗[F] E) * projV v (projFin X) a i) ∈ latV b v

theorem **mem_box_iff** (N : ℕ) (r : ℝ) (X : Fin n → Fin M → AF F ⊗[F] E) : X ∈ box b N r ↔ X ∈ intT b N ∧ sizeTup b (projInf X) ≤ r

theorem **exists_intT** (X : Fin n → Fin M → AF F ⊗[F] E) : ∃ N : ℕ, 0 < N ∧ X ∈ intT b N

theorem **intT_mono** {N N' : ℕ} (h : N ∣ N') : intT (n

theorem **exhaust** (C : Set (Fin n → Fin M → AF F ⊗[F] E)) (hC : IsCompact C) : ∃ (N : ℕ) (r : ℝ), 0 < N ∧ 0 < r ∧ C ⊆ box b N r

theorem **cinf_cpt** (r : ℝ) : IsCompact {Y : Fin n → Fin M → Ainf F ⊗[F] E | sizeTup b Y ≤ r}

theorem **cv_cpt** (N : ℕ) (hN : 0 < N) (v : HeightOneSpectrum (𝒪 F)) (hL : IsCompact (latV b v : Set (Fin n → Fin M → Fv F v ⊗[F] E))) : IsCompact {Y : Fin n → Fin M → Fv F v ⊗[F] E | (fun a i => (N : Fv F v ⊗[F] E) * Y a i) ∈ latV b v}

theorem **cv_cof** (N : ℕ) (hN : 0 < N) : ∀ᶠ v in Filter.cofinite, ∀ Y : Fin n → Fin M → Fv F v ⊗[F] E, (fun a i => (N : Fv F v ⊗[F] E) * Y a i) ∈ latV b v → Y ∈ latV b v

theorem **projFin_scale** (t : Ainf F) (X : Fin n → Fin M → AF F ⊗[F] E) : projFin ((pairA F t 1 ⊗ₜ[F] (1 : E)) • X) = projFin X

theorem **size_scale** (t : Ainf F) (c : ℝ) (ht : ∀ w : InfinitePlace F, ‖t w‖ = c) (X : Fin n → Fin M → AF F ⊗[F] E) : sizeTup b (projInf ((pairA F t 1 ⊗ₜ[F] (1 : E)) • X)) = c * sizeTup b (projInf X)

theorem **box_facts** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (n M : ℕ), (∀ (N : ℕ) (r : ℝ), 0 < N → IsCompact (box b N r : Set (Fin n → Fin M → AF F ⊗[F] E))) ∧ (∀ C : Set (Fin n → Fin M → AF F ⊗[F] E), IsCompact C → ∃ (N : ℕ) (r : ℝ), 0 < N ∧ 0 < r ∧ C ⊆ box b N r) ∧ ∀ τ : Multiplicative ℝ →* (AF F ⊗[F] E)ˣ, IsFlow τ → ∀ (N : ℕ) (r x : ℝ) (X : Fin n → Fin M → AF F ⊗[F] E), ((τ (Multiplicative.ofAdd (x)) : (AF F ⊗[F] E)ˣ) : AF F ⊗[F] E) • X ∈ box b N (Real.exp x * r) ↔ X ∈ box b N r

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.AdelicBoxesCompactExhaustStatement

theorem **_root_.GlobalTheory.T.adelic_boxes_compact_exhaust** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (n M : ℕ), (∀ (N : ℕ) (r : ℝ), 0 < N → IsCompact (box b N r : Set (Fin n → Fin M → AF F ⊗[F] E))) ∧ (∀ C : Set (Fin n → Fin M → AF F ⊗[F] E), IsCompact C → ∃ (N : ℕ) (r : ℝ), 0 < N ∧ 0 < r ∧ C ⊆ box b N r) ∧ ∀ τ : Multiplicative ℝ →* (AF F ⊗[F] E)ˣ, IsFlow τ → ∀ (N : ℕ) (r x : ℝ) (X : Fin n → Fin M → AF F ⊗[F] E), ((τ (Multiplicative.ofAdd (x)) : (AF F ⊗[F] E)ˣ) : AF F ⊗[F] E) • X ∈ box b N (Real.exp x * r) ↔ X ∈ box b N r

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.AdelicCharacterExponentialFormula

theorem **DM_mulVec_emb_apply** {F : Type} [Field F] [NumberField F] {ι : Type} [Fintype ι] [DecidableEq ι] [DecidableEq (InfinitePlace F)] (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (y : ι → F) (i : ι) (w : InfinitePlace F) : (DM B ρ *ᵥ emb ρ y) (i, w) = ρ w ((B *ᵥ y) i)

theorem **emb_dotProduct_DM** {F : Type} [Field F] [NumberField F] {ι : Type} [Fintype ι] [DecidableEq ι] [DecidableEq (InfinitePlace F)] (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (a y : ι → F) : emb ρ a ⬝ᵥ (DM B ρ *ᵥ emb ρ y) = ∑ w, ρ w (a ⬝ᵥ (B *ᵥ y))

theorem **toA_dotProduct_Bf** {F : Type} [Field F] [NumberField F] {ι : Type} [Fintype ι] (B : Matrix ι ι F) (a y : ι → F) : toA a ⬝ᵥ (Bf B *ᵥ toA y) = algebraMap F (A F) (a ⬝ᵥ (B *ᵥ y))

theorem **chiB_toA_eq_exp** (F : Type) [Field F] [NumberField F] (ψ : AddChar (A F) ℂ) (hψ : IsStdChar F ψ) (ι : Type) [Fintype ι] [DecidableEq ι] [DecidableEq (InfinitePlace F)] (M : ℕ) (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (P : InfinitePlace F → Matrix ι ι ℝ) (hQ : QSetup M B ρ P) (a y : ι → F) : chiB ψ (Bf B) (toA a) (toA y) = Complex.exp (-(2 * (Real.pi : ℂ) * Complex.I) * ((emb ρ a ⬝ᵥ (DM B ρ *ᵥ emb ρ y) : ℝ) : ℂ))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.AdelicCharacterExponentialFormulaStatement

theorem **_root_.GlobalTheory.T.adelic_character_exponential_formula** : ∀ (F : Type) [Field F] [NumberField F] (ψ : AddChar (A F) ℂ), IsStdChar F ψ → ∀ (ι : Type) [Fintype ι] [DecidableEq ι] [DecidableEq (InfinitePlace F)] (M : ℕ) (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (P : InfinitePlace F → Matrix ι ι ℝ), QSetup M B ρ P → ∀ a y : ι → F, chiB ψ (Bf B) (toA a) (toA y) = Complex.exp (-(2 * (Real.pi : ℂ) * Complex.I) * ((emb ρ a ⬝ᵥ (DM B ρ *ᵥ emb ρ y) : ℝ) : ℂ))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.AdelicHaarInvariantUnitaryAction

theorem **map_eq_of_conj** {V W : Type} [TopologicalSpace V] [MeasurableSpace V] [BorelSpace V] [TopologicalSpace W] [MeasurableSpace W] [BorelSpace W] (e : V ≃ₜ W) (μ : Measure V) (T : V → V) (G : W → W) (hG : Measurable G) (hc : ∀ x, e (T x) = G (e x)) (hinv : (μ.map e).map G = μ.map e) : μ.map T = μ

theorem **GlobalTheory.CompanionTheory.SW.HaarInv.actW_map_eq** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (μ : Measure (Fin 1 → Fin M → AF F ⊗[F] E)), μ.IsAddHaarMeasure → ∀ h : HA (AF F) σ J, μ.map (actW (AF F) σ J h) = μ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.AdelicHaarInvariantUnitaryActionStatement

theorem **_root_.GlobalTheory.T.adelic_haar_invariant_unitary_action** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (μ : Measure (Fin 1 → Fin M → AF F ⊗[F] E)), μ.IsAddHaarMeasure → ∀ h : HA (AF F) σ J, μ.map (actW (AF F) σ J h) = μ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.AdelicHaarMeasureLinearChange

theorem **continuous_mulVec_left** (M : Matrix ι ι R) : Continuous (fun x : ι → R => M *ᵥ x)

theorem **image_mulVec_eq_preimage** (M N : Matrix ι ι R) (h1 : M * N = 1) (h2 : N * M = 1) (s : Set (ι → R)) : (fun x => M *ᵥ x) '' s = (fun x => N *ᵥ x) ⁻¹' s

theorem **pi_image_mul** (lf : Measure R) [SigmaFinite lf] (u v : ι → R) (huv : ∀ i, u i * v i = 1) (c : ι → ℝ≥0) (hc : ∀ i (s : Set R), MeasurableSet s → lf ((fun x => u i * x) '' s) = (c i : ℝ≥0∞) * lf s) (s : Set (ι → R)) (hs : MeasurableSet s) : (Measure.pi fun _ : ι => lf) ((fun x i => u i * x i) '' s) = (∏ i, (c i : ℝ≥0∞)) * (Measure.pi fun _ : ι => lf) s

theorem **measurePreserving_transvection** (lf : Measure R) [SigmaFinite lf] [lf.IsAddLeftInvariant] (i j : ι) (hij : i ≠ j) (c : R) : MeasurePreserving (fun x : ι → R => transvection i j c *ᵥ x) (Measure.pi fun _ : ι => lf) (Measure.pi fun _ : ι => lf)

theorem **Bf_mul** (g h : Matrix ι ι F) : Bf (g * h) = Bf g * Bf h

theorem **Bf_one** : Bf (1 : Matrix ι ι F) = 1

theorem **Bf_inv_mul** (g : Matrix ι ι F) (hg : g.det ≠ 0) : Bf g⁻¹ * Bf g = 1

theorem **Bf_mul_inv** (g : Matrix ι ι F) (hg : g.det ≠ 0) : Bf g * Bf g⁻¹ = 1

theorem **pi_scale** : ∀ (F : Type) [Field F] [NumberField F], ∀ [MeasurableSpace (A F)] [BorelSpace (A F)] (lf : Measure (A F)), lf.IsAddHaarMeasure → lf (Oh F) = 1 → ∀ (ι : Type) [Fintype ι] [DecidableEq ι], (∀ g : Matrix ι ι F, g.det ≠ 0 → ∀ s : Set (ι → A F), MeasurableSet s → (Measure.pi fun _ : ι => lf) ((fun x => Bf g *ᵥ x) '' s) = ENNReal.ofReal (|((Algebra.norm ℚ g.det : ℚ) : ℝ)|⁻¹) * (Measure.pi fun _ : ι => lf) s) ∧ ∀ (a : (A F)ˣ) (s : Set (ι → A F)), MeasurableSet s → (Measure.pi fun _ : ι => lf) ((fun x => (a : A F) • x) '' s) = lf ((fun x => (a : A F) * x) '' Oh F) ^ Fintype.card ι * (Measure.pi fun _ : ι => lf) s

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.AdelicHaarMeasureLinearChangeStatement

theorem **_root_.GlobalTheory.T.adelic_haar_measure_linear_change** : ∀ (F : Type) [Field F] [NumberField F], ∀ [MeasurableSpace (A F)] [BorelSpace (A F)] (lf : Measure (A F)), lf.IsAddHaarMeasure → lf (Oh F) = 1 → ∀ (ι : Type) [Fintype ι] [DecidableEq ι], (∀ g : Matrix ι ι F, g.det ≠ 0 → ∀ s : Set (ι → A F), MeasurableSet s → (Measure.pi fun _ : ι => lf) ((fun x => Bf g *ᵥ x) '' s) = ENNReal.ofReal (|((Algebra.norm ℚ g.det : ℚ) : ℝ)|⁻¹) * (Measure.pi fun _ : ι => lf) s) ∧ ∀ (a : (A F)ˣ) (s : Set (ι → A F)), MeasurableSet s → (Measure.pi fun _ : ι => lf) ((fun x => (a : A F) • x) '' s) = lf ((fun x => (a : A F) * x) '' Oh F) ^ Fintype.card ι * (Measure.pi fun _ : ι => lf) s

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.AdelicHaarMeasureScaling

lemma **one_mem_Oh** : (1 : A F) ∈ Oh F

lemma **add_mem_Oh** {x y : A F} (hx : x ∈ Oh F) (hy : y ∈ Oh F) : x + y ∈ Oh F

lemma **neg_mem_Oh** {x : A F} (hx : x ∈ Oh F) : -x ∈ Oh F

lemma **zero_mem_Oh** : (0 : A F) ∈ Oh F

lemma **ι_mem** (r : 𝒪 F) : ι r ∈ Oh F

lemma **ι_sub** (r s : 𝒪 F) : ι r - ι s = ι (r - s)

theorem **mass_of_subgroup** [MeasurableSpace (A F)] [BorelSpace (A F)] (lf : Measure (A F)) (hlf : lf.IsAddHaarMeasure) (h1 : lf (Oh F) = 1) (H : AddSubgroup (A F)) (hHo : IsOpen (H : Set (A F))) (hHs : ∀ x ∈ H, x ∈ Oh F) (I : Ideal (𝒪 F)) (hI : ∀ r : 𝒪 F, r ∈ I ↔ ι r ∈ H) (hI0 : I ≠ ⊥) : lf (H : Set (A F)) = ((Ideal.absNorm I : ℝ≥0∞))⁻¹

lemma **mul_mem_Oh** {x y : A F} (hx : x ∈ Oh F) (hy : y ∈ Oh F) : x * y ∈ Oh F

lemma **uF_val** (c : F) (hc : c ≠ 0) : ((uF c hc : (A F)ˣ) : A F) = algebraMap F (A F) c

lemma **continuous_mulEquiv** (a : (A F)ˣ) : Continuous (mulEquiv a)

lemma **continuous_mulEquiv_symm** (a : (A F)ˣ) : Continuous (mulEquiv a).symm

lemma **isOpen_image_mul** (a : (A F)ˣ) {s : Set (A F)} (hs : IsOpen s) : IsOpen ((fun x => (a : A F) * x) '' s)

theorem **scale_unit** [MeasurableSpace (A F)] [BorelSpace (A F)] (lf : Measure (A F)) (hlf : lf.IsAddHaarMeasure) (h1 : lf (Oh F) = 1) (a : (A F)ˣ) (s : Set (A F)) (hs : MeasurableSet s) : lf ((fun x => (a : A F) * x) '' s) = lf ((fun x => (a : A F) * x) '' Oh F) * lf s

theorem **mass_mul_int** [MeasurableSpace (A F)] [BorelSpace (A F)] (lf : Measure (A F)) (hlf : lf.IsAddHaarMeasure) (h1 : lf (Oh F) = 1) (r : 𝒪 F) (hr : (r : F) ≠ 0) : lf ((fun x => algebraMap F (A F) (r : F) * x) '' Oh F) = (((Algebra.norm ℤ r).natAbs : ℝ≥0∞))⁻¹

lemma **isOpen_denSub** (b : F) : IsOpen (denSub b : Set (A F))

lemma **mem_denI** (b : F) (r : 𝒪 F) : r ∈ denI b ↔ ∃ t : 𝒪 F, (t : F) = (r : F) * b

lemma **denI_ne_bot** (b : F) : denI b ≠ ⊥

lemma **mem_denI_iff_sub** (b : F) (r : 𝒪 F) : r ∈ denI b ↔ ι r ∈ denSub b

theorem **mass_den** [MeasurableSpace (A F)] [BorelSpace (A F)] (lf : Measure (A F)) (hlf : lf.IsAddHaarMeasure) (h1 : lf (Oh F) = 1) (b : F) : lf {x | x ∈ Oh F ∧ algebraMap F (A F) b * x ∈ Oh F} = ((den b : ℝ≥0∞))⁻¹

theorem **scale_field** [MeasurableSpace (A F)] [BorelSpace (A F)] (lf : Measure (A F)) (hlf : lf.IsAddHaarMeasure) (h1 : lf (Oh F) = 1) (c : F) (hc : c ≠ 0) (s : Set (A F)) (hs : MeasurableSet s) : lf ((fun x => algebraMap F (A F) c * x) '' s) = ENNReal.ofReal (|((Algebra.norm ℚ c : ℚ) : ℝ)|⁻¹) * lf s

theorem **lf_scale** : ∀ (F : Type) [Field F] [NumberField F], ∀ [MeasurableSpace (A F)] [BorelSpace (A F)] (lf : Measure (A F)), lf.IsAddHaarMeasure → lf (Oh F) = 1 → (∀ (c : F), c ≠ 0 → ∀ s : Set (A F), MeasurableSet s → lf ((fun x => algebraMap F (A F) c * x) '' s) = ENNReal.ofReal (|((Algebra.norm ℚ c : ℚ) : ℝ)|⁻¹) * lf s) ∧ (∀ b : F, lf {x | x ∈ Oh F ∧ algebraMap F (A F) b * x ∈ Oh F} = ((den b : ℝ≥0∞))⁻¹) ∧ ∀ (a : (A F)ˣ) (s : Set (A F)), MeasurableSet s → lf ((fun x => (a : A F) * x) '' s) = lf ((fun x => (a : A F) * x) '' Oh F) * lf s

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.AdelicHaarMeasureScalingStatement

theorem **_root_.GlobalTheory.T.adelic_haar_measure_scaling** : ∀ (F : Type) [Field F] [NumberField F], ∀ [MeasurableSpace (A F)] [BorelSpace (A F)] (lf : Measure (A F)), lf.IsAddHaarMeasure → lf (Oh F) = 1 → (∀ (c : F), c ≠ 0 → ∀ s : Set (A F), MeasurableSet s → lf ((fun x => algebraMap F (A F) c * x) '' s) = ENNReal.ofReal (|((Algebra.norm ℚ c : ℚ) : ℝ)|⁻¹) * lf s) ∧ (∀ b : F, lf {x | x ∈ Oh F ∧ algebraMap F (A F) b * x ∈ Oh F} = ((den b : ℝ≥0∞))⁻¹) ∧ ∀ (a : (A F)ˣ) (s : Set (A F)), MeasurableSet s → lf ((fun x => (a : A F) * x) '' s) = lf ((fun x => (a : A F) * x) '' Oh F) * lf s

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.AdelicHaarScalingByRealIdele

theorem **map_eq_smul_of_scale** {V W G : Type*} [NormedAddCommGroup V] [NormedSpace ℝ V] [FiniteDimensional ℝ V] [MeasurableSpace V] [BorelSpace V] [AddCommGroup W] [TopologicalSpace W] [IsTopologicalAddGroup W] [LocallyCompactSpace W] [SecondCountableTopology W] [MeasurableSpace W] [BorelSpace W] [AddCommGroup G] [TopologicalSpace G] [IsTopologicalAddGroup G] [MeasurableSpace G] [BorelSpace G] (Φ : G ≃ₜ+ V × W) (φ : G → G) (t : ℝ) (ht : t ≠ 0) (h : ∀ x, Φ (φ x) = (t • (Φ x).1, (Φ x).2)) (μ : Measure G) [μ.IsAddHaarMeasure] : μ.map φ = ENNReal.ofReal |(t ^ Module.finrank ℝ V)⁻¹| • μ

theorem **coord_theta** [IsTotallyReal F] (θ : ℝ →+* InfiniteAdeleRing F) (t : ℝ) (w : InfinitePlace F) : InfinitePlace.Completion.ringEquivRealOfIsReal (IsTotallyReal.isReal w) (θ t w) = t

theorem **card_places** [IsTotallyReal F] : Fintype.card (InfinitePlace F) = Module.finrank ℚ F

theorem **map_scalar** : ∀ (F : Type) [Field F] [NumberField F], IsTotallyReal F → ∀ θ : ℝ →+* InfiniteAdeleRing F, Continuous θ → ∀ [MeasurableSpace (AdeleRing (𝒪 F) F)] [BorelSpace (AdeleRing (𝒪 F) F)] (ι : Type) [Fintype ι] (μ : Measure (ι → AdeleRing (𝒪 F) F)) [μ.IsAddHaarMeasure] (t : ℝ), 0 < t → μ.map (fun x i => (show AdeleRing (𝒪 F) F from (θ t, 1)) * x i) = ENNReal.ofReal ((t ^ (Module.finrank ℚ F * Fintype.card ι))⁻¹) • μ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.AdelicHaarScalingByRealIdeleStatement

theorem **_root_.GlobalTheory.T.adelic_haar_scaling_by_real_idele** : ∀ (F : Type) [Field F] [NumberField F], IsTotallyReal F → ∀ θ : ℝ →+* InfiniteAdeleRing F, Continuous θ → ∀ [MeasurableSpace (AdeleRing (𝒪 F) F)] [BorelSpace (AdeleRing (𝒪 F) F)] (ι : Type) [Fintype ι] (μ : Measure (ι → AdeleRing (𝒪 F) F)) [μ.IsAddHaarMeasure] (t : ℝ), 0 < t → μ.map (fun x i => (show AdeleRing (𝒪 F) F from (θ t, 1)) * x i) = ENNReal.ofReal ((t ^ (Module.finrank ℚ F * Fintype.card ι))⁻¹) • μ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.AdelicLatticeMeasureFormula

theorem **muF_latN** : ∀ (F : Type) [Field F] [NumberField F] [MeasurableSpace (A F)] [BorelSpace (A F)] (lf : Measure (A F)), lf.IsAddHaarMeasure → lf (Oh F) = 1 → ∀ (ι : Type) [Fintype ι] [DecidableEq ι] (B : Matrix ι ι F) (N : ℕ), 0 < N → (muF B lf (latN F ι N : Set (ι → A F))).toReal = kap F ^ Fintype.card ι * (Real.sqrt |((Algebra.norm ℚ B.det : ℚ) : ℝ)|)⁻¹ * (((N : ℝ) ^ Module.finrank ℚ F)⁻¹) ^ Fintype.card ι

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.AdelicLatticeMeasureFormulaStatement

theorem **_root_.GlobalTheory.T.adelic_lattice_measure_formula** : ∀ (F : Type) [Field F] [NumberField F] [MeasurableSpace (A F)] [BorelSpace (A F)] (lf : Measure (A F)), lf.IsAddHaarMeasure → lf (Oh F) = 1 → ∀ (ι : Type) [Fintype ι] [DecidableEq ι] (B : Matrix ι ι F) (N : ℕ), 0 < N → (muF B lf (latN F ι N : Set (ι → A F))).toReal = kap F ^ Fintype.card ι * (Real.sqrt |((Algebra.norm ℚ B.det : ℚ) : ℝ)|)⁻¹ * (((N : ℝ) ^ Module.finrank ℚ F)⁻¹) ^ Fintype.card ι

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.AdelicLevelLatticeCompactOpen

theorem **latN_coe_eq_pi** (F : Type) [Field F] [NumberField F] (ι : Type) [Fintype ι] [DecidableEq ι] (N : ℕ) : (latN F ι N : Set (ι → A F)) = Set.pi Set.univ (fun _ => {x : A F | ∃ y ∈ Oh F, x = (N : A F) * y})

theorem **isCompact_mul_oh** (F : Type) [Field F] [NumberField F] (N : ℕ) : IsCompact {x : A F | ∃ y ∈ Oh F, x = (N : A F) * y}

theorem **main** : ∀ (F : Type) [Field F] [NumberField F] (ι : Type) [Fintype ι] [DecidableEq ι] (N : ℕ), 0 < N → IsCompact (latN F ι N : Set (ι → A F)) ∧ IsOpen (latN F ι N : Set (ι → A F)) ∧ ∀ a : ι → F, SB (indC N a)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.AdelicLevelLatticeCompactOpenStatement

theorem **_root_.GlobalTheory.T.adelic_level_lattice_compact_open** : ∀ (F : Type) [Field F] [NumberField F] (ι : Type) [Fintype ι] [DecidableEq ι] (N : ℕ), 0 < N → IsCompact (latN F ι N : Set (ι → A F)) ∧ IsOpen (latN F ι N : Set (ι → A F)) ∧ ∀ a : ι → F, SB (indC N a)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.AdelicOrbitOfRationalTuple

theorem **sigR_inclR** (σ : E →ₐ[F] E) (e : E) : sigR R σ (inclR R e) = inclR R (σ e)

theorem **cT_map_inclR** (σ : E →ₐ[F] E) {n : ℕ} (g : Matrix (Fin n) (Fin n) E) : cT (sigR R σ) (g.map (inclR R)) = (cT (σ : E →+* E) g).map (inclR R)

theorem **gramR_ratTup** (σ : E →ₐ[F] E) {M n : ℕ} (J : Matrix (Fin M) (Fin M) E) (X : Fin n → Fin M → E) : gramR R σ J (ratTup R X) = (gramE σ J X).map (inclR R)

theorem **gramE_herm** (σ : E →ₐ[F] E) (hσ : ∀ x : E, σ (σ x) = x) {M n : ℕ} (J : Matrix (Fin M) (Fin M) E) (hJ : ∀ i j, σ (J j i) = J i j) (X : Fin n → Fin M → E) (a b : Fin n) : σ (gramE σ J X b a) = gramE σ J X a b

theorem **inclR_algebraMap** (d : F) : inclR (E

theorem **trans_uc** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (R : Type) [CommRing R] [Algebra F R], PIW.AdelicOrbitU.UnitChoice R → ∀ (n M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (X : Fin n → Fin M → E) (Y : Fin n → Fin M → R ⊗[F] E), (gramE σ J X).det ≠ 0 → gramR R σ J Y = (gramE σ J X).map (inclR R) → ∃ h : HA R σ J, actW R σ J h (ratTup R X) = Y

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.AdelicOrbitOfRationalTupleStatement

theorem **_root_.GlobalTheory.T.adelic_orbit_of_rational_tuple** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (R : Type) [CommRing R] [Algebra F R], PIW.AdelicOrbitU.UnitChoice R → ∀ (n M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (X : Fin n → Fin M → E) (Y : Fin n → Fin M → R ⊗[F] E), (gramE σ J X).det ≠ 0 → gramR R σ J Y = (gramE σ J X).map (inclR R) → ∃ h : HA R σ J, actW R σ J h (ratTup R X) = Y

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.AdelicPrimitiveVectorCompactRescale

theorem **unimodular_compact_mod_units** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ), ∃ Ω : Set (Fin 1 → Fin M → AF F ⊗[F] E), IsCompact Ω ∧ ∀ y : Fin 1 → Fin M → AF F ⊗[F] E, (∃ z : Fin M → AF F ⊗[F] E, z ·ᵥ y 0 = 1) → ∃ (a : (AF F ⊗[F] E)ˣ) (z : Fin 1 → Fin M → AF F ⊗[F] E), (a : AF F ⊗[F] E) • y ∈ Ω ∧ z ∈ Ω ∧ z 0 ·ᵥ ((a : AF F ⊗[F] E) • y) 0 = 1

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.AdelicPrimitiveVectorCompactRescaleStatement

theorem **_root_.GlobalTheory.T.adelic_primitive_vector_compact_rescale** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ), ∃ Ω : Set (Fin 1 → Fin M → AF F ⊗[F] E), IsCompact Ω ∧ ∀ y : Fin 1 → Fin M → AF F ⊗[F] E, (∃ z : Fin M → AF F ⊗[F] E, z ·ᵥ y 0 = 1) → ∃ (a : (AF F ⊗[F] E)ˣ) (z : Fin 1 → Fin M → AF F ⊗[F] E), (a : AF F ⊗[F] E) • y ∈ Ω ∧ z ∈ Ω ∧ z 0 ·ᵥ ((a : AF F ⊗[F] E) • y) 0 = 1

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.AdelicTupleCompactSets

theorem **isOpen_intAd** : IsOpen (intAd F)

theorem **isOpen_latN** (N : ℕ) : IsOpen (latN F N)

theorem **isCompact_latN** (N : ℕ) (hN : 0 < N) : IsCompact (latN F N)

theorem **natCast_apply** (c : ℕ) (y : Af F) (v : HeightOneSpectrum (𝒪 F)) : ((c : Af F) * y) v = (c : v.adicCompletion F) * y v

theorem **mem_latN_iff** (N : ℕ) (y : Af F) : y ∈ latN F N ↔ ∀ v, (N : v.adicCompletion F) * y v ∈ v.adicCompletionIntegers F

theorem **latN_mono** {N N' : ℕ} (h : N ∣ N') : latN F N ≤ latN F N'

theorem **exists_nat_mul_mem** (v : HeightOneSpectrum (𝒪 F)) (x : v.adicCompletion F) : ∃ N : ℕ, 0 < N ∧ (N : v.adicCompletion F) * x ∈ v.adicCompletionIntegers F

theorem **exists_mem_latN** (y : Af F) : ∃ N : ℕ, 0 < N ∧ y ∈ latN F N

theorem **exists_latN_of_isCompact** {L : Set (Af F)} (hL : IsCompact L) : ∃ N : ℕ, 0 < N ∧ L ⊆ latN F N

theorem **repr_map** {d : ℕ} (b : Module.Basis (Fin d) F E) {R R' : Type} [CommRing R] [CommRing R'] [Algebra F R] [Algebra F R'] (f : R →ₐ[F] R') (y : R ⊗[F] E) (k : Fin d) : (Algebra.TensorProduct.basis R' b).repr (Algebra.TensorProduct.map f (AlgHom.id F E) y) k = f ((Algebra.TensorProduct.basis R b).repr y k)

theorem **repr_natCast_mul** {d : ℕ} (b : Module.Basis (Fin d) F E) {R : Type} [CommRing R] [Algebra F R] (N : ℕ) (y : R ⊗[F] E) (k : Fin d) : (Algebra.TensorProduct.basis R b).repr ((N : R ⊗[F] E) * y) k = (N : R) * (Algebra.TensorProduct.basis R b).repr y k

theorem **fin_iff** {d n M : ℕ} (b : Module.Basis (Fin d) F E) (N : ℕ) (X : Fin n → Fin M → AF F ⊗[F] E) : (∀ v, (fun a i => (N : Fv F v ⊗[F] E) * projV v (projFin X) a i) ∈ latV b v) ↔ ∀ a i k, sndA F (coordA b X a i k) ∈ latN F N

theorem **size_eq** {d n M : ℕ} (b : Module.Basis (Fin d) F E) (X : Fin n → Fin M → AF F ⊗[F] E) : sizeTup b (projInf X) = sizeA (coordA b X)

theorem **continuous_fstA** : Continuous (fstA F)

theorem **continuous_sndA** : Continuous (sndA F)

theorem **continuous_coord** {d n M : ℕ} (a : Fin n) (i : Fin M) (k : Fin d) : Continuous fun Z : Fin n → Fin M → Fin d → AF F => Z a i k

theorem **continuous_sizeA** {d n M : ℕ} : Continuous (sizeA : (Fin n → Fin M → Fin d → AF F) → ℝ)

theorem **norm_le_sizeA** {d n M : ℕ} (Z : Fin n → Fin M → Fin d → AF F) (a i k) (v : InfinitePlace F) : ‖fstA F (Z a i k) v‖ ≤ sizeA Z

theorem **isCompact_boxT** (d n M N : ℕ) (r : ℝ) (hN : 0 < N) : IsCompact (boxT F d n M N r)

theorem **compact_tuples** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (n M : ℕ), (∀ (N : ℕ) (r : ℝ), 0 < N → IsCompact {X : (Fin n → Fin M → AF F ⊗[F] E) | (∀ v, (fun a i => (N : Fv F v ⊗[F] E) * projV v (projFin X) a i) ∈ latV b v) ∧ sizeTup b (projInf X) ≤ r}) ∧ ∀ K : Set (Fin n → Fin M → AF F ⊗[F] E), IsCompact K → ∃ (N : ℕ) (r : ℝ), 0 < N ∧ 0 < r ∧ ∀ X ∈ K, (∀ v, (fun a i => (N : Fv F v ⊗[F] E) * projV v (projFin X) a i) ∈ latV b v) ∧ sizeTup b (projInf X) ≤ r

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.AdelicTupleCompactSetsStatement

theorem **_root_.GlobalTheory.T.adelic_tuple_compact_sets** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (n M : ℕ), (∀ (N : ℕ) (r : ℝ), 0 < N → IsCompact {X : (Fin n → Fin M → AF F ⊗[F] E) | (∀ v, (fun a i => (N : Fv F v ⊗[F] E) * projV v (projFin X) a i) ∈ latV b v) ∧ sizeTup b (projInf X) ≤ r}) ∧ ∀ K : Set (Fin n → Fin M → AF F ⊗[F] E), IsCompact K → ∃ (N : ℕ) (r : ℝ), 0 < N ∧ 0 < r ∧ ∀ X ∈ K, (∀ v, (fun a i => (N : Fv F v ⊗[F] E) * projV v (projFin X) a i) ∈ latV b v) ∧ sizeTup b (projInf X) ≤ r

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.AdelicTupleLocalGlobalProperties

theorem **repr_map** {R R' : Type} [CommRing R] [CommRing R'] [Algebra F R] [Algebra F R'] (φ : R →ₐ[F] R') {d : ℕ} (b : Module.Basis (Fin d) F E) (y : R ⊗[F] E) (k : Fin d) : (Algebra.TensorProduct.basis R' b).repr (Algebra.TensorProduct.map φ (AlgHom.id F E) y) k = φ ((Algebra.TensorProduct.basis R b).repr y k)

theorem **continuous_repr** {d : ℕ} (b : Module.Basis (Fin d) F E) (k : Fin d) : Continuous fun y : R ⊗[F] E => (Algebra.TensorProduct.basis R b).repr y k

theorem **continuous_map** {R' : Type} [CommRing R'] [Algebra F R'] [TopologicalSpace R'] [IsTopologicalRing R'] (φ : R →ₐ[F] R') (hφ : Continuous φ) {d : ℕ} (b : Module.Basis (Fin d) F E) : Continuous fun y : R ⊗[F] E => Algebra.TensorProduct.map φ (AlgHom.id F E) y

theorem **continuous_pV** (v : HeightOneSpectrum (𝒪 F)) : Continuous (pV (F

theorem **continuous_fstA** : Continuous (fstA F)

theorem **crd_inf** (X : Fin n → Fin M → AF F ⊗[F] E) (a : Fin n) (i : Fin M) (k : Fin d) : (Algebra.TensorProduct.basis (Ainf F) b).repr (projInf X a i) k = fstA F ((Algebra.TensorProduct.basis (AF F) b).repr (X a i) k)

theorem **crd_v** (v : HeightOneSpectrum (𝒪 F)) (X : Fin n → Fin M → AF F ⊗[F] E) (a : Fin n) (i : Fin M) (k : Fin d) : (Algebra.TensorProduct.basis (Fv F v) b).repr (projV v (projFin X) a i) k = pV v ((Algebra.TensorProduct.basis (AF F) b).repr (X a i) k)

theorem **tuple_ext** (X Y : Fin n → Fin M → AF F ⊗[F] E) (hinf : projInf X = projInf Y) (hv : ∀ v, projV v (projFin X) = projV v (projFin Y)) : X = Y

theorem **eventually_lat** (X : Fin n → Fin M → AF F ⊗[F] E) : ∀ᶠ v in Filter.cofinite, projV v (projFin X) ∈ latV b v

theorem **exists_glue** (Xinf : Fin n → Fin M → Ainf F ⊗[F] E) (Xv : ∀ v : HeightOneSpectrum (𝒪 F), Fin n → Fin M → Fv F v ⊗[F] E) (h : ∀ᶠ v in Filter.cofinite, Xv v ∈ latV b v) : ∃ X : Fin n → Fin M → AF F ⊗[F] E, projInf X = Xinf ∧ ∀ v, projV v (projFin X) = Xv v

theorem **isCompact_box** {D : ∀ v : HeightOneSpectrum (𝒪 F), Set (Fv F v)} (hc : ∀ v, IsCompact (D v)) (hf : ∀ᶠ v in Filter.cofinite, D v ⊆ (v.adicCompletionIntegers F : Set (Fv F v))) : IsCompact {x : Af F | ∀ v, x v ∈ D v}

theorem **isCompact_glue** (Cinf : Set (Fin n → Fin M → Ainf F ⊗[F] E)) (Cv : ∀ v : HeightOneSpectrum (𝒪 F), Set (Fin n → Fin M → Fv F v ⊗[F] E)) (hinf : IsCompact Cinf) (hv : ∀ v, IsCompact (Cv v)) (hlat : ∀ᶠ v in Filter.cofinite, Cv v ⊆ latV b v) : IsCompact {X : Fin n → Fin M → AF F ⊗[F] E | projInf X ∈ Cinf ∧ ∀ v, projV v (projFin X) ∈ Cv v}

theorem **glue_main** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (n M : ℕ), (∀ X Y : Fin n → Fin M → AF F ⊗[F] E, projInf X = projInf Y → (∀ v, projV v (projFin X) = projV v (projFin Y)) → X = Y) ∧ (∀ X : Fin n → Fin M → AF F ⊗[F] E, ∀ᶠ v in Filter.cofinite, projV v (projFin X) ∈ latV b v) ∧ (∀ (Xinf : Fin n → Fin M → Ainf F ⊗[F] E) (Xv : ∀ v : HeightOneSpectrum (𝒪 F), Fin n → Fin M → Fv F v ⊗[F] E), (∀ᶠ v in Filter.cofinite, Xv v ∈ latV b v) → ∃ X : Fin n → Fin M → AF F ⊗[F] E, projInf X = Xinf ∧ ∀ v, projV v (projFin X) = Xv v) ∧ ∀ (Cinf : Set (Fin n → Fin M → Ainf F ⊗[F] E)) (Cv : ∀ v : HeightOneSpectrum (𝒪 F), Set (Fin n → Fin M → Fv F v ⊗[F] E)), IsCompact Cinf → (∀ v, IsCompact (Cv v)) → (∀ᶠ v in Filter.cofinite, Cv v ⊆ latV b v) → IsCompact {X : Fin n → Fin M → AF F ⊗[F] E

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.AdelicTupleLocalGlobalPropertiesStatement

theorem **_root_.GlobalTheory.T.adelic_tuple_local_global_properties** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (n M : ℕ), (∀ X Y : Fin n → Fin M → AF F ⊗[F] E, projInf X = projInf Y → (∀ v, projV v (projFin X) = projV v (projFin Y)) → X = Y) ∧ (∀ X : Fin n → Fin M → AF F ⊗[F] E, ∀ᶠ v in Filter.cofinite, projV v (projFin X) ∈ latV b v) ∧ (∀ (Xinf : Fin n → Fin M → Ainf F ⊗[F] E) (Xv : ∀ v : HeightOneSpectrum (𝒪 F), Fin n → Fin M → Fv F v ⊗[F] E), (∀ᶠ v in Filter.cofinite, Xv v ∈ latV b v) → ∃ X : Fin n → Fin M → AF F ⊗[F] E, projInf X = Xinf ∧ ∀ v, projV v (projFin X) = Xv v) ∧ ∀ (Cinf : Set (Fin n → Fin M → Ainf F ⊗[F] E)) (Cv : ∀ v : HeightOneSpectrum (𝒪 F), Set (Fin n → Fin M → Fv F v ⊗[F] E)), IsCompact Cinf → (∀ v, IsCompact (Cv v)) → (∀ᶠ v in Filter.cofinite, Cv v ⊆ latV

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.AdelicTupleProjections

theorem **repr_map** {d : ℕ} (b : Module.Basis (Fin d) F E) (f : R →ₐ[F] S) (y : R ⊗[F] E) (k : Fin d) : (Algebra.TensorProduct.basis S b).repr (Algebra.TensorProduct.map f (AlgHom.id F E) y) k = f ((Algebra.TensorProduct.basis R b).repr y k)

theorem **continuous_mapTens** {d : ℕ} (b : Module.Basis (Fin d) F E) (f : R →ₐ[F] S) (hf : Continuous f) : Continuous (Algebra.TensorProduct.map f (AlgHom.id F E) : R ⊗[F] E → S ⊗[F] E)

theorem **continuous_mapTup** {d n M : ℕ} (b : Module.Basis (Fin d) F E) (f : R →ₐ[F] S) (hf : Continuous f) : Continuous (fun (X : Fin n → Fin M → R ⊗[F] E) (a : Fin n) (i : Fin M) => Algebra.TensorProduct.map f (AlgHom.id F E) (X a i))

theorem **continuous_fstA** : Continuous (fstA F)

theorem **continuous_sndA** : Continuous (sndA F)

theorem **continuous_evalV** (v : HeightOneSpectrum (𝒪 F)) : Continuous (evalV F v)

theorem **coord_projInf** {d n M : ℕ} (b : Module.Basis (Fin d) F E) (X : Fin n → Fin M → AF F ⊗[F] E) (a : Fin n) (i : Fin M) (k : Fin d) : (Algebra.TensorProduct.basis (Ainf F) b).repr (projInf X a i) k = ((Algebra.TensorProduct.basis (AF F) b).repr (X a i) k).1

theorem **coord_projFin** {d n M : ℕ} (b : Module.Basis (Fin d) F E) (X : Fin n → Fin M → AF F ⊗[F] E) (a : Fin n) (i : Fin M) (k : Fin d) : (Algebra.TensorProduct.basis (Af F) b).repr (projFin X a i) k = ((Algebra.TensorProduct.basis (AF F) b).repr (X a i) k).2

theorem **coord_projV** {d n M : ℕ} (b : Module.Basis (Fin d) F E) (v : HeightOneSpectrum (𝒪 F)) (X : Fin n → Fin M → Af F ⊗[F] E) (a : Fin n) (i : Fin M) (k : Fin d) : (Algebra.TensorProduct.basis (Fv F v) b).repr (projV v X a i) k = ((Algebra.TensorProduct.basis (Af F) b).repr (X a i) k) v

theorem **latV_eq** {d n M : ℕ} (b : Module.Basis (Fin d) F E) (v : HeightOneSpectrum (𝒪 F)) : (latV b v : Set (Fin n → Fin M → Fv F v ⊗[F] E)) = tupHomeo b n M ⁻¹' (Set.pi Set.univ fun _ => Set.pi Set.univ fun _ => Set.pi Set.univ fun _ => (v.adicCompletionIntegers F : Set (Fv F v)))

theorem **isCompact_latV** {d n M : ℕ} (b : Module.Basis (Fin d) F E) (v : HeightOneSpectrum (𝒪 F)) : IsCompact (latV b v : Set (Fin n → Fin M → Fv F v ⊗[F] E))

theorem **isOpen_latV** {d n M : ℕ} (b : Module.Basis (Fin d) F E) (v : HeightOneSpectrum (𝒪 F)) : IsOpen (latV b v : Set (Fin n → Fin M → Fv F v ⊗[F] E))

theorem **projV_mem_latV_iff** {d n M : ℕ} (b : Module.Basis (Fin d) F E) (v : HeightOneSpectrum (𝒪 F)) (X : Fin n → Fin M → Af F ⊗[F] E) : projV v X ∈ (latV b v : Set (Fin n → Fin M → Fv F v ⊗[F] E)) ↔ ∀ a i k, ((Algebra.TensorProduct.basis (Af F) b).repr (X a i) k) v ∈ v.adicCompletionIntegers F

theorem **isOpen_intOff** {d n M : ℕ} (b : Module.Basis (Fin d) F E) (S : Finset (HeightOneSpectrum (𝒪 F))) : IsOpen (intOff b S : Set (Fin n → Fin M → AF F ⊗[F] E))

theorem **eventually_mem_latV** {d n M : ℕ} (b : Module.Basis (Fin d) F E) (X : Fin n → Fin M → Af F ⊗[F] E) : ∀ᶠ v in Filter.cofinite, projV v X ∈ (latV b v : Set (Fin n → Fin M → Fv F v ⊗[F] E))

theorem **ext_proj** {d n M : ℕ} (b : Module.Basis (Fin d) F E) (X Y : Fin n → Fin M → AF F ⊗[F] E) (h1 : projInf X = projInf Y) (h2 : ∀ v, projV v (projFin X) = projV v (projFin Y)) : X = Y

theorem **exists_glue** {d n M : ℕ} (b : Module.Basis (Fin d) F E) (Y : Fin n → Fin M → Ainf F ⊗[F] E) (Z : ∀ v : HeightOneSpectrum (𝒪 F), (Fin n → Fin M → Fv F v ⊗[F] E)) (hZ : ∀ᶠ v in Filter.cofinite, Z v ∈ (latV b v : Set (Fin n → Fin M → Fv F v ⊗[F] E))) : ∃ X : (Fin n → Fin M → AF F ⊗[F] E), projInf X = Y ∧ ∀ v, projV v (projFin X) = Z v

theorem **continuous_sizeTup** {d n M : ℕ} (b : Module.Basis (Fin d) F E) : Continuous (sizeTup b : (Fin n → Fin M → Ainf F ⊗[F] E) → ℝ)

theorem **proj_props** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (n M : ℕ), Continuous (projInf : (Fin n → Fin M → AF F ⊗[F] E) → _) ∧ Continuous (projFin : (Fin n → Fin M → AF F ⊗[F] E) → _) ∧ (∀ v, Continuous (projV v : (Fin n → Fin M → Af F ⊗[F] E) → _)) ∧ (∀ v, IsCompact (latV b v : Set (Fin n → Fin M → Fv F v ⊗[F] E))) ∧ IsOpen (latV b v : Set (Fin n → Fin M → Fv F v ⊗[F] E))) ∧ (∀ S, IsOpen (intOff b S : Set (Fin n → Fin M → AF F ⊗[F] E))) ∧ (∀ X : (Fin n → Fin M → Af F ⊗[F] E), ∀ᶠ v in Filter.cofinite, projV v X ∈ latV b v) ∧ (∀ X Y : (Fin n → Fin M → AF F ⊗[F] E), projInf X = projInf Y → (∀ v, projV v (projFin X) = projV v (projFin Y)) → X = Y) ∧ (∀ (Y : (Fin n → Fin M → Ainf F ⊗[F] E)) (Z : ∀ v : HeightOneSpectrum (𝒪 F), (Fin n → Fin M → Fv F v ⊗[F] E)), (∀ᶠ v in Filter.

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theorem **_root_.GlobalTheory.T.adelic_tuple_projections** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (n M : ℕ), Continuous (projInf : (Fin n → Fin M → AF F ⊗[F] E) → _) ∧ Continuous (projFin : (Fin n → Fin M → AF F ⊗[F] E) → _) ∧ (∀ v, Continuous (projV v : (Fin n → Fin M → Af F ⊗[F] E) → _)) ∧ (∀ v, IsCompact (latV b v : Set (Fin n → Fin M → Fv F v ⊗[F] E)) ∧ IsOpen (latV b v : Set (Fin n → Fin M → Fv F v ⊗[F] E))) ∧ (∀ S, IsOpen (intOff b S : Set (Fin n → Fin M → AF F ⊗[F] E))) ∧ (∀ X : (Fin n → Fin M → Af F ⊗[F] E), ∀ᶠ v in Filter.cofinite, projV v X ∈ latV b v) ∧ (∀ X Y : (Fin n → Fin M → AF F ⊗[F] E), projInf X = projInf Y → (∀ v, projV v (projFin X) = projV v (projFin Y)) → X = Y) ∧ (∀ (Y : (Fin n → Fin M → Ainf F ⊗[F] E)) (Z : ∀ v : HeightOneSpectrum (𝒪 F), (Fin n → Fin M

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theorem **map_consTup** {K L : Type} [Zero K] [One K] [Zero L] [One L] (f : K → L) (h0 : f 0 = 0) (h1 : f 1 = 1) {M n : ℕ} (X : Fin n → Fin M → K) : (fun a i => f (consTup X a i)) = consTup (fun a i => f (X a i))

theorem **consTup_empty** {K : Type} [Zero K] [One K] {M : ℕ} (X : Fin 0 → Fin M → K) : (consTup X : Fin (0 + 1) → Fin (M + 1) → K) = fun _ : Fin 1 => Fin.cons 1 (fun _ => 0)

theorem **projInf_consTup** {M n : ℕ} (X : Fin n → Fin M → AF F ⊗[F] E) : projInf (consTup X) = consTup (projInf X)

theorem **projFin_consTup** {M n : ℕ} (X : Fin n → Fin M → AF F ⊗[F] E) : projFin (consTup X) = consTup (projFin X)

theorem **projV_consTup** (v : HeightOneSpectrum (𝒪 F)) {M n : ℕ} (X : Fin n → Fin M → Af F ⊗[F] E) : projV v (consTup X) = consTup (projV v X)

theorem **consTup_mem_latV** {d : ℕ} (b : Module.Basis (Fin d) F E) (v : HeightOneSpectrum (𝒪 F)) {M n : ℕ} (h0 : ∀ X₀ : Fin 0 → Fin M → Fv F v ⊗[F] E, consTup X₀ ∈ latV b v) (X : Fin n → Fin M → Fv F v ⊗[F] E) : consTup X ∈ latV b v ↔ X ∈ latV b v

theorem **prod_union_split** {ι : Type} [DecidableEq ι] (S Sh : Finset ι) (A B : ι → ℝ≥0∞) (P : ι → Prop) [DecidablePred P] (hS : ∀ v, P v ↔ v ∈ S) : ∏ v ∈ S ∪ Sh, (if P v then A v else B v) = (∏ v ∈ S, A v) * ∏ v ∈ Sh \ S, B v

theorem **pureL_act** (hinf : ∀ (n : ℕ) (X : Fin n → Fin (M + 1) → AF F ⊗[F] E), projInf (actW (AF F) σ Jp h X) = actW (Ainf F) σ Jp ginf (projInf X)) (hfin : ∀ (n : ℕ) (X : Fin n → Fin (M + 1) → AF F ⊗[F] E) (v : HeightOneSpectrum (𝒪 F)), projV v (projFin (actW (AF F) σ Jp h X)) = actW (Fv F v) σ Jp (g v) (projV v (projFin X))) (hSh : ∀ v, v ∉ Sh → ∀ (n : ℕ) (X : Fin n → Fin (M + 1) → Fv F v ⊗[F] E), actW (Fv F v) σ Jp (g v) X ∈ latV b v ↔ X ∈ latV b v) (hS : ∀ v, v ∉ S → ∀ X₀ : Fin 0 → Fin M → Fv F v ⊗[F] E, consTup X₀ ∈ latV b v) {n : ℕ} (φinf : (Fin (n + 1) → Fin (M + 1) → Ainf F ⊗[F] E) → ℝ≥0∞) (φ : ∀ v : HeightOneSpectrum (𝒪 F), (Fin (n + 1) → Fin (M + 1) → Fv F v ⊗[F] E) → ℝ≥0∞) (X : Fin n → Fin M → AF F ⊗[F] E) : pureL b S φinf φ (actW (AF F) σ Jp h (consTup X)) = pureL b (S ∪ Sh) (fun Y => φinf (actW (Ainf F) σ Jp ginf (consTup Y))) (fun v Y => if v ∈ S then φ v (actW (Fv F v) σ Jp

theorem **pureL_act_vec** (hinf : ∀ (n : ℕ) (X : Fin n → Fin (M + 1) → AF F ⊗[F] E), projInf (actW (AF F) σ Jp h X) = actW (Ainf F) σ Jp ginf (projInf X)) (hfin : ∀ (n : ℕ) (X : Fin n → Fin (M + 1) → AF F ⊗[F] E) (v : HeightOneSpectrum (𝒪 F)), projV v (projFin (actW (AF F) σ Jp h X)) = actW (Fv F v) σ Jp (g v) (projV v (projFin X))) (hSh : ∀ v, v ∉ Sh → ∀ (n : ℕ) (X : Fin n → Fin (M + 1) → Fv F v ⊗[F] E), actW (Fv F v) σ Jp (g v) X ∈ latV b v ↔ X ∈ latV b v) (hS : ∀ v, v ∉ S → ∀ X₀ : Fin 0 → Fin M → Fv F v ⊗[F] E, consTup X₀ ∈ latV b v) (Ψinf : (Fin 1 → Fin (M + 1) → Ainf F ⊗[F] E) → ℝ≥0∞) (Ψ : ∀ v : HeightOneSpectrum (𝒪 F), (Fin 1 → Fin (M + 1) → Fv F v ⊗[F] E) → ℝ≥0∞) : pureL b S Ψinf Ψ (actW (AF F) σ Jp h (fun _ : Fin 1 => Fin.cons 1 (fun _ => 0))) = (Ψinf (actW (Ainf F) σ Jp ginf (fun _ : Fin 1 => Fin.cons 1 (fun _ => 0))) * ∏ v ∈ S, Ψ v (actW (Fv F v) σ Jp (g v) (fun _ : Fin 1 => Fin.co

theorem **pure_h** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] [DecidableEq (HeightOneSpectrum (𝒪 F))] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ t : F, t ≠ 0 → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E), ∃ S₁ : Finset (HeightOneSpectrum (𝒪 F)), ∀ h : HA (AF F) σ (blockOne (algebraMap F E t) J), ∃ (ginf : HA (Ainf F) σ (blockOne (algebraMap F E t) J)) (g : ∀ v : HeightOneSpectrum (𝒪 F), HA (Fv F v) σ (blockOne (algebraMap F E t) J)) (Sh : Finset (HeightOneSpectrum (𝒪 F))), ∀ S, S₁ ⊆ S → (∀ (n : ℕ) (φinf : (Fin (n + 1) → Fin (M + 1) → Ainf F ⊗[F] E) → ℝ≥0∞) (φ : ∀ v : HeightOneSpectrum (𝒪 F), (Fin (n + 1) → Fin (M + 1) → Fv F v ⊗[F] E) → ℝ≥0∞) (X : Fin n → Fin M → AF F ⊗[F] E), pureL b S φinf φ (actW (AF F) σ _ h (consTup X)) = pureL b (S ∪ Sh) (fun Y =>

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theorem **_root_.GlobalTheory.T.adelic_unitary_finite_set_decomposition** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] [DecidableEq (HeightOneSpectrum (𝒪 F))] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ t : F, t ≠ 0 → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E), ∃ S₁ : Finset (HeightOneSpectrum (𝒪 F)), ∀ h : HA (AF F) σ (blockOne (algebraMap F E t) J), ∃ (ginf : HA (Ainf F) σ (blockOne (algebraMap F E t) J)) (g : ∀ v : HeightOneSpectrum (𝒪 F), HA (Fv F v) σ (blockOne (algebraMap F E t) J)) (Sh : Finset (HeightOneSpectrum (𝒪 F))), ∀ S, S₁ ⊆ S → (∀ (n : ℕ) (φinf : (Fin (n + 1) → Fin (M + 1) → Ainf F ⊗[F] E) → ℝ≥0∞) (φ : ∀ v : HeightOneSpectrum (𝒪 F), (Fin (n + 1) → Fin (M + 1) → Fv F v ⊗[F] E) → ℝ≥0∞) (X : Fin n → Fin M → AF F ⊗[F] E), pureL b S φinf φ (actW (AF F) σ _ h (consTup X)) = pureL b (S ∪ Sh) (fun Y =>

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theorem **hform_mulVec** {A : Type} [CommRing A] {m : ℕ} (σ : A →+* A) (J g : Matrix (Fin m) (Fin m) A) (hg : cT σ g * J * g = J) (x y : Fin m → A) : hform σ J (g *ᵥ x) (g *ᵥ y) = hform σ J x y

theorem **isClosed_HR** [IsTopologicalRing (R ⊗[F] E)] [T2Space (R ⊗[F] E)] (σ : E →ₐ[F] E) (hs : Continuous (sigR R σ)) {M : ℕ} (J : Matrix (Fin M) (Fin M) E) : IsClosed (HR R σ J : Set (GL (Fin M) (R ⊗[F] E)))

theorem **topH_props** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [IsTopologicalRing R] [LocallyCompactSpace R] [SecondCountableTopology R] [T2Space R], IsTopologicalGroup (HA R σ J) ∧ LocallyCompactSpace (HA R σ J) ∧ SecondCountableTopology (HA R σ J) ∧ T2Space (HA R σ J) ∧ BorelSpace (HA R σ J) ∧ ∀ n : ℕ, (Continuous fun p : HA R σ J × (Fin n → Fin M → R ⊗[F] E) => actW R σ J p.1 p.2) ∧ (∀ X : Fin n → Fin M → R ⊗[F] E, actW R σ J 1 X = X) ∧ (∀ (g h : HA R σ J) (X : Fin n → Fin M → R ⊗[F] E), actW R σ J (g * h) X = actW R σ J g (actW R σ J h X)) ∧ ∀ (h : HA R σ J) (X : Fin n → Fin M → R ⊗[F] E), gramR R σ J (actW R σ J h X) = gramR R σ J X

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theorem **_root_.GlobalTheory.T.adelic_unitary_group_locally_compact** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [IsTopologicalRing R] [LocallyCompactSpace R] [SecondCountableTopology R] [T2Space R], IsTopologicalGroup (HA R σ J) ∧ LocallyCompactSpace (HA R σ J) ∧ SecondCountableTopology (HA R σ J) ∧ T2Space (HA R σ J) ∧ BorelSpace (HA R σ J) ∧ ∀ n : ℕ, (Continuous fun p : HA R σ J × (Fin n → Fin M → R ⊗[F] E) => actW R σ J p.1 p.2) ∧ (∀ X : Fin n → Fin M → R ⊗[F] E, actW R σ J 1 X = X) ∧ (∀ (g h : HA R σ J) (X : Fin n → Fin M → R ⊗[F] E), actW R σ J (g * h) X = actW R σ J g (actW R σ J h X)) ∧ ∀ (h : HA R σ J) (X : Fin n → Fin M → R ⊗[F] E), gramR R σ J (actW R σ J h X) = gramR R σ J X

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theorem **tm_comp** (φ : R →ₐ[F] R') (ψ : R' →ₐ[F] R'') (x : R ⊗[F] E) : tm E (ψ.comp φ) x = tm E ψ (tm E φ x)

theorem **tm_sig** (σ : E →ₐ[F] E) (φ : R →ₐ[F] R') (x : R ⊗[F] E) : tm E φ (sigR R σ x) = sigR R' σ (tm E φ x)

theorem **tm_incl** (φ : R →ₐ[F] R') (e : E) : tm E φ (inclR R e) = inclR R' e

theorem **map_cT** {A B m p : Type} [CommRing A] [CommRing B] (f : A →+* B) (s : A →+* A) (s' : B →+* B) (hs : ∀ x, f (s x) = s' (f x)) (G : Matrix m p A) : (cT s G).map f = cT s' (G.map f)

theorem **map_JR** {M : ℕ} (J : Matrix (Fin M) (Fin M) E) (φ : R →ₐ[F] R') : (JR R J).map (tm E φ) = JR R' J

theorem **map_unitary** (σ : E →ₐ[F] E) {M : ℕ} (J : Matrix (Fin M) (Fin M) E) (φ : R →ₐ[F] R') (G : Matrix (Fin M) (Fin M) (R ⊗[F] E)) : (cT (sigR R σ) G * JR R J * G).map (tm E φ) = cT (sigR R' σ) (G.map (tm E φ)) * JR R' J * G.map (tm E φ)

theorem **hMap_mat** (σ : E →ₐ[F] E) {M : ℕ} (J : Matrix (Fin M) (Fin M) E) (φ : R →ₐ[F] R') (h : HA R σ J) : ((HA.mat R' σ J (hMap σ J φ h) : GL _ _) : Matrix (Fin M) (Fin M) (R' ⊗[F] E)) = ((HA.mat R σ J h : GL _ _) : Matrix (Fin M) (Fin M) (R ⊗[F] E)).map (tm E φ)

theorem **mat_mul** (σ : E →ₐ[F] E) {M : ℕ} (J : Matrix (Fin M) (Fin M) E) (g h : HA R σ J) : HA.mat R σ J (g * h) = HA.mat R σ J g * HA.mat R σ J h

theorem **mat_one** (σ : E →ₐ[F] E) {M : ℕ} (J : Matrix (Fin M) (Fin M) E) : HA.mat R σ J 1 = 1

theorem **ha_ext** (σ : E →ₐ[F] E) {M : ℕ} (J : Matrix (Fin M) (Fin M) E) {g h : HA R σ J} (e : ((HA.mat R σ J g : GL _ _) : Matrix (Fin M) (Fin M) (R ⊗[F] E)) = (HA.mat R σ J h : GL _ _)) : g = h

theorem **tm_actW** (σ : E →ₐ[F] E) {M n : ℕ} (J : Matrix (Fin M) (Fin M) E) (φ : R →ₐ[F] R') (h : HA R σ J) (X : Fin n → Fin M → R ⊗[F] E) : (fun a i => tm E φ (actW R σ J h X a i)) = actW R' σ J (hMap σ J φ h) (fun a i => tm E φ (X a i))

theorem **mem_latV** {M n : ℕ} (b : Module.Basis (Fin d) F E) (v : HeightOneSpectrum (𝒪 F)) (Y : Fin n → Fin M → Fv F v ⊗[F] E) : Y ∈ latV b v ↔ ∀ a i, Y a i ∈ Lv b v

theorem **ev_int** (c : F) : ∀ᶠ v : HeightOneSpectrum (𝒪 F) in Filter.cofinite, algebraMap F (Fv F v) c ∈ v.adicCompletionIntegers F

theorem **repr_one_tmul** (b : Module.Basis (Fin d) F E) (v : HeightOneSpectrum (𝒪 F)) (e : E) (k : Fin d) : (Algebra.TensorProduct.basis (Fv F v) b).repr ((1 : Fv F v) ⊗ₜ[F] e) k = algebraMap F (Fv F v) (b.repr e k)

theorem **Lv_zero** (b : Module.Basis (Fin d) F E) (v : HeightOneSpectrum (𝒪 F)) : (0 : Fv F v ⊗[F] E) ∈ Lv b v

theorem **Lv_sum** (b : Module.Basis (Fin d) F E) (v : HeightOneSpectrum (𝒪 F)) {ι : Type} (s : Finset ι) (f : ι → Fv F v ⊗[F] E) (hf : ∀ i ∈ s, f i ∈ Lv b v) : ∑ i ∈ s, f i ∈ Lv b v

theorem **eventually_one** (b : Module.Basis (Fin d) F E) : ∀ᶠ v : HeightOneSpectrum (𝒪 F) in Filter.cofinite, (1 : Fv F v ⊗[F] E) ∈ Lv b v

theorem **eventually_mul** (b : Module.Basis (Fin d) F E) : ∀ᶠ v : HeightOneSpectrum (𝒪 F) in Filter.cofinite, ∀ y z : Fv F v ⊗[F] E, y ∈ Lv b v → z ∈ Lv b v → y * z ∈ Lv b v

theorem **act_mem** {M n : ℕ} (b : Module.Basis (Fin d) F E) (v : HeightOneSpectrum (𝒪 F)) (hm : ∀ y z : Fv F v ⊗[F] E, y ∈ Lv b v → z ∈ Lv b v → y * z ∈ Lv b v) (G : Matrix (Fin M) (Fin M) (Fv F v ⊗[F] E)) (hG : ∀ i j, G i j ∈ Lv b v) (X : Fin n → Fin M → Fv F v ⊗[F] E) (hX : X ∈ latV b v) : (fun a => G.mulVec (X a)) ∈ latV b v

theorem **act_inv** (R : Type) [CommRing R] [Algebra F R] (σ : E →ₐ[F] E) {M n : ℕ} (J : Matrix (Fin M) (Fin M) E) (g : HA R σ J) (X : Fin n → Fin M → R ⊗[F] E) : actW R σ J g⁻¹ (actW R σ J g X) = X

theorem **projV_projFin** {M n : ℕ} (v : HeightOneSpectrum (𝒪 F)) (X : Fin n → Fin M → AF F ⊗[F] E) : projV v (projFin X) = fun a i => tm E (evA v) (X a i)

theorem **map_eqs** {R R' : Type} [CommRing R] [Algebra F R] [CommRing R'] [Algebra F R'] (σ : E →ₐ[F] E) {M : ℕ} (J : Matrix (Fin M) (Fin M) E) (φ : R →ₐ[F] R') (A A' : Matrix (Fin M) (Fin M) (R ⊗[F] E)) (g : HA R' σ J) (hA : A.map (tm E φ) = ((HA.mat R' σ J g : GL _ _) : Matrix (Fin M) (Fin M) (R' ⊗[F] E))) (hA' : A'.map (tm E φ) = ((HA.mat R' σ J g⁻¹ : GL _ _) : Matrix (Fin M) (Fin M) (R' ⊗[F] E))) : (A * A').map (tm E φ) = (1 : Matrix (Fin M) (Fin M) (R ⊗[F] E)).map (tm E φ) ∧ (cT (sigR R σ) A * JR R J * A).map (tm E φ) = (JR R J).map (tm E φ)

theorem **components** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∃ (pinf : HA (AF F) σ J →* HA (Ainf F) σ J) (p : ∀ v : HeightOneSpectrum (𝒪 F), HA (AF F) σ J →* HA (Fv F v) σ J), (∀ (n : ℕ) (h : HA (AF F) σ J) (X : (Fin n → Fin M → AF F ⊗[F] E)), projInf (actW (AF F) σ J h X) = actW (Ainf F) σ J (pinf h) (projInf X) ∧ ∀ v, projV v (projFin (actW (AF F) σ J h X)) = actW (Fv F v) σ J (p v h) (projV v (projFin X))) ∧ (∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (h : HA (AF F) σ J), ∀ᶠ v in Filter.cofinite, ∀ (n : ℕ) (X : (Fin n → Fin M → Fv F v ⊗[F] E)), actW (Fv F v) σ J (p v h) X ∈ latV b v ↔ X ∈ latV b v) ∧ ∀ (S : Finset (HeightOneSpectrum (𝒪 F))) (ginf : HA (Ainf F) σ J) (g : ∀ v : HeightOneSpectrum (𝒪 F), HA (Fv F v) σ J), ∃ h : HA (AF F) σ J, pinf h = ginf ∧ (∀

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.AdelicUnitaryGroupProjectionsStatement

theorem **_root_.GlobalTheory.T.adelic_unitary_group_projections** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∃ (pinf : HA (AF F) σ J →* HA (Ainf F) σ J) (p : ∀ v : HeightOneSpectrum (𝒪 F), HA (AF F) σ J →* HA (Fv F v) σ J), (∀ (n : ℕ) (h : HA (AF F) σ J) (X : (Fin n → Fin M → AF F ⊗[F] E)), projInf (actW (AF F) σ J h X) = actW (Ainf F) σ J (pinf h) (projInf X) ∧ ∀ v, projV v (projFin (actW (AF F) σ J h X)) = actW (Fv F v) σ J (p v h) (projV v (projFin X))) ∧ (∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (h : HA (AF F) σ J), ∀ᶠ v in Filter.cofinite, ∀ (n : ℕ) (X : (Fin n → Fin M → Fv F v ⊗[F] E)), actW (Fv F v) σ J (p v h) X ∈ latV b v ↔ X ∈ latV b v) ∧ ∀ (S : Finset (HeightOneSpectrum (𝒪 F))) (ginf : HA (Ainf F) σ J) (g : ∀ v : HeightOneSpectrum (𝒪 F), HA (Fv F v) σ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.AdelicUnitsDecompositionFlowCompact

theorem **isCompact_units_setOf** {Ω W : Set K} (hΩ : IsCompact Ω) (hW : IsCompact W) : IsCompact {u : Kˣ | (u : K) ∈ Ω ∧ ((u⁻¹ : Kˣ) : K) ∈ W}

theorem **continuous_act** : Continuous fun p : Kˣ × (Fin 1 → Fin 1 → K) => act p.1 p.2

theorem **chi_flow** {V : Type} [TopologicalSpace V] [AddCommGroup V] [IsTopologicalAddGroup V] [LocallyCompactSpace V] [MeasurableSpace V] [BorelSpace V] (μ : Measure V) [μ.IsAddHaarMeasure] {c : ℝ≥0} {r : ℝ} (hr : 0 ≤ r) (h : c • μ = ENNReal.ofReal r • μ) : (c : ℝ) = r

theorem **core** {E : Type} [Field E] [Countable E] [LocallyCompactSpace K] [SecondCountableTopology K] (ι : E →+* K) [MeasurableSpace (Fin 1 → Fin 1 → K)] [BorelSpace (Fin 1 → Fin 1 → K)] (μ : Measure (Fin 1 → Fin 1 → K)) [μ.IsAddHaarMeasure] (hinf : μ Set.univ = ⊤) (hcov : ∃ C : Set (Fin 1 → Fin 1 → K), IsCompact C ∧ ∀ y, ∃ X : Fin 1 → Fin 1 → E, y − (fun a i => ι (X a i)) ∈ C) (hfin : ∀ C : Set (Fin 1 → Fin 1 → K), IsCompact C → {X : Fin 1 → Fin 1 → E | (fun a i => ι (X a i)) ∈ C}.Finite) (τ : Multiplicative ℝ →* Kˣ) (hτ : Continuous τ) (D : ℝ) (hD : 0 < D) (hflow : ∀ x : ℝ, μ.map (fun X => ((τ (Multiplicative.ofAdd x) : Kˣ) : K) • X) = ENNReal.ofReal (Real.exp (−(D * x))) • μ) : ∃ ω : Set Kˣ, IsCompact ω ∧ ∀ a : Kˣ, ∃ (l : Eˣ) (a' c : Kˣ) (j : ℤ), (a' : K) = ι (l : E) ∧ c ∈ ω ∧ a = a' * τ (Multiplicative.ofAdd (j : ℝ)) * c

theorem **units_decomp** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ τ : Multiplicative ℝ →* (AF F ⊗[F] E)ˣ, IsFlow τ → ∃ ω : Set (AF F ⊗[F] E)ˣ, IsCompact ω ∧ ∀ a : (AF F ⊗[F] E)ˣ, ∃ (l : Eˣ) (a' : (AF F ⊗[F] E)ˣ) (c : (AF F ⊗[F] E)ˣ) (j : ℤ), (a' : AF F ⊗[F] E) = inclR (AF F) (l : E) ∧ c ∈ ω ∧ a = a' * τ (Multiplicative.ofAdd (j : ℝ)) * c

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.AdelicUnitsDecompositionFlowCompactStatement

theorem **_root_.GlobalTheory.T.adelic_units_decomposition_flow_compact** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ τ : Multiplicative ℝ →* (AF F ⊗[F] E)ˣ, IsFlow τ → ∃ ω : Set (AF F ⊗[F] E)ˣ, IsCompact ω ∧ ∀ a : (AF F ⊗[F] E)ˣ, ∃ (l : Eˣ) (a' : (AF F ⊗[F] E)ˣ) (c : (AF F ⊗[F] E)ˣ) (j : ℤ), (a' : AF F ⊗[F] E) = inclR (AF F) (l : E) ∧ c ∈ ω ∧ a = a' * τ (Multiplicative.ofAdd (j : ℝ)) * c

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.AnnihilatorOfAnnihilatorLe

theorem **card_le_of_pairing** {A B : Type} [AddCommGroup A] [AddCommGroup B] [Finite A] [Finite B] (b : A → B → ℂ) (hn : ∀ x y, ‖b x y‖ = 1) (hl : ∀ x x' y, b (x + x') y = b x y * b x' y) (hr : ∀ x y y', b x (y + y') = b x y * b x y') (hk : ∀ x, (∀ y, b x y = 1) → x = 0) : Nat.card A ≤ Nat.card B

theorem **index_eq_of_pairing** {X Y : Type} [AddCommGroup X] [AddCommGroup Y] (c : X → Y → ℂ) (hn : ∀ x y, ‖c x y‖ = 1) (hl : ∀ x x' y, c (x + x') y = c x y * c x' y) (hr : ∀ x y y', c x (y + y') = c x y * c x y') (N : AddSubgroup X) (M : AddSubgroup Y) (hN : ∀ x, x ∈ N ↔ ∀ y, c x y = 1) (hM : ∀ y, y ∈ M ↔ ∀ x, c x y = 1) [N.FiniteIndex] [M.FiniteIndex] : N.index = M.index

theorem **relIndex_ne_zero_of_isOpen** {V : Type} [AddCommGroup V] [TopologicalSpace V] [IsTopologicalAddGroup V] (S T : AddSubgroup V) (hS : IsOpen (S : Set V)) (hT : IsCompact (T : Set V)) : S.relIndex T ≠ 0

theorem **measure_eq_relIndex_mul** {V : Type} [AddCommGroup V] [TopologicalSpace V] [IsTopologicalAddGroup V] [MeasurableSpace V] [BorelSpace V] (μ : Measure V) [μ.IsAddLeftInvariant] (S T : AddSubgroup V) (hST : S ≤ T) (hS : IsOpen (S : Set V)) (hf : S.relIndex T ≠ 0) : μ T = S.relIndex T * μ S

theorem **relIndex_eq_of_pairing** {V : Type} [AddCommGroup V] (χ : V → V → ℂ) (hs : ∀ x y, χ x y = χ y x) (hn : ∀ x y, ‖χ x y‖ = 1) (ha : ∀ x x' y, χ (x + x') y = χ x y * χ x' y) (X₀ X Y₀ Y : AddSubgroup V) (hX : ∀ x ∈ X, (x ∈ X₀ ↔ ∀ y ∈ Y, χ x y = 1)) (hY : ∀ y ∈ Y, (y ∈ Y₀ ↔ ∀ x ∈ X, χ x y = 1)) (fX : X₀.relIndex X ≠ 0) (fY : Y₀.relIndex Y ≠ 0) : X₀.relIndex X = Y₀.relIndex Y

theorem **selfDual_of_between** : ∀ (V : Type) [AddCommGroup V] [TopologicalSpace V] [IsTopologicalAddGroup V] [MeasurableSpace V] [BorelSpace V] (χ : V → V → ℂ) (μ : Measure V), μ.IsAddHaarMeasure → (∀ x y, χ x y = χ y x) → (∀ x y, ‖χ x y‖ = 1) → (∀ x x' y, χ (x + x') y = χ x y * χ x' y) → ∀ L₁ L L₂ : AddSubgroup V, L₁ ≤ L → L ≤ L₂ → IsOpen (L₁ : Set V) → IsCompact (L₂ : Set V) → IsCompact (perp χ L₁) → IsOpen (perp χ L₂) → perp χ (perp χ L₁) ≤ L₁ → perp χ (perp χ L₂) ≤ L₂ → μ L₁ * μ (perp χ L₁) = 1 → μ L₂ * μ (perp χ L₂) = 1 → IsCompact (perp χ L) ∧ IsOpen (perp χ L) ∧ perp χ (perp χ L) ≤ L ∧ μ L * μ (perp χ L) = 1

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.AnnihilatorOfAnnihilatorLeStatement

theorem **_root_.GlobalTheory.T.annihilator_of_annihilator_le** : ∀ (V : Type) [AddCommGroup V] [TopologicalSpace V] [IsTopologicalAddGroup V] [MeasurableSpace V] [BorelSpace V] (χ : V → V → ℂ) (μ : Measure V), μ.IsAddHaarMeasure → (∀ x y, χ x y = χ y x) → (∀ x y, ‖χ x y‖ = 1) → (∀ x x' y, χ (x + x') y = χ x y * χ x' y) → ∀ L₁ L L₂ : AddSubgroup V, L₁ ≤ L → L ≤ L₂ → IsOpen (L₁ : Set V) → IsCompact (L₂ : Set V) → IsCompact (perp χ L₁) → IsOpen (perp χ L₂) → perp χ (perp χ L₁) ⊆ L₁ → perp χ (perp χ L₂) ⊆ L₂ → μ L₁ * μ (perp χ L₁) = 1 → μ L₂ * μ (perp χ L₂) = 1 → IsCompact (perp χ L) ∧ IsOpen (perp χ L) ∧ perp χ (perp χ L) ⊆ L ∧ μ L * μ (perp χ L) = 1

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ArchimedeanCoordinateMeasureTransport

theorem **map_swap_volume** (ι κ : Type) [Fintype ι] [Fintype κ] : (volume : Measure (ι → κ → ℝ)).map (fun f k i => f i k) = (volume : Measure (κ → ι → ℝ))

theorem **pi_smul_const** {ι α : Type} [Fintype ι] [MeasurableSpace α] (ν : Measure α) [SigmaFinite ν] (c : ℝ≥0) : Measure.pi (fun _ : ι => (c : ℝ≥0∞) • ν) = ((c : ℝ≥0∞) ^ Fintype.card ι) • Measure.pi fun _ : ι => ν

theorem **map_coord_pi** {ι κ A : Type} [Fintype ι] [Fintype κ] [MeasurableSpace A] (lam : Measure A) [SigmaFinite lam] (e : A → κ → ℝ) (he : Measurable e) (c : ℝ≥0) (hlam : lam.map e = (c : ℝ≥0∞) • (volume : Measure (κ → ℝ))) : (Measure.pi fun _ : ι => lam).map (fun y w p => e (y p) w) = ((c : ℝ≥0∞) ^ Fintype.card ι) • (volume : Measure (κ → ι → ℝ))

theorem **GlobalTheory.CompanionTheory.SW.BB.infPush** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M d : ℕ) (b : Module.Basis (Fin d) F E) (e : Ainf F ≃+* (InfinitePlace F → ℝ)), IsHomeomorph e → ∀ (laminf : Measure (Ainf F)), laminf.IsAddHaarMeasure → laminf.map e = ENNReal.ofReal (Real.sqrt |(discr F : ℝ)|)⁻¹ • (volume : Measure (InfinitePlace F → ℝ)) → (coordW (Ainf F) b laminf 1 M).map (fun X w => xw e b X w) = ENNReal.ofReal ((Real.sqrt |(discr F : ℝ)|)⁻¹ ^ (M * d)) • (volume : Measure (InfinitePlace F → Fin M × Fin d → ℝ))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ArchimedeanCoordinateMeasureTransportStatement

theorem **_root_.GlobalTheory.T.archimedean_coordinate_measure_transport** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M d : ℕ) (b : Module.Basis (Fin d) F E) (e : Ainf F ≃+* (InfinitePlace F → ℝ)), IsHomeomorph e → ∀ (laminf : Measure (Ainf F)), laminf.IsAddHaarMeasure → laminf.map e = ENNReal.ofReal (Real.sqrt |(discr F : ℝ)|)⁻¹ • (volume : Measure (InfinitePlace F → ℝ)) → (coordW (Ainf F) b laminf 1 M).map (fun X w => xw e b X w) = ENNReal.ofReal ((Real.sqrt |(discr F : ℝ)|)⁻¹ ^ (M * d)) • (volume : Measure (InfinitePlace F → Fin M × Fin d → ℝ))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ArchimedeanGaussianFourier

theorem **gauss_eq_prod** (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (P : InfinitePlace F → Matrix ι ι ℝ) (τ : InfinitePlace F → ℂ) (x : InfinitePlace F → ι → ℝ) : gauss B ρ P τ x = ∏ w, g1 (B.map (ρ w)) (P w) (τ w) (x w)

theorem **neg_inv_im_pos** {τ : ℂ} (h : 0 < τ.im) : 0 < (-τ⁻¹).im

theorem **abs_norm_eq_prod** (hF : IsTotallyReal F) (ρ : InfinitePlace F → (F →+* ℝ)) (hρ : ∀ w x, ((ρ w x : ℝ) : ℂ) = w.embedding x) (a : F) : |((Algebra.norm ℚ a : ℚ) : ℝ)| = ∏ w, |ρ w a|

theorem **sqrt_prod_abs** {α : Type} (s : Finset α) (f : α → ℝ) : Real.sqrt (∏ w ∈ s, |f w|) = ∏ w ∈ s, Real.sqrt |f w|

theorem **sqrt_norm_det** (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (hF : IsTotallyReal F) (hρ : ∀ w x, ((ρ w x : ℝ) : ℂ) = w.embedding x) : Real.sqrt |((Algebra.norm ℚ B.det : ℚ) : ℝ)| = ∏ w, Real.sqrt |(B.map (ρ w)).det|

theorem **gauss_fourier** : ∀ (F : Type) [Field F] [NumberField F], ∀ (ι : Type) [Fintype ι] [DecidableEq ι] (M : ℕ) (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (P : InfinitePlace F → Matrix ι ι ℝ), QSetup M B ρ P → ∀ τ : InfinitePlace F → ℂ, (∀ w, 0 < (τ w).im) → Integrable (gauss B ρ P τ) ∧ Integrable (fun x => ‖cG B ρ P τ x‖) → ∫ x, gauss B ρ P τ x = cG B ρ P (fun w => -(τ w)⁻¹) = 1 ∧ ∀ y : InfinitePlace F → ι → ℝ, ((Real.sqrt |((Algebra.norm ℚ B.det : ℚ) : ℝ)| : ℝ) : ℂ) = ∫ x, gauss B ρ P τ x * Complex.exp (2 * (Real.pi : ℂ) * Complex.I * ((∑ w, x w * (B.map (ρ w) y w) : ℝ) : ℂ)) = cG B ρ P τ = gauss B ρ P (fun w => -(τ w)⁻¹) y

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ArchimedeanGaussianFourierStatement

theorem **_root_.GlobalTheory.T.archimedean_gaussian_fourier** : ∀ (F : Type) [Field F] [NumberField F], ∀ (ι : Type) [Fintype ι] [DecidableEq ι] (M : ℕ) (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (P : InfinitePlace F → Matrix ι ι ℝ), QSetup M B ρ P → ∀ τ : InfinitePlace F → ℂ, (∀ w, 0 < (τ w).im) → Integrable (gauss B ρ P τ) ∧ Integrable (fun x => ‖cG B ρ P τ x‖) = (∏ w, ‖τ w‖ ^ M)⁻¹ ∧ cG B ρ P τ = (∏ w, ‖τ w‖ ^ M)⁻¹ ∧ cG B ρ P (fun w => -(τ w)⁻¹) = 1 ∧ ∀ y : InfinitePlace F → ι → ℝ, ((Real.sqrt |((Algebra.norm ℚ B.det : ℚ) : ℝ)| : ℝ) : ℂ) = ∫ x, gauss B ρ P τ x * Complex.exp (2 * (Real.pi : ℂ) * Complex.I * ((∑ w, x w * (B.map (ρ w) y w) : ℝ) : ℂ)) = cG B ρ P τ = gauss B ρ P (fun w => -(τ w)⁻¹) y

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ArchimedeanGramFiberContinuousDensity

theorem **fibre_lintegral_eq_density** {V X Y : Type} [TopologicalSpace V] [LocallyCompactSpace V] [SecondCountableTopology V] [T2Space V] [MeasurableSpace V] [BorelSpace V] [TopologicalSpace X] [LocallyCompactSpace X] [SecondCountableTopology X] [T2Space X] [MeasurableSpace X] [BorelSpace X] (μ : Measure V) (ν : Measure X) [IsLocallyFiniteMeasure ν] [ν.IsOpenPosMeasure] (f : V → Y) (g : V → X) (ι : X → Y) (hι : Function.Injective ι) (hg : Continuous g) (hf : ∀ v, f v = ι (g v)) (B : Set X) (fib : X → Measure V) (hfib : Formalism.MainNotions.GlobalTheoryTheory.GenusMassVolumes.IsFibreFamily μ f ι (g ⁻¹' B) ν B fib) (b₀ : X) (O : Set X) (hO : IsOpen O) (hOB : O ⊆ B) (hb₀ : b₀ ∈ O) (T : X → V → V) (κ : X → ℝ≥0) (hT : ContinuousOn (fun p : X × V => T p.1 p.2) (O ×ˢ Set.univ)) (hκ : ContinuousOn κ O) (hκ0 : ∀ b ∈ O, κ b ≠ 0) (hm : ∀ b ∈ O, fib b = (κ b : ℝ≥0∞) • (fib b₀).map (T b)) (w u : V → ℝ≥

theorem **ofReal_inv_pow_le** {a b c : ℝ} (ha : 0 < a) (hb : 0 < b) (hc : 0 ≤ c) (hab : a ≤ c * b) (N : ℕ) : ENNReal.ofReal ((b ^ N)⁻¹) ≤ ((c ^ N).toNNReal : ℝ≥0∞) * ENNReal.ofReal ((a ^ N)⁻¹)

theorem **fibre_rapid_eq_density** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (μ : Measure (Fin 1 → Fin M → Ainf F ⊗[F] E)) (ν : Measure (HermR (Ainf F) σ 1)) m, μ.IsAddHaarMeasure → ν.IsAddHaarMeasure → IsGramFib (Ainf F) σ J 1 μ ν m → ∀ φ : (Fin 1 → Fin M → Ainf F ⊗[F] E) → ℝ, Continuous φ → (∀ X, 0 ≤ φ X) → RapidDecay b (fun X => (φ X : ℂ)) → ∀ D : HermR (Ainf F) σ 1 → ℝ, Continuous D → (∀ U : Set (HermR (Ainf F) σ 1), MeasurableSet U → ∫⁻ X in gramR (Ainf F) σ J ⁻¹' (Subtype.val '' U), ENNReal.ofReal (φ X) ∂μ = ∫⁻ S in U, ENNReal.ofReal (D S) ∂ν) → ∀ S : HermR (Ainf F) σ 1, IsUnit (S : Matrix (Fin 1) (Fin 1) (Ainf F ⊗[F] E)).det → ∫⁻ X, ENNReal.ofReal (φ X) ∂(m S) = ENNReal.ofReal (D S)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ArchimedeanGramFiberContinuousDensityStatement

theorem **_root_.GlobalTheory.T.archimedean_gram_fiber_continuous_density** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (μ : Measure (Fin 1 → Fin M → Ainf F ⊗[F] E)) (ν : Measure (HermR (Ainf F) σ 1)) m, μ.IsAddHaarMeasure → ν.IsAddHaarMeasure → IsGramFib (Ainf F) σ J 1 μ ν m → ∀ φ : (Fin 1 → Fin M → Ainf F ⊗[F] E) → ℝ, Continuous φ → (∀ X, 0 ≤ φ X) → RapidDecay b (fun X => (φ X : ℂ)) → ∀ D : HermR (Ainf F) σ 1 → ℝ, Continuous D → (∀ U : Set (HermR (Ainf F) σ 1), MeasurableSet U → ∫⁻ X in gramR (Ainf F) σ J ⁻¹' (Subtype.val '' U), ENNReal.ofReal (φ X) ∂μ = ∫⁻ S in U, ENNReal.ofReal (D S) ∂ν) → ∀ S : HermR (Ainf F) σ 1, IsUnit (S : Matrix (Fin 1) (Fin 1) (Ainf F ⊗[F] E)).det → ∫⁻ X, ENNReal.ofReal (φ X) ∂(m S) = ENNReal.ofRea

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ArchimedeanLocalDensityFormula

theorem **sigR_sigR** (hs : ∀ x, σ (σ x) = x) (z : R ⊗[F] E) : sigR R σ (sigR R σ z) = z

theorem **sigR_inclR** (y : E) : sigR R σ (inclR R y) = inclR R (σ y)

theorem **sigR_hR** (hs : ∀ x, σ (σ x) = x) {M : ℕ} {J : Matrix (Fin M) (Fin M) E} (hJ : ∀ i j, σ (J j i) = J i j) (x y : Fin M → R ⊗[F] E) : sigR R σ (hR R σ J x y) = hR R σ J y x

theorem **gramR_mem** (hs : ∀ x, σ (σ x) = x) {M n : ℕ} {J : Matrix (Fin M) (Fin M) E} (hJ : ∀ i j, σ (J j i) = J i j) (X : Fin n → Fin M → R ⊗[F] E) : gramR R σ J X ∈ HermR R σ n

theorem **e_half_trace** {σ : E →ₐ[F] E} {M d : ℕ} {J : Matrix (Fin M) (Fin M) E} (b : Module.Basis (Fin d) F E) (B : Matrix (Fin M × Fin d) (Fin M × Fin d) F) (hB : ∀ x y : Fin M → Ainf F ⊗[F] E, Algebra.trace (Ainf F) (Ainf F ⊗[F] E) (hR (Ainf F) σ J x y) = cv (Ainf F) b x ·ᵥ (B.map (algebraMap F (Ainf F)) *ᵥ cv (Ainf F) b y)) (ρ : InfinitePlace F → (F →+* ℝ)) (e : Ainf F ≃+* (InfinitePlace F → ℝ)) (hρ : ∀ x w, (algebraMap F (Ainf F) x) w = ρ w x) (X : Fin 1 → Fin M → Ainf F ⊗[F] E) (w : InfinitePlace F) : e (algebraMap F (Ainf F) (1 / 2) * Algebra.trace (Ainf F) (Ainf F ⊗[F] E) (gramR (Ainf F) σ J X 0 0)) w = (1 / 2 : ℝ) * (xw e b X w ·ᵥ (B.map (ρ w) *ᵥ xw e b X w))

theorem **infDens** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → 3 ≤ M → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (B : Matrix (Fin M × Fin d) (Fin M × Fin d) F), (∀ x y : Fin M → Ainf F ⊗[F] E, Algebra.trace (Ainf F) (Ainf F ⊗[F] E) (hR (Ainf F) σ J x y) = cv (Ainf F) b x ·ᵥ (B.map (algebraMap F (Ainf F)) *ᵥ cv (Ainf F) b y)) → ∀ (ρ : InfinitePlace F → (F →+* ℝ)) (e : Ainf F ≃+*

(InfinitePlace F → ℝ)), IsHomeomorph e → (∀ x w, e (algebraMap F (Ainf F) x) w = ρ w x) → ∀ (Pm : InfinitePlace F → Matrix (Fin M × Fin d) (Fin M × Fin d) ℝ) (laminf : Measure (Ainf F)), laminf.IsAddHaarMeasure → laminf.map e = ENNReal.ofReal (Real.sqrt |(discr F : ℝ)|)⁻¹ • (volume : Measure (InfinitePlace F → ℝ)) → ∀ Dinf : (InfinitePlace F → ℝ) → ℝ, Continuous Dinf → (∀ y, 0 ≤ Dinf y) → (∀

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ArchimedeanLocalDensityFormulaStatement

theorem **_root_.GlobalTheory.T.archimedean_local_density_formula** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → 3 ≤ M → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (B : Matrix (Fin M × Fin d) (Fin M × Fin d) F), (∀ x y : Fin M → Ainf F ⊗[F] E, Algebra.trace (Ainf F) (Ainf F ⊗[F] E) (hR (Ainf F) σ J x y) = cv (Ainf F) b x ·ᵥ (B.map (algebraMap F (Ainf F)) *ᵥ cv (Ainf F) b y)) → ∀ (ρ : InfinitePlace F → (F →+* ℝ)) (e : Ainf F ≃+* (InfinitePlace F → ℝ)), IsHomeomorph e → (∀ x w, e (algebraMap F (Ainf F) x) w = ρ w x) → ∀ (Pm : InfinitePlace F → Matrix (Fin M × Fin d) (Fin M × Fin d) ℝ) (laminf : Measure (Ainf F)), laminf.IsAddHaarMeasure → laminf.map e = ENNReal.ofReal (Real.sqrt |(discr F : ℝ)|)⁻¹ • (volume : Measure (InfinitePlace F → ℝ)) → ∀ Dinf : (InfinitePlace F →

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ArchimedeanPrimitiveVectorCompactRescale

theorem **reduce_of_equiv** {K : Type} [CommRing K] [TopologicalSpace K] {W : Type} [Fintype W] (Φ : K ≃+* (W → ℂ)) (hΦ : IsHomeomorph Φ) (M : ℕ) : ∃ Ω : Set (Fin 1 → Fin M → K), IsCompact Ω ∧ ∀ y : Fin 1 → Fin M → K, (∃ z : Fin M → K, z ·ᵥ y 0 = 1) → ∃ (a : Kˣ) (z : Fin 1 → Fin M → K), (a : K) • y ∈ Ω ∧ z ∈ Ω ∧ z 0 ·ᵥ ((a : K) • y) 0 = 1

theorem **exists_delta** (σ : E →ₐ[F] E) (h : IsCMExt F E σ) : ∃ δ : E, σ δ = -δ ∧ δ ≠ 0

theorem **exists_basis** (σ : E →ₐ[F] E) (h : IsCMExt F E σ) {δ : E} (hδ : σ δ = -δ) (h0 : δ ≠ 0) : ∃ b : Module.Basis (Fin 2) F E, b 0 = 1 ∧ b 1 = δ

theorem **exists_tau** (w : InfinitePlace F) : ∃ τ : E →+* ℂ, τ.comp (algebraMap F E) = w.embedding

theorem **exists_psi** (ρ : ∀ w : InfinitePlace F, w.Completion ≃+* ℝ) (hpc : ∀ w, Continuous (ρ w)) (hps : ∀ w, Continuous (ρ w).symm) (t : InfinitePlace F → ℂ) (htre : ∀ w, (t w).re = 0) (htim : ∀ w, (t w).im ≠ 0) : ∃ Ψ : (Fin 2 → Ainf F) ≃ₜ (InfinitePlace F → ℂ), ∀ p w, Ψ p w = ((ρ w (p 0 w) : ℝ) : ℂ) + ((ρ w (p 1 w) : ℝ) : ℂ) * t w

theorem **exists_equiv** (σ : E →ₐ[F] E) (h : IsCMExt F E σ) : ∃ Φ : (Ainf F ⊗[F] E) ≃+* (InfinitePlace F → ℂ), IsHomeomorph Φ

theorem **unimodular_mod_units_inf** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ), ∃ Ω : Set (Fin 1 → Fin M → Ainf F ⊗[F] E), IsCompact Ω ∧ ∀ y : Fin 1 → Fin M → Ainf F ⊗[F] E, (∃ z : Fin M → Ainf F ⊗[F] E, z ·ᵥ y 0 = 1) → ∃ (a : (Ainf F ⊗[F] E)ˣ) (z : Fin 1 → Fin M → Ainf F ⊗[F] E), (a : Ainf F ⊗[F] E) • y ∈ Ω ∧ z ∈ Ω ∧ z 0 ·ᵥ ((a : Ainf F ⊗[F] E) • y) 0 = 1

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ArchimedeanPrimitiveVectorCompactRescaleStatement

theorem **_root_.GlobalTheory.T.archimedean_primitive_vector_compact_rescale** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ), ∃ Ω : Set (Fin 1 → Fin M → Ainf F ⊗[F] E), IsCompact Ω ∧ ∀ y : Fin 1 → Fin M → Ainf F ⊗[F] E, (∃ z : Fin M → Ainf F ⊗[F] E, z ·ᵥ y 0 = 1) → ∃ (a : (Ainf F ⊗[F] E)ˣ) (z : Fin 1 → Fin M → Ainf F ⊗[F] E), (a : Ainf F ⊗[F] E) • y ∈ Ω ∧ z ∈ Ω ∧ z 0 ·ᵥ ((a : Ainf F ⊗[F] E) • y) 0 = 1

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ArchimedeanSelfdualMeasureDiscriminantScaled

theorem **real_coord_eq_re** {F : Type} [Field F] [NumberField F] (ρ : InfinitePlace F → (F →+* ℝ)) (e : Ainf F ≃+* (InfinitePlace F → ℝ)) (hρ : ∀ w x, ((ρ w x : ℝ) : ℂ) = w.embedding x) (hh : IsHomeomorph e) (he : ∀ x w, e (algebraMap F (Ainf F) x) w = ρ w x) (x : Ainf F) (w : InfinitePlace F) : e x w = (InfinitePlace.Completion.extensionEmbedding w (x w)).re

theorem **psiInf_std_mul** {F : Type} [Field F] [NumberField F] (hF : IsTotallyReal F) (ρ : InfinitePlace F → (F →+* ℝ)) (e : Ainf F ≃+* (InfinitePlace F → ℝ)) (hp : ∀ w x, ((ρ w x : ℝ) : ℂ) = w.embedding x) (hh : IsHomeomorph e) (he : ∀ x w, e (algebraMap F (Ainf F) x) w = ρ w x) (s t : Ainf F) : psiInf F (Circle.coeHom.compAddChar (GroundTheory.Global.adelicChar F)) (s * t) = Complex.exp (2 * Real.pi * Complex.I * (((-1 : ℝ) * (e s ·ᵥ e t) : ℝ) : ℂ))

theorem **selfDual_psiInf_std** (F : Type) [Field F] [NumberField F] (hF : IsTotallyReal F) (laminf : Measure (Ainf F)) (hl : IsStdInf laminf) : SelfDual (Real.toNNReal (Real.sqrt |(discr F : ℝ)|) • laminf) (fun s t => psiInf F (Circle.coeHom.compAddChar (GroundTheory.Global.adelicChar F)) (s * t))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ArchimedeanSelfdualMeasureDiscriminantScaledStatement

theorem **_root_.GlobalTheory.T.archimedean_selfdual_measure_discriminant_scaled** : ∀ (F : Type) [Field F] [NumberField F], IsTotallyReal F → ∀ laminf : Measure (Ainf F), IsStdInf laminf → SelfDual (Real.toNNReal (Real.sqrt |(discr F : ℝ)|) • laminf) (fun s t => psiInf F (Circle.coeHom.compAddChar (GroundTheory.Global.adelicChar F)) (s * t))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ArchimedeanSelfdualMeasurePi

theorem **exists_form** (e : A ≃+* (P → ℝ)) (he' : Continuous e.symm) (χ : A → ℂ) (hχc : Continuous χ) (hχ1 : ∀ x, ‖χ x‖ = 1) (hχa : ∀ x y, χ (x + y) = χ x * χ y) (hnd : ∀ x, (∀ y, χ (x * y) = 1) → x = 0) : ∃ a : P → ℝ, (∀ w, a w ≠ 0) ∧ ∀ x, χ x = Complex.exp (2 * Real.pi * Complex.I * ((∑ w, a w * e x w : ℝ) : ℂ))

theorem **eqv_apply** (e : A ≃+* (P → ℝ)) (he : Continuous e) (he' : Continuous e.symm) (x : A) : eqv e he he' x = e x

theorem **eqvPi_apply** (e : A ≃+* (P → ℝ)) (he : Continuous e) (he' : Continuous e.symm) (ι : Type) (x : ι → A) (q : ι × P) : eqvPi e he he' ι x q = e (x q.1) q.2

theorem **map_pi_eq** (e : A ≃+* (P → ℝ)) (he : Continuous e) (he' : Continuous e.symm) (k : Measure A) [SigmaFinite k] (c₀ : ℝ≥0∞) (h0 : c₀ ≠ 0) (ht : c₀ ≠ ⊤) (hm : k.map (eqv e he he') = c₀ • (volume : Measure (P → ℝ))) (ι : Type) [Fintype ι] : (Measure.pi fun _ : ι => k).map (eqvPi e he he' ι) = (∏ _i : ι, c₀) • (volume : Measure (ι × P → ℝ))

theorem **selfDual_pi_of_form** [LocallyCompactSpace A] (e : A ≃+* (P → ℝ)) (he : Continuous e) (he' : Continuous e.symm) (a : P → ℝ) (ha : ∀ w, a w ≠ 0) (χ : A → ℂ) (hχ : ∀ x, χ x = Complex.exp (2 * Real.pi * Complex.I * ((∑ w, a w * e x w : ℝ) : ℂ))) (k : Measure A) (hk : SelfDual k fun s t => χ (s * t)) (ι : Type) [Fintype ι] : SelfDual (Measure.pi fun _ : ι => k) (fun x y => χ (x ·ᵥ y))

theorem **GlobalTheory.CompanionTheory.SW.selfDual_pi_inf** : ∀ (F : Type) [Field F] [NumberField F], IsTotallyReal F → ∀ (ψ : AddChar (AF F) ℂ), IsCharAF F ψ → ∀ k : Measure (Ainf F), SelfDual k (fun s t => psiInf F ψ (s * t)) → ∀ (ι : Type) [Fintype ι], SelfDual (Measure.pi fun _ : ι => k) (fun x y => psiInf F ψ (x ·ᵥ y))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ArchimedeanSelfdualMeasurePiStatement

theorem **_root_.GlobalTheory.T.archimedean_selfdual_measure_pi** : ∀ (F : Type) [Field F] [NumberField F], IsTotallyReal F → ∀ (ψ : AddChar (AF F) ℂ), IsCharAF F ψ → ∀ k : Measure (Ainf F), SelfDual k (fun s t => psiInf F ψ (s * t)) → ∀ (ι : Type) [Fintype ι], SelfDual (Measure.pi fun _ : ι => k) (fun x y => psiInf F ψ (x ·ᵥ y))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ArchimedeanSelfdualSymmetricFormUnique

theorem **selfDual_matrix_unique_of_equiv** {R P ι : Type} [CommRing R] [TopologicalSpace R] [IsTopologicalRing R] [SecondCountableTopology R] [MeasurableSpace R] [BorelSpace R] [Fintype P] [Fintype ι] [DecidableEq ι] (a : R ≃+ (P → ℝ)) (ha : Continuous a) (ha' : Continuous a.symm) (χ : R → ℂ) (hc : Continuous χ) (h1 : ∀ x, ‖χ x‖ = 1) (hadd : ∀ x y, χ (x + y) = χ x * χ y) (hnd : ∀ x, (∀ y, χ (x * y) = 1) → x = 0) (AR : Matrix ι ι R) (hAR : ARᵀ = AR) (hu : IsUnit AR.det) {μ ν : Measure (ι → R)} (hμ : SelfDual μ (fun x y => χ (x ·ᵥ (AR *ᵥ y)))) (hν : SelfDual ν (fun x y => χ (x ·ᵥ (AR *ᵥ y)))) : μ = ν

theorem **selfDual_rationalForm_Ainf_unique** : ∀ (F : Type) [Field F] [NumberField F], IsTotallyReal F → ∀ (ψ : AddChar (AF F) ℂ), IsCharAF F ψ → ∀ k : Measure (Ainf F), SelfDual k (fun s t => psiInf F ψ (s * t)) → ∀ (ι : Type) [Fintype ι] [DecidableEq ι] (A : Matrix ι ι F), A.IsSymm → A.det ≠ 0 → ∀ μ ν : Measure (ι → Ainf F), SelfDual μ (fun x y => psiInf F ψ (x ·ᵥ (A.map (algebraMap F (Ainf F)) *ᵥ y))) → SelfDual ν (fun x y => psiInf F ψ (x ·ᵥ (A.map (algebraMap F (Ainf F)) *ᵥ y))) → μ = ν

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ArchimedeanSelfdualSymmetricFormUniqueStatement

theorem **_root_.GlobalTheory.T.archimedean_selfdual_symmetric_form_unique** : ∀ (F : Type) [Field F] [NumberField F], IsTotallyReal F → ∀ (ψ : AddChar (AF F) ℂ), IsCharAF F ψ → ∀ k : Measure (Ainf F), SelfDual k (fun s t => psiInf F ψ (s * t)) → ∀ (ι : Type) [Fintype ι] [DecidableEq ι] (A : Matrix ι ι F), A.IsSymm → A.det ≠ 0 → ∀ μ ν : Measure (ι → Ainf F), SelfDual μ (fun x y => psiInf F ψ (x ·ᵥ (A.map (algebraMap F (Ainf F)) *ᵥ y))) → SelfDual ν (fun x y => psiInf F ψ (x ·ᵥ (A.map (algebraMap F (Ainf F)) *ᵥ y))) → μ = ν

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ArithmeticUnitaryGroupDiscrete

theorem **rational_countable_discrete** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → Countable (HFinIn (AF F) σ J) ∧ ∃ U ∈ 𝒩 (1 : HA (AF F) σ J), ∀ h ∈ U, HA.mat (AF F) σ J h ∈ HFin (AF F) σ J → h = 1

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ArithmeticUnitaryGroupDiscreteStatement

theorem **_root_.GlobalTheory.T.arithmetic_unitary_group_discrete** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → Countable (HFinIn (AF F) σ J) ∧ ∃ U ∈ 𝒩 (1 : HA (AF F) σ J), ∀ h ∈ U, HA.mat (AF F) σ J h ∈ HFin (AF F) σ J → h = 1

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.BlockOneDetHermitianGram

theorem **blockOne_zero_zero** : blockOne t A 0 0 = t

theorem **blockOne_zero_succ** (j : Fin M) : blockOne t A 0 j.succ = 0

theorem **blockOne_succ_zero** (i : Fin M) : blockOne t A i.succ 0 = 0

theorem **blockOne_succ_succ** (i j : Fin M) : blockOne t A i.succ j.succ = A i j

theorem **det_blockOne** {K : Type} [CommRing K] {M : ℕ} (t : K) (A : Matrix (Fin M) (Fin M) K) : (blockOne t A).det = t * A.det

theorem **hE_blockOne_cons** (σ : E →ₐ[F] E) {M : ℕ} (J : Matrix (Fin M) (Fin M) E) (t a b : E) (x y : Fin M → E) : hE σ (blockOne t J) (Fin.cons a x : Fin (M + 1) → E) (Fin.cons b y : Fin (M + 1) → E) = σ a * t * b + hE σ J x y

theorem **hE_zero_left** (σ : E →ₐ[F] E) {M : ℕ} (J : Matrix (Fin M) (Fin M) E) (y : Fin M → E) : hE σ J 0 y = 0

theorem **hE_zero_right** (σ : E →ₐ[F] E) {M : ℕ} (J : Matrix (Fin M) (Fin M) E) (x : Fin M → E) : hE σ J x 0 = 0

theorem **gramE_consTup** (σ : E →ₐ[F] E) {M : ℕ} (J : Matrix (Fin M) (Fin M) E) (t : E) {n : ℕ} (X : Fin n → Fin M → E) : gramE σ (blockOne t J) (consTup X) = blockOne t (gramE σ J X)

theorem **exists_consTup_of_gramE** (σ : E →ₐ[F] E) {M : ℕ} (J : Matrix (Fin M) (Fin M) E) {t : E} (ht : t ≠ 0) {n : ℕ} (T : Matrix (Fin n) (Fin n) E) (Y : Fin (n + 1) → Fin (M + 1) → E) (h0 : Y 0 = Fin.cons 1 (fun _ => 0)) (hY : gramE σ (blockOne t J) Y = blockOne t T) : ∃ X, gramE σ J X = T ∧ Y = consTup X

theorem **block_alg** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E) (t : F), (blockOne (algebraMap F E t) J).det = algebraMap F E t * J.det ∧ ((∀ a b, σ (J b a) = J a b) → ∀ a b, σ (blockOne (algebraMap F E t) J b a) = blockOne (algebraMap F E t) J a b) ∧ (∀ (n : ℕ) (X : Fin n → Fin M → E), gramE σ (blockOne (algebraMap F E t) J) (consTup X) = blockOne (algebraMap F E t) (gramE σ J X)) ∧ (t ≠ 0 → ∀ (n : ℕ) (T : Matrix (Fin n) (Fin n) E) (Y : Fin (n + 1) → Fin (M + 1) → E), Y 0 = Fin.cons 1 (fun _ => 0) → gramE σ (blockOne (algebraMap F E t) J) Y = blockOne (algebraMap F E t) T → ∃ X, gramE σ J X = T ∧ Y = consTup X)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.BlockOneDetHermitianGramStatement

theorem **_root_.GlobalTheory.T.block_one_det_hermitian_gram** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E) (t : F), (blockOne (algebraMap F E t) J).det = algebraMap F E t * J.det ∧ ((∀ a b, σ (J b a) = J a b) → ∀ a b, σ (blockOne (algebraMap F E t) J b a) = blockOne (algebraMap F E t) J a b) ∧ (∀ (n : ℕ) (X : Fin n → Fin M → E), gramE σ (blockOne (algebraMap F E t) J) (consTup X) = blockOne (algebraMap F E t) (gramE σ J X)) ∧ (t ≠ 0 → ∀ (n : ℕ) (T : Matrix (Fin n) (Fin n) E) (Y : Fin (n + 1) → Fin (M + 1) → E), Y 0 = Fin.cons 1 (fun _ => 0) → gramE σ (blockOne (algebraMap F E t) J) Y = blockOne (algebraMap F E t) T → ∃ X, gramE σ J X = T ∧ Y = consTup X)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.BlockOneRightInjective

@[simp] lemma **blockOne_zero_zero** (t : K) (A : Matrix (Fin M) (Fin M) K) : blockOne t A 0 0 = t

@[simp] lemma **blockOne_zero_succ** (t : K) (A : Matrix (Fin M) (Fin M) K) (j : Fin M) : blockOne t A 0 j.succ = 0

@[simp] lemma **blockOne_succ_zero** (t : K) (A : Matrix (Fin M) (Fin M) K) (i : Fin M) : blockOne t A i.succ 0 = 0

@[simp] lemma **blockOne_succ_succ** (t : K) (A : Matrix (Fin M) (Fin M) K) (i j : Fin M) : blockOne t A i.succ j.succ = A i j

lemma **blockOne_mul** (a b : K) (A B : Matrix (Fin M) (Fin M) K) : blockOne a A * blockOne b B = blockOne (a * b) (A * B)

lemma **blockOne_one** : blockOne (1 : K) (1 : Matrix (Fin M) (Fin M) K) = 1

lemma **blockOne_right_injective** {a b : K} {A B : Matrix (Fin M) (Fin M) K} (h : blockOne a A = blockOne b B) : A = B

lemma **map_blockOne** {L : Type} [CommRing L] (φ : K →+* L) (a : K) (A : Matrix (Fin M) (Fin M) K) : (blockOne a A).map φ = blockOne (φ a) (A.map φ)

lemma **cT_blockOne** (σ : K →+* K) (a : K) (A : Matrix (Fin M) (Fin M) K) : cT σ (blockOne a A) = blockOne (σ a) (cT σ A)

lemma **coe_blockGL** (g : GL (Fin M) K) : ((blockGL g : GL (Fin (M + 1)) K) : Matrix (Fin (M + 1)) (Fin (M + 1)) K) = blockOne 1 (g : Matrix (Fin M) (Fin M) K)

lemma **blockGL_injective** : Function.Injective (blockGL : GL (Fin M) K → GL (Fin (M + 1)) K)

lemma **blockGL_mem_UJ** (σ : K →+* K) (J : Matrix (Fin M) (Fin M) K) (s : K) {g : GL (Fin M) K} (hg : g ∈ UJ σ J) : blockGL g ∈ UJ σ (blockOne s J)

lemma **of_blockGL_mem_UJ** (σ : K →+* K) (J : Matrix (Fin M) (Fin M) K) (s : K) {g : GL (Fin M) K} (hg : blockGL g ∈ UJ σ (blockOne s J)) : g ∈ UJ σ J

lemma **eq_blockOne_of_col** (σ : K →+* K) (J : Matrix (Fin M) (Fin M) K) {s : K} (hs : IsUnit s) {h : Matrix (Fin (M + 1)) (Fin (M + 1)) K} (hm : cT σ h * blockOne s J * h = blockOne s J) (h00 : h 0 0 = 1) (hi0 : ∀ i : Fin M, h i.succ 0 = 0) : h = blockOne 1 (h.submatrix Fin.succ Fin.succ)

lemma **exists_blockGL** (σ : K →+* K) (J : Matrix (Fin M) (Fin M) K) {s : K} (hs : IsUnit s) {h : GL (Fin (M + 1)) K} (hm : h ∈ UJ σ (blockOne s J)) (h00 : (h : Matrix (Fin (M + 1)) (Fin (M + 1)) K) 0 0 = 1) (hi0 : ∀ i : Fin M, (h : Matrix (Fin (M + 1)) (Fin (M + 1)) K) i.succ 0 = 0) : ∃ g : GL (Fin M) K, blockGL g = h

lemma **bc_blockGL** {L : Type} [CommRing L] (φ : K →+* L) (g : GL (Fin M) K) : bc φ (blockGL g) = blockGL (bc φ g)

lemma **mulVec_first** (A : Matrix (Fin (M + 1)) (Fin (M + 1)) K) (a : Fin (M + 1)) : A.mulVec (Fin.cons 1 (fun _ => 0)) a = A a 0

lemma **blockOne_mulVec_cons** (A : Matrix (Fin M) (Fin M) K) (x : Fin M → K) : (blockOne 1 A).mulVec (Fin.cons 0 x) = Fin.cons 0 (A.mulVec x)

lemma **continuous_blockOne** [TopologicalSpace K] : Continuous (fun A : Matrix (Fin M) (Fin M) K => blockOne (1 : K) A)

lemma **JR_blockOne** : JR (F

lemma **blockGL_mem_HR** {g : GL (Fin M) (R ⊗[F] E)} (hg : g ∈ HR R σ J) : blockGL g ∈ HR R σ (blockOne s J)

lemma **of_blockGL_mem_HR** {g : GL (Fin M) (R ⊗[F] E)} (hg : blockGL g ∈ HR R σ (blockOne s J)) : g ∈ HR R σ J

lemma **mat_mem** (u : HA R σ J) : HA.mat R σ J u ∈ HR R σ J

lemma **mat_injective** : Function.Injective (HA.mat R σ J)

lemma **mat_iotaHom** (u : HA R σ J) : HA.mat R σ _ (iotaHom R σ J s u) = blockGL (HA.mat R σ J u)

lemma **act_first_iff** {n : ℕ} (h : HA R σ (blockOne s J)) : actW R σ _ h (fun _ : Fin n.succ => Fin.cons 1 (fun _ => 0)) = (fun _ : Fin n.succ => Fin.cons 1 (fun _ => 0)) ↔ ((HA.mat R σ _ h : Matrix (Fin (M + 1)) (Fin (M + 1)) (R ⊗[F] E)) 0 0 = 1 ∧ ∀ i : Fin M, (HA.mat R σ _ h : Matrix (Fin (M + 1)) (Fin (M + 1)) (R ⊗[F] E)) i.succ 0 = 0)

lemma **mem_range_iff** (hs : s ≠ 0) (h : HA R σ (blockOne s J)) : h ∈ (iotaHom R σ J s).range ↔ ((HA.mat R σ _ h : Matrix (Fin (M + 1)) (Fin (M + 1)) (R ⊗[F] E)) 0 0 = 1 ∧ ∀ i : Fin M, (HA.mat R σ _ h : Matrix (Fin (M + 1)) (Fin (M + 1)) (R ⊗[F] E)) i.succ 0 = 0)

lemma **mem_HFin_iff** (hs : s ≠ 0) (u : HA R σ J) : HA.mat R σ _ (iotaHom R σ J s u) ∈ HFin R σ (blockOne s J) ↔ HA.mat R σ J u ∈ HFin R σ J

lemma **act_iota** {n : ℕ} (u : HA R σ J) (X : Fin n → Fin M → R ⊗[F] E) : actW R σ _ (iotaHom R σ J s u) (consTup X) = consTup (actW R σ J u X)

lemma **isEmbedding_pairMap** {N : ℕ} (J' : Matrix (Fin N) (Fin N) E) : IsEmbedding (pairMap R σ J')

lemma **continuous_iotaHom** : Continuous (iotaHom R σ J s)

lemma **isEmbedding_iotaHom** : IsEmbedding (iotaHom R σ J s)

lemma **isClosed_range_iotaHom** [T2Space (R ⊗[F] E)] (hs : s ≠ 0) : IsClosed (Set.range (iotaHom R σ J s))

lemma **t2_tensor_product** {F E : Type} [Field F] [Field E] [Algebra F E] [Module.Finite F E] (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [IsTopologicalRing R] [T2Space R] : T2Space (R ⊗[F] E)

theorem **stab_first** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (t : F), t ≠ 0 → ∀ (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [IsTopologicalRing R] [T2Space R], ∃ ι : HA R σ J →* HA R σ (blockOne (algebraMap F E t) J), IsClosedEmbedding ι ∧ (∀ u, (HA.mat R σ _ (ι u) : Matrix (Fin (M + 1)) (Fin (M + 1)) (R ⊗[F] E)) = blockOne 1 (HA.mat R σ J u : Matrix (Fin M) (Fin M) (R ⊗[F] E))) ∧ (∀ h, h ∈ ι.range ↔ actW R σ _ h (fun _ : Fin 1 => Fin.cons 1 (fun _ => 0)) = (fun _ : Fin 1 => Fin.cons 1 (fun _ => 0))) ∧ (∀ u, HA.mat R σ _ (ι u) ∈ HFin R σ (blockOne (algebraMap F E t) J) ↔ HA.mat R σ J u ∈ HFin R σ J) ∧ ∀ (u : HA R σ J) (n : ℕ) (X : Fin n → Fin M → R ⊗[F] E), actW R σ _ (ι u) (consTup X) = consTup (actW R σ J u X)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.BlockOneScalarPreservesNondegenerate

theorem **det_blockOne_eq_mul** {K : Type} [CommRing K] {M : ℕ} (t : K) (A : Matrix (Fin M) (Fin M) K) : (blockOne t A).det = t * A.det

theorem **isHermND_blockOne_algebraMap** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E) (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ t : F, t ≠ 0 → IsHermND σ (blockOne (algebraMap F E t) J)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.BlockOneScalarPreservesNondegenerateStatement

theorem **_root_.GlobalTheory.T.block_one_scalar_preserves_nondegenerate** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E) (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ t : F, t ≠ 0 → IsHermND σ (blockOne (algebraMap F E t) J)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ClosedSubgroupQuotientMeasure

theorem **exists_weight_mass_one_right_coset** : ∀ (G : Type) [Group G] [TopologicalSpace G] [IsTopologicalGroup G] [T2Space G] [LocallyCompactSpace G] [SecondCountableTopology G] [MeasurableSpace G] [BorelSpace G] (G₁ : Type) [Group G₁] [TopologicalSpace G₁] [IsTopologicalGroup G₁] [LocallyCompactSpace G₁] [SecondCountableTopology G₁] [MeasurableSpace G₁] [BorelSpace G₁] (ι : G₁ →* G), IsClosedEmbedding ι → ∀ (μ₁ : Measure G₁) [μ₁.IsHaarMeasure] [μ₁.IsMulRightInvariant], ∃ w : G → ℝ≥0∞, Measurable w ∧ ∀ g, ∫⁻ u, w (ι u * g) ∂μ₁ = 1

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ClosedSubgroupQuotientMeasureStatement

theorem **_root_.GlobalTheory.T.closed_subgroup_quotient_measure** : ∀ (G : Type) [Group G] [TopologicalSpace G] [IsTopologicalGroup G] [T2Space G] [LocallyCompactSpace G] [SecondCountableTopology G] [MeasurableSpace G] [BorelSpace G] (G₁ : Type) [Group G₁] [TopologicalSpace G₁] [IsTopologicalGroup G₁] [LocallyCompactSpace G₁] [SecondCountableTopology G₁] [MeasurableSpace G₁] [BorelSpace G₁] (ι : G₁ →* G), IsClosedEmbedding ι → ∀ (μ₁ : Measure G₁) [μ₁.IsHaarMeasure] [μ₁.IsMulRightInvariant], ∃ w : G → ℝ≥0∞, Measurable w ∧ ∀ g, ∫⁻ u, w (ι u * g) ∂μ₁ = 1

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.CompactPartClosedOrbit

theorem **compact_part_closed_orbit** : ∀ (G : Type) [Group G] [TopologicalSpace G] [IsTopologicalGroup G] [T2Space G] [LocallyCompactSpace G] [SecondCountableTopology G] (V : Type) [TopologicalSpace V] [T2Space V] [LocallyCompactSpace V] (a : G → V → V), (Continuous fun p : G × V => a p.1 p.2) → (∀ y, a 1 y = y) → (∀ g h y, a (g * h) y = a g (a h y)) → ∀ (Z : Set V) (x : V), IsClosed Z → (∀ y, y ∈ Z ↔ ∃ g, a g x = y) → ∀ Ω : Set V, IsCompact Ω → ∃ C : Set G, IsCompact C ∧ ∀ y ∈ Ω, y ∈ Z → ∃ c ∈ C, a c x = y

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.CompactSubgroupAndLevel

theorem **compact_open_sup** {V : Type} [AddCommGroup V] [TopologicalSpace V] [IsTopologicalAddGroup V] {L₁ L₂ : AddSubgroup V} (h₁ : IsCompact (L₁ : Set V)) (o₁ : IsOpen (L₁ : Set V)) (h₂ : IsCompact (L₂ : Set V)) : IsCompact ((L₁ ⊔ L₂ : AddSubgroup V) : Set V) ∧ IsOpen ((L₁ ⊔ L₂ : AddSubgroup V) : Set V)

theorem **compact_subgroup_and_level** : ∀ (V : Type) [AddCommGroup V] [TopologicalSpace V] [MeasurableSpace V] (χ : V → V → ℂ) (μ : Measure V), NADual V χ μ → (∀ K : Set V, IsCompact K → ∃ L : AddSubgroup V, IsCompact (L : Set V) ∧ IsOpen (L : Set V) ∧ K ⊆ L) ∧ ∀ φ : V → ℂ, SB φ → ∃ L L' : AddSubgroup V, IsCompact (L : Set V) ∧ IsOpen (L : Set V) ∧ IsCompact (L' : Set V) ∧ IsOpen (L' : Set V) ∧ L' ≤ L ∧ (∀ x, x ∉ L → φ x = 0) ∧ ∀ x, ∀ l ∈ L', φ (x + l) = φ x

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.CompletionIsLocalRing

theorem **isOpenEmbedding_units_val** (K : Type) [Field K] [TopologicalSpace K] [ContinuousInv₀ K] [ContinuousMul K] [T1Space K] : IsOpenEmbedding (Units.val : Kˣ → K)

theorem **isLocR_of_ultrametric** (K : Type) [NontriviallyNormedField K] [IsUltrametricDist K] [ProperSpace K] (h2 : (2 : K) ≠ 0) : IsLocR K

theorem **isLocR_Fv** : ∀ (F : Type) [Field F] [NumberField F] (v : HeightOneSpectrum (𝒪 F)), IsLocR (Fv F v)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.CompletionIsLocalRingStatement

theorem **_root_.GlobalTheory.T.completion_is_local_ring** : ∀ (F : Type) [Field F] [NumberField F] (v : HeightOneSpectrum (𝒪 F)), IsLocR (Fv F v)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ContDiffOnChartGlobalWhittakerCoefficient

theorem **latV_compact_open** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (n M : ℕ) (v : HeightOneSpectrum (𝒪 F)), IsCompact (latV b v : Set (Fin n → Fin M → Fv F v ⊗[F] E)) ∧ IsOpen (latV b v : Set (Fin n → Fin M → Fv F v ⊗[F] E))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ContDiffOnChartGlobalWhittakerCoefficientStatement

theorem **_root_.GlobalTheory.T.contDiffOn_chart_globalWhittakerCoefficient** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (n M : ℕ) (v : HeightOneSpectrum (𝒪 F)), IsCompact (latV b v : Set (Fin n → Fin M → Fv F v ⊗[F] E)) ∧ IsOpen (latV b v : Set (Fin n → Fin M → Fv F v ⊗[F] E))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ContinuousDensitiesEqOfIntegralsOnCompacts

theorem **not_lt_of_setLIntegral_compact_eq** {X : Type} [TopologicalSpace X] [LocallyCompactSpace X] [T2Space X] [MeasurableSpace X] [BorelSpace X] (ν : Measure X) [IsLocallyFiniteMeasure ν] [ν.IsOpenPosMeasure] {B : Set X} (hB : IsOpen B) {D D' : X → ℝ≥0∞} (hD : ContinuousOn D B) (hD' : ContinuousOn D' B) (h : ∀ S : Set X, IsCompact S → S ⊆ B → ∫⁻ b in S, D b ∂ν = ∫⁻ b in S, D' b ∂ν) {b : X} (hb : b ∈ B) : ¬ D b < D' b

theorem **eq_of_setLIntegral_compact_eq** : ∀ (X : Type) [TopologicalSpace X] [LocallyCompactSpace X] [SecondCountableTopology X] [T2Space X] [MeasurableSpace X] [BorelSpace X] (ν : Measure X) [IsLocallyFiniteMeasure ν] [ν.IsOpenPosMeasure] (B : Set X), IsOpen B → ∀ D D' : X → ℝ≥0∞, ContinuousOn D B → ContinuousOn D' B → (∀ S : Set X, IsCompact S → S ⊆ B → ∫⁻ b in S, D b ∂ν = ∫⁻ b in S, D' b ∂ν) → ∀ b ∈ B, D b = D' b

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ContinuousDensitiesEqOfIntegralsOnCompactsStatement

theorem **_root_.GlobalTheory.T.continuous_densities_eq_of_integrals_on_compacts** : ∀ (X : Type) [TopologicalSpace X] [LocallyCompactSpace X] [SecondCountableTopology X] [T2Space X] [MeasurableSpace X] [BorelSpace X] (ν : Measure X) [IsLocallyFiniteMeasure ν] [ν.IsOpenPosMeasure] (B : Set X), IsOpen B → ∀ D D' : X → ℝ≥0∞, ContinuousOn D B → ContinuousOn D' B → (∀ S : Set X, IsCompact S → S ⊆ B → ∫⁻ b in S, D b ∂ν = ∫⁻ b in S, D' b ∂ν) → ∀ b ∈ B, D b = D' b

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ContinuousHermScal

theorem **sigma_sigma** {σ : E →ₐ[F] E} (hσ : IsCMExt F E σ) (x : E) : σ (σ x) = x

theorem **exists_basis** {σ : E →ₐ[F] E} (hσ : IsCMExt F E σ) : ∃ bb : Module.Basis (Fin 2) F E, bb 0 = 1 ∧ σ (bb 1) = - bb 1

theorem **half_add** (r : R) : algebraMap F R (1 / 2) * (r + r) = r

theorem **fixed_eq** {σ : E →ₐ[F] E} (hσ : IsCMExt F E σ) {y : R ⊗[F] E} (hy : sigR R σ y = y) : ∃ r : R, y = r ⊗ₜ[F] (1 : E)

theorem **trace_tmul_one** {σ : E →ₐ[F] E} (hσ : IsCMExt F E σ) (r : R) : Algebra.trace R (R ⊗[F] E) (r ⊗ₜ[F] (1 : E)) = r + r

theorem **continuous_tmul_one** : Continuous fun s : R => s ⊗ₜ[F] (1 : E)

theorem **continuous_trace** : Continuous (Algebra.trace R (R ⊗[F] E))

theorem **hermScal_hermHalf** {σ : E →ₐ[F] E} (hσ : IsCMExt F E σ) (S : HermR R σ 1) : hermScal R σ (hermHalf R σ S) = S

theorem **hermHalf_hermScal** {σ : E →ₐ[F] E} (hσ : IsCMExt F E σ) (s : R) : hermHalf R σ (hermScal R σ s) = s

theorem **continuous_hermScal** (σ : E →ₐ[F] E) : Continuous (hermScal R σ)

theorem **continuous_hermHalf** (σ : E →ₐ[F] E) : Continuous (hermHalf R σ)

theorem **map_hermScal** {σ : E →ₐ[F] E} (hσ : IsCMExt F E σ) {d : ℕ} (b : Module.Basis (Fin d) F E) (lam : Measure R) [lam.IsAddHaarMeasure] : lam.map (hermScal R σ) = coordHerm R σ b lam 1

theorem **GlobalTheory.CompanionTheory.SW.hermScal_main** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [IsTopologicalRing R] [LocallyCompactSpace R] [SecondCountableTopology R] [T2Space R] [MeasurableSpace R] [BorelSpace R], ∀ (d : ℕ) (b : Module.Basis (Fin d) F E), IsHomeomorph (hermScal R σ) ∧ (∀ S : HermR R σ 1, hermScal R σ (algebraMap F R (1 / 2) * Algebra.trace R (R ⊗[F] E) ((S : Matrix (Fin 1) (Fin 1) (R ⊗[F] E)) 0 0)) = S) ∧ ∀ (lam : Measure R), lam.IsAddHaarMeasure → lam.map (hermScal R σ) = coordHerm R σ b lam 1

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.CoordinateBijectionTraceForm

theorem **repr_ofF** {F E : Type} [Field F] [Field E] [Algebra F E] {d M : ℕ} (b : Module.Basis (Fin d) F E) (x : Fin M × Fin d → F) (u : Fin 1) (i : Fin M) (k : Fin d) : b.repr (ofF b x u i) k = x (i, k)

theorem **ofF_bijective** {F E : Type} [Field F] [Field E] [Algebra F E] {d M : ℕ} (b : Module.Basis (Fin d) F E) : Function.Bijective (ofF b : (Fin M × Fin d → F) → Fin 1 → Fin M → E)

theorem **ofF_zero** {F E : Type} [Field F] [Field E] [Algebra F E] {d M : ℕ} (b : Module.Basis (Fin d) F E) : ofF b (0 : Fin M × Fin d → F) = 0

theorem **exists_algebraMap_of_fixed** {F E : Type} [Field F] [Field E] [Algebra F E] (h2 : Module.finrank F E = 2) {σ : E →ₐ[F] E} (hne : σ ≠ AlgHom.id F E) {h : E} (hh : σ h = h) : ∃ c : F, h = algebraMap F E c

theorem **sigma_ne_id** {F E : Type} [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] {σ : E →ₐ[F] E} (h : IsCMExt F E σ) : σ ≠ AlgHom.id F E

theorem **sigma_sigma** {F E : Type} [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] {σ : E →ₐ[F] E} (h : IsCMExt F E σ) (x : E) : σ (σ x) = x

theorem **sigma_hE_self** {F E : Type} [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] {σ : E →ₐ[F] E} (hσ : IsCMExt F E σ) {M : ℕ} {J : Matrix (Fin M) (Fin M) E} (hJ : IsHermND σ J) (y : Fin M → E) : σ (hE σ J y y) = hE σ J y y

theorem **ofF_gram_iff** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E), Function.Bijective (ofF b : (Fin M × Fin d → F) → Fin 1 → Fin M → E) ∧ (∀ x : Fin M × Fin d → F, ofF b x ≠ 0 ↔ x ≠ 0) ∧ ∀ (B : Matrix (Fin M × Fin d) (Fin M × Fin d) F), (∀ x : Fin M → E, Algebra.trace F E (hE σ J x x) = (fun p : Fin M × Fin d => b.repr (x p.1) p.2) ⬝ᵥ (B *ᵥ fun p : Fin M × Fin d => b.repr (x p.1) p.2)) → ∀ (β : F) (x : Fin M × Fin d → F), gramE σ J (ofF b x) = (fun _ _ => algebraMap F E β) ↔ (1 / 2 : F) * (x ⬝ᵥ (B *ᵥ x)) = β

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.CoordinateBijectionTraceFormStatement

theorem **_root_.GlobalTheory.T.coordinate_bijection_trace_form** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E), Function.Bijective (ofF b : (Fin M × Fin d → F) → Fin 1 → Fin M → E) ∧ (∀ x : Fin M × Fin d → F, ofF b x ≠ 0 ↔ x ≠ 0) ∧ ∀ (B : Matrix (Fin M × Fin d) (Fin M × Fin d) F), (∀ x : Fin M → E, Algebra.trace F E (hE σ J x x) = (fun p : Fin M ×

Fin d => b.repr (x p.1) p.2) ·ᵥ (B *ᵥ fun p : Fin M × Fin d => b.repr (x p.1) p.2)) → ∀ (β : F) (x : Fin M × Fin d → F), gramE σ J (ofF b x) = (fun _ _ => algebraMap F E β) ↔ (1 / 2 : F) * (x ·ᵥ (B *ᵥ x)) = β

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.CoordinateHaarMeasure

theorem **haar_comap** {G H : Type*} [AddGroup G] [TopologicalSpace G] [IsTopologicalAddGroup G] [MeasurableSpace G] [BorelSpace G] [AddGroup H] [TopologicalSpace H] [IsTopologicalAddGroup H] [MeasurableSpace H] [BorelSpace H] (f : G → H) (hadd : ∀ x y, f (x + y) = f x + f y) (hf : IsHomeomorph f) (μ : Measure H) [μ.IsAddHaarMeasure] : (Measure.comap f μ).IsAddHaarMeasure

theorem **conj_facts** [NumberField F] [NumberField E] {σ : E →ₐ[F] E} (h : IsCMExt F E σ) : (∀ x, σ (σ x) = x) ∧ ∃ δ : E, δ ≠ 0 ∧ σ δ = -δ

theorem **sigR_tmul** (σ : E →ₐ[F] E) (r : R) (e : E) : sigR R σ (r ⊗ₜ[F] e) = r ⊗ₜ[F] σ e

theorem **sigR_sigR** {σ : E →ₐ[F] E} (hinv : ∀ x, σ (σ x) = x) (y : R ⊗[F] E) : sigR R σ (sigR R σ y) = y

theorem **half_add_self** [CharZero F] (r : R) : algebraMap F R (1 / 2) * (r + r) = r

theorem **fixed_eq_tmul** [CharZero F] {σ : E →ₐ[F] E} (hinv : ∀ x, σ (σ x) = x) (h2 : Module.finrank F E = 2) {δ : E} (hδ : δ ≠ 0) (hσδ : σ δ = -δ) (y : R ⊗[F] E) (hy : sigR R σ y = y) : ∃ r : R, y = r ⊗ₜ[F] (1 : E)

theorem **half_trace_tmul** [CharZero F] (h2 : Module.finrank F E = 2) (s : R) : algebraMap F R (1 / 2) * Algebra.trace R (R ⊗[F] E) (s ⊗ₜ[F] (1 : E)) = s

theorem **hermMat_mem** {σ : E →ₐ[F] E} (hinv : ∀ x, σ (σ x) = x) (n : ℕ) (x : (Fin n → R) × ({p : Fin n × Fin n // p.1 < p.2} → R ⊗[F] E)) : hermMat R σ n x ∈ HermR R σ n

theorem **hermCoord_add** (σ : E →ₐ[F] E) (n : ℕ) (S T : HermR R σ n) : hermCoord R σ n (S + T) = hermCoord R σ n S + hermCoord R σ n T

theorem **hermCoord_isHomeomorph** [IsTopologicalRing R] [CharZero F] {σ : E →ₐ[F] E} (hinv : ∀ x, σ (σ x) = x) (h2 : Module.finrank F E = 2) {δ : E} (hδ : δ ≠ 0) (hσδ : σ δ = -δ) (n : ℕ) [ContinuousAdd (R ⊗[F] E)] [ContinuousSMul R (R ⊗[F] E)] (hsig : Continuous (sigR R σ)) (htr : Continuous (Algebra.trace R (R ⊗[F] E))) : IsHomeomorph (hermCoord R σ n)

theorem **coord_measures_haar** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [IsTopologicalRing R] [LocallyCompactSpace R] [SecondCountableTopology R] [T2Space R] [MeasurableSpace R] [BorelSpace R] (d : ℕ) (b : Module.Basis (Fin d) F E) (lam : Measure R), lam.IsAddHaarMeasure → (coordK R b lam).IsAddHaarMeasure ∧ (∀ n M : ℕ, (coordW R b lam n M).IsAddHaarMeasure) ∧ ∀ n : ℕ, (coordHerm R σ b lam n).IsAddHaarMeasure

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.CoordinateHaarMeasureStatement

theorem **_root_.GlobalTheory.T.coordinate_haar_measure** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [IsTopologicalRing R] [LocallyCompactSpace R] [SecondCountableTopology R] [T2Space R] [MeasurableSpace R] [BorelSpace R] (d : ℕ) (b : Module.Basis (Fin d) F E) (lam : Measure R), lam.IsAddHaarMeasure → (coordK R b lam).IsAddHaarMeasure ∧ (∀ n M : ℕ, (coordW R b lam n M).IsAddHaarMeasure) ∧ ∀ n : ℕ, (coordHerm R σ b lam n).IsAddHaarMeasure

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.CoordinateHomeomorphRankOneTuples

theorem **map_curry_symm_pi** {R : Type} [MeasurableSpace R] (lam : Measure R) [SigmaFinite lam] (M d : ℕ) : (Measure.pi fun _ : Fin M => Measure.pi fun _ : Fin d => lam).map (MeasurableEquiv.curry (Fin M) (Fin d) R).symm = Measure.pi fun _ : Fin M × Fin d => lam

theorem **coordK_eq_map** {F E : Type} [Field F] [Field E] [Algebra F E] {R : Type} [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] {d : ℕ} (b : Module.Basis (Fin d) F E) (lam : Measure R) (e : R ⊗[F] E ≃ᵐ (Fin d → R)) (he : ∀ y k, e y k = (Algebra.TensorProduct.basis R b).repr y k) : coordK R b lam = (Measure.pi fun _ => lam).map e.symm

theorem **coordMap_one** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [IsTopologicalRing R] [LocallyCompactSpace R] [SecondCountableTopology R] [T2Space R] [MeasurableSpace R] [BorelSpace R] (d : ℕ) (b : Module.Basis (Fin d) F E) (lam : Measure R), lam.IsAddHaarMeasure → ∀ M : ℕ, IsHomeomorph (fun X : Fin 1 → Fin M → R ⊗[F] E => cv R b (X 0)) ∧ MeasurableEmbedding (fun X : Fin 1 → Fin M → R ⊗[F] E => cv R b (X 0)) ∧ (coordW R b lam 1 M).map (fun X : Fin 1 → Fin M → R ⊗[F] E => cv R b (X 0)) = Measure.pi fun _ : Fin M × Fin d => lam

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.CoordinateHomeomorphRankOneTuplesStatement

theorem **_root_.GlobalTheory.T.coordinate_homeomorph_rank_one_tuples** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [IsTopologicalRing R] [LocallyCompactSpace R] [SecondCountableTopology R] [T2Space R] [MeasurableSpace R] [BorelSpace R] (d : ℕ) (b : Module.Basis (Fin d) F E) (lam : Measure R), lam.IsAddHaarMeasure → ∀ M : ℕ, IsHomeomorph (fun X : Fin 1 → Fin M → R ⊗[F] E => cv R b (X 0)) ∧ MeasurableEmbedding (fun X : Fin 1 → Fin M → R ⊗[F] E => cv R b (X 0)) ∧ (coordW R b lam 1 M).map (fun X : Fin 1 → Fin M → R ⊗[F] E => cv R b (X 0)) = Measure.pi fun _ : Fin M × Fin d => lam

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.CoordinateHomeomorphTensorProductMeasure

theorem **pairMap_bijective** (n : ℕ) : Function.Bijective (pairMap n)

theorem **splitE_symm_fst_zero** {R K : Type} {n : ℕ} (q : R × (Fin n → K) × Coords R K n) : ((splitE R K n).symm q).1 0 = q.1

theorem **splitE_symm_fst_succ** {R K : Type} {n : ℕ} (q : R × (Fin n → K) × Coords R K n) (i : Fin n) : ((splitE R K n).symm q).1 i.succ = q.2.2.1 i

theorem **splitE_symm_snd_inl** {R K : Type} {n : ℕ} (q : R × (Fin n → K) × Coords R K n) (j : Fin n) : ((splitE R K n).symm q).2 (pairEquiv n (.inl j)) = q.2.1 j

theorem **splitE_symm_snd_inr** {R K : Type} {n : ℕ} (q : R × (Fin n → K) × Coords R K n) (p : Pairs n) : ((splitE R K n).symm q).2 (pairEquiv n (.inr p)) = q.2.2.2 p

theorem **shuffle_mp** {A B C D : Type} [MeasurableSpace A] [MeasurableSpace B] [MeasurableSpace C] [MeasurableSpace D] (a : Measure A) (b : Measure B) (c : Measure C) (d : Measure D) [SFinite a] [SFinite b] [SFinite c] [SFinite d] : MeasurePreserving (fun x : (A × B) × (C × D) => (x.1.1, x.2.1, x.1.2, x.2.2)) ((a.prod b).prod (c.prod d)) (a.prod (c.prod (b.prod d)))

theorem **splitE_mp** {R K : Type} [MeasurableSpace R] [MeasurableSpace K] (lam : Measure R) (κ : Measure K) [SigmaFinite lam] [SigmaFinite κ] (n : ℕ) : MeasurePreserving (splitE R K n) ((Measure.pi fun _ : Fin (n + 1) => lam).prod (Measure.pi fun _ : Pairs (n + 1) => κ)) (lam.prod ((Measure.pi fun _ : Fin n => κ).prod ((Measure.pi fun _ : Fin n => lam).prod (Measure.pi fun _ : Pairs n => κ))))

theorem **hermMat_mem** (hσ : IsCMExt F E σ) (n : ℕ) (c : Coords R (R ⊗[F] E) n) : hermMat R σ n c ∈ HermR R σ n

theorem **continuous_algebraMapK** : Continuous (algebraMap R (R ⊗[F] E))

theorem **hermMat_zero_zero** (n : ℕ) (q : R × (Fin n → R ⊗[F] E) × Coords R (R ⊗[F] E) n) : hermMat R σ (n + 1) ((splitE R (R ⊗[F] E) n).symm q) 0 0 = q.1 ⊗ₜ[F] (1 : E)

theorem **hermMat_zero_succ** (n : ℕ) (q : R × (Fin n → R ⊗[F] E) × Coords R (R ⊗[F] E) n) (j : Fin n) : hermMat R σ (n + 1) ((splitE R (R ⊗[F] E) n).symm q) 0 j.succ = q.2.1 j

theorem **hermMat_succ_succ** (n : ℕ) (q : R × (Fin n → R ⊗[F] E) × Coords R (R ⊗[F] E) n) (i j : Fin n) : hermMat R σ (n + 1) ((splitE R (R ⊗[F] E) n).symm q) i.succ j.succ = hermMat R σ n q.2.2 i j

theorem **herm_split** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [IsTopologicalRing R] [LocallyCompactSpace R] [SecondCountableTopology R] [T2Space R] [MeasurableSpace R] [BorelSpace R], ∀ (d : ℕ) (b : Module.Basis (Fin d) F E), ∀ (lam : Measure R), lam.IsAddHaarMeasure → ∀ n : ℕ, ∃ e : (R × (Fin n → R ⊗[F] E) × HermR R σ n) ≃ₜ HermR R σ (n + 1), (∀ p, (e p : Matrix (Fin (n + 1)) (Fin (n + 1)) (R ⊗[F] E)) 0 0 = p.1 ⊗ₜ[F] (1 : E)) ∧ (∀ p j, (e p : Matrix (Fin (n + 1)) (Fin (n + 1)) (R ⊗[F] E)) 0 (Fin.succ j) = p.2.1 j) ∧ (∀ p i j, (e p : Matrix (Fin (n + 1)) (Fin (n + 1)) (R ⊗[F] E)) (Fin.succ i) (Fin.succ j) = (p.2.2 : Matrix (Fin n) (Fin n) (R ⊗[F] E)) i j) ∧ (lam.prod ((Measure.pi fun _ => coordK R b lam).prod (coordHerm R σ b lam n))).map e = coordHerm R σ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.CoordinateHomeomorphTensorProductMeasureStatement

theorem **_root_.GlobalTheory.T.coordinate_homeomorph_tensor_product_measure** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [IsTopologicalRing R] [LocallyCompactSpace R] [SecondCountableTopology R] [T2Space R] [MeasurableSpace R] [BorelSpace R], ∀ (d : ℕ) (b : Module.Basis (Fin d) F E), ∀ (lam : Measure R), lam.IsAddHaarMeasure → ∀ n : ℕ, ∃ e : (R × (Fin n → R ⊗[F] E) × HermR R σ n) ≃ₜ HermR R σ (n + 1), (∀ p, (e p : Matrix (Fin (n + 1)) (Fin (n + 1)) (R ⊗[F] E)) 0 0 = p.1 ⊗ₜ[F] (1 : E)) ∧ (∀ p j, (e p : Matrix (Fin (n + 1)) (Fin (n + 1)) (R ⊗[F] E)) 0 (Fin.succ j) = p.2.1 j) ∧ (∀ p i j, (e p : Matrix (Fin (n + 1)) (Fin (n + 1)) (R ⊗[F] E)) (Fin.succ i) (Fin.succ j) = (p.2.2 : Matrix (Fin n) (Fin n) (R ⊗[F] E)) i j) ∧ (lam.prod ((Measure.pi fun _ => coordK R b l

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.CoordinatePolynomialsFormMemKernelDifferential

private lemma **exists_sub_iota_mem_of_sub_involution_mem** (A : Type) [CommRing A] [Finite (MaximalSpectrum A)] (y : MaximalSpectrum A → A) : ∃ a : A, ∀ M : MaximalSpectrum A, a - y M ∈ M.asIdeal

theorem **exists_isUnit_add** (A : Type) [CommRing A] [Finite (MaximalSpectrum A)] (m : ℕ) (x : Fin (m + 1) → A) (hx : ∃ c : Fin (m + 1) → A, ∑ i, c i * x i = 1) : ∃ a : Fin m → A, IsUnit (x 0 + ∑ k, a k * x k.succ)

theorem **exists_invertible_first_column** : ∀ (A : Type) [CommRing A] [Finite (MaximalSpectrum A)] (m : ℕ) (x : Fin (m + 1) → A), (∃ c : Fin (m + 1) → A, ∑ i, c i * x i = 1) → ∃ g : Matrix (Fin (m + 1)) (Fin (m + 1)) A, IsUnit g.det ∧ ∀ i, g i 0 = x i

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.CountingRationalVectorsBoxBound

theorem **hform_eq_dot** {A : Type} [CommRing A] {n : Type} [Fintype n] [DecidableEq n] (s : A →+* A) (J : Matrix n n A) (x y : n → A) : hform s J x y = (fun i => s (x i)) ⬝ᵥ (J *ᵥ y)

theorem **hform_mulVec** {A : Type} [CommRing A] {n : Type} [Fintype n] [DecidableEq n] (s : A →+* A) (J g : Matrix n n A) (x y : n → A) : hform s J (g *ᵥ x) (g *ᵥ y) = hform s (cT s g * J * g) x y

theorem **gramE_mulVec** (σ : E →ₐ[F] E) {M n : ℕ} (J g : Matrix (Fin M) (Fin M) E) (X : Fin n → Fin M → E) : gramE σ J (fun a => g *ᵥ X a) = gramE σ (cT (σ : E →+* E) g * J * g) X

theorem **isotropic_preimage** (σ : E →ₐ[F] E) {M : ℕ} (J : Matrix (Fin M) (Fin M) E) (g : GL (Fin M) E) : {X : Fin 1 → Fin M → E | (fun a => (g : Matrix (Fin M) (Fin M) E) *ᵥ X a) ∈ {X : Fin 1 → Fin M → E | X ≠ 0 ∧ gramE σ J X = 0}} = {X : Fin 1 → Fin M → E | X ≠ 0 ∧ gramE σ (cT (σ : E →+* E) (g : Matrix (Fin M) (Fin M) E) * J * (g : Matrix (Fin M) (Fin M) E)) X = 0}

theorem **actTup_smul** (R : Type) [CommRing R] [Algebra F R] {M n : ℕ} (g : GL (Fin M) (R ⊗[F] E)) (c : R ⊗[F] E) (Y : Fin n → Fin M → R ⊗[F] E) : actTup R g (c • Y) = c • actTup R g Y

theorem **continuous_actTup** (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [IsTopologicalRing (R ⊗[F] E)] {M n : ℕ} (g : GL (Fin M) (R ⊗[F] E)) : Continuous (actTup R g : (Fin n → Fin M → R ⊗[F] E) → _)

theorem **mean_isotropic_box** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → 3 ≤ M → ∀ (μ : Measure (HA (AF F) σ J)) (𝓕 : Set (HA (AF F) σ J)), IsHaarH F E σ M J μ → IsFDH F E σ M J μ 𝓕 → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (N : ℕ), 0 < N → ∃ C : ℝ, ∀ r : ℝ, 0 < r → cntI σ J μ 𝓕 {X : Fin 1 → Fin M → E | X ≠ 0 ∧ gramE σ J X = 0} (Set.indicator (box b N r) (fun _ => (1 : ℝ≥0∞))) ≤ ENNReal.ofReal (C * r ^ (2 * Module.finrank ℚ F * (M - 1)))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.CountingRationalVectorsBoxBoundStatement

theorem **_root_.GlobalTheory.T.isotropic_vector_count_box_bound** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → 3 ≤ M → ∀ (μ : Measure (HA (AF F) σ J)) (𝓕 : Set (HA (AF F) σ J)), IsHaarH F E σ M J μ → IsFDH F E σ M J μ 𝓕 → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (N : ℕ), 0 < N → ∃ C : ℝ, ∀ r : ℝ, 0 < r → cntI σ J μ 𝓕 {X : Fin 1 → Fin M → E | X ≠ 0 ∧ gramE σ J X = 0} (Set.indicator (box b N r) (fun _ => (1 : ℝ≥0∞))) ≤ ENNReal.ofReal (C * r ^ (2 * Module.finrank ℚ F * (M - 1)))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.CountingVectorsBoundedLargeRadius

theorem **count_le** (hpack : ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (M : ℕ) (Λ : Type) [AddCommGroup Λ] (e : Λ →+ (Fin 1 → Fin M → AF F ⊗[F] E)) (N : ℕ) (r R : ℝ), 0 < r → r ≤ R → ∑' l : Λ, (Set.indicator (box b N R) (fun _ => (1 : ℝ≥0∞))) (e l) ≤ ENNReal.ofReal ((5 * R / r) ^ (2 * Module.finrank ℚ F * M)) * ∑' l : Λ, (Set.indicator (box b N r) (fun _ => (1 : ℝ≥0∞))) (e l)) (σ : E →ₐ[F] E) {M : ℕ} (J : Matrix (Fin M) (Fin M) E) (h : HA (AF F) σ J) {d : ℕ} (b : Module.Basis (Fin d) F E) (N : ℕ) {r₀ r : ℝ} (h0 : 0 < r₀) (hr : r₀ ≤ r) : ∑' X : {X : Fin 1 → Fin M → E | X ≠ 0}, (Set.indicator (box b N r) (fun _ => (1 : ℝ≥0∞))) (actW (AF F) σ J h (ratTup (AF F) (X : Fin 1 → Fin M → E))) ≤ ENNReal.ofReal ((5 * r / r₀) ^ (2 * Module.finrank ℚ F * M)) * (1 + ∑' X : {X : Fin 1 → Fin M → E | X ≠ 0}, (Set.indicator (box b N r₀) (fun _ => (1 : ℝ≥0∞))) (actW (AF F) σ J h (ratTup (AF F) (X : Fin 1 → Fin M → E))

theorem **cnt_large** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → 3 ≤ M → ∀ (μ : Measure (HA (AF F) σ J)) (𝓕 : Set (HA (AF F) σ J)), IsHaarH F E σ M J μ → IsFDH F E σ M J μ 𝓕 → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (N : ℕ) (r₀ : ℝ), 0 < N → 0 < r₀ → ∃ C : ℝ, ∀ r : ℝ, r₀ ≤ r → cntI σ J μ 𝓕 {X : Fin 1 → Fin M → E | X ≠ 0} (Set.indicator (box b N r) (fun _ => (1 : ℝ≥0∞))) ≤ ENNReal.ofReal (C * r ^ (2 * Module.finrank ℚ F * M))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.CountingVectorsBoundedLargeRadiusStatement

theorem **_root_.GlobalTheory.T.counting_vectors_bounded_large_radius** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → 3 ≤ M → ∀ (μ : Measure (HA (AF F) σ J)) (𝓕 : Set (HA (AF F) σ J)), IsHaarH F E σ M J μ → IsFDH F E σ M J μ 𝓕 → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (N : ℕ) (r₀ : ℝ), 0 < N → 0 < r₀ → ∃ C : ℝ, ∀ r : ℝ, r₀ ≤ r → cntI σ J μ 𝓕 {X : Fin 1 → Fin M → E | X ≠ 0} (Set.indicator (box b N r) (fun _ => (1 : ℝ≥0∞))) ≤ ENNReal.ofReal (C * r ^ (2 * Module.finrank ℚ F * M))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.CuspRepresentativesFinite

theorem **integral_of_mem** {A : FractionalIdeal (𝒪 F)⁰ F} (hA : A ≠ 0) {x y : F} (hx : x ∈ A) (hy : y ∈ A⁻¹) : ∃ z : 𝒪 F, (z : F) = x * y

theorem **exists_bezout** {A : FractionalIdeal (𝒪 F)⁰ F} (hA : A ≠ 0) {p q : F} (h : A = FractionalIdeal.spanSingleton _ p + FractionalIdeal.spanSingleton _ q) : ∃ a ∈ A⁻¹, ∃ b ∈ A⁻¹, a * q - b * p = 1

theorem **exists_adapted** {A : FractionalIdeal (𝒪 F)⁰ F} (hA : A ≠ 0) {p q : F} (h : A = FractionalIdeal.spanSingleton _ p + FractionalIdeal.spanSingleton _ q) : ∃ g : SL(2, F), Adapted A g ∧ g 1 0 = p ∧ g 1 1 = q

theorem **integral_inv_mul** {A : FractionalIdeal (𝒪 F)⁰ F} (hA : A ≠ 0) {r g : SL(2, F)} (hr : Adapted A r) (hg : Adapted A g) (i j : Fin 2) : ∃ z : 𝒪 F, (z : F) = (r⁻¹ * g) i j

theorem **exists_scale** (J I : (Ideal (𝒪 F))⁰) (h : ClassGroup.mk0 I = ClassGroup.mk0 J) {c d : 𝒪 F} (hI : (I : Ideal (𝒪 F)) = Ideal.span {c, d}) : ∃ l : F, l ≠ 0 ∧ |((Algebra.norm ℚ l : ℚ) : ℝ)| ≤ (Ideal.absNorm (J : Ideal (𝒪 F)) : ℝ) ∧ ((J : Ideal (𝒪 F)) : FractionalIdeal (𝒪 F)⁰ F) = FractionalIdeal.spanSingleton _ (l * (c : F)) + FractionalIdeal.spanSingleton _ (l * (d : F))

theorem **GlobalTheory.CompanionTheory.SW.Core.exists_finset_row_completion** : ∀ (F : Type) [Field F] [NumberField F], ∃ (s : Finset SL(2, F)) (C : ℝ), ∀ c d : 𝒪 F, (c ≠ 0 ∨ d ≠ 0) → ∃ l : F, l ≠ 0 ∧ |((Algebra.norm ℚ l : ℚ) : ℝ)| ≤ C ∧ ∀ u : (𝒪 F)ˣ, ∃ r ∈ s, ∃ k : SL(2, F), (∀ i j, ∃ z : 𝒪 F, (z : F) = k i j) ∧ (r * k) 1 0 = ((u : 𝒪 F) : F) * l * (c : F) ∧ (r * k) 1 1 = ((u : 𝒪 F) : F) * l * (d : F)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.CuspRepresentativesFiniteStatement

theorem **_root_.GlobalTheory.T.cusp_representatives_finite** : ∀ (F : Type) [Field F] [NumberField F], ∃ (s : Finset SL(2, F)) (C : ℝ), ∀ c d : 𝒪 F, (c ≠ 0 ∨ d ≠ 0) → ∃ l : F, l ≠ 0 ∧ |((Algebra.norm ℚ l : ℚ) : ℝ)| ≤ C ∧ ∀ u : (𝒪 F)ˣ, ∃ r ∈ s, ∃ k : SL(2, F), (∀ i j, ∃ z : 𝒪 F, (z : F) = k i j) ∧ (r * k) 1 0 = ((u : 𝒪 F) : F) * l * (c : F) ∧ (r * k) 1 1 = ((u : 𝒪 F) : F) * l * (d : F)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.DenominatorPowerSummable

theorem **den_eq** (b : F) : den b = Ideal.absNorm (dI b)

theorem **mem_dI** {b : F} {x : 𝒪 F} : x ∈ dI b ↔ ∃ t : 𝒪 F, (t : F) = (x : F) * b

theorem **dI_add** (b : F) (r : 𝒪 F) : dI (b + (r : F)) = dI b

theorem **dI_ne_bot** (b : F) : dI b ≠ ⊥

theorem **fibre_bound** (N : ℕ) (hN : 0 < N) (s : Set F) (hs : ∀ b ∈ s, ∀ b' ∈ s, (∃ r : 𝒪 F, b - b' = (N : F) * (r : F)) → b = b') (I : Ideal (𝒪 F)) (hI : I ≠ ⊥) : Finite {b : s // dI (b : F) = I} ∧ Nat.card {b : s // dI (b : F) = I} ≤ N ^ Module.finrank ℤ (𝒪 F) * Ideal.absNorm I

theorem **summable_den** : ∀ (F : Type) [Field F] [NumberField F], ∀ M : ℕ, 3 ≤ M → ∀ N : ℕ, 0 < N → (∀ (b : F) (r : 𝒪 F), den (b + (r : F)) = den b) ∧ (∀ b : F, 0 < den b) ∧ ∀ s : Set F, (∀ b ∈ s, ∀ b' ∈ s, (∃ r : 𝒪 F, b - b' = (N : F) * (r : F)) → b = b') → Summable fun b : s => ((den (b : F) : ℝ) ^ M)⁻¹

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.DenominatorPowerSummableStatement

theorem **_root_.GlobalTheory.T.denominator_power_summable** : ∀ (F : Type) [Field F] [NumberField F], ∀ M : ℕ, 3 ≤ M → ∀ N : ℕ, 0 < N → (∀ (b : F) (r : 𝒪 F), den (b + (r : F)) = den b) ∧ (∀ b : F, 0 < den b) ∧ ∀ s : Set F, (∀ b ∈ s, ∀ b' ∈ s, (∃ r : 𝒪 F, b - b' = (N : F) * (r : F)) → b = b') → Summable fun b : s => ((den (b : F) : ℝ) ^ M)⁻¹

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.DiscriminantDifferentProductFormula

theorem **differentExp_finite** (F : Type) [Field F] [NumberField F] : {v : HeightOneSpectrum (𝒪 F) | GroundTheory.Global.differentExp v ≠ 0}.Finite

private theorem **differentIdeal_int_ne_zero** (F : Type) [Field F] [NumberField F] : differentIdeal ℤ (𝒪 F) ≠ 0

private theorem **differentExp_eq_count_differentIdeal** (F : Type) [Field F] [NumberField F] (v : HeightOneSpectrum (𝒪 F)) : GroundTheory.Global.differentExp v = ((Associates.mk v.asIdeal).count (Associates.mk (differentIdeal ℤ (𝒪 F))).factors : ℤ)

private theorem **prod_absNorm_pow_count_differentIdeal_eq_natAbs_discr** (F : Type) [Field F] [NumberField F] : ∏ v ∈ (differentExp_finite F).toFinset, Ideal.absNorm v.asIdeal ^ ((Associates.mk v.asIdeal).count (Associates.mk (differentIdeal ℤ (𝒪 F))).factors) = (discr F).natAbs

theorem **sqrt_discr_mul_finprod_different** (F : Type) [Field F] [NumberField F] : (∀ᶠ v in Filter.cofinite, (Ideal.absNorm (v : HeightOneSpectrum (𝒪 F)).asIdeal : ℝ≥0) ^ (((-GroundTheory.Global.differentExp v : ℤ) : ℝ) / 2) = 1) ∧ Real.toNNReal (Real.sqrt |(discr F : ℝ)|) * ∏ᶠ v : HeightOneSpectrum (𝒪 F), (Ideal.absNorm v.asIdeal : ℝ≥0) ^ (((-GroundTheory.Global.differentExp v : ℤ) : ℝ) / 2) = 1

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.DiscriminantDifferentProductFormulaStatement

theorem **_root_.GlobalTheory.T.discriminant_different_product_formula** : ∀ (F : Type) [Field F] [NumberField F], (∀ᶠ v in Filter.cofinite, (Ideal.absNorm (v : HeightOneSpectrum (𝒪 F)).asIdeal : ℝ≥0) ^ (((-GroundTheory.Global.differentExp v : ℤ) : ℝ) / 2) = 1) ∧ Real.toNNReal (Real.sqrt |(discr F : ℝ)|) * ∏ᶠ v : HeightOneSpectrum (𝒪 F), (Ideal.absNorm v.asIdeal : ℝ≥0) ^ (((-GroundTheory.Global.differentExp v : ℤ) : ℝ) / 2) = 1

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.DisintegrationDensityContinuous

theorem **exists_bump** [LocallyCompactSpace V] [T2Space V] {K O : Set V} (hK : IsCompact K) (hO : IsOpen O) (hKO : K ⊆ O) : ∃ φ : V → ℝ, Continuous φ ∧ HasCompactSupport φ ∧ tsupport φ ⊆ O ∧ (∀ v, 0 ≤ φ v) ∧ (∀ v, φ v ≤ 1) ∧ ∀ v ∈ K, φ v = 1

theorem **exists_exhaustion** [LocallyCompactSpace V] [SecondCountableTopology V] [T2Space V] {U : Set V} (hU : IsOpen U) : ∃ K W : ℕ → Set V, (∀ n, IsCompact (K n)) ∧ (∀ n, IsOpen (W n)) ∧ (∀ n, W n ⊆ K n) ∧ (∀ n, K n ⊆ U) ∧ Monotone K ∧ Monotone W ∧ (⋃ n, K n) = U ∧ (⋃ n, W n) = U

theorem **le_lintegral_bump** [T2Space V] (m : Measure V) {K : Set V} (hK : IsCompact K) {φ : V → ℝ} (h0 : ∀ v, 0 ≤ φ v) (h1 : ∀ v ∈ K, φ v = 1) : m K ≤ ∫⁻ v, ENNReal.ofReal (φ v) ∂m

theorem **lintegral_bump_le** (m : Measure V) {O : Set V} (hO : IsOpen O) {φ : V → ℝ} (h1 : ∀ v, φ v ≤ 1) (hs : tsupport φ ⊆ O) : ∫⁻ v, ENNReal.ofReal (φ v) ∂m ≤ m O

theorem **BumpFamily.measurable_lintegral** {U : Set V} {B : Set X} {fib : X → Measure V} (h : BumpFamily U B fib) (hB : IsOpen B) {φ : V → ℝ} (hc : Continuous φ) (hcs : HasCompactSupport φ) (hs : tsupport φ ⊆ U) (h0 : ∀ v, 0 ≤ φ v) : Measurable fun b => ∫⁻ v, ENNReal.ofReal (φ v) ∂(fib b)

theorem **BumpFamily.measurable_open** [LocallyCompactSpace V] [SecondCountableTopology V] [T2Space V] {U : Set V} {B : Set X} {fib : X → Measure V} (h : BumpFamily U B fib) (hB : IsOpen B) {O : Set V} (hO : IsOpen O) (hOU : O ⊆ U) : Measurable fun b => fib b O

theorem **BumpFamily.lt_top** [LocallyCompactSpace V] [T2Space V] {U : Set V} {B : Set X} {fib : X → Measure V} (h : BumpFamily U B fib) (hU : IsOpen U) {K : Set V} (hK : IsCompact K) (hKU : K ⊆ U) (b : X) : fib b K < ∞

theorem **BumpFamily.measurable_apply** [LocallyCompactSpace V] [SecondCountableTopology V] [T2Space V] {U : Set V} {B : Set X} {fib : X → Measure V} (h : BumpFamily U B fib) (hB : IsOpen B) (hU : IsOpen U) (hc : ∀ b, fib b Uᶜ = 0) {A : Set V} (hA : MeasurableSet A) : Measurable fun b => fib b A

theorem **fibre_family_basic** : ∀ (V X Y : Type) [TopologicalSpace V] [LocallyCompactSpace V] [SecondCountableTopology V] [T2Space V] [MeasurableSpace V] [BorelSpace V] [TopologicalSpace X] [LocallyCompactSpace X] [SecondCountableTopology X] [T2Space X] [MeasurableSpace X] [BorelSpace X] (μ : Measure V) (ν : Measure X) [IsLocallyFiniteMeasure μ] [IsLocallyFiniteMeasure ν] [ν.IsOpenPosMeasure] (f : V → Y) (g : V → X) (ι : X → Y), Function.Injective ι → Continuous g → (∀ v, f v = ι (g v)) → ∀ (B : Set X), IsOpen B → ∀ fib : X → Measure V, Formalism.MainNotions.GlobalTheoryTheory.GenusMassVolumes.IsFibreFamily μ f ι (g ⁻¹' B) ν B fib → (∀ b, IsLocallyFiniteMeasure (fib b)) ∧ (∀ b, b ∉ B → fib b = 0) ∧ (∀ A : Set V, MeasurableSet A → Measurable fun b => fib b A) ∧ ∀ (C : Set V) (D : Set X), IsCompact C → IsCompact D → D ⊆ B → ∃ c : ℝ≥0, ∀ b ∈ D, fib b C ≤ c

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.DisintegrationDensityContinuousStatement

theorem **_root_.GlobalTheory.T.fibre_family_locallyFinite_measurable_bdd_on_compact** : ∀ (V X Y : Type) [TopologicalSpace V] [LocallyCompactSpace V] [SecondCountableTopology V] [T2Space V] [MeasurableSpace V] [BorelSpace V] [TopologicalSpace X] [LocallyCompactSpace X] [SecondCountableTopology X] [T2Space X] [MeasurableSpace X] [BorelSpace X] (μ : Measure V) (ν : Measure X) [IsLocallyFiniteMeasure μ] [IsLocallyFiniteMeasure ν] [ν.IsOpenPosMeasure] (f : V → Y) (g : V → X) (ι : X → Y), Function.Injective ι → Continuous g → (∀ v, f v = ι (g v)) → ∀ (B : Set X), IsOpen B → ∀ fib : X → Measure V, Formalism.MainNotions.GlobalTheoryTheory.GenusMassVolumes.IsFibreFamily μ f ι (g ⁻¹' B) ν B fib → (∀ b, IsLocallyFiniteMeasure (fib b)) ∧ (∀ b, b ∉ B → fib b = 0) ∧ (∀ A : Set V, MeasurableSet A → Measurable fun b => fib b A) ∧ ∀ (C : Set V) (D : Set X), IsCompact C → IsCompact D → D ⊆ B → ∃ c : ℝ≥0, ∀

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.DisintegrationDensityExists

theorem **measure_eq_of_lintegral_eq** {V : Type} [TopologicalSpace V] [LocallyCompactSpace V] [SecondCountableTopology V] [T2Space V] [MeasurableSpace V] [BorelSpace V] (m m' : Measure V) (O : Set V) (hO : IsOpen O) (hm : m Oᶜ = 0) (hm' : m' Oᶜ = 0) (h : ∀ φ : V → ℝ, Continuous φ → HasCompactSupport φ → tsupport φ ⊆ O → (∀ v, 0 ≤ φ v) → Integrable φ m ∧ ∫⁻ v, ENNReal.ofReal (φ v) ∂m = ∫⁻ v, ENNReal.ofReal (φ v) ∂m') : m = m'

theorem **ae_eq_of_isFibreFamily** {V X Y : Type} [TopologicalSpace V] [MeasurableSpace V] [TopologicalSpace X] [MeasurableSpace X] {μ : Measure V} {ν : Measure X} {f : V → Y} {g : V → X} {ι : X → Y} (hι : Function.Injective ι) (hf : ∀ v, f v = ι (g v)) {U : Set V} {B : Set X} {fib : X → Measure V} (h : Formalism.MainNotions.GlobalTheoryTheory.GenusMassVolumes.IsFibreFamily μ f ι U ν B fib) (b : X) : ∀ᵐ v ∂(fib b), g v = b ∧ v ∈ U

theorem **lintegral_preimage** {V X Y : Type} [TopologicalSpace V] [MeasurableSpace V] [BorelSpace V] [TopologicalSpace X] [MeasurableSpace X] [BorelSpace X] {μ : Measure V} {ν : Measure X} {f : V → Y} {g : V → X} {ι : X → Y} (hι : Function.Injective ι) (hg : Continuous g) (hf : ∀ v, f v = ι (g v)) {O U : Set V} {B : Set X} {fib : X → Measure V} (hO : IsOpen O) (hU : O ⊆ U) (h : Formalism.MainNotions.GlobalTheoryTheory.GenusMassVolumes.IsFibreFamily μ f ι U ν B fib) (φ : V → ℝ≥0∞) (hφ : Measurable φ) (hs : ∀ v, v ∉ O → φ v = 0) (S : Set X) (hS : MeasurableSet S) : ∫⁻ v in O, (g ⁻¹' S).indicator φ v ∂μ = ∫⁻ b in S, ∫⁻ v, φ v ∂(fib b) ∂ν

theorem **fibreFamily_eq** (V X Y : Type) [TopologicalSpace V] [LocallyCompactSpace V] [SecondCountableTopology V] [T2Space V] [MeasurableSpace V] [BorelSpace V] [TopologicalSpace X] [LocallyCompactSpace X] [SecondCountableTopology X] [T2Space X] [MeasurableSpace X] [BorelSpace X] (μ : Measure V) (ν : Measure X) [IsLocallyFiniteMeasure μ] [IsLocallyFiniteMeasure ν] [ν.IsOpenPosMeasure] (f : V → Y) (g : V → X) (ι : X → Y) (hι : Function.Injective ι) (hg : Continuous g) (hf : ∀ v, f v = ι (g v)) (B B' : Set X) (hB : IsOpen B) (hB' : IsOpen B') (hBB : B' ⊆ B) (fib fib' : X → Measure V) (h : Formalism.MainNotions.GlobalTheoryTheory.GenusMassVolumes.IsFibreFamily μ f ι (g ⁻¹' B) ν B fib) (h' : Formalism.MainNotions.GlobalTheoryTheory.GenusMassVolumes.IsFibreFamily μ f ι (g ⁻¹' B') ν B' fib') (b : X) (hb : b ∈ B') : fib' b = fib b

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.DisintegrationDensityExistsStatement

theorem **_root_.GlobalTheory.T.fibre_family_eq_on_open_subset** : ∀ (V X Y : Type) [TopologicalSpace V] [LocallyCompactSpace V] [SecondCountableTopology V] [T2Space V] [MeasurableSpace V] [BorelSpace V] [TopologicalSpace X] [LocallyCompactSpace X] [SecondCountableTopology X] [T2Space X] [MeasurableSpace X] [BorelSpace X] (μ : Measure V) (ν : Measure X) [IsLocallyFiniteMeasure μ] [IsLocallyFiniteMeasure ν] [ν.IsOpenPosMeasure] (f : V → Y) (g : V → X) (ι : X → Y), Function.Injective ι → Continuous g → (∀ v, f v = ι (g v)) → ∀ (B B' : Set X), IsOpen B → IsOpen B' → B' ⊆ B → ∀ fib fib' : X → Measure V, Formalism.MainNotions.GlobalTheoryTheory.GenusMassVolumes.IsFibreFamily μ f ι (g ⁻¹' B) ν B fib → Formalism.MainNotions.GlobalTheoryTheory.GenusMassVolumes.IsFibreFamily μ f ι (g ⁻¹' B') ν B' fib' → ∀ b ∈ B', fib' b = fib b

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.DisintegrationDensityFormula

theorem **comap_map_of_intertwine** {W V : Type} [MeasurableSpace W] [MeasurableSpace V] {T : W → V} (hT : MeasurableEmbedding T) {L L' : W → W} {A A' : V → V} (hL : Measurable L) (hA : Measurable A) (hLL' : ∀ w, L (L' w) = w) (hAA' : ∀ v, A' (A v) = v) (hint : ∀ w, T (L w) = A (T w)) (hint' : ∀ w, T (L' w) = A' (T w)) (μ : Measure V) (c : ℝ≥0∞) (hμ : μ.map A = c • μ) : (μ.comap T).map L = c • μ.comap T

theorem **withDensity_map_of_scale** {W : Type} [MeasurableSpace W] {L : W → W} (hL : Measurable L) (ρ : Measure W) (c k : ℝ≥0∞) (hρ : ρ.map L = c • ρ) {d : W → ℝ≥0∞} (hd : Measurable d) (hk : ∀ w, d w = k * d (L w)) (hck : k * c = 1) : (ρ.withDensity d).map L = ρ.withDensity d

theorem **comap_isLocallyFinite** {W V : Type} [TopologicalSpace W] [MeasurableSpace W] [TopologicalSpace V] [MeasurableSpace V] [BorelSpace W] [BorelSpace V] {T : W → V} (hT : IsOpenEmbedding T) (μ : Measure V) [IsLocallyFiniteMeasure μ] : IsLocallyFiniteMeasure (μ.comap T)

theorem **continuous_theta** (g : V → X) (b₀ : X) : Continuous (theta (Q

theorem **theta_isOpenEmbedding** [T2Space X] {g : V → X} (hg : Continuous g) (heq : ∀ (q : Q) (v : V), g (q • v) = q • g v) {b₀ : X} (ho : IsOpenEmbedding (fun q : Q => q • b₀)) : IsOpenEmbedding (theta (Q

theorem **fibFam_orbit** {g : V → X} {b₀ : X} (w : Q → ℝ≥0) (m : Measure (g ⁻¹' {b₀})) (ho : Function.Injective (fun q : Q => q • b₀)) (q : Q) : fibFam g b₀ w m (q • b₀) = (w q : ℝ≥0∞) • m.map (fun z : (g ⁻¹' {b₀}) => q • (z : V))

theorem **fibFam_off** {g : V → X} {b₀ : X} (w : Q → ℝ≥0) (m : Measure (g ⁻¹' {b₀})) {b : X} (hb : b ∉ Set.range (fun q : Q => q • b₀)) : fibFam g b₀ w m b = 0

theorem **continuous_smul_fib** (g : V → X) (b₀ : X) (q : Q) : Continuous (fun z : (g ⁻¹' {b₀}) => q • (z : V))

theorem **fibFam_support** {Y : Type} {f : V → Y} {g : V → X} {ι : X → Y} (hι : Function.Injective ι) (hg : Continuous g) (hf : ∀ v, f v = ι (g v)) (heq : ∀ (q : Q) (v : V), g (q • v) = q • g v) {b₀ : X} (ho : IsOpenEmbedding (fun q : Q => q • b₀)) (w : Q → ℝ≥0) (m : Measure (g ⁻¹' {b₀})) (b : X) : fibFam g b₀ w m b (f ⁻¹' {ι b} ∩ g ⁻¹' Set.range (fun q : Q => q • b₀))ᶜ = 0

theorem **exists_compact_fib** [T2Space V] {g : V → X} (hg : Continuous g) (b₀ : X) {K : Set Q} (hK : IsCompact K) {S : Set V} (hS : IsCompact S) : ∃ k : Set (g ⁻¹' {b₀}), IsCompact k ∧ ∀ q ∈ K, ∀ z : (g ⁻¹' {b₀}), q • (z : V) ∈ S → z ∈ k

theorem **integrable_and_continuous** [T2Space V] {g : V → X} (hg : Continuous g) {b₀ : X} (ho : IsOpenEmbedding (fun q : Q => q • b₀)) {w : Q → ℝ≥0} (hw : Continuous w) (m : Measure (g ⁻¹' {b₀})) [IsLocallyFiniteMeasure m] {φ : V → ℝ} (hφ : Continuous φ) (hφc : HasCompactSupport φ) : (∀ b ∈ Set.range (fun q : Q => q • b₀), Integrable φ (fibFam g b₀ w m b)) ∧ ContinuousOn (fun b => ∫ v, φ v ∂(fibFam g b₀ w m b)) (Set.range (fun q : Q => q • b₀))

theorem **exists_slice_measure** [T2Space V] [LocallyCompactSpace V] {g : V → X} (hg : Continuous g) (heq : ∀ (q : Q) (v : V), g (q • v) = q • g v) {b₀ : X} (ho : IsOpenEmbedding (fun q : Q => q • b₀)) (μ : Measure V) (ν : Measure X) [IsLocallyFiniteMeasure μ] [IsLocallyFiniteMeasure ν] [ν.IsOpenPosMeasure] (χ χ' : Q →* ℝ≥0) (hχ : Continuous χ) (hχ' : Continuous χ') (hμ : ∀ q : Q, μ.map (fun v => q • v) = χ q • μ) (hν : ∀ q : Q, ν.map (fun x => q • x) = χ' q • ν) : ∃ m : Measure (g ⁻¹' {b₀}), IsLocallyFiniteMeasure m ∧ (μ.comap (theta (Q

theorem **integral_fibFam** [T2Space V] {g : V → X} (hg : Continuous g) (heq : ∀ (q : Q) (v : V), g (q • v) = q • g v) {b₀ : X} (ho : IsOpenEmbedding (fun q : Q => q • b₀)) (μ : Measure V) (ν : Measure X) (χ χ' : Q →* ℝ≥0) (hχ : Continuous χ) (hχ' : Continuous χ') (m : Measure (g ⁻¹' {b₀})) [SFinite m] [SFinite ((ν.comap (fun q : Q => q •

b₀)).withDensity (fun q => (χ' q : ℝ≥0∞)))] (hm : (μ.comap (theta (Q

theorem **exists_isFibreFamily_openOrbit** : ∀ (V X Y : Type) [TopologicalSpace V] [LocallyCompactSpace V] [SecondCountableTopology V] [T2Space V] [MeasurableSpace V] [BorelSpace V] [TopologicalSpace X] [LocallyCompactSpace X] [SecondCountableTopology X] [T2Space X] [MeasurableSpace X] [BorelSpace X] (μ : Measure V) (ν : Measure X) [IsLocallyFiniteMeasure μ] [IsLocallyFiniteMeasure ν] [ν.IsOpenPosMeasure] (f : V → Y) (g : V → X) (ι : X → Y), Function.Injective ι → Continuous g → (∀ v, f v = ι (g v)) → ∀ (Q : Type) [Group Q] [TopologicalSpace Q] [IsTopologicalGroup Q] [LocallyCompactSpace Q] [SecondCountableTopology Q] [T2Space Q] [MulAction Q V] [ContinuousSMul Q V] [MulAction Q X] [ContinuousSMul Q X], (∀ (q : Q) (v : V), g (q • v) = q • g v) → ∀ (χ χ' : Q →* ℝ≥0), Continuous χ → Continuous χ' → (∀ q : Q, μ.map (fun v => q • v) = χ q • μ) → (∀ q : Q, ν.map (fun x => q • x) = χ' q • ν) → ∀ b

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.DisintegrationDensityFormulaStatement

theorem **_root_.GlobalTheory.T.exists_fibre_family_on_open_orbit** : ∀ (V X Y : Type) [TopologicalSpace V] [LocallyCompactSpace V] [SecondCountableTopology V] [T2Space V] [MeasurableSpace V] [BorelSpace V] [TopologicalSpace X] [LocallyCompactSpace X] [SecondCountableTopology X] [T2Space X] [MeasurableSpace X] [BorelSpace X] (μ : Measure V) (ν : Measure X) [IsLocallyFiniteMeasure μ] [IsLocallyFiniteMeasure ν] [ν.IsOpenPosMeasure] (f : V → Y) (g : V → X) (ι : X → Y), Function.Injective ι → Continuous g → (∀ v, f v = ι (g v)) → ∀ (Q : Type) [Group Q] [TopologicalSpace Q] [IsTopologicalGroup Q] [LocallyCompactSpace Q] [SecondCountableTopology Q] [T2Space Q] [MulAction Q V] [ContinuousSMul Q V] [MulAction Q X] [ContinuousSMul Q X], (∀ (q : Q) (v : V), g (q • v) = q • g v) → ∀ (χ χ' : Q →* ℝ≥0), Continuous χ → Continuous χ' → (∀ q : Q, μ.map (fun v => q • v) = χ q • μ) → (∀ q : Q, ν.map (fun x =>

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.DisintegrationDensityUnique

lemma **ae_eq_point** (hι : Function.Injective ι) (hf : ∀ v, f v = ι (g v)) (h : Formalism.MainNotions.GlobalTheoryTheory.GenusMassVolumes.IsFibreFamily μ f ι (g ⁻¹' B) ν B fib) (b : X) : ∀ᵐ v ∂(fib b), g v = b

lemma **integral_preimage** (hι : Function.Injective ι) (hf : ∀ v, f v = ι (g v)) (hg : Continuous g) (h : Formalism.MainNotions.GlobalTheoryTheory.GenusMassVolumes.IsFibreFamily μ f ι (g ⁻¹' B) ν B fib) {S : Set X} (hS : IsCompact S) (hSB : S ⊆ B) {u : V → ℝ≥0∞} (hu : Measurable u) : ∫⁻ v in g ⁻¹' S, u v ∂μ = ∫⁻ b in S, ∫⁻ v, u v ∂(fib b) ∂ν

lemma **integral_preimage_nonneg** [IsLocallyFiniteMeasure ν] [IsLocallyFiniteMeasure μ] (hfin : ∀ b, IsLocallyFiniteMeasure (fib b)) (hι : Function.Injective ι) (hf : ∀ v, f v = ι (g v)) (hg : Continuous g) (h : Formalism.MainNotions.GlobalTheoryTheory.GenusMassVolumes.IsFibreFamily μ f ι (g ⁻¹' B) ν B fib) {S : Set X} (hS : IsCompact S) (hSB : S ⊆ B) {u : V → ℝ} (hc : Continuous u) (hs : HasCompactSupport u) (hsupp : tsupport u ⊆ g ⁻¹' B) (h0 : ∀ v, 0 ≤ u v) : ∫ v in g ⁻¹' S, u v ∂μ = ∫ b in S, ∫ v, u v ∂(fib b) ∂ν

lemma **integral_preimage_real** [IsLocallyFiniteMeasure ν] [IsLocallyFiniteMeasure μ] (hfin : ∀ b, IsLocallyFiniteMeasure (fib b)) (hι : Function.Injective ι) (hf : ∀ v, f v = ι (g v)) (hg : Continuous g) (h : Formalism.MainNotions.GlobalTheoryTheory.GenusMassVolumes.IsFibreFamily μ f ι (g ⁻¹' B) ν B fib) {S : Set X} (hS : IsCompact S) (hSB : S ⊆ B) {u : V → ℝ} (hc : Continuous u) (hs : HasCompactSupport u) (hsupp : tsupport u ⊆ g ⁻¹' B) : ∫ v in g ⁻¹' S, u v ∂μ = ∫ b in S, ∫ v, u v ∂(fib b) ∂ν

lemma **integral_fib_eq** (hfin : ∀ b, IsLocallyFiniteMeasure (fib b)) {ψ : V → ℂ} (hc : Continuous ψ) (hs : HasCompactSupport ψ) (b : X) : ∫ v, ψ v ∂(fib b) = ((∫ v, (ψ v).re ∂(fib b) : ℝ) : ℂ) + ((∫ v, (ψ v).im ∂(fib b) : ℝ) : ℂ) * Complex.I

lemma **re_facts** {ψ : V → ℂ} (hc : Continuous ψ) (hs : HasCompactSupport ψ) {U : Set V} (hsupp : tsupport ψ ⊆ U) : Continuous (fun v => (ψ v).re) ∧ HasCompactSupport (fun v => (ψ v).re) ∧ tsupport (fun v => (ψ v).re) ⊆ U

lemma **im_facts** {ψ : V → ℂ} (hc : Continuous ψ) (hs : HasCompactSupport ψ) {U : Set V} (hsupp : tsupport ψ ⊆ U) : Continuous (fun v => (ψ v).im) ∧ HasCompactSupport (fun v => (ψ v).im) ∧ tsupport (fun v => (ψ v).im) ⊆ U

lemma **continuousOn_fib** (hfin : ∀ b, IsLocallyFiniteMeasure (fib b)) (h : Formalism.MainNotions.GlobalTheoryTheory.GenusMassVolumes.IsFibreFamily μ f ι (g ⁻¹' B) ν B fib) {ψ : V → ℂ} (hc : Continuous ψ) (hs : HasCompactSupport ψ) (hsupp : tsupport ψ ⊆ g ⁻¹' B) : ContinuousOn (fun b => ∫ v, ψ v ∂(fib b)) B

lemma **integral_preimage_complex** [IsLocallyFiniteMeasure ν] [IsLocallyFiniteMeasure μ] (hfin : ∀ b, IsLocallyFiniteMeasure (fib b)) (hι : Function.Injective ι) (hf : ∀ v, f v = ι (g v)) (hg : Continuous g) (h : Formalism.MainNotions.GlobalTheoryTheory.GenusMassVolumes.IsFibreFamily μ f ι (g ⁻¹' B) ν B fib) {S : Set X} (hS : IsCompact S) (hSB : S ⊆ B) {ψ : V → ℂ} (hc : Continuous ψ) (hs : HasCompactSupport ψ) (hsupp : tsupport ψ ⊆ g ⁻¹' B) : ∫ v in g ⁻¹' S, ψ v ∂μ = ∫ b in S, ∫ v, ψ v ∂(fib b) ∂ν

lemma **exists_cutoff** (hg : Continuous g) (hB : IsOpen B) {K : Set X} (hK : IsCompact K) (hKB : K ⊆ B) {φ : V → ℂ} (hc : Continuous φ) (hs : HasCompactSupport φ) : ∃ ψ : V → ℂ, Continuous ψ ∧ HasCompactSupport ψ ∧ tsupport ψ ⊆ g ⁻¹' B ∧ ∀ v, g v ∈ K → ψ v = φ v

theorem **isContDensityOn_fibreIntegral** : ∀ (V X Y : Type) [TopologicalSpace V] [LocallyCompactSpace V] [SecondCountableTopology V] [T2Space V] [MeasurableSpace V] [BorelSpace V] [TopologicalSpace X] [LocallyCompactSpace X] [SecondCountableTopology X] [T2Space X] [MeasurableSpace X] [BorelSpace X] (μ : Measure V) (ν : Measure X) [IsLocallyFiniteMeasure μ] [IsLocallyFiniteMeasure ν] [ν.IsOpenPosMeasure] (f : V → Y) (g : V → X) (ι : X → Y), Function.Injective ι → Continuous g → (∀ v, f v = ι (g v)) → ∀ (B : Set X), IsOpen B → ∀ fib : X → Measure V, Formalism.MainNotions.GlobalTheoryTheory.GenusMassVolumes.IsFibreFamily μ f ι (g ⁻¹' B) ν B fib → ∀ φ : V → ℂ, Continuous φ → HasCompactSupport φ → HeckeCharacterZeroLocus.SiegelWeilU.IsContDensityOn μ f ι ν B φ (fun b => ∫ v, φ v ∂(fib b)) ∧ ∀ b, Integrable φ (fib b)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.DisintegrationDensityUniqueStatement

theorem **_root_.GlobalTheory.T.fibre_family_integral_isContDensityOn** : ∀ (V X Y : Type) [TopologicalSpace V] [LocallyCompactSpace V] [SecondCountableTopology V] [T2Space V] [MeasurableSpace V] [BorelSpace V] [TopologicalSpace X] [LocallyCompactSpace X] [SecondCountableTopology X] [T2Space X] [MeasurableSpace X] [BorelSpace X] (μ : Measure V) (ν : Measure X) [IsLocallyFiniteMeasure μ] [IsLocallyFiniteMeasure ν] [ν.IsOpenPosMeasure] (f : V → Y) (g : V → X) (ι : X → Y), Function.Injective ι → Continuous g → (∀ v, f v = ι (g v)) → ∀ (B : Set X), IsOpen B → ∀ fib : X → Measure V, Formalism.MainNotions.GlobalTheoryTheory.GenusMassVolumes.IsFibreFamily μ f ι (g ⁻¹' B) ν B fib → ∀ φ : V → ℂ, Continuous φ → HasCompactSupport φ → HeckeCharacterZeroLocus.SiegelWeilU.IsContDensityOn μ f ι ν B φ (fun b => ∫ v, φ v ∂(fib b)) ∧ ∀ b, Integrable φ (fib b)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.DualLatticeMembershipTrace

theorem **Bf_mulVec_toA** {F : Type} [Field F] [NumberField F] {ι : Type} [Fintype ι] [DecidableEq ι] (B : Matrix ι ι F) (y : ι → F) : Bf B *ᵥ toA y = toA (B *ᵥ y)

theorem **trace_int_of_basis** {F : Type} [Field F] [NumberField F] (b : Module.Basis (InfinitePlace F) ℤ (𝓞 F)) (N : ℕ) (c : F) (h : ∀ v, ∃ m : ℤ, (N : ℚ) * Algebra.trace ℚ F ((b v : F) * c) = m) (r : 𝓞 F) : ∃ n : ℤ, Algebra.trace ℚ F ((N : F) * c * (r : F)) = n

theorem **toA_mem_perp_latN_iff** : ∀ (F : Type) [Field F] [NumberField F] (ψ : AddChar (A F) ℂ), IsStdChar F ψ → ∀ (ι : Type) [Fintype ι] [DecidableEq ι] (B : Matrix ι ι F) (b : Module.Basis (InfinitePlace F) ℤ (𝓞 F)) (N : ℕ), 0 < N → ∀ y : ι → F, toA y ∈ perp (chiB ψ (Bf B)) (latN F ι N : Set (ι → A F)) ↔ ∀ i v, ∃ m : ℤ, (N : ℚ) * Algebra.trace ℚ F ((b v : F) * (B *ᵥ y) i) = m

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.DualLatticeMembershipTraceStatement

theorem **_root_.GlobalTheory.T.dual_lattice_membership_trace** : ∀ (F : Type) [Field F] [NumberField F] (ψ : AddChar (A F) ℂ), IsStdChar F ψ → ∀ (ι : Type) [Fintype ι] [DecidableEq ι] (B : Matrix ι ι F) (b : Module.Basis (InfinitePlace F) ℤ (𝓞 F)) (N : ℕ), 0 < N → ∀ y : ι → F, toA y ∈ perp (chiB ψ (Bf B)) (latN F ι N : Set (ι → A F)) ↔ ∀ i v, ∃ m : ℤ, (N : ℚ) * Algebra.trace ℚ F ((b v : F) * (B *ᵥ y) i) = m

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.DualLatticeParametrization

theorem **ratBasis_apply** {F : Type} [Field F] [NumberField F] {κ : Type} [Fintype κ] (b : Module.Basis κ ℤ (𝓞 F)) (v : κ) : ratBasis b v = (b v : F)

theorem **exists_trace_dual** {F : Type} [Field F] [NumberField F] {κ : Type} [Fintype κ] [DecidableEq κ] (b : Module.Basis κ ℤ (𝓞 F)) : ∃ d : κ → F, ∀ v u, Algebra.trace ℚ F ((b v : F) * d u) = if v = u then 1 else 0

theorem **eq_of_trace_eq** {F : Type} [Field F] [NumberField F] {κ : Type} [Fintype κ] (b : Module.Basis κ ℤ (𝓞 F)) {x y : F} (h : ∀ v, Algebra.trace ℚ F ((b v : F) * x) = Algebra.trace ℚ F ((b v : F) * y)) : x = y

theorem **exists_dual_param** (F : Type) [Field F] [NumberField F] (ι : Type) [Fintype ι] [DecidableEq ι] (B : Matrix ι ι F) (b : Module.Basis (InfinitePlace F) ℤ (𝓞 F)) (N : ℕ) (hB : B.det ≠ 0) (hN : 0 < N) : ∃ e : (ι × InfinitePlace F → ℤ) → ι → F, (∀ n i v, (N : ℚ) * Algebra.trace ℚ F ((b v : F) * (B *ᵥ e n) i) = n (i, v)) ∧ ∀ y : ι → F, (∀ i v, ∃ m : ℤ, (N : ℚ) * Algebra.trace ℚ F ((b v : F) * (B *ᵥ y) i) = m) → ∃ n, e n = y

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.DualLatticeParametrizationStatement

theorem **_root_.GlobalTheory.T.dual_lattice_parametrization** : ∀ (F : Type) [Field F] [NumberField F] (ι : Type) [Fintype ι] [DecidableEq ι] (B : Matrix ι ι F) (b : Module.Basis (InfinitePlace F) ℤ (𝓞 F)) (N : ℕ), B.det ≠ 0 → 0 < N → ∃ e : (ι × InfinitePlace F → ℤ) → ι → F, (∀ n i v, (N : ℚ) * Algebra.trace ℚ F ((b v : F) * (B *ᵥ e n) i) = n (i, v)) ∧ ∀ y : ι → F, (∀ i v, ∃ m : ℤ, (N : ℚ) * Algebra.trace ℚ F ((b v : F) * (B *ᵥ y) i) = m) → ∃ n, e n = y

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.EisensteinLatticeSumBound

theorem **hasSum_tele** {y : ℝ} (hy : 0 < y) : HasSum (fun n : ℕ => 1 / ((n : ℝ) + y) − 1 / ((n : ℝ) + y + 1)) (1 / y)

theorem **pt_bound** {t u y : ℝ} (ht : 0 < t) (hy : t ≤ y) (hu : y ≤ u) (m : ℕ) : (u ^ (m + 2))⁻¹ ≤ (1 + 1 / t) * (y ^ m)⁻¹ * (1 / u − 1 / (u + 1))

theorem **sum_int** {t y : ℝ} (ht : 0 < t) (hy : t ≤ y) (m : ℕ) : Summable (fun k : ℤ => ((|(k : ℝ)| + y) ^ (m + 2))⁻¹) ∧ ∑' k : ℤ, ((|(k : ℝ)| + y) ^ (m + 2))⁻¹ ≤ 2 * (1 + 1 / t) * (y * (y ^ (m + 2))⁻¹)

theorem **norm_lower** {t : ℝ} (ht : 0 < t) (z : ℂ) (hz : t ≤ z.im) (s : ℝ) : (2 + 1 / t)⁻¹ * (|((⌊z.re + s⌋ : ℤ) : ℝ)| + z.im) ≤ ‖z + (s : ℂ)‖

theorem **finite_small** (F : Type) [Field F] [NumberField F] (ρ : InfinitePlace F → (F →+* ℝ)) (hp : ∀ w x, ((ρ w x : ℝ) : ℂ) = w.embedding x) : {α : 𝓞 F | ∀ w, |ρ w (α : F)| < 1}.Finite

theorem **sum_le_of_fibres** {α ι : Type} [Fintype ι] (a : α → ℝ) (φ : α → ι → ℤ) (g : ι → ℤ → ℝ) (K : ℕ) (ha : ∀ r, 0 ≤ a r) (hg : ∀ w k, 0 ≤ g w k) (hgs : ∀ w, Summable (g w)) (hle : ∀ r, a r ≤ ∏ w, g w (φ r w)) (hfib : ∀ (T : Finset α) (n : ι → ℤ), (T.filter fun r => φ r = n).card ≤ K) : Summable a ∧ ∑' r, a r ≤ K * ∏ w, ∑' k, g w k

theorem **fibre_card** (F : Type) [Field F] [NumberField F] (ρ : InfinitePlace F → (F →+* ℝ)) (S : Finset (𝓞 F)) (hS : ∀ α : 𝓞 F, (∀ w, |ρ w (α : F)| < 1) → α ∈ S) (x : InfinitePlace F → ℝ) (T : Finset (𝓞 F)) (n : InfinitePlace F → ℤ) : (T.filter fun r : 𝓞 F => (fun w => |x w + ρ w (r : F)|) = n).card ≤ S.card

theorem **lattice_sum** : ∀ (F : Type) [Field F] [NumberField F], IsTotallyReal F → ∀ ρ : InfinitePlace F → (F →+* ℝ), (∀ w x, ((ρ w x : ℝ) : ℂ) = w.embedding x) → ∀ M : ℕ, 2 ≤ M → ∀ t : ℝ, 0 < t → ∃ C : ℝ, ∀ τ : InfinitePlace F → ℂ, (∀ w, t ≤ (τ w).im) → Summable (fun r : 𝓞 F => (∏ w, ‖τ w + (ρ w (r : F) : ℝ)‖ ^ M)⁻¹) ∧ ∑' r : 𝓞 F, (∏ w, ‖τ w + (ρ w (r : F) : ℝ)‖ ^ M)⁻¹ ≤ C * ∏ w, (τ w).im * ((τ w).im ^ M)⁻¹

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.EisensteinLatticeSumBoundStatement

theorem **_root_.GlobalTheory.T.eisenstein_lattice_sum_bound** : ∀ (F : Type) [Field F] [NumberField F], IsTotallyReal F → ∀ ρ : InfinitePlace F → (F →+* ℝ), (∀ w x, ((ρ w x : ℝ) : ℂ) = w.embedding x) → ∀ M : ℕ, 2 ≤ M → ∀ t : ℝ, 0 < t → ∃ C : ℝ, ∀ τ : InfinitePlace F → ℂ, (∀ w, t ≤ (τ w).im) → Summable (fun r : 𝓞 F => (∏ w, ‖τ w + (ρ w (r : F) : ℝ)‖ ^ M)⁻¹) ∧ ∑' r : 𝓞 F, (∏ w, ‖τ w + (ρ w (r : F) : ℝ)‖ ^ M)⁻¹ ≤ C * ∏ w, (τ w).im * ((τ w).im ^ M)⁻¹

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.EtaleAwayQuotientEqLocalization

theorem **trace_tmul** [FiniteDimensional F E] (R : Type) [CommRing R] [Algebra F R] (a : R) (e : E) : Algebra.trace R (R ⊗[F] E) (a ⊗ₜ[F] e) = a * algebraMap F R (Algebra.trace F E e)

theorem **expand** {d : ℕ} (b : Module.Basis (Fin d) F E) (R : Type) [CommRing R] [Algebra F R] (u : R ⊗[F] E) : u = ∑ k, ((Algebra.TensorProduct.basis R b).repr u k) ⊗ₜ[F] b k

theorem **trace_entry** [FiniteDimensional F E] (σ : E →ₐ[F] E) {d : ℕ} (b : Module.Basis (Fin d) F E) (R : Type) [CommRing R] [Algebra F R] (u v : R ⊗[F] E) (e : E) : Algebra.trace R (R ⊗[F] E) (sigR R σ u * ((1 : R) ⊗ₜ[F] e) * v) = ∑ k, ∑ l, (Algebra.TensorProduct.basis R b).repr u k * (algebraMap F R (Algebra.trace F E (σ (b k) * e * b l)) * (Algebra.TensorProduct.basis R b).repr v l)

theorem **sig_sig** {σ : E →ₐ[F] E} (h : IsCMExt F E σ) (x : E) : σ (σ x) = x

theorem **trace_sig** {σ : E →ₐ[F] E} (h : IsCMExt F E σ) (x : E) : Algebra.trace F E (σ x) = Algebra.trace F E x

theorem **trGram_isSymm** {σ : E →ₐ[F] E} (h : IsCMExt F E σ) {M d : ℕ} {J : Matrix (Fin M) (Fin M) E} (hJ : IsHermND σ J) (b : Module.Basis (Fin d) F E) : (trGram σ J b).IsSymm

theorem **trGram_det_ne_zero** {σ : E →ₐ[F] E} (h : IsCMExt F E σ) {M d : ℕ} {J : Matrix (Fin M) (Fin M) E} (hJ : IsHermND σ J) (b : Module.Basis (Fin d) F E) : (trGram σ J b).det ≠ 0

theorem **trace_form_coords** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E), ∃ B : Matrix (Fin M × Fin d) (Fin M × Fin d) F, B.IsSymm ∧ B.det ≠ 0 ∧ ∀ (R : Type) [CommRing R] [Algebra F R] (x y : Fin M → R ⊗[F] E), Algebra.trace R (R ⊗[F] E) (hR R σ J x y) = (fun p : Fin M × Fin d => (Algebra.TensorProduct.basis R b).repr (x p.1) p.2) ⬝ᵥ (B.map (algebraMap F R) *ᵥ fun p : Fin M × Fin d => (Algebra.TensorProduct.basis R b).repr (y p.1) p.2)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.EulerProductLocalDensitiesConverges

theorem **mass_fibre_summable** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (n : ℕ), n = 0 ∨ n + 2 ≤ M → ∀ (T : Matrix (Fin n) (Fin n) E) (hT : ∀ a b, σ (T b a) = T a b), T.det ≠ 0 → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (lam : ∀ v : HeightOneSpectrum (𝓞 F), Measure (Fv F v)), (∀ v, IsStdV v (lam v)) → ∀ (m : ∀ v : HeightOneSpectrum (𝓞 F), HermR (Fv F v) σ n → Measure (Fin n → Fin M → Fv F v ⊗[F] E)), (∀ v, IsGramFib (Fv F v) σ J n (coordW (Fv F v) b (lam v) n M) (coordHerm (Fv F v) σ b (lam v) n) (m v)) → (∀ v, m v (hermOf (Fv F v) σ T hT) (latV b v) < ⊤) ∧ Summable fun v => |(m v (hermOf (Fv F v) σ T hT) (latV b v)).toReal − 1|

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.EulerProductLocalDensitiesConvergesStatement

theorem **_root_.GlobalTheory.T.euler_product_local_densities_converges** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (n : ℕ), n = 0 ∨ n + 2 ≤ M → ∀ (T : Matrix (Fin n) (Fin n) E) (hT : ∀ a b, σ (T b a) = T a b), T.det ≠ 0 → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (lam : ∀ v : HeightOneSpectrum (𝓞 F), Measure (Fv F v)), (∀ v, IsStdV v (lam v)) → ∀ (m : ∀ v : HeightOneSpectrum (𝓞 F), HermR (Fv F v) σ n → Measure (Fin n → Fin M → Fv F v ⊗[F] E)), (∀ v, IsGramFib (Fv F v) σ J n (coordW (Fv F v) b (lam v) n M) (coordHerm (Fv F v) σ b (lam v) n) (m v)) → (∀ v, m v (hermOf (Fv F v) σ T hT) (latV b v) < ⊤) ∧ Summable fun v => |(m v (hermOf (Fv F v) σ T hT) (latV b v)).toReal − 1|

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.EulerProductLocallyFinite

theorem **map_eq_prod** {X Y ι : Type*} [MeasurableSpace X] [MeasurableSpace Y] [Fintype ι] {Z : ι → Type*} [∀ i, MeasurableSpace (Z i)] (I : Measure X) (μ₀ : Measure Y) (μ : ∀ i, Measure (Z i)) [SigmaFinite μ₀] [∀ i, SigmaFinite (μ i)] (c : ℝ≥0) (P : X → Y × ∀ i, Z i) (hP : Measurable P) (h : ∀ (A : Set Y) (B : ∀ i, Set (Z i)), MeasurableSet A → (∀ i, MeasurableSet (B i)) → I (P ⁻¹' (A ×ˢ Set.univ.pi B)) = (c : ℝ≥0∞) * (μ₀ A * ∏ i, μ i (B i))) : I.map P = (c • μ₀).prod (Measure.pi μ)

theorem **integral_box** {X Y ι : Type*} [MeasurableSpace X] [MeasurableSpace Y] [Fintype ι] {Z : ι → Type*} [∀ i, MeasurableSpace (Z i)] (I : Measure X) (μ₀ : Measure Y) (μ : ∀ i, Measure (Z i)) [SigmaFinite μ₀] [∀ i, SigmaFinite (μ i)] (c : ℝ≥0) (P : X → Y × ∀ i, Z i) (hP : Measurable P) (h : ∀ (A : Set Y) (B : ∀ i, Set (Z i)), MeasurableSet A → (∀ i, MeasurableSet (B i)) → I (P ⁻¹' (A ×ˢ Set.univ.pi B)) = (c : ℝ≥0∞) * (μ₀ A * ∏ i, μ i (B i))) (f₀ : Y → ℂ) (f : ∀ i, Z i → ℂ) (hf₀ : Integrable f₀ μ₀) (hf : ∀ i, Integrable (f i) (μ i)) : Integrable (fun x => f₀ (P x).1 * ∏ i, f i ((P x).2 i)) I ∧ ∫ x, f₀ (P x).1 * ∏ i, f i ((P x).2 i) ∂I = (c : ℂ) * ((∫ y, f₀ y ∂μ₀) * ∏ i, ∫ z, f i z ∂(μ i))

theorem **infinite_adeles_t2Space** : T2Space (Ainf F)

theorem **sc_ainf** (h : SecondCountableTopology (AF F)) : SecondCountableTopology (Ainf F)

theorem **finprod_eq_indicator** {F E : Type} [Field F] [NumberField F] [Field E] [Algebra F E] {d n M : ℕ} (b : Module.Basis (Fin d) F E) (Φv : ∀ v : HeightOneSpectrum (𝓞 F), (Fin n → Fin M → Fv F v ⊗[F] E) → ℂ) (S : Finset (HeightOneSpectrum (𝓞 F))) (hS : ∀ v, v ∉ S → Φv v = (latV b v).indicator 1) (hae : ∀ X : (Fin n → Fin M → Af F ⊗[F] E), ∀ᶠ v in Filter.cofinite, projV v X ∈ latV b v) (X : Fin n → Fin M → AF F ⊗[F] E) : ∏ᶠ v, Φv v (projV v (projFin X)) = (intOff b S).indicator (fun Y => ∏ v ∈ S, Φv v (projV v (projFin Y))) X

theorem **GlobalTheory.CompanionTheory.SW.EulerInt.euler_int** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (n M : ℕ) (I : Measure (Fin n → Fin M → AF F ⊗[F] E)) (minf : Measure (Fin n → Fin M → Ainf F ⊗[F] E)) (m : ∀ v : HeightOneSpectrum (𝓞 F), Measure (Fin n → Fin M → Fv F v ⊗[F] E)), IsEulerProd b I 1 minf m → IsLocallyFiniteMeasure minf → (∀ v, IsLocallyFiniteMeasure (m v)) → ∀ (Φ : (Fin n → Fin M → AF F ⊗[F] E) → ℂ) (Φinf : (Fin n → Fin M → Ainf F ⊗[F] E) → ℂ) (Φf : (Fin n → Fin M → Af F ⊗[F] E) → ℂ) (Φv : ∀ v : HeightOneSpectrum (𝓞 F), (Fin n → Fin M → Fv F v ⊗[F] E) → ℂ), Continuous Φinf → HasCompactSupport Φinf → IsPureFin b Φf Φv → IsProdInfFin Φ Φinf Φf → Measurable Φ ∧ Integrable Φ I ∧ Multipliable (fun v => ∫ X, Φv v X ∂(m v)) ∧ ∫ X, Φ X ∂I = (∫ X, Φinf X ∂minf) * ∏' v

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.EulerProductLocallyFiniteStatement

theorem **_root_.GlobalTheory.T.integral_pure_tensor_eq_prod_of_isEulerProd** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (n M : ℕ) (I : Measure (Fin n → Fin M → AF F ⊗[F] E)) (minf : Measure (Fin n → Fin M → Ainf F ⊗[F] E)) (m : ∀ v : HeightOneSpectrum (𝓞 F), Measure (Fin n → Fin M → Fv F v ⊗[F] E)), IsEulerProd b I 1 minf m → IsLocallyFiniteMeasure minf → (∀ v, IsLocallyFiniteMeasure (m v)) → ∀ (Φ : (Fin n → Fin M → AF F ⊗[F] E) → ℂ) (Φinf : (Fin n → Fin M → Ainf F ⊗[F] E) → ℂ) (Φf : (Fin n → Fin M → Af F ⊗[F] E) → ℂ) (Φv : ∀ v : HeightOneSpectrum (𝓞 F), (Fin n → Fin M → Fv F v ⊗[F] E) → ℂ), Continuous Φinf → HasCompactSupport Φinf → IsPureFin b Φf Φv → IsProdInfFin Φ Φinf Φf → Measurable Φ ∧ Integrable Φ I ∧ Multipliable (fun v => ∫ X, Φv v X ∂(m v)) ∧ ∫ X, Φ X ∂I = (∫ X, Φinf

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.EulerProductMeasureIntegral

theorem **pureL_indicator_eq** {d n M : ℕ} (b : Module.Basis (Fin d) F E) (S : Finset (HeightOneSpectrum (𝓞 F))) (φinf : (Fin n → Fin M → Ainf F ⊗[F] E) → ℝ≥0∞) (A : ∀ v : HeightOneSpectrum (𝓞 F), Set (Fin n → Fin M → Fv F v ⊗[F] E)) (hS : ∀ v, v ∉ S → A v = latV b v) (X : Fin n → Fin M → AF F ⊗[F] E) : pureL b S φinf (fun v => (A v).indicator 1) X = {X : Fin n → Fin M → AF F ⊗[F] E | ∀ v, projV v (projFin X) ∈ A v}.indicator (fun X => φinf (projInf X)) X

theorem **box_eq** {d n M : ℕ} (b : Module.Basis (Fin d) F E) (S : Finset (HeightOneSpectrum (𝓞 F))) (A : ∀ v : HeightOneSpectrum (𝓞 F), Set (Fin n → Fin M → Fv F v ⊗[F] E)) (hS : ∀ v, v ∉ S → A v = latV b v) : {X : Fin n → Fin M → AF F ⊗[F] E | ∀ v, projV v (projFin X) ∈ A v} = intOff b S ∩ ⋂ v ∈ S, (fun X : Fin n → Fin M → AF F ⊗[F] E => projV v (projFin X)) ⁻¹' A v

theorem **measurableSet_box** (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E) (hσ : IsCMExt F E σ) {d : ℕ} (b : Module.Basis (Fin d) F E) (n M : ℕ) (S : Finset (HeightOneSpectrum (𝓞 F))) (A : ∀ v : HeightOneSpectrum (𝓞 F), Set (Fin n → Fin M → Fv F v ⊗[F] E)) (hA : ∀ v, MeasurableSet (A v)) (hS : ∀ v, v ∉ S → A v = latV b v) : MeasurableSet {X : Fin n → Fin M → AF F ⊗[F] E | ∀ v, projV v (projFin X) ∈ A v}

theorem **euler_box** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (n M : ℕ) (I : Measure (Fin n → Fin M → AF F ⊗[F] E)) (c : ℝ≥0) (minf : Measure (Fin n → Fin M → Ainf F ⊗[F] E)) (m : ∀ v : HeightOneSpectrum (𝓞 F), Measure (Fin n → Fin M → Fv F v ⊗[F] E)), IsEulerProd b I c minf m → ∀ (φinf : (Fin n → Fin M → Ainf F ⊗[F] E) → ℝ≥0∞) (A : ∀ v : HeightOneSpectrum (𝓞 F), Set (Fin n → Fin M → Fv F v ⊗[F] E)), Measurable φinf → (∀ v, MeasurableSet (A v)) → (∀ v, m v (A v) < ⊤) → (∀ᶠ v in Filter.cofinite, A v = latV b v) → Multipliable (fun v => (m v (A v)).toReal) ∧ ∫⁻ X in {X | ∀ v, projV v (projFin X) ∈ A v}, φinf (projInf X) ∂I = (c : ℝ≥0∞) * (∫⁻ X, φinf X ∂minf) * ENNReal.ofReal (∏' v, (m v (A v)).toReal)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.EulerProductMeasureIntegralStatement

theorem **_root_.GlobalTheory.T.euler_product_measure_integral** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (n M : ℕ) (I : Measure (Fin n → Fin M → AF F ⊗[F] E)) (c : ℝ≥0) (minf : Measure (Fin n → Fin M → Ainf F ⊗[F] E)) (m : ∀ v : HeightOneSpectrum (𝒪 F), Measure (Fin n → Fin M → Fv F v ⊗[F] E)), IsEulerProd b I c minf m → ∀ (φinf : (Fin n → Fin M → Ainf F ⊗[F] E) → ℝ≥0∞) (A : ∀ v : HeightOneSpectrum (𝒪 F), Set (Fin n → Fin M → Fv F v ⊗[F] E)), Measurable φinf → (∀ v, MeasurableSet (A v)) → (∀ v, m v (A v) < ⊤) → (∀ᶠ v in Filter.cofinite, A v = latV b v) → Multipliable (fun v => (m v (A v)).toReal) ∧ ∫⁻ X in {X | ∀ v, projV v (projFin X) ∈ A v}, φinf (projInf X) ∂I = (c : ℝ≥0∞) * (∫⁻ X, φinf X ∂minf) * ENNReal.ofReal (∏' v, (m v (A v)).toReal)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.EulerProductMeasureTransport

theorem **borel_ainf** (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E) (hσ : IsCMExt F E σ) (n M : ℕ) : BorelSpace (Fin n → Fin M → Ainf F ⊗[F] E)

theorem **borel_af** (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E) (hσ : IsCMExt F E σ) (n M : ℕ) : BorelSpace (Fin n → Fin M → AF F ⊗[F] E)

theorem **borel_fv** (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E) (hσ : IsCMExt F E σ) (n M : ℕ) (v : HeightOneSpectrum (𝒪 F)) : BorelSpace (Fin n → Fin M → Fv F v ⊗[F] E)

theorem **euler_move** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (n M : ℕ) (I : Measure (Fin n → Fin M → AF F ⊗[F] E)) (minf minf' : Measure (Fin n → Fin M → Ainf F ⊗[F] E)) (m m' : ∀ v : HeightOneSpectrum (𝒪 F), Measure (Fin n → Fin M → Fv F v ⊗[F] E)) (e : (Fin n → Fin M → AF F ⊗[F] E) → (Fin n → Fin M → AF F ⊗[F] E)) (einf : (Fin n → Fin M → Ainf F ⊗[F] E) → (Fin n → Fin M → Ainf F ⊗[F] E)) (ev : ∀ v : HeightOneSpectrum (𝒪 F), (Fin n → Fin M → Fv F v ⊗[F] E) → (Fin n → Fin M → Fv F v ⊗[F] E)), Measurable e → Measurable einf → (∀ v, Measurable (ev v)) → (∀ X, projInf (e X) = einf (projInf X) ∧ ∀ v, projV v (projFin (e X)) = ev v (projV v (projFin X))) → (∀ᶠ v in Filter.cofinite, ∀ X, ev v X ∈ latV b v ↔ X ∈ latV b v) → ∀ (cinf : ℝ≥0) (c : HeightOneSpectrum (𝒪 F) → ℝ≥0), IsUnit

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.EulerProductMeasureTransportStatement

theorem **_root_.GlobalTheory.T.euler_product_measure_transport** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (n M : ℕ) (I : Measure (Fin n → Fin M → AF F ⊗[F] E)) (minf minf' : Measure (Fin n → Fin M → Ainf F ⊗[F] E)) (m m' : ∀ v : HeightOneSpectrum (𝒪 F), Measure (Fin n → Fin M → Fv F v ⊗[F] E)) (e : (Fin n → Fin M → AF F ⊗[F] E) → (Fin n → Fin M → AF F ⊗[F] E)) (einf : (Fin n → Fin M → Ainf F ⊗[F] E) → (Fin n → Fin M → Ainf F ⊗[F] E)) (ev : ∀ v : HeightOneSpectrum (𝒪 F), (Fin n → Fin M → Fv F v ⊗[F] E) → (Fin n → Fin M → Fv F v ⊗[F] E)), Measurable e → Measurable einf → (∀ v, Measurable (ev v)) → (∀ X, projInf (e X) = einf (projInf X) ∧ ∀ v, projV v (projFin (e X)) = ev v (projV v (projFin X))) → (∀ᶠ v in Filter.cofinite, ∀ X, ev v X ∈ latV b v ↔ X ∈ latV b v) → ∀ (cinf : ℝ≥0)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.EulerProductMeasureZero

theorem **exists_tail_pos** {ι : Type} {a : ι → ℝ} (h : Summable (fun v => |a v - 1|)) (S₀ : Finset ι) : ∃ S : Finset ι, S₀ ⊆ S ∧ 0 < ∏' v : {v // v ∉ S}, a v.1

theorem **countable_places** (F : Type) [Field F] [NumberField F] : Countable (HeightOneSpectrum (𝒪 F))

theorem **exists_measure_intOff** {d M n : ℕ} {b : Module.Basis (Fin d) F E} {I : Measure (Fin n → Fin M → AF F ⊗[F] E)} {c : ℝ≥0} {minf : Measure (Fin n → Fin M → Ainf F ⊗[F] E)} {m : ∀ v : HeightOneSpectrum (𝒪 F), Measure (Fin n → Fin M → Fv F v ⊗[F] E)} (h : IsEulerProd b I c minf m) (hmeas : ∀ S, MeasurableSet (intOff (n

theorem **GlobalTheory.CompanionTheory.SW.EulerZero.euler_zero** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (n M : ℕ) (I : Measure (Fin n → Fin M → AF F ⊗[F] E)) (c : ℝ≥0) (minf : Measure (Fin n → Fin M → Ainf F ⊗[F] E)) (m : ∀ v : HeightOneSpectrum (𝒪 F), Measure (Fin n → Fin M → Fv F v ⊗[F] E)), (IsEulerProd b I c minf m → I = 0 → c = 0 ∨ minf = 0 ∨ ∃ v, m v = 0) ∧ (IsEulerProd b I c minf m → (minf = 0 ∨ ∃ v, m v = 0) → I = 0) ∧ ((∀ v, m v (latV b v) < ⊤) → Summable (fun v => |(m v (latV b v)).toReal - 1|) → (minf = 0 ∨ ∃ v, m v = 0) → IsEulerProd b 0 c minf m)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.EulerProductMeasureZeroStatement

theorem **_root_.GlobalTheory.T.euler_product_measure_zero** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (n M : ℕ) (I : Measure (Fin n → Fin M → AF F ⊗[F] E)) (c : ℝ≥0) (minf : Measure (Fin n → Fin M → Ainf F ⊗[F] E)) (m : ∀ v : HeightOneSpectrum (𝒪 F), Measure (Fin n → Fin M → Fv F v ⊗[F] E)), (IsEulerProd b I c minf m → I = 0 → c = 0 ∨ minf = 0 ∨ ∃ v, m v = 0) ∧ (IsEulerProd b I c minf m → (minf = 0 ∨ ∃ v, m v = 0) → I = 0) ∧ ((∀ v, m v (latV b v) < ⊤) → Summable (fun v => |(m v (latV b v)).toReal - 1|) → (minf = 0 ∨ ∃ v, m v = 0) → IsEulerProd b 0 c minf m)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ExistsCompactForMeasurePreservingEndos

theorem **lattice_pigeonhole_measure_preserving** : ∀ (V : Type) [AddCommGroup V] [TopologicalSpace V] [IsTopologicalAddGroup V] [T2Space V] [LocallyCompactSpace V] [SecondCountableTopology V] [MeasurableSpace V] [BorelSpace V] (μ : Measure V) [μ.IsAddHaarMeasure], μ Set.univ = ⊤ → ∀ (Λ : Type) [AddCommGroup Λ] [Countable Λ] (j : Λ →+ V) (C : Set V), IsCompact C → (∀ y, ∃ l, y - j l ∈ C) → ∃ Ω : Set V, IsCompact Ω ∧ ∀ e : V →+ V, Continuous e → μ.map e = μ → ∃ l, l ≠ 0 ∧ e (j l) ∈ Ω

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ExistsCompactForMeasurePreservingEndosStatement

theorem **_root_.GlobalTheory.T.exists_compact_for_measure_preserving_endos** : ∀ (V : Type) [AddCommGroup V] [TopologicalSpace V] [IsTopologicalAddGroup V] [T2Space V] [LocallyCompactSpace V] [SecondCountableTopology V] [MeasurableSpace V] [BorelSpace V] (μ : Measure V) [μ.IsAddHaarMeasure], μ Set.univ = ⊤ → ∀ (Λ : Type) [AddCommGroup Λ] [Countable Λ] (j : Λ →+ V) (C : Set V), IsCompact C → (∀ y, ∃ l, y - j l ∈ C) → ∃ Ω : Set V, IsCompact Ω ∧ ∀ e : V →+ V, Continuous e → μ.map e = μ → ∃ l, l ≠ 0 ∧ e (j l) ∈ Ω

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ExistsCuspReductionBound

theorem **exists_realBasis** {F : Type} [Field F] [NumberField F] (hF : IsTotallyReal F) (ρ : InfinitePlace F → (F →+* ℝ)) (hρ : ∀ w x, ((ρ w x : ℝ) : ℂ) = w.embedding x) : ∃ (κ : Type) (_ : Fintype κ) (b : Module.Basis κ ℝ (InfinitePlace F → ℝ)), ∀ i, ∃ a : 𝒪 F, ∀ w, b i w = ρ w (a : F)

theorem **det_block** (τ : ℂ) (h : 0 < τ.im) : (block τ).det = 1

theorem **bigM_mulVec** {ι : Type} [Fintype ι] [DecidableEq ι] (τ : ι → ℂ) (v : Fin 2 × ι → ℝ) (k : Fin 2) (w : ι) : (bigM τ *ᵥ v) (k, w) = ∑ j, block (τ w) k j * v (j, w)

theorem **det_bigM** {ι : Type} [Fintype ι] [DecidableEq ι] (τ : ι → ℂ) (h : ∀ w, 0 < (τ w).im) : (bigM τ).det = 1

theorem **exists_small** {ι : Type} [Fintype ι] [DecidableEq ι] {κ : Type} [Fintype κ] [Nonempty ι] (b : Module.Basis κ ℝ (ι → ℝ)) (S : AddSubgroup (ι → ℝ)) (hS : ∀ i, b i ∈ S) : ∃ R : ℝ, 0 ≤ R ∧ ∀ τ : ι → ℂ, (∀ w, 0 < (τ w).im) → ∃ u ∈ S, ∃ v ∈ S, (u ≠ 0 ∨ v ≠ 0) ∧ ∀ w, |√(τ w).im * u w| ≤ R ∧ |(τ w).re / √(τ w).im * u w + 1 / √(τ w).im * v w| ≤ R

theorem **exists_small_pair** : ∀ (F : Type) [Field F] [NumberField F], IsTotallyReal F → ∀ ρ : InfinitePlace F → (F →+* ℝ), (∀ w x, ((ρ w x : ℝ) : ℂ) = w.embedding x) → ∃ C : ℝ, ∀ τ : InfinitePlace F → ℂ, (∀ w, 0 < (τ w).im) → ∃ c d : 𝒪 F, (c ≠ 0 ∨ d ≠ 0) ∧ ∀ w, ‖((ρ w (c : F) : ℝ) : ℂ) * τ w + (ρ w (d : F) : ℝ)‖ ^ 2 ≤ C * (τ w).im

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ExistsCuspReductionBoundStatement

theorem **_root_.GlobalTheory.T.exists_cusp_reduction_bound** : ∀ (F : Type) [Field F] [NumberField F], IsTotallyReal F → ∀ ρ : InfinitePlace F → (F →+* ℝ), (∀ w x, ((ρ w x : ℝ) : ℂ) = w.embedding x) → ∃ C : ℝ, ∀ τ : InfinitePlace F → ℂ, (∀ w, 0 < (τ w).im) → ∃ c d : 𝒪 F, (c ≠ 0 ∨ d ≠ 0) ∧ ∀ w, ‖((ρ w (c : F) : ℝ) : ℂ) * τ w + (ρ w (d : F) : ℝ)‖ ^ 2 ≤ C * (τ w).im

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ExistsEqAdelicCharMul

theorem **coe_toCircle** (ψ : AddChar (AF F) ℂ) (h : ∀ x, ‖ψ x‖ = 1) (x : AF F) : ((toCircle ψ h x : Circle) : ℂ) = ψ x

theorem **continuous_toCircle** (ψ : AddChar (AF F) ℂ) (h : ∀ x, ‖ψ x‖ = 1) (hc : Continuous ψ) : Continuous (toCircle ψ h)

theorem **exists_eq_adelicChar_mul** (ψ : AddChar (AF F) ℂ) (hψ : IsCharAF F ψ) : ∃ ξ : F, ξ ≠ 0 ∧ ∀ x : AF F, ψ x = ((GroundTheory.Global.adelicChar F (algebraMap F (AF F) ξ * x) : Circle) : ℂ)

theorem **exists_scalar** (ψ ψ' : AddChar (AF F) ℂ) (hψ : IsCharAF F ψ) (hψ' : IsCharAF F ψ') : ∃ a : F, a ≠ 0 ∧ ∀ x, ψ' x = ψ (algebraMap F (AF F) a * x)

theorem **GlobalTheory.CompanionTheory.SW.char_dual** : ∀ (F : Type) [Field F] [NumberField F] (ψ ψ' : AddChar (AF F) ℂ), IsCharAF F ψ → IsCharAF F ψ' → ∃ a : F, a ≠ 0 ∧ ∀ x, ψ' x = ψ (algebraMap F (AF F) a * x)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ExistsFiniteAdeleHaarNormalized

theorem **exists_haar_integral_one** : ∀ (F : Type) [Field F] [NumberField F], ∃ lf : Measure (Af F), lf.IsAddHaarMeasure ∧ lf {x | ∀ v, x v ∈ v.adicCompletionIntegers F} = 1

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ExistsFiniteAdeleHaarNormalizedStatement

theorem **_root_.GlobalTheory.T.exists_finite_adele_haar_normalized** : ∀ (F : Type) [Field F] [NumberField F], ∃ lf : Measure (Af F), lf.IsAddHaarMeasure ∧ lf {x | ∀ v, x v ∈ v.adicCompletionIntegers F} = 1

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ExistsGramFiberMeasures

theorem **exists_gramFib** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R], IsLocR R → ∀ (n : ℕ) (μ : Measure (Fin n → Fin M → R ⊗[F] E)) (ν : Measure (HermR R σ n)), μ.IsAddHaarMeasure → ν.IsAddHaarMeasure → ∃ m, IsGramFib R σ J n μ ν m

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ExistsGramFiberMeasuresStatement

theorem **_root_.GlobalTheory.T.exists_gram_fiber_measures** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R], IsLocR R → ∀ (n : ℕ) (μ : Measure (Fin n → Fin M → R ⊗[F] E)) (ν : Measure (HermR R σ n)), μ.IsAddHaarMeasure → ν.IsAddHaarMeasure → ∃ m, IsGramFib R σ J n μ ν m

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ExistsIsCharAF

theorem **exists_isCharAF** : ∀ (F : Type) [Field F] [NumberField F], ∃ ψ, IsCharAF F ψ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ExistsNormOneUnitShift

theorem **conj_ne_id** {F E : Type} [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] {σ : E →ₐ[F] E} (h : IsCMExt F E σ) : σ ≠ AlgHom.id F E

theorem **fixed_iff** {F E : Type} [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] {σ : E →ₐ[F] E} (h : IsCMExt F E σ) (a : E) : σ a = a ↔ a ∈ (algebraMap F E).range

theorem **exists_norm_one_sub_isUnit** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (R : Type) [CommRing R] [Algebra F R], PIW.AdelicOrbitU.UnitChoice R → ∀ (t : R ⊗[F] E) (β : Rˣ), ∃ lam : R ⊗[F] E, lam * sigR R σ lam = 1 ∧ IsUnit (algebraMap R (R ⊗[F] E) (β : R) - sigR R σ lam * t)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ExistsNormOneUnitShiftStatement

theorem **_root_.GlobalTheory.T.exists_norm_one_unit_shift** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (R : Type) [CommRing R] [Algebra F R], PIW.AdelicOrbitU.UnitChoice R → ∀ (t : R ⊗[F] E) (β : Rˣ), ∃ lam : R ⊗[F] E, lam * sigR R σ lam = 1 ∧ IsUnit (algebraMap R (R ⊗[F] E) (β : R) - sigR R σ lam * t)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ExistsRealFlowAdelicUnits

theorem **continuous_tmul_one** (F E : Type) [Field F] [NumberField F] [Field E] [Algebra F E] : Continuous (fun a : AF F => a ⊗ₜ[F] (1 : E))

theorem **exists_flow** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∃ τ : Multiplicative ℝ →* (AF F ⊗[F] E)ˣ, IsFlow τ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ExistsRealFlowAdelicUnitsStatement

theorem **_root_.GlobalTheory.T.exists_real_flow_adelic_units** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∃ τ : Multiplicative ℝ →* (AF F ⊗[F] E)ˣ, IsFlow τ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ExistsSizeMulLe

theorem **continuous_size** (b : Module.Basis (Fin d) F E) : Continuous (sizeInf b : Ainf F ⊗[F] E → ℝ)

theorem **size_nonneg** (b : Module.Basis (Fin d) F E) (y : Ainf F ⊗[F] E) : 0 ≤ sizeInf b y

theorem **tup_nonneg** {M n : ℕ} (b : Module.Basis (Fin d) F E) (X : Fin n → Fin M → Ainf F ⊗[F] E) : 0 ≤ sizeTup b X

theorem **size_add_le** (b : Module.Basis (Fin d) F E) (x y : Ainf F ⊗[F] E) : sizeInf b (x + y) ≤ sizeInf b x + sizeInf b y

theorem **size_zero** (b : Module.Basis (Fin d) F E) : sizeInf b (0 : Ainf F ⊗[F] E) = 0

theorem **size_sum_le** {ι : Type} (b : Module.Basis (Fin d) F E) (s : Finset ι) (f : ι → Ainf F ⊗[F] E) : sizeInf b (∑ c ∈ s, f c) ≤ ∑ c ∈ s, sizeInf b (f c)

theorem **coord_le_size** (b : Module.Basis (Fin d) F E) (y : Ainf F ⊗[F] E) (j : Fin d) (v : InfinitePlace F) : ‖(Algebra.TensorProduct.basis (Ainf F) b).repr y j v‖ ≤ sizeInf b y

theorem **repr_mul** (b : Module.Basis (Fin d) F E) (x y : Ainf F ⊗[F] E) (k : Fin d) : (Algebra.TensorProduct.basis (Ainf F) b).repr (x * y) k = ∑ i, ∑ j, (Algebra.TensorProduct.basis (Ainf F) b).repr x i * (Algebra.TensorProduct.basis (Ainf F) b).repr y j * algebraMap F (Ainf F) (b.repr (b i * b j) k)

theorem **exists_size_mul_le** (b : Module.Basis (Fin d) F E) : ∃ K : ℝ, 0 ≤ K ∧ ∀ x y : Ainf F ⊗[F] E, sizeInf b (x * y) ≤ K * sizeInf b x * sizeInf b y

theorem **size_rmul_le** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E), ∀ (n M : ℕ) (C : Set (Matrix (Fin n) (Fin n) (Ainf F ⊗[F] E))), IsCompact C → ∃ c : ℝ, 0 < c ∧ ∀ g ∈ C, ∀ X : Fin n → Fin M → Ainf F ⊗[F] E, sizeTup b (rmul (Ainf F) X g) ≤ c * sizeTup b X

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ExistsStandardLocalMeasures

theorem **exists_isStdV** (v : HeightOneSpectrum (𝒪 F)) : ∃ lam : Measure (Fv F v), IsStdV v lam

theorem **continuous_toMixed** : Continuous (InfiniteAdeleRing.ringEquiv_mixedSpace F)

theorem **continuous_ofMixed** : Continuous (InfiniteAdeleRing.ringEquiv_mixedSpace F).symm

theorem **mem_span_iff** (x : mixedSpace F) : x ∈ Submodule.span ℤ (Set.range (latticeBasis F)) ↔ (InfiniteAdeleRing.ringEquiv_mixedSpace F).symm x ∈ ((algebraMap F (Ainf F)).comp (algebraMap (𝒪 F) F)).toAddMonoidHom.range

theorem **exists_isStdInf** : ∃ laminf : Measure (Ainf F), IsStdInf laminf

theorem **exists_std** : ∀ (F : Type) [Field F] [NumberField F], (∃ laminf : Measure (Ainf F), IsStdInf laminf) ∧ ∀ v : HeightOneSpectrum (𝒪 F), ∃ lam : Measure (Fv F v), IsStdV v lam

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ExistsStandardLocalMeasuresStatement

theorem **_root_.GlobalTheory.T.exists_standard_local_measures** : ∀ (F : Type) [Field F] [NumberField F], (∃ laminf : Measure (Ainf F), IsStdInf laminf) ∧ ∀ v : HeightOneSpectrum (𝒪 F), ∃ lam : Measure (Fv F v), IsStdV v lam

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ExistsTraceFormGramMatrixInBasis

theorem **trace_hform** (s : A →+* A) (hs : ∀ (r : R) (a : A), s (r • a) = r • s a) (β : Module.Basis (Fin d) R A) (J : Matrix (Fin M) (Fin M) A) (x y : Fin M → A) : Algebra.trace R A (hform s J x y) = coord β x ⬝ᵥ (gramB s β J *ᵥ coord β y)

theorem **coord_mulVec** (β : Module.Basis (Fin d) R A) (H : Matrix (Fin M) (Fin M) A) (g : Matrix (Fin M × Fin d) (Fin M × Fin d) R) (hg : ∀ p q, g p q = β.repr (H p.1 q.1 * β q.2) p.2) (x : Fin M → A) : coord β (H *ᵥ x) = g *ᵥ coord β x

theorem **coord_surj** (β : Module.Basis (Fin d) R A) (u : Fin M × Fin d → R) : ∃ x : Fin M → A, coord β x = u

theorem **hform_eq** (s : A →+* A) (J : Matrix (Fin M) (Fin M) A) (x y : Fin M → A) : hform s J x y = (s ∘ x) ⬝ᵥ (J *ᵥ y)

theorem **dot_mulVec_left** {n : Type} [Fintype n] (P : Matrix n n R) (u w : n → R) : (P *ᵥ u) ⬝ᵥ w = u ⬝ᵥ (Pᵀ *ᵥ w)

theorem **hform_mulVec** (s : A →+* A) (J H : Matrix (Fin M) (Fin M) A) (hH : cT s H * J * H = J) (x y : Fin M → A) : hform s J (H *ᵥ x) (H *ᵥ y) = hform s J x y

theorem **mat_ext** {n : Type} [Fintype n] [DecidableEq n] (P Q : Matrix n n R) (h : ∀ u v : n → R, u ⬝ᵥ (P *ᵥ v) = u ⬝ᵥ (Q *ᵥ v)) : P = Q

theorem **sigR_smul** (R : Type) [CommRing R] [Algebra F R] (σ : E →ₐ[F] E) (r : R) (a : R ⊗[F] E) : sigR R σ (r • a) = r • sigR R σ a

theorem **trace_one_tmul** (R : Type) [CommRing R] [Algebra F R] [Module.Finite F E] (e : E) : Algebra.trace R (R ⊗[F] E) ((1 : R) ⊗ₜ[F] e) = algebraMap F R (Algebra.trace F E e)

theorem **gramB_tensor** (R : Type) [CommRing R] [Algebra F R] [Module.Finite F E] {d M : ℕ} (σ : E →ₐ[F] E) (b : Module.Basis (Fin d) F E) (J : Matrix (Fin M) (Fin M) E) : gramB (sigR R σ) (Algebra.TensorProduct.basis R b) (JR R J) = (gramB (σ : E →+* E) b J).map (algebraMap F R)

theorem **sig_smul** (σ : E →ₐ[F] E) (r : F) (a : E) : (σ : E →+* E) (r • a) = r • (σ : E →+* E) a

theorem **sig_sig** (σ : E →ₐ[F] E) (hσ : IsCMExt F E σ) (x : E) : σ (σ x) = x

theorem **gramB_symm** (σ : E →ₐ[F] E) (hσ : IsCMExt F E σ) {M d : ℕ} (J : Matrix (Fin M) (Fin M) E) (hJ : IsHermND σ J) (b : Module.Basis (Fin d) F E) : (gramB (σ : E →+* E) b J).IsSymm

theorem **gramB_det** (σ : E →ₐ[F] E) (hσ : IsCMExt F E σ) {M d : ℕ} (J : Matrix (Fin M) (Fin M) E) (hJ : IsHermND σ J) (b : Module.Basis (Fin d) F E) : (gramB (σ : E →+* E) b J).det ≠ 0

theorem **exists_trace_coord** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E), ∃ B : Matrix (Fin M × Fin d) (Fin M × Fin d) F, B.IsSymm ∧ B.det ≠ 0 ∧ (∀ x : Fin M → E, Algebra.trace F E (hE σ J x x) = (fun p : Fin M × Fin d => b.repr (x p.1) p.2) ⬝ᵥ (B *ᵥ fun p : Fin M × Fin d => b.repr (x p.1) p.2)) ∧ ∀ (R : Type) [CommRing R] [Algebra F R], (∀ x y : Fin M → R ⊗[F] E, Algebra.trace R (R ⊗[F] E) (hR R σ J x y) = (fun p : Fin M × Fin d => (Algebra.TensorProduct.basis R b).repr (x p.1) p.2) ⬝ᵥ (B.map (algebraMap F R) *ᵥ fun p : Fin M × Fin d => (Algebra.TensorProduct.basis R b).repr (y p.1) p.2)) ∧ ∀ h : HA R σ J, ∀ g : Matrix (Fin M × Fin d) (Fin M × Fin d) R, (∀ p q, g p q = (Algebra.TensorProduct.basis R b).repr ((HA.mat R σ J

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ExistsTraceFormGramMatrixInBasisStatement

theorem **_root_.GlobalTheory.T.exists_trace_form_gram_matrix_in_basis** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E), ∃ B : Matrix (Fin M × Fin d) (Fin M × Fin d) F, B.IsSymm ∧ B.det ≠ 0 ∧ (∀ x : Fin M → E, Algebra.trace F E (hE σ J x x) = (fun p : Fin M × Fin d => b.repr (x p.1) p.2) ⬝ᵥ (B *ᵥ fun p : Fin M × Fin d => b.repr (x p.1) p.2)) ∧ ∀ (R : Type) [CommRing R] [Algebra F R], (∀ x y : Fin M → R ⊗[F] E, Algebra.trace R (R ⊗[F] E) (hR R σ J x y) = (fun p : Fin M × Fin d => (Algebra.TensorProduct.basis R b).repr (x p.1) p.2) ⬝ᵥ (B.map (algebraMap F R) *ᵥ fun p : Fin M × Fin d => (Algebra.TensorProduct.basis R b).repr (y p.1) p.2)) ∧ ∀ h : HA R σ J, ∀ g : Matrix (Fin M × Fin d) (Fin M × Fin d) R, (∀ p q, g p q = (Algebra.Te

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ExistsUnipotentInvariantCompactlySupportedWeight

theorem **pi_smul** {X : Type} [MeasurableSpace X] (lam : Measure X) [SigmaFinite lam] (c : ℝ≥0) : Measure.pi (fun _ : ι => c • lam) = c ^ Fintype.card ι • Measure.pi fun _ : ι => lam

theorem **selfDual_mat** (ψR : R → ℂ) (M : Matrix ι ι R) (hM : M.IsSymm) (hu : IsUnit M.det) (μ : Measure (ι → R)) (h : SelfDual μ (fun x y => ψR (x ⬝ᵥ y))) (c : ℝ≥0) (hc : μ.map (fun x => M *ᵥ x) = c • μ) : SelfDual ((NNReal.sqrt c)⁻¹ • μ) (fun x y => ψR (x ⬝ᵥ (M *ᵥ y)))

theorem **iff_of** (ψR : R → ℂ) (M : Matrix ι ι R) (hM : M.IsSymm) (hu : IsUnit M.det) (k : Measure R) [SigmaFinite k] (lam : Measure R) [SigmaFinite lam] (c₀ c : ℝ≥0) (hk : k = c₀ • lam) (h : SelfDual (Measure.pi fun _ : ι => k) (fun x y => ψR (x ⬝ᵥ y))) (hc : (Measure.pi fun _ : ι => k).map (fun x => M *ᵥ x) = c • Measure.pi fun _ : ι => k) (hU : ∀ μ ν : Measure (ι → R), SelfDual μ (fun x y => ψR (x ⬝ᵥ (M *ᵥ y))) → SelfDual ν (fun x y => ψR (x ⬝ᵥ (M *ᵥ y))) → μ = ν) (μ : Measure (ι → R)) : SelfDual μ (fun x y => ψR (x ⬝ᵥ (M *ᵥ y))) ↔ μ = ((NNReal.sqrt c)⁻¹ * c₀ ^ Fintype.card ι) • Measure.pi fun _ : ι => lam

theorem **dot_smul** {F R : Type} [Field F] [CommRing R] [Algebra F R] {ι : Type} [Fintype ι] (a : F) (B : Matrix ι ι F) (x y : ι → R) : x ⬝ᵥ ((a • B).map (algebraMap F R) *ᵥ y) = algebraMap F R a * (x ⬝ᵥ (B.map (algebraMap F R) *ᵥ y))

theorem **isUnit_det_map** {F R : Type} [Field F] [CommRing R] [Algebra F R] {ι : Type} [Fintype ι] [DecidableEq ι] (A : Matrix ι ι F) (h : A.det ≠ 0) : IsUnit (A.map (algebraMap F R)).det

theorem **scale_inf** (ψ ψ₀ : AddChar (AF F) ℂ) (a : F) (h : ∀ x, ψ x = ψ₀ (algebraMap F (AF F) a * x)) (s : Ainf F) : psiInf F ψ s = psiInf F ψ₀ (algebraMap F (Ainf F) a * s)

theorem **scale_v** (ψ ψ₀ : AddChar (AF F) ℂ) (a : F) (h : ∀ x, ψ x = ψ₀ (algebraMap F (AF F) a * x)) (v : HeightOneSpectrum (𝒪 F)) (s : Fv F v) : psiV F ψ v s = psiV F ψ₀ v (algebraMap F (Fv F v) a * s)

theorem **selfDual_coord** : ∀ (F : Type) [Field F] [NumberField F], IsTotallyReal F → ∀ (laminf : Measure (Ainf F)) (lam : ∀ v : HeightOneSpectrum (𝒪 F), Measure (Fv F v)), IsStdInf laminf → (∀ v, IsStdV v (lam v)) → ∀ ψ, IsCharAF F ψ → ∀ (ι : Type) [Fintype ι] [DecidableEq ι] (B : Matrix ι ι F), B.IsSymm → B.det ≠ 0 → ∃ (cinf : ℝ≥0) (c : HeightOneSpectrum (𝒪 F) → ℝ≥0), IsUnitProd cinf c ∧ (∀ μ : Measure (ι → Ainf F), SelfDual μ (fun x y => psiInf F ψ (x ⬝ᵥ (B.map (algebraMap F (Ainf F)) *ᵥ y))) ↔ μ = cinf • Measure.pi fun _ => laminf) ∧ ∀ v : HeightOneSpectrum (𝒪 F), (∀ μ : Measure (ι → Fv F v), SelfDual μ (fun x y => psiV F ψ v (x ⬝ᵥ (B.map (algebraMap F (Fv F v)) *ᵥ y))) ↔ μ = c v • Measure.pi fun _ => lam v)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ExistsUnipotentInvariantCompactlySupportedWeightStatement

theorem **_root_.GlobalTheory.T.exists_unipotentInvariant_compactlySupported_weight** : ∀ (F : Type) [Field F] [NumberField F], IsTotallyReal F → ∀ (laminf : Measure (Ainf F)) (lam : ∀ v : HeightOneSpectrum (𝒪 F), Measure (Fv F v)), IsStdInf laminf → (∀ v, IsStdV v (lam v)) → ∀ ψ, IsCharAF F ψ → ∀ (ι : Type) [Fintype ι] [DecidableEq ι] (B : Matrix ι ι F), B.IsSymm → B.det ≠ 0 → ∃ (cinf : ℝ≥0) (c : HeightOneSpectrum (𝒪 F) → ℝ≥0), IsUnitProd cinf c ∧ (∀ μ : Measure (ι → Ainf F), SelfDual μ (fun x y => psiInf F ψ (x ⬝ᵥ (B.map (algebraMap F (Ainf F)) *ᵥ y))) ↔ μ = cinf • Measure.pi fun _ => laminf) ∧ ∀ v : HeightOneSpectrum (𝒪 F), (∀ μ : Measure (ι → Fv F v), SelfDual μ (fun x y => psiV F ψ v (x ⬝ᵥ (B.map (algebraMap F (Fv F v)) *ᵥ y))) ↔ μ = c v • Measure.pi fun _ => lam v)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ExistsUnitScalingSelfdualAdelicMeasures

theorem **coordK_preimage** (hc : IsHomeomorph (cf (F

theorem **map_coordW** (hCM : IsCMExt F E σ) (hc : IsHomeomorph (cf (F

theorem **BR_eq** (hc : IsHomeomorph (cf (F

theorem **sdW_iff** (hCM : IsCMExt F E σ) {M : ℕ} (J : Matrix (Fin M) (Fin M) E) (B : Matrix (Fin M × Fin d) (Fin M × Fin d) F) (hB : ∀ x y : Fin M → R ⊗[F] E, Algebra.trace R (R ⊗[F] E) (hR R σ J x y) = (fun p : Fin M × Fin d => (Algebra.TensorProduct.basis R b).repr (x p.1) p.2) ⬝ᵥ (B.map (algebraMap F R) *ᵥ fun p : Fin M × Fin d => (Algebra.TensorProduct.basis R b).repr (y p.1) p.2)) (lam : Measure R) (hlam : lam.IsAddHaarMeasure) (ψR : R → ℂ) (n : ℕ) (cc : ℝ≥0) (hmain : ∀ μ : Measure ((Fin M × Fin d) × Fin n → R), SelfDual μ (fun x y => ψR (x ⬝ᵥ ((Matrix.blockDiagonal fun _ : Fin n => B).map (algebraMap F R) *ᵥ y))) ↔ μ = cc • Measure.pi fun _ => lam) (μ : Measure (Fin n → Fin M → R ⊗[F] E)) : SdW F E σ M J R ψR n μ ↔ μ = cc • coordW R b lam n M

theorem **sdW_coord** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (laminf : Measure (Ainf F)) (lam : ∀ v : HeightOneSpectrum (𝒪 F), Measure (Fv F v)), IsStdInf laminf → (∀ v, IsStdV v (lam v)) → ∀ ψ, IsCharAF F ψ → ∀ n : ℕ, ∃ (cinf : ℝ≥0) (c : HeightOneSpectrum (𝒪 F) → ℝ≥0), IsUnitProd cinf c ∧ (∀ μ, SdW F E σ M J (Ainf F) (psiInf F ψ) n μ ↔ μ = cinf • coordW (Ainf F) b laminf n M) ∧ ∀ v : HeightOneSpectrum (𝒪 F), (∀ μ, SdW F E σ M J (Fv F v) (psiV F ψ v) n μ ↔ μ = c v • coordW (Fv F v) b (lam v) n M)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ExistsUnitScalingSelfdualAdelicMeasuresStatement

theorem **_root_.GlobalTheory.T.exists_unit_scaling_selfdual_adelic_measures** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (laminf : Measure (Ainf F)) (lam : ∀ v : HeightOneSpectrum (𝒪 F), Measure (Fv F v)), IsStdInf laminf → (∀ v, IsStdV v (lam v)) → ∀ ψ, IsCharAF F ψ → ∀ n : ℕ, ∃ (cinf : ℝ≥0) (c : HeightOneSpectrum (𝒪 F) → ℝ≥0), IsUnitProd cinf c ∧ (∀ μ, SdW F E σ M J (Ainf F) (psiInf F ψ) n μ ↔ μ = cinf • coordW (Ainf F) b laminf n M) ∧ ∀ v : HeightOneSpectrum (𝒪 F), (∀ μ, SdW F E σ M J (Fv F v) (psiV F ψ v) n μ ↔ μ = c v • coordW (Fv F v) b (lam v) n M)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ExistsUnitScalingSelfdualHermMeasures

theorem **sum_square** {M : Type*} [AddCommMonoid M] {n : ℕ} (f : Fin n → Fin n → M) : ∑ a, ∑ b, f a b = ∑ a, f a a + ∑ p : {p : Fin n × Fin n // p.1 < p.2}, (f p.1.1 p.1.2 + f p.1.2 p.1.1)

theorem **trace_tmul** [FiniteDimensional F E] (R : Type) [CommRing R] [Algebra F R] (r : R) (x : E) : Algebra.trace R (R ⊗[F] E) (r ⊗ₜ[F] x) = r * algebraMap F R (Algebra.trace F E x)

theorem **trace_sigma** {σ : E →ₐ[F] E} (hinv : ∀ x, σ (σ x) = x) (x : E) : Algebra.trace F E (σ x) = Algebra.trace F E x

theorem **trace_sigR** [FiniteDimensional F E] (R : Type) [CommRing R] [Algebra F R] {σ : E →ₐ[F] E} (hinv : ∀ x, σ (σ x) = x) (y : R ⊗[F] E) : Algebra.trace R (R ⊗[F] E) (sigR R σ y) = Algebra.trace R (R ⊗[F] E) y

theorem **gramB_isSymm** {σ : E →ₐ[F] E} (hinv : ∀ x, σ (σ x) = x) {d : ℕ} (b : Module.Basis (Fin d) F E) : (gramB σ b).IsSymm

theorem **gramB_det** [FiniteDimensional F E] [Algebra.IsSeparable F E] {σ : E →ₐ[F] E} (hinv : ∀ x, σ (σ x) = x) {d : ℕ} (b : Module.Basis (Fin d) F E) : (gramB σ b).det ≠ 0

theorem **formB_isSymm** {σ : E →ₐ[F] E} (hinv : ∀ x, σ (σ x) = x) {d : ℕ} (b : Module.Basis (Fin d) F E) (n : ℕ) : (formB σ b n).IsSymm

theorem **formB_det** [FiniteDimensional F E] [Algebra.IsSeparable F E] {σ : E →ₐ[F] E} (hinv : ∀ x, σ (σ x) = x) {d : ℕ} (b : Module.Basis (Fin d) F E) (n : ℕ) : (formB σ b n).det ≠ 0

theorem **formB_mulVec_inl** (σ : E →ₐ[F] E) {d : ℕ} (b : Module.Basis (Fin d) F E) (n : ℕ) (y : Idx n d → R) (i : Fin n) : (((formB σ b n).map (algebraMap F R)) *ᵥ y) (Sum.inl i) = y (Sum.inl i)

theorem **formB_mulVec_inr** (σ : E →ₐ[F] E) {d : ℕ} (b : Module.Basis (Fin d) F E) (n : ℕ) (y : Idx n d → R) (p : {p : Fin n × Fin n // p.1 < p.2}) (k : Fin d) : (((formB σ b n).map (algebraMap F R)) *ᵥ y) (Sum.inr (p, k)) = ∑ l, algebraMap F R (gramB σ b k l) * y (Sum.inr (p, l))

theorem **off_diag** [FiniteDimensional F E] [CharZero F] {σ : E →ₐ[F] E} (hinv : ∀ x, σ (σ x) = x) {d : ℕ} (b : Module.Basis (Fin d) F E) (x y : R ⊗[F] E) : algebraMap F R (1 / 2) * Algebra.trace R (R ⊗[F] E) (x * sigR R σ y + sigR R σ x * y) = ∑ k, (Algebra.TensorProduct.basis R b).repr x k * ∑ l, algebraMap F R (gramB σ b k l) * (Algebra.TensorProduct.basis R b).repr y l

theorem **pair_eq** [FiniteDimensional F E] [CharZero F] {σ : E →ₐ[F] E} (hinv : ∀ x, σ (σ x) = x) (h2 : Module.finrank F E = 2) {δ : E} (hδ : δ ≠ 0) (hσδ : σ δ = -δ) {d : ℕ} (b : Module.Basis (Fin d) F E) (n : ℕ) (S T : HermR R σ n) : pairHerm R σ n S T = hermVec R σ b n S ⬝ᵥ (((formB σ b n).map (algebraMap F R)) *ᵥ hermVec R σ b n T)

theorem **curry_mp** (ι κ A : Type) [Fintype ι] [Fintype κ] [MeasurableSpace A] (μ : Measure A) [SigmaFinite μ] : MeasurePreserving (MeasurableEquiv.curry ι κ A) (Measure.pi fun _ => μ) (Measure.pi fun _ => Measure.pi fun _ => μ)

theorem **mp_comap** {X Y : Type*} [MeasurableSpace X] [MeasurableSpace Y] (h : X ≃ᵐ Y) (ρ : Measure Y) : MeasurePreserving h (Measure.comap h ρ) ρ

theorem **toVec_isHomeomorph** {d : ℕ} (b : Module.Basis (Fin d) F E) (n : ℕ) : IsHomeomorph (toVec R b n)

theorem **mp_toVec** {d : ℕ} (b : Module.Basis (Fin d) F E) (n : ℕ) (lam : Measure R) [lam.IsAddHaarMeasure] : MeasurePreserving (toVec R b n) ((Measure.pi fun _ : Fin n => lam).prod (Measure.pi fun _ : {p : Fin n × Fin n // p.1 < p.2} => coordK R b lam)) (Measure.pi fun _ : Idx n d => lam)

theorem **mp_hermVec** {d : ℕ} (b : Module.Basis (Fin d) F E) (n : ℕ) (lam : Measure R) [lam.IsAddHaarMeasure] : MeasurePreserving (hermVec R σ b n) (coordHerm R σ b lam n) (Measure.pi fun _ : Idx n d => lam)

theorem **hermVec_add** {d : ℕ} (b : Module.Basis (Fin d) F E) (n : ℕ) (S T : HermR R σ n) : hermVec R σ b n (S + T) = hermVec R σ b n S + hermVec R σ b n T

theorem **hermVec_isHomeomorph** {d : ℕ} (b : Module.Basis (Fin d) F E) (n : ℕ) : IsHomeomorph (hermVec R σ b n)

theorem **sd_core** {d : ℕ} (b : Module.Basis (Fin d) F E) (n : ℕ) (lam : Measure R) [lam.IsAddHaarMeasure] (ψR : R → ℂ) (c : ℝ≥0) (h : ∀ μ : Measure (Idx n d → R), SelfDual μ (fun x y => ψR (x ⬝ᵥ ((formB σ b n).map (algebraMap F R) *ᵥ y))) ↔ μ = c • Measure.pi fun _ => lam) (ν : Measure (HermR R σ n)) : SdHerm F E σ R ψR n ν ↔ ν = c • coordHerm R σ b lam n

theorem **sdHerm_eq_coord** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (laminf : Measure (Ainf F)) (lam : ∀ v : HeightOneSpectrum (𝒪 F), Measure (Fv F v)), IsStdInf laminf → (∀ v, IsStdV v (lam v)) → ∀ ψ, IsCharAF F ψ → ∀ n : ℕ, ∃ (cinf : ℝ≥0) (c : HeightOneSpectrum (𝒪 F) → ℝ≥0), IsUnitProd cinf c ∧ (∀ ν, SdHerm F E σ (Ainf F) (psiInf F ψ) n ν ↔ ν = cinf • coordHerm (Ainf F) σ b laminf n) ∧ ∀ v : HeightOneSpectrum (𝒪 F), (∀ ν, SdHerm F E σ (Fv F v) (psiV F ψ v) n ν ↔ ν = c v • coordHerm (Fv F v) σ b (lam v) n)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ExistsUnitScalingSelfdualHermMeasuresStatement

theorem **_root_.GlobalTheory.T.exists_unit_scaling_selfdual_herm_measures** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (laminf : Measure (Ainf F)) (lam : ∀ v : HeightOneSpectrum (𝒪 F), Measure (Fv F v)), IsStdInf laminf → (∀ v, IsStdV v (lam v)) → ∀ ψ, IsCharAF F ψ → ∀ n : ℕ, ∃ (cinf : ℝ≥0) (c : HeightOneSpectrum (𝒪 F) → ℝ≥0), IsUnitProd cinf c ∧ (∀ ν, SdHerm F E σ (Ainf F) (psiInf F ψ) n ν ↔ ν = cinf • coordHerm (Ainf F) σ b laminf n) ∧ ∀ v : HeightOneSpectrum (𝒪 F), (∀ ν, SdHerm F E σ (Fv F v) (psiV F ψ v) n ν ↔ ν = c v • coordHerm (Fv F v) σ b (lam v) n)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.FibreFamilySmul

theorem **isFibreFamily_rescale** (V X Y : Type) [TopologicalSpace V] [MeasurableSpace V] [TopologicalSpace X] [MeasurableSpace X] (μ : Measure V) (ν : Measure X) (f : V → Y) (ι : X → Y) (U : Set V) (B : Set X) (fib : X → Measure V) (h : Formalism.MainNotions.GlobalTheoryTheory.GenusMassVolumes.IsFibreFamily μ f ι U ν B fib) (a c : ℝ≥0) (hc : c ≠ 0) : Formalism.MainNotions.GlobalTheoryTheory.GenusMassVolumes.IsFibreFamily (a • μ) f ι U (c • ν) B (fun b => (a / c) • fib b)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.FiniteAdeleHaarEqProductLocalMeasures

theorem **cK_map** {R R' : Type} [CommRing R] [Algebra F R] [CommRing R'] [Algebra F R'] {d : ℕ} (b : Module.Basis (Fin d) F E) (f : R →ₐ[F] R') (y : R ⊗[F] E) (k : Fin d) : cK R' b (Algebra.TensorProduct.map f (AlgHom.id F E) y) k = f (cK R b y k)

theorem **eW_apply** (hK : IsHomeomorph (cK (F

theorem **coordK_eq** (hK : IsHomeomorph (cK (F

theorem **sigmaFinite_coordK** (hK : IsHomeomorph (cK (F

theorem **map_coordW** (hK : IsHomeomorph (cK (F

theorem **coordW_box** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (lam : ∀ v : HeightOneSpectrum (𝒪 F), Measure (Fv F v)), (∀ v, IsStdV v (lam v)) → ∀ (lf : Measure (Af F)), lf.IsAddHaarMeasure → lf {x | ∀ v, x v ∈ v.adicCompletionIntegers F} = 1 → ∀ (n M : ℕ) (S : Finset (HeightOneSpectrum (𝒪 F))) (A : ∀ v : HeightOneSpectrum (𝒪 F), Set (Fin n → Fin M → Fv F v ⊗[F] E)), (∀ v, MeasurableSet (A v)) → coordW (Af F) b lf n M {X | (∀ v ∈ S, projV v X ∈ A v) ∧ ∀ v, v ∉ S → projV v X ∈ latV b v} = ∏ v ∈ S, coordW (Fv F v) b (lam v) n M (A v)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.FiniteAdeleHaarEqProductLocalMeasuresStatement

theorem **_root_.GlobalTheory.T.finite_adele_haar_eq_product_local_measures** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (lam : ∀ v : HeightOneSpectrum (𝒪 F), Measure (Fv F v)), (∀ v, IsStdV v (lam v)) → ∀ (lf : Measure (Af F)), lf.IsAddHaarMeasure → lf {x | ∀ v, x v ∈ v.adicCompletionIntegers F} = 1 → ∀ (n M : ℕ) (S : Finset (HeightOneSpectrum (𝒪 F))) (A : ∀ v : HeightOneSpectrum (𝒪 F), Set (Fin n → Fin M → Fv F v ⊗[F] E)), (∀ v, MeasurableSet (A v)) → coordW (Af F) b lf n M {X | (∀ v ∈ S, projV v X ∈ A v) ∧ ∀ v, v ∉ S → projV v X ∈ latV b v} = ∏ v ∈ S, coordW (Fv F v) b (lam v) n M (A v)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.FiniteAdeleHaarPiRestrictedProduct

theorem **eq_of_invariant_of_open** {G : Type} [AddCommGroup G] [TopologicalSpace G] [IsTopologicalAddGroup G] [LocallyCompactSpace G] [SecondCountableTopology G] [MeasurableSpace G] [BorelSpace G] (μ ν : Measure G) [ν.IsAddHaarMeasure] [μ.IsAddLeftInvariant] {C : Set G} (hC : IsOpen C) (h0 : (0 : G) ∈ C) (hμ : μ C = 1) (hν : ν C = 1) : μ = ν

theorem **map_restrict_invariant** {G T : Type} [AddCommGroup G] [MeasurableSpace G] [MeasurableAdd G] [AddCommGroup T] [MeasurableSpace T] [MeasurableAdd T] (m : Measure G) [m.IsAddLeftInvariant] (H : AddSubgroup G) (hH : MeasurableSet (H : Set G)) (ρ : G →+ T) (hρ : Measurable ρ) (hs : ∀ t, ∃ h ∈ H, ρ h = t) : ((m.restrict H).map ρ).IsAddLeftInvariant

theorem **measure_inter_preimage** {G T : Type} [AddCommGroup G] [MeasurableSpace G] [MeasurableAdd G] [AddCommGroup T] [TopologicalSpace T] [IsTopologicalAddGroup T] [LocallyCompactSpace T] [SecondCountableTopology T] [MeasurableSpace T] [BorelSpace T] (m : Measure G) [m.IsAddLeftInvariant] (H : AddSubgroup G) (hH : MeasurableSet (H : Set G)) (ρ : G →+ T) (hρ : Measurable ρ) (hs : ∀ t, ∃ h ∈ H, ρ h = t) (ν : Measure T) [ν.IsAddHaarMeasure] {C : Set T} (hC : IsOpen C) (h0 : (0 : T) ∈ C) (hm : m ((H : Set G) ∩ ρ ⁻¹' C) = 1) (hν : ν C = 1) {B : Set T} (hB : MeasurableSet B) : m ((H : Set G) ∩ ρ ⁻¹' B) = ν B

theorem **continuous_compS** (ι : Type) (S : Finset (HeightOneSpectrum (𝒪 F))) : Continuous (compS (F

theorem **evalV_liftS_of_mem** {S : Finset (HeightOneSpectrum (𝒪 F))} (t : ∀ v : S, Fv F v.1) (v : S) : evalV F v.1 (liftS t) = t v

theorem **evalV_liftS_of_notMem** {S : Finset (HeightOneSpectrum (𝒪 F))} (t : ∀ v : S, Fv F v.1) {v : HeightOneSpectrum (𝒪 F)} (hv : v ∉ S) : evalV F v (liftS t) = 0

theorem **exists_offS_compS** (ι : Type) (S : Finset (HeightOneSpectrum (𝒪 F))) (t : ∀ v : S, ι → Fv F v.1) : ∃ h ∈ offS (F

theorem **isOpen_integral** : IsOpen {x : Af F | ∀ v, x v ∈ v.adicCompletionIntegers F}

theorem **isOpen_offS** (ι : Type) [Finite ι] (S : Finset (HeightOneSpectrum (𝒪 F))) : IsOpen (offS (F

theorem **pi_box** : ∀ (F : Type) [Field F] [NumberField F] (lam : ∀ v : HeightOneSpectrum (𝒪 F), Measure (Fv F v)), (∀ v, IsStdV v (lam v)) → ∀ (lf : Measure (Af F)), lf.IsAddHaarMeasure → lf {x | ∀ v, x v ∈ v.adicCompletionIntegers F} = 1 → ∀ (ι : Type) [Fintype ι] (S : Finset (HeightOneSpectrum (𝒪 F))) (A : ∀ v : HeightOneSpectrum (𝒪 F), Set (ι → Fv F v)), (∀ v, MeasurableSet (A v)) → (Measure.pi fun _ : ι => lf) {x | (∀ v ∈ S, (fun i => evalV F v (x i)) ∈ A v) ∧ ∀ v, v ∉ S → ∀ i, evalV F v (x i) ∈ v.adicCompletionIntegers F} = ∏ v ∈ S, (Measure.pi fun _ : ι => lam v) (A v)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.FiniteAdeleHaarPiRestrictedProductStatement

theorem **_root_.GlobalTheory.T.finite_adele_haar_pi_restricted_product** : ∀ (F : Type) [Field F] [NumberField F] (lam : ∀ v : HeightOneSpectrum (𝒪 F), Measure (Fv F v)), (∀ v, IsStdV v (lam v)) → ∀ (lf : Measure (Af F)), lf.IsAddHaarMeasure → lf {x | ∀ v, x v ∈ v.adicCompletionIntegers F} = 1 → ∀ (ι : Type) [Fintype ι] (S : Finset (HeightOneSpectrum (𝒪 F))) (A : ∀ v : HeightOneSpectrum (𝒪 F), Set (ι → Fv F v)), (∀ v, MeasurableSet (A v)) → (Measure.pi fun _ : ι => lf) {x | (∀ v ∈ S, (fun i => evalV F v (x i)) ∈ A v) ∧ ∀ v, v ∉ S → ∀ i, evalV F v (x i) ∈ v.adicCompletionIntegers F} = ∏ v ∈ S, (Measure.pi fun _ : ι => lam v) (A v)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.FiniteAdeleTopology

theorem **isCompact_Oh** : IsCompact (Oh F)

theorem **isOpen_Oh** : IsOpen (Oh F)

theorem **natCast_mul_mem_Oh** (N : ℕ) {x : A F} (hx : x ∈ Oh F) : (N : A F) * x ∈ Oh F

theorem **algebraMap_mem_Oh_iff** (b : F) : algebraMap F (A F) b ∈ Oh F ↔ ∃ r : 𝒪 F, (r : F) = b

theorem **exists_mul_Oh_subset** (U : Set (A F)) (hU : U ∈ 𝒩 (0 : A F)) : ∃ N : ℕ, 0 < N ∧ ∀ x ∈ Oh F, (N : A F) * x ∈ U

theorem **exists_mul_mem_Oh_of_isCompact** (K : Set (A F)) (hK : IsCompact K) : ∃ N : ℕ, 0 < N ∧ ∀ x ∈ K, (N : A F) * x ∈ Oh F

theorem **isOpen_mul_Oh** (N : ℕ) (hN : 0 < N) : IsOpen {x : A F | ∃ y ∈ Oh F, x = (N : A F) * y}

theorem **denseRange_algebraMap** : DenseRange (algebraMap F (A F))

theorem **finiteAdele_topology** : ∀ (F : Type) [Field F] [NumberField F], LocallyCompactSpace (A F) ∧ SecondCountableTopology (A F) ∧ T2Space (A F) ∧ IsCompact (Oh F) ∧ IsOpen (Oh F) ∧ DenseRange (algebraMap F (A F)) ∧ (∀ b : F, algebraMap F (A F) b ∈ Oh F ↔ ∃ r : 𝒪 F, (r : F) = b) ∧ (∀ U ∈ 𝒩 (0 : A F), ∃ N : ℕ, 0 < N ∧ ∀ x ∈ Oh F, (N : A F) * x ∈ U) ∧ (∀ K : Set (A F), IsCompact K → ∃ N : ℕ, 0 < N ∧ ∀ x ∈ K, (N : A F) * x ∈ Oh F) ∧ ∀ N : ℕ, 0 < N → IsOpen {x : A F | ∃ y ∈ Oh F, x = (N : A F) * y}

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.FiniteAdeleTopologyStatement

theorem **_root_.GlobalTheory.T.mem_Oh_iff_and_nat_mul_properties** : ∀ (F : Type) [Field F] [NumberField F], LocallyCompactSpace (A F) ∧ SecondCountableTopology (A F) ∧ T2Space (A F) ∧ IsCompact (Oh F) ∧ IsOpen (Oh F) ∧ DenseRange (algebraMap F (A F)) ∧ (∀ b : F, algebraMap F (A F) b ∈ Oh F ↔ ∃ r : 𝒪 F, (r : F) = b) ∧ (∀ U ∈ 𝒩 (0 : A F), ∃ N : ℕ, 0 < N ∧ ∀ x ∈ Oh F, (N : A F) * x ∈ U) ∧ (∀ K : Set (A F), IsCompact K → ∃ N : ℕ, 0 < N ∧ ∀ x ∈ K, (N : A F) * x ∈ Oh F) ∧ ∀ N : ℕ, 0 < N → IsOpen {x : A F | ∃ y ∈ Oh F, x = (N : A F) * y}

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.FiniteAdelicLatticeMembershipIff

theorem **repr_map_evalV** {F E : Type} [Field F] [NumberField F] [Field E] [Algebra F E] {d : ℕ} (b : Module.Basis (Fin d) F E) (v : HeightOneSpectrum (𝒪 F)) (y : Af F ⊗[F] E) (k : Fin d) : (Algebra.TensorProduct.basis (Fv F v) b).repr (Algebra.TensorProduct.map (evalV F v) (AlgHom.id F E) y) k = evalV F v ((Algebra.TensorProduct.basis (Af F) b).repr y k)

theorem **repr_natCast_mul** {F E R : Type} [Field F] [Field E] [Algebra F E] [CommRing R] [Algebra F R] {d : ℕ} (b : Module.Basis (Fin d) F E) (a : ℕ) (y : R ⊗[F] E) (k : Fin d) : (Algebra.TensorProduct.basis R b).repr ((a : R ⊗[F] E) * y) k = (a : R) * (Algebra.TensorProduct.basis R b).repr y k

theorem **natCast_mul_mem_latV_iff** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [Algebra F E] (d M : ℕ) (b : Module.Basis (Fin d) F E) (a : ℕ) (X : Fin 1 → Fin M → Af F ⊗[F] E), (∀ v, (fun p i => (a : Fv F v ⊗[F] E) * projV v X p i) ∈ latV b v) ↔ ∀ i v, ((a : Af F) * cv (Af F) b (X 0) i) v ∈ v.adicCompletionIntegers F

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.FiniteAdelicLatticeMembershipIffStatement

theorem **_root_.GlobalTheory.T.finite_adelic_lattice_membership_iff** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [Algebra F E] (d M : ℕ) (b : Module.Basis (Fin d) F E) (a : ℕ) (X : Fin 1 → Fin M → Af F ⊗[F] E), (∀ v, (fun p i => (a : Fv F v ⊗[F] E) * projV v X p i) ∈ latV b v) ↔ ∀ i v, ((a : Af F) * cv (Af F) b (X 0) i) v ∈ v.adicCompletionIntegers F

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.FiniteAdelicTraceFormCoordinates

theorem **finDensity_coord** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (B : Matrix (Fin M × Fin d) (Fin M × Fin d) F), (∀ x y : Fin M → Af F ⊗[F] E, Algebra.trace (Af F) (Af F ⊗[F] E) (hR (Af F) σ J x y) = cv (Af F) b x ⬝ᵥ (B.map (algebraMap F (Af F)) *ᵥ cv (Af F) b y)) → ∀ (lf : Measure (Af F)), lf.IsAddHaarMeasure → ∀ (a : ℕ) (U : Set (Af F)), coordW (Af F) b lf 1 M {X | (∀ v, projV v X ∈ {X : (Fin 1 → Fin M → Fv F v ⊗[F] E) | (fun p i => (a : Fv F v ⊗[F] E) * X p i) ∈ latV b v}) ∧ algebraMap F (Af F) (1 / 2) * Algebra.trace (Af F) (Af F ⊗[F] E) (gramR (Af F) σ J X 0 0) ∈ U} = (Measure.pi fun _ : Fin M × Fin d => lf) {x | (∀ i v, ((a : Af F) * x i) v ∈ v.adicCompletionIntegers F) ∧ algebraMap F (Af F) (1 / 2) * (x ⬝ᵥ (B.

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.FiniteAdelicTraceFormCoordinatesStatement

theorem **_root_.GlobalTheory.T.finite_adelic_trace_form_coordinates** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (B : Matrix (Fin M × Fin d) (Fin M × Fin d) F), (∀ x y : Fin M → Af F ⊗[F] E, Algebra.trace (Af F) (Af F ⊗[F] E) (hR (Af F) σ J x y) = cv (Af F) b x ⬝ᵥ (B.map (algebraMap F (Af F)) *ᵥ cv (Af F) b y)) → ∀ (lf : Measure (Af F)), lf.IsAddHaarMeasure → ∀ (a : ℕ) (U : Set (Af F)), coordW (Af F) b lf 1 M {X | (∀ v, projV v X ∈ {X : (Fin 1 → Fin M → Fv F v ⊗[F] E) | (fun p i => (a : Fv F v ⊗[F] E) * X p i) ∈ latV b v}) ∧ algebraMap F (Af F) (1 / 2) * Algebra.trace (Af F) (Af F ⊗[F] E) (gramR (Af F) σ J X 0 0) ∈ U} = (Measure.pi fun _ : Fin M × Fin d => lf) {x | (∀ i v, ((a : Af F) * x i) v ∈ v.adicCompletionIntegers F)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.FiniteCovolumeUnimodular

theorem **unimodular_of_isFundamentalDomain** : ∀ (G : Type) [Group G] [TopologicalSpace G] [IsTopologicalGroup G] [T2Space G] [LocallyCompactSpace G] [SecondCountableTopology G] [MeasurableSpace G] [BorelSpace G] (Γ : Subgroup G) [Countable Γ] (μ : Measure G) [μ.IsHaarMeasure] (𝓕 : Set G), IsFundamentalDomain Γ 𝓕 μ → μ 𝓕 < ⊤ → μ 𝓕 ≠ 0 ∧ μ.IsMulRightInvariant ∧ μ.IsInvInvariant

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.FiniteCovolumeUnimodularStatement

theorem **_root_.GlobalTheory.T.finite_covolume_unimodular** : ∀ (G : Type) [Group G] [TopologicalSpace G] [IsTopologicalGroup G] [T2Space G] [LocallyCompactSpace G] [SecondCountableTopology G] [MeasurableSpace G] [BorelSpace G] (Γ : Subgroup G) [Countable Γ] (μ : Measure G) [μ.IsHaarMeasure] (𝓕 : Set G), IsFundamentalDomain Γ 𝓕 μ → μ 𝓕 < ⊤ → μ 𝓕 ≠ 0 ∧ μ.IsMulRightInvariant ∧ μ.IsInvInvariant

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.FiniteFieldQuadraticFormPointCount

theorem **quadForm_add** (h2 : (2 : k) ≠ 0) {B : Matrix ι ι k} (hB : B.IsSymm) (y z : ι → k) : quadForm B (y + z) = quadForm B y + quadForm B z + y ⬝ᵥ (B *ᵥ z)

theorem **sum_char_dot** (ψ : AddChar k ℂ) (hψ : ψ ≠ 1) {w : ι → k} (hw : w ≠ 0) : ∑ y : ι → k, ψ (y ⬝ᵥ w) = 0

theorem **gauss_mul_conj** (ψ : AddChar k ℂ) (hψ : ψ ≠ 1) (h2 : (2 : k) ≠ 0) {B : Matrix ι ι k} (hB : B.IsSymm) (hdet : B.det ≠ 0) {t : k} (ht : t ≠ 0) : (∑ x, ψ (t * quadForm B x)) * conj (∑ x, ψ (t * quadForm B x)) = (Fintype.card k : ℂ) ^ Fintype.card ι

theorem **norm_gauss** (ψ : AddChar k ℂ) (hψ : ψ ≠ 1) (h2 : (2 : k) ≠ 0) {B : Matrix ι ι k} (hB : B.IsSymm) (hdet : B.det ≠ 0) {M : ℕ} (hM : Fintype.card ι = 2 * M) {t : k} (ht : t ≠ 0) : ‖∑ x, ψ (t * quadForm B x)‖ = (Fintype.card k : ℝ) ^ M

theorem **count_mul_card** (ψ : AddChar k ℂ) (hψ : ψ ≠ 1) (B : Matrix ι ι k) (s : k) : (Nat.card {x : ι → k // quadForm B x = s} : ℂ) * (Fintype.card k : ℂ) = ∑ t : k, ψ (-(t * s)) * ∑ x, ψ (t * quadForm B x)

theorem **card_quadric_even** (k : Type) [Field k] [Fintype k] (hchar : ringChar k ≠ 2) (ι : Type) [Fintype ι] [DecidableEq ι] (M : ℕ) (hM : Fintype.card ι = 2 * M) (hM1 : 1 ≤ M) (B : Matrix ι ι k) (hB : B.IsSymm) (hdet : B.det ≠ 0) (s : k) : |(Nat.card {x : ι → k // (2 : k)⁻¹ * (x ⬝ᵥ (B *ᵥ x)) = s} : ℝ) - (Fintype.card k : ℝ) ^ (2 * M - 1)| ≤ (Fintype.card k : ℝ) ^ M

theorem **quadric_count_even** : ∀ (k : Type) [Field k] [Fintype k], ringChar k ≠ 2 → ∀ (ι : Type) [Fintype ι] [DecidableEq ι] (M : ℕ), Fintype.card ι = 2 * M → 1 ≤ M → ∀ (B : Matrix ι ι k), B.IsSymm → B.det ≠ 0 → ∀ s : k, |(Nat.card {x : ι → k // (2 : k)⁻¹ * (x ⬝ᵥ (B *ᵥ x)) = s} : ℝ) - (Fintype.card k : ℝ) ^ (2 * M - 1)| ≤ (Fintype.card k : ℝ) ^ M

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.FiniteFieldQuadraticFormPointCountStatement

theorem **_root_.GlobalTheory.T.finite_field_quadratic_form_point_count** : ∀ (k : Type) [Field k] [Fintype k], ringChar k ≠ 2 → ∀ (ι : Type) [Fintype ι] [DecidableEq ι] (M : ℕ), Fintype.card ι = 2 * M → 1 ≤ M → ∀ (B : Matrix ι ι k), B.IsSymm → B.det ≠ 0 → ∀ s : k, |(Nat.card {x : ι → k // (2 : k)⁻¹ * (x ⬝ᵥ (B *ᵥ x)) = s} : ℝ) - (Fintype.card k : ℝ) ^ (2 * M - 1)| ≤ (Fintype.card k : ℝ) ^ M

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.FiniteVolumeBlockOneExtension

theorem **mean_lt_top** {G G₁ : Type} [Group G] [TopologicalSpace G] [IsTopologicalGroup G] [T2Space G] [LocallyCompactSpace G] [SecondCountableTopology G] [MeasurableSpace G] [BorelSpace G] [Group G₁] [TopologicalSpace G₁] [IsTopologicalGroup G₁] [T2Space G₁] [LocallyCompactSpace G₁] [SecondCountableTopology G₁] [MeasurableSpace G₁] [BorelSpace G₁] (μ : Measure G) [μ.IsHaarMeasure] (μ₁ : Measure G₁) [μ₁.IsHaarMeasure] (ι : G₁ →* G) (hι : IsClosedEmbedding ι) (Γ : Subgroup G) [Countable Γ] (Γ₁ : Subgroup G₁) (hΓ : ∀ u ∈ Γ₁, ι u ∈ Γ) (𝓕 : Set G) (h𝓕 : IsFundamentalDomain Γ 𝓕 μ) (𝓕₁ : Set G₁) (hm₁ : MeasurableSet 𝓕₁) (hfin : μ₁ 𝓕₁ < ⊤) (hcov : ∀ u, ∃ γ ∈ Γ₁, γ * u ∈ 𝓕₁) (Q : Type) (r : Q → G) (hr : ∀ q, r q ∈ Γ) (hQ : ∀ γ ∈ Γ, ∃! q, γ * (r q)⁻¹ ∈ Γ ⊓ ι.range) (f : G → ℝ≥0∞) (hf : Measurable f) (hf1 : ∀ x, f x ≤ 1) (hinv : ∀ p ∈ ι.range, ∀ x, f (p * x) = f x) (C : Set G) (hC : IsCompact C) (hsu

theorem **bc_mem_HR** (g : GL (Fin M) E) (hg : g ∈ UJ (σ : E →+* E) J) : bc (inclR (F

theorem **mat_ratH** (g : GL (Fin M) E) (hg : g ∈ UJ (σ : E →+* E) J) : HA.mat R σ J (ratH R σ J g hg) = bc (inclR (F

theorem **act_rat** (γ : HA R σ J) (g : GL (Fin M) E) (hγ : HA.mat R σ J γ = bc (inclR (F

theorem **count_lt_top** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → FinVol F E σ M J → ∀ t : F, t ≠ 0 → ∀ (μ : Measure (HA (AF F) σ (blockOne (algebraMap F E t) J))) (𝓕 : Set (HA (AF F) σ (blockOne (algebraMap F E t) J))), IsHaarH F E σ (M + 1) (blockOne (algebraMap F E t) J) μ → IsFDH F E σ (M + 1) (blockOne (algebraMap F E t) J) μ 𝓕 → ∀ Ω : Set (Fin 1 → Fin (M + 1) → AF F ⊗[F] E), IsCompact Ω → cntI σ _ μ 𝓕 {X | gramE σ (blockOne (algebraMap F E t) J) X = fun _ _ => algebraMap F E t} (Set.indicator Ω (fun _ => (1 : ℝ≥0∞))) < ⊤

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.FiniteVolumeBlockOneExtensionStatement

theorem **_root_.GlobalTheory.T.finite_volume_block_one_extension** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → FinVol F E σ M J → ∀ t : F, t ≠ 0 → ∀ (μ : Measure (HA (AF F) σ (blockOne (algebraMap F E t) J))) (𝓕 : Set (HA (AF F) σ (blockOne (algebraMap F E t) J))), IsHaarH F E σ (M + 1) (blockOne (algebraMap F E t) J) μ → IsFDH F E σ (M + 1) (blockOne (algebraMap F E t) J) μ 𝓕 → ∀ Ω : Set (Fin 1 → Fin (M + 1) → AF F ⊗[F] E), IsCompact Ω → cntI σ _ μ 𝓕 {X | gramE σ (blockOne (algebraMap F E t) J) X = fun _ _ => algebraMap F E t} (Set.indicator Ω (fun _ => (1 : ℝ≥0∞))) < ⊤

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.FiniteVolumeInductionCompactSum

theorem **hE_eq_dot** (σ : E →ₐ[F] E) {M : ℕ} (A : Matrix (Fin M) (Fin M) E) (x y : Fin M → E) : hE σ A x y = ((σ : E →+* E) ∘ x) ⬝ᵥ (A *ᵥ y)

theorem **hE_mulVec** (σ : E →ₐ[F] E) {M : ℕ} (J g : Matrix (Fin M) (Fin M) E) (x y : Fin M → E) : hE σ J (g *ᵥ x) (g *ᵥ y) = hE σ (cT (σ : E →+* E) g * J * g) x y

theorem **gramE_mulVec** (σ : E →ₐ[F] E) {M n : ℕ} (J g : Matrix (Fin M) (Fin M) E) (X : Fin n → Fin M → E) : gramE σ J (fun a => g *ᵥ X a) = gramE σ (cT (σ : E →+* E) g * J * g) X

theorem **continuous_actTup** (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [IsTopologicalRing (R ⊗[F] E)] {M n : ℕ} (g : GL (Fin M) (R ⊗[F] E)) : Continuous (actTup R g : (Fin n → Fin M → R ⊗[F] E) → _)

theorem **isCompact_preimage_actTup** (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [IsTopologicalRing (R ⊗[F] E)] {M n : ℕ} (g : GL (Fin M) (R ⊗[F] E)) {Ω : Set (Fin n → Fin M → R ⊗[F] E)} (hΩ : IsCompact Ω) : IsCompact (actTup R g ⁻¹' Ω)

theorem **finite_count** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → (∀ (M' : ℕ), M' < M → ∀ J' : Matrix (Fin M') (Fin M') E, IsHermND σ J' → FinVol F E σ M' J') → ∀ t : F, t ≠ 0 → ∀ (μ : Measure (HA (AF F) σ J)) (𝓕 : Set (HA (AF F) σ J)), IsHaarH F E σ M J μ → IsFDH F E σ M J μ 𝓕 → ∀ Ω : Set (Fin 1 → Fin M → AF F ⊗[F] E), IsCompact Ω → cntI σ J μ 𝓕 {X | gramE σ J X = fun _ _ => algebraMap F E t} (Set.indicator Ω (fun _ => (1 : ℝ≥0∞))) < ⊤

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.FiniteVolumeInductionCompactSumStatement

theorem **_root_.GlobalTheory.T.finite_volume_induction_compact_sum** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → (∀ (M' : ℕ), M' < M → ∀ J' : Matrix (Fin M') (Fin M') E, IsHermND σ J' → FinVol F E σ M' J') → ∀ t : F, t ≠ 0 → ∀ (μ : Measure (HA (AF F) σ J)) (𝓕 : Set (HA (AF F) σ J)), IsHaarH F E σ M J μ → IsFDH F E σ M J μ 𝓕 → ∀ Ω : Set (Fin 1 → Fin M → AF F ⊗[F] E), IsCompact Ω → cntI σ J μ 𝓕 {X | gramE σ J X = fun _ _ => algebraMap F E t} (Set.indicator Ω (fun _ => (1 : ℝ≥0∞))) < ⊤

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.FlowLatticeCountBounded

lemma **box_mono** {d M n : ℕ} (b : Module.Basis (Fin d) F E) (N : ℕ) {r r' : ℝ} (h : r ≤ r') : (box b N r : Set (Fin n → Fin M → AF F ⊗[F] E)) ⊆ box b N r'

lemma **sizeInf_nonneg** {d : ℕ} (b : Module.Basis (Fin d) F E) (y : Ainf F ⊗[F] E) : 0 ≤ sizeInf b y

lemma **repr_projInf_ratTup** {d M n : ℕ} (b : Module.Basis (Fin d) F E) (X : Fin n → Fin M → E) (a : Fin n) (i : Fin M) (k : Fin d) : (Algebra.TensorProduct.basis (Ainf F) b).repr (projInf (ratTup (AF F) X) a i) k = algebraMap F (Ainf F) (b.repr (X a i) k)

lemma **sizeTup_ratTup_pos** {d M n : ℕ} (b : Module.Basis (Fin d) F E) (X : Fin n → Fin M → E) (hX : X ≠ 0) : 0 < sizeTup b (projInf (ratTup (AF F) X))

lemma **exists_pos_lt** {α : Type} {S : Set α} (hS : S.Finite) (f : α → ℝ) (hf : ∀ x ∈ S, 0 < f x) : ∃ r : ℝ, 0 < r ∧ r ≤ 1 ∧ ∀ x ∈ S, r < f x

theorem **count_le** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ τ : Multiplicative ℝ →* (AF F ⊗[F] E)ˣ, IsFlow τ → ∀ (M : ℕ) (Ω : Set (Fin 1 → Fin M → AF F ⊗[F] E)), IsCompact Ω → ∃ x₀ C : ℝ, ∀ x : ℝ, ∑' X : {X : Fin 1 → Fin M → E // X ≠ 0}, (Set.indicator Ω (fun _ => (1 : ℝ≥0∞))) (((τ (Multiplicative.ofAdd (-x))) : (AF F ⊗[F] E)ˣ) : AF F ⊗[F] E) • ratTup (AF F) X.1) ≤ if x < x₀ then 0 else ENNReal.ofReal (C * Real.exp (2 * Module.finrank ℚ F * M * x))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.FlowLatticeCountBoundedStatement

theorem **_root_.GlobalTheory.T.flow_lattice_count_bounded** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ τ : Multiplicative ℝ →* (AF F ⊗[F] E)ˣ, IsFlow τ → ∀ (M : ℕ) (Ω : Set (Fin 1 → Fin M → AF F ⊗[F] E)), IsCompact Ω → ∃ x₀ C : ℝ, ∀ x : ℝ, ∑' X : {X : Fin 1 → Fin M → E // X ≠ 0}, (Set.indicator Ω (fun _ => (1 : ℝ≥0∞))) (((τ (Multiplicative.ofAdd (-x))) : (AF F ⊗[F] E)ˣ) : AF F ⊗[F] E) • ratTup (AF F) X.1) ≤ if x < x₀ then 0 else ENNReal.ofReal (C * Real.exp (2 * Module.finrank ℚ F * M * x))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.FourierCoefficientGrowthBound

theorem **norm_fstar_le_den** : ∀ (F : Type) [Field F] [NumberField F] [MeasurableSpace (A F)] [BorelSpace (A F)] (lf : Measure (A F)), lf.IsAddHaarMeasure → lf (Oh F) = 1 → ∀ ψ : AddChar (A F) ℂ, IsStdChar F ψ → ∀ (ι : Type) [Fintype ι] [DecidableEq ι] (M : ℕ) (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (P : InfinitePlace F → Matrix ι ι ℝ), QSetup M B ρ P → ∀ φ : (ι → A F) → ℂ, SB φ → ∃ C : ℝ, ∀ b : F, ‖fstar ψ (hf F) (Bf B) (muF B lf) φ (algebraMap F (A F) b)‖ ≤ C * ((den b : ℝ) ^ M)⁻¹

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.FourierCoefficientGrowthBoundStatement

theorem **_root_.GlobalTheory.T.fourier_coefficient_growth_bound** : ∀ (F : Type) [Field F] [NumberField F] [MeasurableSpace (A F)] [BorelSpace (A F)] (lf : Measure (A F)), lf.IsAddHaarMeasure → lf (Oh F) = 1 → ∀ ψ : AddChar (A F) ℂ, IsStdChar F ψ → ∀ (ι : Type) [Fintype ι] [DecidableEq ι] (M : ℕ) (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (P : InfinitePlace F → Matrix ι ι ℝ), QSetup M B ρ P → ∀ φ : (ι → A F) → ℂ, SB φ → ∃ C : ℝ, ∀ b : F, ‖fstar ψ (hf F) (Bf B) (muF B lf) φ (algebraMap F (A F) b)‖ ≤ C * ((den b : ℝ) ^ M)⁻¹

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.FourierInversionSchwartzBruhat

theorem **fou_sb** : ∀ (V : Type) [AddCommGroup V] [TopologicalSpace V] [MeasurableSpace V] (χ : V → V → ℂ) (μ : Measure V), NADual V χ μ → ∀ φ : V → ℂ, SB φ → SB (fou χ μ φ) ∧ ∀ x, fou χ μ (fou χ μ φ) x = φ (-x)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.FourierInversionSchwartzBruhatStatement

theorem **_root_.GlobalTheory.T.fourier_inversion_schwartz_bruhat** : ∀ (V : Type) [AddCommGroup V] [TopologicalSpace V] [MeasurableSpace V] (χ : V → V → ℂ) (μ : Measure V), NADual V χ μ → ∀ φ : V → ℂ, SB φ → SB (fou χ μ φ) ∧ ∀ x, fou χ μ (fou χ μ φ) x = φ (-x)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.FourierTransformLatticeIndicator

theorem **fou_indicator_coset** : ∀ (V : Type) [AddCommGroup V] [TopologicalSpace V] [MeasurableSpace V] (χ : V → V → ℂ) (μ : Measure V), NADual V χ μ → ∀ L : AddSubgroup V, IsCompact (L : Set V) → IsOpen (L : Set V) → ∀ a y : V, fou χ μ (Set.indicator {x | x - a ∈ L} (fun _ => (1 : ℂ))) y = ((μ L).toReal : ℂ) * χ a y * Set.indicator (perp χ L) (fun _ => (1 : ℂ)) y

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.FourierTransformLatticeIndicatorStatement

theorem **_root_.GlobalTheory.T.fourier_transform_lattice_indicator** : ∀ (V : Type) [AddCommGroup V] [TopologicalSpace V] [MeasurableSpace V] (χ : V → V → ℂ) (μ : Measure V), NADual V χ μ → ∀ L : AddSubgroup V, IsCompact (L : Set V) → IsOpen (L : Set V) → ∀ a y : V, fou χ μ (Set.indicator {x | x - a ∈ L} (fun _ => (1 : ℂ))) y = ((μ L).toReal : ℂ) * χ a y * Set.indicator (perp χ L) (fun _ => (1 : ℂ)) y

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.FundamentalDomainOfSubgroupFromCosetReps

theorem **unfold_coset_sum_fundamentalDomain** : ∀ (G : Type) [Group G] [TopologicalSpace G] [IsTopologicalGroup G] [T2Space G] [LocallyCompactSpace G] [SecondCountableTopology G] [MeasurableSpace G] [BorelSpace G] (μ : Measure G) [μ.IsMulLeftInvariant] (Γ Γ' : Subgroup G) [Countable Γ], Γ' ≤ Γ → ∀ 𝓕 : Set G, IsFundamentalDomain Γ 𝓕 μ → ∀ (Q : Type) (r : Q → G), (∀ q, r q ∈ Γ) → (∀ γ ∈ Γ, ∃! q, γ * (r q)⁻¹ ∈ Γ') → ∃ 𝓕' : Set G, IsFundamentalDomain Γ' 𝓕' μ ∧ ∀ f : G → ℝ≥0∞, Measurable f → ∫⁻ h in 𝓕, ∑' q, f (r q * h) ∂μ = ∫⁻ h in 𝓕', f h ∂μ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.FundamentalDomainOfSubgroupFromCosetRepsStatement

theorem **_root_.GlobalTheory.T.fundamental_domain_of_subgroup_from_coset_reps** : ∀ (G : Type) [Group G] [TopologicalSpace G] [IsTopologicalGroup G] [T2Space G] [LocallyCompactSpace G] [SecondCountableTopology G] [MeasurableSpace G] [BorelSpace G] (μ : Measure G) [μ.IsMulLeftInvariant] (Γ Γ' : Subgroup G) [Countable Γ], Γ' ≤ Γ → ∀ 𝓕 : Set G, IsFundamentalDomain Γ 𝓕 μ → ∀ (Q : Type) (r : Q → G), (∀ q, r q ∈ Γ) → (∀ γ ∈ Γ, ∃! q, γ * (r q)⁻¹ ∈ Γ') → ∃ 𝓕' : Set G, IsFundamentalDomain Γ' 𝓕' μ ∧ ∀ f : G → ℝ≥0∞, Measurable f → ∫⁻ h in 𝓕, ∑' q, f (r q * h) ∂μ = ∫⁻ h in 𝓕', f h ∂μ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.FundamentalDomainTransportCongruentForm

theorem **actTup_ratTup** {M n : ℕ} (g : GL (Fin M) E) (X : Fin n → Fin M → E) : actTup R (bc (inclR (F

theorem **actTup_mul** {M n : ℕ} (a b : GL (Fin M) (R ⊗[F] E)) (X : Fin n → Fin M → R ⊗[F] E) : actTup R (a * b) X = actTup R a (actTup R b X)

theorem **isTopologicalGroup_HA** [IsTopologicalRing R] [Module.Finite F E] (σ : E →ₐ[F] E) {M : ℕ} (J : Matrix (Fin M) (Fin M) E) : IsTopologicalGroup (HA R σ J)

theorem **borelSpace_HA** (σ : E →ₐ[F] E) {M : ℕ} (J : Matrix (Fin M) (Fin M) E) : BorelSpace (HA R σ J)

theorem **GlobalTheory.CompanionTheory.SW.FTransfer.cntI_transfer** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (g : GL (Fin M) E) (J' : Matrix (Fin M) (Fin M) E), J' = cT (σ : E →+* E) (g : Matrix (Fin M) (Fin M) E) * J * (g : Matrix (Fin M) (Fin M) E) → ∀ (μ : Measure (HA (AF F) σ J)) (𝓕 : Set (HA (AF F) σ J)), IsHaarH F E σ M J μ → IsFDH F E σ M J μ 𝓕 → ∃ (μ' : Measure (HA (AF F) σ J')) (𝓕' : Set (HA (AF F) σ J')), IsHaarH F E σ M J' μ' ∧ IsFDH F E σ M J' μ' 𝓕' ∧ μ' 𝓕' = μ 𝓕 ∧ ∀ (n : ℕ) (S : Set (Fin n → Fin M → E)) (Φ : (Fin n → Fin M → AF F ⊗[F] E) → ℝ≥0∞), cntI σ J μ 𝓕 S Φ = cntI σ J' μ' 𝓕' {X | (fun a => (g : Matrix (Fin M) (Fin M) E) *ᵥ X a) ∈ S} (fun Y => Φ (actTup (AF F) (bc (inclR (AF F)) g) Y))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.FundamentalDomainTransportCongruentFormStatement

theorem **_root_.GlobalTheory.T.fundamental_domain_transport_congruent_form** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (g : GL (Fin M) E) (J' : Matrix (Fin M) (Fin M) E), J' = cT (σ : E →+* E) (g : Matrix (Fin M)

(Fin M) E) ∗ J ∗ (g : Matrix (Fin M) (Fin M) E) → ∀ (μ : Measure (HA (AF F) σ J)) (ℱ : Set (HA (AF F) σ J)), IsHaarH F E σ M J μ → IsFDH F E σ M J μ ℱ → ∃ (μ' : Measure (HA (AF F) σ J')) (ℱ' : Set (HA (AF F) σ J')), IsHaarH F E σ M J' μ' ∧ IsFDH F E σ M J' μ' ℱ' ∧ μ' ℱ' = μ ℱ ∧ ∀ (n : ℕ) (S : Set (Fin n → Fin M → E)) (Φ : (Fin n → Fin M → AF F ⊗[F] E) → ℝ≥0∞), cntI σ J μ ℱ S Φ = cntI σ J' μ' ℱ' {X | (fun a => (g : Matrix (Fin M) (Fin M) E) *ᵥ X a) ∈ S} (fun Y => Φ (actTup (AF F) (bc (inclR (AF F)) g) Y))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.GaussianChangeOfVariablesIsometry

theorem **quad_mulVec** {ι : Type} [Fintype ι] [DecidableEq ι] (g M : Matrix ι ι ℝ) (x : ι → ℝ) : (g *ᵥ x) ⬝ᵥ (M *ᵥ (g *ᵥ x)) = x ⬝ᵥ ((gᵀ * M * g) *ᵥ x)

theorem **abs_det_eq_one** {ι : Type} [Fintype ι] [DecidableEq ι] {g M : Matrix ι ι ℝ} (hM : M.det ≠ 0) (h : gᵀ * M * g = M) : |g.det| = 1

theorem **measurePreserving_mulVec** {ι : Type} [Fintype ι] [DecidableEq ι] {g : Matrix ι ι ℝ} (h : |g.det| = 1) : MeasurePreserving (fun x : ι → ℝ => g *ᵥ x) volume volume

theorem **measurableEmbedding_mulVec** {W ι : Type} [Fintype W] [Fintype ι] [DecidableEq ι] {g : W → Matrix ι ι ℝ} (h : ∀ w, (g w).det ≠ 0) : MeasurableEmbedding (fun (x : W → ι → ℝ) (w : W) => g w *ᵥ x w)

theorem **gauss_orth** {F : Type} [Field F] [NumberField F] {ι : Type} [Fintype ι] [DecidableEq ι] (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (P : InfinitePlace F → Matrix ι ι ℝ) (gi : InfinitePlace F → Matrix ι ι ℝ) (hB : B.det ≠ 0) (hg : ∀ w, (gi w)ᵀ * B.map (ρ w) * gi w = B.map (ρ w)) : (∀ (τ : InfinitePlace F → ℂ) (x : InfinitePlace F → ι → ℝ), gauss B ρ P τ (fun w => gi w *ᵥ x w) = gauss B ρ (fun w => (gi w)ᵀ * P w * gi w) τ x) ∧ ∀ τ : InfinitePlace F → ℂ, cG B ρ (fun w => (gi w)ᵀ * P w * gi w) τ = cG B ρ P τ

theorem **GlobalTheory.CompanionTheory.SW.ThW.gauss_orth_closed** : ∀ (F : Type) [Field F] [NumberField F] (ι : Type) [Fintype ι] [DecidableEq ι] (B : Matrix ι ι F) (ρ : NumberField.InfinitePlace F → (F →+* ℝ)) (P : NumberField.InfinitePlace F → Matrix ι ι ℝ) (gi : NumberField.InfinitePlace F → Matrix ι ι ℝ), B.det ≠ 0 → (∀ w, (gi w).transpose ∗ B.map (ρ w) ∗ gi w = B.map (ρ w)) → (∀ (τ : NumberField.InfinitePlace F → ℂ) (x : NumberField.InfinitePlace F → ι → ℝ), GlobalTheory.CompanionTheory.SW.Core.gauss B ρ P τ (fun w => (gi w).mulVec (x w)) = GlobalTheory.CompanionTheory.SW.Core.gauss B ρ (fun w => (gi w).transpose ∗ P w ∗ gi w) τ x) ∧ ∀ τ : NumberField.InfinitePlace F → ℂ, GlobalTheory.CompanionTheory.SW.Core.cG B ρ (fun w => (gi w).transpose ∗ P w ∗ gi w) τ = GlobalTheory.CompanionTheory.SW.Core.cG B ρ P τ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.GaussianChangeOfVariablesIsometryStatement

theorem **_root_.GlobalTheory.T.gaussian_change_of_variables_isometry** : ∀ (F : Type) [Field F] [NumberField F] (ι : Type) [Fintype ι] [DecidableEq ι] (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (P : InfinitePlace F → Matrix ι ι ℝ) (gi : InfinitePlace F → Matrix ι ι ℝ), B.det ≠ 0 → (∀ w, (gi w)ᵀ * B.map (ρ w) * gi w = B.map (ρ w)) → (∀ (τ : InfinitePlace F → ℂ) (x : InfinitePlace F → ι → ℝ), gauss B ρ P τ (fun w => gi w *ᵥ x w) = gauss B ρ (fun w => (gi w)ᵀ * P w * gi w) τ x) ∧ ∀ τ : InfinitePlace F → ℂ, cG B ρ (fun w => (gi w)ᵀ * P w * gi w) τ = cG B ρ P τ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.GaussianContinuousRapidDecay

theorem **exists_lower_sup** {n : Type} [Fintype n] [DecidableEq n] {P : Matrix n n ℝ} (hP : P.PosDef) : ∃ c : ℝ, 0 < c ∧ ∀ x : n → ℝ, c * ‖x‖ ^ 2 ≤ x ⬝ᵥ (P *ᵥ x)

theorem **posdef_quadratic_lower_bound_l1** {n : Type} [Fintype n] [DecidableEq n] {P : Matrix n n ℝ} (hP : P.PosDef) : ∃ c : ℝ, 0 < c ∧ ∀ x : n → ℝ, c * (∑ p, |x p|) ^ 2 ≤ x ⬝ᵥ (P *ᵥ x)

theorem **exists_lower_family** {ι n : Type} [Fintype ι] [Fintype n] [DecidableEq n] {P : ι → Matrix n n ℝ} (hP : ∀ i, (P i).PosDef) : ∃ c : ℝ, 0 < c ∧ ∀ y : ι → n → ℝ, c * (∑ i, ∑ p, |y i p|) ^ 2 ≤ ∑ i, y i ⬝ᵥ (P i *ᵥ y i)

theorem **exp_mul_pow_le** {a : ℝ} (ha : 0 < a) (N : ℕ) {s : ℝ} (hs : 0 ≤ s) {q : ℝ} (hq : a * s ^ 2 ≤ q) : Real.exp (-q) * (1 + s) ^ N ≤ Real.exp ((N : ℝ) ^ 2 / (4 * a))

theorem **sizeTup_eq_sum_xw** {F E : Type} [Field F] [NumberField F] [Field E] [Algebra F E] {M d : ℕ} (b : Module.Basis (Fin d) F E) (e : Ainf F ≃+* (InfinitePlace F → ℝ)) (he : ∀ y w, |e y w| = ‖y w‖) (X : Fin 1 → Fin M → Ainf F ⊗[F] E) : sizeTup b X = ∑ w, ∑ p, |xw e b X w p|

theorem **gaussQ_continuous_rapidDecay** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M d : ℕ) (b : Module.Basis (Fin d) F E) (e : Ainf F ≃+* (InfinitePlace F → ℝ)), IsHomeomorph e → (∀ y w, |e y w| = ‖y w‖) → ∀ (Pm : InfinitePlace F → Matrix (Fin M × Fin d) (Fin M × Fin d) ℝ), Continuous (gaussQ e b Pm) ∧ ((∀ w, (Pm w).PosDef) → RapidDecay b (fun X => (gaussQ e b Pm X : ℂ)))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.GaussianContinuousRapidDecayStatement

theorem **_root_.GlobalTheory.T.gaussian_continuous_rapid_decay** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M d : ℕ) (b : Module.Basis (Fin d) F E) (e : Ainf F ≃+* (InfinitePlace F → ℝ)), IsHomeomorph e → (∀ y w, |e y w| = ‖y w‖) → ∀ (Pm : InfinitePlace F → Matrix (Fin M × Fin d) (Fin M × Fin d) ℝ), Continuous (gaussQ e b Pm) ∧ ((∀ w, (Pm w).PosDef) → RapidDecay b (fun X => (gaussQ e b Pm X : ℂ)))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.GaussianFourierTransformFormula

theorem **swapCurry_measurePreserving** (α β : Type) [Fintype α] [Fintype β] : MeasurePreserving (swapCurry α β) volume volume

theorem **integral_cur** (f : (InfinitePlace F → ι → ℝ) → ℂ) : ∫ x : ι × InfinitePlace F → ℝ, f (cur x) = ∫ z, f z

theorem **dot_DM** [DecidableEq (InfinitePlace F)] (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (x y : ι × InfinitePlace F → ℝ) : x ⬝ᵥ (DM B ρ *ᵥ y) = ∑ w, cur x w ⬝ᵥ (B.map (ρ w) *ᵥ cur y w)

theorem **gauss_fourier_flat** : ∀ (F : Type) [Field F] [NumberField F] (ι : Type) [Fintype ι] [DecidableEq ι] [DecidableEq (InfinitePlace F)] (M : ℕ) (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (P : InfinitePlace F → Matrix ι ι ℝ), QSetup M B ρ P → ∀ τ : InfinitePlace F → ℂ, (∀ w, 0 < (τ w).im) → ∀ y : ι × InfinitePlace F → ℝ, ((Real.sqrt |((Algebra.norm ℚ B.det : ℚ) : ℝ)| : ℝ) : ℂ) * ∫ x : ι × InfinitePlace F → ℝ, gauss B ρ P τ (cur x) * Complex.exp (2 * (Real.pi : ℂ) * Complex.I * ((x ⬝ᵥ (DM B ρ *ᵥ y) : ℝ) : ℂ)) = cG B ρ P τ * gauss B ρ P (fun w => -(τ w)⁻¹) (cur y)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.GaussianFourierTransformFormulaStatement

theorem **_root_.GlobalTheory.T.gaussian_fourier_transform_formula** : ∀ (F : Type) [Field F] [NumberField F] (ι : Type) [Fintype ι] [DecidableEq ι] [DecidableEq (InfinitePlace F)] (M : ℕ) (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (P : InfinitePlace F → Matrix ι ι ℝ), QSetup M B ρ P → ∀ τ : InfinitePlace F → ℂ, (∀ w, 0 < (τ w).im) → ∀ y : ι × InfinitePlace F → ℝ, ((Real.sqrt |((Algebra.norm ℚ B.det : ℚ) : ℝ)| : ℝ) : ℂ) * ∫ x : ι × InfinitePlace F → ℝ, gauss B ρ P τ (cur x) * Complex.exp (2 * (Real.pi : ℂ) * Complex.I * ((x ⬝ᵥ (DM B ρ *ᵥ y) : ℝ) : ℂ)) = cG B ρ P τ * gauss B ρ P (fun w => -(τ w)⁻¹) (cur y)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.GaussianSumOfCounting

lemma **wt_nonneg** (c : ℝ) (j : ℕ) : 0 ≤ wt c j

lemma **exists_dyadic** {c u : ℝ} (hc : 0 < c) (hu : 0 ≤ u) : ∃ j : ℕ, u ≤ 2 ^ j ∧ Real.exp (-(c * u ^ 2)) ≤ wt c j

lemma **summable_wt** {c : ℝ} (hc : 0 < c) (n : ℕ) : Summable fun j : ℕ => wt c j * ((4 : ℝ) ^ n) ^ j

lemma **radius_bound** (n j : ℕ) {t₀ t : ℝ} (ht₀ : 0 < t₀) (ht : t₀ ≤ t) : (2 ^ j / Real.sqrt t) ^ n + (2 ^ j / Real.sqrt t) ^ (2 * n) ≤ ((4 : ℝ) ^ n) ^ j * ((1 + (Real.sqrt t₀ ^ n)⁻¹) * (Real.sqrt t ^ n)⁻¹)

lemma **gaussian_le_tsum** {X : Type} (c t : ℝ) (hc : 0 < c) (ht : 0 < t) (s : X → ℝ) (p : X → Prop) (hs : ∀ x, 0 ≤ s x) (x : X) : Set.indicator {x | p x} (fun x => ENNReal.ofReal (Real.exp (-(c * t * s x ^ 2)))) x ≤ ∑' j : ℕ, ENNReal.ofReal (wt c j) * Set.indicator {x | p x ∧ s x ≤ 2 ^ j / Real.sqrt t} (fun _ => (1 : ℝ≥0∞)) x

theorem **gaussian_sum_le** (n : ℕ) (c t₀ C : ℝ) (hc : 0 < c) (ht₀ : 0 < t₀) : ∀ (X Ω : Type) [MeasurableSpace Ω] [Countable X] (Pr : Measure Ω) (s : Ω → X → ℝ) (p : Ω → X → Prop), (∀ x, Measurable fun ω => s ω x) → (∀ x, MeasurableSet {ω | p ω x}) → (∀ ω x, 0 ≤ s ω x) → (∀ R : ℝ, 0 < R → ∫⁻ ω, ∑' x, Set.indicator {x | p ω x ∧ s ω x ≤ R} (fun _ => (1 : ℝ≥0∞)) x ∂Pr ≤ ENNReal.ofReal (C * (R ^ n + R ^ (2 * n)))) → ∀ t : ℝ, t₀ ≤ t → ∫⁻ ω, ∑' x, Set.indicator {x | p ω x} (fun x => ENNReal.ofReal (Real.exp (-(c * t * s ω x ^ 2)))) x ∂Pr ≤ ENNReal.ofReal ((|C| * (1 + (Real.sqrt t₀ ^ n)⁻¹) * ∑' j : ℕ, wt c j * ((4 : ℝ) ^ n) ^ j) * (Real.sqrt t ^ n)⁻¹)

theorem **gaussian_sum_of_counting** : ∀ (n : ℕ) (c t₀ C : ℝ), 0 < c → 0 < t₀ → ∃ C' : ℝ, ∀ (X Ω : Type) [MeasurableSpace Ω] [Countable X] (Pr : Measure Ω) (s : Ω → X → ℝ) (p : Ω → X → Prop), (∀ x, Measurable fun ω => s ω x) → (∀ x, MeasurableSet {ω | p ω x}) → (∀ ω x, 0 ≤ s ω x) → (∀ R : ℝ, 0 < R → ∫⁻ ω, ∑' x, Set.indicator {x | p ω x ∧ s ω x ≤ R} (fun _ => (1 : ℝ≥0∞)) x ∂Pr ≤ ENNReal.ofReal (C * (R ^ n + R ^ (2 * n)))) → ∀ t : ℝ, t₀ ≤ t → ∫⁻ ω, ∑' x, Set.indicator {x | p ω x} (fun x => ENNReal.ofReal (Real.exp (-(c * t * s ω x ^ 2)))) x ∂Pr ≤ ENNReal.ofReal (C' * (Real.sqrt t ^ n)⁻¹)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.GeomSumIntLe

theorem **geom_sum_int_le** (D M : ℕ) (hD : 0 < D) (hM : 1 ≤ M) (x₀ C : ℝ) : ∃ C' : ℝ, ∀ r : ℝ, 0 < r → ∑' j : ℤ, (if (j : ℝ) + Real.log r < x₀ then 0 else ENNReal.ofReal (C * Real.exp (2 * D * ((j : ℝ) + Real.log r)))) * ENNReal.ofReal (Real.exp (-(2 * D * (M + 1) * (j : ℝ)))) ≤ ENNReal.ofReal (C' * r ^ (2 * D * (M + 1)))

theorem **geom_sum_int** : ∀ (D M : ℕ), 0 < D → 1 ≤ M → ∀ (x₀ C : ℝ), ∃ C' : ℝ, ∀ r : ℝ, 0 < r → ∑' j : ℤ, (if (j : ℝ) + Real.log r < x₀ then 0 else ENNReal.ofReal (C * Real.exp (2 * D * ((j : ℝ) + Real.log r)))) * ENNReal.ofReal (Real.exp (-(2 * D * (M + 1) * (j : ℝ)))) ≤ ENNReal.ofReal (C' * r ^ (2 * D * (M + 1)))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.GermConstantsEqAlongAffineSpaceSquare

theorem **hform_eq_dot** (x y : n → A) : hform σ J x y = (σ ∘ x) ⬝ᵥ J *ᵥ y

theorem **comp_mulVec'** (P : Matrix n n A) (x : n → A) : σ ∘ (P *ᵥ x) = P.map σ *ᵥ (σ ∘ x)

theorem **hform_mulVec'** (P Q : Matrix n n A) (x y : n → A) : hform σ J (P *ᵥ x) (Q *ᵥ y) = hform σ (cT σ P * J * Q) x y

theorem **hform_sub_left** (x d y : n → A) (c : A) : hform σ J (x - c • d) y = hform σ J x y - σ c * hform σ J d y

theorem **hform_sub_right** (x d y : n → A) (c : A) : hform σ J x (y − c • d) = hform σ J x y − c ∗ hform σ J x d

theorem **hform_smul_right'** (d y : n → A) (c : A) : hform σ J y (c • d) = c ∗ hform σ J y d

theorem **hform_conj_symm** (hσ : ∀ a, σ (σ a) = a) (hJ : ∀ i j, σ (J j i) = J i j) (x y : n → A) : σ (hform σ J x y) = hform σ J y x

theorem **qrMat_mulVec** (d : n → A) (c : A) (z : n → A) : qrMat σ J d c *ᵥ z = z − (c ∗ hform σ J d z) • d

theorem **ext_mulVec'** {P Q : Matrix n n A} (h : ∀ z, P *ᵥ z = Q *ᵥ z) : P = Q

theorem **qrMat_mul** (d : n → A) {c c' : A} (hc : c + c' = c ∗ c' ∗ hform σ J d d) : qrMat σ J d c ∗ qrMat σ J d c' = 1

theorem **hform_single'** (i j : n) : hform σ J (Pi.single i 1) (Pi.single j 1) = J i j

theorem **qrMat_isometry** (hσ : ∀ a, σ (σ a) = a) (hJ : ∀ i j, σ (J j i) = J i j) (d : n → A) (c : A) (hc : c + σ c = c ∗ σ c ∗ hform σ J d d) : cT σ (qrMat σ J d c) ∗ J ∗ qrMat σ J d c = J

theorem **exists_unitary_carry** (A : Type) [CommRing A] (σ : A →+* A) (hσ : ∀ y, σ (σ y) = y) (M : ℕ) (J : Matrix (Fin M) (Fin M) A) (hJ : ∀ i j, σ (J j i) = J i j) (x y : Fin M → A) (ε : A) (hxy : hform σ J x x = hform σ J y y) (hβ : IsUnit (hform σ J x x)) (hε : ε ∗ σ ε = 1) (hu : IsUnit (hform σ J x x − σ ε ∗ hform σ J y x)) : ∃ g ∈ UJ σ J, (g : Matrix (Fin M) (Fin M) A) *ᵥ x = y ∧ ∀ z, hform σ J x z = 0 → hform σ J y z = 0 → (g : Matrix (Fin M) (Fin M) A) *ᵥ z = z

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.GermFormPulledBackCombinationsExpandMonomials

theorem **continuousOn_integral_of_bounded** {V X P : Type} [TopologicalSpace V] [T2Space V] [MeasurableSpace V] [BorelSpace V] [TopologicalSpace X] [LocallyCompactSpace X] [TopologicalSpace P] (fib : X → Measure V) (B : Set X) (hB : IsOpen B) (hloc : ∀ b, IsLocallyFiniteMeasure (fib b)) (hbd : ∀ (C : Set V) (D : Set X), IsCompact C → IsCompact D → D ⊆ B → ∃ c : ℝ≥0, ∀ b ∈ D, fib b C ≤ c) (hcont : ∀ φ : V → ℝ, Continuous φ → HasCompactSupport φ → ContinuousOn (fun b => ∫ v, φ v ∂(fib b)) B) (W : Set P) (Φ : P → V → ℝ) (hΦ : ContinuousOn (fun p : P × V => Φ p.1 p.2) (W ×ˢ Set.univ)) (hW : IsOpen W) (C : Set V) (hC : IsCompact C) (hsupp : ∀ p ∈ W, ∀ v, v ∉ C → Φ p v = 0) : ContinuousOn (fun q : P × X => ∫ v, Φ q.1 v ∂(fib q.2)) (W ×ˢ B)

theorem **continuousOn_fibre_integral** : ∀ (V X Y : Type) [TopologicalSpace V] [LocallyCompactSpace V] [SecondCountableTopology V] [T2Space V] [MeasurableSpace V] [BorelSpace V] [TopologicalSpace X] [LocallyCompactSpace X] [SecondCountableTopology X] [T2Space X] [MeasurableSpace X] [BorelSpace X] (μ : Measure V) (ν : Measure X) [IsLocallyFiniteMeasure μ] [IsLocallyFiniteMeasure ν] [ν.IsOpenPosMeasure] (f : V → Y) (g : V → X) (ι : X → Y), Function.Injective ι → Continuous g → (∀ v, f v = ι (g v)) → ∀ (B : Set X), IsOpen B → ∀ fib : X → Measure V, Formalism.MainNotions.GlobalTheoryTheory.GenusMassVolumes.IsFibreFamily μ f ι (g ⁻¹' B) ν B fib → ∀ (P : Type) [TopologicalSpace P] (W : Set P) (Φ : P → V → ℝ), ContinuousOn (fun p : P × V => Φ p.1 p.2) (W ×ˢ Set.univ) → IsOpen W → ∀ C : Set V, IsCompact C → (∀ p ∈ W, ∀ v, v ∉ C → Φ p v = 0) → ContinuousOn (fun q : P × X => ∫ v, Φ q.1 v ∂(fib q.2))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.GlobalProductGramFiberRankOne

theorem **prod_close** {ι : Type} (S : Finset ι) (a : ι → ℝ) {ε : ℝ} (hε : 0 < ε) : ∃ δ > 0, ∀ r : ι → ℝ, (∀ v ∈ S, |r v − a v| ≤ δ) → |∏ v ∈ S, r v − ∏ v ∈ S, a v| ≤ ε

theorem **avg_bound** {α : Type} [MeasurableSpace α] (μ : Measure α) {U : Set α} (hU : MeasurableSet U) (hfin : μ U ≠ ∞) (D : α → ℝ) {c ε : ℝ} (hc : 0 ≤ c) (hε : 0 ≤ ε) (hD : ∀ y ∈ U, |D y − c| ≤ ε) : |(∫⁻ y in U, ENNReal.ofReal (D y) ∂μ).toReal − c ∗ (μ U).toReal| ≤ ε ∗ (μ U).toReal

theorem **isOpen_int** (v : HeightOneSpectrum (𝒪 F)) : IsOpen (v.adicCompletionIntegers F : Set (Fv F v))

theorem **exists_box** (x : Af F) {W : Set (Af F)} (hW : W ∈ 𝒩 x) : ∃ (S₀ : Finset (HeightOneSpectrum (𝒪 F))) (N : ∀ v : HeightOneSpectrum (𝒪 F), Set (Fv F v)), (∀ v, N v ∈ 𝒩 (x v)) ∧ (∀ v, v ∉ S₀ → N v = (v.adicCompletionIntegers F : Set (Fv F v))) ∧ ∀ y : Af F, (∀ v, y v ∈ N v) → y ∈ W

theorem **trace_one_tmul'** [FiniteDimensional F E] (R : Type) [CommRing R] [Algebra F R] (z : E) : Algebra.trace R (R ⊗[F] E) ((1 : R) ⊗ₜ[F] z) = algebraMap F R (Algebra.trace F E z)

theorem **trace_tmul'** [FiniteDimensional F E] {R : Type} [CommRing R] [Algebra F R] (r : R) (e : E) : Algebra.trace R (R ⊗[F] E) (r ⊗ₜ[F] e) = r ∗ algebraMap F R (Algebra.trace F E e)

theorem **trace_map** [FiniteDimensional F E] (φ : R →ₐ[F] R') (y : R ⊗[F] E) : φ (Algebra.trace R (R ⊗[F] E) y) = Algebra.trace R' (R' ⊗[F] E) (Algebra.TensorProduct.map φ (AlgHom.id F E) y)

theorem **map_sigR** (σ : E →ₐ[F] E) (φ : R →ₐ[F] R') (z : R ⊗[F] E) : Algebra.TensorProduct.map φ (AlgHom.id F E) (sigR R σ z) = sigR R' σ (Algebra.TensorProduct.map φ (AlgHom.id F E) z)

theorem **map_gram** (σ : E →ₐ[F] E) {M n : ℕ} (J : Matrix (Fin M) (Fin M) E) (φ : R →ₐ[F] R') (X : Fin n → Fin M → R ⊗[F] E) (a c : Fin n) : Algebra.TensorProduct.map φ (AlgHom.id F E) (gramR R σ J X a c) = gramR R' σ J (fun a i => Algebra.TensorProduct.map φ (AlgHom.id F E) (X a i)) a c

theorem **map_qF** [FiniteDimensional F E] (σ : E →ₐ[F] E) {M : ℕ} (J : Matrix (Fin M) (Fin M) E) (φ : R →ₐ[F] R') (X : Fin 1 → Fin M → R ⊗[F] E) : φ (qF R σ J X) = qF R' σ J (fun a i => Algebra.TensorProduct.map φ (AlgHom.id F E) (X a i))

theorem **sig_expand** {d : ℕ} (b : Module.Basis (Fin d) F E) (σ : E →ₐ[F] E) (p q : R ⊗[F] E) (e : E) : sigR R σ p ∗ inclR R e ∗ q = ∑ k, ∑ l, ((Algebra.TensorProduct.basis R b).repr p k ∗ (Algebra.TensorProduct.basis R b).repr q l) ⊗ₜ[F] (σ (b k) ∗ e ∗ b l)

theorem **q_expand** [FiniteDimensional F E] {d M : ℕ} (b : Module.Basis (Fin d) F E) (σ : E →ₐ[F] E) (J : Matrix (Fin M) (Fin M) E) (X : Fin 1 → Fin M → R ⊗[F] E) : qF R σ J X = ∑ i, ∑ j, ∑ k, ∑ l, (Algebra.TensorProduct.basis R b).repr (X 0 i) k ∗ (Algebra.TensorProduct.basis R b).repr (X 0 j) l ∗ algebraMap F R (1 / 2 ∗ Algebra.trace F E (σ (b k) ∗ J i j ∗ b l))

theorem **scale_repr** {d : ℕ} (b : Module.Basis (Fin d) F E) (a : ℕ) (y : R ⊗[F] E) (k : Fin d) : (Algebra.TensorProduct.basis R b).repr ((a : R ⊗[F] E) ∗ y) k = (a : R) ∗ (Algebra.TensorProduct.basis R b).repr y k

theorem **int_ev** (c : F) : ∀ᶠ v : HeightOneSpectrum (𝒪 F) in Filter.cofinite, algebraMap F (Fv F v) c ∈ v.adicCompletionIntegers F

theorem **q_int_ev** [FiniteDimensional F E] {d M : ℕ} (b : Module.Basis (Fin d) F E) (σ : E →ₐ[F] E) (J : Matrix (Fin M) (Fin M) E) : ∀ᶠ v : HeightOneSpectrum (𝒪 F) in Filter.cofinite, ∀ X : Fin 1 → Fin M → Fv F v ⊗[F] E, X ∈ latV b v → qF (Fv F v) σ J X ∈ v.adicCompletionIntegers F

theorem **scale_ev** {d M : ℕ} (b : Module.Basis (Fin d) F E) {a : ℕ} (ha : 0 < a) : ∀ᶠ v : HeightOneSpectrum (𝒪 F) in Filter.cofinite, ∀ X : Fin 1 → Fin M → Fv F v ⊗[F] E, X ∈ sA b v a ↔ X ∈ latV b v

theorem **nice_nbhd** {v : HeightOneSpectrum (𝒪 F)} {lam : Measure (Fv F v)} (h : IsStdV v lam) {s : Fv F v} {n : Set (Fv F v)} (hn : n ∈ 𝒩 s) : ∃ V : Set (Fv F v), IsOpen V ∧ V ⊆ n ∧ 0 < lam V ∧ lam V < ⊤

theorem **sA_cpt_open** (σ : E →ₐ[F] E) (hcm : IsCMExt F E σ) {d M : ℕ} (b : Module.Basis (Fin d) F E) (v : HeightOneSpectrum (𝒪 F)) {a : ℕ} (ha : 0 < a) : IsCompact (sA (M

theorem **qF_cont** (σ : E →ₐ[F] E) (hcm : IsCMExt F E σ) {M : ℕ} (J : Matrix (Fin M) (Fin M) E) (v : HeightOneSpectrum (𝒪 F)) : Continuous (qF (Fv F v) σ J : (Fin 1 → Fin M → Fv F v ⊗[F] E) → Fv F v)

theorem **box_open** (S : Finset (HeightOneSpectrum (𝒪 F))) (V : ∀ v : HeightOneSpectrum (𝒪 F), Set (Fv F v)) (hV : ∀ v, IsOpen (V v)) : IsOpen (box S V)

theorem **box_meas** (lam : ∀ v : HeightOneSpectrum (𝒪 F), Measure (Fv F v)) (hlam : ∀ v, IsStdV v (lam v)) (lf : Measure (Af F)) (hlf : lf.IsAddHaarMeasure) (hlf1 : lf {x | ∀ v, x v ∈ v.adicCompletionIntegers F} = 1) (S : Finset (HeightOneSpectrum (𝒪 F))) (V : ∀ v : HeightOneSpectrum (𝒪 F), Set (Fv F v)) (hV : ∀ v, IsOpen (V v)) : lf (box S V) = ∏ v ∈ S, lam v (V v)

theorem **box_tuple** (σ : E →ₐ[F] E) (hcm : IsCMExt F E σ) {M : ℕ} (J : Matrix (Fin M) (Fin M) E) {d : ℕ} (b : Module.Basis (Fin d) F E) (lam : ∀ v : HeightOneSpectrum (𝒪 F), Measure (Fv F v)) (hlam : ∀ v, IsStdV v (lam v)) (lf : Measure (Af F)) (hlf : lf.IsAddHaarMeasure) (hlf1 : lf {x | ∀ v, x v ∈ v.adicCompletionIntegers F} = 1) {a : ℕ} (ha : 0 < a) (S : Finset (HeightOneSpectrum (𝒪 F))) (V : ∀ v : HeightOneSpectrum (𝒪 F), Set (Fv F v)) (hV : ∀ v, IsOpen (V v)) (hun : ∀ v, v ∉ S → ∀ X : Fin 1 → Fin M → Fv F v ⊗[F] E, X ∈ sA b v a ↔ X ∈ latV b v) (hint : ∀ v, v ∉ S → ∀ X : Fin 1 → Fin M → Fv F v ⊗[F] E, X ∈ latV b v → qF (Fv F v) σ J X ∈ v.adicCompletionIntegers F) : coordW (Af F) b lf 1 M {X | (∀ v, projV v X ∈ sA b v a) ∧ qF (Af F) σ J X ∈ box S V} = ∏ v ∈ S, coordW (Fv F v) b (lam v) 1 M (sA b v a ∩ {X | qF (Fv F v) σ J X ∈ V v})

theorem **hasProd_fibre_mass** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (lam : ∀ v : HeightOneSpectrum (𝒪 F), Measure (Fv F v)), (∀ v, IsStdV v (lam v)) → ∀ (m : ∀ v : HeightOneSpectrum (𝒪 F), HermR (Fv F v) σ 1 → Measure (Fin 1 → Fin M → Fv F v ⊗[F] E)), (∀ v, IsGramFib (Fv F v) σ J 1 (coordW (Fv F v) b (lam v) 1 M) (coordHerm (Fv F v) σ b (lam v) 1) (m v)) → ∀ (lf : Measure (Af F)), lf.IsAddHaarMeasure → lf {x | ∀ v, x v ∈ v.adicCompletionIntegers F} = 1 → ∀ a : ℕ, 0 < a → ∀ D : Af F → ℝ, Continuous D → (∀ y, 0 ≤ D y) → (∀ U : Set (Af F), MeasurableSet U → coordW (Af F) b lf 1 M {X | (∀ v, projV v X ∈ {X : (Fin 1 → Fin M → Fv F v ⊗[F] E) | (fun p i => (a : Fv F v ⊗[F] E) ∗ X p i) ∈ latV b v}) ∧ algebraMap F (Af F) (1 /

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.GlobalProductGramFiberRankOneStatement

theorem **_root_.GlobalTheory.T.global_product_gram_fiber_rank_one** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (lam : ∀ v : HeightOneSpectrum (𝒪 F), Measure (Fv F v)), (∀ v, IsStdV v (lam v)) → ∀ (m : ∀ v : HeightOneSpectrum (𝒪 F), HermR (Fv F v) σ 1 → Measure (Fin 1 → Fin M → Fv F v ⊗[F] E)), (∀ v, IsGramFib (Fv F v) σ J 1 (coordW (Fv F v) b (lam v) 1 M) (coordHerm (Fv F v) σ b (lam v) 1) (m v)) → ∀ (lf : Measure (Af F)), lf.IsAddHaarMeasure → lf {x | ∀ v, x v ∈ v.adicCompletionIntegers F} = 1 → ∀ a : ℕ, 0 < a → ∀ D : Af F → ℝ, Continuous D → (∀ y, 0 ≤ D y) → (∀ U : Set (Af F), MeasurableSet U → coordW (Af F) b lf 1 M {X | (∀ v, projV v X ∈ {X : (Fin 1 → Fin M → Fv F v ⊗[F] E) | (fun p i => (a : Fv F v ⊗[F] E) ∗ X p i) ∈

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.GlobalUnitScalingHaarProductFormula

theorem **map_mul_left_infinite** {m : MeasurableSpace (InfiniteAdeleRing F)} (hm : @BorelSpace _ _ m) (b : (InfiniteAdeleRing F)ˣ) (μ : Measure (InfiniteAdeleRing F)) (hμ : μ.IsAddHaarMeasure) : μ.map (fun y => (b : InfiniteAdeleRing F) * y) = ENNReal.ofReal ‖((b⁻¹ : (InfiniteAdeleRing F)ˣ) : InfiniteAdeleRing F)‖ • μ

theorem **norm_algebraMap_infinite** (a : F) : ‖algebraMap F (InfiniteAdeleRing F) a‖ = ∏ w : InfinitePlace F, w a ^ w.mult

theorem **distribHaarChar_eq_nnnorm** (v : HeightOneSpectrum (𝒪 F)) (a : Fv F v) (ha : a ≠ 0) : distribHaarChar (Fv F v) (Units.mk0 a ha) = ‖a‖₊

theorem **map_mul_left_finite** (v : HeightOneSpectrum (𝒪 F)) (a : Fv F v) (ha : a ≠ 0) (μ : Measure (Fv F v)) (hμ : μ.IsAddHaarMeasure) : μ.map (fun x => a * x) = (‖a‖₊)⁻¹ • μ

theorem **unitProd_scalar** : ∀ (F : Type) [Field F] [NumberField F] (a : F), a ≠ 0 → ∃ (cinf : ℝ≥0) (c : HeightOneSpectrum (𝒪 F) → ℝ≥0), IsUnitProd cinf c ∧ (∀ μ : Measure (Ainf F), μ.IsAddHaarMeasure → μ.map (fun x => algebraMap F (Ainf F) a * x) = cinf • μ) ∧ ∀ v : HeightOneSpectrum (𝒪 F), ∀ μ : Measure (Fv F v), μ.IsAddHaarMeasure → μ.map (fun x => algebraMap F (Fv F v) a * x) = c v • μ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.GlobalUnitScalingHaarProductFormulaStatement

theorem **_root_.GlobalTheory.T.global_unit_scaling_haar_product_formula** : ∀ (F : Type) [Field F] [NumberField F] (a : F), a ≠ 0 → ∃ (cinf : ℝ≥0) (c : HeightOneSpectrum (𝒪 F) → ℝ≥0), IsUnitProd cinf c ∧ (∀ μ : Measure (Ainf F), μ.IsAddHaarMeasure → μ.map (fun x => algebraMap F (Ainf F) a * x) = cinf • μ) ∧ ∀ v : HeightOneSpectrum (𝒪 F), ∀ μ : Measure (Fv F v), μ.IsAddHaarMeasure → μ.map (fun x => algebraMap F (Fv F v) a * x) = c v • μ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.GramBlockOneFirstVectorFormula

theorem **hR_blockOne_cons** (R : Type) [CommRing R] [Algebra F R] (σ : E →ₐ[F] E) {M : ℕ} (J : Matrix (Fin M) (Fin M) E) (t : E) (x₀ y₀ : R ⊗[F] E) (x y : Fin M → R ⊗[F] E) : hR R σ (blockOne t J) (Fin.cons x₀ x : Fin (M + 1) → R ⊗[F] E) (Fin.cons y₀ y) = sigR R σ x₀ * inclR R t * y₀ + hR R σ J x y

theorem **hR_zero_left** (R : Type) [CommRing R] [Algebra F R] (σ : E →ₐ[F] E) {M : ℕ} (J : Matrix (Fin M) (Fin M) E) (y : Fin M → R ⊗[F] E) : hR R σ J (fun _ => 0) y = 0

theorem **sigR_tmul_one** (R : Type) [CommRing R] [Algebra F R] (σ : E →ₐ[F] E) (u : R) : sigR R σ (u ⊗ₜ[F] (1 : E)) = u ⊗ₜ[F] (1 : E)

theorem **tmul_one_mul_inclR** (R : Type) [CommRing R] [Algebra F R] (u : R) (e : E) : (u ⊗ₜ[F] (1 : E)) * inclR R e = u ⊗ₜ[F] e

theorem **tmul_algebraMap** (R : Type) [CommRing R] [Algebra F R] (u : R) (t : F) : u ⊗ₜ[F] (algebraMap F E t) = (u * algebraMap F R t) ⊗ₜ[F] (1 : E)

theorem **gramR_vec** (F E : Type) [Field F] [Field E] [Algebra F E] (σ : E →ₐ[F] E) (M : ℕ) (J : Matrix (Fin M) (Fin M) E) (t : F) (n : ℕ) (R : Type) [CommRing R] [Algebra F R] (u : R) (c : Fin n → R ⊗[F] E) (X : Fin n → Fin M → R ⊗[F] E) : gramR R σ (blockOne (algebraMap F E t) J) (vec (u ⊗ₜ[F] (1 : E)) c X) 0 0 = (u ^ 2 * algebraMap F R t) ⊗ₜ[F] (1 : E) ∧ (∀ j : Fin n, gramR R σ (blockOne (algebraMap F E t) J) (vec (u ⊗ₜ[F] (1 : E)) c X) 0 j.succ = (u ⊗ₜ[F] (algebraMap F E t)) * c j) ∧ ∀ i j : Fin n, gramR R σ (blockOne (algebraMap F E t) J) (vec (u ⊗ₜ[F] (1 : E)) c X) i.succ j.succ = sigR R σ (c i) * inclR R (algebraMap F E t) * c j + gramR R σ J X i j

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.GramBlockOneFirstVectorFormulaStatement

theorem **_root_.GlobalTheory.T.gram_block_one_first_vector_formula** : ∀ (F E : Type) [Field F] [Field E] [Algebra F E] (σ : E →ₐ[F] E) (M : ℕ) (J : Matrix (Fin M) (Fin M) E) (t : F) (n : ℕ) (R : Type) [CommRing R] [Algebra F R] (u : R) (c : Fin n → R ⊗[F] E) (X : Fin n → Fin M → R ⊗[F] E), gramR R σ (blockOne (algebraMap F E t) J) (vec (u ⊗ₜ[F] (1 : E)) c X) 0 0 = (u ^ 2 * algebraMap F R t) ⊗ₜ[F] (1 : E) ∧ (∀ j : Fin n, gramR R σ (blockOne (algebraMap F E t) J) (vec (u ⊗ₜ[F] (1 : E)) c X) 0 j.succ = (u ⊗ₜ[F] (algebraMap F E t)) * c j) ∧ ∀ i j : Fin n, gramR R σ (blockOne (algebraMap F E t) J) (vec (u ⊗ₜ[F] (1 : E)) c X) i.succ j.succ = sigR R σ (c i) * inclR R (algebraMap F E t) * c j + gramR R σ J X i j

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.GramCoordinatesMeasurable

theorem **continuous_gmat_entry** {F E : Type} [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E) (hσ : IsCMExt F E σ) {M : ℕ} (J : Matrix (Fin M) (Fin M) E) (hJ : IsHermND σ J) {d : ℕ} (b : Module.Basis (Fin d) F E) (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [IsTopologicalRing R] [LocallyCompactSpace R] [SecondCountableTopology R] [T2Space R] (p q : Fin M × Fin d) : Continuous fun h : HA R σ J => gmat R σ J b h p q

theorem **ginf_gfin_measurable** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (e : Ainf F ≃+* (InfinitePlace F → ℝ)), IsHomeomorph e → (∀ w p q, Measurable fun ω => ginf e σ J b ω w p q) ∧ (∀ p q, Measurable fun ω => gfin σ J b ω p q)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.GramCoordinatesMeasurableStatement

theorem **_root_.GlobalTheory.T.gram_coordinates_measurable** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (e : Ainf F ≃+* (InfinitePlace F → ℝ)), IsHomeomorph e → (∀ w p q, Measurable fun ω => ginf e σ J b ω w p q) ∧ (∀ p q, Measurable fun ω => gfin σ J b ω p q)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.GramFiberConsProductDensity

theorem **measurable_vec** {K : Type} [Zero K] [MeasurableSpace K] {M n : ℕ} (α : K) : Measurable (fun p : (Fin n → K) × (Fin n → Fin M → K) => vec α p.1 p.2)

theorem **swap_through_fibres** {Y C X H : Type} [MeasurableSpace Y] [MeasurableSpace C] [MeasurableSpace X] [MeasurableSpace H] (μ₁ : Measure Y) (μc : Measure C) (μ : Measure X) (ν : Measure H) [SFinite μ₁] [SFinite μc] [SFinite μ] (U : Set X) (W : Set H) (hW : MeasurableSet W) (m : H → Measure X) (hm : ∀ S, SFinite (m S)) (hoff : ∀ S, S ∉ W → m S = 0) (hfib : ∀ ψ : X → ℝ≥0∞, Measurable ψ → ∫⁻ v in U, ψ v ∂μ = ∫⁻ S, ∫⁻ v, ψ v ∂(m S) ∂ν) (f : Y → C → X → ℝ≥0∞) (hf : Measurable fun p : Y × C × X => f p.1 p.2.1 p.2.2) : ∫⁻ y, ∫⁻ c, ∫⁻ S in W, ∫⁻ v, f y c v ∂(m S) ∂ν ∂μc ∂μ₁ = ∫⁻ c, ∫⁻ S in W, ∫⁻ y, ∫⁻ v, f y c v ∂(m S) ∂μ₁ ∂ν ∂μc

theorem **lintegral_Uset_eq_Mid** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E) (M : ℕ) (J : Matrix (Fin M) (Fin M) E) (t : F) (n : ℕ) (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R] (d : ℕ) (b : Module.Basis (Fin d) F E) (lam : Measure R) (m₁ : HermR R σ 1 → Measure (Fin 1 → Fin (M + 1) → R ⊗[F] E)) (m' : HermR R σ n → Measure (Fin n → Fin M → R ⊗[F] E)) (s : (Fin 1 → Fin (M + 1) → R ⊗[F] E) → HA R σ (blockOne (algebraMap F E t) J)) (B₀ : Set R) (a : R → Rˣ) (κ₁ cm : Rˣ → ℝ≥0), StageData σ J t n b lam m₁ m' s B₀ a κ₁ cm → ∀ φ : (Fin (n + 1) → Fin (M + 1) → R ⊗[F] E) → ℝ≥0∞, Measurable φ → ∫⁻ v in (Uset σ J t B₀ a : Set (Fin (n + 1) → Fin (M + 1) → R ⊗[F] E)), φ v ∂(coordW R b lam (n + 1) (M + 1)) = Mid σ J t m₁ m' s B₀ a κ₁ b lam φ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.GramFiberConsProductDensityStatement

theorem **_root_.GlobalTheory.T.setLIntegral_Uset_coordW_eq_Mid** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E) (M : ℕ) (J : Matrix (Fin M) (Fin M) E) (t : F) (n : ℕ) (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R] (d : ℕ) (b : Module.Basis (Fin d) F E) (lam : Measure R) (m₁ : HermR R σ 1 → Measure (Fin 1 → Fin (M + 1) → R ⊗[F] E)) (m' : HermR R σ n → Measure (Fin n → Fin M → R ⊗[F] E)) (s : (Fin 1 → Fin (M + 1) → R ⊗[F] E) → HA R σ (blockOne (algebraMap F E t) J)) (B₀ : Set R) (a : R → Rˣ) (κ₁ cm : Rˣ → ℝ≥0), StageData σ J t n b lam m₁ m' s B₀ a κ₁ cm → ∀ φ : (Fin (n + 1) → Fin (M + 1) → R ⊗[F] E) → ℝ≥0∞, Measurable φ → ∫⁻ v in (Uset σ J t B₀ a : Set (Fin (n + 1) → Fin (M + 1) → R ⊗[F] E)), φ v ∂(coordW R b lam (n + 1) (M + 1)) = Mid σ J t m₁ m' s B₀ a κ₁ b lam φ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.GramFiberConsProductDisintegration

theorem **lintegral_fibS_of_mem** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E) (M : ℕ) (J : Matrix (Fin M) (Fin M) E) (t : F) (n : ℕ) (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R] (d : ℕ) (b : Module.Basis (Fin d) F E) (lam : Measure R) (m₁ : HermR R σ 1 → Measure (Fin 1 → Fin (M + 1) → R ⊗[F] E)) (m' : HermR R σ n → Measure (Fin n → Fin M → R ⊗[F] E)) (s : (Fin 1 → Fin (M + 1) → R ⊗[F] E) → HA R σ (blockOne (algebraMap F E t) J)) (B₀ : Set R) (a : R → Rˣ) (κ₁ cm : Rˣ → ℝ≥0), StageData σ J t n b lam m₁ m' s B₀ a κ₁ cm → ∀ φ : (Fin (n + 1) → Fin (M + 1) → R ⊗[F] E) → ℝ≥0∞, Measurable φ → ∀ S ∈ (Bset R σ t a B₀ : Set (HermR R σ (n + 1))), ∫⁻ v, φ v ∂(fibS σ J t m₁ m' s B₀ a κ₁ cm S) = (κ₁ (a (s0 R (S : Matrix (Fin (n + 1)) (Fin (n + 1)) (R ⊗[F] E)))) : ℝ≥0∞) * (cm (a (s0 R (S : Matrix (Fin (n

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.GramFiberConsProductDisintegrationStatement

theorem **_root_.GlobalTheory.T.gram_fiber_cons_lintegral_eq_scale_mul_Pint** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E) (M : ℕ) (J : Matrix (Fin M) (Fin M) E) (t : F) (n : ℕ) (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R] (d : ℕ) (b : Module.Basis (Fin d) F E) (lam : Measure R) (m₁ : HermR R σ 1 → Measure (Fin 1 → Fin (M + 1) → R ⊗[F] E)) (m' : HermR R σ n → Measure (Fin n → Fin M → R ⊗[F] E)) (s : (Fin 1 → Fin (M + 1) → R ⊗[F] E) → HA R σ (blockOne (algebraMap F E t) J)) (B₀ : Set R) (a : R → Rˣ) (κ₁ cm : Rˣ → ℝ≥0), StageData σ J t n b lam m₁ m' s B₀ a κ₁ cm → ∀ φ : (Fin (n + 1) → Fin (M + 1) → R ⊗[F] E) → ℝ≥0∞, Measurable φ → ∀ S ∈ (Bset R σ t a B₀ : Set (HermR R σ (n + 1))), ∫⁻ v, φ v ∂(fibS σ J t m₁ m' s B₀ a κ₁ cm S) = (κ₁ (a (s0 R (S : Matrix (Fin (n + 1)) (Fin (n + 1)) (R ⊗[F] E))))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.GramFiberConsProductFold

theorem **sigR_scalar_tmul** (σ : E →ₐ[F] E) (α : R) : sigR R σ (α ⊗ₜ[F] (1 : E)) = α ⊗ₜ[F] (1 : E)

theorem **gramR_scl_tmul** (σ : E →ₐ[F] E) {N : ℕ} (J : Matrix (Fin N) (Fin N) E) (α : R) (y : Fin 1 → Fin N → R ⊗[F] E) (i j : Fin 1) : gramR R σ J (scl (α ⊗ₜ[F] (1 : E)) y) i j = (α ⊗ₜ[F] (1 : E)) * (α ⊗ₜ[F] (1 : E)) * gramR R σ J y i j

theorem **corner_half_trace_of_pure_tensor** [CharZero F] {d : ℕ} (b : Module.Basis (Fin d) F E) (hd : Module.finrank F E = 2) (x : R) (S : Matrix (Fin (0 + 1)) (Fin (0 + 1)) (R ⊗[F] E)) (h : S 0 0 = x ⊗ₜ[F] (1 : E)) : s0 R S = x

theorem **chi_at_fibre** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E) (M : ℕ) (J : Matrix (Fin M) (Fin M) E) (t : F) (n : ℕ) (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R] (d : ℕ) (b : Module.Basis (Fin d) F E) (lam : Measure R) (m₁ : HermR R σ 1 → Measure (Fin 1 → Fin (M + 1) → R ⊗[F] E)) (m' : HermR R σ n → Measure (Fin n → Fin M → R ⊗[F] E)) (s : (Fin 1 → Fin (M + 1) → R ⊗[F] E) → HA R σ (blockOne (algebraMap F E t) J)) (B₀ : Set R) (a : R → Rˣ) (κ₁ cm : Rˣ → ℝ≥0), StageData σ J t n b lam m₁ m' s B₀ a κ₁ cm → ∀ φ : (Fin (n + 1) → Fin (M + 1) → R ⊗[F] E) → ℝ≥0∞, Measurable φ → ∀ x ∈ B₀, ∀ y₀ : Fin 1 → Fin (M + 1) → R ⊗[F] E, gramR R σ (blockOne (algebraMap F E t) J) y₀ = (hermScal R σ (algebraMap F R t) : HermR R σ 1) → chi σ J t s a b lam ((Uset σ J t B₀ a : Set (Fin (n + 1) → Fin (M + 1)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.GramFiberConsProductFoldStatement

theorem **_root_.GlobalTheory.T.gram_fiber_cons_product_fold** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E) (M : ℕ) (J : Matrix (Fin M) (Fin M) E) (t : F) (n : ℕ) (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R] (d : ℕ) (b : Module.Basis (Fin d) F E) (lam : Measure R) (m₁ : HermR R σ 1 → Measure (Fin 1 → Fin (M + 1) → R ⊗[F] E)) (m' : HermR R σ n → Measure (Fin n → Fin M → R ⊗[F] E)) (s : (Fin 1 → Fin (M + 1) → R ⊗[F] E) → HA R σ (blockOne (algebraMap F E t) J)) (B₀ : Set R) (a : R → Rˣ) (κ₁ cm : Rˣ → ℝ≥0), StageData σ J t n b lam m₁ m' s B₀ a κ₁ cm → ∀ φ : (Fin (n + 1) → Fin (M + 1) → R ⊗[F] E) → ℝ≥0∞, Measurable φ → ∀ x ∈ B₀, ∀ y₀ : Fin 1 → Fin (M + 1) → R ⊗[F] E, gramR R σ (blockOne (algebraMap F E t) J) y₀ = (hermScal R σ (algebraMap F R t) : HermR R σ 1) → chi σ J t s a b lam ((Uset σ J t

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.GramFiberConsProductFubini

theorem **isSFiniteKernel_of_forall_univ_lt_top** {α β : Type} [MeasurableSpace α] [MeasurableSpace β] (η : Kernel α β) (h : ∀ a, η a Set.univ < ⊤) : IsSFiniteKernel η

theorem **isSFiniteKernel_of_locallyFinite** {α β : Type} [MeasurableSpace α] [TopologicalSpace β] [MeasurableSpace β] [BorelSpace β] [T2Space β] [SigmaCompactSpace β] (κ : Kernel α β) (h : ∀ a, IsLocallyFiniteMeasure (κ a)) : IsSFiniteKernel κ

theorem **measurable_vec_triple** {K : Type} [Zero K] [MeasurableSpace K] {M n : ℕ} : Measurable (fun p : K × (Fin n → K) × (Fin n → Fin M → K) => (vec p.1 p.2.1 p.2.2 : Fin (n + 1) → Fin (M + 1) → K))

theorem **measurable_Pint** (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E) (M : ℕ) (J : Matrix (Fin M) (Fin M) E) (t : F) (n : ℕ) (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R] (d : ℕ) (b : Module.Basis (Fin d) F E) (lam : Measure R) (m₁ : HermR R σ 1 → Measure (Fin 1 → Fin (M + 1) → R ⊗[F] E)) (m' : HermR R σ n → Measure (Fin n → Fin M → R ⊗[F] E)) (s : (Fin 1 → Fin (M + 1) → R ⊗[F] E) → HA R σ (blockOne (algebraMap F E t) J)) (B₀ : Set R) (a : R → Rˣ) (κ₁ cm : Rˣ → ℝ≥0) (D : StageData σ J t n b lam m₁ m' s B₀ a κ₁ cm) (φ : (Fin (n + 1) → Fin (M + 1) → R ⊗[F] E) → ℝ≥0∞) (hφ : Measurable φ) : Measurable (fun z : R × (Fin n → R ⊗[F] E) × HermR R σ n => Pint σ J t m₁ m' s φ z.1 z.2.1 z.2.2)

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theorem **_root_.GlobalTheory.T.measurable_Pint'** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E) (M : ℕ) (J : Matrix (Fin M) (Fin M) E) (t : F) (n : ℕ) (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R] (d : ℕ) (b : Module.Basis (Fin d) F E) (lam : Measure R) (m₁ : HermR R σ 1 → Measure (Fin 1 → Fin (M + 1) → R ⊗[F] E)) (m' : HermR R σ n → Measure (Fin n → Fin M → R ⊗[F] E)) (s : (Fin 1 → Fin (M + 1) → R ⊗[F] E) → HA R σ (blockOne (algebraMap F E t) J)) (B₀ : Set R) (a : R → Rˣ) (κ₁ cm : Rˣ → ℝ≥0), StageData σ J t n b lam m₁ m' s B₀ a κ₁ cm → ∀ φ : (Fin (n + 1) → Fin (M + 1) → R ⊗[F] E) → ℝ≥0∞, Measurable φ → Measurable (fun z : R × (Fin n → R ⊗[F] E) × HermR R σ n => Pint σ J t m₁ m' s φ z.1 z.2.1 z.2.2)

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theorem **schurM_mem** {F E : Type} [Field F] [Field E] [Algebra F E] (R : Type) [CommRing R] [Algebra F R] (σ : E →ₐ[F] E) (hinv : ∀ y : R ⊗[F] E, sigR R σ (sigR R σ y) = y) {n : ℕ} (t : F) (a : R → Rˣ) (S : HermR R σ (n + 1)) : schurM R σ t a (S : Matrix (Fin (n + 1)) (Fin (n + 1)) (R ⊗[F] E)) ∈ HermR R σ n

theorem **coords** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E) (M : ℕ) (J : Matrix (Fin M) (Fin M) E) (t : F) (n : ℕ) (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R] (d : ℕ) (b : Module.Basis (Fin d) F E) (lam : Measure R) (m₁ : HermR R σ 1 → Measure (Fin 1 → Fin (M + 1) → R ⊗[F] E)) (m' : HermR R σ n → Measure (Fin n → Fin M → R ⊗[F] E)) (s : (Fin 1 → Fin (M + 1) → R ⊗[F] E) → HA R σ (blockOne (algebraMap F E t) J)) (B₀ : Set R) (a : R → Rˣ) (κ₁ cm : Rˣ → ℝ≥0), StageData σ J t n b lam m₁ m' s B₀ a κ₁ cm → Continuous (fun S : HermR R σ (n + 1) => s0 R (S : Matrix (Fin (n + 1)) (Fin (n + 1)) (R ⊗[F] E))) ∧ ContinuousOn (fun S : HermR R σ (n + 1) => (a (s0 R (S : Matrix (Fin (n + 1)) (Fin (n + 1)) (R ⊗[F] E))))) {S | s0 R (S : Matrix (Fin (n + 1)) (Fin (n + 1)) (R ⊗[F] E)) ∈ B₀} ∧ ContinuousOn

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theorem **_root_.GlobalTheory.T.continuousOn_schur_coordinates_and_isOpen_Bset** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E) (M : ℕ) (J : Matrix (Fin M) (Fin M) E) (t : F) (n : ℕ) (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R] (d : ℕ) (b : Module.Basis (Fin d) F E) (lam : Measure R) (m₁ : HermR R σ 1 → Measure (Fin 1 → Fin (M + 1) → R ⊗[F] E)) (m' : HermR R σ n → Measure (Fin n → Fin M → R ⊗[F] E)) (s : (Fin 1 → Fin (M + 1) → R ⊗[F] E) → HA R σ (blockOne (algebraMap F E t) J)) (B₀ : Set R) (a : R → Rˣ) (κ₁ cm : Rˣ → ℝ≥0), StageData σ J t n b lam m₁ m' s B₀ a κ₁ cm → Continuous (fun S : HermR R σ (n + 1) => s0 R (S : Matrix (Fin (n + 1)) (Fin (n + 1)) (R ⊗[F] E))) ∧ ContinuousOn (fun S : HermR R σ (n + 1) => (a (s0 R (S : Matrix (Fin (n + 1)) (Fin (n + 1)) (R ⊗[F] E))))) {S | s0 R (S : Matrix

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theorem **det_eq_corner_mul** {K : Type} [CommRing K] {n : ℕ} (S : Matrix (Fin (n + 1)) (Fin (n + 1)) K) (w : K) (hw : S 0 0 * w = 1) : S.det = S 0 0 * (Matrix.of fun i j : Fin n => S i.succ j.succ − S i.succ 0 * w * S 0 j.succ).det

theorem **half_trace_tmul** (h2F : (2 : F) ≠ 0) (b : Module.Basis (Fin 2) F E) (r : R) : algebraMap F R (1 / 2) * Algebra.trace R (R ⊗[F] E) (r ⊗ₜ[F] (1 : E)) = r

theorem **corner_mul_inv** {t : F} (ht : t ≠ 0) (u : Rˣ) : (((u : R) ^ 2 * algebraMap F R t) ⊗ₜ[F] (1 : E)) * ((((u⁻¹ : Rˣ) : R) ⊗ₜ[F] (algebraMap F E t⁻¹)) * inclR R (algebraMap F E t) * (((u⁻¹ : Rˣ) : R) ⊗ₜ[F] (algebraMap F E t⁻¹))) = 1

theorem **isUnit_det_of_mem_Bset** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E) (M : ℕ) (J : Matrix (Fin M) (Fin M) E) (t : F) (n : ℕ) (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R] (d : ℕ) (b : Module.Basis (Fin d) F E) (lam : Measure R) (m₁ : HermR R σ 1 → Measure (Fin 1 → Fin (M + 1) → R ⊗[F] E)) (m' : HermR R σ n → Measure (Fin n → Fin M → R ⊗[F] E)) (s : (Fin 1 → Fin (M + 1) → R ⊗[F] E) → HA R σ (blockOne (algebraMap F E t) J)) (B₀ : Set R) (a : R → Rˣ) (κ₁ cm : Rˣ → ℝ≥0), StageData σ J t n b lam m₁ m' s B₀ a κ₁ cm → ∀ S ∈ (Bset R σ t a B₀ : Set (HermR R σ (n + 1))), IsUnit (S : Matrix (Fin (n + 1)) (Fin (n + 1)) (R ⊗[F] E)).det

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theorem **_root_.GlobalTheory.T.isUnit_det_of_mem_Bset'** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E) (M : ℕ) (J : Matrix (Fin M) (Fin M) E) (t : F) (n : ℕ) (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R] (d : ℕ) (b : Module.Basis (Fin d) F E) (lam : Measure R) (m₁ : HermR R σ 1 → Measure (Fin 1 → Fin (M + 1) → R ⊗[F] E)) (m' : HermR R σ n → Measure (Fin n → Fin M → R ⊗[F] E)) (s : (Fin 1 → Fin (M + 1) → R ⊗[F] E) → HA R σ (blockOne (algebraMap F E t) J)) (B₀ : Set R) (a : R → Rˣ) (κ₁ cm : Rˣ → ℝ≥0), StageData σ J t n b lam m₁ m' s B₀ a κ₁ cm → ∀ S ∈ (Bset R σ t a B₀ : Set (HermR R σ (n + 1))), IsUnit (S : Matrix (Fin (n + 1)) (Fin (n + 1)) (R ⊗[F] E)).det

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theorem **corner_half_trace_of_pure_tensor** (hfr : Module.finrank F E = 2) {n : ℕ} (S : Matrix (Fin (n + 1)) (Fin (n + 1)) (R ⊗[F] E)) (r : R) (h : S 0 0 = r ⊗ₜ[F] (1 : E)) : s0 R S = r

theorem **gram_symm** (σ : E →ₐ[F] E) (hinv : ∀ y, sigR R σ (sigR R σ y) = y) {M n : ℕ} {J : Matrix (Fin M) (Fin M) E} (hJ : ∀ i j, σ (J j i) = J i j) (X : Fin n → Fin M → R ⊗[F] E) (p q : Fin n) : sigR R σ (gramR R σ J X q p) = gramR R σ J X p q

theorem **schurM_mem** (σ : E →ₐ[F] E) (hinv : ∀ y, sigR R σ (sigR R σ y) = y) {n : ℕ} (t : F) (a : R → Rˣ) (S : HermR R σ (n + 1)) : schurM R σ t a (S : Matrix (Fin (n + 1)) (Fin (n + 1)) (R ⊗[F] E)) ∈ HermR R σ n

theorem **unit_tmul** {F E : Type} [Field F] [Field E] [Algebra F E] {R : Type} [CommRing R] [Algebra F R] {t : F} (ht : t ≠ 0) (u : Rˣ) : ((((u⁻¹ : Rˣ) : R) ⊗ₜ[F] (algebraMap F E t⁻¹)) * (((u : Rˣ) : R) ⊗ₜ[F] (algebraMap F E t)) : R ⊗[F] E) = 1

theorem **coords** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E) (M : ℕ) (J : Matrix (Fin M) (Fin M) E) (t : F) (n : ℕ) (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R] (d : ℕ) (b : Module.Basis (Fin d) F E) (lam : Measure R) (m₁ : HermR R σ 1 → Measure (Fin 1 → Fin (M + 1) → R ⊗[F] E)) (m' : HermR R σ n → Measure (Fin n → Fin M → R ⊗[F] E)) (s : (Fin 1 → Fin (M + 1) → R ⊗[F] E) → HA R σ (blockOne (algebraMap F E t) J)) (B₀ : Set R) (a : R → Rˣ) (κ₁ cm : Rˣ → ℝ≥0), StageData σ J t n b lam m₁ m' s B₀ a κ₁ cm → (∀ x ∈ B₀, ∀ (c : Fin n → R ⊗[F] E) (X : Fin n → Fin M → R ⊗[F] E) (S : HermR R σ (n + 1)), (S : Matrix (Fin (n + 1)) (Fin (n + 1)) (R ⊗[F] E)) = gramR R σ (blockOne (algebraMap F E t) J) (vec (((a x : Rˣ) : R) ⊗ₜ[F] (1 : E)) c X) → s0 R (S : Matrix (Fin (n + 1)) (Fin (n + 1)) (R ⊗[F] E)) =

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theorem **_root_.GlobalTheory.T.gram_block_one_schur_coordinates** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E) (M : ℕ) (J : Matrix (Fin M) (Fin M) E) (t : F) (n : ℕ) (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R] (d : ℕ) (b : Module.Basis (Fin d) F E) (lam : Measure R) (m₁ : HermR R σ 1 → Measure (Fin 1 → Fin (M + 1) → R ⊗[F] E)) (m' : HermR R σ n → Measure (Fin n → Fin M → R ⊗[F] E)) (s : (Fin 1 → Fin (M + 1) → R ⊗[F] E) → HA R σ (blockOne (algebraMap F E t) J)) (B₀ : Set R) (a : R → Rˣ) (κ₁ cm : Rˣ → ℝ≥0), StageData σ J t n b lam m₁ m' s B₀ a κ₁ cm → (∀ x ∈ B₀, ∀ (c : Fin n → R

⊗[F] E) (X : Fin n → Fin M → R ⊗[F] E) (S : HermR R σ (n + 1)), (S : Matrix (Fin (n + 1)) (Fin (n + 1)) (R ⊗[F] E)) = gramR R σ (blockOne (algebraMap F E t) J) (vec (((a x : Rˣ) : R) ⊗ₜ[F] (1 : E)) c X) → s0 R (S :

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theorem **measurable_vec** {K : Type} [Zero K] [MeasurableSpace K] {M n : ℕ} (α : K) (c : Fin n → K) : Measurable (fun X : Fin n → Fin M → K => vec α c X)

theorem **fibS_support** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E) (M : ℕ) (J : Matrix (Fin M) (Fin M) E) (t : F) (n : ℕ) (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R] (d : ℕ) (b : Module.Basis (Fin d) F E) (lam : Measure R) (m₁ : HermR R σ 1 → Measure (Fin 1 → Fin (M + 1) → R ⊗[F] E)) (m' : HermR R σ n → Measure (Fin n → Fin M → R ⊗[F] E)) (s : (Fin 1 → Fin (M + 1) → R ⊗[F] E) → HA R σ (blockOne (algebraMap F E t) J)) (B₀ : Set R) (a : R → Rˣ) (κ₁ cm : Rˣ → ℝ≥0), StageData σ J t n b lam m₁ m' s B₀ a κ₁ cm → ∀ S : HermR R σ (n + 1), fibS σ J t m₁ m' s B₀ a κ₁ cm S (gramR R σ (blockOne (algebraMap F E t) J) ⁻¹' {(S : Matrix (Fin (n + 1)) (Fin (n + 1)) (R ⊗[F] E))} ∩ gramR R σ (blockOne (algebraMap F E t) J) ⁻¹' (Subtype.val '' (Bset R σ t a B₀ : Set (HermR R σ (n + 1))))ᶜ = 0

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theorem **_root_.GlobalTheory.T.gram_fiber_cons_concentrated_on_fiber** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E) (M : ℕ) (J : Matrix (Fin M) (Fin M) E) (t : F) (n : ℕ) (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R] (d : ℕ) (b : Module.Basis (Fin d) F E) (lam : Measure R) (m₁ : HermR R σ 1 → Measure (Fin 1 → Fin (M + 1) → R ⊗[F] E)) (m' : HermR R σ n → Measure (Fin n → Fin M → R ⊗[F] E)) (s : (Fin 1 → Fin (M + 1) → R ⊗[F] E) → HA R σ (blockOne (algebraMap F E t) J)) (B₀ : Set R) (a : R → Rˣ) (κ₁ cm : Rˣ → ℝ≥0), StageData σ J t n b lam m₁ m' s B₀ a κ₁ cm → ∀ S : HermR R σ (n + 1), fibS σ J t m₁ m' s B₀ a κ₁ cm S (gramR R σ (blockOne (algebraMap F E t) J) ⁻¹' {(S : Matrix (Fin (n + 1)) (Fin (n + 1)) (R ⊗[F] E))} ∩ gramR R σ (blockOne (algebraMap F E t) J) ⁻¹' (Subtype.val '' (Bset R σ t a B₀

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theorem **sigR_inclT** (t : F) : sigR R σ (inclR R (algebraMap F E t)) = inclR R (algebraMap F E t)

theorem **qM_mem** (hσ : IsCMExt F E σ) (t : F) (c : Fin n → R ⊗[F] E) : qM R σ t c ∈ HermR R σ n

theorem **schurM_eq** (t : F) (a : R → Rˣ) (S : Matrix (Fin (n + 1)) (Fin (n + 1)) (R ⊗[F] E)) (i j : Fin n) : schurM R σ t a S i j = S i.succ j.succ − qM R σ t (cvec R t a S) i j

theorem **schur_split** (hσ : IsCMExt F E σ) (t : F) (a : R → Rˣ) (S : HermR R σ (n + 1)) (S'' : HermR R σ n) (h : ∀ i j, (S : Matrix (Fin (n + 1)) (Fin (n + 1)) (R ⊗[F] E)) i.succ j.succ = (S'' : Matrix (Fin n) (Fin n) (R ⊗[F] E)) i j) : schur R σ t a S = S'' − (qM R σ t (cvec R t a (S : Matrix _ _ _)), qM_mem R σ hσ t _)

theorem **corner_half_trace_of_cm_pure_tensor** (hσ : IsCMExt F E σ) (S : Matrix (Fin (n + 1)) (Fin (n + 1)) (R ⊗[F] E)) (x : R) (h : S 0 0 = x ⊗ₜ[F] (1 : E)) : s0 R S = x

theorem **fold** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E) (M : ℕ) (J : Matrix (Fin M) (Fin M) E) (t : F) (n : ℕ) (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R] (d : ℕ) (b : Module.Basis (Fin d) F E) (lam : Measure R) (m₁ : HermR R σ 1 → Measure (Fin 1 → Fin (M + 1) → R ⊗[F] E)) (m' : HermR R σ n → Measure (Fin n → Fin M → R ⊗[F] E)) (s : (Fin 1 → Fin (M + 1) → R ⊗[F] E) → HA R σ (blockOne (algebraMap F E t) J)) (B₀ : Set R) (a : R → Rˣ) (κ₁ cm : Rˣ → ℝ≥0), StageData σ J t n b lam m₁ m' s B₀ a κ₁ cm → ∀ φ : (Fin (n + 1) → Fin (M + 1) → R ⊗[F] E) → ℝ≥0∞, Measurable φ → ∫⁻ S, ∫⁻ v, φ v ∂(fibS σ J t m₁ m' s B₀ a κ₁ cm S) ∂(coordHerm R σ b lam (n + 1)) = Mid σ J t m₁ m' s B₀ a κ₁ b lam φ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.GramFiberConsProductPushforwardStatement

theorem **_root_.GlobalTheory.T.gram_fiber_cons_iterated_lintegral_eq_Mid** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E) (M : ℕ) (J : Matrix (Fin M) (Fin M) E) (t : F) (n : ℕ) (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R] (d : ℕ) (b : Module.Basis (Fin d) F E) (lam : Measure R) (m₁ : HermR R σ 1 → Measure (Fin 1 → Fin (M + 1) → R ⊗[F] E)) (m' : HermR R σ n → Measure (Fin n → Fin M → R ⊗[F] E)) (s : (Fin 1 → Fin (M + 1) → R ⊗[F] E) → HA R σ (blockOne (algebraMap F E t) J)) (B₀ : Set R) (a : R → Rˣ) (κ₁ cm : Rˣ → ℝ≥0), StageData σ J t n b lam m₁ m' s B₀ a κ₁ cm → ∀ φ : (Fin (n + 1) → Fin (M + 1) → R ⊗[F] E) → ℝ≥0∞, Measurable φ → ∫⁻ S, ∫⁻ v, φ v ∂(fibS σ J t m₁ m' s B₀ a κ₁ cm S) ∂(coordHerm R σ b lam (n + 1)) = Mid σ J t m₁ m' s B₀ a κ₁ b lam φ

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theorem **GlobalTheory.CompanionTheory.SW.PiLocalization₂.fibS_integrable_continuousOn** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E) (M : ℕ) (J : Matrix (Fin M) (Fin M) E) (t : F) (n : ℕ) (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R] (d : ℕ) (b : Module.Basis (Fin d) F E) (lam : Measure R) (m₁ : HermR R σ 1 → Measure (Fin 1 → Fin (M + 1) → R ⊗[F] E)) (m' : HermR R σ n → Measure (Fin n → Fin M → R ⊗[F] E)) (s : (Fin 1 → Fin (M + 1) → R ⊗[F] E) → HA R σ (blockOne (algebraMap F E t) J)) (B₀ : Set R) (a : R → Rˣ) (κ₁ cm : Rˣ → ℝ≥0), StageData σ J t n b lam m₁ m' s B₀ a κ₁ cm → ∀ φ : (Fin (n + 1) → Fin (M + 1) → R ⊗[F] E) → ℝ, Continuous φ → HasCompactSupport φ → (∀ S ∈ (Bset R σ t a B₀ : Set (HermR R σ (n + 1))), Integrable φ (fibS σ J t m₁ m' s B₀ a κ₁ cm S)) ∧ ContinuousOn (fun S => ∫ v, φ v

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theorem **_root_.GlobalTheory.T.gram_fiber_cons_integrable_and_continuousOn_integral** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E) (M : ℕ) (J : Matrix (Fin M) (Fin M) E) (t : F) (n : ℕ) (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R] (d : ℕ) (b : Module.Basis (Fin d) F E) (lam : Measure R) (m₁ : HermR R σ 1 → Measure (Fin 1 → Fin (M + 1) → R ⊗[F] E)) (m' : HermR R σ n → Measure (Fin n → Fin M → R ⊗[F] E)) (s : (Fin 1 → Fin (M + 1) → R ⊗[F] E) → HA R σ (blockOne (algebraMap F E t) J)) (B₀ : Set R) (a : R → Rˣ) (κ₁ cm : Rˣ → ℝ≥0), StageData σ J t n b lam m₁ m' s B₀ a κ₁ cm → ∀ φ : (Fin (n + 1) → Fin (M + 1) → R ⊗[F] E) → ℝ, Continuous φ → HasCompactSupport φ → (∀ S ∈ (Bset R σ t a B₀ : Set (HermR R σ (n + 1))), Integrable φ (fibS σ J t m₁ m' s B₀ a κ₁ cm S)) ∧ ContinuousOn (fun S => ∫ v, φ v ∂(

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.GramFiberConsProductScaling

theorem **integrable_map_prod_of_compact** {A B W : Type} [TopologicalSpace A] [MeasurableSpace A] [OpensMeasurableSpace A] [T2Space A] [TopologicalSpace B] [MeasurableSpace B] [OpensMeasurableSpace B] [T2Space B] [TopologicalSpace W] [MeasurableSpace W] [BorelSpace W] (μ : Measure A) (ν : Measure B) [SFinite μ] [SFinite ν] [IsFiniteMeasureOnCompacts μ] [IsFiniteMeasureOnCompacts ν] (f : A × B → W) (hf : Measurable f) (φ : W → ℝ) (hφ : Continuous φ) (hc : HasCompactSupport φ) (C₁ : Set A) (C₂ : Set B) (h₁ : IsCompact C₁) (h₂ : IsCompact C₂) (hs : ∀ᵐ y ∂μ, ∀ X, φ (f (y, X)) ≠ 0 → y ∈ C₁ ∧ X ∈ C₂) (k : ℝ≥0) : Integrable φ (k • (μ.prod ν).map f) ∧ ∫ v, φ v ∂(k • (μ.prod ν).map f) = (k : ℝ) ∗ ∫ y, ∫ X, φ (f (y, X)) ∂ν ∂μ

theorem **vec_zero** {K : Type} [CommRing K] {M n : ℕ} (u : K) (c : Fin n → K) (X : Fin n → Fin M → K) : vec u c X 0 = u • (Fin.cons 1 (fun _ => 0) : Fin (M + 1) → K)

theorem **continuous_vec** {K : Type} [CommRing K] [TopologicalSpace K] [Zero K] {M n : ℕ} (u : K) (c : Fin n → K) : Continuous (fun X : Fin n → Fin M → K => vec u c X)

theorem **integrable_fibS_and_integral** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E) (M : ℕ) (J : Matrix (Fin M) (Fin M) E) (t : F) (n : ℕ) (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R] (d : ℕ) (b : Module.Basis (Fin d) F E) (lam : Measure R) (m₁ : HermR R σ 1 → Measure (Fin 1 → Fin (M + 1) → R ⊗[F] E)) (m' : HermR R σ n → Measure (Fin n → Fin M → R ⊗[F] E)) (s : (Fin 1 → Fin (M + 1) → R ⊗[F] E) → HA R σ (blockOne (algebraMap F E t) J)) (B₀ : Set R) (a : R → Rˣ) (κ₁ cm : Rˣ → ℝ≥0), StageData σ J t n b lam m₁ m' s B₀ a κ₁ cm → ∀ φ : (Fin (n + 1) → Fin (M + 1) → R ⊗[F] E) → ℝ, Continuous φ → HasCompactSupport φ → ∀ S ∈ (Bset R σ t a B₀ : Set (HermR R σ (n + 1))), Integrable φ (fibS σ J t m₁ m' s B₀ a κ₁ cm S) ∧ ∫ v, φ v ∂(fibS σ J t m₁ m' s B₀ a κ₁ cm S) = ((κ₁ (a (s0 R (S : Matrix (Fin (n +

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.GramFiberConsProductScalingStatement

theorem **_root_.GlobalTheory.T.gram_fiber_cons_integrable_and_integral_eq** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E) (M : ℕ) (J : Matrix (Fin M) (Fin M) E) (t : F) (n : ℕ) (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R] (d : ℕ) (b : Module.Basis (Fin d) F E) (lam : Measure R) (m₁ : HermR R σ 1 → Measure (Fin 1 → Fin (M + 1) → R ⊗[F] E)) (m' : HermR R σ n → Measure (Fin n → Fin M → R ⊗[F] E)) (s : (Fin 1 → Fin (M + 1) → R ⊗[F] E) → HA R σ (blockOne (algebraMap F E t) J)) (B₀ : Set R) (a : R → Rˣ) (κ₁ cm : Rˣ → ℝ≥0), StageData σ J t n b lam m₁ m' s B₀ a κ₁ cm → ∀ φ : (Fin (n + 1) → Fin (M + 1) → R ⊗[F] E) → ℝ, Continuous φ → HasCompactSupport φ → ∀ S ∈ (Bset R σ t a B₀ : Set (HermR R σ (n + 1))), Integrable φ (fibS σ J t m₁ m' s B₀ a κ₁ cm S) ∧ ∫ v, φ v ∂(fibS σ J t m₁ m' s B₀ a κ₁ cm S) =

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.GramFiberConsProductSupport

theorem **one_tmul_algebraMap** (t : F) : (1 : R) ⊗ₜ[F] (algebraMap F E t) = (algebraMap F R t) ⊗ₜ[F] (1 : E)

theorem **corner_half_trace_eq_of_pure_tensor** [CharZero F] (hfr : Module.finrank F E = 2) {n : ℕ} (S : Matrix (Fin (n + 1)) (Fin (n + 1)) (R ⊗[F] E)) {x : R} (h : S 0 0 = x ⊗ₜ[F] (1 : E)) : s0 R S = x

theorem **vec_one_zero** {M n : ℕ} (X : Fin n → Fin M → R ⊗[F] E) : vec ((1 : R) ⊗ₜ[F] (1 : E)) (0 : Fin n → R ⊗[F] E) X = consTup X

theorem **corner_half_trace_block_one** [CharZero F] (hfr : Module.finrank F E = 2) (T : Matrix (Fin n) (Fin n) E) : s0 R ((blockOne (algebraMap F E t) T).map (inclR (F

theorem **cvec_block** (T : Matrix (Fin n) (Fin n) E) : cvec R t a ((blockOne (algebraMap F E t) T).map (inclR R)) = 0

theorem **schurM_block** (T : Matrix (Fin n) (Fin n) E) : schurM R σ t a ((blockOne (algebraMap F E t) T).map (inclR R)) = T.map (inclR R)

theorem **schur_block** (T : Matrix (Fin n) (Fin n) E) (hT : ∀ a b, σ (T b a) = T a b) (hT' : ∀ a b, σ (blockOne (algebraMap F E t) T b a) = blockOne (algebraMap F E t) T a b) : schur R σ t a (hermOf R σ _ hT') = hermOf R σ T hT

theorem **isUnit_det_map** (T : Matrix (Fin n) (Fin n) E) (hdet : T.det ≠ 0) : IsUnit (T.map (inclR (F

theorem **family_at_block** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E) (M : ℕ) (J : Matrix (Fin M) (Fin M) E) (t : F) (n : ℕ) (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R] (d : ℕ) (b : Module.Basis (Fin d) F E) (lam : Measure R) (m₁ : HermR R σ 1 → Measure (Fin 1 → Fin (M + 1) → R ⊗[F] E)) (m' : HermR R σ n → Measure (Fin n → Fin M → R ⊗[F] E)) (s : (Fin 1 → Fin (M + 1) → R ⊗[F] E) → HA R σ (blockOne (algebraMap F E t) J)) (B₀ : Set R) (a : R → Rˣ) (κ₁ cm : Rˣ → ℝ≥0), StageData σ J t n b lam m₁ m' s B₀ a κ₁ cm → ∀ (T : Matrix (Fin n) (Fin n) E) (hT : ∀ a b, σ (T b a) = T a b), T.det ≠ 0 → ∀ (hT' : ∀ a b, σ (blockOne (algebraMap F E t) T b a) = blockOne (algebraMap F E t) T a b), hermOf R σ _ hT' ∈ (Bset R σ t a B₀ : Set (HermR R σ (n + 1))) ∧ fibS σ J t m₁ m' s B₀ a κ₁ cm (hermOf R σ _ hT'

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theorem **_root_.GlobalTheory.T.gram_fiber_cons_at_block_one_eq_map_prod** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E) (M : ℕ) (J : Matrix (Fin M) (Fin M) E) (t : F) (n : ℕ) (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R] (d : ℕ) (b : Module.Basis (Fin d) F E) (lam : Measure R) (m₁ : HermR R σ 1 → Measure (Fin 1 → Fin (M + 1) → R ⊗[F] E)) (m' : HermR R σ n → Measure (Fin n → Fin M → R ⊗[F] E)) (s : (Fin 1 → Fin (M + 1) → R ⊗[F] E) → HA R σ (blockOne (algebraMap F E t) J)) (B₀ : Set R) (a : R → Rˣ) (κ₁ cm : Rˣ → ℝ≥0), StageData σ J t n b lam m₁ m' s B₀ a κ₁ cm → ∀ (T : Matrix (Fin n) (Fin n) E) (hT : ∀ a b, σ (T b a) = T a b), T.det ≠ 0 → ∀ (hT' : ∀ a b, σ (blockOne (algebraMap F E t) T b a) = blockOne (algebraMap F E t) T a b), hermOf R σ _ hT' ∈ (Bset R σ t a B₀ : Set (HermR R σ (n + 1))) ∧

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theorem **measurable_fin_cons** {Ω A : Type} [MeasurableSpace Ω] [MeasurableSpace A] {n : ℕ} {f : Ω → A} {g : Ω → Fin n → A} (hf : Measurable f) (hg : Measurable g) : Measurable fun p => (Fin.cons (f p) (g p) : Fin (n + 1) → A)

theorem **measurable_scl** {K : Type} [CommRing K] [TopologicalSpace K] [IsTopologicalRing K] [SecondCountableTopology K] [MeasurableSpace K] [BorelSpace K] {N m : ℕ} : Measurable fun p : K × (Fin m → Fin N → K) => scl p.1 p.2

theorem **corner_half_trace_gram_cons** (σ : E →ₐ[F] E) {N n : ℕ} (J₂ : Matrix (Fin N) (Fin N) E) (y : Fin 1 → Fin N → R ⊗[F] E) (Z : Fin n → Fin N → R ⊗[F] E) : s0 R (gramR R σ J₂ (Fin.cons (y 0) Z : Fin (n + 1) → Fin N → R ⊗[F] E)) = s0 R (gramR R σ J₂ y)

theorem **isUnit_tmul_one** {x : R} (hx : IsUnit x) : IsUnit (x ⊗ₜ[F] (1 : E))

theorem **tmul_one_eq_smul** (x : R) : x ⊗ₜ[F] (1 : E) = x • (1 : R ⊗[F] E)

theorem **continuous_tmul_one** [TopologicalSpace R] : Continuous fun x : R => x ⊗ₜ[F] (1 : E)

theorem **measurable_inv_a** (D : StageData σ J t n b lam m₁ m' s B₀ a κ₁ cm) : Measurable fun x : R => ((((a x)⁻¹ : Rˣ) : R))

theorem **measurable_sTil** (D : StageData σ J t n b lam m₁ m' s B₀ a κ₁ cm) : Measurable (sTil σ J t s a)

theorem **measurable_chi** (D : StageData σ J t n b lam m₁ m' s B₀ a κ₁ cm) {φ : (Fin (n + 1) → Fin (M + 1) → R ⊗[F] E) → ℝ≥0∞} (hφ : Measurable φ) : Measurable (chi σ J t s a b lam φ)

theorem **chi_eq_zero** (D : StageData σ J t n b lam m₁ m' s B₀ a κ₁ cm) (φ : (Fin (n + 1) → Fin (M + 1) → R ⊗[F] E) → ℝ≥0∞) {y : Fin 1 → Fin (M + 1) → R ⊗[F] E} (hy : s0 R (gramR R σ (blockOne (algebraMap F E t) J) y) ∉ B₀) : chi σ J t s a b lam ((Uset σ J t B₀ a : Set (Fin (n + 1) → Fin (M + 1) → R ⊗[F] E)).indicator φ) y = 0

theorem **isUnit_gram** (D : StageData σ J t n b lam m₁ m' s B₀ a κ₁ cm) {y : Fin 1 → Fin (M + 1) → R ⊗[F] E} (hy : s0 R (gramR R σ (blockOne (algebraMap F E t) J) y) ∈ B₀) : IsUnit (gramR R σ (blockOne (algebraMap F E t) J) y).det

theorem **first_step** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E) (M : ℕ) (J : Matrix (Fin M) (Fin M) E) (t : F) (n : ℕ) (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R] (d : ℕ) (b : Module.Basis (Fin d) F E) (lam : Measure R) (m₁ : HermR R σ 1 → Measure (Fin 1 → Fin (M + 1) → R ⊗[F] E)) (m' : HermR R σ n → Measure (Fin n → Fin M → R ⊗[F] E)) (s : (Fin 1 → Fin (M + 1) → R ⊗[F] E) → HA R σ (blockOne (algebraMap F E t) J)) (B₀ : Set R) (a : R → Rˣ) (κ₁ cm : Rˣ → ℝ≥0), StageData σ J t n b lam m₁ m' s B₀ a κ₁ cm → ∀ φ : (Fin (n + 1) → Fin (M + 1) → R ⊗[F] E) → ℝ≥0∞, Measurable φ → ∫⁻ v in (Uset σ J t B₀ a : Set (Fin (n + 1) → Fin (M + 1) → R ⊗[F] E)), φ v ∂(coordW R b lam (n + 1) (M + 1)) = ∫⁻ x in B₀, (κ₁ (a x) : ℝ≥0∞) * ∫⁻ y₀, chi σ J t s a b lam ((Uset σ J t B₀ a : Set (Fin (n + 1) → Fin (M +

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theorem **_root_.GlobalTheory.T.gram_fiber_cons_product_unfold** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E) (M : ℕ) (J : Matrix (Fin M) (Fin M) E) (t : F) (n : ℕ) (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R] (d : ℕ) (b : Module.Basis (Fin d) F E) (lam : Measure R) (m₁ : HermR R σ 1 → Measure (Fin 1 → Fin (M + 1) → R ⊗[F] E)) (m' : HermR R σ n → Measure (Fin n → Fin M → R ⊗[F] E)) (s : (Fin 1 → Fin (M + 1) → R ⊗[F] E) → HA R σ (blockOne (algebraMap F E t) J)) (B₀ : Set R) (a : R → Rˣ) (κ₁ cm : Rˣ → ℝ≥0), StageData σ J t n b lam m₁ m' s B₀ a κ₁ cm → ∀ φ : (Fin (n + 1) → Fin (M + 1) → R ⊗[F] E) → ℝ≥0∞, Measurable φ → ∫⁻ v in (Uset σ J t B₀ a : Set (Fin (n + 1) → Fin (M + 1) → R ⊗[F] E)), φ v ∂(coordW R b lam (n + 1) (M + 1)) = ∫⁻ x in B₀, (κ₁ (a x) : ℝ≥0∞) * ∫⁻ y₀, chi σ J t s a b lam ((Uset

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theorem **tl_vec** {K : Type} [Zero K] {M n : ℕ} (α : K) (c : Fin n → K) (X : Fin n → Fin M → K) : tl (vec α c X) = X

theorem **continuous_tl** {K : Type} [TopologicalSpace K] {M n : ℕ} : Continuous (tl : (Fin (n + 1) → Fin (M + 1) → K) → Fin n → Fin M → K)

theorem **continuous_vec** {K : Type} [Zero K] [TopologicalSpace K] {M n : ℕ} : Continuous (fun p : K × (Fin n → K) × (Fin n → Fin M → K) => vec p.1 p.2.1 p.2.2)

theorem **actW_vec_zero** {F E : Type} [Field F] [Field E] [Algebra F E] (σ : E →ₐ[F] E) {N n : ℕ} (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] (J' : Matrix (Fin (N + 1)) (Fin (N + 1)) E) (h : HA R σ J') (A : R ⊗[F] E) (c : Fin n → R ⊗[F] E) (X : Fin n → Fin N → R ⊗[F] E) (i : Fin (N + 1)) : actW R σ J' h (vec A c X) 0 i = actW R σ J' h (fun _ : Fin 1 => Fin.cons 1 (fun _ => 0)) 0 i * A

theorem **continuous_tmul_one** {F E : Type} [Field F] [Field E] [Algebra F E] (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [IsTopologicalRing R] : Continuous (fun α : R => α ⊗ₜ[F] (1 : E))

theorem **cont_double** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E) (M : ℕ) (J : Matrix (Fin M) (Fin M) E) (t : F) (n : ℕ) (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R] (d : ℕ) (b : Module.Basis (Fin d) F E) (lam : Measure R) (m₁ : HermR R σ 1 → Measure (Fin 1 → Fin (M + 1) → R ⊗[F] E)) (m' : HermR R σ n → Measure (Fin n → Fin M → R ⊗[F] E)) (s : (Fin 1 → Fin (M + 1) → R ⊗[F] E) → HA R σ (blockOne (algebraMap F E t) J)) (B₀ : Set R) (a : R → Rˣ) (κ₁ cm : Rˣ → ℝ≥0), StageData σ J t n b lam m₁ m' s B₀ a κ₁ cm → ∀ φ : (Fin (n + 1) → Fin (M + 1) → R ⊗[F] E) → ℝ, Continuous φ → HasCompactSupport φ → ContinuousOn (fun z : R × (Fin n → R ⊗[F] E) × HermR R σ n => Preal σ J t m₁ m' s φ z.1 z.2.1 z.2.2) ({α : R | IsUnit α} ×ˢ (Set.univ ×ˢ Unt σ R n))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.GramFiberConsProductUniquenessStatement

theorem **_root_.GlobalTheory.T.continuousOn_preal_units_of_hasCompactSupport** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E) (M : ℕ) (J : Matrix (Fin M) (Fin M) E) (t : F) (n : ℕ) (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R] (d : ℕ) (b : Module.Basis (Fin d) F E) (lam : Measure R) (m₁ : HermR R σ 1 → Measure (Fin 1 → Fin (M + 1) → R ⊗[F] E)) (m' : HermR R σ n → Measure (Fin n → Fin M → R ⊗[F] E)) (s : (Fin 1 → Fin (M + 1) → R ⊗[F] E) → HA R σ (blockOne (algebraMap F E t) J)) (B₀ : Set R) (a : R → Rˣ) (κ₁ cm : Rˣ → ℝ≥0), StageData σ J t n b lam m₁ m' s B₀ a κ₁ cm → ∀ φ : (Fin (n + 1) → Fin (M + 1) → R ⊗[F] E) → ℝ, Continuous φ → HasCompactSupport φ → ContinuousOn (fun z : R × (Fin n → R ⊗[F] E) × HermR R σ n => Preal σ J t m₁ m' s φ z.1 z.2.1 z.2.2) ({α : R | IsUnit α} ×ˢ (Set.univ ×ˢ Unt σ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.GramFiberMeasureConsProduct

theorem **pi_const_smul** {ι X : Type} [Fintype ι] [MeasurableSpace X] (κ : Measure X) [SigmaFinite κ] (c : ℝ≥0) : Measure.pi (fun _ : ι => c • κ) = ((c : ℝ≥0∞) ^ Fintype.card ι) • Measure.pi (fun _ : ι => κ)

theorem **subst_row_translate** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E) (M : ℕ) (J : Matrix (Fin M) (Fin M) E) (t : F) (n : ℕ) (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R] (d : ℕ) (b : Module.Basis (Fin d) F E) (lam : Measure R) (m₁ : HermR R σ 1 → Measure (Fin 1 → Fin (M + 1) → R ⊗[F] E)) (m' : HermR R σ n → Measure (Fin n → Fin M → R ⊗[F] E)) (s : (Fin 1 → Fin (M + 1) → R ⊗[F] E) → HA R σ (blockOne (algebraMap F E t) J)) (B₀ : Set R) (a : R → Rˣ) (κ₁ cm : Rˣ → ℝ≥0), StageData σ J t n b lam m₁ m' s B₀ a κ₁ cm → ∀ (u : Rˣ) (w : (Fin n → R ⊗[F] E) → HermR R σ n), Measurable w → ∀ f : (Fin n → R ⊗[F] E) × HermR R σ n → ℝ≥0∞, Measurable f → ∫⁻ c, ∫⁻ S', f (c, S') ∂(coordHerm R σ b lam n) ∂(Measure.pi fun _ => coordK R b lam) = (cm u : ℝ≥0∞) ^ n * ∫⁻ r, ∫⁻ S'', f (fun j => ((((u⁻¹ : Rˣ) :

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theorem **_root_.GlobalTheory.T.lintegral_coordK_coordHerm_scale_shear** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E) (M : ℕ) (J : Matrix (Fin M) (Fin M) E) (t : F) (n : ℕ) (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R] (d : ℕ) (b : Module.Basis (Fin d) F E) (lam : Measure R) (m₁ : HermR R σ 1 → Measure (Fin 1 → Fin (M + 1) → R ⊗[F] E)) (m' : HermR R σ n → Measure (Fin n → Fin M → R ⊗[F] E)) (s : (Fin 1 → Fin (M + 1) → R ⊗[F] E) → HA R σ (blockOne (algebraMap F E t) J)) (B₀ : Set R) (a : R → Rˣ) (κ₁ cm : Rˣ → ℝ≥0), StageData σ J t n b lam m₁ m' s B₀ a κ₁ cm → ∀ (u : Rˣ) (w : (Fin n → R ⊗[F] E) → HermR R σ n), Measurable w → ∀ f : (Fin n → R ⊗[F] E) × HermR R σ n → ℝ≥0∞, Measurable f → ∫⁻ c, ∫⁻ S', f (c, S') ∂(coordHerm R σ b lam n) ∂(Measure.pi fun _ => coordK R b lam) = (cm u : ℝ≥0∞) ^ n * ∫

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theorem **fibre_on_orbit** (V X Y : Type) [AddCommGroup V] [TopologicalSpace V] [LocallyCompactSpace V] [SecondCountableTopology V] [T2Space V] [MeasurableSpace V] [BorelSpace V] [IsTopologicalAddGroup V] [AddCommGroup X] [TopologicalSpace X] [LocallyCompactSpace X] [SecondCountableTopology X] [T2Space X] [MeasurableSpace X] [BorelSpace X] [IsTopologicalAddGroup X] (μ : Measure V) (ν : Measure X) [μ.IsAddHaarMeasure] [ν.IsAddHaarMeasure] (f : V → Y) (g : V → X) (ι : X → Y) (hι : Function.Injective ι) (hg : Continuous g) (hf : ∀ v, f v = ι (g v)) (Q : Type) [Group Q] [TopologicalSpace Q] [IsTopologicalGroup Q] [LocallyCompactSpace Q] [SecondCountableTopology Q] [T2Space Q] [DistribMulAction Q V] [ContinuousSMul Q V] [DistribMulAction Q X] [ContinuousSMul Q X] (heq : ∀ (q : Q) (v : V), g (q • v) = q • g v) (b₀ : X) (ho : IsOpenEmbedding (fun q : Q => q • b₀)) : ∃ fib : X → Measure V, Formalism

theorem **exists_fibre_near** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R], IsLocR R → ∀ (n : ℕ) (μ : Measure (Fin n → Fin M → R ⊗[F] E)) (ν : Measure (HermR R σ n)), μ.IsAddHaarMeasure → ν.IsAddHaarMeasure → ∀ S : HermR R σ n, IsUnit (S : Matrix (Fin n) (Fin n) (R ⊗[F] E)).det → ∃ B : Set (HermR R σ n), IsOpen B ∧ S ∈ B ∧ (∀ S' ∈ B, IsUnit (S' : Matrix (Fin n) (Fin n) (R ⊗[F] E)).det) ∧ ∃ fib, Formalism.MainNotions.GlobalTheoryTheory.GenusMassVolumes.IsFibreFamily μ (gramR R σ J) Subtype.val (gramR R σ J ⁻¹' (Subtype.val '' B)) ν B fib

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.GramFiberMeasureDisintegrationFormulaStatement

theorem **_root_.GlobalTheory.T.gram_fiber_measure_disintegration_formula** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R], IsLocR R → ∀ (n : ℕ) (μ : Measure (Fin n → Fin M → R ⊗[F] E)) (ν : Measure (HermR R σ n)), μ.IsAddHaarMeasure → ν.IsAddHaarMeasure → ∀ S : HermR R σ n, IsUnit (S : Matrix (Fin n) (Fin n) (R ⊗[F] E)).det → ∃ B : Set (HermR R σ n), IsOpen B ∧ S ∈ B ∧ (∀ S' ∈ B, IsUnit (S' : Matrix (Fin n) (Fin n) (R ⊗[F] E)).det) ∧ ∃ fib, Formalism.MainNotions.GlobalTheoryTheory.GenusMassVolumes.IsFibreFamily μ (gramR R σ J) Subtype.val (gramR R σ J ⁻¹' (Subtype.val '' B)) ν B fib

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theorem **gramFib_basic** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (N n : ℕ) (J₂ : Matrix (Fin N) (Fin N) E), IsHermND σ J₂ → ∀ (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R], IsLocR R → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (lam : Measure R), lam.IsAddHaarMeasure → ∀ m : HermR R σ n → Measure (Fin n → Fin N → R ⊗[F] E), IsGramFib R σ J₂ n (coordW R b lam n N) (coordHerm R σ b lam n) m → IsOpen (Unt σ R n) ∧ (∀ S, IsLocallyFiniteMeasure (m S)) ∧ (∀ S, S ∉ Unt σ R n → m S = 0) ∧ (∀ A : Set (Fin n → Fin N → R ⊗[F] E), MeasurableSet A → Measurable fun S => m S A) ∧ (∀ (C : Set (Fin n → Fin N → R ⊗[F] E)) (D : Set (HermR R σ n)), IsCompact C → IsCompact D → D ⊆ Unt σ R n → ∃ c : ℝ≥0, ∀ S ∈ D, m S C ≤ c) ∧ ∀ S : HermR R σ n, ∀ᵐ X ∂(m S), gramR R σ J₂ X = (S : Matrix (Fin

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.GramFiberMeasureGeneralRankExtensionStatement

theorem **_root_.GlobalTheory.T.gram_fiber_measure_locally_finite_measurable_supported** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (N n : ℕ) (J₂ : Matrix (Fin N) (Fin N) E), IsHermND σ J₂ → ∀ (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R], IsLocR R → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (lam : Measure R), lam.IsAddHaarMeasure → ∀ m : HermR R σ n → Measure (Fin n → Fin N → R ⊗[F] E), IsGramFib R σ J₂ n (coordW R b lam n N) (coordHerm R σ b lam n) m → IsOpen (Unt σ R n) ∧ (∀ S, IsLocallyFiniteMeasure (m S)) ∧ (∀ S, S ∉ Unt σ R n → m S = 0) ∧ (∀ A : Set (Fin n → Fin N → R ⊗[F] E), MeasurableSet A → Measurable fun S => m S A) ∧ (∀ (C : Set (Fin n → Fin N → R ⊗[F] E)) (D : Set (HermR R σ n)), IsCompact C → IsCompact D → D ⊆ Unt σ R n → ∃ c : ℝ≥0, ∀ S ∈ D, m S C ≤ c) ∧ ∀ S

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theorem **measurable_cons_tail** {A : Type} [MeasurableSpace A] {n : ℕ} (x : A) : Measurable (fun Y : Fin n → A => (Fin.cons x Y : Fin (n + 1) → A))

theorem **piFinSuccAbove_symm_apply** {A : Type} [MeasurableSpace A] {n : ℕ} (x : A) (Y : Fin n → A) : (MeasurableEquiv.piFinSuccAbove (fun _ : Fin (n + 1) => A) 0).symm (x, Y) = Fin.cons x Y

theorem **lintegral_tuple_succ** (m : Measure T) [SigmaFinite m] (n N : ℕ) (ψ : (Fin (n + 1) → Fin N → T) → ℝ≥0∞) (hψ : Measurable ψ) : ∫⁻ X, ψ X ∂(Measure.pi fun _ : Fin (n + 1) => Measure.pi fun _ : Fin N => m) = ∫⁻ y : Fin 1 → Fin N → T, ∫⁻ Y : Fin n → Fin N → T, ψ (Fin.cons (y 0) Y) ∂(Measure.pi fun _ : Fin n => Measure.pi fun _ : Fin N => m) ∂(Measure.pi fun _ : Fin 1 => Measure.pi fun _ : Fin N => m)

theorem **lintegral_tuple_first_coord** (m : Measure T) [SigmaFinite m] (n M : ℕ) (ψ : (Fin n → Fin (M + 1) → T) → ℝ≥0∞) (hψ : Measurable ψ) : ∫⁻ Y, ψ Y ∂(Measure.pi fun _ : Fin n => Measure.pi fun _ : Fin (M + 1) => m) = ∫⁻ c : Fin n → T, ∫⁻ X : Fin n → Fin M → T, ψ (fun a => Fin.cons (c a) (X a)) ∂(Measure.pi fun _ : Fin n => Measure.pi fun _ : Fin M => m) ∂(Measure.pi fun _ : Fin n => m)

theorem **lintegral_tuple_moving_frame** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin (M + 1)) (Fin (M + 1)) E), IsHermND σ J → ∀ (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R], IsLocR R → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E), ∀ (lam : Measure R), lam.IsAddHaarMeasure → ∀ (n : ℕ) (s : (Fin 1 → Fin (M + 1) → R ⊗[F] E) → HA R σ J), Measurable s → ∀ φ : (Fin (n + 1) → Fin (M + 1) → R ⊗[F] E) → ℝ≥0∞, Measurable φ → ∫⁻ X, φ X ∂(coordW R b lam (n + 1) (M + 1)) = ∫⁻ y, ∫⁻ c : Fin n → R ⊗[F] E, ∫⁻ X : Fin n → Fin M → R ⊗[F] E, φ (Fin.cons (y 0) (actW R σ J (s y) (fun a => Fin.cons (c a) (X a)))) ∂(coordW R b lam n M) ∂(Measure.pi fun _ => coordK R b lam) ∂(coordW R b lam 1 (M + 1))

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theorem **_root_.GlobalTheory.T.lintegral_coordW_succ_eq_iterated_of_unitary_section** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin (M + 1)) (Fin (M + 1)) E), IsHermND σ J → ∀ (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R], IsLocR R → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E), ∀ (lam : Measure R), lam.IsAddHaarMeasure → ∀ (n : ℕ) (s : (Fin 1 → Fin (M + 1) → R ⊗[F] E) → HA R σ J), Measurable s → ∀ φ : (Fin (n + 1) → Fin (M + 1) → R ⊗[F] E) → ℝ≥0∞, Measurable φ → ∫⁻ X, φ X ∂(coordW R b lam (n + 1) (M + 1)) = ∫⁻ y, ∫⁻ c : Fin n → R ⊗[F] E, ∫⁻ X : Fin n → Fin M → R ⊗[F] E, φ (Fin.cons (y 0) (actW R σ J (s y) (fun a => Fin.cons (c a) (X a)))) ∂(coordW R b lam n M) ∂(Measure.pi fun _ => coordK R b lam) ∂(coordW R b lam 1 (M + 1))

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theorem **fibre_map_isometry** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (J' : Matrix (Fin M) (Fin M) E), IsHermND σ J' → ∀ (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R], IsLocR R → ∀ (n : ℕ), ∀ (μ : Measure (Fin n → Fin M → R ⊗[F] E)) (ν : Measure (HermR R σ n)) (m : HermR R σ n → Measure (Fin n → Fin M → R ⊗[F] E)), μ.IsAddHaarMeasure → ν.IsAddHaarMeasure → IsGramFib R σ J n μ ν m → ∀ (μ' : Measure (Fin n → Fin M → R ⊗[F] E)) m', μ'.IsAddHaarMeasure → IsGramFib R σ J' n μ' ν m' → ∀ g : GL (Fin M) (R ⊗[F] E), JR R J' = cT (sigR R σ) (g : Matrix (Fin M) (Fin M) (R ⊗[F] E)) ∗ JR R J ∗ (g : Matrix (Fin M) (Fin M) (R ⊗[F] E)) → ∀ a : ℝ≥0, μ'.map (actTup R g) = a • μ → ∀ S, (m' S).map (actTup R g) = a • m S

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theorem **_root_.GlobalTheory.T.gram_fiber_measures_change_of_form** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (J' : Matrix (Fin M) (Fin M) E), IsHermND σ J' → ∀ (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R], IsLocR R → ∀ (n : ℕ), ∀ (μ : Measure (Fin n → Fin M → R ⊗[F] E)) (ν : Measure (HermR R σ n)) (m : HermR R σ n → Measure (Fin n → Fin M → R ⊗[F] E)), μ.IsAddHaarMeasure → ν.IsAddHaarMeasure → IsGramFib R σ J n μ ν m → ∀ (μ' : Measure (Fin n → Fin M → R ⊗[F] E)) m', μ'.IsAddHaarMeasure → IsGramFib R σ J' n μ' ν m' → ∀ g : GL (Fin M) (R ⊗[F] E), JR R J' = cT (sigR R σ) (g : Matrix (Fin M) (Fin M) (R ⊗[F] E)) ∗ JR R J ∗ (g : Matrix (Fin M) (Fin M) (R ⊗[F] E)) → ∀ a : ℝ≥0, μ'.map (actTup R g) = a • μ → ∀ S, (m' S)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.GramFiberMeasuresContinuity

theorem **null_of_fibres_null** {V X Y : Type} [TopologicalSpace V] [MeasurableSpace V] [TopologicalSpace X] [MeasurableSpace X] {μ : Measure V} {f : V → Y} {ι : X → Y} {U : Set V} {ν : Measure X} {B : Set X} {fib : X → Measure V} (h : Formalism.MainNotions.GlobalTheoryTheory.GenusMassVolumes.IsFibreFamily μ f ι U ν B fib) {W : Set V} (hW : MeasurableSet W) (h0 : ∀ b, fib b W = 0) : μ (W ∩ U) = 0

theorem **fibre_loc_fin** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R], IsLocR R → ∀ (n : ℕ), ∀ (μ : Measure (Fin n → Fin M → R ⊗[F] E)) (ν : Measure (HermR R σ n)) (m : HermR R σ n → Measure (Fin n → Fin M → R ⊗[F] E)), μ.IsAddHaarMeasure → ν.IsAddHaarMeasure → IsGramFib R σ J n μ ν m → ∀ S : HermR R σ n, IsLocallyFiniteMeasure (m S) ∧ ∀ U : Set (Fin n → Fin M → R ⊗[F] E), IsOpen U → (∃ X ∈ U, gramR R σ J X = S ∧ IsUnit (gramR R σ J X).det) → m S U ≠ 0

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.GramFiberMeasuresContinuityStatement

theorem **_root_.GlobalTheory.T.gram_fiber_measure_locallyFinite_and_ne_zero_of_isOpen** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R], IsLocR R → ∀ (n : ℕ), ∀ (μ : Measure (Fin n → Fin M → R ⊗[F] E)) (ν : Measure (HermR R σ n)) (m : HermR R σ n → Measure (Fin n → Fin M → R ⊗[F] E)), μ.IsAddHaarMeasure → ν.IsAddHaarMeasure → IsGramFib R σ J n μ ν m → ∀ S : HermR R σ n, IsLocallyFiniteMeasure (m S) ∧ ∀ U : Set (Fin n → Fin M → R ⊗[F] E), IsOpen U → (∃ X ∈ U, gramR R σ J X = S ∧ IsUnit (gramR R σ J X).det) → m S U ≠ 0

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theorem **tupR_tupR** (X : Fin n → Fin M → K) (g h : Matrix (Fin n) (Fin n) K) : tupR (tupR X g) h = tupR X (g ∗ h)

theorem **tupR_one** (X : Fin n → Fin M → K) : tupR X 1 = X

theorem **cT_one'** (s : K →+* K) : cT s (1 : Matrix (Fin n) (Fin n) K) = 1

theorem **cT_mul'** (s : K →+* K) (A B : Matrix (Fin n) (Fin n) K) : cT s (A ∗ B) = cT s B ∗ cT s A

theorem **continuous_inv_val** : Continuous fun q : GL (Fin n) K => ((q⁻¹ : GL (Fin n) K) : Matrix (Fin n) (Fin n) K)

theorem **tup_continuousSMul** : @ContinuousSMul (GL (Fin n) K) (Fin n → Fin M → K) (tupAction (K

theorem **herm_continuousSMul** (s : K →+* K) (hs : Continuous s) (H : AddSubgroup (Matrix (Fin n) (Fin n) K)) (hmem : ∀ (S : H) (g : Matrix (Fin n) (Fin n) K), cT s g ∗ (S : Matrix (Fin n) (Fin n) K) ∗ g ∈ H) : @ContinuousSMul (GL (Fin n) K) H (hermAction s H hmem).toSMul _ _

theorem **fibre_scale_continuous** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R], IsLocR R → ∀ (n : ℕ), ∀ (μ : Measure (Fin n → Fin M → R ⊗[F] E)) (ν : Measure (HermR R σ n)) (m : HermR R σ n → Measure (Fin n → Fin M → R ⊗[F] E)), μ.IsAddHaarMeasure → ν.IsAddHaarMeasure → IsGramFib R σ J n μ ν m → ∃ κ : GL (Fin n) (R ⊗[F] E) → ℝ≥0, Continuous κ ∧ (∀ g, κ g ≠ 0) ∧ κ 1 = 1 ∧ ∀ (g : GL (Fin n) (R ⊗[F] E)) (S S' : HermR R σ n), (S' : Matrix (Fin n) (Fin n) (R ⊗[F] E)) = cT (sigR R σ) (g : Matrix (Fin n) (Fin n) (R ⊗[F] E)) ∗ (S : Matrix (Fin n) (Fin n) (R ⊗[F] E)) ∗ (g : Matrix (Fin n) (Fin n) (R ⊗[F] E)) → m S' = κ g • (m S).map (fun X => rmul R X (g : Matrix (Fin n) (Fin n)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.GramFiberMeasuresIntegralFormulaStatement

theorem **_root_.GlobalTheory.T.gram_fiber_measures_gl_equivariant** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R], IsLocR R → ∀ (n : ℕ), ∀ (μ : Measure (Fin n → Fin M → R ⊗[F] E)) (ν : Measure (HermR R σ n)) (m : HermR R σ n → Measure (Fin n → Fin M → R ⊗[F] E)), μ.IsAddHaarMeasure → ν.IsAddHaarMeasure → IsGramFib R σ J n μ ν m → ∃ κ : GL (Fin n) (R ⊗[F] E) → ℝ≥0, Continuous κ ∧ (∀ g, κ g ≠ 0) ∧ κ 1 = 1 ∧ ∀ (g : GL (Fin n) (R ⊗[F] E)) (S S' : HermR R σ n), (S' : Matrix (Fin n) (Fin n) (R ⊗[F] E)) = cT (sigR R σ) (g : Matrix (Fin n) (Fin n) (R ⊗[F] E)) ∗ (S : Matrix (Fin n) (Fin n) (R ⊗[F] E)) ∗ (g : Matrix (Fin n) (Fin n) (R ⊗[F] E)) → m S' = κ g • (m S).map (fun X => rmu

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theorem **map_actW_fibre** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R], IsLocR R → ∀ (n : ℕ), ∀ (μ : Measure (Fin n → Fin M → R ⊗[F] E)) (ν : Measure (HermR R σ n)) (m : HermR R σ n → Measure (Fin n → Fin M → R ⊗[F] E)), μ.IsAddHaarMeasure → ν.IsAddHaarMeasure → IsGramFib R σ J n μ ν m → ∀ (h : HA R σ J) (S : HermR R σ n), (m S).map (actW R σ J h) = m S

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.GramFiberMeasuresInvariantStatement

theorem **_root_.GlobalTheory.T.gram_fiber_measures_invariant** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R], IsLocR R → ∀ (n : ℕ), ∀ (μ : Measure (Fin n → Fin M → R ⊗[F] E)) (ν : Measure (HermR R σ n)) (m : HermR R σ n → Measure (Fin n → Fin M → R ⊗[F] E)), μ.IsAddHaarMeasure → ν.IsAddHaarMeasure → IsGramFib R σ J n μ ν m → ∀ (h : HA R σ J) (S : HermR R σ n), (m S).map (actW R σ J h) = m S

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.GramFiberMeasuresScaling

theorem **isGramFib_rescale** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R], IsLocR R → ∀ (n : ℕ), ∀ (μ : Measure (Fin n → Fin M → R ⊗[F] E)) (ν : Measure (HermR R σ n)) (m : HermR R σ n → Measure (Fin n → Fin M → R ⊗[F] E)), μ.IsAddHaarMeasure → ν.IsAddHaarMeasure → IsGramFib R σ J n μ ν m → ∀ a c : ℝ≥0, a ≠ 0 → c ≠ 0 → IsGramFib R σ J n (a • μ) (c • ν) (fun S => (a / c) • m S)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.GramFiberMeasuresScalingStatement

theorem **_root_.GlobalTheory.T.gram_fiber_measures_scaling** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R], IsLocR R → ∀ (n : ℕ), ∀ (μ : Measure (Fin n → Fin M → R ⊗[F] E)), μ.IsAddHaarMeasure → ν.IsAddHaarMeasure → IsGramFib R σ J n μ ν m → ∀ a c : ℝ≥0, a ≠ 0 → c ≠ 0 → IsGramFib R σ J n (a • μ) (c • ν) (fun S => (a / c) • m S)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.GramFiberMeasuresSupport

theorem **fib_rmul** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R], IsLocR R → ∀ (n : ℕ), ∀ (μ : Measure (Fin n → Fin M → R ⊗[F] E)) (ν : Measure (HermR R σ n)) (m : HermR R σ n → Measure (Fin n → Fin M → R ⊗[F] E)), μ.IsAddHaarMeasure → ν.IsAddHaarMeasure → IsGramFib R σ J n μ ν m → ∀ (g : Matrix (Fin n) (Fin n) (R ⊗[F] E)), IsUnit g.det → ∀ τ : HermR R σ n → HermR R σ n, (∀ S, (τ S : Matrix (Fin n) (Fin n) (R ⊗[F] E)) = cT (sigR R σ) g ∗ S ∗ g) → ∀ a c : ℝ≥0, μ.map (fun X => rmul R X g) = a • μ → ν.map τ = c • ν → ∀ S, a • m (τ S) = c • (m S).map (fun X => rmul R X g)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.GramFiberMeasuresSupportStatement

theorem **_root_.GlobalTheory.T.gram_fiber_measures_rmul_equivariant** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R], IsLocR R → ∀ (n : ℕ), ∀ (μ : Measure (Fin n → Fin M → R ⊗[F] E)) (ν : Measure (HermR R σ n)) (m : HermR R σ n → Measure (Fin n → Fin M → R ⊗[F] E)), μ.IsAddHaarMeasure → ν.IsAddHaarMeasure → IsGramFib R σ J n μ ν m → ∀ (g : Matrix (Fin n) (Fin n) (R ⊗[F] E)), IsUnit g.det → ∀ τ : HermR R σ n → HermR R σ n, (∀ S, (τ S : Matrix (Fin n) (Fin n) (R ⊗[F] E)) = cT (sigR R σ) g ∗ S ∗ g) → ∀ a c : ℝ≥0, μ.map (fun X => rmul R X g) = a • μ → ν.map τ = c • ν → ∀ S, a • m (τ S) = c • (m S).map (fun X => rmul R X g)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.GramFiberMeasuresTransitivity

theorem **isContDensityOn_gram** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R], IsLocR R → ∀ (n : ℕ), ∀ (μ : Measure (Fin n → Fin M → R ⊗[F] E)) (ν : Measure (HermR R σ n)) (m : HermR R σ n → Measure (Fin n → Fin M → R ⊗[F] E)), μ.IsAddHaarMeasure → ν.IsAddHaarMeasure → IsGramFib R σ J n μ ν m → ∀ φ : (Fin n → Fin M → R ⊗[F] E) → ℂ, Continuous φ → HasCompactSupport φ → IsContDensityOn μ (gramR R σ J) Subtype.val ν {S | IsUnit (S : Matrix (Fin n) (Fin n) (R ⊗[F] E)).det} φ (fun S => ∫ X, φ X ∂(m S)) ∧ ∀ S, Integrable φ (m S)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.GramFiberMeasuresTransitivityStatement

theorem **_root_.GlobalTheory.T.gram_fiber_integral_isContDensityOn** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R], IsLocR R → ∀ (n : ℕ), ∀ (μ : Measure (Fin n → Fin M → R ⊗[F] E)) (ν : Measure (HermR R σ n)) (m : HermR R σ n → Measure (Fin n → Fin M → R ⊗[F] E)), μ.IsAddHaarMeasure → ν.IsAddHaarMeasure → IsGramFib R σ J n μ ν m → ∀ φ : (Fin n → Fin M → R ⊗[F] E) → ℂ, Continuous φ → HasCompactSupport φ → IsContDensityOn μ (gramR R σ J) Subtype.val ν {S | IsUnit (S : Matrix (Fin n) (Fin n) (R ⊗[F] E)).det} φ (fun S => ∫ X, φ X ∂(m S)) ∧ ∀ S, Integrable φ (m S)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.GramFiberMeasuresUniqueAe

theorem **fibre_unique** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R], IsLocR R → ∀ (n : ℕ), ∀ (μ : Measure (Fin n → Fin M → R ⊗[F] E)) (ν : Measure (HermR R σ n)) (m : HermR R σ n → Measure (Fin n → Fin M → R ⊗[F] E)), μ.IsAddHaarMeasure → ν.IsAddHaarMeasure → IsGramFib R σ J n μ ν m → ∀ m', IsGramFib R σ J n μ ν m' → ∀ S, m S = m' S

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.GramFiberMeasuresUniqueAeStatement

theorem **_root_.GlobalTheory.T.gram_fiber_measures_unique** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R], IsLocR R → ∀ (n : ℕ), ∀ (μ : Measure (Fin n → Fin M → R ⊗[F] E)) (ν : Measure (HermR R σ n)) (m : HermR R σ n → Measure (Fin n → Fin M → R ⊗[F] E)), μ.IsAddHaarMeasure → ν.IsAddHaarMeasure → IsGramFib R σ J n μ ν m → ∀ m', IsGramFib R σ J n μ ν m' → ∀ S, m S = m' S

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.GramMatrixLocalGlobal

theorem **bcE_inclR** (f : R →ₐ[F] S) (e : E) : bcE E f (inclR R e) = inclR S e

theorem **bcE_sigR** (f : R →ₐ[F] S) (σ : E →ₐ[F] E) (z : R ⊗[F] E) : bcE E f (sigR R σ z) = sigR S σ (bcE E f z)

theorem **bcE_hR** (f : R →ₐ[F] S) (σ : E →ₐ[F] E) {M : ℕ} (J : Matrix (Fin M) (Fin M) E) (x y : Fin M → R ⊗[F] E) : bcE E f (hR R σ J x y) = hR S σ J (fun i => bcE E f (x i)) (fun i => bcE E f (y i))

theorem **bcE_gramR** (f : R →ₐ[F] S) (σ : E →ₐ[F] E) {M n : ℕ} (J : Matrix (Fin M) (Fin M) E) (X : Fin n → Fin M → R ⊗[F] E) : gramR S σ J (fun a i => bcE E f (X a i)) = fun a b => bcE E f (gramR R σ J X a b)

theorem **repr_bcE** {ι : Type} [Fintype ι] [DecidableEq ι] (b : Module.Basis ι F E) (f : R →ₐ[F] S) (z : R ⊗[F] E) (k : ι) : (Algebra.TensorProduct.basis S b).repr (bcE E f z) k = f ((Algebra.TensorProduct.basis R b).repr z k)

theorem **gramR_ratTup** (σ : E →ₐ[F] E) {M n : ℕ} (J : Matrix (Fin M) (Fin M) E) (Y : Fin n → Fin M → E) : gramR R σ J (ratTup R Y) = (gramE σ J Y).map (inclR R)

theorem **adele_ext** (x y : AF F) (h1 : fstA F x = fstA F y) (h2 : ∀ v, evalV F v (sndA F x) = evalV F v (sndA F y)) : x = y

theorem **tensor_ext** [Module.Finite F E] (z w : AF F ⊗[F] E) (h1 : bcE E (fstA F) z = bcE E (fstA F) w) (h2 : ∀ v, bcE E (evalV F v) (bcE E (sndA F) z) = bcE E (evalV F v) (bcE E (sndA F) w)) : z = w

theorem **gram_local** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (n : ℕ) (X : (Fin n → Fin M → AF F ⊗[F] E)) (T : Matrix (Fin n) (Fin n) E), (gramR (AF F) σ J X = T.map (inclR (AF F)) ↔ gramR (Ainf F) σ J (projInf X) = T.map (inclR (Ainf F)) ∧ ∀ v, gramR (Fv F v) σ J (projV v (projFin X)) = T.map (inclR (Fv F v))) ∧ ∀ Y : Fin n → Fin M → E, gramR (AF F) σ J (ratTup (AF F) Y) = (gramE σ J Y).map (inclR (AF F))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.GramMatrixLocalGlobalStatement

theorem **_root_.GlobalTheory.T.gram_matrix_local_global** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (n : ℕ) (X : (Fin n → Fin M → AF F ⊗[F] E)) (T : Matrix (Fin n) (Fin n) E), (gramR (AF F) σ J X = T.map (inclR (AF F)) ↔ gramR (Ainf F) σ J (projInf X) = T.map (inclR (Ainf F)) ∧ ∀ v, gramR (Fv F v) σ J (projV v (projFin X)) = T.map (inclR (Fv F v))) ∧ ∀ Y : Fin n → Fin M → E, gramR (AF F) σ J (ratTup (AF F) Y) = (gramE σ J Y).map (inclR (AF F))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.GramOrbitCompactRepresentative

theorem **compact_gram_fibre** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (n : ℕ) (X : Fin n → Fin M → E), (gramE σ J X).det ≠ 0 → ∀ Ω : Set (Fin n → Fin M → AF F ⊗[F] E), IsCompact Ω → ∃ C : Set (HA (AF F) σ J), IsCompact C ∧ ∀ Y ∈ Ω, gramR (AF F) σ J Y = (gramE σ J X).map (inclR (AF F)) → ∃ c ∈ C, actW (AF F) σ J c (ratTup (AF F) X) = Y

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.GramOrbitCompactRepresentativeStatement

theorem **_root_.GlobalTheory.T.gram_orbit_compact_representative** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (n : ℕ) (X : Fin n → Fin M → E), (gramE σ J X).det ≠ 0 → ∀ Ω : Set (Fin n → Fin M → AF F ⊗[F] E), IsCompact Ω → ∃ C : Set (HA (AF F) σ J), IsCompact C ∧ ∀ Y ∈ Ω, gramR (AF F) σ J Y = (gramE σ J X).map (inclR (AF F)) → ∃ c ∈ C, actW (AF F) σ J c (ratTup (AF F) X) = Y

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.GramRightActionIdentities

theorem **colM_injective** : Function.Injective (colM : (Fin n → Fin M → A) → _)

theorem **hform_eq_entry** (s : A →+* A) (J : Matrix (Fin M) (Fin M) A) (X : Fin n → Fin M → A) (a b : Fin n) : hform s J (X a) (X b) = (cT s (colM X) * J * colM X) a b

theorem **mul_shape** {p q r s t u : Type} [Fintype q] [Fintype r] [Fintype s] [Fintype t] (a : Matrix p q A) (b : Matrix q r A) (c : Matrix r s A) (d : Matrix s t A) (e : Matrix t u A) : (((a * b : Matrix p r A) * c : Matrix p s A) * (d * e : Matrix s u A) : Matrix p u A) = ((a * ((b * c : Matrix q s A) * d : Matrix q t A) : Matrix p t A) * e : Matrix p u A)

theorem **cT_cT** (s : A →+* A) (hs : ∀ x, s (s x) = x) {m p : Type} (B : Matrix m p A) : cT s (cT s B) = B

theorem **sigma_invol** [NumberField F] [NumberField E] {σ : E →ₐ[F] E} (hσ : IsCMExt F E σ) (x : E) : σ (σ x) = x

theorem **sigR_invol** {σ : E →ₐ[F] E} (h : ∀ x, σ (σ x) = x) (x : R ⊗[F] E) : sigR R σ (sigR R σ x) = x

theorem **gramR_eq** (σ : E →ₐ[F] E) (J : Matrix (Fin M) (Fin M) E) (X : Fin n → Fin M → R ⊗[F] E) : gramR R σ J X = (cT (sigR R σ) (colM X) * JR R J : Matrix (Fin n) (Fin M) (R ⊗[F] E)) * colM X

theorem **colM_rmul** (X : Fin n → Fin M → R ⊗[F] E) (g : Matrix (Fin n) (Fin n) (R ⊗[F] E)) : colM (rmul R X g) = colM X * g

theorem **colM_actTup** (g : GL (Fin M) (R ⊗[F] E)) (X : Fin n → Fin M → R ⊗[F] E) : colM (actTup R g X) = (g : Matrix (Fin M) (Fin M) (R ⊗[F] E)) * colM X

theorem **gram_algebra** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (R : Type) [CommRing R] [Algebra F R] (n : ℕ), (∀ (X : Fin n → Fin M → R ⊗[F] E) (g : Matrix (Fin n) (Fin n) (R ⊗[F] E)), gramR R σ J (rmul R X g) = cT (sigR R σ) g * gramR R σ J X * g) ∧ (∀ (X : Fin n → Fin M → R ⊗[F] E) (g g' : Matrix (Fin n) (Fin n) (R ⊗[F] E)), rmul R (rmul R X g) g' = rmul R X (g * g')) ∧ (∀ X : Fin n → Fin M → R ⊗[F] E, rmul R X 1 = X) ∧ (∀ (S : HermR R σ n) (g : Matrix (Fin n) (Fin n) (R ⊗[F] E)), cT (sigR R σ) g * (S : Matrix (Fin n) (Fin n) (R ⊗[F] E)) * g ∈ HermR R σ n) ∧ ∀ (J' : Matrix (Fin M) (Fin M) E) (g : GL (Fin M) (R ⊗[F] E)), JR R J' = cT (sigR R σ) (g : Matrix (Fin M) (Fin M) (R ⊗[F] E)) * JR R J * (g : Matrix (Fin M) (Fin M) (R ⊗[F] E)) → ∀ X : Fin n → Fin M

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.GramRightActionIdentitiesStatement

theorem **_root_.GlobalTheory.T.gram_right_action_identities** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (R : Type) [CommRing R] [Algebra F R] (n : ℕ), (∀ (X : Fin n → Fin M → R ⊗[F] E) (g : Matrix (Fin n) (Fin n) (R ⊗[F] E)), gramR R σ J (rmul R X g) = cT (sigR R σ) g * gramR R σ J X * g) ∧ (∀ (X : Fin n → Fin M → R ⊗[F] E) (g g' : Matrix (Fin n) (Fin n) (R ⊗[F] E)), rmul R (rmul R X g) g' = rmul R X (g * g')) ∧ (∀ X : Fin n → Fin M → R ⊗[F] E, rmul R X 1 = X) ∧ (∀ (S : HermR R σ n) (g : Matrix (Fin n) (Fin n) (R ⊗[F] E)), cT (sigR R σ) g * (S : Matrix (Fin n) (Fin n) (R ⊗[F] E)) * g ∈ HermR R σ n) ∧ ∀ (J' : Matrix (Fin M) (Fin M) E) (g : GL (Fin M) (R ⊗[F] E)), JR R J' = cT (sigR R σ) (g : Matrix (Fin M) (Fin M) (R ⊗[F] E)) * JR R J * (g : Matrix (Fin M) (Fin

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.GramRightMulActionLaws

theorem **mul_regroup** {A : Type} [CommRing A] {m n : Type} [Fintype m] [Fintype n] (P : Matrix n n A) (B : Matrix n m A) (J : Matrix m m A) (C : Matrix m n A) (g : Matrix n n A) : (P * B) * J * (C * g) = P * ((B * J : Matrix n m A) * C) * g

theorem **colsM_injective** {M n : ℕ} {X Y : Fin n → Fin M → R ⊗[F] E} (h : colsM R X = colsM R Y) : X = Y

theorem **gramR_eq_mul** (σ : E →ₐ[F] E) {M n : ℕ} (J : Matrix (Fin M) (Fin M) E) (X : Fin n → Fin M → R ⊗[F] E) : gramR R σ J X = (cT (sigR R σ) (colsM R X) * JR R J : Matrix (Fin n) (Fin M) (R ⊗[F] E)) * colsM R X

theorem **colsM_rmul** {M n : ℕ} (X : Fin n → Fin M → R ⊗[F] E) (g : Matrix (Fin n) (Fin n) (R ⊗[F] E)) : colsM R (rmul R X g) = colsM R X * g

theorem **gramR_rmul** (σ : E →ₐ[F] E) {M n : ℕ} (J : Matrix (Fin M) (Fin M) E) (X : Fin n → Fin M → R ⊗[F] E) (g : Matrix (Fin n) (Fin n) (R ⊗[F] E)) : gramR R σ J (rmul R X g) = cT (sigR R σ) g * gramR R σ J X * g

theorem **rmul_rmul** {M n : ℕ} (X : Fin n → Fin M → R ⊗[F] E) (g g' : Matrix (Fin n) (Fin n) (R ⊗[F] E)) : rmul R (rmul R X g) g' = rmul R X (g * g')

theorem **rmul_one** {M n : ℕ} (X : Fin n → Fin M → R ⊗[F] E) : rmul R X 1 = X

theorem **actTup_rmul** {M n : ℕ} (k : GL (Fin M) (R ⊗[F] E)) (X : Fin n → Fin M → R ⊗[F] E) (g : Matrix (Fin n) (Fin n) (R ⊗[F] E)) : actTup R k (rmul R X g) = rmul R (actTup R k X) g

theorem **rmul_gram_facts** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (R : Type) [CommRing R] [Algebra F R] (M n : ℕ) (J : Matrix (Fin M) (Fin M) E) (X : Fin n → Fin M → R ⊗[F] E) (g g' : Matrix (Fin n) (Fin n) (R ⊗[F] E)), gramR R σ J (rmul R X g) = cT (sigR R σ) g * gramR R σ J X * g ∧ rmul R (rmul R X g) g' = rmul R X (g * g') ∧ rmul R X 1 = X ∧ ∀ k : GL (Fin M) (R ⊗[F] E), actTup R k (rmul R X g) = rmul R (actTup R k X) g

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.GramRightMulActionLawsStatement

theorem **_root_.GlobalTheory.T.gram_right_mul_action_laws** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (R : Type) [CommRing R] [Algebra F R] (M n : ℕ) (J : Matrix (Fin M) (Fin M) E) (X : Fin n → Fin M → R ⊗[F] E) (g g' : Matrix (Fin n) (Fin n) (R ⊗[F] E)), gramR R σ J (rmul R X g) = cT (sigR R σ) g * gramR R σ J X * g ∧ rmul R (rmul R X g) g' = rmul R X (g * g') ∧ rmul R X 1 = X ∧ ∀ k : GL (Fin M) (R ⊗[F] E), actTup R k (rmul R X g) = rmul R (actTup R k X) g

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.GramTransposeDualTraceIdentity

theorem **DM_mulVec_emb** {F : Type} [Field F] [NumberField F] {ι : Type} [Fintype ι] [DecidableEq ι] [DecidableEq (InfinitePlace F)] (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (y : ι → F) : DM B ρ *ᵥ emb ρ y = emb ρ (B *ᵥ y)

theorem **gM_transpose_mulVec** {F : Type} [Field F] [NumberField F] {ι : Type} [Fintype ι] [DecidableEq ι] [DecidableEq (InfinitePlace F)] (ρ : InfinitePlace F → (F →+* ℝ)) (b : InfinitePlace F → 𝒪 F) (N : ℕ) (z : ι × InfinitePlace F → ℝ) (i : ι) (v : InfinitePlace F) : ((gM ρ b N)ᵀ *ᵥ z) (i, v) = (N : ℝ) * ∑ w, ρ w (b v : F) * z (i, w)

theorem **gM_DM_emb_eq_trace** : ∀ (F : Type) [Field F] [NumberField F] (ι : Type) [Fintype ι] [DecidableEq ι] [DecidableEq (InfinitePlace F)] (M : ℕ) (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (P : InfinitePlace F → Matrix ι ι ℝ), QSetup M B ρ P → ∀ (b : InfinitePlace F → 𝒪 F) (N : ℕ) (y : ι → F), (gM ρ b N)ᵀ *ᵥ (DM B ρ *ᵥ emb ρ y) =

fun p => (((N : ℚ) * Algebra.trace ℚ F ((b p.2 : F) * (B *ᵥ y) p.1) : ℚ) : ℝ)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.GramTransposeDualTraceIdentityStatement

theorem **_root_.GlobalTheory.T.gram_transpose_dual_trace_identity** : ∀ (F : Type) [Field F] [NumberField F] (ι : Type) [Fintype ι] [DecidableEq ι] [DecidableEq (InfinitePlace F)] (M : ℕ) (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (P : InfinitePlace F → Matrix ι ι ℝ), QSetup M B ρ P → ∀ (b : InfinitePlace F → 𝒪 F) (N : ℕ) (y : ι → F), (gM ρ b N)ᵀ *ᵥ (DM B ρ *ᵥ emb ρ y) = fun p => (((N : ℚ) * Algebra.trace ℚ F ((b p.2 : F) * (B *ᵥ y) p.1) : ℚ) : ℝ)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.GroupActionHaarScalingContinuous

theorem **exists_continuous_modulus** : ∀ (V : Type) [AddCommGroup V] [TopologicalSpace V] [IsTopologicalAddGroup V] [T2Space V] [LocallyCompactSpace V] [SecondCountableTopology V] [MeasurableSpace V] [BorelSpace V] (μ : Measure V) [μ.IsAddHaarMeasure] (T : Type) [Group T] [TopologicalSpace T] [FirstCountableTopology T] [LocallyCompactSpace T] (a : T → V →+ V), (Continuous fun p : T × V => a p.1 p.2) → Continuous (fun t : T => t⁻¹) → (∀ y, a 1 y = y) → (∀ s t y, a (s * t) y = a s (a t y)) → ∃ χ : T →* ℝ≥0, Continuous χ ∧ ∀ t, μ.map (a t) = χ t • μ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.GroupActionHaarScalingContinuousStatement

theorem **_root_.GlobalTheory.T.group_action_haar_scaling_continuous** : ∀ (V : Type) [AddCommGroup V] [TopologicalSpace V] [IsTopologicalAddGroup V] [T2Space V] [LocallyCompactSpace V] [SecondCountableTopology V] [MeasurableSpace V] [BorelSpace V] (μ : Measure V) [μ.IsAddHaarMeasure] (T : Type) [Group T] [TopologicalSpace T] [FirstCountableTopology T] [LocallyCompactSpace T] (a : T → V →+ V), (Continuous fun p : T × V => a p.1 p.2) → Continuous (fun t : T => t⁻¹) → (∀ y, a 1 y = y) → (∀ s t y, a (s * t) y = a s (a t y)) → ∃ χ : T →* ℝ≥0, Continuous χ ∧ ∀ t, μ.map (a t) = χ t • μ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.HaarFundamentalDomainExistsUnitary

theorem **haar_fundamental_exist** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → (∃ μ, IsHaarH F E σ M J μ) ∧ ∀ μ, IsHaarH F E σ M J μ → (∃ 𝓕, IsFDH F E σ M J μ 𝓕) ∧ (∃ 𝓕, MeasurableSet 𝓕 ∧ IsFDH F E σ M J μ 𝓕 ∧ ∀ h : HA (AF F) σ J, ∃ γ : HA (AF F) σ J, HA.mat (AF F) σ J γ ∈ HFin (AF F) σ J ∧ γ * h ∈ 𝓕) ∧ ∀ 𝓕, IsFDH F E σ M J μ 𝓕 → μ 𝓕 ≠ 0

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.HaarFundamentalDomainExistsUnitaryStatement

theorem **_root_.GlobalTheory.T.haar_fundamental_domain_exists_unitary** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → (∃ μ, IsHaarH F E σ M J μ) ∧ ∀ μ, IsHaarH F E σ M J μ → (∃ 𝓕, IsFDH F E σ M J μ 𝓕) ∧ (∃ 𝓕, MeasurableSet 𝓕 ∧ IsFDH F E σ M J μ 𝓕 ∧ ∀ h : HA (AF F) σ J, ∃ γ : HA (AF F) σ J, HA.mat (AF F) σ J γ ∈ HFin (AF F) σ J ∧ γ * h ∈ 𝓕) ∧ ∀ 𝓕, IsFDH F E σ M J μ 𝓕 → μ 𝓕 ≠ 0

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.HaarFundamentalDomainOrthogonalSum

theorem **hform_eq_dot** {A : Type} [CommRing A] (s : A →+* A) {M : ℕ} (J : Matrix (Fin M) (Fin M) A) (x y : Fin M → A) : hform s J x y = (fun i => s (x i)) ⬝ᵥ (J *ᵥ y)

theorem **hform_act** {A : Type} [CommRing A] (s : A →+* A) {M : ℕ} (J g : Matrix (Fin M) (Fin M) A) (hg : cT s g * J * g = J) (x y : Fin M → A) : hform s J (g *ᵥ x) (g *ᵥ y) = hform s J x y

theorem **gramE_act** (σ : E →ₐ[F] E) {M n : ℕ} (J : Matrix (Fin M) (Fin M) E) (g : UJ (σ : E →+* E) J) (X : Fin n → Fin M → E) : gramE σ J (fun a => ((g : GL (Fin M) E) : Matrix (Fin M) (Fin M) E) *ᵥ X a) = gramE σ J X

theorem **bc_mem_HR** (σ : E →ₐ[F] E) {M : ℕ} (J : Matrix (Fin M) (Fin M) E) (g : UJ (σ : E →+* E) J) : bc (inclR R) (g : GL (Fin M) E) ∈ HR R σ J

theorem **mat_ratH** (σ : E →ₐ[F] E) {M : ℕ} (J : Matrix (Fin M) (Fin M) E) (g : UJ (σ : E →+* E) J) : HA.mat R σ J (ratH R σ J g) = bc (inclR R) (g : GL (Fin M) E)

theorem **ratH_inv** (σ : E →ₐ[F] E) {M : ℕ} (J : Matrix (Fin M) (Fin M) E) (g : UJ (σ : E →+* E) J) : (ratH R σ J g)⁻¹ = ratH R σ J g⁻¹

theorem **actW_ratH** (σ : E →ₐ[F] E) {M n : ℕ} (J : Matrix (Fin M) (Fin M) E) (g : UJ (σ : E →+* E) J) (X : Fin n → Fin M → E) : actW R σ J (ratH R σ J g) (ratTup R X) = ratTup R (fun a => ((g : GL (Fin M) E) : Matrix (Fin M) (Fin M) E) *ᵥ X a)

theorem **ratTup_consTup** {M n : ℕ} (X : Fin n → Fin M → E) : ratTup (F

theorem **ratTup_first** {M : ℕ} : ratTup (F

theorem **gact_inv_gact** (σ : E →ₐ[F] E) {M n : ℕ} {J : Matrix (Fin M) (Fin M) E} (g : UJ (σ : E →+* E) J) (X : Fin n → Fin M → E) : gact σ g⁻¹ (gact σ g X) = X

theorem **gact_gact_inv** (σ : E →ₐ[F] E) {M n : ℕ} {J : Matrix (Fin M) (Fin M) E} (g : UJ (σ : E →+* E) J) (X : Fin n → Fin M → E) : gact σ g (gact σ g⁻¹ X) = X

theorem **gramE_gact** (σ : E →ₐ[F] E) {M n : ℕ} {J : Matrix (Fin M) (Fin M) E} (g : UJ (σ : E →+* E) J) (X : Fin n → Fin M → E) : gramE σ J (gact σ g X) = gramE σ J X

theorem **tsum_gact** (σ : E →ₐ[F] E) {M n : ℕ} {J : Matrix (Fin M) (Fin M) E} (g : UJ (σ : E →+* E) J) (T : Matrix (Fin n) (Fin n) E) (f : (Fin n → Fin M → E) → ℝ≥0∞) : ∑' X : {X : Fin n → Fin M → E | gramE σ J X = T}, f (gact σ g X.1) = ∑' X : {X : Fin n → Fin M → E | gramE σ J X = T}, f X.1

theorem **consTup_injective** {K : Type} [Zero K] [One K] {M n : ℕ} : Function.Injective (consTup : (Fin n → Fin M → K) → Fin (n + 1) → Fin (M + 1) → K)

theorem **continuous_consTup** {K : Type} [Zero K] [One K] [TopologicalSpace K] {M n : ℕ} : Continuous (consTup : (Fin n → Fin M → K) → Fin (n + 1) → Fin (M + 1) → K)

theorem **gram_first** (σ : E →ₐ[F] E) {M : ℕ} (J : Matrix (Fin M) (Fin M) E) (t : E) : gramE σ (blockOne t J) (fun _ : Fin 1 => (Fin.cons 1 (fun _ => 0) : Fin (M + 1) → E)) = fun _ _ => t

theorem **tsum_block** (σ : E →ₐ[F] E) {M n : ℕ} (J : Matrix (Fin M) (Fin M) E) {t : E} (ht : t ≠ 0) (T : Matrix (Fin n) (Fin n) E) (rr : {y : Fin 1 → Fin (M + 1) → E | gramE σ (blockOne t J) y = fun _ _ => t} → UJ (σ : E →+* E) (blockOne t J)) (hrr : ∀ q, gact σ (rr q) q.1 = fun _ => Fin.cons 1 (fun _ => 0)) (f : (Fin (n + 1) → Fin (M + 1) → E) → ℝ≥0∞) : ∑' Y : {Y : Fin (n + 1) → Fin (M + 1) → E | gramE σ (blockOne t J) Y = blockOne t T}, f Y.1 = ∑' q : {y : Fin 1 → Fin (M + 1) → E | gramE σ (blockOne t J) y = fun _ _ => t}, ∑' X : {X : Fin n → Fin M → E | gramE σ J X = T}, f (gact σ (rr q)⁻¹ (consTup X.1))

theorem **mem_HFinIn_iff** (σ : E →ₐ[F] E) {M : ℕ} (J : Matrix (Fin M) (Fin M) E) (h : HA R σ J) : h ∈ HFinIn R σ J ↔ HA.mat R σ J h ∈ HFin R σ J

theorem **ratH_mem** (σ : E →ₐ[F] E) {M : ℕ} (J : Matrix (Fin M) (Fin M) E) (g : UJ (σ : E →+* E) J) : ratH R σ J g ∈ HFinIn R σ J

theorem **exists_ratH** (σ : E →ₐ[F] E) {M : ℕ} (J : Matrix (Fin M) (Fin M) E) (h : HA R σ J) (hh : h ∈ HFinIn R σ J) : ∃ g, h = ratH R σ J g

theorem **actW_ratH'** (σ : E →ₐ[F] E) {M n : ℕ} (J : Matrix (Fin M) (Fin M) E) (g : UJ (σ : E →+* E) J) (X : Fin n → Fin M → E) : actW R σ J (ratH R σ J g) (ratTup R X) = ratTup R (gact σ g X)

theorem **ratTup_adele_injective** {F E : Type} [Field F] [NumberField F] [Field E] [Algebra F E] {M n : ℕ} : Function.Injective (ratTup (F

theorem **theta_stage** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ t : F, t ≠ 0 → ∀ (n : ℕ) (T : Matrix (Fin n) (Fin n) E) (μ : Measure (HA (AF F) σ (blockOne (algebraMap F E t) J))) 𝓕, IsHaarH F E σ (M + 1) _ μ → IsFDH F E σ (M + 1) _ μ 𝓕 → ∀ (μ₁ : Measure (HA (AF F) σ J)) 𝓕₁, IsHaarH F E σ M J μ₁ → IsFDH F E σ M J μ₁ 𝓕₁ → ∀ (Φ : (Fin (n + 1) → Fin (M + 1) → AF F ⊗[F] E) → ℝ≥0∞) (Ψ : (Fin 1 → Fin (M + 1) → AF F ⊗[F] E) → ℝ≥0∞), Measurable Φ → Measurable Ψ → (∀ h : HA (AF F) σ (blockOne (algebraMap F E t) J), Ψ (actW (AF F) σ _ h (fun _ => Fin.cons 1 (fun _ => 0))) = ∫⁻ X, Φ (actW (AF F) σ _ h (consTup X)) ∂(thetaMeas σ J μ₁ 𝓕₁ {X | gramE σ J X = T})) → ∫⁻ X, Φ X ∂(thetaMeas σ _ μ 𝓕 {X | gramE σ (blockOne (algebraMap F E t) J) X = blockOne (algebraMap F E t) T})

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.HaarFundamentalDomainOrthogonalSumStatement

theorem **_root_.GlobalTheory.T.lintegral_thetaMeas_block_one_eq_rank_one** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ t : F, t ≠ 0 → ∀ (n : ℕ) (T : Matrix (Fin n) (Fin n) E) (μ : Measure (HA (AF F) σ (blockOne (algebraMap F E t) J))) 𝓕, IsHaarH F E σ (M + 1) _ μ → IsFDH F E σ (M + 1) _ μ 𝓕 → ∀ (μ₁ : Measure (HA (AF F) σ J)) 𝓕₁, IsHaarH F E σ M J μ₁ → IsFDH F E σ M J μ₁ 𝓕₁ → ∀ (Φ : (Fin (n + 1) → Fin (M + 1) → AF F ⊗[F] E) → ℝ≥0∞) (Ψ : (Fin 1 → Fin (M + 1) → AF F ⊗[F] E) → ℝ≥0∞), Measurable Φ → Measurable Ψ → (∀ h : HA (AF F) σ (blockOne (algebraMap F E t) J), Ψ (actW (AF F) σ _ h (fun _ => Fin.cons 1 (fun _ => 0))) = ∫⁻ X, Φ (actW (AF F) σ _ h (consTup X)) ∂(thetaMeas σ J μ₁ 𝓕₁ {X | gramE σ J X = T})) → ∫⁻ X, Φ X ∂(thetaMeas σ _ μ 𝓕 {X | gramE σ (blockOne (alge

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.HaarFundamentalDomainRankOneDensity

theorem **bound_hermitian** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → 3 ≤ M → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (laminf : Measure (Ainf F)) (lam : ∀ v : HeightOneSpectrum (𝒪 F), Measure (Fv F v)), IsStdInf laminf → (∀ v, IsStdV v (lam v)) → ∀ (μ : Measure (HA (AF F) σ J)) (𝓕 : Set (HA (AF F) σ J)), IsHaarH F E σ M J μ → IsFDH F E σ M J μ 𝓕 → ∀ (minf : HermR (Ainf F) σ 1 → Measure (Fin 1 → Fin M → Ainf F ⊗[F] E)), IsGramFib (Ainf F) σ J 1 (coordW (Ainf F) b laminf 1 M) (coordHerm (Ainf F) σ b laminf 1) minf → ∀ (m : ∀ v : HeightOneSpectrum (𝒪 F), HermR (Fv F v) σ 1 → Measure (Fin 1 → Fin M → Fv F v ⊗[F] E)), (∀ v, IsGramFib (Fv F v) σ J 1 (coordW (Fv F v) b (lam v) 1 M) (coordHerm (Fv F v) σ b (lam v) 1) (m v)) → ∀ β : F, β ≠ 0 → ∃ (φ : (Fin 1 → Fin M → Ain

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.HaarFundamentalDomainRankOneDensityStatement

theorem **_root_.GlobalTheory.T.thetaMeas_rank_one_sub_density_product_le** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → 3 ≤ M → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (laminf : Measure (Ainf F)) (lam : ∀ v : HeightOneSpectrum (𝒪 F), Measure (Fv F v)), IsStdInf laminf → (∀ v, IsStdV v (lam v)) → ∀ (μ : Measure (HA (AF F) σ J)) (𝓕 : Set (HA (AF F) σ J)), IsHaarH F E σ M J μ → IsFDH F E σ M J μ 𝓕 → ∀ (minf : HermR (Ainf F) σ 1 → Measure (Fin 1 → Fin M → Ainf F ⊗[F] E)), IsGramFib (Ainf F) σ J 1 (coordW (Ainf F) b laminf 1 M) (coordHerm (Ainf F) σ

b laminf 1) minf → ∀ (m : ∀ v : HeightOneSpectrum (𝒪 F), HermR (Fv F v) σ 1 → Measure (Fin 1 → Fin M → Fv F v ⊗[F] E)), (∀ v, IsGramFib (Fv F v) σ J 1 (coordW (Fv F v) b (lam v) 1 M) (coordHerm (Fv F v) σ b (lam v) 1) (m v)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.HaarInvariantUnderTransvection

theorem **toMatrix_mulVec_apply** {K ι : Type} [CommRing K] [Fintype ι] [DecidableEq ι] (t : Matrix.TransvectionStruct ι K) (x : ι → K) (k : ι) : (t.toMatrix *ᵥ x) k = if k = t.i then x k + t.c * x t.j else x k

theorem **measurePreserving_pi** {K : Type} [CommRing K] [TopologicalSpace K] [IsTopologicalRing K] [SecondCountableTopology K] [MeasurableSpace K] [BorelSpace K] {ι : Type} [Fintype ι] [DecidableEq ι] (ν : Measure K) [SigmaFinite ν] [ν.IsAddLeftInvariant] (t : Matrix.TransvectionStruct ι K) : MeasurePreserving (fun x : ι → K => t.toMatrix *ᵥ x) (Measure.pi fun _ : ι => ν) (Measure.pi fun _ : ι => ν)

theorem **map_toMatrix_mulVec** (K : Type) [CommRing K] [TopologicalSpace K] [IsTopologicalRing K] [LocallyCompactSpace K] [SecondCountableTopology K] [T2Space K] [MeasurableSpace K] [BorelSpace K] (ι : Type) [Fintype ι] [DecidableEq ι] (μ : Measure (ι → K)) [μ.IsAddHaarMeasure] (t : Matrix.TransvectionStruct ι K) : μ.map (fun x => t.toMatrix *ᵥ x) = μ

theorem **GlobalTheory.CompanionTheory.SW.transvection_map_addHaar** : ∀ (K : Type) [CommRing K] [TopologicalSpace K] [IsTopologicalRing K] [LocallyCompactSpace K] [SecondCountableTopology K] [T2Space K] [MeasurableSpace K] [BorelSpace K] (ι : Type) [Fintype ι] [DecidableEq ι] (μ : Measure (ι → K)) [μ.IsAddHaarMeasure] (t : Matrix.TransvectionStruct ι K), μ.map (fun x => t.toMatrix *ᵥ x) = μ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.HaarInvariantUnderTransvectionStatement

theorem **_root_.GlobalTheory.T.haar_invariant_under_transvection** : ∀ (K : Type) [CommRing K] [TopologicalSpace K] [IsTopologicalRing K] [LocallyCompactSpace K] [SecondCountableTopology K] [T2Space K] [MeasurableSpace K] [BorelSpace K] (ι : Type) [Fintype ι] [DecidableEq ι] (μ : Measure (ι → K)) [μ.IsAddHaarMeasure] (t : Matrix.TransvectionStruct ι K), μ.map (fun x => t.toMatrix *ᵥ x) = μ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.HaarMapLinearScalingByDeterminant

theorem **measurable_mulVec** (A : Matrix ι ι R) : Measurable (fun x : ι → R => A *ᵥ x)

theorem **char_smul_map** (lam : Measure R) [lam.IsAddHaarMeasure] (u : Rˣ) : distribHaarChar R u • lam.map (fun x => (u : R) * x) = lam

theorem **map_transvection** (lam : Measure R) [lam.IsAddHaarMeasure] (i j : ι) (c : R) (hij : i ≠ j) : (Measure.pi fun _ : ι => lam).map (fun x => transvection i j c *ᵥ x) = Measure.pi fun _ : ι => lam

theorem **map_diagonal** (lam : Measure R) [lam.IsAddHaarMeasure] (d : ι → Rˣ) : distribHaarChar R (∏ i, d i) • (Measure.pi fun _ : ι => lam).map (fun x => diagonal (fun i => (d i : R)) *ᵥ x) = Measure.pi fun _ : ι => lam

theorem **smul_map_pi** {F : Type} [Field F] {R : Type} [CommRing R] [Algebra F R] [TopologicalSpace R] [IsTopologicalRing R] [LocallyCompactSpace R] [SecondCountableTopology R] [T2Space R] [MeasurableSpace R] [BorelSpace R] {ι : Type} [Fintype ι] [DecidableEq ι] (lam : Measure R) [lam.IsAddHaarMeasure] (A : Matrix ι ι F) (h : A.det ≠ 0) : distribHaarChar R (Units.map (algebraMap F R : F →* R) (Units.mk0 A.det h)) • (Measure.pi fun _ : ι => lam).map (fun x => A.map (algebraMap F R) *ᵥ x) = Measure.pi fun _ : ι => lam

theorem **map_mulVec_eq_smul** : ∀ (F : Type) [Field F] (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [IsTopologicalRing R] [LocallyCompactSpace R] [SecondCountableTopology R] [T2Space R] [MeasurableSpace R] [BorelSpace R] (ι : Type) [Fintype ι] [DecidableEq ι] (A : Matrix ι ι F), A.det ≠ 0 → ∀ (lam : Measure R), lam.IsAddHaarMeasure → ∀ c : ℝ≥0, lam.map (fun x => algebraMap F R A.det * x) = c • lam → ∀ μ : Measure (ι → R), μ.IsAddHaarMeasure → μ.map (fun x => A.map (algebraMap F R) *ᵥ x) = c • μ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.HaarMapLinearScalingByDeterminantStatement

theorem **_root_.GlobalTheory.T.haar_map_linear_scaling_by_determinant** : ∀ (F : Type) [Field F] (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [IsTopologicalRing R] [LocallyCompactSpace R] [SecondCountableTopology R] [T2Space R] [MeasurableSpace R] [BorelSpace R] (ι : Type) [Fintype ι] [DecidableEq ι] (A : Matrix ι ι F), A.det ≠ 0 → ∀ (lam : Measure R), lam.IsAddHaarMeasure → ∀ c : ℝ≥0, lam.map (fun x => algebraMap F R A.det * x) = c • lam → ∀ μ : Measure (ι → R), μ.IsAddHaarMeasure → μ.map (fun x => A.map (algebraMap F R) *ᵥ x) = c • μ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.HaarMeasureIwasawaDecomposition

theorem **tripleProduct_map_leftInvariant** : ∀ (G N H T : Type) [Group G] [MeasurableSpace G] [MeasurableMul₂ G] [MeasurableSpace N] [Group H] [MeasurableSpace H] [MeasurableMul H] [CommGroup T] [MeasurableSpace T] [MeasurableMul T] (μN : Measure N) [SFinite μN] (μH : Measure H) [SFinite μH] [μH.IsMulLeftInvariant] (μT : Measure T) [SFinite μT] [μT.IsMulLeftInvariant] (nn : N → G) (ι : H →* G) (mm : T →* G) (χ : T →* ℝ≥0), Measurable nn → Measurable ι → Measurable mm → Measurable χ → (∀ p, ∃ L : N → N, MeasurePreserving L μN μN ∧ ∀ x, nn p * nn x = nn (L x)) → (∀ u, ∃ L : N → N, MeasurePreserving L μN μN ∧ ∀ x, ι u * nn x = nn (L x) * ι u) → (∀ t, ∃ L : N → N, Measurable L ∧ μN.map L = χ t • μN ∧ ∀ x, mm t * nn x = nn (L x) * mm t) → (∀ u t, mm t * ι u = ι u * mm t) → ∀ (p : N) (u : H) (t : T), ((μN.prod (μH.prod (μT.withDensity fun s => (χ s : ℝ≥0∞))))).map fun z => nn z.1 * ι z.2.1 * mm z.

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.HaarMeasureIwasawaDecompositionStatement

theorem **_root_.GlobalTheory.T.triple_product_measure_map_mul_left_invariant** : ∀ (G N H T : Type) [Group G] [MeasurableSpace G] [MeasurableMul₂ G] [MeasurableSpace N] [Group H] [MeasurableSpace H] [MeasurableMul H] [CommGroup T] [MeasurableSpace T] [MeasurableMul T] (μN : Measure N) [SFinite μN] (μH : Measure H) [SFinite μH] [μH.IsMulLeftInvariant] (μT : Measure T) [SFinite μT] [μT.IsMulLeftInvariant] (nn : N → G) (ι : H →* G) (mm : T →* G) (χ : T →* ℝ≥0), Measurable nn → Measurable ι → Measurable mm → Measurable χ → (∀ p, ∃ L : N → N, MeasurePreserving L μN μN ∧ ∀ x, nn p * nn x = nn (L x)) → (∀ u, ∃ L : N → N, MeasurePreserving L μN μN ∧ ∀ x, ι u * nn x = nn (L x) * ι u) → (∀ t, ∃ L : N → N, Measurable L ∧ μN.map L = χ t • μN ∧ ∀ x, mm t * nn x = nn (L x) * mm t) → (∀ u t, mm t * ι u = ι u * mm t) → ∀ (p : N) (u : H) (t : T), ((μN.prod (μH.prod (μT.withDensity fun s => (χ s : ℝ≥0∞))))).m

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.HaarMeasureProdOfAddGroups

theorem **map_addEquiv_haar** (V V' : Type) [AddCommGroup V] [TopologicalSpace V] [LocallyCompactSpace V] [SecondCountableTopology V] [T2Space V] [MeasurableSpace V] [BorelSpace V] [IsTopologicalAddGroup V] [AddCommGroup V'] [TopologicalSpace V'] [LocallyCompactSpace V'] [SecondCountableTopology V'] [T2Space V'] [MeasurableSpace V'] [BorelSpace V'] [IsTopologicalAddGroup V'] (μ : Measure V) [μ.IsAddHaarMeasure] (μ' : Measure V') [μ'.IsAddHaarMeasure] (e : V ≃+ V') (he : Continuous e) (hes : Continuous e.symm) : (∃ a : ℝ≥0, a ≠ 0 ∧ μ.map e = a • μ') ∧ ∀ (L : Set V) (L' : Set V'), IsCompact L → IsOpen L → L.Nonempty → (∀ v, e v ∈ L' ↔ v ∈ L) → μ L = μ' L' → μ.map e = μ'

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.HaarMeasureProdOfAddGroupsStatement

theorem **_root_.GlobalTheory.T.map_addHaar_addEquiv_eq_smul_and_eq_of_compact_open** : ∀ (V V' : Type) [AddCommGroup V] [TopologicalSpace V] [LocallyCompactSpace V] [SecondCountableTopology V] [T2Space V] [MeasurableSpace V] [BorelSpace V] [IsTopologicalAddGroup V] [AddCommGroup V'] [TopologicalSpace V'] [LocallyCompactSpace V'] [SecondCountableTopology V'] [T2Space V'] [MeasurableSpace V'] [BorelSpace V'] [IsTopologicalAddGroup V'] (μ : Measure V) [μ.IsAddHaarMeasure] (μ' : Measure V') [μ'.IsAddHaarMeasure] (e : V ≃+ V'), Continuous e → Continuous e.symm → (∃ a : ℝ≥0, a ≠ 0 ∧ μ.map e = a • μ') ∧ ∀ (L : Set V) (L' : Set V'), IsCompact L → IsOpen L → L.Nonempty → (∀ v, e v ∈ L' ↔ v ∈ L) → μ L = μ' L' → μ.map e = μ'

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.HaarModularCharacterOfContinuousAction

theorem **exists_continuous_modulus_of_continuous_action** : ∀ (V : Type) [AddCommGroup V] [TopologicalSpace V] [LocallyCompactSpace V] [SecondCountableTopology V] [T2Space V] [MeasurableSpace V] [BorelSpace V] [IsTopologicalAddGroup V] (μ : Measure V) [μ.IsAddHaarMeasure] (Q : Type) [Group Q] [TopologicalSpace Q] [IsTopologicalGroup Q] [DistribMulAction Q V] [ContinuousSMul Q V], ∃ χ : Q →* ℝ≥0, Continuous χ ∧ ∀ q : Q, μ.map (fun v => q • v) = χ q • μ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.HaarModularCharacterOfContinuousActionStatement

theorem **_root_.GlobalTheory.T.haar_modular_character_of_continuous_action** : ∀ (V : Type) [AddCommGroup V] [TopologicalSpace V] [LocallyCompactSpace V] [SecondCountableTopology V] [T2Space V] [MeasurableSpace V] [BorelSpace V] [IsTopologicalAddGroup V] (μ : Measure V) [μ.IsAddHaarMeasure] (Q : Type) [Group Q] [TopologicalSpace Q] [IsTopologicalGroup Q] [DistribMulAction Q V] [ContinuousSMul Q V], ∃ χ : Q →* ℝ≥0, Continuous χ ∧ ∀ q : Q, μ.map (fun v => q • v) = χ q • μ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.HaarScalingUnderFlow

theorem **map_eq_of_conj** {G H : Type} [AddCommGroup G] [TopologicalSpace G] [IsTopologicalAddGroup G] [MeasurableSpace G] [BorelSpace G] [AddCommGroup H] [TopologicalSpace H] [IsTopologicalAddGroup H] [MeasurableSpace H] [BorelSpace H] (e : G ≃+ H) (he : Continuous e) (hs : Continuous e.symm) (L : G → G) (L' : H → H) (hL' : Continuous L') (hc : ∀ X, e (L X) = L' (e X)) (c : ℝ≥0∞) (hH : ∀ ν : Measure H, ν.IsAddHaarMeasure → ν.map L' = c • ν) (μ : Measure G) [μ.IsAddHaarMeasure] : μ.map L = c • μ

theorem **coordEquiv_smul** {d : ℕ} (b : Module.Basis (Fin d) F E) (n M : ℕ) (a : A) (X : Fin n → Fin M → A ⊗[F] E) : coordEquiv b n M ((a ⊗ₜ[F] (1 : E)) • X) = fun p => a ∗ coordEquiv b n M X p

theorem **modulus_flow** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ τ : Multiplicative ℝ →* (AF F ⊗[F] E)ˣ, IsFlow τ → ∀ (n M : ℕ) (μ : Measure (Fin n → Fin M → AF F ⊗[F] E)), μ.IsAddHaarMeasure → ∀ x : ℝ, μ.map (fun X => ((τ (Multiplicative.ofAdd (x)) : (AF F ⊗[F] E)ˣ) : AF F ⊗[F] E) • X) = ENNReal.ofReal (Real.exp (−(2 * Module.finrank ℚ F * n * M * x))) • μ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.HaarScalingUnderFlowStatement

theorem **_root_.GlobalTheory.T.haar_scaling_under_flow** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ τ : Multiplicative ℝ →* (AF F ⊗[F] E)ˣ, IsFlow τ → ∀ (n M : ℕ) (μ : Measure (Fin n → Fin M → AF F ⊗[F] E)), μ.IsAddHaarMeasure → ∀ x : ℝ, μ.map (fun X => ((τ (Multiplicative.ofAdd (x)) : (AF F ⊗[F] E)ˣ) : AF F ⊗[F] E) • X) = ENNReal.ofReal (Real.exp (-(2 * Module.finrank ℚ F * n * M * x))) • μ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.HarrisTaylorCardKilledByIdealEqAbsNormPow

theorem **psi_ne** (hn : ∀ x, ‖ψ x‖ = 1) (x : R) : ψ x ≠ 0

theorem **psi_zero** (ha : ∀ x y, ψ (x + y) = ψ x * ψ y) (hn : ∀ x, ‖ψ x‖ = 1) : ψ 0 = 1

theorem **psi_sum** (ha : ∀ x y, ψ (x + y) = ψ x * ψ y) (hn : ∀ x, ‖ψ x‖ = 1) {ι : Type} (s : Finset ι) (f : ι → R) : ψ (∑ i ∈ s, f i) = ∏ i ∈ s, ψ (f i)

theorem **psi_dot** (ha : ∀ x y, ψ (x + y) = ψ x * ψ y) (hn : ∀ x, ‖ψ x‖ = 1) {ι : Type} [Fintype ι] (x y : ι → R) : ψ (x ⬝ᵥ y) = ∏ i, ψ (x i * y i)

theorem **integral_subgroup_eq_zero** (lam : Measure R) [lam.IsAddHaarMeasure] (A : AddSubgroup R) (hA : MeasurableSet (A : Set R)) (ha : ∀ x y, ψ (x + y) = ψ x * ψ y) (t a : R) (haA : a ∈ A) (hne : ψ (a * t) ≠ 1) : ∫ x in (A : Set R), ψ (x * t) ∂lam = 0

theorem **integral_subgroup_eq_real** (lam : Measure R) (A : Set R) (hA : MeasurableSet A) (t : R) (h : ∀ x ∈ A, ψ (x * t) = 1) : ∫ x in A, ψ (x * t) ∂lam = (lam.real A : ℂ)

theorem **dual_facts** (hc : Continuous ψ) (ha : ∀ x y, ψ (x + y) = ψ x * ψ y) (hn : ∀ x, ‖ψ x‖ = 1) (lam : Measure R) (hsd : SelfDual lam (fun s t => ψ (s * t))) (L : AddSubgroup R) (hLc : IsCompact (L : Set R)) (hLo : IsOpen (L : Set R)) (hP : IsCompact (ann ha hn L : Set R)) : lam.real (L : Set R) * lam.real (ann ha hn L : Set R) = 1 ∧ (∀ z, z ∈ ann ha hn (ann ha hn L) → z ∈ L) ∧ IsOpen (ann ha hn L : Set R)

theorem **integral_ann_eq** (lam : Measure R) [lam.IsAddHaarMeasure] (L P : AddSubgroup R) (hPm : MeasurableSet (P : Set R)) (ha : ∀ x y, ψ (x + y) = ψ x * ψ y) (hLP : ∀ s ∈ L, ∀ t ∈ P, ψ (s * t) = 1) (hPL : ∀ z, (∀ t ∈ P, ψ (t * z) = 1) → z ∈ L) (s : R) : ∫ t in (P : Set R), ψ (s * t) ∂lam = if s ∈ L then (lam.real (P : Set R) : ℂ) else 0

theorem **integral_box** (lam : Measure R) [lam.IsAddHaarMeasure] [SigmaFinite lam] (L P : AddSubgroup R) (hPm : MeasurableSet (P : Set R)) (ha : ∀ x y, ψ (x + y) = ψ x * ψ y) (hn : ∀ x, ‖ψ x‖ = 1) (hLP : ∀ s ∈ L, ∀ t ∈ P, ψ (s * t) = 1) (hPL : ∀ z, (∀ t ∈ P, ψ (t * z) = 1) → z ∈ L) (w : ι → R) : ∫ y in box ι (P : Set R), ψ (w ⬝ᵥ y) ∂(Measure.pi fun _ : ι => lam) = (box ι (L : Set R)).indicator (fun _ => (lam.real (P : Set R) : ℂ) ^ Fintype.card ι) w

theorem **continuous_psi_dot** (hc : Continuous ψ) {X : Type} [TopologicalSpace X] {f g : X → ι → R} (hf : Continuous f) (hg : Continuous g) : Continuous fun p => ψ (f p ⬝ᵥ g p)

theorem **fubini_box** (hc : Continuous ψ) (ha : ∀ x y, ψ (x + y) = ψ x * ψ y) (hn : ∀ x, ‖ψ x‖ = 1) (μ : Measure (ι → R)) [SFinite μ] (B : Set (ι → R)) (hBf : μ B < ⊤) (Φ : (ι → R) → ℂ) (Φc : Continuous Φ) (hi : Integrable Φ μ) (z : ι → R) : ∫ y in B, (∫ x, Φ x * ψ (x ⬝ᵥ y) ∂μ) * ψ (y ⬝ᵥ z) ∂μ = ∫ x, Φ x * ∫ y in B, ψ ((x + z) ⬝ᵥ y) ∂μ ∂μ

theorem **box_formula** (hc : Continuous ψ) (ha : ∀ x y, ψ (x + y) = ψ x * ψ y) (hn : ∀ x, ‖ψ x‖ = 1) (lam : Measure R) [lam.IsAddHaarMeasure] [SigmaFinite lam] (L P : AddSubgroup R) (hLm : MeasurableSet (L : Set R)) (hPm : MeasurableSet (P : Set R)) (hPf : lam (P : Set R) < ⊤) (hLP : ∀ s ∈ L, ∀ t ∈ P, ψ (s * t) = 1) (hPL : ∀ z, (∀ t ∈ P, ψ (t * z) = 1) → z ∈ L) (Φ : (ι → R) → ℂ) (Φc : Continuous Φ) (hi : Integrable Φ (Measure.pi fun _ : ι => lam)) (z : ι → R) : ∫ y in box ι (P : Set R), (∫ x, Φ x * ψ (x ⬝ᵥ y) ∂(Measure.pi fun _ : ι => lam)) * ψ (y ⬝ᵥ z) ∂(Measure.pi fun _ : ι => lam) = (lam.real (P : Set R) : ℂ) ^ Fintype.card ι * ∫ u in box ι (L : Set R), Φ (u - z) ∂(Measure.pi fun _ : ι => lam)

theorem **mean_close** (lam : Measure R) [lam.IsAddHaarMeasure] [SigmaFinite lam] (A : Set R) (hAm : MeasurableSet A) (hAf : lam A < ⊤) (c : ℝ) (hc0 : 0 ≤ c) (hcm : lam.real A * c = 1) (f : (ι → R) → ℂ) (hf : IntegrableOn f (box ι A) (Measure.pi fun _ : ι => lam)) (v : ℂ) (ε : ℝ) (h : ∀ u ∈ box ι A, ‖f u - v‖ ≤ ε) : ‖(c : ℂ) ^ Fintype.card ι * ∫ u in box ι A, f u ∂(Measure.pi fun _ : ι => lam) - v‖ ≤ ε

theorem **selfDual_pi** : ∀ (R : Type) [CommRing R] [TopologicalSpace R] [IsTopologicalRing R] [LocallyCompactSpace R] [SecondCountableTopology R] [T2Space R] [MeasurableSpace R] [BorelSpace R] (ψ : R → ℂ), Continuous ψ → (∀ x y, ψ (x + y) = ψ x * ψ y) → (∀ x, ‖ψ x‖ = 1) → ∀ lam : Measure R, SelfDual lam (fun s t => ψ (s * t)) → (∃ L : AddSubgroup R, IsCompact (L : Set R) ∧ IsOpen (L : Set R)) → (∀ L : AddSubgroup R, IsCompact (L : Set R) → IsOpen (L : Set R) → IsCompact {t | ∀ s ∈ L, ψ (s * t) = 1}) → (∀ K : Set R, IsCompact K → ∃ L : AddSubgroup R, IsCompact (L : Set R) ∧ IsOpen (L : Set R) ∧ K ⊆ L) → (∀ U ∈ 𝓝 (0 : R), ∃ L : AddSubgroup R, IsCompact (L : Set R) ∧ IsOpen (L : Set R) ∧ (L : Set R) ⊆ U) → ∀ (ι : Type) [Fintype ι], SelfDual (Measure.pi fun _ : ι => lam) (fun x y => ψ (x ⬝ᵥ y))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.HermitianDiagonalizeOverReducedTensor

theorem **isUnit_of_forall_notMem** {a : B} (h : ∀ m : MaximalSpectrum B, a ∉ m.asIdeal) : IsUnit a

theorem **exists_crt** [Finite (MaximalSpectrum B)] (y : MaximalSpectrum B → B) : ∃ c : B, ∀ m : MaximalSpectrum B, c - y m ∈ m.asIdeal

theorem **exists_unit_comb** [Finite (MaximalSpectrum B)] (i0 : ι) (x l : ι → B) (h : ∑ i, l i * x i = 1) : ∃ c : ι → B, c i0 = 1 ∧ IsUnit (∑ i, c i * x i)

theorem **exists_col** [Finite (MaximalSpectrum B)] (i0 : ι) (x l : ι → B) (h : ∑ i, l i * x i = 1) : ∃ g : Matrix ι ι B, IsUnit g.det ∧ ∀ i, g i i0 = x i

theorem **bf_add_left** (S : Matrix ι ι A) (x x' y : ι → A) : bf s S (x + x') y = bf s S x y + bf s S x' y

theorem **bf_add_right** (S : Matrix ι ι A) (x y y' : ι → A) : bf s S x (y + y') = bf s S x y + bf s S x y'

theorem **bf_smul_left** (S : Matrix ι ι A) (a : A) (x y : ι → A) : bf s S (a • x) y = s a * bf s S x y

theorem **bf_smul_right** (S : Matrix ι ι A) (a : A) (x y : ι → A) : bf s S x (a • y) = a * bf s S x y

theorem **bf_sum_left** {κ : Type} (T : Finset κ) (S : Matrix ι ι A) (f : κ → ι → A) (y : ι → A) : bf s S (∑ k ∈ T, f k) y = ∑ k ∈ T, bf s S (f k) y

theorem **bf_sum_right** {κ : Type} (T : Finset κ) (S : Matrix ι ι A) (x : ι → A) (f : κ → ι → A) : bf s S x (∑ k ∈ T, f k) = ∑ k ∈ T, bf s S x (f k)

theorem **bf_single** (S : Matrix ι ι A) (i j : ι) : bf s S (Pi.single i 1) (Pi.single j 1) = S i j

theorem **cT_apply** {l m : Type} (S : Matrix l m A) (i : m) (j : l) : cT s S i j = s (S j i)

theorem **bf_conj** (hs : ∀ z, s (s z) = z) {S : Matrix ι ι A} (hS : cT s S = S) (x y : ι → A) : s (bf s S x y) = bf s S y x

theorem **entry_mem** (I : Ideal A) {θ : A} (hθ : IsUnit θ) (hsθ : s θ = -θ) (h2 : IsUnit (2 : A)) (S : Matrix ι ι A) (hv : ∀ x, bf s S x x ∈ I) (i j : ι) : S i j ∈ I

theorem **det_cT** (S : Matrix ι ι A) : (cT s S).det = s S.det

theorem **isUnit_det_congr** (S g : Matrix ι ι A) (hS : IsUnit S.det) (hg : IsUnit g.det) : IsUnit (cT s g * S * g).det

theorem **cT_congr** (hs : ∀ z, s (s z) = z) {S : Matrix ι ι A} (hS : cT s S = S) (g : Matrix ι ι A) : cT s (cT s g * S * g) = cT s g * S * g

theorem **congr_mul** (S g g' : Matrix ι ι A) : cT s (g * g') * S * (g * g') = cT s g' * (cT s g * S * g) * g'

theorem **congr_apply** (S g : Matrix ι ι A) (i : ι) : (cT s g * S * g) i i = bf s S (fun k => g k i) (fun k => g k i)

theorem **exists_value_notMem** [Nonempty ι] {S : Matrix ι ι A} (hS : cT s S = S) (hdet : IsUnit S.det) (m : MaximalSpectrum R) : ∃ x, t (bf s S x x) ∉ m.asIdeal

theorem **exists_value_unit** [Nonempty ι] [Finite (MaximalSpectrum R)] {S : Matrix ι ι A} (hS : cT s S = S) (hdet : IsUnit S.det) : ∃ (x : ι → A) (r : Rˣ), bf s S x x = algebraMap R A r

theorem **cT_fromBlocks** {l m : Type} (a : Matrix l l A) (b : Matrix l m A) (c : Matrix m l A) (d : Matrix m m A) : cT s (fromBlocks a b c d) = fromBlocks (cT s a) (cT s c) (cT s b) (cT s d)

theorem **DiagProp.of_equiv** {κ : Type} [Fintype κ] [DecidableEq κ] (e : ι ≃ κ) (h : DiagProp (R

theorem **split_step** (ih : DiagProp (R

theorem **diagProp_succ** [Finite (MaximalSpectrum R)] [Finite (MaximalSpectrum A)] (ih : DiagProp (R

theorem **diagProp_fin** [Finite (MaximalSpectrum R)] [Finite (MaximalSpectrum A)] (n : ℕ) : DiagProp (R

theorem **exists_diagonal_congr** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (R : Type) [CommRing R] [Algebra F R] [IsArtinianRing R] [IsReduced R] (n : ℕ) (S : Matrix (Fin n) (Fin n) (R ⊗[F] E)), cT (sigR R σ) S = S → IsUnit S.det → ∃ (g : GL (Fin n) (R ⊗[F] E)) (D : Fin n → Rˣ), cT (sigR R σ) (g : Matrix (Fin n) (Fin n) (R ⊗[F] E)) * S * (g : Matrix (Fin n) (Fin n) (R ⊗[F] E)) = Matrix.diagonal fun i => algebraMap R (R ⊗[F] E) (D i : R)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.HermitianFormDiagonalizesOverField

theorem **blockOne_mul** {K : Type} [CommRing K] {M : ℕ} (a b : K) (A B : Matrix (Fin M) (Fin M) K) : blockOne a A * blockOne b B = blockOne (a * b) (A * B)

theorem **blockOne_one** {K : Type} [CommRing K] {M : ℕ} : blockOne (1 : K) (1 : Matrix (Fin M) (Fin M) K) = 1

theorem **diag_hermitian** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (n : ℕ) (T : Matrix (Fin n) (Fin n) E), IsHermND σ T → ∃ (g : GL (Fin n) E) (D : Fin n → Fˣ), cT (σ : E →+* E) (g : Matrix (Fin n) (Fin n) E) * T * g = Matrix.diagonal fun i => algebraMap F E (D i : F)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.HermitianFormDiagonalizesOverFieldStatement

theorem **_root_.GlobalTheory.T.hermitian_form_diagonalizes_over_field** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (n : ℕ) (T : Matrix (Fin n) (Fin n) E), IsHermND σ T → ∃ (g : GL (Fin n) E) (D : Fin n → Fˣ), cT (σ : E →+* E) (g : Matrix (Fin n) (Fin n) E) * T * g = Matrix.diagonal fun i => algebraMap F E (D i : F)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.HermitianFormsIntegrallyEquivalentAlmostAll

theorem **exists_gl_to_one** (A : Type) [CommRing A] [Finite (MaximalSpectrum A)] (σ : A →+* A) (hσ : ∀ y, σ (σ y) = y) (h2 : IsUnit (2 : A)) (hδ : ∃ δ : A, IsUnit δ ∧ σ δ = -δ) (hN : ∀ β : A, IsUnit β → σ β = β → ∃ a : A, a * σ a = β) (m : ℕ) (J : Matrix (Fin m) (Fin m) A) (hJ : ∀ i j, σ (J j i) = J i j) (hd : IsUnit J.det) : ∃ h : GL (Fin m) A, cT σ (h : Matrix (Fin m) (Fin m) A) * J * (h : Matrix (Fin m) (Fin m) A) = 1

theorem **exists_gl_equiv** (A : Type) [CommRing A] [Finite (MaximalSpectrum A)] (σ : A →+* A) (hσ : ∀ y, σ (σ y) = y) (h2 : IsUnit (2 : A)) (hδ : ∃ δ : A, IsUnit δ ∧ σ δ = -δ) (hN : ∀ β : A, IsUnit β → σ β = β → ∃ a : A, a * σ a = β) (m : ℕ) (J J' : Matrix (Fin m) (Fin m) A) (hJ : ∀ i j, σ (J j i) = J i j) (hd : IsUnit J.det) (hJ' : ∀ i j, σ (J' j i) = J' i j) (hd' : IsUnit J'.det) : ∃ G : GL (Fin m) A, J' = cT σ (G : Matrix (Fin m) (Fin m) A) * J * (G : Matrix (Fin m) (Fin m) A)

theorem **exists_gl_sub** {R : Type} [CommRing R] (s : R →+* R) (hs : ∀ y, s (s y) = y) (O : Subring R) (hO : ∀ y ∈ O, s y ∈ O) [Finite (MaximalSpectrum O)] (h2 : IsUnit (2 : O)) (hδ : ∃ δ : O, IsUnit δ ∧ s (δ : R) = -(δ : R)) (hN : ∀ β : O, IsUnit β → s (β : R) = β → ∃ a : O, (a : R) * s a = β) (m : ℕ) (K K' : Matrix (Fin m) (Fin m) R) (hK : ∀ i j, s (K j i) = K i j) (hK' : ∀ i j, s (K' j i) = K' i j) (iK : ∀ i j, K i j ∈ O) (iK' : ∀ i j, K' i j ∈ O) (wK : ∃ w ∈ O, K.det * w = 1) (wK' : ∃ w ∈ O, K'.det * w = 1) : ∃ g : GL (Fin m) R, K' = cT s (g : Matrix (Fin m) (Fin m) R) * K * (g : Matrix (Fin m) (Fin m) R) ∧ (∀ i j, (g : Matrix (Fin m) (Fin m) R) i j ∈ O) ∧ ∀ i j, ((g⁻¹ : GL (Fin m) R) : Matrix (Fin m) (Fin m) R) i j ∈ O

theorem **sigma_sigma** {F E : Type} [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E) (h : IsCMExt F E σ) (x : E) : σ (σ x) = x

theorem **sigR_sigR** {F E : Type} [Field F] [Field E] [Algebra F E] (R : Type) [CommRing R] [Algebra F R] (σ : E →ₐ[F] E) (h : ∀ x, σ (σ x) = x) (y : R ⊗[F] E) : sigR R σ (sigR R σ y) = y

theorem **mulVec_mem** {R : Type} [CommRing R] (O : Subring R) {m : ℕ} (P : Matrix (Fin m) (Fin m) R) (hP : ∀ i j, P i j ∈ O) (x : Fin m → R) (hx : ∀ i, x i ∈ O) (i : Fin m) : P.mulVec x i ∈ O

theorem **good_iso** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (J' : Matrix (Fin M) (Fin M) E), IsHermND σ J' → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E), ∀ᶠ v : HeightOneSpectrum (𝒪 F) in Filter.cofinite, ∃ g : GL (Fin M) (Fv F v ⊗[F] E), JR (Fv F v) J' = cT (sigR (Fv F v) σ) (g : Matrix (Fin M) (Fin M) (Fv F v ⊗[F] E)) * JR (Fv F v) J * (g : Matrix (Fin M) (Fin M) (Fv F v ⊗[F] E)) ∧ ∀ (n : ℕ) (X : Fin n → Fin M → Fv F v ⊗[F] E), actTup (Fv F v) g X ∈ latV b v ↔ X ∈ latV b v

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theorem **_root_.GlobalTheory.T.hermitian_forms_integrally_equivalent_almost_all** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (J' : Matrix (Fin M) (Fin M) E), IsHermND σ J' → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E), ∀ᶠ v : HeightOneSpectrum (𝒪 F) in Filter.cofinite, ∃ g : GL (Fin M) (Fv F v ⊗[F] E), JR (Fv F v) J' = cT (sigR (Fv F v) σ) (g : Matrix (Fin M) (Fin M) (Fv F v ⊗[F] E)) * JR (Fv F v) J * (g : Matrix (Fin M) (Fin M) (Fv F v ⊗[F] E)) ∧ ∀ (n : ℕ) (X : Fin n → Fin M → Fv F v ⊗[F] E), actTup (Fv F v) g X ∈ latV b v ↔ X ∈ latV b v

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theorem **exists_local_section** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R], IsLocR R → ∀ (n : ℕ) (S : HermR R σ n), IsUnit (S : Matrix (Fin n) (Fin n) (R ⊗[F] E)).det → ∃ (B : Set (HermR R σ n)) (g : HermR R σ n → GL (Fin n) (R ⊗[F] E)), IsOpen B ∧ S ∈ B ∧ ContinuousOn g B ∧ g S = 1 ∧ ∀ S' ∈ B, (S' : Matrix (Fin n) (Fin n) (R ⊗[F] E)) = cT (sigR R σ) (g S' : Matrix (Fin n) (Fin n) (R ⊗[F] E)) * (S : Matrix (Fin n) (Fin n) (R ⊗[F] E)) * (g S' : Matrix (Fin n) (Fin n) (R ⊗[F] E))

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theorem **_root_.GlobalTheory.T.hermitian_local_continuous_congruence_section** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R], IsLocR R → ∀ (n : ℕ) (S : HermR R σ n), IsUnit (S : Matrix (Fin n) (Fin n) (R ⊗[F] E)).det → ∃ (B : Set (HermR R σ n)) (g : HermR R σ n → GL (Fin n) (R ⊗[F] E)), IsOpen B ∧ S ∈ B ∧ ContinuousOn g B ∧ g S = 1 ∧ ∀ S' ∈ B, (S' : Matrix (Fin n) (Fin n) (R ⊗[F] E)) = cT (sigR R σ) (g S' : Matrix (Fin n) (Fin n) (R ⊗[F] E)) * (S : Matrix (Fin n) (Fin n) (R ⊗[F] E)) * (g S' : Matrix (Fin n) (Fin n) (R ⊗[F] E))

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theorem **sigma_sigma** {F E : Type} [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] {σ : E →ₐ[F] E} (h : IsCMExt F E σ) (x : E) : σ (σ x) = x

theorem **sigR_sigR** {σ : E →ₐ[F] E} (h : ∀ x, σ (σ x) = x) (y : R ⊗[F] E) : sigR R σ (sigR R σ y) = y

theorem **sigR_inclR** (σ : E →ₐ[F] E) (x : E) : sigR R σ (inclR R x) = inclR R (σ x)

theorem **gramR_mem** {σ : E →ₐ[F] E} (h : ∀ x, σ (σ x) = x) {n M : ℕ} {J : Matrix (Fin M) (Fin M) E} (hJ : ∀ i j, σ (J j i) = J i j) (X : Fin n → Fin M → R ⊗[F] E) : gramR R σ J X ∈ HermR R σ n

theorem **isModTop** : IsModuleTopology R (R ⊗[F] E)

theorem **isTopRing** [Module.Finite F E] : IsTopologicalRing (R ⊗[F] E)

theorem **exists_homeo** [Module.Finite F E] : Nonempty ((R ⊗[F] E) ≃ₜ (Fin (Module.finrank F E) → R))

theorem **continuous_sigR** [Module.Finite F E] (σ : E →ₐ[F] E) : Continuous (sigR R σ)

theorem **herm_top** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [IsTopologicalRing R] [LocallyCompactSpace R] [SecondCountableTopology R] [T2Space R] [MeasurableSpace R] [BorelSpace R], ∀ n : ℕ, LocallyCompactSpace (HermR R σ n) ∧ SecondCountableTopology (HermR R σ n) ∧ T2Space (HermR R σ n) ∧ BorelSpace (HermR R σ n) ∧ ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∃ g : (Fin n → Fin M → R ⊗[F] E) → HermR R σ n, Continuous g ∧ ∀ X, gramR R σ J X = (g X : Matrix (Fin n) (Fin n) (R ⊗[F] E))

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theorem **_root_.GlobalTheory.T.hermitian_matrices_locally_compact_borel** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [IsTopologicalRing R] [LocallyCompactSpace R] [SecondCountableTopology R] [T2Space R] [MeasurableSpace R] [BorelSpace R], ∀ n : ℕ, LocallyCompactSpace (HermR R σ n) ∧ SecondCountableTopology (HermR R σ n) ∧ T2Space (HermR R σ n) ∧ BorelSpace (HermR R σ n) ∧ ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∃ g : (Fin n → Fin M → R ⊗[F] E) → HermR R σ n, Continuous g ∧ ∀ X, gramR R σ J X = (g X : Matrix (Fin n) (Fin n) (R ⊗[F] E))

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theorem **sigR_smul** (σ : E →ₐ[F] E) (r : R) (y : R ⊗[F] E) : sigR R σ (r • y) = r • sigR R σ y

theorem **psi_lt** (σ : E →ₐ[F] E) (b : Module.Basis (Fin d) F E) (x : Idx n d → R) {a c : Fin n} (h : a < c) : psi R σ b x a c = offE R b x ⟨(a, c), h⟩

theorem **psi_gt** (σ : E →ₐ[F] E) (b : Module.Basis (Fin d) F E) (x : Idx n d → R) {a c : Fin n} (h : c < a) : psi R σ b x a c = sigR R σ (offE R b x ⟨(c, a), h⟩)

theorem **psi_eq** (σ : E →ₐ[F] E) (b : Module.Basis (Fin d) F E) (x : Idx n d → R) (a : Fin n) : psi R σ b x a a = x (Sum.inl a) ⊗ₜ[F] (1 : E)

theorem **offE_add** (b : Module.Basis (Fin d) F E) (x y : Idx n d → R) (p) : offE R b (x + y) p = offE R b x p + offE R b y p

theorem **offE_smul** (b : Module.Basis (Fin d) F E) (r : R) (x : Idx n d → R) (p) : offE R b (r • x) p = r • offE R b x p

theorem **psi_mem** {σ : E →ₐ[F] E} (hinv : ∀ x, σ (σ x) = x) (b : Module.Basis (Fin d) F E) (x : Idx n d → R) : psi R σ b x ∈ HermR R σ n

theorem **coord_psi** [CharZero F] (h2 : Module.finrank F E = 2) (σ : E →ₐ[F] E) (b : Module.Basis (Fin d) F E) (x : Idx n d → R) : coordOf R b (psi R σ b x) = x

theorem **psi_coord** [CharZero F] {σ : E →ₐ[F] E} (hinv : ∀ x, σ (σ x) = x) (h2 : Module.finrank F E = 2) {δ : E} (hδ : δ ≠ 0) (hσδ : σ δ = -δ) (b : Module.Basis (Fin d) F E) {S : Matrix (Fin n) (Fin n) (R ⊗[F] E)} (hS : S ∈ HermR R σ n) : psi R σ b (coordOf R b S) = S

theorem **mg_mem** {σ : E →ₐ[F] E} (hinv : ∀ x, σ (σ x) = x) (G : Matrix (Fin n) (Fin n) (R ⊗[F] E)) {S : Matrix (Fin n) (Fin n) (R ⊗[F] E)} (hS : S ∈ HermR R σ n) : mg R σ G S ∈ HermR R σ n

theorem **phi_tmul** (r : F) (e : E) : phi R (r ⊗ₜ[F] e) = algebraMap F R r ⊗ₜ[F] e

theorem **phi_sig** (σ : E →ₐ[F] E) (y : F ⊗[F] E) : phi R (sigR F σ y) = sigR R σ (phi R y)

theorem **phi_incl** (e : E) : phi R (inclR F e) = inclR R e

theorem **offE_map** (b : Module.Basis (Fin d) F E) (x : Idx n d → F) (p) : phi R (offE F b x p) = offE R b (fun i => algebraMap F R (x i)) p

theorem **psi_map** (σ : E →ₐ[F] E) (b : Module.Basis (Fin d) F E) (x : Idx n d → F) : (psi F σ b x).map (phi R) = psi R σ b (fun i => algebraMap F R (x i))

theorem **mg_map** (σ : E →ₐ[F] E) (g : Matrix (Fin n) (Fin n) E) (S : Matrix (Fin n) (Fin n) (F ⊗[F] E)) : (mg F σ (g.map (inclR F)) S).map (phi R) = mg R σ (g.map (inclR R)) (S.map (phi R))

theorem **Lmap_apply** (σ : E →ₐ[F] E) (b : Module.Basis (Fin d) F E) (g : Matrix (Fin n) (Fin n) E) (x : Idx n d → R) : Lmap R σ b g x = coordOf R b (mg R σ (g.map (inclR R)) (psi R σ b x))

theorem **Lmap_bc** [CharZero F] {σ : E →ₐ[F] E} (hinv : ∀ x, σ (σ x) = x) (h2 : Module.finrank F E = 2) {δ : E} (hδ : δ ≠ 0) (hσδ : σ δ = -δ) (b : Module.Basis (Fin d) F E) (g : Matrix (Fin n) (Fin n) E) (x : Idx n d → F) : Lmap R σ b g (fun i => algebraMap F R (x i)) = fun i => algebraMap F R (Lmap F σ b g x i)

theorem **Amat_spec** [CharZero F] {σ : E →ₐ[F] E} (hinv : ∀ x, σ (σ x) = x) (h2 : Module.finrank F E = 2) {δ : E} (hδ : δ ≠ 0) (hσδ : σ δ = -δ) (b : Module.Basis (Fin d) F E) (g : Matrix (Fin n) (Fin n) E) (x : Idx n d → R) : (Amat σ b g).map (algebraMap F R) *ᵥ x = Lmap R σ b g x

theorem **Lmap_inv** [CharZero F] {σ : E →ₐ[F] E} (hinv : ∀ x, σ (σ x) = x) (h2 : Module.finrank F E = 2) {δ : E} (hδ : δ ≠ 0) (hσδ : σ δ = -δ) (b : Module.Basis (Fin d) F E) {g g' : Matrix (Fin n) (Fin n) E} (hgg : g' * g = 1) (x : Idx n d → R) : Lmap R σ b g (Lmap R σ b g' x) = x

theorem **Amat_det** [CharZero F] {σ : E →ₐ[F] E} (hinv : ∀ x, σ (σ x) = x) (h2 : Module.finrank F E = 2) {δ : E} (hδ : δ ≠ 0) (hσδ : σ δ = -δ) (b : Module.Basis (Fin d) F E) {g : Matrix (Fin n) (Fin n) E} (hg : g.det ≠ 0) : (Amat σ b g).det ≠ 0

theorem **continuous_tmul** [IsTopologicalRing R] [ContinuousAdd (R ⊗[F] E)] [ContinuousSMul R (R ⊗[F] E)] (e : E) : Continuous fun r : R => r ⊗ₜ[F] e

theorem **continuous_offE** [IsTopologicalRing R] [ContinuousAdd (R ⊗[F] E)] [ContinuousSMul R (R ⊗[F] E)] (b : Module.Basis (Fin d) F E) (p) : Continuous fun x : Idx n d → R => offE R b x p

theorem **continuous_psi** [IsTopologicalRing R] [ContinuousAdd (R ⊗[F] E)] [ContinuousSMul R (R ⊗[F] E)] {σ : E →ₐ[F] E} (hsig : Continuous (sigR R σ)) (b : Module.Basis (Fin d) F E) : Continuous (psi R σ b : (Idx n d → R) → Matrix (Fin n) (Fin n) (R ⊗[F] E))

theorem **continuous_coordOf** [IsTopologicalRing R] (b : Module.Basis (Fin d) F E) (htr : Continuous (Algebra.trace R (R ⊗[F] E))) (hrepr : IsHomeomorph fun (y : R ⊗[F] E) (k : Fin d) => (Algebra.TensorProduct.basis R b).repr y k) : Continuous (coordOf R b : Matrix (Fin n) (Fin n) (R ⊗[F] E) → Idx n d → R)

theorem **map_eq_of_coord** {σ : E →ₐ[F] E} (hcm : IsCMExt F E σ) (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [IsTopologicalRing R] [LocallyCompactSpace R] [SecondCountableTopology R] [T2Space R] [MeasurableSpace R] [BorelSpace R] {d n : ℕ} (b : Module.Basis (Fin d) F E) (g : Matrix (Fin n) (Fin n) E) (c : ℝ≥0) (hc : ∀ μ : Measure (Idx n d → R), μ.IsAddHaarMeasure → μ.map (fun x => (Amat σ b g).map (algebraMap F R) *ᵥ x) = c • μ) (ν : Measure (HermR R σ n)) (hν : ν.IsAddHaarMeasure) (τ : HermR R σ n → HermR R σ n) (hτ : ∀ S, (τ S : Matrix (Fin n) (Fin n) (R ⊗[F] E)) = cT (sigR R σ) (g.map (inclR R)) * (S : Matrix (Fin n) (Fin n) (R ⊗[F] E)) * g.map (inclR R)) : ν.map τ = c • ν

theorem **pf_herm** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (n : ℕ) (g : Matrix (Fin n) (Fin n) E), g.det ≠ 0 → ∃ (cinf : ℝ≥0) (c : HeightOneSpectrum (𝒪 F) → ℝ≥0), IsUnitProd cinf c ∧ (∀ ν : Measure (HermR (Ainf F) σ n), ν.IsAddHaarMeasure → ∀ τ : HermR (Ainf F) σ n → HermR (Ainf F) σ n, (∀ S, (τ S : Matrix (Fin n) (Fin n) (Ainf F ⊗[F] E)) = cT (sigR (Ainf F) σ) (g.map (inclR (Ainf F))) * (S : Matrix (Fin n) (Fin n) (Ainf F ⊗[F] E)) * g.map (inclR (Ainf F))) → ν.map τ = cinf • ν) ∧ ∀ ν : HeightOneSpectrum (𝒪 F), (∀ ν : Measure (HermR (Fv F v) σ n), ν.IsAddHaarMeasure → ∀ τ : HermR (Fv F v) σ n → HermR (Fv F v) σ n, (∀ S, (τ S : Matrix (Fin n) (Fin n) (Fv F v ⊗[F] E)) = cT (sigR (Fv F v) σ) (g.map (inclR (Fv F v))) * (S : Matrix (Fin n) (Fin n) (Fv F v ⊗[F] E)) * g.map (inclR (Fv F v))) → ν.map τ = c v • ν)

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theorem **_root_.GlobalTheory.T.hermitian_matrix_euler_product** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (n : ℕ) (g : Matrix (Fin n) (Fin n) E), g.det ≠ 0 → ∃ (cinf : ℝ≥0) (c : HeightOneSpectrum (𝒪 F) → ℝ≥0), IsUnitProd cinf c ∧ (∀ ν : Measure (HermR (Ainf F) σ n), ν.IsAddHaarMeasure → ∀ τ : HermR (Ainf F) σ n → HermR (Ainf F) σ n, (∀ S, (τ S : Matrix (Fin n) (Fin n) (Ainf F ⊗[F] E)) = cT (sigR (Ainf F) σ) (g.map (inclR (Ainf F))) * (S : Matrix (Fin n) (Fin n) (Ainf F ⊗[F] E)) * g.map (inclR (Ainf F))) → ν.map τ = cinf • ν) ∧ ∀ ν : HeightOneSpectrum (𝒪 F), (∀ ν : Measure (HermR (Fv F v) σ n), ν.IsAddHaarMeasure → ∀ τ : HermR (Fv F v) σ n → HermR (Fv F v) σ n, (∀ S, (τ S : Matrix (Fin n) (Fin n) (Fv F v ⊗[F] E)) = cT (sigR (Fv F v) σ) (g.map (inclR (Fv F v))) * (S : Matrix (Fin n) (Fin n) (Fv F v ⊗[F] E)) * g.map (i

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theorem **sig_sig** (h : IsCMExt F E σ) (x : E) : σ (σ x) = x

theorem **exists_of_sig_fixed** (h : IsCMExt F E σ) {x : E} (hx : σ x = x) : ∃ t : F, algebraMap F E t = x

theorem **hE_add_left** (x x' y : Fin M → E) : hE σ T (x + x') y = hE σ T x y + hE σ T x' y

theorem **hE_add_right** (x y y' : Fin M → E) : hE σ T x (y + y') = hE σ T x y + hE σ T x y'

theorem **hE_single** (i j : Fin M) (a b : E) : hE σ T (Pi.single i a) (Pi.single j b) = σ a * T i j * b

theorem **exists_repr** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (n : ℕ) (T : Matrix (Fin (n + 1)) (Fin (n + 1)) E), IsHermND σ T → ∃ (t : F) (x : Fin (n + 1) → E), t ≠ 0 ∧ hE σ T x x = algebraMap F E t

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theorem **exists_ne** (h : IsCMExt F E σ) : ∃ x : E, σ x ≠ x

theorem **exists_eq_algebraMap** (h : IsCMExt F E σ) {x : E} (hx : σ x = x) : ∃ β : F, x = algebraMap F E β

theorem **hermOne_eq_scal** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (T : Matrix (Fin 1) (Fin 1) E) (hT : ∀ a b, σ (T b a) = T a b), ∃ β : F, T = (fun _ _ => algebraMap F E β) ∧ (T.det ≠ 0 → β ≠ 0) ∧ ∀ (R : Type) [CommRing R] [Algebra F R], hermOf R σ T hT = hermScal R σ (algebraMap F R β)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.HermitianOneByOneIsScalarStatement

theorem **_root_.GlobalTheory.T.hermitian_one_by_one_is_scalar** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (T : Matrix (Fin 1) (Fin 1) E) (hT : ∀ a b, σ (T b a) = T a b), ∃ β : F, T = (fun _ _ => algebraMap F E β) ∧ (T.det ≠ 0 → β ≠ 0) ∧ ∀ (R : Type) [CommRing R] [Algebra F R], hermOf R σ T hT = hermScal R σ (algebraMap F R β)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.HermitianOpenChartDiagonalUnipotent

theorem **up_snd_ne** {n : ℕ} (p : Up (n + 1)) : p.1.2 ≠ 0

theorem **joinU_head_tail** {n : ℕ} (u : Up (n + 1) → K) : joinU (headU u) (tailU u) = u

theorem **headU_join** {n : ℕ} (c : Fin n → K) (v : Up n → K) : headU (joinU c v) = c

theorem **tailU_join** {n : ℕ} (c : Fin n → K) (v : Up n → K) : tailU (joinU c v) = v

theorem **unip_zero_zero** {n : ℕ} (u : Up (n + 1) → K) : unip u 0 0 = 1

theorem **unip_succ_zero** {n : ℕ} (u : Up (n + 1) → K) (k : Fin n) : unip u k.succ 0 = 0

theorem **unip_zero_succ** {n : ℕ} (u : Up (n + 1) → K) (j : Fin n) : unip u 0 j.succ = headU u j

theorem **unip_succ_succ** {n : ℕ} (u : Up (n + 1) → K) (i j : Fin n) : unip u i.succ j.succ = unip (tailU u) i j

theorem **gram_zero_zero** {n : ℕ} (d : Fin (n + 1) → Rˣ) (u : Up (n + 1) → K) : gram ι τ d u 0 0 = ι (d 0)

theorem **gram_zero_succ** {n : ℕ} (d : Fin (n + 1) → Rˣ) (u : Up (n + 1) → K) (j : Fin n) : gram ι τ d u 0 j.succ = ι (d 0) * headU u j

theorem **gram_succ_zero** {n : ℕ} (d : Fin (n + 1) → Rˣ) (u : Up (n + 1) → K) (i : Fin n) : gram ι τ d u i.succ 0 = τ (headU u i) * ι (d 0)

theorem **gram_succ_succ** {n : ℕ} (d : Fin (n + 1) → Rˣ) (u : Up (n + 1) → K) (i j : Fin n) : gram ι τ d u i.succ j.succ = gram ι τ (Fin.tail d) (tailU u) i j + τ (headU u i) * ι (d 0) * headU u j

theorem **gram_split** (hτι : ∀ r, τ (ι r) = ι r) {n : ℕ} (d : Fin (n + 1) → Rˣ) (u : Up (n + 1) → K) : gram ι τ d u = blk τ (ι (d 0)) (fun j => ι (d 0) * headU u j) (fun a b => gram ι τ (Fin.tail d) (tailU u) a b + τ (headU u a) * ι (d 0) * headU u b)

theorem **gram_mem** (hττ : ∀ x, τ (τ x) = x) (hτι : ∀ r, τ (ι r) = ι r) {n : ℕ} (d : Fin n → Rˣ) (u : Up n → K) : gram ι τ d u ∈ Hm τ n

theorem **blk_mem** (hττ : ∀ x, τ (τ x) = x) {n : ℕ} {A : K} (hA : τ A = A) (c : Fin n → K) {S : Matrix (Fin n) (Fin n) K} (hS : S ∈ Hm τ n) : blk τ A c S ∈ Hm τ (n + 1)

theorem **gram_eq** (ι : R →+* K) (τ : K →+* K) {n : ℕ} (d : Fin n → Rˣ) (u : Up n → K) : gram ι τ d u = cT τ (unip u) * Matrix.diagonal (fun i => ι (d i)) * unip u

theorem **oH_isOpenEmbedding** {lev : K → R} (hι : Continuous ι) (hτ : Continuous τ) (hlev : Continuous lev) (hττ : ∀ x, τ (τ x) = x) (hτι : ∀ r, τ (ι r) = ι r) (hlι : ∀ r, lev (ι r) = r) (hfix : ∀ y, τ y = y → ∃ r, y = ι r) (hU : IsOpenEmbedding (Units.val : Rˣ → R)) (n : ℕ) : IsOpenEmbedding (oH ι τ hττ hτι n)

theorem **ldl_isOpenEmbedding** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R], IsLocR R → ∀ n : ℕ, ∃ o : (Fin n → Rˣ) × ({p : Fin n × Fin n // p.1 < p.2} → R ⊗[F] E) → HermR R σ n, IsOpenEmbedding o ∧ ∀ d u, ∀ N : Matrix (Fin n) (Fin n) (R ⊗[F] E), (N = Matrix.of fun i j => if i = j then 1 else if h : i < j then u ((i, j), h) else 0) → (o (d, u) : Matrix (Fin n) (Fin n) (R ⊗[F] E)) = cT (sigR R σ) N * Matrix.diagonal (fun i => ((d i : R) ⊗ₜ[F] (1 : E) : R ⊗[F] E)) * N

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.HermitianOpenChartDiagonalUnipotentStatement

theorem **_root_.GlobalTheory.T.hermitian_open_chart_diagonal_unipotent** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R], IsLocR R → ∀ n : ℕ, ∃ o : (Fin n → Rˣ) × ({p : Fin n × Fin n // p.1 < p.2} → R ⊗[F] E) → HermR R σ n, IsOpenEmbedding o ∧ ∀ d u, ∀ N : Matrix (Fin n) (Fin n) (R ⊗[F] E), (N = Matrix.of fun i j => if i = j then 1 else if h : i < j then u ((i, j), h) else 0) → (o (d, u) : Matrix (Fin n) (Fin n) (R ⊗[F] E)) = cT (sigR R σ) N * Matrix.diagonal (fun i => ((d i : R) ⊗ₜ[F] (1 : E) : R ⊗[F] E)) * N

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.HermitianOrbitLocalSectionSubgroup

theorem **tri_mul_diag** {A B : Matrix (Fin n) (Fin n) K} (hA : A.BlockTriangular id) (hB : B.BlockTriangular id) (i : Fin n) : (A * B) i i = A i i * B i i

theorem **nMat_tri** (m : {p : Fin n × Fin n // p.1 < p.2} → K) : (nMat m).BlockTriangular id

theorem **nMat_det** (m : {p : Fin n × Fin n // p.1 < p.2} → K) : (nMat m).det = 1

theorem **aMat_apply** (ι : R →+* K) {U : Subgroup Rˣ} (u : Fin n → U) (m : {p : Fin n × Fin n // p.1 < p.2} → K) (i j : Fin n) : aMat ι u m i j = ι ((u i : Rˣ) : R) * nMat m i j

theorem **aMat_tri** (ι : R →+* K) {U : Subgroup Rˣ} (u : Fin n → U) (m : {p : Fin n × Fin n // p.1 < p.2} → K) : (aMat ι u m).BlockTriangular id

theorem **aMat_det** (ι : R →+* K) {U : Subgroup Rˣ} (u : Fin n → U) (m : {p : Fin n × Fin n // p.1 < p.2} → K) : (aMat ι u m).det = ι ((∏ i, (u i : Rˣ) : Rˣ) : R)

theorem **cInv_mul** (ι : R →+* K) {U : Subgroup Rˣ} (u : Fin n → U) (m : {p : Fin n × Fin n // p.1 < p.2} → K) : cInv ι u * (aMat ι u m).det = 1

theorem **aGL_mem** (ι : R →+* K) {U : Subgroup Rˣ} (u : Fin n → U) (m : {p : Fin n × Fin n // p.1 < p.2} → K) : aGL ι u m ∈ triSub ι U n

theorem **uOf_spec** (ι : R →+* K) {U : Subgroup Rˣ} (x : triSub ι U n) (i : Fin n) : ((x : GL (Fin n) K) : Matrix (Fin n) (Fin n) K) i i = ι ((uOf ι x i : Rˣ) : R)

theorem **unit_cancel** (ι : R →+* K) (u : Rˣ) (a : K) : ι ((u⁻¹ : Rˣ) : R) * (ι (u : R) * a) = a

theorem **unit_cancel'** (ι : R →+* K) (u : Rˣ) (a : K) : ι (u : R) * (ι ((u⁻¹ : Rˣ) : R) * a) = a

theorem **aGL_coords** (ι : R →+* K) {U : Subgroup Rˣ} (x : triSub ι U n) : aGL ι (uOf ι x) (mOf ι x) = (x : GL (Fin n) K)

theorem **coords_aGL** (ι : R →+* K) (hι : Function.Injective ι) {U : Subgroup Rˣ} (u : Fin n → U) (m : {p : Fin n × Fin n // p.1 < p.2} → K) : uOf ι (aGL ι u m, aGL_mem ι u m) = u ∧ mOf ι (aGL ι u m, aGL_mem ι u m) = m

theorem **aMat_gram** (ι : R →+* K) (sig : K →+* K) (hs : ∀ r, sig (ι r) = ι r) {U : Subgroup Rˣ} (u : Fin n → U) (m : {p : Fin n × Fin n // p.1 < p.2} → K) (d : Fin n → Rˣ) : cT sig (aMat ι u m) * Matrix.diagonal (fun i => ι (d i : R)) * aMat ι u m = cT sig (nMat m) * Matrix.diagonal (fun i => ι ((((u i : Rˣ) ^ 2 * d i : Rˣ)) : R)) * nMat m

theorem **nMat_cont** : Continuous (nMat : ({p : Fin n × Fin n // p.1 < p.2} → K) → _)

theorem **aMat_cont** (ι : R →+* K) (hι : Continuous ι) (U : Subgroup Rˣ) : Continuous fun p : (Fin n → U) × ({p : Fin n × Fin n // p.1 < p.2} → K) => aMat ι p.1 p.2

theorem **aGL_cont** (ι : R →+* K) (hι : Continuous ι) (U : Subgroup Rˣ) : Continuous fun p : (Fin n → U) × ({p : Fin n × Fin n // p.1 < p.2} → K) => aGL ι p.1 p.2

theorem **entry_cont** (ι : R →+* K) (U : Subgroup Rˣ) (i j : Fin n) : Continuous fun x : triSub ι U n => ((x : GL (Fin n) K) : Matrix (Fin n) (Fin n) K) i j

theorem **uOf_cont** (ι : R →+* K) (π : K → R) (hπ : Continuous π) (hπι : ∀ r, π (ι r) = r) (hv : IsInducing (Units.val : Rˣ → R)) (U : Subgroup Rˣ) (i : Fin n) : Continuous fun x : triSub ι U n => (uOf ι x i : Rˣ)

theorem **sq_open** (U : Subgroup Rˣ) (hU : IsOpenEmbedding (fun u : U => ((u : Rˣ) ^ 2 : Rˣ))) (d : Fin n → Rˣ) : IsOpenEmbedding (fun p : (Fin n → U) × ({p : Fin n × Fin n // p.1 < p.2} → K) => ((fun i => (p.1 i : Rˣ) ^ 2 * d i : Fin n → Rˣ), p.2))

theorem **conj_case** (sig : K →+* K) (Hm : AddSubgroup (Matrix (Fin n) (Fin n) K)) (hH : ∀ S ∈ Hm, ∀ g : Matrix (Fin n) (Fin n) K, cT sig g * S * g ∈ Hm) (G : GL (Fin n) K) (Q₀ : Subgroup (GL (Fin n) K)) (o₀ : Q₀ → Hm) (ho₀ : IsOpenEmbedding o₀) (D : Matrix (Fin n) (Fin n) K) (hf : ∀ x : Q₀, (o₀ x : Matrix (Fin n) (Fin n) K) = cT sig ((x : GL (Fin n) K) : Matrix (Fin n) (Fin n) K) * D * ((x : GL (Fin n) K) : Matrix (Fin n) (Fin n) K)) : ∃ (Q : Subgroup (GL (Fin n) K)) (_ : Q ≃ₜ Q₀) (o : Q → Hm), IsOpenEmbedding o ∧ ∀ x : Q, (o x : Matrix (Fin n) (Fin n) K) = cT sig ((x : GL (Fin n) K) : Matrix (Fin n) (Fin n) K) * (cT sig (G : Matrix (Fin n) (Fin n) K) * D * (G : Matrix (Fin n) (Fin n) K)) * ((x : GL (Fin n) K) : Matrix (Fin n) (Fin n) K)

theorem **half_trace_tmul** {F E : Type} [Field F] [NumberField F] [Field E] [Algebra F E] (h2 : Module.finrank F E = 2) (R : Type) [CommRing R] [Algebra F R] (r : R) : algebraMap F R (1 / 2) * Algebra.trace R (R ⊗[F] E) (r ⊗ₜ[F] (1 : E)) = r

theorem **tri_slice** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R], IsLocR R → ∀ (n : ℕ) (S : HermR R σ n), IsUnit (S : Matrix (Fin n) (Fin n) (R ⊗[F] E)).det → ∃ (Q : Subgroup (GL (Fin n) (R ⊗[F] E))) (o : Q → HermR R σ n), LocallyCompactSpace Q ∧ SecondCountableTopology Q ∧ IsOpenEmbedding o ∧ ∀ x : Q, (o x : Matrix (Fin n) (Fin n) (R ⊗[F] E)) = cT (sigR R σ) ((x : GL (Fin n) (R ⊗[F] E)) : Matrix (Fin n) (Fin n) (R ⊗[F] E)) * (S : Matrix (Fin n) (Fin n) (R ⊗[F] E)) * ((x : GL (Fin n) (R ⊗[F] E)) : Matrix (Fin n) (Fin n) (R ⊗[F] E))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.HermitianOrbitLocalSectionSubgroupStatement

theorem **_root_.GlobalTheory.T.hermitian_orbit_local_section_subgroup** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R], IsLocR R → ∀ (n : ℕ) (S : HermR R σ n), IsUnit (S : Matrix (Fin n) (Fin n) (R ⊗[F] E)).det → ∃ (Q : Subgroup (GL (Fin n) (R ⊗[F] E))) (o : Q → HermR R σ n), LocallyCompactSpace Q ∧ SecondCountableTopology Q ∧ IsOpenEmbedding o ∧ ∀ x : Q, (o x : Matrix (Fin n) (Fin n) (R ⊗[F] E)) = cT (sigR R σ) ((x : GL (Fin n) (R ⊗[F] E)) : Matrix (Fin n) (Fin n) (R ⊗[F] E)) * (S : Matrix (Fin n) (Fin n) (R ⊗[F] E)) * ((x : GL (Fin n) (R ⊗[F] E)) : Matrix (Fin n) (Fin n) (R ⊗[F] E))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.HermitianRankOneHomeomorphBase

theorem **sig_sig** (hcm : IsCMExt F E σ) (x : E) : σ (σ x) = x

theorem **exists_sig_ne** (hcm : IsCMExt F E σ) : ∃ x : E, σ x ≠ x

theorem **trace_sig** (hcm : IsCMExt F E σ) (y : E) : Algebra.trace F E (σ y) = Algebra.trace F E y

theorem **add_sig_eq_trace** (hcm : IsCMExt F E σ) (y : E) : y + σ y = algebraMap F E (Algebra.trace F E y)

theorem **trace_tmul** (r : R) (e : E) : Algebra.trace R (R ⊗[F] E) (r ⊗ₜ[F] e) = r * algebraMap F R (Algebra.trace F E e)

theorem **add_sigR_eq_trace** (hcm : IsCMExt F E σ) (z : R ⊗[F] E) : z + sigR R σ z = (Algebra.trace R (R ⊗[F] E) z) ⊗ₜ[F] (1 : E)

theorem **tH_hermScal** (hcm : IsCMExt F E σ) (r : R) : tH R σ (hermScal R σ r) = r

theorem **hermScal_tH** (hcm : IsCMExt F E σ) (S : HermR R σ 1) : hermScal R σ (tH R σ S) = S

theorem **map_coordHerm** (hcm : IsCMExt F E σ) {d : ℕ} (b : Module.Basis (Fin d) F E) (lam : Measure R) [SigmaFinite lam] : (coordHerm R σ b lam 1).map (hermHomeo R hcm) = lam

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.HermitianUnitFormDiagonalizes

@[simp] lemma **bd_zero_zero** (a : A) (M : Matrix (Fin m) (Fin m) A) : bd a M 0 0 = a

@[simp] lemma **bd_zero_succ** (a : A) (M : Matrix (Fin m) (Fin m) A) (j : Fin m) : bd a M 0 j.succ = 0

@[simp] lemma **bd_succ_zero** (a : A) (M : Matrix (Fin m) (Fin m) A) (i : Fin m) : bd a M i.succ 0 = 0

@[simp] lemma **bd_succ_succ** (a : A) (M : Matrix (Fin m) (Fin m) A) (i j : Fin m) : bd a M i.succ j.succ = M i j

lemma **bd_mul** (a b : A) (M N : Matrix (Fin m) (Fin m) A) : bd a M * bd b N = bd (a * b) (M * N)

lemma **det_bd** (a : A) (M : Matrix (Fin m) (Fin m) A) : (bd a M).det = a * M.det

lemma **cT_bd** (σ : A →+* A) (a : A) (M : Matrix (Fin m) (Fin m) A) : cT σ (bd a M) = bd (σ a) (cT σ M)

lemma **bd_diagonal** (a : A) (d : Fin m → A) : bd a (Matrix.diagonal d) = Matrix.diagonal (Fin.cons a d : Fin (m + 1) → A)

@[simp] lemma **uni_zero_zero** (r : Fin m → A) : uni r 0 0 = 1

@[simp] lemma **uni_zero_succ** (r : Fin m → A) (j : Fin m) : uni r 0 j.succ = r j

@[simp] lemma **uni_succ_zero** (r : Fin m → A) (i : Fin m) : uni r i.succ 0 = 0

@[simp] lemma **uni_succ_succ** (r : Fin m → A) (i j : Fin m) : uni r i.succ j.succ = (1 : Matrix (Fin m) (Fin m) A) i j

lemma **det_uni** (r : Fin m → A) : (uni r).det = 1

lemma **mul_uni** (K : Matrix (Fin (m + 1)) (Fin (m + 1)) A) (r : Fin m → A) (i : Fin (m + 1)) : (K * uni r) i 0 = K i 0 ∧ ∀ j : Fin m, (K * uni r) i j.succ = K i j.succ + K i 0 * r j

lemma **cT_uni_mul** (σ : A →+* A) (P : Matrix (Fin (m + 1)) (Fin (m + 1)) A) (r : Fin m → A) (j : Fin (m + 1)) : (cT σ (uni r) * P) 0 j = P 0 j ∧ ∀ i : Fin m, (cT σ (uni r) * P) i.succ j = P i.succ j + σ (r i) * P 0 j

lemma **schur_step** (σ : A →+* A) (K : Matrix (Fin (m + 1)) (Fin (m + 1)) A) (hK : ∀ i j, σ (K j i) = K i j) (b : A) (hb : b * K 0 0 = 1) (hσb : σ b = b) : cT σ (uni fun j => -(b * K 0 j.succ)) * K * (uni fun j => -(b * K 0 j.succ)) = bd (K 0 0) (schur K b)

lemma **det_cT** (σ : A →+* A) (g : Matrix (Fin m) (Fin m) A) : (cT σ g).det = σ g.det

lemma **cT_cT** (σ : A →+* A) (hσ : ∀ y, σ (σ y) = y) {p q : Type} (M : Matrix p q A) : cT σ (cT σ M) = M

lemma **cT_congr_herm** (σ : A →+* A) {n : ℕ} (hσ : ∀ y, σ (σ y) = y) (J g : Matrix (Fin n) (Fin n) A) (hJ : ∀ i j, σ (J j i) = J i j) : ∀ i j, σ ((cT σ g * J * g) j i) = (cT σ g * J * g) i j

lemma **corner** (σ : A →+* A) {n : ℕ} (J g : Matrix (Fin (n + 1)) (Fin (n + 1)) A) (x : Fin (n + 1) → A) (hg : ∀ i, g i 0 = x i) : (cT σ g * J * g) 0 0 = hform σ J x x

lemma **unimodular** (σ : A →+* A) {n : ℕ} (J : Matrix (Fin n) (Fin n) A) (x : Fin n → A) (b : A) (hb : b * hform σ J x x = 1) : ∃ c : Fin n → A, ∑ i, c i * x i = 1

theorem **diagonalise** : ∀ (A : Type) [CommRing A] [Finite (MaximalSpectrum A)] (σ : A →+* A), (∀ y, σ (σ y) = y) → (∀ (m : ℕ) (J : Matrix (Fin (m + 1)) (Fin (m + 1)) A), (∀ i j, σ (J j i) = J i j) → IsUnit J.det → ∃ x, IsUnit (HeckeCharacterZeroLocus.SiegelWeilU.hform σ J x x)) → ∀ (m : ℕ) (J : Matrix (Fin m) (Fin m) A), (∀ i j, σ (J j i) = J i j) → IsUnit J.det → ∃ (g : Matrix (Fin m) (Fin m) A) (d : Fin m → A), IsUnit g.det ∧ (∀ i, IsUnit (d i) ∧ σ (d i) = d i) ∧ HeckeCharacterZeroLocus.SiegelWeilU.cT σ g * J * g = Matrix.diagonal d

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.HermitianUnitFormDiagonalizesStatement

theorem **_root_.GlobalTheory.T.hermitian_unit_form_diagonalizes** : ∀ (A : Type) [CommRing A] [Finite (MaximalSpectrum A)] (σ : A →+* A), (∀ y, σ (σ y) = y) → (∀ (m : ℕ) (J : Matrix (Fin (m + 1)) (Fin (m + 1)) A), (∀ i j, σ (J j i) = J i j) → IsUnit J.det → ∃ x, IsUnit (HeckeCharacterZeroLocus.SiegelWeilU.hform σ J x x)) → ∀ (m : ℕ) (J : Matrix (Fin m) (Fin m) A), (∀ i j, σ (J j i) = J i j) → IsUnit J.det → ∃ (g : Matrix (Fin m) (Fin m) A) (d : Fin m → A), IsUnit g.det ∧ (∀ i, IsUnit (d i) ∧ σ (d i) = d i) ∧ HeckeCharacterZeroLocus.SiegelWeilU.cT σ g * J * g = Matrix.diagonal d

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.HermitianUnitFormRepresentsUnit

theorem **hform_add_left** (σ : A →+* A) (J : Matrix n n A) (x x' y : n → A) : hform σ J (x + x') y = hform σ J x y + hform σ J x' y

theorem **hform_add_right** (σ : A →+* A) (J : Matrix n n A) (x y y' : n → A) : hform σ J x (y + y') = hform σ J x y + hform σ J x y'

theorem **hform_smul_left'** (σ : A →+* A) (J : Matrix n n A) (a : A) (x y : n → A) : hform σ J (a • x) y = σ a * hform σ J x y

theorem **hform_smul_right'** (σ : A →+* A) (J : Matrix n n A) (a : A) (x y : n → A) : hform σ J x (a • y) = a * hform σ J x y

theorem **hform_single_single** (σ : A →+* A) (J : Matrix n n A) (i j : n) : hform σ J (Pi.single i 1) (Pi.single j 1) = J i j

theorem **hform_self_fixed** (σ : A →+* A) (hσ : ∀ y, σ (σ y) = y) (J : Matrix n n A) (hJ : ∀ i j, σ (J j i) = J i j) (x : n → A) : σ (hform σ J x x) = hform σ J x x

theorem **hform_self_congr** (σ : A →+* A) (J : Matrix n n A) (P : Ideal A) (x z : n → A) (h₁ : ∀ i, z i − x i ∈ P) (h₂ : ∀ i, σ (z i − x i) ∈ P) : hform σ J z z − hform σ J x x ∈ P

theorem **exists_hform_not_mem** (σ : A →+* A) (hσ : ∀ y, σ (σ y) = y) (h2 : IsUnit (2 : A)) (hδ : ∃ δ : A, IsUnit δ ∧ σ δ = −δ) (m : ℕ) (J : Matrix (Fin (m + 1)) (Fin (m + 1)) A) (hdet : IsUnit J.det) (P : Ideal A) (hP : P.IsMaximal) : ∃ y, hform σ J y y ∉ P

theorem **comap_isMaximal** (σ : A →+* A) (hσ : ∀ y, σ (σ y) = y) (P : Ideal A) (hP : P.IsMaximal) : (P.comap σ).IsMaximal

theorem **isCoprime_of_mem_not_mem** {P Q : Ideal A} (hP : P.IsMaximal) (hQ : Q.IsMaximal) {a : A} (ha : a ∈ P) (ha' : a ∉ Q) : IsCoprime P Q

theorem **exists_good_on_finset** [Finite (MaximalSpectrum A)] (σ : A →+* A) (hσ : ∀ y, σ (σ y) = y) (h2 : IsUnit (2 : A)) (hδ : ∃ δ : A, IsUnit δ ∧ σ δ = −δ) (m : ℕ) (J : Matrix (Fin (m + 1)) (Fin (m + 1)) A) (hJ : ∀ i j, σ (J j i) = J i j) (hdet : IsUnit J.det) (T : Finset (MaximalSpectrum A)) : ∃ x, ∀ p ∈ T, hform σ J x x ∉ p.asIdeal

theorem **exists_isUnit_hform** : ∀ (A : Type) [CommRing A] [Finite (MaximalSpectrum A)] (σ : A →+* A), (∀ y, σ (σ y) = y) → IsUnit (2 : A) → (∃ δ : A, IsUnit δ ∧ σ δ = −δ) → ∀ (m : ℕ) (J : Matrix (Fin (m + 1)) (Fin (m + 1)) A), (∀ i j, σ (J j i) = J i j) → IsUnit J.det → ∃ x, IsUnit (HeckeCharacterZeroLocus.SiegelWeilU.hform σ J x x)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.HermitianUnitFormRepresentsUnitStatement

theorem **_root_.GlobalTheory.T.hermitian_unit_form_represents_unit** : ∀ (A : Type) [CommRing A] [Finite (MaximalSpectrum A)] (σ : A →+* A), (∀ y, σ (σ y) = y) → IsUnit (2 : A) → (∃ δ : A, IsUnit δ ∧ σ δ = −δ) → ∀ (m : ℕ) (J : Matrix (Fin (m + 1)) (Fin (m + 1)) A), (∀ i j, σ (J j i) = J i j) → IsUnit J.det → ∃ x, IsUnit (HeckeCharacterZeroLocus.SiegelWeilU.hform σ J x x)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.HyperbolicAndBlockOneNondegenerate

theorem **blockOne_zero_zero** {K : Type} [Zero K] {M : ℕ} (t : K) (A : Matrix (Fin M) (Fin M) K) : blockOne t A 0 0 = t

theorem **blockOne_zero_succ** {K : Type} [Zero K] {M : ℕ} (t : K) (A : Matrix (Fin M) (Fin M) K) (j : Fin M) : blockOne t A 0 j.succ = 0

theorem **blockOne_succ_zero** {K : Type} [Zero K] {M : ℕ} (t : K) (A : Matrix (Fin M) (Fin M) K) (i : Fin M) : blockOne t A i.succ 0 = 0

theorem **blockOne_succ_succ** {K : Type} [Zero K] {M : ℕ} (t : K) (A : Matrix (Fin M) (Fin M) K) (i j : Fin M) : blockOne t A i.succ j.succ = A i j

theorem **det_blockOne** {K : Type} [CommRing K] {M : ℕ} (t : K) (A : Matrix (Fin M) (Fin M) K) : (blockOne t A).det = t * A.det

theorem **hypJ_zero_zero** {K : Type} [Zero K] [One K] {M : ℕ} (A : Matrix (Fin M) (Fin M) K) : hypJ A 0 0 = 0

theorem **hypJ_zero_one** {K : Type} [Zero K] [One K] {M : ℕ} (A : Matrix (Fin M) (Fin M) K) : hypJ A 0 1 = 1

theorem **hypJ_zero_succ_succ** {K : Type} [Zero K] [One K] {M : ℕ} (A : Matrix (Fin M) (Fin M) K) (j : Fin M) : hypJ A 0 j.succ.succ = 0

theorem **hypJ_one_zero** {K : Type} [Zero K] [One K] {M : ℕ} (A : Matrix (Fin M) (Fin M) K) : hypJ A 1 0 = 1

theorem **hypJ_one_one** {K : Type} [Zero K] [One K] {M : ℕ} (A : Matrix (Fin M) (Fin M) K) : hypJ A 1 1 = 0

theorem **hypJ_one_succ_succ** {K : Type} [Zero K] [One K] {M : ℕ} (A : Matrix (Fin M) (Fin M) K) (j : Fin M) : hypJ A 1 j.succ.succ = 0

theorem **hypJ_succ_succ_zero** {K : Type} [Zero K] [One K] {M : ℕ} (A : Matrix (Fin M) (Fin M) K) (i : Fin M) : hypJ A i.succ.succ 0 = 0

theorem **hypJ_succ_succ_one** {K : Type} [Zero K] [One K] {M : ℕ} (A : Matrix (Fin M) (Fin M) K) (i : Fin M) : hypJ A i.succ.succ 1 = 0

theorem **hypJ_succ_succ_succ_succ** {K : Type} [Zero K] [One K] {M : ℕ} (A : Matrix (Fin M) (Fin M) K) (i j : Fin M) : hypJ A i.succ.succ j.succ.succ = A i j

theorem **det_hypJ** {K : Type} [CommRing K] {M : ℕ} (A : Matrix (Fin M) (Fin M) K) : (hypJ A).det = −A.det

theorem **isHermND_hypJ** (σ : E →ₐ[F] E) {M : ℕ} {J : Matrix (Fin M) (Fin M) E} (hJ : IsHermND σ J) : IsHermND σ (hypJ J)

theorem **isHermND_blockOne** (σ : E →ₐ[F] E) {M : ℕ} {J : Matrix (Fin M) (Fin M) E} (hJ : IsHermND σ J) {t : F} (ht : t ≠ 0) : IsHermND σ (blockOne (algebraMap F E t) J)

theorem **blockForms_isHermND** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → IsHermND σ (hypJ J) ∧ ∀ t : F, t ≠ 0 → IsHermND σ (blockOne (algebraMap F E t) J)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.HyperbolicAndBlockOneNondegenerateStatement

theorem **_root_.GlobalTheory.T.hyperbolic_and_block_one_nondegenerate** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → IsHermND σ (hypJ J) ∧ ∀ t : F, t ≠ 0 → IsHermND σ (blockOne (algebraMap F E t) J)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.HyperbolicFiniteVolumeIntegralBound

theorem **actW_smul** {F E : Type} [Field F] [Field E] [Algebra F E] (R : Type) [CommRing R] [Algebra F R] (σ : E →ₐ[F] E) {M n : ℕ} (J : Matrix (Fin M) (Fin M) E) (h : HA R σ J) (a : R ⊗[F] E) (X : Fin n → Fin M → R ⊗[F] E) : actW R σ J h (a • X) = a • actW R σ J h X

theorem **mean_isotropic_count_le** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → 1 ≤ M → FinVol F E σ M J → ∀ τ : Multiplicative ℝ →* (AF F ⊗[F] E)ˣ, IsFlow τ → ∀ (μ : Measure (HA (AF F) σ (hypJ J))) (𝒯 : Set (HA (AF F) σ (hypJ J))), IsHaarH F E σ (M + 2) (hypJ J) μ → IsFDH F E σ (M + 2) (hypJ J) μ 𝒯 → ∀ Ω : Set (Fin 1 → Fin (M + 2) → AF F ⊗[F] E), IsCompact Ω → ∃ C : ℝ, ∀ r : ℝ, 0 < r → cntI σ _ μ 𝒯 {X : Fin 1 → Fin (M + 2) → E | X ≠ 0 ∧ gramE σ (hypJ J) X = 0} (fun Y => (Set.indicator Ω (fun _ => (1 : ℝ≥0∞))) (((τ (Multiplicative.ofAdd (−Real.log r)) : (AF F ⊗[F] E)ˣ) : AF F ⊗[F] E) • Y)) ≤ ENNReal.ofReal (C * r ^ (2 * Module.finrank ℚ F * (M + 1)))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.HyperbolicFiniteVolumeIntegralBoundStatement

theorem **_root_.GlobalTheory.T.hyperbolic_finite_volume_integral_bound** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → 1 ≤ M → FinVol F E σ M J → ∀ τ : Multiplicative ℝ →* (AF F ⊗[F] E)ˣ, IsFlow τ → ∀ (μ : Measure (HA (AF F) σ (hypJ J))) (𝒯 : Set (HA (AF F) σ (hypJ J))), IsHaarH F E σ (M + 2) (hypJ J) μ → IsFDH F E σ (M + 2) (hypJ J) μ 𝒯 → ∀ Ω : Set (Fin 1 → Fin (M + 2) → AF F ⊗[F] E), IsCompact Ω → ∃ C : ℝ, ∀ r : ℝ, 0 < r → cntI σ _ μ 𝒯 {X : Fin 1 → Fin (M + 2) → E | X ≠ 0 ∧ gramE σ (hypJ J) X = 0} (fun Y => (Set.indicator Ω (fun _ => (1 : ℝ≥0∞))) (((τ (Multiplicative.ofAdd (−Real.log r)) : (AF F ⊗[F] E)ˣ) : AF F ⊗[F] E) • Y)) ≤ ENNReal.ofReal (C * r ^ (2 * Module.finrank ℚ F * (M + 1)))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.HyperbolicFundamentalDomainCover

theorem **actW_smul** {F E : Type} [Field F] [Field E] [Algebra F E] (R : Type) [CommRing R] [Algebra F R] (σ : E →ₐ[F] E) {M n : ℕ} (J : Matrix (Fin M) (Fin M) E) (h : HA R σ J) (a : R ⊗[F] E) (X : Fin n → Fin M → R ⊗[F] E) : actW R σ J h (a • X) = a • actW R σ J h X

theorem **integral_isotropic_lines_le_flow_sum** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → FinVol F E σ M J → ∀ τ : Multiplicative ℝ →* (AF F ⊗[F] E)ˣ, IsFlow τ → ∀ (μ : Measure (HA (AF F) σ (hypJ J))) (𝒯 : Set (HA (AF F) σ (hypJ J))), IsHaarH F E σ (M + 2) (hypJ J) μ → IsFDH F E σ (M + 2) (hypJ J) μ 𝒯 → ∃ (Y : Set ((Fin 1 → Fin (M + 2) → AF F ⊗[F] E) × (Fin 1 → Fin (M + 2) → AF F ⊗[F] E))) (Kc : ℝ≥0∞), IsCompact Y ∧ Kc < ⊤ ∧ (∀ p ∈ Y, hR (AF F) σ (hypJ J) (p.2 0) (p.1 0) = 1) ∧ ∀ Φ : (Fin 1 → Fin (M + 2) → AF F ⊗[F] E) → ℝ≥0∞, Measurable Φ → (∀ (l : E) y, l ≠ 0 → Φ ((inclR (F

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.HyperbolicFundamentalDomainCoverStatement

theorem **_root_.GlobalTheory.T.hyperbolic_fundamental_domain_cover** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → FinVol F E σ M J → ∀ τ : Multiplicative ℝ →* (AF F ⊗[F] E)ˣ, IsFlow τ → ∀ (μ : Measure (HA (AF F) σ (hypJ J))) (𝒯 : Set (HA (AF F) σ (hypJ J))), IsHaarH F E σ (M + 2) (hypJ J) μ → IsFDH F E σ (M + 2) (hypJ J) μ 𝒯 → ∃ (Y : Set ((Fin 1 → Fin (M + 2) → AF F ⊗[F] E) × (Fin 1 → Fin (M + 2) → AF F ⊗[F] E))) (Kc : ℝ≥0∞), IsCompact Y ∧ Kc < ⊤ ∧ (∀ p ∈ Y, hR (AF F) σ (hypJ J) (p.2 0) (p.1 0) = 1) ∧ ∀ Φ : (Fin 1 → Fin (M + 2) → AF F ⊗[F] E) → ℝ≥0∞, Measurable Φ → (∀ (l : E) y, l ≠ 0 → Φ ((inclR (F

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.HyperbolicParabolicSiegelSetBound

theorem **exists_nu_siegel** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → FinVol F E σ M J → ∀ τ : Multiplicative ℝ →* (AF F ⊗[F] E)ˣ, IsFlow τ → ∃ (P : Subgroup (HA (AF F) σ (hypJ J))) (ν : Measure (HA (AF F) σ (hypJ J))) (S : ℤ → Set (HA (AF F) σ (hypJ J))) (ω : Set (AF F ⊗[F] E)ˣ) (Kc : ℝ≥0∞), (∀ h, h ∈ P ↔ ∃ a : (AF F ⊗[F] E)ˣ, actW (AF F) σ _ h (eOne _ (M + 1)) = (a : AF F ⊗[F] E) • eOne _ (M + 1)) ∧ SFinite ν ∧ (∀ p ∈ P, ν.map (fun g => p * g) = ν) ∧ (∀ U ∈ 𝒩 (1 : HA (AF F) σ (hypJ J)), ν U ≠ 0) ∧ IsCompact ω ∧ Kc < ⊤ ∧ (∀ j, MeasurableSet (S j)) ∧ (∀ j : ℤ, ν (S j) ≤ Kc * ENNReal.ofReal (Real.exp (-(2 * Module.finrank ℚ F * (M + 1) * (j : ℝ))))) ∧ (∀ j : ℤ, ∀ p ∈ S j, p ∈ P ∧ ∃ c ∈ ω, actW (AF F) σ _ p (eOne _ (M + 1)) = ((τ (Multiplicative.ofAdd (j : ℝ)) * c

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.HyperbolicParabolicSiegelSetBoundStatement

theorem **_root_.GlobalTheory.T.hyperbolic_parabolic_siegel_set_bound** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → FinVol F E σ M J → ∀ τ : Multiplicative ℝ →* (AF F ⊗[F] E)ˣ, IsFlow τ → ∃ (P : Subgroup (HA (AF F) σ (hypJ J))) (ν : Measure (HA (AF F) σ (hypJ J))) (S : ℤ → Set (HA (AF F) σ (hypJ J))) (ω : Set (AF F ⊗[F] E)ˣ) (Kc : ℝ≥0∞), (∀ h, h ∈ P ↔ ∃ a : (AF F ⊗[F] E)ˣ, actW (AF F) σ _ h (eOne _ (M + 1)) = (a : AF F ⊗[F] E) • eOne _ (M + 1)) ∧ SFinite ν ∧ (∀ p ∈ P, ν.map (fun g => p * g) = ν) ∧ (∀ U ∈ 𝒩 (1 : HA (AF F) σ (hypJ J)), ν U ≠ 0) ∧ IsCompact ω ∧ Kc < ⊤ ∧ (∀ j, MeasurableSet (S j)) ∧ (∀ j : ℤ, ν (S j) ≤ Kc * ENNReal.ofReal (Real.exp (-(2 * Module.finrank ℚ F * (M + 1) * (j : ℝ))))) ∧ (∀ j : ℤ, ∀ p ∈ S j, p ∈ P ∧ ∃ c ∈ ω, actW (AF F) σ _ p (eOne _ (M + 1

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.HyperbolicUnitaryStabilizerCocompact

theorem **hform_eq** (s : K →+* K) (L : Matrix (Fin N) (Fin N) K) (x y : Fin N → K) : hform s L x y = (fun i => s (x i)) ⬝ᵥ (L *ᵥ y)

theorem **hform_sub_right** (s : K →+* K) (L : Matrix (Fin N) (Fin N) K) (x y w : Fin N → K) : hform s L x (y - w) = hform s L x y - hform s L x w

theorem **hform_smul_right** (s : K →+* K) (L : Matrix (Fin N) (Fin N) K) (c : K) (x y : Fin N → K) : hform s L x (c • y) = c * hform s L x y

theorem **hform_sub_left** (s : K →+* K) (L : Matrix (Fin N) (Fin N) K) (x w y : Fin N → K) : hform s L (x - w) y = hform s L x y - hform s L w y

theorem **hform_smul_left** (s : K →+* K) (L : Matrix (Fin N) (Fin N) K) (c : K) (x y : Fin N → K) : hform s L (c • x) y = s c * hform s L x y

theorem **hform_conj** (s : K →+* K) (L : Matrix (Fin N) (Fin N) K) (hs : ∀ x, s (s x) = x) (hL : ∀ i j, s (L j i) = L i j) (x y : Fin N → K) : s (hform s L x y) = hform s L y x

theorem **hform_dual** (s : K →+* K) (L Li : Matrix (Fin N) (Fin N) K) (hs : ∀ x, s (s x) = x) (hLi : Li * L = 1) (z w : Fin N → K) : hform s L (dualV s Li z) w = z ⬝ᵥ w

theorem **partner_spec** (s : K →+* K) (L Li : Matrix (Fin N) (Fin N) K) (half : K) (hs : ∀ x, s (s x) = x) (hL : ∀ i j, s (L j i) = L i j) (hLi : Li * L = 1) (hh : s half = half) (h2 : 2 * half = 1) (z w : Fin N → K) (hw : hform s L w w = 0) (hz : z ⬝ᵥ w = 1) : hform s L w (partner s L Li half z w) = 1 ∧ hform s L (partner s L Li half z w) w = 1 ∧ hform s L (partner s L Li half z w) (partner s L Li half z w) = 0

theorem **hform_single** (s : K →+* K) (L : Matrix (Fin N) (Fin N) K) (i j : Fin N) : hform s L (Pi.single i 1) (Pi.single j 1) = L i j

theorem **continuous_hform** (s : K →+* K) (hs : Continuous s) (L : Matrix (Fin N) (Fin N) K) : Continuous fun p : (Fin N → K) × (Fin N → K) => hform s L p.1 p.2

theorem **continuous_dualV** (s : K →+* K) (hs : Continuous s) (Li : Matrix (Fin N) (Fin N) K) : Continuous (dualV s Li)

theorem **continuous_partner** (s : K →+* K) (hs : Continuous s) (L Li : Matrix (Fin N) (Fin N) K) (half : K) : Continuous fun p : (Fin N → K) × (Fin N → K) => partner s L Li half p.1 p.2

theorem **sigR_invol** (R : Type) [CommRing R] [Algebra F R] (σ : E →ₐ[F] E) (hσ : ∀ x, σ (σ x) = x) (a : R ⊗[F] E) : sigR R σ (sigR R σ a) = a

theorem **sigR_incl** (R : Type) [CommRing R] [Algebra F R] (σ : E →ₐ[F] E) (e : E) : sigR R σ (inclR R e) = inclR R (σ e)

theorem **gramR_ratTup** (R : Type) [CommRing R] [Algebra F R] (σ : E →ₐ[F] E) {N n : ℕ} (J : Matrix (Fin N) (Fin N) E) (X : Fin n → Fin N → E) : gramR R σ J (ratTup R X) = (gramE σ J X).map (inclR R)

theorem **actW_smul** (R : Type) [CommRing R] [Algebra F R] (σ : E →ₐ[F] E) {N n : ℕ} (J : Matrix (Fin N) (Fin N) E) (g : HA R σ J) (a : R ⊗[F] E) (X : Fin n → Fin N → R ⊗[F] E) : actW R σ J g (a • X) = a • actW R σ J g X

theorem **gram_frame** {F E : Type} [Field F] [Field E] [Algebra F E] (σ : E →ₐ[F] E) {M : ℕ} (J : Matrix (Fin M) (Fin M) E) : gramE σ (hypJ J) (frame E M) = !![0, 1; 1, 0]

theorem **eOne_eq_frame** {F E : Type} [Field F] [Field E] [Algebra F E] (R : Type) [CommRing R] [Algebra F R] (M : ℕ) (a : Fin 1) : eOne (R ⊗[F] E) (M + 1) a = ratTup R (frame E M) 0

theorem **continuous_pair** (s : K →+* K) (hs : Continuous s) (L Li : Matrix (Fin N) (Fin N) K) (half : K) : Continuous fun p : (Fin 1 → Fin N → K) × (Fin 1 → Fin N → K) => (![p.1 0, partner s L Li half (p.2 0) (p.1 0)] : Fin 2 → Fin N → K)

theorem **gram_pair** (s : K →+* K) (L : Matrix (Fin N) (Fin N) K) (w p : Fin N → K) (h00 : hform s L w w = 0) (h01 : hform s L w p = 1) (h10 : hform s L p w = 1) (h11 : hform s L p p = 0) (a b : Fin 2) : hform s L ((![w, p] : Fin 2 → Fin N → K) a) ((![w, p] : Fin 2 → Fin N → K) b) = (!![0, 1; 1, 0] : Matrix (Fin 2) (Fin 2) K) a b

theorem **iwa_core** {H : Type} [Group H] [TopologicalSpace H] [IsTopologicalGroup H] (s : K →+* K) (hsc : Continuous s) (hs : ∀ x, s (s x) = x) (L Li : Matrix (Fin N) (Fin N) K) (hL : ∀ i j, s (L j i) = L i j) (hLi : Li * L = 1) (half : K) (hh : s half = half) (h2 : 2 * half = 1) (act : H → (Fin N → K) → Fin N → K) (hmul : ∀ g k v, act (g * k) v = act g (act k v)) (hone : ∀ v, act 1 v = v) (hsmul : ∀ g (a : K) v, act g (a • v) = a • act g v) (hpres : ∀ g x y, hform s L (act g x) (act g y) = hform s L x y) (X : Fin 2 → Fin N → K) (hX : ∀ a b, hform s L (X a) (X b) = (!![0, 1; 1, 0] : Matrix (Fin 2) (Fin 2) K) a b) (Ω : Set (Fin 1 → Fin N → K)) (hΩ : IsCompact Ω) (hprim : ∀ y : Fin 1 → Fin N → K, (∃ z : Fin N → K, z ⬝ᵥ y 0 = 1) → ∃ (a : Kˣ) (z : Fin 1 → Fin N → K), (a : K) • y ∈ Ω ∧ z ∈ Ω ∧ z 0 ⬝ᵥ ((a : K) • y) 0 = 1) (hproper : ∀ Ω₂ : Set (Fin 2 → Fin N → K), IsCompact Ω₂ → ∃ C : Set H, IsCo

theorem **map_hyp** {E K : Type} [Field E] [CommRing K] (φ : E →+* K) : (!![0, 1; 1, 0] : Matrix (Fin 2) (Fin 2) E).map φ = !![0, 1; 1, 0]

theorem **iwasawa** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∃ C : Set (HA (AF F) σ (hypJ J)), IsCompact C ∧ ∀ h : HA (AF F) σ (hypJ J), ∃ c ∈ C, ∃ a : (AF F ⊗[F] E)ˣ, actW (AF F) σ _ (h * c⁻¹) (eOne _ (M + 1)) = (a : AF F ⊗[F] E) • eOne _ (M + 1)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.HyperbolicUnitaryStabilizerCocompactStatement

theorem **_root_.GlobalTheory.T.hyperbolic_unitary_stabilizer_cocompact** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∃ C : Set (HA (AF F) σ (hypJ J)), IsCompact C ∧ ∀ h : HA (AF F) σ (hypJ J), ∃ c ∈ C, ∃ a : (AF F ⊗[F] E)ˣ, actW (AF F) σ _ (h * c⁻¹) (eOne _ (M + 1)) = (a : AF F ⊗[F] E) • eOne _ (M + 1)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.Imports

theorem **siegelWeilUnitaryConvergent** : HeckeCharacterZeroLocus.SiegelWeilU.SiegelWeilUnitaryConvergent

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.InfiniteAdeleHaarMapEqScaledVolume

theorem **coord_apply** (y : Ainf F) (w : InfinitePlace F) : coord F y w = ringEquivRealOfIsReal (IsTotallyReal.isReal w) (y w)

theorem **coordHomeo_coe** : ⇑(coordHomeo F) = ⇑(coord F)

theorem **isHomeomorph_coord** : IsHomeomorph (coord F)

theorem **abs_coord** (y : Ainf F) (w : InfinitePlace F) : |coord F y w| = ‖y w‖

theorem **rho_coe** (w : InfinitePlace F) (x : F) : ((rho F w x : ℝ) : ℂ) = w.embedding x

theorem **coord_algebraMap** (x : F) (w : InfinitePlace F) : coord F (algebraMap F (Ainf F) x) w = rho F w x

theorem **toMixed_add** (y z : InfinitePlace F → ℝ) : toMixed F (y + z) = toMixed F y + toMixed F z

theorem **toMixed_injective** : Function.Injective (toMixed F)

theorem **toMixed_coord_algebraMap** (x : F) : toMixed F (coord F (algebraMap F (Ainf F) x)) = mixedEmbedding F x

theorem **measurePreserving_toMixed** : MeasurePreserving (toMixed F) volume volume

theorem **coord_measurable** : Measurable (coord F)

theorem **map_coord_eq** (lam : Measure (Ainf F)) (hl : IsStdInf lam) : lam.map (coord F) = ENNReal.ofReal (Real.sqrt |(discr F : ℝ)|)⁻¹ • (volume : Measure (InfinitePlace F → ℝ))

theorem **GlobalTheory.CompanionTheory.SW.real_iso** : ∀ (F : Type) [Field F] [NumberField F], IsTotallyReal F → ∃ (ρ : InfinitePlace F → (F →+* ℝ)) (e : Ainf F ≃+* (InfinitePlace F → ℝ)), (∀ w x, ((ρ w x : ℝ) : ℂ) = w.embedding x) ∧ IsHomeomorph e ∧ (∀ x w, e (algebraMap F (Ainf F) x) w = ρ w x) ∧ (∀ y w, |e y w| = ‖y w‖) ∧ ∀ lam : Measure (Ainf F), IsStdInf lam → lam.map e = ENNReal.ofReal (Real.sqrt |(discr F : ℝ)|)⁻¹ • (volume : Measure (InfinitePlace F → ℝ))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.InfiniteAdeleHaarMapEqScaledVolumeStatement

theorem **_root_.GlobalTheory.T.infinite_adele_haar_map_eq_scaled_volume** : ∀ (F : Type) [Field F] [NumberField F], IsTotallyReal F → ∃ (ρ : InfinitePlace F → (F →+* ℝ)) (e : Ainf F ≃+* (InfinitePlace F → ℝ)), (∀ w x, ((ρ w x : ℝ) : ℂ) = w.embedding x) ∧ IsHomeomorph e ∧ (∀ x w, e (algebraMap F (Ainf F) x) w = ρ w x) ∧ (∀ y w, |e y w| = ‖y w‖) ∧ ∀ lam : Measure (Ainf F), IsStdInf lam → lam.map e = ENNReal.ofReal (Real.sqrt |(discr F : ℝ)|)⁻¹ • (volume : Measure (InfinitePlace F → ℝ))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.InfiniteAdelesLocalRingTotallyReal

theorem **isOpen_pos** (ι : Type) [Finite ι] : IsOpen {x : ι → ℝ | ∀ i, 0 < x i}

theorem **isOpen_posU** (ι : Type) [Fintype ι] : IsOpen (posU ι : Set (ι → ℝ)ˣ)

theorem **isOpenEmbedding_sq** (ι : Type) [Fintype ι] : IsOpenEmbedding (fun u : posU ι => ((u : (ι → ℝ)ˣ) ^ 2 : (ι → ℝ)ˣ))

theorem **continuous_eInf** (h : IsTotallyReal F) : Continuous (eInf F h)

theorem **continuous_eInf_symm** (h : IsTotallyReal F) : Continuous (eInf F h).symm

theorem **GlobalTheory.CompanionTheory.SW.isLocR_Ainf** : ∀ (F : Type) [Field F] [NumberField F], IsTotallyReal F → IsLocR (Ainf F)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.InfiniteAdelesLocalRingTotallyRealStatement

theorem **_root_.GlobalTheory.T.infinite_adeles_local_ring_totally_real** : ∀ (F : Type) [Field F] [NumberField F], IsTotallyReal F → IsLocR (Ainf F)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.InfiniteComponentCoordinatesAction

theorem **repr_map** {R S : Type} [CommRing R] [Algebra F R] [CommRing S] [Algebra F S] (f : R →ₐ[F] S) {d : ℕ} (b : Module.Basis (Fin d) F E) (y : R ⊗[F] E) (k : Fin d) : (Algebra.TensorProduct.basis S b).repr (Algebra.TensorProduct.map f (AlgHom.id F E) y) k = f ((Algebra.TensorProduct.basis R b).repr y k)

theorem **cv_ratTup** (R : Type) [CommRing R] [Algebra F R] {d M : ℕ} (b : Module.Basis (Fin d) F E) (x : Fin M × Fin d → F) : cv R b (ratTup R (ofF b x) 0) = fun j => algebraMap F R (x j)

theorem **cv_act** (R : Type) [CommRing R] [Algebra F R] (σ : E →ₐ[F] E) {d M : ℕ} (J : Matrix (Fin M) (Fin M) E) (b : Module.Basis (Fin d) F E) (h : HA R σ J) (x : Fin M × Fin d → F) : cv R b (actW R σ J h (ratTup R (ofF b x)) 0) = gmat R σ J b h *ᵥ fun j => algebraMap F R (x j)

theorem **coords_ratTup** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (ρ : InfinitePlace F → (F →+* ℝ)) (e : Ainf F ≃+* (InfinitePlace F → ℝ)), (∀ x w, e (algebraMap F (Ainf F) x) w = ρ w x) → ∀ (ω : HA (AF F) σ J) (x : Fin M × Fin d → F), (∀ w, xw e b (projInf (actW (AF F) σ J ω⁻¹ (ratTup (AF F) (ofF b x)))) w = ginf e σ J b ω w *ᵥ fun j => ρ w (x j)) ∧ cv (Af F) b (projFin (actW (AF F) σ J ω⁻¹ (ratTup (AF F) (ofF b x))) 0) = gfin σ J b ω *ᵥ fun j => algebraMap F (Af F) (x j)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.InfiniteComponentCoordinatesActionStatement

theorem **_root_.GlobalTheory.T.infinite_component_coordinates_action** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (ρ : InfinitePlace F → (F →+* ℝ)) (e : Ainf F ≃+* (InfinitePlace F → ℝ)), (∀ x w, e (algebraMap F (Ainf F) x) w = ρ w x) → ∀ (ω : HA (AF F) σ J) (x : Fin M × Fin d → F), (∀ w, xw e b (projInf (actW (AF F) σ J ω⁻¹ (ratTup (AF F) (ofF b x)))) w = ginf e σ J b ω w *ᵥ fun j => ρ w (x j)) ∧ cv (Af F) b (projFin (actW (AF F) σ J ω⁻¹ (ratTup (AF F) (ofF b x))) 0) = gfin σ J b ω *ᵥ fun j => algebraMap F (Af F) (x j)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.InfiniteProductMultipliable

theorem **multipliable_of_summable_abs_sub_one** {ι : Type} {a : ι → ℝ} (h : Summable (fun v => |a v - 1|)) : Multipliable a

theorem **tprod_nonneg_of_nonneg** {ι : Type} {a : ι → ℝ} (hm : Multipliable a) (h0 : ∀ v, 0 ≤ a v) : 0 ≤ ∏' v, a v

theorem **tprod_ne_zero_of_summable** {ι : Type} {a : ι → ℝ} (h : Summable (fun v => |a v - 1|)) (h0 : ∀ v, a v ≠ 0) : ∏' v, a v ≠ 0

theorem **tprod_eq_prod_mul_tprod_compl** {ι : Type} {a : ι → ℝ} (h : Summable (fun v => |a v - 1|)) (S : Finset ι) : Multipliable (fun v : {v // v ∉ S} => a v.1) ∧ ∏' v, a v = (∏ v ∈ S, a v) * ∏' v : {v // v ∉ S}, a v.1

theorem **abs_tprod** : ∀ (ι : Type) (a : ι → ℝ), Summable (fun v => |a v - 1|) → Multipliable a ∧ (∀ S : Finset ι, Multipliable (fun v : {v // v ∉ S} => a v.1) ∧ ∏' v, a v = (∏ v ∈ S, a v) * ∏' v : {v // v ∉ S}, a v.1) ∧ ((∀ v, 0 < a v) → 0 < ∏' v, a v) ∧ ((∀ v, 0 ≤ a v) → 0 ≤ ∏' v, a v)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.InfiniteProductMultipliableStatement

theorem **_root_.GlobalTheory.T.infinite_product_multipliable** : ∀ (ι : Type) (a : ι → ℝ), Summable (fun v => |a v - 1|) → Multipliable a ∧ (∀ S : Finset ι, Multipliable (fun v : {v // v ∉ S} => a v.1) ∧ ∏' v, a v = (∏ v ∈ S, a v) * ∏' v : {v // v ∉ S}, a v.1) ∧ ((∀ v, 0 < a v) → 0 < ∏' v, a v) ∧ ((∀ v, 0 ≤ a v) → 0 ≤ ∏' v, a v)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.InfiniteProductRenormalization

theorem **tail_eq_mul_tail_insert** {ι : Type} [DecidableEq ι] (a : ι → ℝ) (hs : Summable (fun v => |a v - 1|)) (S : Finset ι) (v : ι) (hv : v ∉ S) : ∏' w : {w // w ∉ S}, a w.1 = a v * ∏' w : {w // w ∉ insert v S}, a w.1

theorem **tail_pos** {ι : Type} (a : ι → ℝ) (hs : Summable (fun v => |a v - 1|)) (S : Finset ι) (hpos : ∀ v, v ∉ S → 0 < a v) : 0 < ∏' w : {w // w ∉ S}, a w.1

theorem **exists_const_mul_tail** : ∀ (ι : Type) [DecidableEq ι] (a : ι → ℝ) (c : Finset ι → ℝ≥0) (S₁ : Finset ι), (∀ v, 0 ≤ a v) → Summable (fun v => |a v - 1|) → (∀ S, S₁ ⊆ S → ∀ v, v ∉ S → (c S : ℝ) = c (insert v S) * a v) → ∃ (c₀ : ℝ≥0) (S₀ : Finset ι), S₁ ⊆ S₀ ∧ ∀ S, S₀ ⊆ S → (0 < ∏' v : {v // v ∉ S}, a v.1) ∧ (c S : ℝ) = c₀ * ∏' v : {v // v ∉ S}, a v.1

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.InfiniteProductRenormalizationStatement

theorem **_root_.GlobalTheory.T.infinite_product_renormalization** : ∀ (ι : Type) [DecidableEq ι] (a : ι → ℝ) (c : Finset ι → ℝ≥0) (S₁ : Finset ι), (∀ v, 0 ≤ a v) → Summable (fun v => |a v - 1|) → (∀ S, S₁ ⊆ S → ∀ v, v ∉ S → (c S : ℝ) = c (insert v S) * a v) → ∃ (c₀ : ℝ≥0) (S₀ : Finset ι), S₁ ⊆ S₀ ∧ ∀ S, S₀ ⊆ S → (0 < ∏' v : {v // v ∉ S}, a v.1) ∧ (c S : ℝ) = c₀ * ∏' v : {v // v ∉ S}, a v.1

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.IntegralAlmostAllPlaces

theorem **valued_algebraMap** {F : Type} [Field F] [NumberField F] (v : HeightOneSpectrum (𝒪 F)) (x : F) : Valued.v (algebraMap F (Fv F v) x) = v.valuation F x

theorem **eventually_le_one** {F : Type} [Field F] [NumberField F] (x : F) : ∀ᶠ v : HeightOneSpectrum (𝒪 F) in Filter.cofinite, v.valuation F x ≤ 1

theorem **eventually_eq_one** {F : Type} [Field F] [NumberField F] (x : F) (hx : x ≠ 0) : ∀ᶠ v : HeightOneSpectrum (𝒪 F) in Filter.cofinite, v.valuation F x = 1

theorem **repr_inclR** {F E : Type} [Field F] [Field E] [Algebra F E] {d : ℕ} (b : Module.Basis (Fin d) F E) (R : Type) [CommRing R] [Algebra F R] (x : E) (k : Fin d) : (Algebra.TensorProduct.basis R b).repr (inclR R x) k = algebraMap F R (b.repr x k)

theorem **rational_integral_units** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E), (∀ x : E, ∀ᶠ v : HeightOneSpectrum (𝒪 F) in Filter.cofinite, ∀ k, (Algebra.TensorProduct.basis (Fv F v) b).repr (inclR (Fv F v) x) k ∈ v.adicCompletionIntegers F) ∧ ∀ x : F, x ≠ 0 → ∀ᶠ v : HeightOneSpectrum (𝒪 F) in Filter.cofinite, Valued.v (algebraMap F (Fv F v) x) = 1

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.IntegralAlmostAllPlacesStatement

theorem **_root_.GlobalTheory.T.integral_almost_all_places** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E), (∀ x : E, ∀ᶠ v : HeightOneSpectrum (𝒪 F) in Filter.cofinite, ∀ k, (Algebra.TensorProduct.basis (Fv F v) b).repr (inclR (Fv F v) x) k ∈ v.adicCompletionIntegers F) ∧ ∀ x : F, x ≠ 0 → ∀ᶠ v : HeightOneSpectrum (𝒪 F) in Filter.cofinite, Valued.v (algebraMap F (Fv F v) x) = 1

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.InvariantMeasureCompactCoverBound

theorem **integral_fundamentalDomain_le_siegel** : ∀ (G : Type) [Group G] [TopologicalSpace G] [IsTopologicalGroup G] [T2Space G] [LocallyCompactSpace G] [SecondCountableTopology G] [MeasurableSpace G] [BorelSpace G] (μ : Measure G) [μ.IsHaarMeasure] (P : Subgroup G) (ν : Measure G) [SFinite ν], (∀ p ∈ P, ν.map (fun g => p * g) = ν) → (∀ U ∈ 𝒩 (1 : G), ν U ≠ 0) → ∀ C : Set G, IsCompact C → ∃ (C' : Set G) (K : ℝ≥0∞), IsCompact C' ∧ K < ⊤ ∧ ∀ (Γ : Subgroup G) [Countable Γ], Γ ≤ P → ∀ 𝓕 : Set G, IsFundamentalDomain Γ 𝓕 μ → ∀ S : Set G, MeasurableSet S → (∀ᵐ p ∂ν, ∃ γ ∈ Γ, γ * p ∈ S) → ∀ f g : G → ℝ≥0∞, Measurable f → Measurable g → (∀ γ ∈ Γ, ∀ x, f (γ * x) = f x) → (∀ x, f x ≠ 0 → ∃ p ∈ P, ∃ c ∈ C, x = p * c) → (∀ p ∈ S, ∀ c ∈ C', f (p * c) ≤ g p) → ∫⁻ x in 𝓕, f x ∂μ ≤ K * ∫⁻ p in S, g p ∂ν

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.InvariantMeasureCompactCoverBoundStatement

theorem **_root_.GlobalTheory.T.invariant_measure_compact_cover_bound** : ∀ (G : Type) [Group G] [TopologicalSpace G] [IsTopologicalGroup G] [T2Space G] [LocallyCompactSpace G] [SecondCountableTopology G] [MeasurableSpace G] [BorelSpace G] (μ : Measure G) [μ.IsHaarMeasure] (P : Subgroup G) (ν : Measure G) [SFinite ν], (∀ p ∈ P, ν.map (fun g => p *

g) = ν) → (∀ U ∈ 𝒩 (1 : G), ν U ≠ 0) → ∀ C : Set G, IsCompact C → ∃ (C' : Set G) (K : ℝ≥0∞), IsCompact C' ∧ K < ⊤ ∧ ∀ (Γ : Subgroup G) [Countable Γ], Γ ≤ P → ∀ 𝓕 : Set G, IsFundamentalDomain Γ 𝓕 μ → ∀ S : Set G, MeasurableSet S → (∀ᵐ p ∂ν, ∃ γ ∈ Γ, γ ∗ p ∈ S) → ∀ f g : G → ℝ≥0∞, Measurable f → Measurable g → (∀ γ ∈ Γ, ∀ x, f (γ ∗ x) = f x) → (∀ x, f x ≠ 0 → ∃ p ∈ P, ∃ c ∈ C, x = p ∗ c) → (∀ p ∈ S, ∀ c ∈ C', f (p ∗ c) ≤ g p) → ∫⁻ x in 𝓕, f x ∂μ ≤ K ∗ ∫⁻ p in S, g p ∂ν

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.IsFibreFamilyTransport

theorem **isFibreFamily_transport** : ∀ (V X Y : Type) [TopologicalSpace V] [LocallyCompactSpace V] [SecondCountableTopology V] [T2Space V] [MeasurableSpace V] [BorelSpace V] [TopologicalSpace X] [LocallyCompactSpace X] [SecondCountableTopology X] [T2Space X] [MeasurableSpace X] [BorelSpace X] (μ : Measure V) (ν : Measure X) [IsLocallyFiniteMeasure μ] [IsLocallyFiniteMeasure ν] [ν.IsOpenPosMeasure] (f : V → Y) (g : V → X) (ι : X → Y), Function.Injective ι → Continuous g → (∀ v, f v = ι (g v)) → ∀ (V' : Type) [TopologicalSpace V'] [LocallyCompactSpace V'] [SecondCountableTopology V'] [T2Space V'] [MeasurableSpace V'] [BorelSpace V'] (f' : V' → Y) (g' : V' → X), Continuous g' → (∀ v, f' v = ι (g' v)) → ∀ (e : V ≃ₜ V') (τ : X ≃ₜ X), (∀ v, g' (e v) = τ (g v)) → ∀ (B : Set X), IsOpen B → ∀ fib : X → Measure V, Formalism.MainNotions.GlobalTheoryTheory.GenusMassVolumes.IsFibreFamily μ f ι (g ⁻¹' B)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.IsotropicVectorSplitsHyperbolicPlane

theorem **hE_eq_dot** (u v : Fin M → E) : hE σ J u v = Matrix.vecMul (fun i => σ (u i)) J ⬝ᵥ v

@[simp] theorem **hLin_apply** (u v : Fin M → E) : hLin σ J u v = hE σ J u v

theorem **hE_conj** (hσ : ∀ a, σ (σ a) = a) (hJ : ∀ i j, σ (J j i) = J i j) (u v : Fin M → E) : σ (hE σ J u v) = hE σ J v u

theorem **exists_partner** (hσ : ∀ a, σ (σ a) = a) (hJ : IsHermND σ J) (h2 : (2 : E) ≠ 0) (x : Fin M → E) (hx : x ≠ 0) (hxx : hE σ J x x = 0) : ∃ y, hE σ J x y = 1 ∧ hE σ J y y = 0

theorem **finrank_perp** (hσ : ∀ a, σ (σ a) = a) (hJ : ∀ i j, σ (J j i) = J i j) {x y : Fin M → E} (hxx : hE σ J x x = 0) (hyy : hE σ J y y = 0) (hxy : hE σ J x y = 1) : Module.finrank E (LinearMap.ker ((hLin σ J x).prod (hLin σ J y))) + 2 = M

theorem **gram_eq** {n : ℕ} (c : Fin n → Fin M → E) : cT (σ : E →+* E) (Matrix.of c)ᵀ ∗ J ∗ (Matrix.of c)ᵀ = gramE σ J c

theorem **det_ne_zero_of_hypJ** {M₁ : ℕ} (A : Matrix (Fin M₁) (Fin M₁) E) (h : (hypJ A).det ≠ 0) : A.det ≠ 0

theorem **selfDualLattice_projection_mem_of_unit_length** (hσ : ∀ a, σ (σ a) = a) (h2 : (2 : E) ≠ 0) {M₁ : ℕ} {J : Matrix (Fin (M₁ + 2)) (Fin (M₁ + 2)) E} (hJ : IsHermND σ J) {x y : Fin (M₁ + 2) → E} (hxx : hE σ J x x = 0) (hyy : hE σ J y y = 0) (hxy : hE σ J x y = 1) : ∃ (g : GL (Fin (M₁ + 2)) E) (J₁ : Matrix (Fin M₁) (Fin M₁) E), IsHermND σ J₁ ∧ hypJ J₁ = cT (σ : E →+* E) (g : Matrix (Fin (M₁ + 2)) (Fin (M₁ + 2)) E) ∗ J ∗ (g : Matrix (Fin (M₁ + 2)) (Fin (M₁ + 2)) E)

theorem **sigma_invol** {F E : Type} [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E) (h : IsCMExt F E σ) (a : E) : σ (σ a) = a

theorem **splitHyp** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ x : Fin M → E, x ≠ 0 → hE σ J x x = 0 → ∃ (M₁ : ℕ) (hM : M = M₁ + 2) (g : GL (Fin (M₁ + 2)) E) (J₁ : Matrix (Fin M₁) (Fin M₁) E), IsHermND σ J₁ ∧ hypJ J₁ = cT (σ : E →+* E) (g : Matrix (Fin (M₁ + 2)) (Fin (M₁ + 2)) E) ∗ (hM ▸ J) ∗ (g : Matrix (Fin (M₁ + 2)) (Fin (M₁ + 2)) E)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.IsotropicVectorSplitsHyperbolicPlaneStatement

theorem **_root_.GlobalTheory.T.isotropic_vector_splits_hyperbolic_plane** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ x : Fin M → E, x ≠ 0 → hE σ J x x = 0 → ∃ (M₁ : ℕ) (hM : M = M₁ + 2) (g : GL (Fin (M₁ + 2)) E) (J₁ : Matrix (Fin M₁) (Fin M₁) E), IsHermND σ J₁ ∧ hypJ J₁ = cT (σ : E →+* E) (g : Matrix (Fin (M₁ + 2)) (Fin (M₁ + 2)) E) ∗ (hM ▸ J) ∗ (g : Matrix (Fin (M₁ + 2)) (Fin (M₁ + 2)) E)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.IwasawaDecompositionAdelicSL2

theorem **mul_at** (x y : A F) (v : HeightOneSpectrum (𝒪 F)) : (x ∗ y) v = x v ∗ y v

theorem **add_at** (x y : A F) (v : HeightOneSpectrum (𝒪 F)) : (x + y) v = x v + y v

theorem **sub_at** (x y : A F) (v : HeightOneSpectrum (𝒪 F)) : (x − y) v = x v − y v

theorem **one_at** (v : HeightOneSpectrum (𝒪 F)) : (1 : A F) v = 1

theorem **idem_int** {b : F} {v : HeightOneSpectrum (𝒪 F)} (h : IntAt b v) : idem b v = 1

theorem **idem_not** {b : F} {v : HeightOneSpectrum (𝒪 F)} (h : ¬ IntAt b v) : idem b v = 0

theorem **scale_int** {b : F} {v : HeightOneSpectrum (𝒪 F)} (h : IntAt b v) : scale b v = 1

theorem **scale_not** {b : F} {v : HeightOneSpectrum (𝒪 F)} (h : ¬ IntAt b v) : scale b v = (algebraMap F (A F) b v)⁻¹

theorem **scaleInv_int** {b : F} {v : HeightOneSpectrum (𝒪 F)} (h : IntAt b v) : scaleInv b v = 1

theorem **scaleInv_not** {b : F} {v : HeightOneSpectrum (𝒪 F)} (h : ¬ IntAt b v) : scaleInv b v = algebraMap F (A F) b v

theorem **ne_zero_of_not_int** {b : F} {v : HeightOneSpectrum (𝒪 F)} (h : ¬ IntAt b v) : algebraMap F (A F) b v ≠ 0

theorem **scale_mul_scaleInv** (b : F) : scale b ∗ scaleInv b = 1

theorem **scale_combo** (b : F) : scale b ∗ idem b + (1 − idem b) ∗ (scale b ∗ algebraMap F (A F) b) = 1

theorem **idem_mem** (b : F) : idem b ∈ Oh F

theorem **scale_mem** (b : F) : scale b ∈ Oh F

theorem **Oh_one** : (1 : A F) ∈ Oh F

theorem **Oh_neg** {x : A F} (hx : x ∈ Oh F) : −x ∈ Oh F

theorem **Oh_sub** {x y : A F} (hx : x ∈ Oh F) (hy : y ∈ Oh F) : x − y ∈ Oh F

theorem **Oh_mul** {x y : A F} (hx : x ∈ Oh F) (hy : y ∈ Oh F) : x ∗ y ∈ Oh F

theorem **scale_mul_mem** (b : F) : scale b ∗ algebraMap F (A F) b ∈ Oh F

theorem **image_scale** (b : F) : (fun x => scale b ∗ x) '' Oh F = {x | x ∈ Oh F ∧ algebraMap F (A F) b ∗ x ∈ Oh F}

theorem **w_n_eq** {R : Type} [CommRing R] (β e : R) (u : Rˣ) (k : SL(2, R)) (h00 : k 0 0 = e − 1) (h01 : k 0 1 = e) (h10 : k 1 0 = −(u : R)) (h11 : k 1 1 = −((u : R) ∗ β)) (h : (u : R) ∗ e + (1 − e) ∗ ((u : R) ∗ β) = 1) : wMat ∗ nMat β = nMat ((e − 1) ∗ (u : R)) ∗ mMat u ∗ k

theorem **iwasawa_w_n** : ∀ (F : Type) [Field F] [NumberField F], ∀ b : F, ∃ (y : A F) (a : (A F)ˣ) (k : SL(2, A F)), (∀ i j, k i j ∈ Oh F) ∧ wMat ∗ nMat (algebraMap F (A F) b) = nMat y ∗ mMat a ∗ k ∧ (fun x => (a : A F) ∗ x) '' Oh F = {x | x ∈ Oh F ∧ algebraMap F (A F) b ∗ x ∈ Oh F}

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.IwasawaDecompositionAdelicSL2Statement

theorem **_root_.GlobalTheory.T.iwasawa_decomposition_adelic_SL2** : ∀ (F : Type) [Field F] [NumberField F], ∀ b : F, ∃ (y : A F) (a : (A F)ˣ) (k : SL(2, A F)), (∀ i j, k i j ∈ Oh F) ∧ wMat ∗ nMat (algebraMap F (A F) b) = nMat y ∗ mMat a ∗ k ∧ (fun x => (a : A F) ∗ x) '' Oh F = {x | x ∈ Oh F ∧ algebraMap F (A F) b ∗ x ∈ Oh F}

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.LatticeBallCountRatioBound

theorem **Sep.card_le** {e : Λ →+ V} {r R : ℝ} (hr : 0 < r) (hrR : r ≤ R) {s : Finset Λ} (hs : Sep e r R s) : (s.card : ℝ) ≤ (3 ∗ R / r) ^ Module.finrank ℝ V

theorem **exists_max_sep** (e : Λ →+ V) {r R : ℝ} (hr : 0 < r) (hrR : r ≤ R) : ∃ s : Finset Λ, Sep e r R s ∧ ∀ t : Finset Λ, Sep e r R t → t.card ≤ s.card

theorem **Sep.exists_near** {e : Λ →+ V} {r R : ℝ} (hr : 0 < r) {s : Finset Λ} (hs : Sep e r R s) (hmax : ∀ t : Finset Λ, Sep e r R t → t.card ≤ s.card) {l : Λ} (hl : ‖e l‖ ≤ R) : ∃ t ∈ s, ‖e (l − t)‖ ≤ r

theorem **GlobalTheory.CompanionTheory.Adelic.tsum_indicator_norm_le_packing** : ∀ (V : Type) [NormedAddCommGroup V] [NormedSpace ℝ V] [FiniteDimensional ℝ V] (Λ : Type) [AddCommGroup Λ] (e : Λ →+ V) (r R : ℝ), 0 < r → r ≤ R → ∑' l : Λ, Set.indicator {l : Λ | ‖e l‖ ≤ R} (fun _ => (1 : ℝ≥0∞)) l ≤ ENNReal.ofReal ((5 ∗ R / r) ^ Module.finrank ℝ V) ∗ ∑' l : Λ, Set.indicator {l : Λ | ‖e l‖ ≤ r} (fun _ => (1 : ℝ≥0∞)) l

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.LatticeBallCountRatioBoundStatement

theorem **_root_.GlobalTheory.T.lattice_ball_count_ratio_bound** : ∀ (V : Type) [NormedAddCommGroup V] [NormedSpace ℝ V] [FiniteDimensional ℝ V] (Λ : Type) [AddCommGroup Λ] (e : Λ →+ V) (r R : ℝ), 0 < r → r ≤ R → ∑' l : Λ, Set.indicator {l : Λ | ‖e l‖ ≤ R} (fun _ => (1 : ℝ≥0∞)) l ≤ ENNReal.ofReal ((5 ∗ R / r) ^ Module.finrank ℝ V) ∗ ∑' l : Λ, Set.indicator {l : Λ | ‖e l‖ ≤ r} (fun _ => (1 : ℝ≥0∞)) l

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.LatticeCosetParametrization

theorem **rational_coset_param** : ∀ (F : Type) [Field F] [NumberField F] (ι : Type) [Fintype ι] [DecidableEq ι] [DecidableEq (InfinitePlace F)] (ρ : InfinitePlace F → (F →+* ℝ)) (b : Module.Basis (InfinitePlace F) ℤ (𝒪 F)) (a : ι → F) (N : ℕ), 0 < N → ∃ e : (ι × InfinitePlace F → ℤ) → ι → F, Function.Injective e ∧ (∀ x : ι → F, toA x − toA a ∈ latN F ι N ↔ x ∈ Set.range e) ∧ ∀ n, emb ρ (e n) = emb ρ a + gM ρ (fun v => b v) N ∗ᵥ fun p => (n p : ℝ)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.LatticeCosetParametrizationStatement

theorem **_root_.GlobalTheory.T.lattice_coset_parametrization** : ∀ (F : Type) [Field F] [NumberField F] (ι : Type) [Fintype ι] [DecidableEq ι] [DecidableEq (InfinitePlace F)] (ρ : InfinitePlace F → (F →+* ℝ)) (b : Module.Basis (InfinitePlace F) ℤ (𝒪 F)) (a : ι → F) (N : ℕ), 0 < N → ∃ e : (ι × InfinitePlace F → ℤ) → ι → F, Function.Injective e ∧ (∀ x : ι → F, toA x - toA a ∈ latN F ι N ↔ x ∈ Set.range e) ∧ ∀ n, emb ρ (e n) = emb ρ a + gM ρ (fun v => b v) N *ᵥ fun p => (n p : ℝ)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.LatticeFundamentalDomainVolume

theorem **latR_eq_range** (ρ : InfinitePlace F → (F →+* ℝ)) (N : ℕ) : latR ρ N = (scaleMap ρ N).range

theorem **mem_latR_iff** (ρ : InfinitePlace F → (F →+* ℝ)) (N : ℕ) (y : InfinitePlace F → ℝ) : y ∈ latR ρ N ↔ ∃ r : 𝒪 F, y = fun w => ρ w ((N : F) * (r : F))

theorem **embedding_bijective** (hF : IsTotallyReal F) : Function.Bijective (fun w : InfinitePlace F => (w.embedding : F →+* ℂ))

theorem **card_places** (hF : IsTotallyReal F) : Fintype.card (InfinitePlace F) = Module.finrank ℚ F

theorem **exists_integral_basis** [DecidableEq (InfinitePlace F)] (hF : IsTotallyReal F) (ρ : InfinitePlace F → (F →+* ℝ)) (hp : ∀ w x, ((ρ w x : ℝ) : ℂ) = w.embedding x) : ∃ B : Module.Basis (InfinitePlace F) ℤ (𝒪 F), (Matrix.det (Matrix.of fun i w => ρ w ((B i : 𝒪 F) : F))) ^ 2 = (discr F : ℝ)

theorem **exists_real_basis** {ι : Type} [Fintype ι] [DecidableEq ι] [Nonempty ι] (A : Matrix ι ι ℝ) (h : A.det ≠ 0) : ∃ b : Module.Basis ι ℝ (ι → ℝ), ∀ i, b i = A i

theorem **sum_places_int** (hF : IsTotallyReal F) (ρ : InfinitePlace F → (F →+* ℝ)) (hp : ∀ w x, ((ρ w x : ℝ) : ℂ) = w.embedding x) (a : 𝒪 F) : ∃ z : ℤ, ∑ w, ρ w (a : F) = (z : ℝ)

theorem **integral_char_eq_zero** (hF : IsTotallyReal F) (ρ : InfinitePlace F → (F →+* ℝ)) (hp : ∀ w x, ((ρ w x : ℝ) : ℂ) = w.embedding x) (N : ℕ) [Countable (latR ρ N)] (D : Set (InfinitePlace F → ℝ)) (hD : IsAddFundamentalDomain (latR ρ N) D volume) (β : F) (hβ : β ≠ 0) (hint : ∃ r : 𝒪 F, (r : F) = (N : F) * β) : ∫ x in D, Complex.exp ((2 * (Real.pi : ℂ) * Complex.I) * ((∑ w, ρ w β * x w : ℝ) : ℂ)) = 0

theorem **GlobalTheory.CompanionTheory.SW.FD.main** : ∀ (F : Type) [Field F] [NumberField F], IsTotallyReal F → ∀ ρ : InfinitePlace F → (F →+* ℝ), (∀ w x, ((ρ w x : ℝ) : ℂ) = w.embedding x) → ∀ N : ℕ, 0 < N → (∃ D : Set (InfinitePlace F → ℝ), IsAddFundamentalDomain (latR ρ N) D volume) ∧ Countable (latR ρ N) ∧ (∀ y, y ∈ latR ρ N ↔ ∃ r : 𝒪 F, y = fun w => ρ w ((N : F) * (r : F))) ∧ ∀ D : Set (InfinitePlace F → ℝ), IsAddFundamentalDomain (latR ρ N) D volume → volume D = ENNReal.ofReal ((N : ℝ) ^ Module.finrank ℚ F * (kap F)⁻¹) ∧ ∀ β : F, β ≠ 0 → (∃ r : 𝒪 F, (r : F) = (N : F) * β) → ∫ x in D, Complex.exp ((2 * (Real.pi : ℂ) * Complex.I) * ((∑ w, ρ w β * x w : ℝ) : ℂ)) = 0

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.LatticeFundamentalDomainVolumeStatement

theorem **_root_.GlobalTheory.T.lattice_fundamental_domain_volume** : ∀ (F : Type) [Field F] [NumberField F], IsTotallyReal F → ∀ ρ : InfinitePlace F → (F →+* ℝ), (∀ w x, ((ρ w x : ℝ) : ℂ) = w.embedding x) → ∀ N : ℕ, 0 < N → (∃ D : Set (InfinitePlace F → ℝ), IsAddFundamentalDomain (latR ρ N) D volume) ∧ Countable (latR ρ N) ∧ (∀ y, y ∈ latR ρ N ↔ ∃ r : 𝒪 F, y = fun w => ρ w ((N : F) * (r : F))) ∧ ∀ D : Set (InfinitePlace F → ℝ), IsAddFundamentalDomain (latR ρ N) D volume → volume D = ENNReal.ofReal ((N : ℝ) ^ Module.finrank ℚ F * (kap F)⁻¹) ∧ ∀ β : F, β ≠ 0 → (∃ r : 𝒪 F, (r : F) = (N : F) * β) → ∫ x in D, Complex.exp ((2 * (Real.pi : ℂ) * Complex.I) * ((∑ w, ρ w β * x w : ℝ) : ℂ)) = 0

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.LatticeGramScalarAlmostAll

theorem **transit_ring** {T : Type} [CommRing T] (O : Subring T) (s : T →+* T) (hs : ∀ y ∈ O, s y ∈ O) (hss : ∀ y, s (s y) = y) {M : ℕ} (JT : Matrix (Fin M) (Fin M) T) (hJO : ∀ i j, JT i j ∈ O) (hJ : ∀ i j, s (JT j i) = JT i j) (x y : Fin M → T) (hx : ∀ i, x i ∈ O) (hy : ∀ i, y i ∈ O) (hxy : hform s JT x x = hform s JT y y) (w : T) (hw : w ∈ O) (hxw : hform s JT x x * w = 1) (hgood : ∀ p β : O, IsUnit β → s β = β → ∃ ε z : O, (ε : T) * s ε = 1 ∧ ((β : T) - s ε * p) * z = 1) : ∃ g ∈ UJ s JT, (g : Matrix (Fin M) (Fin M) T) *ᵥ x = y ∧ (∀ i j, (g : Matrix (Fin M) (Fin M) T) i j ∈ O) ∧ ∀ i j, ((g⁻¹ : GL (Fin M) T) : Matrix (Fin M) (Fin M) T) i j ∈ O

theorem **mem_latV_iff** {F E : Type} [Field F] [NumberField F] [Field E] [Algebra F E] {d n M : ℕ} (b : Module.Basis (Fin d) F E) (v : HeightOneSpectrum (𝒪 F)) (O : Subring (Fv F v ⊗[F] E)) (hO : ∀ y, y ∈ O ↔ ∀ k, (Algebra.TensorProduct.basis (Fv F v) b).repr y k ∈ v.adicCompletionIntegers F) (X : Fin n → Fin M → Fv F v ⊗[F] E) : X ∈ latV b v ↔ ∀ a i, X a i ∈ O

theorem **mulVec_mem** {T : Type} [CommRing T] (O : Subring T) {M : ℕ} (g : Matrix (Fin M) (Fin M) T) (hg : ∀ i j, g i j ∈ O) (x : Fin M → T) (hx : ∀ i, x i ∈ O) (i : Fin M) : (g *ᵥ x) i ∈ O

theorem **good_trans** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (t : F), t ≠ 0 → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E), ∀ᶠ v : HeightOneSpectrum (𝒪 F) in Filter.cofinite, ∀ x y : Fin 1 → Fin M → Fv F v ⊗[F] E, x ∈ latV b v → y ∈ latV b v → gramR (Fv F v) σ J x = (hermScal (Fv F v) σ (algebraMap F _ t) : HermR (Fv F v) σ 1) → gramR (Fv F v) σ J y = gramR (Fv F v) σ J x → ∃ h : HA (Fv F v) σ J, actW (Fv F v) σ J h x = y ∧ ∀ (n : ℕ) (X : Fin n → Fin M → Fv F v ⊗[F] E), actW (Fv F v) σ J h X ∈ latV b v ↔ X ∈ latV b v

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.LatticeGramScalarAlmostAllStatement

theorem **_root_.GlobalTheory.T.lattice_gram_scalar_almost_all** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (t : F), t ≠ 0 → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E), ∀ᶠ v : HeightOneSpectrum (𝒪 F) in Filter.cofinite, ∀ x y : Fin 1 → Fin M → Fv F v ⊗[F] E, x ∈ latV b v → y ∈ latV b v → gramR (Fv F v) σ J x = (hermScal (Fv F v) σ (algebraMap F _ t) : HermR (Fv F v) σ 1) → gramR (Fv F v) σ J y = gramR (Fv F v) σ J x → ∃ h : HA (Fv F v) σ J, actW (Fv F v) σ J h x = y ∧ ∀ (n : ℕ) (X : Fin n → Fin M → Fv F v ⊗[F] E), actW (Fv F v) σ J h X ∈ latV b v ↔ X ∈ latV b v

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.LatticeMatrixDeterminantFormula

theorem **det_EM_sq** {ρ : InfinitePlace F → (F →+* ℝ)} (hT : IsTotallyReal F) (hp : ∀ w x, ((ρ w x : ℝ) : ℂ) = w.embedding x) (b : Module.Basis (InfinitePlace F) ℤ (𝒪 F)) : (EM ρ (fun v => b v)).det ^ 2 = (discr F : ℝ)

theorem **abs_det_gM** {ρ : InfinitePlace F → (F →+* ℝ)} (hT : IsTotallyReal F) (hp : ∀ w x, ((ρ w x : ℝ) : ℂ) = w.embedding x) (b : Module.Basis (InfinitePlace F) ℤ (𝒪 F)) (N : ℕ) : |(gM (ι

theorem **abs_det_gM_all** : ∀ (F : Type) [Field F] [NumberField F] (ι : Type) [Fintype ι] [DecidableEq ι] [DecidableEq (InfinitePlace F)] (ρ : InfinitePlace F → (F →+* ℝ)), IsTotallyReal F → (∀ w x, ((ρ w x : ℝ) : ℂ) = w.embedding x) → ∀ (b : Module.Basis (InfinitePlace F) ℤ (𝒪 F)) (N : ℕ), |(gM (ι

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.LatticeMatrixDeterminantFormulaStatement

theorem **_root_.GlobalTheory.T.lattice_matrix_determinant_formula** : ∀ (F : Type) [Field F] [NumberField F] (ι : Type) [Fintype ι] [DecidableEq ι] [DecidableEq (InfinitePlace F)] (ρ : InfinitePlace F → (F →+* ℝ)), IsTotallyReal F → (∀ w x, ((ρ w x : ℝ) : ℂ) = w.embedding x) → ∀ (b : Module.Basis (InfinitePlace F) ℤ (𝒪 F)) (N : ℕ), |(gM (ι

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.LatticeStableAlmostEverywhere

theorem **eventually_integral** (x : F) : ∀ᶠ v : HeightOneSpectrum (𝒪 F) in Filter.cofinite, algebraMap F (Fv F v) x ∈ v.adicCompletionIntegers F

theorem **repr_mul** {d : ℕ} (b : Module.Basis (Fin d) F E) (v : HeightOneSpectrum (𝒪 F)) (e : E) (y : Fv F v ⊗[F] E) (l : Fin d) : (Algebra.TensorProduct.basis (Fv F v) b).repr (y * inclR (Fv F v) e) l = ∑ k, (Algebra.TensorProduct.basis (Fv F v) b).repr y k * algebraMap F (Fv F v) (b.repr (b k * e) l)

theorem **mul_mem_latR** {d : ℕ} {b : Module.Basis (Fin d) F E} {v : HeightOneSpectrum (𝒪 F)} {e : E} (he : Good b v e) {y : Fv F v ⊗[F] E} (hy : y ∈ latR b v) : y * inclR (Fv F v) e ∈ latR b v

theorem **sum_mem_latR** {d : ℕ} {b : Module.Basis (Fin d) F E} {v : HeightOneSpectrum (𝒪 F)} {ι : Type} (s : Finset ι) (f : ι → Fv F v ⊗[F] E) (h : ∀ c ∈ s, f c ∈ latR b v) : ∑ c ∈ s, f c ∈ latR b v

theorem **eventually_good** {d : ℕ} (b : Module.Basis (Fin d) F E) (e : E) : ∀ᶠ v : HeightOneSpectrum (𝒪 F) in Filter.cofinite, Good b v e

theorem **eventually_good_matrix** {d m : ℕ} (b : Module.Basis (Fin d) F E) (g : Matrix (Fin m) (Fin m) E) : ∀ᶠ v : HeightOneSpectrum (𝒪 F) in Filter.cofinite, ∀ i j, Good b v (g i j)

theorem **rmul_rmul** (R : Type) [CommRing R] [Algebra F R] {M n : ℕ} (X : Fin n → Fin M → R ⊗[F] E) (A B : Matrix (Fin n) (Fin n) (R ⊗[F] E)) : rmul R (rmul R X A) B = rmul R X (A * B)

theorem **rmul_one** (R : Type) [CommRing R] [Algebra F R] {M n : ℕ} (X : Fin n → Fin M → R ⊗[F] E) : rmul R X 1 = X

theorem **rmul_mem** {d M n : ℕ} {b : Module.Basis (Fin d) F E} {v : HeightOneSpectrum (𝒪 F)} {g : Matrix (Fin n) (Fin n) E} (hg : ∀ i j, Good b v (g i j)) {X : Fin n → Fin M → Fv F v ⊗[F] E} (hX : X ∈ latV b v) : rmul (Fv F v) X (g.map (inclR (Fv F v))) ∈ latV b v

theorem **actTup_mem** {d M n : ℕ} {b : Module.Basis (Fin d) F E} {v : HeightOneSpectrum (𝒪 F)} {g : GL (Fin M) E} (hg : ∀ i j, Good b v ((g : Matrix (Fin M) (Fin M) E) i j)) {X : Fin n → Fin M → Fv F v ⊗[F] E} (hX : X ∈ latV b v) : actTup (Fv F v) (bc (inclR (Fv F v)) g) X ∈ latV b v

theorem **actTup_inv** {M n : ℕ} (R : Type) [CommRing R] [Algebra F R] (g : GL (Fin M) E) (X : Fin n → Fin M → R ⊗[F] E) : actTup R (bc (inclR R) g⁻¹) (actTup R (bc (inclR R) g) X) = X

theorem **one_mem_latR** {d : ℕ} {b : Module.Basis (Fin d) F E} {v : HeightOneSpectrum (𝒪 F)} (h : ∀ k, algebraMap F (Fv F v) (b.repr 1 k) ∈ v.adicCompletionIntegers F) : (1 : Fv F v ⊗[F] E) ∈ latR b v

theorem **consTup_mem_iff** {d M n : ℕ} {b : Module.Basis (Fin d) F E} {v : HeightOneSpectrum (𝒪 F)} (h : ∀ k, algebraMap F (Fv F v) (b.repr 1 k) ∈ v.adicCompletionIntegers F) (X : Fin n → Fin M → Fv F v ⊗[F] E) : consTup X ∈ latV b v ↔ X ∈ latV b v

theorem **latV_rational** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (n M : ℕ), (∀ g : Matrix (Fin n) (Fin n) E, g.det ≠ 0 → ∀ᶠ v : HeightOneSpectrum (𝒪 F) in Filter.cofinite, ∀ X : (Fin n → Fin M → Fv F v ⊗[F] E), rmul (Fv F v) X (g.map (inclR (Fv F v))) ∈ latV b v ↔ X ∈ latV b v) ∧ (∀ g : GL (Fin M) E, ∀ᶠ v : HeightOneSpectrum (𝒪 F) in Filter.cofinite, ∀ X : (Fin n → Fin M → Fv F v ⊗[F] E), actTup (Fv F v) (bc (inclR (Fv F v)) g) X ∈ latV b v ↔ X ∈ latV b v) ∧ ∀ᶠ v : HeightOneSpectrum (𝒪 F) in Filter.cofinite, ∀ X : (Fin n → Fin M → Fv F v ⊗[F] E), consTup X ∈ latV b v ↔ X ∈ latV b v

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.LatticeStableAlmostEverywhereStatement

theorem **_root_.GlobalTheory.T.lattice_stable_almost_everywhere** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (n M : ℕ), (∀ g : Matrix (Fin n) (Fin n) E, g.det ≠ 0 → ∀ᶠ v : HeightOneSpectrum (𝒪 F) in Filter.cofinite, ∀ X : (Fin n → Fin M → Fv F v ⊗[F] E), rmul (Fv F v) X (g.map (inclR (Fv F v))) ∈ latV b v ↔ X ∈ latV b v) ∧ (∀ g : GL (Fin M) E, ∀ᶠ v : HeightOneSpectrum (𝒪 F) in Filter.cofinite, ∀ X : (Fin n → Fin M → Fv F v ⊗[F] E), actTup (Fv F v) (bc (inclR (Fv F v)) g) X ∈ latV b v ↔ X ∈ latV b v) ∧ ∀ᶠ v : HeightOneSpectrum (𝒪 F) in Filter.cofinite, ∀ X : (Fin n → Fin M → Fv F v ⊗[F] E), consTup X ∈ latV b v ↔ X ∈ latV b v

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.LebesgueSelfdualForDotCharacter

theorem **fourierChar_smul** (t : ℝ) (c : ℂ) : (Real.fourierChar t : Circle) • c = c * Complex.exp (2 * Real.pi * Complex.I * (t : ℝ))

theorem **inversion_neg** (ι : Type) [Fintype ι] (Φ : (ι → ℝ) → ℂ) (hc : Continuous Φ) (hs : HasCompactSupport Φ) (hi : Integrable (fun y : ι → ℝ => ∫ x : ι → ℝ, Φ x * Complex.exp (2 * Real.pi * Complex.I * (((-1 : ℝ) * (x ⬝ᵥ y) : ℝ) : ℝ))) volume) (z : ι → ℝ) : ∫ y : ι → ℝ, (∫ x : ι → ℝ, Φ x * Complex.exp (2 * Real.pi * Complex.I * (((-1 : ℝ) * (x ⬝ᵥ y) : ℝ) : ℝ))) * Complex.exp (2 * Real.pi * Complex.I * (((-1 : ℝ) * (y ⬝ᵥ z) : ℝ) : ℝ)) = Φ (-z)

theorem **selfDual_real_coord** : ∀ (ι : Type) [Fintype ι] (ε : ℝ), ε = 1 ∨ ε = -1 → SelfDual (volume : Measure (ι → ℝ)) (fun x y => Complex.exp (2 * Real.pi * Complex.I * (ε * (x ⬝ᵥ y) : ℝ)))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.LinearChangeEulerProduct

theorem **modulus_matrix** : ∀ (F : Type) [Field F] [NumberField F] (ι : Type) [Fintype ι] [DecidableEq ι] (A : Matrix ι ι F), A.det ≠ 0 → ∃ (cinf : ℝ≥0) (c : HeightOneSpectrum (𝒪 F) → ℝ≥0), IsUnitProd cinf c ∧ (∀ μ : Measure (ι → Ainf F), μ.IsAddHaarMeasure → μ.map (fun x => A.map (algebraMap F (Ainf F)) *ᵥ x) = cinf • μ) ∧ ∀ v : HeightOneSpectrum (𝒪 F), (∀ μ : Measure (ι → Fv F v), μ.IsAddHaarMeasure → μ.map (fun x => A.map (algebraMap F (Fv F v)) *ᵥ x) = c v • μ)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.LinearChangeEulerProductStatement

theorem **_root_.GlobalTheory.T.addHaar_map_mulVec_eq_smul_at_all_places** : ∀ (F : Type) [Field F] [NumberField F] (ι : Type) [Fintype ι] [DecidableEq ι] (A : Matrix ι ι F), A.det ≠ 0 → ∃ (cinf : ℝ≥0) (c : HeightOneSpectrum (𝒪 F) → ℝ≥0), IsUnitProd cinf c ∧ (∀ μ : Measure (ι → Ainf F), μ.IsAddHaarMeasure → μ.map (fun x => A.map (algebraMap F (Ainf F)) *ᵥ x) = cinf • μ) ∧ ∀ v : HeightOneSpectrum (𝒪 F), (∀ μ : Measure (ι → Fv F v), μ.IsAddHaarMeasure → μ.map (fun x => A.map (algebraMap F (Fv F v)) *ᵥ x) = c v • μ)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.LocalDensitiesBlockOneAlmostAll

theorem **blockOne_zero_zero** {K : Type} [Zero K] {M : ℕ} (t : K) (A : Matrix (Fin M) (Fin M) K) : blockOne t A 0 0 = t

theorem **blockOne_zero_succ** {K : Type} [Zero K] {M : ℕ} (t : K) (A : Matrix (Fin M) (Fin M) K) (j : Fin M) : blockOne t A 0 j.succ = 0

theorem **blockOne_succ_zero** {K : Type} [Zero K] {M : ℕ} (t : K) (A : Matrix (Fin M) (Fin M) K) (i : Fin M) : blockOne t A i.succ 0 = 0

theorem **blockOne_succ_succ** {K : Type} [Zero K] {M : ℕ} (t : K) (A : Matrix (Fin M) (Fin M) K) (i j : Fin M) : blockOne t A i.succ j.succ = A i j

theorem **det_blockOne** {K : Type} [CommRing K] {M : ℕ} (t : K) (A : Matrix (Fin M) (Fin M) K) : (blockOne t A).det = t * A.det

theorem **isHermND_blockOne** {F E : Type} [Field F] [Field E] [Algebra F E] (σ : E →ₐ[F] E) {M : ℕ} (J : Matrix (Fin M) (Fin M) E) (hJ : IsHermND σ J) (t : F) (ht : t ≠ 0) : IsHermND σ (blockOne (algebraMap F E t) J)

theorem **gram_mulVec_e0** {F E : Type} [Field F] [Field E] [Algebra F E] (R : Type) [CommRing R] [Algebra F R] (σ : E →ₐ[F] E) {M : ℕ} (J : Matrix (Fin M) (Fin M) E) (t : F) : gramR R σ (blockOne (algebraMap F E t) J) (fun _ : Fin 1 => (Fin.cons 1 (fun _ => 0) : Fin (M + 1) → R ⊗[F] E)) = (hermScal R σ (algebraMap F R t) : HermR R σ 1)

theorem **consTup_mem_latV** {d : ℕ} (b : Module.Basis (Fin d) F E) (v : HeightOneSpectrum (𝒪 F)) (h1 : ∀ k, (Algebra.TensorProduct.basis (Fv F v) b).repr (1 : Fv F v ⊗[F] E) k ∈ v.adicCompletionIntegers F) {n M : ℕ} (X : Fin n → Fin M → Fv F v ⊗[F] E) : consTup X ∈ latV b v ↔ X ∈ latV b v

theorem **e0_mem_latV** {d : ℕ} (b : Module.Basis (Fin d) F E) (v : HeightOneSpectrum (𝒪 F)) (h1 : ∀ k, (Algebra.TensorProduct.basis (Fv F v) b).repr (1 : Fv F v ⊗[F] E) k ∈ v.adicCompletionIntegers F) {M : ℕ} : (fun _ : Fin 1 => (Fin.cons 1 (fun _ => 0) : Fin (M + 1) → Fv F v ⊗[F] E)) ∈ latV b v

theorem **incl_mul_eq_smul** (R : Type) [CommRing R] [Algebra F R] (t : F) (y : R ⊗[F] E) : inclR R (algebraMap F E t) * y = algebraMap F R t • y

theorem **coordK_map_unit** [NumberField E] (σ : E →ₐ[F] E) (hCM : IsCMExt F E σ) (v : HeightOneSpectrum (𝒪 F)) (lam : Measure (Fv F v)) (hlam : IsStdV v lam) (t : F) (ht : t ≠ 0) (htv : Valued.v (algebraMap F (Fv F v) t) = 1) (d : ℕ) (b : Module.Basis (Fin d) F E) : (coordK (Fv F v) b lam).map (fun y => inclR (Fv F v) (algebraMap F E t) * y) = (1 : ℝ≥0) • coordK (Fv F v) b lam

theorem **blockFibre_mass** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (t : F), t ≠ 0 → ∀ (n : ℕ) (T : Matrix (Fin n) (Fin n) E) (hT : ∀ a b, σ (T b a) = T a b), T.det ≠ 0 → ∀ (hT' : ∀ a b, σ (blockOne (algebraMap F E t) T b a) = blockOne (algebraMap F E t) T a b), ∀ (d : ℕ) (b : Module.Basis (Fin d) F E), ∀ᶠ v : HeightOneSpectrum (𝒪 F) in Filter.cofinite, ∀ lam, IsStdV v lam → ∀ m m₁ m', IsGramFib (Fv F v) σ (blockOne (algebraMap F E t) J) (n + 1) (coordW (Fv F v) b lam (n + 1) (M + 1)) (coordHerm (Fv F v) σ b lam (n + 1)) m → IsGramFib (Fv F v) σ (blockOne (algebraMap F E t) J) 1 (coordW (Fv F v) b lam 1 (M + 1)) (coordHerm (Fv F v) σ b lam 1) m₁ → IsGramFib (Fv F v) σ J n (coordW (Fv F v) b lam n M) (coordHerm (Fv F v) σ b lam n) m' → m (hermOf (Fv F v) σ _ h

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.LocalDensitiesBlockOneAlmostAllStatement

theorem **_root_.GlobalTheory.T.local_densities_block_one_almost_all** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (t : F), t ≠ 0 → ∀ (n : ℕ) (T : Matrix (Fin n) (Fin n) E) (hT : ∀ a b, σ (T b a) = T a b), T.det ≠ 0 → ∀ (hT' : ∀ a b, σ (blockOne (algebraMap F E t) T b a) = blockOne (algebraMap F E t) T a b), ∀ (d : ℕ) (b : Module.Basis (Fin d) F E), ∀ᶠ v : HeightOneSpectrum (𝒪 F) in Filter.cofinite, ∀ lam, IsStdV v lam → ∀ m m₁ m', IsGramFib (Fv F v) σ (blockOne (algebraMap F E t) J) (n + 1) (coordW (Fv F v) b lam (n + 1) (M + 1)) (coordHerm (Fv F v) σ b lam (n + 1)) m → IsGramFib (Fv F v) σ (blockOne (algebraMap F E t) J) 1 (coordW (Fv F v) b lam 1 (M + 1)) (coordHerm (Fv F v) σ b lam 1) m₁ → IsGramFib (Fv F v) σ J n (coordW (Fv F v) b lam n M) (coordHerm (Fv F

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.LocalDensitiesBoundedAlmostAll

theorem **exists_moved** (hcm : IsCMExt F E σ) : ∃ c : E, σ c ≠ c

theorem **fixed_mem** (hcm : IsCMExt F E σ) (x : E) (hx : σ x = x) : ∃ t : F, algebraMap F E t = x

theorem **hE_add_left** {n : ℕ} (T : Matrix (Fin n) (Fin n) E) (x x' y : Fin n → E) : hE σ T (x + x') y = hE σ T x y + hE σ T x' y

theorem **hE_add_right** {n : ℕ} (T : Matrix (Fin n) (Fin n) E) (x y y' : Fin n → E) : hE σ T x (y + y') = hE σ T x y + hE σ T x y'

theorem **hE_smul_left** {n : ℕ} (T : Matrix (Fin n) (Fin n) E) (c : E) (x y : Fin n → E) : hE σ T (c • x) y = σ c * hE σ T x y

theorem **hE_smul_right** {n : ℕ} (T : Matrix (Fin n) (Fin n) E) (c : E) (x y : Fin n → E) : hE σ T x (c • y) = c * hE σ T x y

theorem **hE_single** {n : ℕ} (T : Matrix (Fin n) (Fin n) E) (i j : Fin n) : hE σ T (Pi.single i 1) (Pi.single j 1) = T i j

theorem **hE_self_fixed** {n : ℕ} (T : Matrix (Fin n) (Fin n) E) (hT : ∀ a b, σ (T b a) = T a b) (x : Fin n → E) (hinv : ∀ y, σ (σ y) = y) : σ (hE σ T x x) = hE σ T x x

theorem **exists_aniso** (hcm : IsCMExt F E σ) {n : ℕ} (T : Matrix (Fin (n + 1)) (Fin (n + 1)) E) (hT : ∀ a b, σ (T b a) = T a b) (hdet : T.det ≠ 0) : ∃ (x : Fin (n + 1) → E) (t : F), t ≠ 0 ∧ hE σ T x x = algebraMap F E t

theorem **blockOne_herm** {M : ℕ} (a : E) (ha : σ a = a) (A : Matrix (Fin M) (Fin M) E) (hA : ∀ i j, σ (A j i) = A i j) : ∀ i j, σ (blockOne a A j i) = blockOne a A i j

theorem **one_hermND** (M : ℕ) : IsHermND σ (1 : Matrix (Fin M) (Fin M) E)

theorem **blockOne_hermND** {M : ℕ} (t : F) (ht : t ≠ 0) : IsHermND σ (blockOne (algebraMap F E t) (1 : Matrix (Fin M) (Fin M) E))

theorem **arith** {x y q C : ℝ} {k : ℕ} (hq : 1 ≤ q) (hk : 2 ≤ k) (hx : |x - 1| ≤ 2 / q ^ k) (hy : |y - 1| ≤ C / q ^ 2) : |x * y - 1| ≤ (3 * |C| + 2) / q ^ 2

theorem **exists_family** (hcm : IsCMExt F E σ) {M : ℕ} {J : Matrix (Fin M) (Fin M) E} (hJ : IsHermND σ J) {d : ℕ} (b : Module.Basis (Fin d) F E) (v : HeightOneSpectrum (𝒪 F)) (lam : Measure (Fv F v)) (hl : IsStdV v lam) (n : ℕ) : ∃ m, IsGramFib (Fv F v) σ J n (coordW (Fv F v) b lam n M) (coordHerm (Fv F v) σ b lam n) m

theorem **absNorm_one_le** (v : HeightOneSpectrum (𝒪 F)) : (1 : ℝ) ≤ (Ideal.absNorm v.asIdeal : ℝ)

theorem **key** (hcm : IsCMExt F E σ) : ∀ (n M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → (n = 0 ∨ n + 2 ≤ M) → ∀ (T : Matrix (Fin n) (Fin n) E) (hT : ∀ a b, σ (T b a) = T a b), T.det ≠ 0 → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E), ∃ C : ℝ, ∀ᶠ v : HeightOneSpectrum (𝒪 F) in Filter.cofinite, ∀ lam, IsStdV v lam → ∀ m, IsGramFib (Fv F v) σ J n (coordW (Fv F v) b lam n M) (coordHerm (Fv F v) σ b lam n) m → |(m (hermOf (Fv F v) σ T hT) (latV b v)).toReal - 1| ≤ C / (Ideal.absNorm v.asIdeal : ℝ) ^ 2

theorem **mass_fibre_good** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (n : ℕ), n = 0 ∨ n + 2 ≤ M → ∀ (T : Matrix (Fin n) (Fin n) E) (hT : ∀ a b, σ (T b a) = T a b), T.det ≠ 0 → ∀ (d : ℕ) (b : Module.Basis (Fin d)

F E), ∃ C : ℝ, ∀ᶠ v : HeightOneSpectrum (𝒪 F) in Filter.cofinite, ∀ lam, IsStdV v lam → ∀ m, IsGramFib (Fv F v) σ J n (coordW (Fv F v) b lam n M) (coordHerm (Fv F v) σ b lam n) m → |(m (hermOf (Fv F v) σ T hT) (latV b v)).toReal − 1| ≤ C / (Ideal.absNorm v.asIdeal : ℝ) ^ 2

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.LocalDensitiesBoundedAlmostAllStatement

theorem **_root_.GlobalTheory.T.local_densities_bounded_almost_all** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (n : ℕ), n = 0 ∨ n + 2 ≤ M → ∀ (T : Matrix (Fin n) (Fin n) E) (hT : ∀ a b, σ (T b a) = T a b), T.det ≠ 0 → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E), ∃ C : ℝ, ∀ᶠ v : HeightOneSpectrum (𝒪 F) in Filter.cofinite, ∀ lam, IsStdV v lam → ∀ m, IsGramFib (Fv F v) σ J n (coordW (Fv F v) b lam n M) (coordHerm (Fv F v) σ b lam n) m → |(m (hermOf (Fv F v) σ T hT) (latV b v)).toReal − 1| ≤ C / (Ideal.absNorm v.asIdeal : ℝ) ^ 2

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.LocalDensitiesCongruentTargetAlmostAll

theorem **map_eq_self** {V : Type} [AddCommGroup V] [TopologicalSpace V] [LocallyCompactSpace V] [SecondCountableTopology V] [T2Space V] [MeasurableSpace V] [BorelSpace V] [IsTopologicalAddGroup V] (μ : Measure V) [μ.IsAddHaarMeasure] (e f : V → V) (he : ∀ x y, e (x + y) = e x + e y) (hef : ∀ x, e (f x) = x) (hfe : ∀ x, f (e x) = x) (hec : Continuous e) (hfc : Continuous f) (L : Set V) (hLc : IsCompact L) (hLo : IsOpen L) (hLn : L.Nonempty) (heL : ∀ x ∈ L, e x ∈ L) (hfL : ∀ x ∈ L, f x ∈ L) : μ.map e = μ

theorem **mul_entry_mem** {K : Type} [CommRing K] (O : Subring K) {n : ℕ} {A B : Matrix (Fin n) (Fin n) K} (hA : ∀ i j, A i j ∈ O) (hB : ∀ i j, B i j ∈ O) (i j : Fin n) : (A * B) i j ∈ O

theorem **rmul_add** (R : Type) [CommRing R] [Algebra F R] {M n : ℕ} (g : Matrix (Fin n) (Fin n) (R ⊗[F] E)) (X Y : Fin n → Fin M → R ⊗[F] E) : rmul R (X + Y) g = rmul R X g + rmul R Y g

theorem **continuous_rmul** (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] (hK : IsTopologicalRing (R ⊗[F] E)) {M n : ℕ} (g : Matrix (Fin n) (Fin n) (R ⊗[F] E)) : Continuous (fun X : Fin n → Fin M → R ⊗[F] E => rmul R X g)

theorem **mass_eq_of_integral** (σ : E →ₐ[F] E) (hCM : IsCMExt F E σ) {M : ℕ} (J : Matrix (Fin M) (Fin M) E) (hJ : IsHermND σ J) (n : ℕ) {d : ℕ} (b : Module.Basis (Fin d) F E) (v : HeightOneSpectrum (𝒪 F)) (O : Subring (Fv F v ⊗[F] E)) (hO : ∀ y, y ∈ O ↔ ∀ k, (Algebra.TensorProduct.basis (Fv F v) b).repr y k ∈ v.adicCompletionIntegers F) (ho : ∀ y ∈ O, sigR (Fv F v) σ y ∈ O) (g gi : Matrix (Fin n) (Fin n) (Fv F v ⊗[F] E)) (hg : ∀ i j, g i j ∈ O) (hgi : ∀ i j, gi i j ∈ O) (hggi : g * gi = 1) (hgig : gi * g = 1) (lam : Measure (Fv F v)) (hlam : IsStdV v lam) (m : HermR (Fv F v) σ n → Measure (Fin n → Fin M → Fv F v ⊗[F] E)) (hm : IsGramFib (Fv F v) σ J n (coordW (Fv F v) b lam n M) (coordHerm (Fv F v) σ b lam n) m) (S S' : HermR (Fv F v) σ n) (hS : (S' : Matrix (Fin n) (Fin n) (Fv F v ⊗[F] E)) = cT (sigR (Fv F v) σ) g * (S : Matrix (Fin n) (Fin n) (Fv F v ⊗[F] E)) * g) : m S' (latV b v) = m S

theorem **mass_eq** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (n : ℕ) (T : Matrix (Fin n) (Fin n) E) (hT : ∀ a b, σ (T b a) = T a b), T.det ≠ 0 → ∀ (g : Matrix (Fin n) (Fin n) E), g.det ≠ 0 → ∀ (T' : Matrix (Fin n) (Fin n) E) (hT' : ∀ a b, σ (T' b a) = T' a b), T' = cT (σ : E →+* E) g * T * g → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E), ∀ᶠ v : HeightOneSpectrum (𝒪 F) in Filter.cofinite, ∀ lam, IsStdV v lam → ∀ m, IsGramFib (Fv F v) σ J n (coordW (Fv F v) b lam n M) (coordHerm (Fv F v) σ b lam n) m → m (hermOf (Fv F v) σ T' hT') (latV b v) = m (hermOf (Fv F v) σ T hT) (latV b v)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.LocalDensitiesCongruentTargetAlmostAllStatement

theorem **_root_.GlobalTheory.T.local_densities_congruent_target_almost_all** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (n : ℕ) (T : Matrix (Fin n) (Fin n) E) (hT : ∀ a b, σ (T b a) = T a b), T.det ≠ 0 → ∀ (g : Matrix (Fin n) (Fin n) E), g.det ≠ 0 → ∀ (T' : Matrix (Fin n) (Fin n) E) (hT' : ∀ a b, σ (T' b a) = T' a b), T' = cT (σ : E →+* E) g * T * g → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E), ∀ᶠ v : HeightOneSpectrum (𝒪 F) in Filter.cofinite, ∀ lam, IsStdV v lam → ∀ m, IsGramFib (Fv F v) σ J n (coordW (Fv F v) b lam n M) (coordHerm (Fv F v) σ b lam n) m → m (hermOf (Fv F v) σ T' hT') (latV b v) = m (hermOf (Fv F v) σ T hT) (latV b v)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.LocalDensityBlockOneEventually

theorem **continuous_act_consTup** {F E : Type} [Field F] [Field E] [Algebra F E] (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] (hring : IsTopologicalRing (R ⊗[F] E)) (σ : E →ₐ[F] E) {M n : ℕ} (J : Matrix (Fin (M + 1)) (Fin (M + 1)) E) (h : HA R σ J) : Continuous fun X : Fin n → Fin M → R ⊗[F] E => actW R σ J h (consTup X)

theorem **good_mass** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ t : F, t ≠ 0 → ∀ (n : ℕ) (T : Matrix (Fin n) (Fin n) E) (hT : ∀ a b, σ (T b a) = T a b), T.det ≠ 0 → ∀ (hT' : ∀ a b, σ (blockOne (algebraMap F E t) T b a) = blockOne (algebraMap F E t) T a b) (d : ℕ) (b : Module.Basis (Fin d) F E), ∀ᶠ v : HeightOneSpectrum (𝒪 F) in Filter.cofinite, ∀ lam : Measure (Fv F v), lam.IsAddHaarMeasure → ∀ m m₁ m', IsGramFib (Fv F v) σ (blockOne (algebraMap F E t) J) (n + 1) (coordW (Fv F v) b lam (n + 1) (M + 1)) (coordHerm (Fv F v) σ b lam (n + 1)) m → IsGramFib (Fv F v) σ (blockOne (algebraMap F E t) J) 1 (coordW (Fv F v) b lam 1 (M + 1)) (coordHerm (Fv F v) σ b lam 1) m₁ → IsGramFib (Fv F v) σ J n (coordW (Fv F v) b lam n M) (coordHerm (Fv F v) σ b lam n) m' → ∀ c : ℝ≥

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.LocalDensityBlockOneEventuallyStatement

theorem **_root_.GlobalTheory.T.local_density_block_one_eventually** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ t : F, t ≠ 0 → ∀ (n : ℕ) (T : Matrix (Fin n) (Fin n) E) (hT : ∀ a b, σ (T b a) = T a b), T.det ≠ 0 → ∀ (hT' : ∀ a b, σ (blockOne (algebraMap F E t) T b a) = blockOne (algebraMap F E t) T a b) (d : ℕ) (b : Module.Basis (Fin d) F E), ∀ᶠ v : HeightOneSpectrum (𝒪 F) in Filter.cofinite, ∀ lam : Measure (Fv F v), lam.IsAddHaarMeasure → ∀ m m₁ m', IsGramFib (Fv F v) σ (blockOne (algebraMap F E t) J) (n + 1) (coordW (Fv F v) b lam (n + 1) (M + 1)) (coordHerm (Fv F v) σ b lam (n + 1)) m → IsGramFib (Fv F v) σ (blockOne (algebraMap F E t) J) 1 (coordW (Fv F v) b lam 1 (M + 1)) (coordHerm (Fv F v) σ b lam 1) m₁ → IsGramFib (Fv F v) σ J n (coordW (Fv F v) b lam n

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.LocalDensityBlockOneProduct

theorem **measurable_consTup** {K : Type} [Zero K] [One K] [MeasurableSpace K] {M n : ℕ} : Measurable (consTup : (Fin n → Fin M → K) → Fin (n + 1) → Fin (M + 1) → K)

theorem **gram_first** (R : Type) [CommRing R] [Algebra F R] (σ : E →ₐ[F] E) {M : ℕ} (J : Matrix (Fin M) (Fin M) E) (s : E) : gramR R σ (blockOne s J) (fun _ : Fin 1 => (Fin.cons 1 (fun _ => 0) : Fin (M + 1) → R ⊗[F] E)) = fun _ _ => inclR R s

theorem **inclR_algebraMap** (R : Type) [CommRing R] [Algebra F R] (t : F) : inclR (F

theorem **fibre_block_stages** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (t : F), t ≠ 0 → ∀ (n : ℕ) (T : Matrix (Fin n) (Fin n) E) (hT : ∀ a b, σ (T b a) = T a b), T.det ≠ 0 → ∀ (hT' : ∀ a b, σ (blockOne (algebraMap F E t) T b a) = blockOne (algebraMap F E t) T a b), ∀ (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R], IsLocR R → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (lam : Measure R), lam.IsAddHaarMeasure → ∀ m m₁ m', IsGramFib R σ (blockOne (algebraMap F E t) J) (n + 1) (coordW R b lam (n + 1) (M + 1)) (coordHerm R σ b lam (n + 1)) m → IsGramFib R σ (blockOne (algebraMap F E t) J) 1 (coordW R b lam 1 (M + 1)) (coordHerm R σ b lam 1) m₁ → IsGramFib R σ J n (coordW R b lam n M) (coordHerm R σ b lam n) m' → ∀ c : ℝ≥

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.LocalDensityBlockOneProductStatement

theorem **_root_.GlobalTheory.T.local_density_block_one_product** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (t : F), t ≠ 0 → ∀ (n : ℕ) (T : Matrix (Fin n) (Fin n) E) (hT : ∀ a b, σ (T b a) = T a b), T.det ≠ 0 → ∀ (hT' : ∀ a b, σ (blockOne (algebraMap F E t) T b a) = blockOne (algebraMap F E t) T a b), ∀ (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R], IsLocR R → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (lam : Measure R), lam.IsAddHaarMeasure → ∀ m m₁ m', IsGramFib R σ (blockOne (algebraMap F E t) J) (n + 1) (coordW R b lam (n + 1) (M + 1)) (coordHerm R σ b lam (n + 1)) m → IsGramFib R σ (blockOne (algebraMap F E t) J) 1 (coordW R b lam 1 (M + 1)) (coordHerm R σ b lam 1) m₁ → IsGramFib R σ J n (coordW R b lam n M) (coordHerm R σ b lam n) m' → ∀ c

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.LocalDensityBlockOneSum

theorem **fibre_family_near_block** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (t : F), t ≠ 0 → ∀ (n : ℕ) (T : Matrix (Fin n) (Fin n) E) (hT : ∀ a b, σ (T b a) = T a b), T.det ≠ 0 → ∀ (hT' : ∀ a b, σ (blockOne (algebraMap F E t) T b a) = blockOne (algebraMap F E t) T a b), ∀ (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R], IsLocR R → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E), ∀ (lam : Measure R), lam.IsAddHaarMeasure → ∀ m₁ m', IsGramFib R σ (blockOne (algebraMap F E t) J) 1 (coordW R b lam 1 (M + 1)) (coordHerm R σ b lam 1) m₁ → IsGramFib R σ J n (coordW R b lam n M) (coordHerm R σ b lam n) m' → ∀ c : ℝ≥0, (coordK R b lam).map (fun y => inclR R (algebraMap F E t) * y) = c • coordK R b lam → ∀ s : (Fin 1 → Fin (M + 1)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.LocalDensityBlockOneSumStatement

theorem **_root_.GlobalTheory.T.exists_fibre_family_block_one_eq_map_prod** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (t : F), t ≠ 0 → ∀ (n : ℕ) (T : Matrix (Fin n) (Fin n) E) (hT : ∀ a b, σ (T b a) = T a b), T.det ≠ 0 → ∀ (hT' : ∀ a b, σ (blockOne (algebraMap F E t) T b a) = blockOne (algebraMap F E t) T a b), ∀ (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R], IsLocR R → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E), ∀ (lam : Measure R), lam.IsAddHaarMeasure → ∀ m₁ m', IsGramFib R σ (blockOne (algebraMap F E t) J) 1 (coordW R b lam 1 (M + 1)) (coordHerm R σ b lam 1) m₁ → IsGramFib R σ J n (coordW R b lam n M) (coordHerm R σ b lam n) m' → ∀ c : ℝ≥0, (coordK R b lam).map (fun y => inclR R (algebraMap F E t) * y) = c • coo

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.LocalFieldSelfdualMeasureExists

theorem **measureReal_dual_mul** (ψ : AddChar K Circle) (k : ℤ) : (selfDualMeasure ψ).real (valBall K (addCharConductor ψ − k)) * (selfDualMeasure ψ).real (valBall K k) = 1

theorem **tendsto_mean** (ψ : AddChar K Circle) {Φ : K → ℂ} (hΦ : Continuous Φ) (z : K) : Tendsto (fun k : ℕ => (((selfDualMeasure ψ).real (valBall K (addCharConductor ψ − (k : ℤ))) : ℝ) : ℂ) * ∫ x in valBall K (k : ℤ), Φ (x − z) ∂(selfDualMeasure ψ)) atTop (𝓝 (Φ (−z)))

theorem **setIntegral_fourier_mul_char** {ψ : AddChar K Circle} (hψ : Continuous ψ) (hψ1 : ψ ≠ 1) {Φ : K → ℂ} (hΦ : Continuous Φ) (hc : HasCompactSupport Φ) (z : K) (k : ℤ) : ∫ y in valBall K (addCharConductor ψ − k), (∫ x, Φ x * (ψ (x * y) : ℂ) ∂(selfDualMeasure ψ)) * (ψ (y * z) : ℂ) ∂(selfDualMeasure ψ) = (((selfDualMeasure ψ).real (valBall K (addCharConductor ψ − k)) : ℝ) : ℂ) * ∫ x in valBall K k, Φ (x − z) ∂(selfDualMeasure ψ)

theorem **iUnion_valBall_sub** (n : ℤ) : ⋃ k : ℕ, (valBall K (n − (k : ℤ)) : Set K) = Set.univ

theorem **inversion** {ψ : AddChar K Circle} (hψ : Continuous ψ) (hψ1 : ψ ≠ 1) {Φ : K → ℂ} (hΦ : Continuous Φ) (hc : HasCompactSupport Φ) (hI : Integrable (fun y => ∫ x, Φ x * (ψ (x * y) : ℂ) ∂(selfDualMeasure ψ)) (selfDualMeasure ψ)) (z : K) : ∫ y, (∫ x, Φ x * (ψ (x * y) : ℂ) ∂(selfDualMeasure ψ)) * (ψ (y * z) : ℂ) ∂(selfDualMeasure ψ) = Φ (−z)

theorem **GlobalTheory.CompanionTheory.SW.selfDual_selfDualMeasure** : ∀ (K : Type) [Field K] [ValuativeRel K] [TopologicalSpace K] [IsNonarchimedeanLocalField K] [MeasurableSpace K] [BorelSpace K] (ψ : AddChar K Circle), Continuous ψ → ψ ≠ 1 → SelfDual (GroundTheory.LocalField.selfDualMeasure ψ) (fun s t => ((ψ (s * t) : Circle) : ℂ))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.LocalFieldSelfdualMeasureExistsStatement

theorem **_root_.GlobalTheory.T.local_field_selfdual_measure_exists** : ∀ (K : Type) [Field K] [ValuativeRel K] [TopologicalSpace K] [IsNonarchimedeanLocalField K] [MeasurableSpace K] [BorelSpace K] (ψ : AddChar K Circle), Continuous ψ → ψ ≠ 1 → SelfDual (GroundTheory.LocalField.selfDualMeasure ψ) (fun s t => ((ψ (s * t) : Circle) : ℂ))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.LocalGramFiberMeasuresAgreeAlmostAll

theorem **lattice_mass_fibre_eq** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (J' : Matrix (Fin M) (Fin M) E), IsHermND σ J' → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E), ∀ n : ℕ, ∀ᶠ v : HeightOneSpectrum (𝒪 F) in Filter.cofinite, ∀ lam, IsStdV v lam → ∀ m m', IsGramFib (Fv F v) σ J n (coordW (Fv F v) b lam n M) (coordHerm (Fv F v) σ b lam n) m → IsGramFib (Fv F v) σ J' n (coordW (Fv F v) b lam n M) (coordHerm (Fv F v) σ b lam n) m' → ∀ S, m S (latV b v) = m' S (latV b v)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.LocalGramFiberMeasuresAgreeAlmostAllStatement

theorem **_root_.GlobalTheory.T.local_gram_fiber_measures_agree_almost_all** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (J' : Matrix (Fin M) (Fin M) E), IsHermND σ J' → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E), ∀ n : ℕ, ∀ᶠ v : HeightOneSpectrum (𝒪 F) in Filter.cofinite, ∀ lam, IsStdV v lam → ∀ m m', IsGramFib (Fv F v) σ J n (coordW (Fv F v) b lam n M) (coordHerm (Fv F v) σ b lam n) m → IsGramFib (Fv F v) σ J' n (coordW (Fv F v) b lam n M) (coordHerm (Fv F v) σ b lam n) m' → ∀ S, m S (latV b v) = m' S (latV b v)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.LocalGramFiberRankOneSetFormula

theorem **conj_invol** (h : IsCMExt F E σ) (x : E) : σ (σ x) = x

theorem **exists_conj_ne** (h : IsCMExt F E σ) : ∃ e : E, σ e ≠ e

theorem **exists_imag** (h : IsCMExt F E σ) : ∃ w : E, σ w = −w ∧ ∀ e : E, ∃ a c : F, a • (1 : E) + c • w = e

theorem **sigR_tmul** (r : R) (e : E) : sigR R σ (r ⊗ₜ[F] e) = r ⊗ₜ[F] σ e

theorem **fixed_tmul** (h : IsCMExt F E σ) (y : R ⊗[F] E) (hy : sigR R σ y = y) : ∃ r : R, y = r ⊗ₜ[F] (1 : E)

theorem **half_trace_tmul** (h : IsCMExt F E σ) (r : R) : algebraMap F R (1 / 2) * Algebra.trace R (R ⊗[F] E) (r ⊗ₜ[F] (1 : E)) = r

theorem **sigR_hR_self** (h : IsCMExt F E σ) (hJ : IsHermND σ J) (x : Fin M → R ⊗[F] E) : sigR R σ (hR R σ J x x) = hR R σ J x x

theorem **qS_hermScal** (h : IsCMExt F E σ) (r : R) : qS R σ (hermScal R σ r) = r

theorem **hermScal_qS** (h : IsCMExt F E σ) (S : HermR R σ 1) : hermScal R σ (qS R σ S) = S

theorem **gram_entry** (h : IsCMExt F E σ) (hJ : IsHermND σ J) (X : Fin 1 → Fin M → R ⊗[F] E) : gramR R σ J X 0 0 = (algebraMap F R (1 / 2) * Algebra.trace R (R ⊗[F] E) (gramR R σ J X 0 0)) ⊗ₜ[F] (1 : E)

theorem **isUnit_tmul** {r : R} (hr : r ≠ 0) : IsUnit (r ⊗ₜ[F] (1 : E))

theorem **continuous_hermScal** [IsTopologicalRing R] : Continuous (hermScal R σ)

theorem **continuous_qS** (htr : Continuous (Algebra.trace R (R ⊗[F] E))) [IsTopologicalRing R] : Continuous (qS R σ)

theorem **coordHerm_qS** (h : IsCMExt F E σ) [IsTopologicalRing R] [SecondCountableTopology R] (htr : Continuous (Algebra.trace R (R ⊗[F] E))) {d : ℕ} (b : Module.Basis (Fin d) F E) (lam : Measure R) [SigmaFinite lam] {V : Set R} (hV : MeasurableSet V) : coordHerm R σ b lam 1 {S | qS R σ S ∈ V} = lam V

theorem **mass_eq_lintegral** (R : Type) [Field R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] (h : IsCMExt F E σ) (hJ : IsHermND σ J) {μ : Measure (Fin 1 → Fin M → R ⊗[F] E)} {ν : Measure (HermR R σ 1)} {m : HermR R σ 1 → Measure (Fin 1 → Fin M → R ⊗[F] E)} (hfib : IsGramFib R σ J 1 μ ν m) {A : Set (Fin 1 → Fin M → R ⊗[F] E)} (hA : MeasurableSet A) {V : Set R} (h0 : (0 : R) ∉ V) (hQ : MeasurableSet {X : Fin 1 → Fin M → R ⊗[F] E | algebraMap F R (1 / 2) * Algebra.trace R (R ⊗[F] E) (gramR R σ J X 0 0) ∈ V}) (hP : MeasurableSet {S | qS R σ S ∈ V}) : μ (A ∩ {X | algebraMap F R (1 / 2) * Algebra.trace R (R ⊗[F] E) (gramR R σ J X 0 0) ∈ V}) = ∫⁻ S in {S | qS R σ S ∈ V}, m S A ∂ν

theorem **ratio_field** (R : Type) [Field R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R] (hloc : IsLocR R) (h : IsCMExt F E σ) (hJ : IsHermND σ J) {d : ℕ} (b : Module.Basis (Fin d) F E) (lam : Measure R) (hlam : lam.IsAddHaarMeasure) (m : HermR R σ 1 → Measure (Fin 1 → Fin M → R ⊗[F] E)) (hfib : IsGramFib R σ J 1 (coordW R b lam 1 M) (coordHerm R σ b lam 1) m) (A : Set (Fin 1 → Fin M → R ⊗[F] E)) (hAc : IsCompact A) (hAo : IsOpen A) (s : R) (hs : s ≠ 0) (ε : ℝ) (hε : 0 < ε) : ∃ U ∈ 𝒩 s, ∀ V : Set R, MeasurableSet V → V ⊆ U → |(coordW R b lam 1 M (A ∩ {X | algebraMap F R (1 / 2) * Algebra.trace R (R ⊗[F] E) (gramR R σ J X 0 0) ∈ V})).toReal − (m (hermScal R σ s) A).toReal * (lam V).toReal| ≤ ε * (lam V).toReal

theorem **fibre_mass_ratio** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (v : HeightOneSpectrum (𝒪 F)) (lam : Measure (Fv F v)), IsStdV v lam → ∀ m, IsGramFib (Fv F v) σ J 1 (coordW (Fv F v) b lam 1 M) (coordHerm (Fv F v) σ b lam 1) m → ∀ A : Set (Fin 1 → Fin M → Fv F v ⊗[F] E), IsCompact A → IsOpen A → ∀ s : Fv F v, s ≠ 0 → ∀ ε : ℝ, 0 < ε → ∃ U ∈ 𝒩 s, ∀ V : Set (Fv F v), MeasurableSet V → V ⊆ U → |(coordW (Fv F v) b lam 1 M (A ∩ {X | algebraMap F (Fv F v) (1 / 2) * Algebra.trace (Fv F v) (Fv F v ⊗[F] E) (gramR (Fv F v) σ J X 0 0) ∈ V})).toReal − (m (hermScal (Fv F v) σ s) A).toReal * (lam V).toReal| ≤ ε * (lam V).toReal

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.LocalGramFiberRankOneSetFormulaStatement

theorem **_root_.GlobalTheory.T.local_gram_fiber_rank_one_set_formula** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (v : HeightOneSpectrum (𝒪 F)) (lam : Measure (Fv F v)), IsStdV v lam → ∀ m, IsGramFib (Fv F v) σ J 1 (coordW (Fv F v) b lam 1 M) (coordHerm (Fv F v) σ b lam 1) m → ∀ A : Set (Fin 1 → Fin M → Fv F v ⊗[F] E), IsCompact A → IsOpen A → ∀ s : Fv F v, s ≠ 0 → ∀ ε : ℝ, 0 < ε → ∃ U ∈ 𝒩 s, ∀ V : Set (Fv F v), MeasurableSet V → V ⊆ U → |(coordW (Fv F v) b lam 1 M (A ∩ {X | algebraMap F (Fv F v) (1 / 2) * Algebra.trace (Fv F v) (Fv F v ⊗[F] E) (gramR (Fv F v) σ J X 0 0) ∈ V})).toReal − (m (hermScal (Fv F v) σ s) A).toReal * (lam V).toReal| ≤ ε * (lam V).toReal

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.LocalHaarQuadraticSubstitutionInvariant

lemma **vball_eq_of_mem** {x y : K} {γ : ℤᵐ⁰} (h : y ∈ vball x γ) : vball y γ = vball x γ

lemma **IsRadius.ne_zero** {γ : ℤᵐ⁰} (h : IsRadius K γ) : γ ≠ 0

lemma **isOpen_ball_of_isRadius** {γ : ℤᵐ⁰} (h : IsRadius K γ) : IsOpen {y : K | Valued.v y < γ}

lemma **isOpen_vball** (x : K) {γ : ℤᵐ⁰} (h : IsRadius K γ) : IsOpen (vball x γ)

lemma **isTopologicalBasis_vball** : TopologicalSpace.IsTopologicalBasis {s : Set K | ∃ (x : K) (γ : ℤᵐ⁰), s = vball x γ ∧ IsRadius K γ}

lemma **isPiSystem_vball** : IsPiSystem {s : Set K | ∃ (x : K) (γ : ℤᵐ⁰), s = vball x γ ∧ IsRadius K γ}

lemma **le_exp_neg_one** {x : ℤᵐ⁰} (h : x < 1) : x ≤ WithZero.exp (−1 : ℤ)

lemma **quad_sub** (a c t t' : K) : (a * t + c * t ^ 2) − (a * t' + c * t' ^ 2) = (t − t') * (a + c * (t + t'))

lemma **v_lin_eq_one** {a c s : K} (ha : Valued.v a = 1) (hc : Valued.v c ≤ 1) (hs : Valued.v s < 1) : Valued.v (a + c * s) = 1

lemma **v_quad_sub** {a c t t' : K} (ha : Valued.v a = 1) (hc : Valued.v c ≤ 1) (ht : Valued.v t < 1) (ht' : Valued.v t' < 1) : Valued.v ((a * t + c * t ^ 2) − (a * t' + c * t' ^ 2)) = Valued.v (t − t')

lemma **v_quad_lt** {a c t : K} (ha : Valued.v a = 1) (hc : Valued.v c ≤ 1) (ht : Valued.v t < 1) : Valued.v (a * t + c * t ^ 2) < 1

lemma **exists_approx** {a c x : K} (ha : Valued.v a = 1) (hc : Valued.v c ≤ 1) (hx : Valued.v x < 1) (n : ℕ) : ∃ t : K, Valued.v t < 1 ∧ Valued.v ((a * t + c * t ^ 2) − x) ≤ WithZero.exp (−(n + 1 : ℤ))

lemma **exists_near** {a c x : K} (ha : Valued.v a = 1) (hc : Valued.v c ≤ 1) (hx : Valued.v x < 1) {γ : ℤᵐ⁰} (hγ : γ ≠ 0) : ∃ t : K, Valued.v t < 1 ∧ Valued.v ((a * t + c * t ^ 2) - x) < γ

lemma **measure_ball** (lam : Measure K) [lam.IsAddLeftInvariant] {a c : K} (ha : Valued.v a = 1) (hc : Valued.v c ≤ 1) (x : K) {γ : ℤᵐ⁰} (hγ : γ ≠ 0) : lam ((fun t => a * t + c * t ^ 2) ⁻¹' vball x γ ∩ {s | Valued.v s < 1}) = lam (vball x γ ∩ {s | Valued.v s < 1})

theorem **measure_quad** [SecondCountableTopology K] [IsTopologicalRing K] (lam : Measure K) [lam.IsAddLeftInvariant] (hfin : lam {s | Valued.v s < 1} ≠ ⊤) {a c : K} (ha : Valued.v a = 1) (hc : Valued.v c ≤ 1) {A : Set K} (hA : MeasurableSet A) : lam {t | Valued.v t < 1 ∧ a * t + c * t ^ 2 ∈ A} = lam (A ∩ {s | Valued.v s < 1})

theorem **GlobalTheory.CompanionTheory.SW.QuadIso.measure_quad_Fv** : ∀ (F : Type) [Field F] [NumberField F] (v : HeightOneSpectrum (𝒪 F)) (lam : Measure (Fv F v)), IsStdV v lam → Valued.v (2 : Fv F v) = 1 → ∀ a c : Fv F v, Valued.v a = 1 → c ∈ v.adicCompletionIntegers F → ∀ A : Set (Fv F v), MeasurableSet A → lam {t | Valued.v t < 1 ∧ a * t + c * t ^ 2 ∈ A} = lam (A ∩ {s | Valued.v s < 1})

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.LocalHaarQuadraticSubstitutionInvariantStatement

theorem **_root_.GlobalTheory.T.local_haar_quadratic_substitution_invariant** : ∀ (F : Type) [Field F] [NumberField F] (v : HeightOneSpectrum (𝒪 F)) (lam : Measure (Fv F v)), IsStdV v lam → Valued.v (2 : Fv F v) = 1 → ∀ a c : Fv F v, Valued.v a = 1 → c ∈ v.adicCompletionIntegers F → ∀ A : Set (Fv F v), MeasurableSet A → lam {t | Valued.v t < 1 ∧ a * t + c * t ^ 2 ∈ A} = lam (A ∩ {s | Valued.v s < 1})

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.LocalHermitianDiagonalizationUnits

theorem **hermR_unit_diagonalisable** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (R : Type) [CommRing R] [Algebra F R] [IsArtinianRing R] [IsReduced R] (n : ℕ) (S : HermR R σ n), IsUnit (S : Matrix (Fin n) (Fin n) (R ⊗[F] E)).det → ∃ (g : Matrix (Fin n) (Fin n) (R ⊗[F] E)) (d : Fin n → Rˣ), IsUnit g.det ∧ (S : Matrix (Fin n) (Fin n) (R ⊗[F] E)) = cT (sigR R σ) g * Matrix.diagonal (fun i => ((d i : R) ⊗ₜ[F] (1 : E) : R ⊗[F] E)) * g

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.LocalHermitianDiagonalizationUnitsStatement

theorem **_root_.GlobalTheory.T.local_hermitian_diagonalization_units** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (R : Type) [CommRing R] [Algebra F R] [IsArtinianRing R] [IsReduced R] (n : ℕ) (S : HermR R σ n), IsUnit (S : Matrix (Fin n) (Fin n) (R ⊗[F] E)).det → ∃ (g : Matrix (Fin n) (Fin n) (R ⊗[F] E)) (d : Fin n → Rˣ), IsUnit g.det ∧ (S : Matrix (Fin n) (Fin n) (R ⊗[F] E)) = cT (sigR R σ) g * Matrix.diagonal (fun i => ((d i : R) ⊗ₜ[F] (1 : E) : R ⊗[F] E)) * g

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.LocalLatticeCompactOpenUnitVolume

theorem **latV_compact_open_mass** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E), ∀ (n M : ℕ) (v : HeightOneSpectrum (𝒪 F)), IsCompact (latV (M

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.LocalLatticeCompactOpenUnitVolumeStatement

theorem **_root_.GlobalTheory.T.local_lattice_compact_open_unit_volume** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E), ∀ (n M : ℕ) (v : HeightOneSpectrum (𝒪 F)), IsCompact (latV (M

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.LocalMeasuresOrthogonalSumAlmostAllPlaces

theorem **gram_first** {F E : Type} [Field F] [Field E] [Algebra F E] (R : Type) [CommRing R] [Algebra F R] (σ : E →ₐ[F] E) {M : ℕ} (J : Matrix (Fin M) (Fin M) E) (t : F) : gramR R σ (blockOne (algebraMap F E t) J) (fun _ : Fin 1 => (Fin.cons 1 (fun _ => 0) : Fin (M + 1) → R ⊗[F] E)) = (hermScal R σ (algebraMap F R t) : HermR R σ 1)

theorem **first_mem** {F E : Type} [Field F] [NumberField F] [Field E] [Algebra F E] {d M n : ℕ} (b : Module.Basis (Fin d) F E) (v : HeightOneSpectrum (𝒪 F)) (σ : E →ₐ[F] E) (J : Matrix (Fin (M + 1)) (Fin (M + 1)) E) (h : HA (Fv F v) σ J) (X : Fin n → Fin M → Fv F v ⊗[F] E) (hX : actW (Fv F v) σ J h (consTup X) ∈ latV b v) : actW (Fv F v) σ J h (fun _ : Fin 1 => Fin.cons 1 (fun _ => 0)) ∈ latV b v

theorem **stage_good** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ t : F, t ≠ 0 → ∀ (n : ℕ) (T : Matrix (Fin n) (Fin n) E) (hT : ∀ a b, σ (T b a) = T a b), T.det ≠ 0 → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E), ∀ᶠ v : HeightOneSpectrum (𝒪 F) in Filter.cofinite, ∀ (μ : Measure (Fin n → Fin M → Fv F v ⊗[F] E)) (ν : Measure (HermR (Fv F v) σ n)) m, μ.IsAddHaarMeasure → ν.IsAddHaarMeasure → IsGramFib (Fv F v) σ J n μ ν m → ∀ h : HA (Fv F v) σ (blockOne (algebraMap F E t) J), m (hermOf (Fv F v) σ T hT) {X | actW (Fv F v) σ _ h (consTup X) ∈ latV b v} = (latV b v).indicator (fun _ => m (hermOf (Fv F v) σ T hT) (latV b v)) (actW (Fv F v) σ _ h (fun _ : Fin 1 => Fin.cons 1 (fun _ => 0)))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.LocalMeasuresOrthogonalSumAlmostAllPlacesStatement

theorem **_root_.GlobalTheory.T.local_measures_orthogonal_sum_almost_all_places** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ t : F, t ≠ 0 → ∀ (n : ℕ) (T : Matrix (Fin n) (Fin n) E) (hT : ∀ a b, σ (T b a) = T a b), T.det ≠ 0 → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E), ∀ᶠ v : HeightOneSpectrum (𝒪 F) in Filter.cofinite, ∀ (μ : Measure (Fin n → Fin M → Fv F v ⊗[F] E)) (ν : Measure (HermR (Fv F v) σ n)) m, μ.IsAddHaarMeasure → ν.IsAddHaarMeasure → IsGramFib (Fv F v) σ J n μ ν m → ∀ h : HA (Fv F v) σ (blockOne (algebraMap F E t) J), m (hermOf (Fv F v) σ T hT) {X | actW (Fv F v) σ _ h (consTup X) ∈ latV b v} = (latV b v).indicator (fun _ => m (hermOf (Fv F v) σ T hT) (latV b v)) (actW (Fv F v) σ _ h (fun _ : Fin 1 => Fin.cons 1 (fun _ => 0)))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.LocalPrimitiveVectorCompactRescale

theorem **primitiveCompact_of_multiplicative** {K : Type*} [CommRing K] [Nontrivial K] [TopologicalSpace K] (M : ℕ) (ν : K → ℝ) (h0 : ν 0 = 0) (h1 : ν 1 = 1) (hm : ∀ x y, ν (x * y) = ν x * ν y) (hn : ∀ x, 0 ≤ ν x) (hu : ∀ x, x ≠ 0 → IsUnit x ∧ 0 < ν x) (hc : IsCompact {x | ν x ≤ 1}) : UniP K M

theorem **primitiveCompact_prod** {A B : Type*} [CommRing A] [CommRing B] [TopologicalSpace A] [TopologicalSpace B] {M : ℕ} (hA : UniP A M) (hB : UniP B M) : UniP (A × B) M

theorem **primitiveCompact_of_ringEquiv** {A B : Type*} [CommRing A] [CommRing B] [TopologicalSpace A] [TopologicalSpace B] {M : ℕ} (e : A ≃+* B) (hc : Continuous e.symm) (h : UniP B M) : UniP A M

theorem **gg_mul** {d : K} {D : L} (hd : d * d = algebraMap L K D) (p q : L × L) : gg d p * gg d q = gg d (p.1 * q.1 + D * (p.2 * q.2), p.1 * q.2 + p.2 * q.1)

theorem **gg_rr** (h0 : bs 0 = 1) (x : K) : gg (bs 1) (rr bs x) = x

theorem **rr_gg** (h0 : bs 0 = 1) (p : L × L) : rr bs (gg (bs 1) p) = p

theorem **rr_mul** (h0 : bs 0 = 1) {D : L} (hd : bs 1 * bs 1 = algebraMap L K D) (x y : K) : rr bs (x * y) = ((rr bs x).1 * (rr bs y).1 + D * ((rr bs x).2 * (rr bs y).2), (rr bs x).1 * (rr bs y).2 + (rr bs x).2 * (rr bs y).1)

theorem **rr_zero** : rr bs (0 : K) = 0

theorem **rr_one** (h0 : bs 0 = 1) : rr bs (1 : K) = (1, 0)

theorem **rr_add** (x y : K) : rr bs (x + y) = rr bs x + rr bs y

theorem **nontrivial_of_basis** (h0 : bs 0 = 1) : Nontrivial K

theorem **Nf_mul** (D : L) (p q : L × L) : Nf D (p.1 * q.1 + D * (p.2 * q.2), p.1 * q.2 + p.2 * q.1) = Nf D p * Nf D q

theorem **Nf_ne** {D : L} (hD : ∀ s, s * s ≠ D) {p : L × L} (hp : p ≠ 0) : Nf D p ≠ 0

theorem **compact_Nf** {D : L} (hD : ∀ s, s * s ≠ D) : IsCompact {p : L × L | ‖Nf D p‖ ≤ 1}

theorem **primitiveCompact_nonsplit** (h0 : bs 0 = 1) {D : L} (hd : bs 1 * bs 1 = algebraMap L K D) (hD : ∀ s, s * s ≠ D) (hg : Continuous (gg (bs 1) : L × L → K)) (M : ℕ) : UniP K M

theorem **primitiveCompact_normedField** (M : ℕ) : UniP L M

theorem **primitiveCompact_split** (h0 : bs 0 = 1) {D : L} (hd : bs 1 * bs 1 = algebraMap L K D) {s : L} (hs : s ≠ 0) (hD : s * s = D) (h2 : (2 : L) ≠ 0) (hg : Continuous (gg (bs 1) : L × L → K)) (M : ℕ) : UniP K M

theorem **basis_of_not_mem** (h2 : Module.finrank F E = 2) {d : E} (hd : d ∉ (algebraMap F E).range) : ∃ b : Module.Basis (Fin 2) F E, b 0 = 1 ∧ b 1 = d

theorem **exists_delta** [CharZero F] (h2 : Module.finrank F E = 2) : ∃ (b : Module.Basis (Fin 2) F E) (Δ : F), b 0 = 1 ∧ b 1 * b 1 = algebraMap F E Δ ∧ Δ ≠ 0

theorem **exists_compact_unimodular_vectors_mod_units** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (v : HeightOneSpectrum (𝒪 F)), ∃ Ω : Set (Fin 1 → Fin M → Fv F v ⊗[F] E), IsCompact Ω ∧ ∀ y : Fin 1 → Fin M → Fv F v ⊗[F] E, (∃ z : Fin M → Fv F v ⊗[F] E, z ᵥ* y 0 = 1) → ∃ (a : (Fv F v ⊗[F] E)ˣ) (z : Fin 1 → Fin M → Fv F v ⊗[F] E), (a : Fv F v ⊗[F] E) • y ∈ Ω ∧ z ∈ Ω ∧ z 0 ᵥ* ((a : Fv F v ⊗[F] E) • y) 0 = 1

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.LocalPrimitiveVectorCompactRescaleStatement

theorem **_root_.GlobalTheory.T.local_primitive_vector_compact_rescale** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (v : HeightOneSpectrum (𝒪 F)), ∃ Ω : Set (Fin 1 → Fin M → Fv F v ⊗[F] E), IsCompact Ω ∧ ∀ y : Fin 1 → Fin M → Fv F v ⊗[F] E, (∃ z : Fin M → Fv F v ⊗[F] E, z ᵥ* y 0 = 1) → ∃ (a : (Fv F v ⊗[F] E)ˣ) (z : Fin 1 → Fin M → Fv F v ⊗[F] E), (a : Fv F v ⊗[F] E) • y ∈ Ω ∧ z ∈ Ω ∧ z 0 ᵥ* ((a : Fv F v ⊗[F] E) • y) 0 = 1

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.LocalQuadraticDensityBoundNonzero

theorem **exists_unif** (v : HeightOneSpectrum (𝒪 F)) : ∃ π : Fv F v, Valued.v π = WithZero.exp (-1 : ℤ)

theorem **le_exp_of_lt_one** {a : WithZero (Multiplicative ℤ)} (h : a < 1) : a ≤ WithZero.exp (-1 : ℤ)

theorem **mem_int** {x : Fv F v} : x ∈ v.adicCompletionIntegers F ↔ Valued.v x ≤ 1

theorem **q_int** (h2 : Valued.v (2 : Fv F v) = 1) {B : Matrix ι ι (Fv F v)} (hB : ∀ i j, B i j ∈ v.adicCompletionIntegers F) {y : ι → Fv F v} (hy : ∀ i, y i ∈ v.adicCompletionIntegers F) : Valued.v ((2 : Fv F v)⁻¹ * (y ·ᵥ (B *ᵥ y))) ≤ 1

theorem **q_smul** (B : Matrix ι ι (Fv F v)) (π : Fv F v) (y : ι → Fv F v) : (2 : Fv F v)⁻¹ * ((fun i => π * y i) ·ᵥ (B *ᵥ fun i => π * y i)) = π * (π * ((2 : Fv F v)⁻¹ * (y ·ᵥ (B *ᵥ y))))

theorem **borel_Fv** : BorelSpace (Fv F v)

private theorem **sigmaFinite_nnreal_smul** {X : Type} [MeasurableSpace X] (ν : Measure X) [SigmaFinite ν] (c : ℝ≥0) : SigmaFinite ((c : ℝ≥0∞) • ν)

theorem **pi_const_smul** (ι : Type) [Fintype ι] {X : Type} [MeasurableSpace X] (ν : Measure X) [SigmaFinite ν] (c : ℝ≥0) : Measure.pi (fun _ : ι => (c : ℝ≥0∞) • ν) = ((c : ℝ≥0∞) ^ Fintype.card ι) • Measure.pi (fun _ : ι => ν)

private theorem **sigmaFinite_of_isStdV** {lam : Measure (Fv F v)} (hstd : IsStdV v lam) : SigmaFinite lam

theorem **pi_scale** {lam : Measure (Fv F v)} (hstd : IsStdV v lam) {π : Fv F v} (hπ : Valued.v π = WithZero.exp (-1 : ℤ)) (T : Set (ι → Fv F v)) : (Measure.pi fun _ : ι => lam) {y | (fun i => π * y i) ∈ T} = (Ideal.absNorm v.asIdeal : ℝ≥0∞) ^ Fintype.card ι * (Measure.pi fun _ : ι => lam) T

theorem **lam_scale** {lam : Measure (Fv F v)} (hstd : IsStdV v lam) {π : Fv F v} (hπ : Valued.v π = WithZero.exp (-1 : ℤ)) {A : Set (Fv F v)} (hA : MeasurableSet A) : MeasurableSet {z | π * (π * z) ∈ A} ∧ lam {z | π * (π * z) ∈ A} = (Ideal.absNorm v.asIdeal : ℝ≥0∞) ^ 2 * lam A

theorem **isOpen_unit_sphere** : IsOpen {x : Fv F v | Valued.v x = 1}

theorem **meas_prim** : MeasurableSet {x : ι → Fv F v | ∃ i, Valued.v (x i) = 1}

theorem **val_inv** {π : Fv F v} (hπ : Valued.v π = WithZero.exp (-1 : ℤ)) : Valued.v π⁻¹ = WithZero.exp (1 : ℤ)

theorem **pi_ne_zero** {π : Fv F v} (hπ : Valued.v π = WithZero.exp (-1 : ℤ)) : π ≠ 0

theorem **div_int** {π : Fv F v} (hπ : Valued.v π = WithZero.exp (-1 : ℤ)) {a : Fv F v} (h1 : Valued.v a ≤ 1) (h2 : Valued.v a ≠ 1) : Valued.v (π⁻¹ * a) ≤ 1

theorem **pi_mul_lt** {π : Fv F v} (hπ : Valued.v π = WithZero.exp (-1 : ℤ)) {a : Fv F v} (h1 : Valued.v a ≤ 1) : Valued.v (π * a) < 1

theorem **val_pp** {π : Fv F v} (hπ : Valued.v π = WithZero.exp (-1 : ℤ)) (a : Fv F v) : Valued.v (π * (π * a)) = WithZero.exp (-2 : ℤ) * Valued.v a

theorem **q_nonprim** (h2 : Valued.v (2 : Fv F v) = 1) {B : Matrix ι ι (Fv F v)} (hB : ∀ i j, B i j ∈ v.adicCompletionIntegers F) {π : Fv F v} (hπ : Valued.v π = WithZero.exp (-1 : ℤ)) {x : ι → Fv F v} (hx : ∀ i, x i ∈ v.adicCompletionIntegers F) (hnp : ¬ ∃ i, Valued.v (x i) = 1) : Valued.v ((2 : Fv F v)⁻¹ * (x ·ᵥ (B *ᵥ x))) ≤ WithZero.exp (-2 : ℤ)

theorem **bound_step** {q : ℝ} {M : ℕ} (hq : 2 ≤ q) (hM : 3 ≤ M) {Dp D' : ℝ} (hD' : |D' - 1| ≤ 2 / q ^ (M - 1)) (hDp : |Dp - 1| ≤ 1 / q ^ (M - 1) + 1 / q ^ (2 * M - 1)) : |Dp + q ^ 2 / q ^ (2 * M) * D' - 1| ≤ 2 / q ^ (M - 1)

theorem **local_quadratic_density_bound_at_level** {lam : Measure (Fv F v)} (hstd : IsStdV v lam) (h2 : Valued.v (2 : Fv F v) = 1) [DecidableEq ι] (B : Matrix ι ι (Fv F v)) (hsym : B.IsSymm) (hB : ∀ i j, B i j ∈ v.adicCompletionIntegers F) (hdet : Valued.v B.det = 1) (M : ℕ) (hc : Fintype.card ι = 2 * M) (hM : 3 ≤ M) (n : ℕ) : ∀ s : Fv F v, s ∈ v.adicCompletionIntegers F → s ≠ 0 → WithZero.exp (-(n : ℤ)) ≤ Valued.v s → ∃ D : ℝ≥0, |(D : ℝ) - 1| ≤ 2 / (Ideal.absNorm v.asIdeal : ℝ) ^ (M - 1) ∧ ∀ A : Set (Fv F v), MeasurableSet A → A ⊆ {z | Valued.v (z - s) < Valued.v s} → (Measure.pi fun _ : ι => lam) {x | (∀ i, x i ∈ v.adicCompletionIntegers F) ∧ (2 : Fv F v)⁻¹ * (x ·ᵥ (B *ᵥ x)) ∈ A} = (D : ℝ≥0∞) * lam A

theorem **density_near_value** : ∀ (F : Type) [Field F] [NumberField F] (v : HeightOneSpectrum (𝒪 F)) (lam : Measure (Fv F v)), IsStdV v lam → Valued.v (2 : Fv F v) = 1 → ∀ (ι : Type) [Fintype ι] [DecidableEq ι] (B : Matrix ι ι (Fv F v)), B.IsSymm → (∀ i j, B i j ∈ v.adicCompletionIntegers F) → Valued.v B.det = 1 → ∀ M : ℕ, Fintype.card ι = 2 * M → 3 ≤ M → ∀ s : Fv F v, s ∈ v.adicCompletionIntegers F → s ≠ 0 → ∃ D : ℝ≥0, |(D : ℝ) - 1| ≤ 2 / (Ideal.absNorm v.asIdeal : ℝ) ^ (M - 1) ∧ ∀ A : Set (Fv F v), MeasurableSet A → A ⊆ {z | Valued.v (z - s) < Valued.v s} → (Measure.pi fun _ : ι => lam) {x | (∀ i, x i ∈ v.adicCompletionIntegers F) ∧ (2 : Fv F v)⁻¹ * (x ·ᵥ (B *ᵥ x)) ∈ A} = (D : ℝ≥0∞) * lam A

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.LocalQuadraticDensityBoundNonzeroStatement

theorem **_root_.GlobalTheory.T.local_quadratic_density_bound_nonzero** : ∀ (F : Type) [Field F] [NumberField F] (v : HeightOneSpectrum (𝒪 F)) (lam : Measure (Fv F v)), IsStdV v lam → Valued.v (2 : Fv F v) = 1 → ∀ (ι : Type) [Fintype ι] [DecidableEq ι] (B : Matrix ι ι (Fv F v)), B.IsSymm → (∀ i j, B i j ∈ v.adicCompletionIntegers F) → Valued.v B.det = 1 → ∀ M : ℕ, Fintype.card ι = 2 * M → 3 ≤ M → ∀ s : Fv F v, s ∈ v.adicCompletionIntegers F → s ≠ 0 → ∃ D : ℝ≥0, |(D : ℝ) - 1| ≤ 2 / (Ideal.absNorm v.asIdeal : ℝ) ^ (M - 1) ∧ ∀ A : Set (Fv F v), MeasurableSet A → A ⊆ {z | Valued.v (z - s) < Valued.v s} → (Measure.pi fun _ : ι => lam) {x | (∀ i, x i ∈ v.adicCompletionIntegers F) ∧ (2 : Fv F v)⁻¹ * (x ·ᵥ (B *ᵥ x)) ∈ A} = (D : ℝ≥0∞) * lam A

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.LocalQuadraticDensityNearOne

theorem **real_bound** (q N Nf : ℝ) (hq : 0 < q) (M : ℕ) (hM : 1 ≤ M) (h : |Nf - q ^ (2 * M - 1)| ≤ q ^ M) (h1 : N ≤ Nf) (h2 : Nf ≤ N + 1) : |N * (q⁻¹) ^ (2 * M - 1) - 1| ≤ 1 / q ^ (M - 1) + 1 / q ^ (2 * M - 1)

theorem **map_quad** {R S ι : Type} [CommRing R] [CommRing S] [Fintype ι] (f : R →+* S) (B : Matrix ι ι R) (x : ι → R) : f (x ·ᵥ (B *ᵥ x)) = (f ∘ x) ·ᵥ ((B.map f) *ᵥ (f ∘ x))

theorem **res_eq_zero_iff** (a : 𝒪) : res a = 0 ↔ Valued.v (a : Fv F v) < 1

theorem **res_eq_iff** (a b : 𝒪) : res a = res b ↔ Valued.v ((a : Fv F v) - b) < 1

theorem **res_lift** (y : 𝓀) : res (lift y) = y

theorem **cover** {ι : Type} (x : ι → Fv F v) : ((∀ i, x i ∈ 𝒪) ∧ ∃ i, Valued.v (x i) = 1) ↔ ∃ y : ι → 𝓀, y ≠ 0 ∧ ∀ i, Valued.v (x i - lf y i) < 1

theorem **eq_of_close** {ι : Type} (x : ι → Fv F v) (y y' : ι → 𝓀) (h : ∀ i, Valued.v (x i - lf y i) < 1) (h' : ∀ i, Valued.v (x i - lf y' i) < 1) : y = y'

theorem **res_Q** {ι : Type} [Fintype ι] (c : 𝒪) (hc : (c : Fv F v) = 2⁻¹) (hr : res c = 2⁻¹) (BO : Matrix ι ι 𝒪) (y : ι → 𝓀) : ∃ a : 𝒪, (a : Fv F v) = (2 : Fv F v)⁻¹ * (lf y ·ᵥ ((BO.map phi) *ᵥ lf y)) ∧ res a = (2 : 𝓀)⁻¹ * (y ·ᵥ ((BO.map res) *ᵥ y))

theorem **core** (lam : Measure (Fv F v)) (hlam : IsStdV v lam) (h2 : Valued.v (2 : Fv F v) = 1) (ι : Type) [Fintype ι] [DecidableEq ι] (B : Matrix ι ι (Fv F v)) (hBs : B.IsSymm) (hBint : ∀ i j, B i j ∈ v.adicCompletionIntegers F) (hBdet : Valued.v B.det = 1) (M : ℕ) (hM : Fintype.card ι = 2 * M) (hM1 : 1 ≤ M) (s : Fv F v) (hs : s ∈ v.adicCompletionIntegers F) : ∃ D : ℝ≥0, |(D : ℝ) - 1| ≤ 1 / (Ideal.absNorm v.asIdeal : ℝ) ^ (M - 1) + 1 / (Ideal.absNorm v.asIdeal : ℝ) ^ (2 * M - 1) ∧ ∀ A : Set (Fv F v), MeasurableSet A → A ⊆ {z | Valued.v (z - s) < 1} → (Measure.pi fun _ : ι => lam) {x | (∀ i, x i ∈ v.adicCompletionIntegers F) ∧ (∃ i, Valued.v (x i) = 1) ∧ (2 : Fv F v)⁻¹ * (x ·ᵥ (B *ᵥ x)) ∈ A} = (D : ℝ≥0∞) * lam A

theorem **density_primitive** : ∀ (F : Type) [Field F] [NumberField F] (v : HeightOneSpectrum (𝒪 F)) (lam : Measure (Fv F v)), IsStdV v lam → Valued.v (2 : Fv F v) = 1 → ∀ (ι : Type) [Fintype ι] [DecidableEq ι] (B : Matrix ι ι (Fv F v)), B.IsSymm → (∀ i j, B i j ∈ v.adicCompletionIntegers F) → Valued.v B.det = 1 → ∀ M : ℕ, Fintype.card ι = 2 * M → 1 ≤ M → ∀ s : Fv F v, s ∈ v.adicCompletionIntegers F → ∃ D : ℝ≥0, |(D : ℝ) - 1| ≤ 1 / (Ideal.absNorm v.asIdeal : ℝ) ^ (M - 1) + 1 / (Ideal.absNorm v.asIdeal : ℝ) ^ (2 * M - 1) ∧ ∀ A : Set (Fv F v), MeasurableSet A → A ⊆ {z | Valued.v (z - s) < 1} → (Measure.pi fun _ : ι => lam) {x | (∀ i, x i ∈ v.adicCompletionIntegers F) ∧ (∃ i, Valued.v (x i) = 1) ∧ (2 : Fv F v)⁻¹ * (x ·ᵥ (B *ᵥ x)) ∈ A} = (D : ℝ≥0∞) * lam A

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.LocalQuadraticDensityNearOneStatement

theorem **_root_.GlobalTheory.T.local_quadratic_density_near_one** : ∀ (F : Type) [Field F] [NumberField F] (v : HeightOneSpectrum (𝒪 F)) (lam : Measure (Fv F v)), IsStdV v lam → Valued.v (2 : Fv F v) = 1 → ∀ (ι : Type) [Fintype ι] [DecidableEq ι] (B : Matrix ι ι (Fv F v)), B.IsSymm → (∀ i j, B i j ∈ v.adicCompletionIntegers F) → Valued.v B.det = 1 → ∀ M : ℕ, Fintype.card ι = 2 * M → 1 ≤ M → ∀ s : Fv F v, s ∈ v.adicCompletionIntegers F → ∃ D : ℝ≥0, |(D : ℝ) - 1| ≤ 1 / (Ideal.absNorm v.asIdeal : ℝ) ^ (M - 1) + 1 / (Ideal.absNorm v.asIdeal : ℝ) ^ (2 * M - 1) ∧ ∀ A : Set (Fv F v), MeasurableSet A → A ⊆ {z | Valued.v (z - s) < 1} → (Measure.pi fun _ : ι => lam) {x | (∀ i, x i ∈ v.adicCompletionIntegers F) ∧ (∃ i, Valued.v (x i) = 1) ∧ (2 : Fv F v)⁻¹ * (x ·ᵥ (B *ᵥ x)) ∈ A} = (D : ℝ≥0∞) * lam A

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.LocalRankOneDensityUniformizerPower

theorem **pi_map_scale** {ι X : Type} [Fintype ι] [MeasurableSpace X] (μ : Measure X) [SigmaFinite μ] (f : X → X) (hf : Measurable f) (c : ℝ≥0) (h : μ.map f = c • μ) : (Measure.pi fun _ : ι => μ).map (fun x i => f (x i)) = (c ^ Fintype.card ι) • Measure.pi fun _ : ι => μ

theorem **map_comap_scale** {X Y : Type} [MeasurableSpace X] [MeasurableSpace Y] (φ : X → Y) (P : Measure Y) (f : X → X) (hf : Measurable f) (D : Y =ᵐ Y) (C : ℝ≥0) (hs : Function.Surjective φ) (hi : Function.Injective φ) (him : ∀ s, MeasurableSet s → NullMeasurableSet (φ '' s) P) (hc : ∀ x, φ (f x) = D (φ x)) (hD : P.map D = C • P) : (Measure.comap φ P).map f = C • Measure.comap φ P

theorem **norm_natCast** (v : HeightOneSpectrum (𝒪 F)) (a e : ℕ) (ha : 0 < a) (h1 : (Ideal.span {(a : 𝒪 F)}) ≤ v.asIdeal ^ e) (h2 : ¬ (Ideal.span {(a : 𝒪 F)}) ≤ v.asIdeal ^ (e + 1)) : ‖(a : Fv F v)‖ = ((Ideal.absNorm v.asIdeal : ℝ))⁻¹ ^ e

theorem **scale_fv** (v : HeightOneSpectrum (𝒪 F)) (lam : Measure (Fv F v)) [lam.IsAddHaarMeasure] (x : Fv F v) (hx : x ≠ 0) (k : ℕ) (hk : ‖x‖ = ((Ideal.absNorm v.asIdeal : ℝ))⁻¹ ^ k) : lam.map (fun y => x * y) = ((Ideal.absNorm v.asIdeal : ℝ≥0) ^ k) • lam

theorem **mulPi_apply** (ι : Type) (t : R) (ht : t ≠ 0) (x : ι → R) (i : ι) : mulPi R ι t ht x i = t * x i

theorem **map_mulPi** {ι : Type} [Fintype ι] (lam : Measure R) [SigmaFinite lam] (t : R) (ht : t ≠ 0) (c : ℝ≥0) (h : lam.map (fun y => t * y) = c • lam) : (Measure.pi fun _ : ι => lam).map (mulPi R ι t ht) = (c ^ Fintype.card ι) • Measure.pi fun _ : ι => lam

theorem **coordK_map** (σ : E →ₐ[F] E) (hσ : IsCMExt F E σ) {d : ℕ} (b : Module.Basis (Fin d) F E) (lam : Measure R) [SigmaFinite lam] (t : R) (ht : t ≠ 0) (c : ℝ≥0) (h : lam.map (fun y => t * y) = c • lam) : (coordK R b lam).map (fun y => algebraMap R (R ⊗[F] E) t * y) = (c ^ d) • coordK R b lam

theorem **cT_scalar** {A : Type} [CommRing A] (s : A →+* A) (c : A) (hs : s c = c) (S : Matrix (Fin 1) (Fin 1) A) (i j : Fin 1) : (cT s (Matrix.of fun _ _ => c : Matrix (Fin 1) (Fin 1) A) * S * (Matrix.of fun _ _ => c : Matrix (Fin 1) (Fin 1) A)) i j = c ^ 2 * S i j

theorem **half_trace_tmul** (σ : E →ₐ[F] E) (hσ : IsCMExt F E σ) (s : R) : algebraMap F R (1 / 2) * Algebra.trace R (R ⊗[F] E) (s ⊗ₜ[F] (1 : E)) = s

theorem **coordHerm_map** (σ : E →ₐ[F] E) (hσ : IsCMExt F E σ) {d : ℕ} (b : Module.Basis (Fin d) F E) (lam : Measure R) [lam.IsAddHaarMeasure] (t : R) (ht : t ≠ 0) (c : ℝ≥0) (h : lam.map (fun y => t * y) = c • lam) (τ : HermR R σ 1 → HermR R σ 1) (hτm : Measurable τ) (hτ : ∀ S i j, (τ S : Matrix (Fin 1) (Fin 1) (R ⊗[F] E)) i j = algebraMap R (R ⊗[F] E) t * (S : Matrix (Fin 1) (Fin 1) (R ⊗[F] E)) i j) : (coordHerm R σ b lam 1).map τ = c • coordHerm R σ b lam 1

theorem **dilation** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (v : HeightOneSpectrum (𝒪 F)) (lam : Measure (Fv F v)), IsStdV v lam → ∀ m, IsGramFib (Fv F v) σ J 1 (coordW (Fv F v) b lam 1 M) (coordHerm (Fv F v) σ b lam 1) m → 1 ≤ M → ∀ (a : ℕ) (e : ℕ), 0 < a → (Ideal.span {(a : 𝒪 F)}) ≤ v.asIdeal ^ e → ¬ (Ideal.span {(a : 𝒪 F)}) ≤ v.asIdeal ^ (e + 1) → ∀ (s : Fv F v) (A : Set (Fin 1 → Fin M → Fv F v ⊗[F] E)), MeasurableSet A → m (hermScal (Fv F v) σ s) {X | (fun p i => (a : Fv F v ⊗[F] E) * X p i) ∈ A} = (Ideal.absNorm v.asIdeal : ℝ≥0∞) ^ (e * (2 * M - 2)) * m (hermScal (Fv F v) σ ((a : Fv F v) ^ 2 * s)) A

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.LocalRankOneDensityUniformizerPowerStatement

theorem **_root_.GlobalTheory.T.local_rank_one_density_uniformizer_power** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (v : HeightOneSpectrum (𝒪 F)) (lam : Measure (Fv F v)), IsStdV v lam → ∀ m, IsGramFib (Fv F v) σ J 1 (coordW (Fv F v) b lam 1 M) (coordHerm (Fv F v) σ b lam 1) m → 1 ≤ M → ∀ (a : ℕ) (e : ℕ), 0 < a → (Ideal.span {(a : 𝒪 F)}) ≤ v.asIdeal ^ e → ¬ (Ideal.span {(a : 𝒪 F)}) ≤ v.asIdeal ^ (e + 1) → ∀ (s : Fv F v) (A : Set (Fin 1 → Fin M → Fv F v ⊗[F] E)), MeasurableSet A → m (hermScal (Fv F v) σ s) {X | (fun p i => (a : Fv F v ⊗[F] E) * X p i) ∈ A} = (Ideal.absNorm v.asIdeal : ℝ≥0∞) ^ (e * (2 * M - 2)) * m (hermScal (Fv F v) σ ((a : Fv F v) ^ 2 * s)) A

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.LocalUnitsContinuousSquareRoot

theorem **exists_sqrt_near_unit** : ∀ (R : Type) [CommRing R] [TopologicalSpace R], IsLocR R → ∀ x : Rˣ, ∃ (B₀ : Set R) (a : R → Rˣ), IsOpen B₀ ∧ (x : R) ∈ B₀ ∧ ContinuousOn a B₀ ∧ (∀ y, y ∉ B₀ → a y = 1) ∧ (∀ y ∈ B₀, y = ((a y : Rˣ) : R) ^ 2 * (x : R)) ∧ a (x : R) = 1

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.LocalUnitsContinuousSquareRootStatement

theorem **_root_.GlobalTheory.T.local_units_continuous_square_root** : ∀ (R : Type) [CommRing R] [TopologicalSpace R], IsLocR R → ∀ x : Rˣ, ∃ (B₀ : Set R) (a : R → Rˣ), IsOpen B₀ ∧ (x : R) ∈ B₀ ∧ ContinuousOn a B₀ ∧ (∀ y, y ∉ B₀ → a y = 1) ∧ (∀ y ∈ B₀, y = ((a y : Rˣ) : R) ^ 2 * (x : R)) ∧ a (x : R) = 1

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.MapMulEqOfNormOne

theorem **char_conj** (σ : K →+* K) (hc : Continuous σ) (hi : ∀ y, σ (σ y) = y) (u v : K) (h : u * v = 1) (h' : σ u * σ v = 1) : addEquivAddHaarChar (mulEquiv (σ u) (σ v) h') = addEquivAddHaarChar (mulEquiv u v h)

theorem **char_mul_eq_one** (u v : K) (h : u * v = 1) (h' : v * u = 1) : addEquivAddHaarChar (mulEquiv u v h) * addEquivAddHaarChar (mulEquiv v u h') = 1

theorem **map_mul_eq_of_norm_one** : ∀ (K : Type) [CommRing K] [TopologicalSpace K] [IsTopologicalRing K] [LocallyCompactSpace K] [SecondCountableTopology K] [T2Space K] [MeasurableSpace K] [BorelSpace K] (σ : K →+* K), Continuous σ → (∀ y, σ (σ y) = y) → ∀ (lam : Measure K) [lam.IsAddHaarMeasure] (u : K), u * σ u = 1 → lam.map (fun y => u * y) = lam

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.MapMulVecEqOfTransposeMulMulEq

theorem **transvection_mulVec_apply** (i j : ι) (c : R) (f : ι → R) (k : ι) : (Matrix.transvection i j c *ᵥ f) k = f k + if k = i then c * f j else 0

theorem **measurePreserving_transvection** (ν : Measure R) [ν.IsAddHaarMeasure] {i j : ι} (hij : i ≠ j) (c : R) : MeasurePreserving (fun x : ι → R => Matrix.transvection i j c *ᵥ x) (Measure.pi fun _ => ν) (Measure.pi fun _ => ν)

theorem **scale_of_isUnit** {A : Matrix ι ι R} (h : IsUnit A.det) : scale A = addEquivAddHaarChar (mulVecEquiv A h)

theorem **scale_mul** {A B : Matrix ι ι R} (hA : IsUnit A.det) (hB : IsUnit B.det) : scale (A * B) = scale A * scale B

theorem **scale_transvection** {i j : ι} (hij : i ≠ j) (c : R) : scale (Matrix.transvection i j c) = 1

theorem **transpose_transvection** (i j : ι) (c : R) : (Matrix.transvection i j c)ᵀ = Matrix.transvection j i c

theorem **scale_transpose** (hsr : ∀ a b : R, IsCoprime a b → ∃ t, IsUnit (a + t * b)) (A : Matrix ι ι R) (hA : IsUnit A.det) : scale Aᵀ = scale A

theorem **map_mulVec_eq_of_transpose_mul_mul_eq** : ∀ (R : Type) [CommRing R] [TopologicalSpace R] [IsTopologicalRing R] [T2Space R] [LocallyCompactSpace R] [SecondCountableTopology R] [MeasurableSpace R] [BorelSpace R], (∀ a b : R, IsCoprime a b → ∃ t, IsUnit (a + t * b)) → ∀ (ι : Type) [Fintype ι] [DecidableEq ι] (μ : Measure (ι → R)) [μ.IsAddHaarMeasure] (B g : Matrix ι ι R), IsUnit B.det → gᵀ * B * g = B → μ.map (fun x => g *ᵥ x) = μ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.MapMulVecEqSmul

theorem **map_mul_unit** (lam : Measure K) [lam.IsAddHaarMeasure] (u : Kˣ) : lam.map (fun y => (u : K) * y) = (distribHaarChar K u)⁻¹ • lam

theorem **pi_smul_const** {ι : Type} [Fintype ι] (lam : Measure K) [SigmaFinite lam] (c : ι → ℝ≥0) : Measure.pi (fun i => c i • lam) = (∏ i, c i) • Measure.pi (fun _ : ι => lam)

theorem **pi_map_diag** {ι : Type} [Fintype ι] (lam : Measure K) [lam.IsAddHaarMeasure] (D : ι → Kˣ) : (Measure.pi fun _ : ι => lam).map (fun x i => (D i : K) * x i) = (∏ i, (distribHaarChar K (D i))⁻¹) • Measure.pi fun _ : ι => lam

theorem **smul_cancel** (lam : Measure K) [lam.IsAddHaarMeasure] {a b : ℝ≥0} (h : a • lam = b • lam) : a = b

theorem **map_transvections** {ι : Type} [Fintype ι] [DecidableEq ι] (μ : Measure (ι → K)) [μ.IsAddHaarMeasure] (L : List (Matrix.TransvectionStruct ι K)) : μ.map (fun x => (L.map Matrix.TransvectionStruct.toMatrix).prod *ᵥ x) = μ

theorem **det_transvections** {ι : Type} [Fintype ι] [DecidableEq ι] (L : List (Matrix.TransvectionStruct ι K)) : ((L.map Matrix.TransvectionStruct.toMatrix).prod).det = 1

theorem **map_mulVec_eq_smul** : ∀ (K : Type) [CommRing K] [TopologicalSpace K] [IsTopologicalRing K] [LocallyCompactSpace K] [SecondCountableTopology K] [T2Space K] [MeasurableSpace K] [BorelSpace K] [IsArtinianRing K] [IsReduced K] (ι : Type) [Fintype ι] [DecidableEq ι] (μ : Measure (ι → K)) [μ.IsAddHaarMeasure] (lam : Measure K) [lam.IsAddHaarMeasure] (A : Matrix ι ι K), IsUnit A.det → ∀ c : ℝ≥0, lam.map (fun y => A.det * y) = c • lam → μ.map (fun x => A *ᵥ x) = c • μ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.MatrixInductionTransvectionDiagonal

lemma **transvection_mem** {i j : ι} (h : i ≠ j) (c : R) : Matrix.transvection i j c ∈ elem R ι

lemma **diagonal_mem** {D : ι → R} (h : ∀ i, IsUnit (D i)) : Matrix.diagonal D ∈ elem R ι

lemma **exists_left_inv** {A : Matrix ι ι R} (hA : A ∈ elem R ι) : ∃ B ∈ elem R ι, B * A = 1

lemma **mem_of_mul_left** {T A : Matrix ι ι R} (hT : T ∈ elem R ι) (h : T * A ∈ elem R ι) : A ∈ elem R ι

lemma **mem_of_mul_right** {T A : Matrix ι ι R} (hT : T ∈ elem R ι) (h : A * T ∈ elem R ι) : A ∈ elem R ι

lemma **isUnit_det_of_mem** {A : Matrix ι ι R} (hA : A ∈ elem R ι) : IsUnit A.det

lemma **rowOp_mem** (k : ι) (c : ι → R) (hc : c k = 0) : rowOp k c ∈ elem R ι

lemma **colOp_mem** (k : ι) (c : ι → R) (hc : c k = 0) : colOp k c ∈ elem R ι

lemma **rowOp_mul_apply** (k : ι) (c : ι → R) (A : Matrix ι ι R) (i j : ι) : (rowOp k c * A) i j = A i j + if i = k then ∑ l, c l * A l j else 0

lemma **colOp_mul_apply** (k : ι) (c : ι → R) (A : Matrix ι ι R) (i j : ι) : (colOp k c * A) i j = A i j + c i * A k j

lemma **mul_rowOp_apply** (k : ι) (c : ι → R) (A : Matrix ι ι R) (i j : ι) : (A * rowOp k c) i j = A i j + A i k * c j

lemma **mem_of_unit_pivot** {s : Finset ι} {k : ι} (ih : ∀ A : Matrix ι ι R, IsUnit A.det → Fixed s A → A ∈ elem R ι) {A : Matrix ι ι R} (hA : IsUnit A.det) (hF : Fixed (insert k s) A) (hk : k ∉ s) (hu : IsUnit (A k k)) : A ∈ elem R ι

lemma **exists_unit_pivot** (hR : ∀ a b : R, IsCoprime a b → ∃ t, IsUnit (a + t * b)) {s : Finset ι} {k : ι} {A : Matrix ι ι R} (hA : IsUnit A.det) (hF : Fixed (insert k s) A) (hk : k ∉ s) : ∃ w : ι → R, w k = 0 ∧ Fixed (insert k s) (rowOp k w * A) ∧ IsUnit ((rowOp k w * A) k k)

lemma **mem_of_fixed** (hR : ∀ a b : R, IsCoprime a b → ∃ t, IsUnit (a + t * b)) (s : Finset ι) : ∀ A : Matrix ι ι R, IsUnit A.det → Fixed s A → A ∈ elem R ι

theorem **matrix_induction** : ∀ (R : Type) [CommRing R], (∀ a b : R, IsCoprime a b → ∃ t, IsUnit (a + t * b)) → ∀ (ι : Type) [Fintype ι] [DecidableEq ι] (p : Matrix ι ι R → Prop), (∀ (i j : ι) (c : R), i ≠ j → p (Matrix.transvection i j c)) → (∀ D : ι → R, (∀ i, IsUnit (D i)) → p (Matrix.diagonal D)) → (∀ A B, IsUnit A.det → IsUnit B.det → p A → p B → p (A * B)) → ∀ A : Matrix ι ι R, IsUnit A.det → p A

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.MatrixInductionTransvectionDiagonalStatement

theorem **_root_.GlobalTheory.T.matrix_induction_transvection_diagonal** : ∀ (R : Type) [CommRing R], (∀ a b : R, IsCoprime a b → ∃ t, IsUnit (a + t * b)) → ∀ (ι : Type) [Fintype ι] [DecidableEq ι] (p : Matrix ι ι R → Prop), (∀ (i j : ι) (c : R), i ≠ j → p (Matrix.transvection i j c)) → (∀ D : ι → R, (∀ i, IsUnit (D i)) → p (Matrix.diagonal D)) → (∀ A B, IsUnit A.det → IsUnit B.det → p A → p B → p (A * B)) → ∀ A : Matrix ι ι R, IsUnit A.det → p A

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.MatrixTransvectionDiagonalDecomposition

lemma **coord_symm_single** (e : K ≃+* ∀ m, F m) (m m' : M) (c : F m) : coord e m' (e.symm (Pi.single m c)) = (Pi.single m c : ∀ m, F m) m'

lemma **mapMatrix_single** (f : K →+* K') (a b : ι) (c : K) : f.mapMatrix (single a b c) = single a b (f c)

lemma **liftT_toMatrix** (e : K ≃+* ∀ m, F m) (m : M) (t : TransvectionStruct ι (F m)) : (liftT e m t).toMatrix = 1 + single t.i t.j (e.symm (Pi.single m t.c))

lemma **mapMatrix_liftT_same** (e : K ≃+* ∀ m, F m) (m : M) (t : TransvectionStruct ι (F m)) : (coord e m).mapMatrix (liftT e m t).toMatrix = t.toMatrix

lemma **mapMatrix_liftT_ne** (e : K ≃+* ∀ m, F m) {m m' : M} (h : m' ≠ m) (t : TransvectionStruct ι (F m)) : (coord e m').mapMatrix (liftT e m t).toMatrix = 1

lemma **mapMatrix_prod_same** (e : K ≃+* ∀ m, F m) (m : M) (T : List (TransvectionStruct ι (F m))) : (coord e m).mapMatrix (((T.map (liftT e m)).map TransvectionStruct.toMatrix).prod) = (T.map TransvectionStruct.toMatrix).prod

lemma **mapMatrix_prod_ne** (e : K ≃+* ∀ m, F m) {m m' : M} (h : m' ≠ m) (T : List (TransvectionStruct ι (F m))) : (coord e m').mapMatrix (((T.map (liftT e m)).map TransvectionStruct.toMatrix).prod) = 1

lemma **mapMatrix_big** (e : K ≃+* ∀ m, F m) (T : ∀ m, List (TransvectionStruct ι (F m))) (m' : M) (l : List M) (hl : l.Nodup) : (coord e m').mapMatrix (((big e T l).map TransvectionStruct.toMatrix).prod) = if m' ∈ l then ((T m').map TransvectionStruct.toMatrix).prod else 1

lemma **matrix_ext_coord** (e : K ≃+* ∀ m, F m) {A B : Matrix ι ι K} (h : ∀ m, (coord e m).mapMatrix A = (coord e m).mapMatrix B) : A = B

theorem **factor_of_pi** [Fintype M] (e : K ≃+* ∀ m, F m) (hF : ∀ (m : M) (A : Matrix ι ι (F m)), ∃ (L L' : List (TransvectionStruct ι (F m))) (D : ι → F m), A = (L.map TransvectionStruct.toMatrix).prod * diagonal D * (L'.map TransvectionStruct.toMatrix).prod) (A : Matrix ι ι K) : ∃ (L L' : List (TransvectionStruct ι K)) (D : ι → K), A = (L.map TransvectionStruct.toMatrix).prod * diagonal D * (L'.map TransvectionStruct.toMatrix).prod

theorem **artinian_reduced_list_transvec_mul_diagonal_mul_list_transvec** (K : Type) [CommRing K] [IsArtinianRing K] [IsReduced K] (ι : Type) [Fintype ι] [DecidableEq ι] (A : Matrix ι ι K) : ∃ (L L' : List (Matrix.TransvectionStruct ι K)) (D : ι → K), A = (L.map Matrix.TransvectionStruct.toMatrix).prod * Matrix.diagonal D * (L'.map Matrix.TransvectionStruct.toMatrix).prod

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.MatrixTransvectionDiagonalDecompositionStatement

theorem **_root_.GlobalTheory.T.matrix_transvection_diagonal_decomposition** : ∀ (K : Type) [CommRing K] [IsArtinianRing K] [IsReduced K] (ι : Type) [Fintype ι] [DecidableEq ι] (A : Matrix ι ι K), ∃ (L L' : List (Matrix.TransvectionStruct ι K)) (D : ι → K), A = (L.map Matrix.TransvectionStruct.toMatrix).prod * Matrix.diagonal D * (L'.map Matrix.TransvectionStruct.toMatrix).prod

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.MeasurableCoordinateMapsTensor

theorem **map_curry_pi_pi** {R : Type} [MeasurableSpace R] (lam : Measure R) [SigmaFinite lam] (M d : ℕ) : (Measure.pi fun _ : Fin M => Measure.pi fun _ : Fin d => lam).map (fun (Y : Fin M → Fin d → R) (p : Fin M × Fin d) => Y p.1 p.2) = Measure.pi fun _ : Fin M × Fin d => lam

theorem **coordW_one_map_coords** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [IsTopologicalRing R] [LocallyCompactSpace R] [SecondCountableTopology R] [T2Space R] [MeasurableSpace R] [BorelSpace R], ∀ (d : ℕ) (b : Module.Basis (Fin d) F E), ∀ (M : ℕ) (lam : Measure R), lam.IsAddHaarMeasure → (Measurable fun (X : Fin 1 → Fin M → R ⊗[F] E) (p : Fin M × Fin d) => (Algebra.TensorProduct.basis R b).repr (X 0 p.1) p.2) ∧ (coordW R b lam 1 M).map (fun X (p : Fin M × Fin d) => (Algebra.TensorProduct.basis R b).repr (X 0 p.1) p.2) = Measure.pi fun _ => lam

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.MeasurableCoordinateMapsTensorStatement

theorem **_root_.GlobalTheory.T.measurable_coordinate_maps_tensor** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [IsTopologicalRing R] [LocallyCompactSpace R] [SecondCountableTopology R] [T2Space R] [MeasurableSpace R] [BorelSpace R], ∀ (d : ℕ) (b : Module.Basis (Fin d) F E), ∀ (M : ℕ) (lam : Measure R), lam.IsAddHaarMeasure → (Measurable fun (X : Fin 1 → Fin M → R ⊗[F] E) (p : Fin M × Fin d) => (Algebra.TensorProduct.basis R b).repr (X 0 p.1) p.2) ∧ (coordW R b lam 1 M).map (fun X (p : Fin M × Fin d) => (Algebra.TensorProduct.basis R b).repr (X 0 p.1) p.2) = Measure.pi fun _ => lam

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.MeasurableFiniteLevelLatticeSet

theorem **measurableSet_natMul_int** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (d M : ℕ) (b : Module.Basis (Fin d) F E) (N : ℕ), MeasurableSet {X : Fin 1 → Fin M → AF F ⊗[F] E | ∀ v, projV v (projFin X) ∈ {X : (Fin 1 → Fin M → Fv F v ⊗[F] E) | (fun p i => (N : Fv F v ⊗[F] E) * X p i) ∈ latV b v}}

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.MeasurableFiniteLevelLatticeSetStatement

theorem **_root_.GlobalTheory.T.measurable_finite_level_lattice_set** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (d M : ℕ) (b : Module.Basis (Fin d) F E) (N : ℕ), MeasurableSet {X : Fin 1 → Fin M → AF F ⊗[F] E | ∀ v, projV v (projFin X) ∈ {X : (Fin 1 → Fin M → Fv F v ⊗[F] E) | (fun p i => (N : Fv F v ⊗[F] E) * X p i) ∈ latV b v}}

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.MeasureEqHaarProdOfLeftInvariant

theorem **exists_const_rect** {Q Z : Type} [Group Q] [TopologicalSpace Q] [LocallyCompactSpace Q] [SecondCountableTopology Q] [T2Space Q] [MeasurableSpace Q] [BorelSpace Q] [IsTopologicalGroup Q] [TopologicalSpace Z] [T2Space Z] [MeasurableSpace Z] [BorelSpace Z] (η : Measure Q) [η.IsHaarMeasure] (ρ : Measure (Q × Z)) [IsLocallyFiniteMeasure ρ] (hinv : ∀ q : Q, ρ.map (fun p => (q * p.1, p.2)) = ρ) {A C : Set Z} (hA : MeasurableSet A) (hC : IsCompact C) (hAC : A ⊆ C) : ∃ c : ℝ≥0, ∀ U : Set Q, MeasurableSet U → ρ (U ×ˢ A) = c * η U

theorem **measure_eq_haar_prod_of_left_invariant** : ∀ (Q Z : Type) [Group Q] [TopologicalSpace Q] [LocallyCompactSpace Q] [SecondCountableTopology Q] [T2Space Q] [MeasurableSpace Q] [BorelSpace Q] [IsTopologicalGroup Q] [TopologicalSpace Z] [LocallyCompactSpace Z] [SecondCountableTopology Z] [T2Space Z] [MeasurableSpace Z] [BorelSpace Z] (η : Measure Q) [η.IsHaarMeasure] (ρ : Measure (Q × Z)) [IsLocallyFiniteMeasure ρ], (∀ q : Q, ρ.map (fun p => (q * p.1, p.2)) = ρ) → ∃ m : Measure Z, IsLocallyFiniteMeasure m ∧ ρ = η.prod m

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.MobiusActionUpperHalfPlane

theorem **den_ne** (a b c d : ℝ) (hdet : a * d - b * c = 1) (τ : ℂ) (h : 0 < τ.im) : (c : ℂ) * τ + d ≠ 0

theorem **im_frac** (a b c d : ℝ) (hdet : a * d - b * c = 1) (τ : ℂ) (h : 0 < τ.im) : (((a : ℂ) * τ + b) / ((c : ℂ) * τ + d)).im = τ.im / ‖(c : ℂ) * τ + d‖ ^ 2

theorem **comp_frac** (a b c d a' b' c' d' : ℝ) (hdet : a * d - b * c = 1) (hdet' : a' * d' - b' * c' = 1) (τ : ℂ) (h : 0 < τ.im) : (((a * a' + b * c' : ℝ) : ℂ) * τ + ((a * b' + b * d' : ℝ) : ℂ)) / (((c * a' + d * c' : ℝ) : ℂ) * τ + ((c * b' + d * d' : ℝ) : ℂ)) = ((a : ℂ) * (((a' : ℂ) * τ + b') / ((c' : ℂ) * τ + d')) + b) / ((c : ℂ) * (((a' : ℂ) * τ + b') / ((c' : ℂ) * τ + d')) + d)

theorem **det_emb** (σ : F →+* ℝ) (γ : SL(2, F)) : σ (γ 0 0) * σ (γ 1 1) - σ (γ 0 1) * σ (γ 1 0) = 1

theorem **main** : ∀ (F : Type) [Field F] [NumberField F], ∀ (ρ : InfinitePlace F → (F →+* ℝ)) (τ : InfinitePlace F → ℂ), (∀ w, 0 < (τ w).im) → (∀ (γ : SL(2, F)) w, (mob ρ γ τ w).im = (τ w).im / ‖((ρ w (γ 1 0) : ℝ) : ℂ) * τ w + (ρ w (γ 1 1) : ℝ)‖ ^ 2 ∧ 0 < (mob ρ γ τ w).im) ∧ (∀ γ γ', SL(2, F), mob ρ (γ * γ') τ = mob ρ γ (mob ρ γ' τ)) ∧ (∀ b : F, mob ρ (nMat b) τ = fun w => τ w + (ρ w b : ℝ)) ∧ (∀ a : Fˣ, mob ρ (mMat a) τ = fun w => ((ρ w (a : F) : ℝ) : ℂ) ^ 2 * τ w) ∧ mob ρ wMat τ = fun w => -(τ w)⁻¹

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.MobiusActionUpperHalfPlaneStatement

theorem **_root_.GlobalTheory.T.mobius_action_upper_half_plane** : ∀ (F : Type) [Field F] [NumberField F], ∀ (ρ : InfinitePlace F → (F →+* ℝ)) (τ : InfinitePlace F → ℂ), (∀ w, 0 < (τ w).im) → (∀ (γ : SL(2, F)) w, (mob ρ γ τ w).im = (τ w).im / ‖((ρ w (γ 1 0) : ℝ) : ℂ) * τ w + (ρ w (γ 1 1) : ℝ)‖ ^ 2 ∧ 0 < (mob ρ γ τ w).im) ∧ (∀ γ γ' : SL(2, F), mob ρ (γ * γ') τ = mob ρ γ (mob ρ γ' τ)) ∧ (∀ b : F, mob ρ (nMat b) τ = fun w => τ w + (ρ w b : ℝ)) ∧ (∀ a : Fˣ, mob ρ (mMat a) τ = fun w => ((ρ w (a : F) : ℝ) : ℂ) ^ 2 * τ w) ∧ mob ρ wMat τ = fun w => -(τ w)⁻¹

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ModulusCharacterUnitActionProduct

theorem **continuous_norm_tensor** {F E : Type} [Field F] [Field E] [Algebra F E] [FiniteDimensional F E] (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [IsTopologicalRing R] [IsTopologicalRing (R ⊗[F] E)] : Continuous (Algebra.norm R : R ⊗[F] E → R)

theorem **norm_tmul_one** {F E : Type} [Field F] [Field E] [Algebra F E] (h2 : Module.finrank F E = 2) (R : Type) [CommRing R] [Nontrivial R] [Algebra F R] (r : R) : Algebra.norm R (r ⊗ₜ[F] (1 : E)) = r ^ 2

theorem **map_scale_adele** {F : Type} [Field F] [NumberField F] (hF : IsTotallyReal F) (θ : ℝ →+* Ainf F) (hθ : Continuous θ) (μA : Measure (AF F)) (hA : μA.IsAddHaarMeasure) (t : ℝ) (ht : 0 < t) : μA.map (fun y => pairA F (θ t) 1 * y) = ENNReal.ofReal ((t ^ Module.finrank ℚ F)⁻¹) • μA

theorem **exists_modulus_chi** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ τ : Multiplicative ℝ →* (AF F ⊗[F] E)ˣ, IsFlow τ → ∀ (M : ℕ) (μW : Measure (Fin 1 → Fin M → AF F ⊗[F] E)) (μA : Measure (AF F)), μW.IsAddHaarMeasure → μA.IsAddHaarMeasure → ∃ χ : (AF F ⊗[F] E)ˣ →* ℝ≥0, Continuous χ ∧ (∀ a : (AF F ⊗[F] E)ˣ, (μW.prod μA).map (fun p => (sigR (AF F) σ (a : AF F ⊗[F] E) • p.1, Algebra.norm (AF F) (a : AF F ⊗[F] E) • p.2)) = χ a • μW.prod μA) ∧ ∀ x : ℝ, (χ (τ (Multiplicative.ofAdd x)) : ℝ) = Real.exp (-(2 * Module.finrank ℚ F * (M + 1) * x))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ModulusCharacterUnitActionProductStatement

theorem **_root_.GlobalTheory.T.modulus_character_unit_action_product** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ τ : Multiplicative ℝ →* (AF F ⊗[F] E)ˣ, IsFlow τ → ∀ (M : ℕ) (μW : Measure (Fin 1 → Fin M → AF F ⊗[F] E)) (μA : Measure (AF F)), μW.IsAddHaarMeasure → μA.IsAddHaarMeasure → ∃ χ : (AF F ⊗[F] E)ˣ →* ℝ≥0, Continuous χ ∧ (∀ a : (AF F ⊗[F] E)ˣ, (μW.prod μA).map (fun p => (sigR (AF F) σ (a : AF F ⊗[F] E) • p.1, Algebra.norm (AF F) (a : AF F ⊗[F] E) • p.2)) = χ a • μW.prod μA) ∧ ∀ x : ℝ, (χ (τ (Multiplicative.ofAdd x)) : ℝ) = Real.exp (-(2 * Module.finrank ℚ F * (M + 1) * x))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.NonarchimedeanSelfdualMeasurePi

theorem **mem_ball** (m : ℤ) (x : Fv F v) : x ∈ ball v m ↔ Valued.v x ≤ WithZero.exp m

theorem **ball_isCompact** (m : ℤ) : IsCompact (ball v m : Set (Fv F v))

theorem **ball_isOpen** (m : ℤ) : IsOpen (ball v m : Set (Fv F v))

theorem **ball_le** {a b : ℤ} (hab : a ≤ b) : (ball v a : Set (Fv F v)) ⊆ ball v b

theorem **exists_mem_ball** (x : Fv F v) : ∃ m, x ∈ ball v m

theorem **exists_ball_of_isCompact** (K : Set (Fv F v)) (hK : IsCompact K) : ∃ m, K ⊆ ball v m

theorem **exists_ball_subset** (U : Set (Fv F v)) (hU : U ∈ 𝓝 (0 : Fv F v)) : ∃ m, (ball v m : Set (Fv F v)) ⊆ U

theorem **psiV_add** (ψ : AddChar (AF F) ℂ) (v : HeightOneSpectrum (𝒪 F)) (x y : Fv F v) : psiV F ψ v (x + y) = psiV F ψ v x * psiV F ψ v y

theorem **ann_isCompact** {χ : Fv F v → ℂ} (hc : Continuous χ) (h0 : χ 0 = 1) {y : Fv F v} (hy : χ y ≠ 1) (L : AddSubgroup (Fv F v)) (hLo : IsOpen (L : Set (Fv F v))) : IsCompact {t | ∀ s ∈ L, χ (s * t) = 1}

theorem **selfDual_pi_v** : ∀ (F : Type) [Field F] [NumberField F] (ψ : AddChar (AF F) ℂ), IsCharAF F ψ → ∀ (v : HeightOneSpectrum (𝒪 F)) (k : Measure (Fv F v)), SelfDual k (fun s t => psiV F ψ v (s * t)) → ∀ (ι : Type) [Fintype ι], SelfDual (Measure.pi fun _ : ι => k) (fun x y => psiV F ψ v (x ⬝ᵥ y))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.NonarchimedeanSelfdualMeasurePiStatement

theorem **_root_.GlobalTheory.T.nonarchimedean_selfdual_measure_pi** : ∀ (F : Type) [Field F] [NumberField F] (ψ : AddChar (AF F) ℂ), IsCharAF F ψ → ∀ (v : HeightOneSpectrum (𝒪 F)) (k : Measure (Fv F v)), SelfDual k (fun s t => psiV F ψ v (s * t)) → ∀ (ι : Type) [Fintype ι], SelfDual (Measure.pi fun _ : ι => k) (fun x y => psiV F ψ v (x ⬝ᵥ y))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.NonarchimedeanSelfdualSymmetricFormUnique

theorem **not_selfDual_one** (V : Type) [AddCommGroup V] [TopologicalSpace V] [IsTopologicalAddGroup V] [T2Space V] [LocallyCompactSpace V] [MeasurableSpace V] [BorelSpace V] [Nontrivial V] (k : Measure V) : ¬ SelfDual k (fun _ _ => (1 : ℂ))

theorem **chiV_apply** (ψ : AddChar (AF F) ℂ) (v : HeightOneSpectrum (𝒪 F)) (t : Fv F v) : chiV F ψ v t = psiV F ψ v t

theorem **selfDual_unique_coord** : ∀ (F : Type) [Field F] [NumberField F] (ψ : AddChar (AF F) ℂ), IsCharAF F ψ → ∀ (v : HeightOneSpectrum (𝒪 F)) (k : Measure (Fv F v)), SelfDual k (fun s t => psiV F ψ v (s * t)) → ∀ (ι : Type) [Fintype ι] [DecidableEq ι] (A : Matrix ι ι F), A.IsSymm → A.det ≠ 0 → ∀ μ ν : Measure (ι → Fv F v), SelfDual μ (fun x y => psiV F ψ v (x ⬝ᵥ (A.map (algebraMap F (Fv F v)) *ᵥ y))) → SelfDual ν (fun x y => psiV F ψ v (x ⬝ᵥ (A.map (algebraMap F (Fv F v)) *ᵥ y))) → μ = ν

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.NonarchimedeanSelfdualSymmetricFormUniqueStatement

theorem **_root_.GlobalTheory.T.nonarchimedean_selfdual_symmetric_form_unique** : ∀ (F : Type) [Field F] [NumberField F] (ψ : AddChar (AF F) ℂ), IsCharAF F ψ → ∀ (v : HeightOneSpectrum (𝒪 F)) (k : Measure (Fv F v)), SelfDual k (fun s t => psiV F ψ v (s * t)) → ∀ (ι : Type) [Fintype ι] [DecidableEq ι] (A : Matrix ι ι F), A.IsSymm → A.det ≠ 0 → ∀ μ ν : Measure (ι → Fv F v), SelfDual μ (fun x y => psiV F ψ v (x ⬝ᵥ (A.map (algebraMap F (Fv F v)) *ᵥ y))) → SelfDual ν (fun x y => psiV F ψ v (x ⬝ᵥ (A.map (algebraMap F (Fv F v)) *ᵥ y))) → μ = ν

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.PoissonSummationGaussian

theorem **quad_mulVec** (P : Matrix κ κ ℝ) (W : Matrix κ κ ℂ) (v : κ → ℝ) : (fun i => (((P *ᵥ v) i : ℝ) : ℂ)) ⬝ᵥ (W *ᵥ fun i => (((P *ᵥ v) i : ℝ) : ℂ)) = (fun i => (v i : ℂ)) ⬝ᵥ (((P.map Complex.ofReal)ᵀ * W * P.map Complex.ofReal) *ᵥ fun i => (v i : ℂ))

theorem **contragredient_congr** {g : Matrix κ κ ℝ} (hg : g.det ≠ 0) (Z : Matrix κ κ ℂ) : (((g⁻¹)ᵀ).map Complex.ofReal)ᵀ * Z⁻¹ * ((g⁻¹)ᵀ).map Complex.ofReal = ((g.map Complex.ofReal)ᵀ * Z * g.map Complex.ofReal)⁻¹

theorem **gaussian_lattice_term** (Z : Matrix κ κ ℂ) {g : Matrix κ κ ℝ} (hg : g.det ≠ 0) (a : κ → ℝ) (n : κ → ℤ) : Complex.exp ((Real.pi : ℂ) * Complex.I * ((fun i => (((a + g *ᵥ fun i => (n i : ℝ)) i : ℝ) : ℂ)) ⬝ᵥ (Z *ᵥ fun i => (((a + g *ᵥ fun i => (n i : ℝ)) i : ℝ) : ℂ)))) = GlobalTheory.riemannThetaTerm (g⁻¹ *ᵥ a) 0 ((g.map Complex.ofReal)ᵀ * Z * g.map Complex.ofReal) 0 n

theorem **gaussian_fourier** {Z : Matrix κ κ ℂ} (hZ : GlobalTheory.IsSiegel Z) (y : κ → ℝ) : ∫ x : κ → ℝ, Complex.exp ((Real.pi : ℂ) * Complex.I * ((fun i => (x i : ℂ)) ⬝ᵥ (Z *ᵥ fun i => (x i : ℂ)))) * Complex.exp (2 * (Real.pi : ℂ) * Complex.I * ((x ⬝ᵥ y : ℝ) : ℂ)) = GlobalTheory.siegelGaussIntegral Z * Complex.exp (-((Real.pi : ℂ) * Complex.I) * ((fun i => (y i : ℂ)) ⬝ᵥ (Z⁻¹ *ᵥ fun i => (y i : ℂ))))

theorem **siegelGaussIntegral_congr** (Z : Matrix κ κ ℂ) {g : Matrix κ κ ℝ} (hg : g.det ≠ 0) : GlobalTheory.siegelGaussIntegral ((g.map Complex.ofReal)ᵀ * Z * g.map Complex.ofReal) = ((|g.det|⁻¹ : ℝ) : ℂ) * GlobalTheory.siegelGaussIntegral Z

theorem **fourier_lattice_term** {Z : Matrix κ κ ℂ} (hZ : GlobalTheory.IsSiegel Z) {g : Matrix κ κ ℝ} (hg : g.det ≠ 0) (n : κ → ℤ) : Complex.exp (-((Real.pi : ℂ) * Complex.I) * ((fun i => ((((g⁻¹)ᵀ *ᵥ fun i => (n i : ℝ)) i : ℝ) : ℂ)) ⬝ᵥ (Z⁻¹ *ᵥ fun i => ((((g⁻¹)ᵀ *ᵥ fun i => (n i : ℝ)) i : ℝ) : ℂ)))) = GlobalTheory.riemannThetaTerm 0 0 (-((g.map Complex.ofReal)ᵀ * Z * g.map Complex.ofReal)⁻¹) 0 n

theorem **phase_lattice** {g : Matrix κ κ ℝ} (a : κ → ℝ) (n : κ → ℤ) : ((a ⬝ᵥ ((g⁻¹)ᵀ *ᵥ fun i => (n i : ℝ)) : ℝ) : ℂ) = (fun i => (n i : ℂ)) ⬝ᵥ fun i => ((g⁻¹ *ᵥ a) i : ℂ)

theorem **poisson_shifted_lattice** : ∀ (κ : Type) [Fintype κ] [DecidableEq κ] (Z : Matrix κ κ ℂ), Z.IsSymm → (Z.map Complex.im).PosDef → ∀ (g : Matrix κ κ ℝ), g.det ≠ 0 → ∀ a : κ → ℝ, let f : (κ → ℝ) → ℂ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.PoissonSummationGaussianStatement

theorem **_root_.GlobalTheory.T.poisson_summation_gaussian** : ∀ (κ : Type) [Fintype κ] [DecidableEq κ] (Z : Matrix κ κ ℂ), Z.IsSymm → (Z.map Complex.im).PosDef → ∀ (g : Matrix κ κ ℝ), g.det ≠ 0 → ∀ a : κ → ℝ, let f : (κ → ℝ) → ℂ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.PositiveDefiniteDensity

theorem **chi_neg_left** {χ : V → V → ℂ} (hs : ∀ x y, χ x y = χ y x) (hn : ∀ x y, ‖χ x y‖ = 1) (ha : ∀ x x' y, χ (x + x') y = χ x y * χ x' y) (x y : V) : χ (-x) y = χ x (-y)

theorem **dens_continuous** [SecondCountableTopology V] [BorelSpace V] {χ : V → V → ℂ} {μ m : Measure V} (hc : Continuous (fun p : V × V => χ p.1 p.2)) (hn : ∀ x y, ‖χ x y‖ = 1) (hint : Integrable (mhat χ m) μ) : Continuous (dens χ μ m)

theorem **coset_isOpen** {L : AddSubgroup V} (hL : IsOpen (L : Set V)) (a : V) : IsOpen (coset L a)

theorem **coset_eq_image** (L : AddSubgroup V) (a : V) : coset L a = (fun x => x + a) '' (L : Set V)

theorem **coset_isCompact** {L : AddSubgroup V} (hL : IsCompact (L : Set V)) (a : V) : IsCompact (coset L a)

theorem **coset_sb** [T2Space V] {L : AddSubgroup V} (hK : IsCompact (L : Set V)) (hO : IsOpen (L : Set V)) (a : V) : SB (Set.indicator (coset L a) (fun _ => (1 : ℂ)))

theorem **integral_mul_dens** {χ : V → V → ℂ} {μ m : Measure V} [μ.IsAddHaarMeasure] [IsFiniteMeasure m] (h : NADual V χ μ) (hc : Continuous (fun p : V × V => χ p.1 p.2)) (hs : ∀ x y, χ x y = χ y x) (hn : ∀ x y, ‖χ x y‖ = 1) (ha : ∀ x x' y, χ (x + x') y = χ x y * χ x' y) (hint : Integrable (mhat χ m) μ) {φ : V → ℂ} (hφ : SB φ) : ∫ x, φ x * dens χ μ m x ∂μ = ∫ z, φ z ∂m

theorem **setIntegral_dens** {χ : V → V → ℂ} {μ m : Measure V} [μ.IsAddHaarMeasure] [IsFiniteMeasure m] (h : NADual V χ μ) (hc : Continuous (fun p : V × V => χ p.1 p.2)) (hs : ∀ x y, χ x y = χ y x) (hn : ∀ x y, ‖χ x y‖ = 1) (ha : ∀ x x' y, χ (x + x') y = χ x y * χ x' y) (hint : Integrable (mhat χ m) μ) {L : AddSubgroup V} (hK : IsCompact (L : Set V)) (hO : IsOpen (L : Set V)) (a : V) : ∫ x in coset L a, dens χ μ m x ∂μ = ((m (coset L a)).toReal : ℂ)

theorem **nonneg_of_cosets** {μ : Measure V} [μ.IsAddHaarMeasure] {f : V → ℝ} (hf : Continuous f) (hb : ∀ U ∈ 𝓝 (0 : V), ∃ L : AddSubgroup V, IsCompact (L : Set V) ∧ IsOpen (L : Set V) ∧ (L : Set V) ⊆ U) (hi : ∀ (L : AddSubgroup V) (a : V), IsCompact (L : Set V) → IsOpen (L : Set V) → 0 ≤ ∫ x in coset L a, f x ∂μ) (x : V) : 0 ≤ f x

theorem **eq_of_cosets** {m m' : Measure V} [IsFiniteMeasure m] (hb : ∀ U ∈ 𝓝 (0 : V), ∃ L : AddSubgroup V, IsCompact (L : Set V) ∧ IsOpen (L : Set V) ∧ (L : Set V) ⊆ U) (hcov : ∀ x : V, ∃ L : AddSubgroup V, IsCompact (L : Set V) ∧ IsOpen (L : Set V) ∧ x ∈ L) (heq : ∀ (L : AddSubgroup V) (a : V), IsCompact (L : Set V) → IsOpen (L : Set V) → m (coset L a) = m' (coset L a)) : m = m'

theorem **exists_density** : ∀ (V : Type) [AddCommGroup V] [TopologicalSpace V] [MeasurableSpace V] (χ : V → V → ℂ) (μ : Measure V), NADual V χ μ → ∀ (m : Measure V), IsFiniteMeasure m → Integrable (fun y => ∫ x, χ x y ∂m) μ → ∃ D : V → ℝ, Continuous D ∧ (∀ x, 0 ≤ D x) ∧ (∀ x, (D x : ℂ) = ∫ y, (∫ z, χ z y ∂m) * χ (-x) y ∂μ) ∧ ∀ U : Set

V, MeasurableSet U → m U = ∫⁻ x in U, ENNReal.ofReal (D x) ∂μ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.PositiveDefiniteDensityStatement

theorem **_root_.GlobalTheory.T.positive_definite_density** : ∀ (V : Type) [AddCommGroup V] [TopologicalSpace V] [MeasurableSpace V] (χ : V → V → ℂ) (μ : Measure V), NADual V χ μ → ∀ (m : Measure V), IsFiniteMeasure m → Integrable (fun y => ∫ x, χ x y ∂m) μ → ∃ D : V → ℝ, Continuous D ∧ (∀ x, 0 ≤ D x) ∧ (∀ x, (D x : ℂ) = ∫ y, (∫ z, χ z y ∂m) * χ (-x) y ∂μ) ∧ ∀ U : Set V, MeasurableSet U → m U = ∫⁻ x in U, ENNReal.ofReal (D x) ∂μ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.PositiveMajorantGaussian

theorem **core_estimate** : ∀ (F : Type) [Field F] [NumberField F], IsTotallyReal F → ∀ (ι : Type) [Fintype ι] [DecidableEq ι] (M : ℕ), Fintype.card ι = 2 * M → 3 ≤ M → ∀ (B : Matrix ι ι F), B.IsSymm → B.det ≠ 0 → ∀ (ρ : InfinitePlace F → (F →+* ℝ)), (∀ w x, ((ρ w x : ℝ) : ℂ) = w.embedding x) → ∀ (Pm : InfinitePlace F → Matrix ι ι ℝ), (∀ w, (Pm w).PosDef) → (∀ w, Pm w * (B.map (ρ w))⁻¹ * Pm w = B.map (ρ w)) → ∀ [MeasurableSpace (FiniteAdeleRing (𝒪 F) F)] [BorelSpace (FiniteAdeleRing (𝒪 F) F)] (lf : Measure (FiniteAdeleRing (𝒪 F) F)), lf.IsAddHaarMeasure → lf {x | ∀ v, x v ∈ v.adicCompletionIntegers F} = 1 → ∀ (Ω : Type) [MeasurableSpace Ω] (Pr : Measure Ω) [IsProbabilityMeasure Pr] (ginf : Ω → InfinitePlace F → Matrix ι ι ℝ) (gf : Ω → Matrix ι ι (FiniteAdeleRing (𝒪 F) F)), (∀ w i j, Measurable fun ω => ginf ω w i j) → (∀ i j, Measurable fun ω => gf ω i j) → (∀ ω w, (ginf ω w)ᵀ * B.map (ρ w)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.PositiveMajorantGaussianStatement

theorem **_root_.GlobalTheory.T.positive_majorant_gaussian** : ∀ (F : Type) [Field F] [NumberField F], IsTotallyReal F → ∀ (ι : Type) [Fintype ι] [DecidableEq ι] (M : ℕ), Fintype.card ι = 2 * M → 3 ≤ M → ∀ (B : Matrix ι ι F), B.IsSymm → B.det ≠ 0 → ∀ (ρ : InfinitePlace F → (F →+* ℝ)), (∀ w x, ((ρ w x : ℝ) : ℂ) = w.embedding x) → ∀ (Pm : InfinitePlace F → Matrix ι ι ℝ), (∀ w, (Pm w).PosDef) → (∀ w, Pm w * (B.map (ρ w))⁻¹ * Pm w = B.map (ρ w)) → ∀ [MeasurableSpace (FiniteAdeleRing (𝒪 F) F)] [BorelSpace (FiniteAdeleRing (𝒪 F) F)] (lf : Measure (FiniteAdeleRing (𝒪 F) F)), lf.IsAddHaarMeasure → lf {x | ∀ v, x v ∈ v.adicCompletionIntegers F} = 1 → ∀ (Ω : Type) [MeasurableSpace Ω] (Pr : Measure Ω) [IsProbabilityMeasure Pr] (ginf : Ω → InfinitePlace F → Matrix ι ι ℝ) (gf : Ω → Matrix ι ι (FiniteAdeleRing (𝒪 F) F)), (∀ w i j, Measurable fun ω => ginf ω w i j) → (∀ i j, Measurable fun ω => gf ω i j) →

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.PrimitiveVectorLatticeRescaleAlmostAll

theorem **val_norm** (hsq : ∀ x : L, w (x - 1) < 1 → IsSquare x) {Δ : L} (hΔ : w Δ = 1) (hn : ¬ IsSquare Δ) (a c : L) : w (a ^ 2 - Δ * c ^ 2) = max (w a) (w c) ^ 2

theorem **repr_coord** (s t : L) : B.repr (s • B 0 + t • B 1) 0 = s ∧ B.repr (s • B 0 + t • B 1) 1 = t

theorem **coord_eq** (h0 : B 0 = 1) (x : K) : x = algebraMap L K (B.repr x 0) + algebraMap L K (B.repr x 1) * B 1

theorem **repr_one** (h0 : B 0 = 1) : B.repr 1 0 = 1 ∧ B.repr 1 1 = 0

theorem **repr_mul** (h0 : B 0 = 1) (h1 : B 1 * B 1 = algebraMap L K Δ) (x y : K) : B.repr (x * y) 0 = B.repr x 0 * B.repr y 0 + Δ * (B.repr x 1 * B.repr y 1) ∧ B.repr (x * y) 1 = B.repr x 0 * B.repr y 1 + B.repr x 1 * B.repr y 0

theorem **nrm_mul** (h0 : B 0 = 1) (h1 : B 1 * B 1 = algebraMap L K Δ) (x y : K) : nrm B Δ (x * y) = nrm B Δ x * nrm B Δ y

theorem **nrm_one** (h0 : B 0 = 1) : nrm B Δ (1 : K) = 1

theorem **exists_inv** (h0 : B 0 = 1) (h1 : B 1 * B 1 = algebraMap L K Δ) (x : K) (hN : nrm B Δ x ≠ 0) : ∃ x', x * x' = 1

theorem **le_one_of_sq** {a : Γ} (h : a ^ 2 ≤ 1) : a ≤ 1

theorem **inert_case** (h0 : B 0 = 1) (h1 : B 1 * B 1 = algebraMap L K Δ) (hsq : ∀ x : L, w (x - 1) < 1 → IsSquare x) (hΔ : w Δ = 1) (hn : ¬ IsSquare Δ) {M : ℕ} (y z : Fin M → K) (hz : z ⬝ᵥ y = 1) : ∃ (u : Kˣ) (z' : Fin M → K), (∀ k l, w (B.repr ((u : K) * y k) l) ≤ 1) ∧ (∀ k l, w (B.repr (z' k) l) ≤ 1) ∧ ∑ k, z' k * ((u : K) * y k) = 1

theorem **mul_ef** {e f : K} (he : e * e = e) (hf : f * f = f) (hef : e * f = 0) (α β γ δ : L) : (α • e + β • f) * (γ • e + δ • f) = (α * γ) • e + (β * δ) • f

theorem **int_ef** {e f : K} (he : ∀ l, w (B.repr e l) ≤ 1) (hf : ∀ l, w (B.repr f l) ≤ 1) {α β : L} (hα : w α ≤ 1) (hβ : w β ≤ 1) (l : Fin 2) : w (B.repr (α • e + β • f) l) ≤ 1

theorem **mul_idm** (h0 : B 0 = 1) (h1 : B 1 * B 1 = algebraMap L K Δ) (h2 : (2 : L) ≠ 0) {s : L} (hs0 : s ≠ 0) (hs : s * s = Δ) (x : K) : x * idm B s = ev B s x • idm B s

theorem **ev_idm** (h2 : (2 : L) ≠ 0) {s : L} (hs0 : s ≠ 0) : ev B s (idm B s) = 1 ∧ ev B s (idm B (-s)) = 0

theorem **idm_add** (h0 : B 0 = 1) (h2 : (2 : L) ≠ 0) {s : L} (hs : s ≠ 0) : idm B s + idm B (-s) = 1

theorem **split_case** (h0 : B 0 = 1) (h1 : B 1 * B 1 = algebraMap L K Δ) (h2 : w (2 : L) = 1) (hΔ : w Δ = 1) {s : L} (hs : s * s = Δ) {M : ℕ} (y z : Fin M → K) (hz : z ⬝ᵥ y = 1) : ∃ (u : Kˣ) (z' : Fin M → K), (∀ k l, w (B.repr ((u : K) * y k) l) ≤ 1) ∧ (∀ k l, w (B.repr (z' k) l) ≤ 1) ∧ ∑ k, z' k * ((u : K) * y k) = 1

theorem **core** (h0 : B 0 = 1) (h1 : B 1 * B 1 = algebraMap L K Δ) (hsq : ∀ x : L, w (x - 1) < 1 → IsSquare x) (h2 : w (2 : L) = 1) (hΔ : w Δ = 1) {M : ℕ} (y z : Fin M → K) (hz : z ⬝ᵥ y = 1) : ∃ (u : Kˣ) (z' : Fin M → K), (∀ k l, w (B.repr ((u : K) * y k) l) ≤ 1) ∧ (∀ k l, w (B.repr (z' k) l) ≤ 1) ∧ ∑ k, z' k * ((u : K) * y k) = 1

theorem **exists_basis** {σ : E →ₐ[F] E} (h : IsCMExt F E σ) : ∃ (b₂ : Module.Basis (Fin 2) F E) (Δ : F), Δ ≠ 0 ∧ b₂ 0 = 1 ∧ b₂ 1 * b₂ 1 = algebraMap F E Δ

theorem **change_basis** {d d' : ℕ} (b : Module.Basis (Fin d) F E) (b' : Module.Basis (Fin d') F E) : ∀ᶠ v : HeightOneSpectrum (𝒪 F) in Filter.cofinite, ∀ x : Fv F v ⊗[F] E, (∀ k, (Algebra.TensorProduct.basis (Fv F v) b).repr x k ∈ v.adicCompletionIntegers F) → ∀ k, (Algebra.TensorProduct.basis (Fv F v) b').repr x k ∈ v.adicCompletionIntegers F

theorem **exists_primitive** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (M : ℕ), ∀ᶠ v : HeightOneSpectrum (𝒪 F) in Filter.cofinite, ∀ y : Fin 1 → Fin M → Fv F v ⊗[F] E, (∃ z : Fin M → Fv F v ⊗[F] E, z ⬝ᵥ y 0 = 1) → ∃ (a : (Fv F v ⊗[F] E)ˣ) (z : Fin 1 → Fin M → Fv F v ⊗[F] E), (a : Fv F v ⊗[F] E) • y ∈ latV b v ∧ z ∈ latV b v ∧ z 0 ⬝ᵥ ((a : Fv F v ⊗[F] E) • y) 0 = 1

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.PrimitiveVectorLatticeRescaleAlmostAllStatement

theorem **_root_.GlobalTheory.T.primitive_vector_lattice_rescale_almost_all** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (M : ℕ), ∀ᶠ v : HeightOneSpectrum (𝒪 F) in Filter.cofinite, ∀ y : Fin 1 → Fin M → Fv F v ⊗[F] E, (∃ z : Fin M → Fv F v ⊗[F] E, z ⬝ᵥ y 0 = 1) → ∃ (a : (Fv F v ⊗[F] E)ˣ) (z : Fin 1 → Fin M → Fv F v ⊗[F] E), (a : Fv F v ⊗[F] E) • y ∈ latV b v ∧ z ∈ latV b v ∧ z 0 ⬝ᵥ ((a : Fv F v ⊗[F] E) • y) 0 = 1

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.PrimitiveVectorQuadraticLevelMeasure

theorem **mem_int** {x : Fv F v} : x ∈ v.adicCompletionIntegers F ↔ Valued.v x ≤ 1

theorem **v_le_of_close** {x y : Fv F v} (hy : Valued.v y ≤ 1) (h : Valued.v (x - y) < 1) : Valued.v x ≤ 1

theorem **mulVec_le** {B : Matrix ι ι (Fv F v)} (hB : ∀ i j, Valued.v (B i j) ≤ 1) {x : ι → Fv F v} (hx : ∀ i, Valued.v (x i) ≤ 1) (k : ι) : Valued.v ((B *ᵥ x) k) ≤ 1

theorem **mulVec_lt** {B : Matrix ι ι (Fv F v)} (hB : ∀ i j, Valued.v (B i j) ≤ 1) {x : ι → Fv F v} (hx : ∀ i, Valued.v (x i) < 1) (k : ι) : Valued.v ((B *ᵥ x) k) < 1

theorem **dot_close** {B : Matrix ι ι (Fv F v)} (hB : ∀ i j, Valued.v (B i j) ≤ 1) {x y : ι → Fv F v} (hy : ∀ i, Valued.v (y i) ≤ 1) (h : ∀ i, Valued.v (x i - y i) < 1) : Valued.v (x ⬝ᵥ (B *ᵥ x) - y ⬝ᵥ (B *ᵥ y)) < 1

theorem **exists_unit_coord** {B : Matrix ι ι (Fv F v)} (hB : ∀ i j, Valued.v (B i j) ≤ 1) (hdet : Valued.v B.det = 1) {x : ι → Fv F v} (hx : ∀ i, Valued.v (x i) ≤ 1) (hp : ∃ i, Valued.v (x i) = 1) : ∃ i, Valued.v ((B *ᵥ x) i) = 1

theorem **dot_add_single** {B : Matrix ι ι (Fv F v)} (hs : B.IsSymm) (x : ι → Fv F v) (i : ι) (s : Fv F v) : (x + Pi.single i s) ⬝ᵥ (B *ᵥ (x + Pi.single i s)) = x ⬝ᵥ (B *ᵥ x) + 2 * (B *ᵥ x) i * s + B i i * s ^ 2

theorem **isOpen_maxIdeal** : IsOpen {t : Fv F v | Valued.v t < 1}

theorem **measurableSet_close** (b : Fv F v) : MeasurableSet {t : Fv F v | Valued.v (t - b) < 1}

theorem **slice** (hstd : IsStdV v lam) (h2 : Valued.v (2 : Fv F v) = 1) {a c : Fv F v} (ha : Valued.v a = 1) (hc : c ∈ v.adicCompletionIntegers F) (q b : Fv F v) {A : Set (Fv F v)} (hA : MeasurableSet A) : lam {t | Valued.v (t - b) < 1 ∧ q + a * (t - b) + c * (t - b) ^ 2 ∈ A} = lam (A ∩ {s | Valued.v (s - q) < 1})

theorem **measurable_form** (B : Matrix ι ι (Fv F v)) : Measurable fun x : ι → Fv F v => (2 : Fv F v)⁻¹ * (x ⬝ᵥ (B *ᵥ x))

theorem **slice_form** (hstd : IsStdV v lam) (h2 : Valued.v (2 : Fv F v) = 1) {B : Matrix ι ι (Fv F v)} (hs : B.IsSymm) (hB : ∀ i j, Valued.v (B i j) ≤ 1) {x₀ : ι → Fv F v} (hx₀ : ∀ i, Valued.v (x₀ i) ≤ 1) {i : ι} (hi : Valued.v ((B *ᵥ x₀) i) = 1) {A : Set (Fv F v)} (hA : MeasurableSet A) {x : ι → Fv F v} (hx : ∀ j, j ≠ i → Valued.v (x j - x₀ j) < 1) : lam {t | Function.update x i t ∈ {y : ι → Fv F v | (∀ j, Valued.v (y j - x₀ j) < 1) ∧ (2 : Fv F v)⁻¹ * (y ⬝ᵥ (B *ᵥ y)) ∈ A}} = lam (A ∩ {s | Valued.v (s - (2 : Fv F v)⁻¹ * (x₀ ⬝ᵥ (B *ᵥ x₀))) < 1})

theorem **pi_eq_mul** [SigmaFinite lam] (i : ι) {S E : Set (ι → Fv F v)} (hS : MeasurableSet S) (hE : MeasurableSet E) (C : ℝ≥0∞) (hC : C ≠ ⊤) (x₀ : ι → Fv F v) (h1 : ∀ x, x ∈ E → lam {t | Function.update x i t ∈ S} = C) (h0 : ∀ x, x ∉ E → lam {t | Function.update x i t ∈ S} = 0) : (Measure.pi fun _ : ι => lam) S = C * (∫⋯∫_(Finset.univ.erase i), E.indicator 1 ∂(fun _ : ι => lam)) x₀

theorem **measure_residue_class** : ∀ (F : Type) [Field F] [NumberField F] (v : HeightOneSpectrum (𝒪 F)) (lam : Measure (Fv F v)), IsStdV v lam → Valued.v (2 : Fv F v) = 1 → ∀ (ι : Type) [Fintype ι] [DecidableEq ι] (B : Matrix ι ι (Fv F v)), B.IsSymm → (∀ i j, B i j ∈ v.adicCompletionIntegers F) → Valued.v B.det = 1 → ∀ x₀ : ι → Fv F v, (∀ i, x₀ i ∈ v.adicCompletionIntegers F) → (∃ i, Valued.v (x₀ i) = 1) → ∀ A : Set (Fv F v), MeasurableSet A → (Measure.pi fun _ : ι => lam) {x | (∀ i, Valued.v (x i - x₀ i) < 1) ∧ (2 : Fv F v)⁻¹ * (x ⬝ᵥ (B *ᵥ x)) ∈ A} = (Ideal.absNorm v.asIdeal : ℝ≥0∞)⁻¹ ^ (Fintype.card ι - 1) * lam (A ∩ {s | Valued.v (s - (2 : Fv F v)⁻¹ * (x₀ ⬝ᵥ (B *ᵥ x₀))) < 1})

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.PrimitiveVectorQuadraticLevelMeasureStatement

theorem **_root_.GlobalTheory.T.primitive_vector_quadratic_level_measure** : ∀ (F : Type) [Field F] [NumberField F] (v : HeightOneSpectrum (𝒪 F)) (lam : Measure (Fv F v)), IsStdV v lam → Valued.v (2 : Fv F v) = 1 → ∀ (ι : Type) [Fintype ι] [DecidableEq ι] (B : Matrix ι ι (Fv F v)), B.IsSymm → (∀ i j, B i j ∈ v.adicCompletionIntegers F) → Valued.v B.det = 1 → ∀ x₀ : ι → Fv F v, (∀ i, x₀ i ∈ v.adicCompletionIntegers F) → (∃ i, Valued.v (x₀ i) = 1) → ∀ A : Set (Fv F v), MeasurableSet A → (Measure.pi fun _ : ι => lam) {x | (∀ i, Valued.v (x i - x₀ i) < 1) ∧ (2 : Fv F v)⁻¹ * (x ·ᵥ (B *ᵥ x)) ∈ A} = (Ideal.absNorm v.asIdeal : ℝ≥0∞)⁻¹ ^ (Fintype.card ι - 1) * lam (A ∩ {s | Valued.v (s - (2 : Fv F v)⁻¹ * (x₀ ·ᵥ (B *ᵥ x₀))) < 1})

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ProperActionOrbitMeasure

theorem **orbitQuot_mk** {G V : Type} [Group G] (act : G → V → V) (h1 : ∀ x, act 1 x = x) (hm : ∀ g h x, act (g * h) x = act g (act h x)) (x₀ : V) (g : G) : orbitQuot act h1 hm x₀ (g : G ⧸ stabilizerOf act h1 hm x₀) = act g x₀

theorem **orbitQuot_injective** {G V : Type} [Group G] (act : G → V → V) (h1 : ∀ x, act 1 x = x) (hm : ∀ g h x, act (g * h) x = act g (act h x)) (x₀ : V) : Function.Injective (orbitQuot act h1 hm x₀)

theorem **orbitQuot_smul** {G V : Type} [Group G] (act : G → V → V) (h1 : ∀ x, act 1 x = x) (hm : ∀ g h x, act (g * h) x = act g (act h x)) (x₀ : V) (g : G) (y : G ⧸ stabilizerOf act h1 hm x₀) : orbitQuot act h1 hm x₀ (g • y) = act g (orbitQuot act h1 hm x₀ y)

theorem **range_orbitQuot** {G V : Type} [Group G] (act : G → V → V) (h1 : ∀ x, act 1 x = x) (hm : ∀ g h x, act (g * h) x = act g (act h x)) (x₀ : V) (Z : Set V) (hx : x₀ ∈ Z) (hinv : ∀ g, ∀ x ∈ Z, act g x ∈ Z) (htr : ∀ x ∈ Z, ∀ y ∈ Z, ∃ g, act g x = y) : Set.range (orbitQuot act h1 hm x₀) = Z

theorem **continuous_orbitQuot** {G V : Type} [Group G] [TopologicalSpace G] [TopologicalSpace V] (act : G → V → V) (hc : Continuous fun p : G × V => act p.1 p.2) (h1 : ∀ x, act 1 x = x) (hm : ∀ g h x, act (g * h) x = act g (act h x)) (x₀ : V) : Continuous (orbitQuot act h1 hm x₀)

theorem **measurableEmbedding_orbitQuot** {G V : Type} [Group G] [TopologicalSpace G] [IsTopologicalGroup G] [T2Space G] [LocallyCompactSpace G] [SecondCountableTopology G] [MeasurableSpace G] [BorelSpace G] [TopologicalSpace V] [T2Space V] [SecondCountableTopology V] [MeasurableSpace V] [BorelSpace V] (act : G → V → V) (hc : Continuous fun p : G × V => act p.1 p.2) (h1 : ∀ x, act 1 x = x) (hm : ∀ g h x, act (g * h) x = act g (act h x)) (x₀ : V) (Z : Set V) (hZ : IsClosed Z) (hx : x₀ ∈ Z) (hinv : ∀ g, ∀ x ∈ Z, act g x ∈ Z) (htr : ∀ x ∈ Z, ∀ y ∈ Z, ∃ g, act g x = y) : MeasurableEmbedding (orbitQuot act h1 hm x₀)

theorem **comap_orbitQuot_invariant** {G V : Type} [Group G] [TopologicalSpace G] [IsTopologicalGroup G] [T2Space G] [LocallyCompactSpace G] [SecondCountableTopology G] [MeasurableSpace G] [BorelSpace G] [TopologicalSpace V] [T2Space V] [SecondCountableTopology V] [MeasurableSpace V] [BorelSpace V] (act : G → V → V) (hc : Continuous fun p : G × V => act p.1 p.2) (h1 : ∀ x, act 1 x = x) (hm : ∀ g h x, act (g * h) x = act g (act h x)) (x₀ : V) (hμ : MeasurableEmbedding (orbitQuot act h1 hm x₀)) (μ : Measure V) (hμ : ∀ g, μ.map (act g) = μ) (g : G) (s : Set (G ⧸ stabilizerOf act h1 hm x₀)) (hs : MeasurableSet s) : μ.comap (orbitQuot act h1 hm x₀) ((fun y => g • y) ⁻¹' s) = μ.comap (orbitQuot act h1 hm x₀) s

theorem **invariant_measure_unique** : ∀ (G : Type) [Group G] [TopologicalSpace G] [IsTopologicalGroup G] [T2Space G] [LocallyCompactSpace G] [SecondCountableTopology G] [MeasurableSpace G] [BorelSpace G] (V : Type) [TopologicalSpace V] [T2Space V] [LocallyCompactSpace V] [SecondCountableTopology V] [MeasurableSpace V] [BorelSpace V] (act : G → V → V), (Continuous fun p : G × V => act p.1 p.2) → (∀ x, act 1 x = x) → (∀ g h x, act (g * h) x = act g (act h x)) → ∀ Z : Set V, IsClosed Z → (∀ g, ∀ x ∈ Z, act g x ∈ Z) → (∀ x ∈ Z, ∀ y ∈ Z, ∃ g, act g x = y) → ∀ (μ ν : Measure V), IsLocallyFiniteMeasure μ → IsLocallyFiniteMeasure ν → μ Zᶜ = 0 → ν Zᶜ = 0 → (∀ g, μ.map (act g) = μ) → (∀ g, ν.map (act g) = ν) → ν ≠ 0 → ∃ c : ℝ≥0, μ = c • ν

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ProperActionOrbitMeasureStatement

theorem **_root_.GlobalTheory.T.proper_action_orbit_measure** : ∀ (G : Type) [Group G] [TopologicalSpace G] [IsTopologicalGroup G] [T2Space G] [LocallyCompactSpace G] [SecondCountableTopology G] [MeasurableSpace G] [BorelSpace G] (V : Type) [TopologicalSpace V] [T2Space V] [LocallyCompactSpace V] [SecondCountableTopology V] [MeasurableSpace V] [BorelSpace V] (act : G → V → V), (Continuous fun p : G × V => act p.1 p.2) → (∀ x, act 1 x = x) → (∀ g h x, act (g * h) x = act g (act h x)) → ∀ Z : Set V, IsClosed Z → (∀ g, ∀ x ∈ Z, act g x ∈ Z) → (∀ x ∈ Z, ∀ y ∈ Z, ∃ g, act g x = y) → ∀ (μ ν : Measure V), IsLocallyFiniteMeasure μ → IsLocallyFiniteMeasure ν → μ Zᶜ = 0 → ν Zᶜ = 0 → (∀ g, μ.map (act g) = μ) → (∀ g, ν.map (act g) = ν) → ν ≠ 0 → ∃ c : ℝ≥0, μ = c • ν

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.PureAdelicFunctionMeasurable

theorem **borelSpace_ainf** (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E) (hσ : IsCMExt F E σ) (n M : ℕ) : BorelSpace (Fin n → Fin M → Ainf F ⊗[F] E)

theorem **borelSpace_af** (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E) (hσ : IsCMExt F E σ) (n M : ℕ) : BorelSpace (Fin n → Fin M → AF F ⊗[F] E)

theorem **borelSpace_fv** (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E) (hσ : IsCMExt F E σ) (n M : ℕ) (v : HeightOneSpectrum (𝒪 F)) : BorelSpace (Fin n → Fin M → Fv F v ⊗[F] E)

theorem **measurable_pureL** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (n M : ℕ) (S : Finset (HeightOneSpectrum (𝒪 F))) (φinf : (Fin n → Fin M → Ainf F ⊗[F] E) → ℝ≥0∞) (φ : ∀ v : HeightOneSpectrum (𝒪 F), (Fin n → Fin M → Fv F v ⊗[F] E) → ℝ≥0∞), Measurable φinf → (∀ v, Measurable (φ v)) → Measurable (pureL b S φinf φ)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.PureAdelicFunctionMeasurableStatement

theorem **_root_.GlobalTheory.T.pure_adelic_function_measurable** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (n M : ℕ) (S : Finset (HeightOneSpectrum (𝒪 F))) (φinf : (Fin n → Fin M → Ainf F ⊗[F] E) → ℝ≥0∞) (φ : ∀ v : HeightOneSpectrum (𝒪 F), (Fin n → Fin M → Fv F v ⊗[F] E) → ℝ≥0∞), Measurable φinf → (∀ v, Measurable (φ v)) → Measurable (pureL b S φinf φ)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.QuadraticLevelSetDensity

theorem **continuous_qMap** (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) : Continuous (qMap B ρ)

theorem **continuous_gW** (P : InfinitePlace F → Matrix ι ι ℝ) : Continuous (gW P)

theorem **gauss_I** (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (P : InfinitePlace F → Matrix ι ι ℝ) (x : InfinitePlace F → ι → ℝ) : gauss B ρ P (fun _ => Complex.I) x = (gW P x : ℂ)

theorem **gauss_shift** (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (P : InfinitePlace F → Matrix ι ι ℝ) (t : InfinitePlace F → ℝ) (v : InfinitePlace F → ι → ℝ) : gauss B ρ P (fun w => (t w : ℂ) + Complex.I) v = (gW P v : ℂ) * Complex.exp ((2 * (Real.pi : ℂ) * Complex.I) * ((∑ w, qMap B ρ v w * t w : ℝ) : ℂ))

theorem **one_dim_integrable** (M : ℕ) (hM : 3 ≤ M) : Integrable (fun t : ℝ => (‖(t : ℂ) + Complex.I‖ ^ M)⁻¹)

theorem **prod_integrable** (M : ℕ) (hM : 3 ≤ M) : Integrable (fun x : InfinitePlace F → ℝ => (∏ w, ‖(x w : ℂ) + Complex.I‖ ^ M)⁻¹)

theorem **density_of_q** : ∀ (F : Type) [Field F] [NumberField F], ∀ (ι : Type) [Fintype ι] [DecidableEq ι] (M : ℕ) (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (P : InfinitePlace F → Matrix ι ι ℝ), QSetup M B ρ P → 3 ≤ M → ∃ Di : (InfinitePlace F → ℝ) → ℝ, Continuous Di ∧ (∀ y, 0 ≤ Di y) ∧ (∀ U : Set (InfinitePlace F → ℝ), MeasurableSet U → ∫⁻ x in {x : InfinitePlace F → ι → ℝ | (fun w => (1 / 2 : ℝ) * (x w ·ᵥ (B.map (ρ w) *ᵥ x w))) ∈ U}, ENNReal.ofReal (Real.exp (-Real.pi * ∑ w, x w ·ᵥ (P w *ᵥ x w))) = ∫⁻ y in U, ENNReal.ofReal (Di y)) ∧ Integrable (fun x : InfinitePlace F → ℝ => cG B ρ P (fun w => (x w : ℂ) + Complex.I)) ∧ ∀ z : InfinitePlace F → ℝ, ∫ x : InfinitePlace F → ℝ, cG B ρ P (fun w => (x w : ℂ) + Complex.I) * Complex.exp (-(2 * (Real.pi : ℂ) * Complex.I) * ((∑ w, z w * x w : ℝ) : ℂ)) = ((Real.sqrt |((Algebra.norm ℚ B.det : ℚ) : ℝ)| * Di z : ℝ) : ℂ)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.QuadraticLevelSetDensityStatement

theorem **_root_.GlobalTheory.T.quadratic_level_set_density** : ∀ (F : Type) [Field F] [NumberField F], ∀ (ι : Type) [Fintype ι] [DecidableEq ι] (M : ℕ) (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (P : InfinitePlace F → Matrix ι ι ℝ), QSetup M B ρ P → 3 ≤ M → ∃ Di : (InfinitePlace F → ℝ) → ℝ, Continuous Di ∧ (∀ y, 0 ≤ Di y) ∧ (∀ U : Set (InfinitePlace F → ℝ), MeasurableSet U → ∫⁻ x in {x : InfinitePlace F → ι → ℝ | (fun w => (1 / 2 : ℝ) * (x w ·ᵥ (B.map (ρ w) *ᵥ x w))) ∈ U}, ENNReal.ofReal (Real.exp (-Real.pi * ∑ w, x w ·ᵥ (P w *ᵥ x w))) = ∫⁻ y in U, ENNReal.ofReal (Di y)) ∧ Integrable (fun x : InfinitePlace F → ℝ => cG B ρ P (fun w => (x w : ℂ) + Complex.I)) ∧ ∀ z : InfinitePlace F → ℝ, ∫ x : InfinitePlace F → ℝ, cG B ρ P (fun w => (x w : ℂ) + Complex.I) * Complex.exp (-(2 * (Real.pi : ℂ) * Complex.I) * ((∑ w, z w * x w : ℝ) : ℂ)) = ((Real.sqrt |((Algebra.norm ℚ B.det : ℚ) : ℝ)| *

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.QuadraticSetupInvariantUnderIsometry

theorem **qsetup_conj_majorant** (F : Type) [Field F] [NumberField F] (ι : Type) [Fintype ι] [DecidableEq ι] (M : ℕ) (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (P : InfinitePlace F → Matrix ι ι ℝ) (gi : InfinitePlace F → Matrix ι ι ℝ) (h : QSetup M B ρ P) (hg : ∀ w, (gi w)ᵀ * B.map (ρ w) * gi w = B.map (ρ w)) : QSetup M B ρ (fun w => (gi w)ᵀ * P w * gi w)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.QuadraticSetupInvariantUnderIsometryStatement

theorem **_root_.GlobalTheory.T.quadratic_setup_invariant_under_isometry** : ∀ (F : Type) [Field F] [NumberField F] (ι : Type) [Fintype ι] [DecidableEq ι] (M : ℕ) (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (P : InfinitePlace F → Matrix ι ι ℝ) (gi : InfinitePlace F → Matrix ι ι ℝ), QSetup M B ρ P → (∀ w, (gi w)ᵀ * B.map (ρ w) * gi w = B.map (ρ w)) → QSetup M B ρ (fun w => (gi w)ᵀ * P w * gi w)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.QuotientMeasureTower

theorem **setLIntegral_mul_right_eq** (μ₁ : Measure G₁) [SFinite μ₁] [μ₁.IsMulLeftInvariant] [μ₁.IsMulRightInvariant] (Γ₁ : Subgroup G₁) [Countable Γ₁] (𝓕₁ : Set G₁) (h𝓕₁ : IsFundamentalDomain Γ₁ 𝓕₁ μ₁) (f : G₁ → ℝ≥0∞) (hf : ∀ γ ∈ Γ₁, ∀ u, f (γ * u) = f u) (u₀ : G₁) : ∫⁻ u in 𝓕₁, f (u * u₀) ∂μ₁ = ∫⁻ u in 𝓕₁, f u ∂μ₁

theorem **bijective_pair** (ι : G₁ →* G) (hι : Function.Injective ι) (Γ : Subgroup G) (Γ₁ : Subgroup G₁) (hΓ : ∀ u ∈ Γ₁, ι u ∈ Γ) {Q : Type} (r : Q → G) (hr : ∀ q, r q ∈ Γ) (hQ : ∀ γ ∈ Γ, ∃! q, ∃ u ∈ Γ₁, γ = ι u * r q) : Function.Bijective (fun p : Q × Γ₁ => ((ι (p.2 : G₁)⁻¹ * r p.1, Γ.mul_mem (hΓ _ (Γ₁.inv_mem p.2.2)) (hr p.1)) : Γ))

theorem **lintegral_sum_cosets_eq** (ι : G₁ →* G) (hιc : Continuous ι) (hι : Function.Injective ι) (μ : Measure G) [SFinite μ] [μ.IsMulLeftInvariant] (μ₁ : Measure G₁) [SFinite μ₁] [μ₁.IsMulLeftInvariant] [μ₁.IsInvInvariant] (Γ : Subgroup G) [Countable Γ] (Γ₁ : Subgroup G₁) [Countable Γ₁] (𝓕 : Set G) (𝓕₁ : Set G₁) (h𝓕 : IsFundamentalDomain Γ 𝓕 μ) (h𝓕₁ : IsFundamentalDomain Γ₁ 𝓕₁ μ₁) (hΓ : ∀ u ∈ Γ₁, ι u ∈ Γ) {Q : Type} [Countable Q] (r : Q → G) (hr : ∀ q, r q ∈ Γ) (hQ : ∀ γ ∈ Γ, ∃! q, ∃ u ∈ Γ₁, γ = ι u * r q) (w : G → ℝ≥0∞) (hw : Measurable w) (hw1 : ∀ g, ∫⁻ u, w (ι u * g) ∂μ₁ = 1) (ψ : G → ℝ≥0∞) (hψ : Measurable ψ) (hinv : ∀ u ∈ Γ₁, ∀ g, ψ (ι u * g) = ψ g) : ∫⁻ h in 𝓕, ∑' q, ψ (r q * h) ∂μ = ∫⁻ g, w g * ∫⁻ u in 𝓕₁, ψ (ι u * g) ∂μ₁ ∂μ

theorem **lintegral_sum_cosets_average** : ∀ (G : Type) [Group G] [TopologicalSpace G] [IsTopologicalGroup G] [T2Space G] [LocallyCompactSpace G] [SecondCountableTopology G] [MeasurableSpace G] [BorelSpace G] (G₁ : Type) [Group G₁] [TopologicalSpace G₁] [IsTopologicalGroup G₁] [LocallyCompactSpace G₁] [SecondCountableTopology G₁] [MeasurableSpace G₁] [BorelSpace G₁] (ι : G₁ →* G), IsClosedEmbedding ι → ∀ (μ : Measure G) [μ.IsHaarMeasure] (μ₁ : Measure G₁) [μ₁.IsHaarMeasure] (Γ : Subgroup G) [Countable Γ] (Γ₁ : Subgroup G₁) [Countable Γ₁] (𝓕 : Set G) (𝓕₁ : Set G₁), IsFundamentalDomain Γ 𝓕 μ → IsFundamentalDomain Γ₁ 𝓕₁ μ₁ → μ₁ 𝓕₁ < ⊤ → (∀ u ∈ Γ₁, ι u ∈ Γ) → ∀ (Q : Type) [Countable Q] (r : Q → G), (∀ q, r q ∈ Γ) → (∀ γ ∈ Γ, ∃! q, ∃ u ∈ Γ₁, γ = ι u * r q) → ∀ φ : G → ℝ≥0∞, Measurable φ → (∀ u ∈ Γ₁, ∀ g, φ (ι u * g) = φ g) → ∫⁻ h in 𝓕, ∑' q, φ (r q * h) ∂μ = ∫⁻ h in 𝓕, ∑' q, (μ₁ 𝓕₁)⁻¹ * ∫⁻ u in 𝓕

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.QuotientMeasureTowerStatement

theorem **_root_.GlobalTheory.T.quotient_measure_tower** : ∀ (G : Type) [Group G] [TopologicalSpace G] [IsTopologicalGroup G] [T2Space G] [LocallyCompactSpace G] [SecondCountableTopology G] [MeasurableSpace G] [BorelSpace G] (G₁ : Type) [Group G₁] [TopologicalSpace G₁] [IsTopologicalGroup G₁] [LocallyCompactSpace G₁] [SecondCountableTopology G₁] [MeasurableSpace G₁] [BorelSpace G₁] (ι : G₁ →* G), IsClosedEmbedding ι → ∀ (μ : Measure G) [μ.IsHaarMeasure] (μ₁ : Measure G₁) [μ₁.IsHaarMeasure] (Γ : Subgroup G) [Countable Γ] (Γ₁ : Subgroup G₁) [Countable Γ₁] (𝓕 : Set G) (𝓕₁ : Set G₁), IsFundamentalDomain Γ 𝓕 μ → IsFundamentalDomain Γ₁ 𝓕₁ μ₁ → μ₁ 𝓕₁ < ⊤ → (∀ u ∈ Γ₁, ι u ∈ Γ) → ∀ (Q : Type) [Countable Q] (r : Q → G), (∀ q, r q ∈ Γ) → (∀ γ ∈ Γ, ∃! q, ∃ u ∈ Γ₁, γ = ι u * r q) → ∀ φ : G → ℝ≥0∞, Measurable φ → (∀ u ∈ Γ₁, ∀ g, φ (ι u * g) = φ g) → ∫⁻ h in 𝓕, ∑' q, φ (r q * h) ∂μ = ∫⁻ h in 𝓕, ∑' q, (μ₁ 𝓕

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.QuotientProbabilityMeasure

theorem **isProbabilityMeasure_prH** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (μ : Measure (HA (AF F) σ J)) (𝓕 : Set (HA (AF F) σ J)), IsHaarH F E σ M J μ → IsFDH F E σ M J μ 𝓕 → IsProbabilityMeasure (prH μ 𝓕)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.QuotientProbabilityMeasureStatement

theorem **_root_.GlobalTheory.T.quotient_probability_measure** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (μ : Measure (HA (AF F) σ J)) (𝓕 : Set (HA (AF F) σ J)), IsHaarH F E σ M J μ → IsFDH F E σ M J μ 𝓕 → IsProbabilityMeasure (prH μ 𝓕)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.RankOneFiberMeasureBlockExtension

theorem **nnreal_eq_of_smul_haar** {V : Type} [AddCommGroup V] [TopologicalSpace V] [LocallyCompactSpace V] [IsTopologicalAddGroup V] [MeasurableSpace V] [BorelSpace V] (μ : Measure V) [μ.IsAddHaarMeasure] {c e : ℝ≥0} (h : c • μ = e • μ) : c = e

theorem **sclMat_apply** (u : R) (i j : Fin 1) : sclMat (F

theorem **unitTens_coe** (u : Rˣ) : ((unitTens (F

theorem **rmul_sclMat** {M : ℕ} (X : Fin 1 → Fin M → R ⊗[F] E) (u : R) : rmul R X (sclMat (F

theorem **cT_sclMat_mul** (σ : E →ₐ[F] E) (u s : R) : ((hermScal R σ (u ^ 2 * s) : HermR R σ 1) : Matrix (Fin 1) (Fin 1) (R ⊗[F] E)) = cT (sigR R σ) (sclMat (F

theorem **fibre_scale_modulus_length_one** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (N : ℕ) (J₁ : Matrix (Fin N) (Fin N) E), IsHermND σ J₁ → ∀ t : F, t ≠ 0 → ∀ (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R], IsLocR R → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (lam : Measure R), lam.IsAddHaarMeasure → ∀ m₁ : HermR R σ 1 → Measure (Fin 1 → Fin N → R ⊗[F] E), IsGramFib R σ J₁ 1 (coordW R b lam 1 N) (coordHerm R σ b lam 1) m₁ → ∃ κ₁ cm : Rˣ → ℝ≥0, Continuous κ₁ ∧ κ₁ 1 = 1 ∧ (∀ u : Rˣ, m₁ (hermScal R σ ((u : R) ^ 2 * algebraMap F R t)) = κ₁ u • (m₁ (hermScal R σ (algebraMap F R t))).map (scl ((u : R) ⊗ₜ[F] (1 : E)))) ∧ Continuous cm ∧ (∀ u : Rˣ, (coordK R b lam).map (fun y => ((u : R) ⊗ₜ[F] (algebraMap F E t)) * y) = cm u • coordK R b lam) ∧ ∀ c : ℝ≥0, (coordK R b lam).map (

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.RankOneFiberMeasureBlockExtensionStatement

theorem **_root_.GlobalTheory.T.coordK_map_mul_eq_smul_continuous_modulus** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (N : ℕ) (J₁ : Matrix (Fin N) (Fin N) E), IsHermND σ J₁ → ∀ t : F, t ≠ 0 → ∀ (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R], IsLocR R → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (lam : Measure R), lam.IsAddHaarMeasure → ∀ m₁ : HermR R σ 1 → Measure (Fin 1 → Fin N → R ⊗[F] E), IsGramFib R σ J₁ 1 (coordW R b lam 1 N) (coordHerm R σ b lam 1) m₁ → ∃ κ₁ cm : Rˣ → ℝ≥0, Continuous κ₁ ∧ κ₁ 1 = 1 ∧ (∀ u : Rˣ, m₁ (hermScal R σ ((u : R) ^ 2 * algebraMap F R t)) = κ₁ u • (m₁ (hermScal R σ (algebraMap F R t))).map (scl ((u : R) ⊗ₜ[F] (1 : E)))) ∧ Continuous cm ∧ (∀ u : Rˣ, (coordK R b lam).map (fun y => ((u : R) ⊗ₜ[F] (algebraMap F E t)) * y) = cm u • coordK R b lam) ∧

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.RankOneGramMeasurableSection

theorem **rowOf_dot** (τ : A →+* A) (J : Matrix n n A) (x y : n → A) : rowOf τ J x ⬝ᵥ y = hform τ J x y

theorem **hform_sub_left** (τ : A →+* A) (J : Matrix n n A) (x y z : n → A) : hform τ J (x − y) z = hform τ J x z − hform τ J y z

theorem **hform_sub_right** (τ : A →+* A) (J : Matrix n n A) (x y z : n → A) : hform τ J x (y − z) = hform τ J x y − hform τ J x z

theorem **map_hform** (τ : A →+* A) (hτ : ∀ a, τ (τ a) = a) (J : Matrix n n A) (hJ : ∀ i j, τ (J j i) = J i j) (x y : n → A) : τ (hform τ J x y) = hform τ J y x

theorem **reflM_mul** (τ : A →+* A) (J : Matrix n n A) (a b : A) (d : n → A) (h : a + b = a * b * hform τ J d d) : reflM τ J a d * reflM τ J b d = 1

theorem **reflM_unitary** (τ : A →+* A) (hτ : ∀ a, τ (τ a) = a) (J : Matrix n n A) (hJ : ∀ i j, τ (J j i) = J i j) (a : A) (d : n → A) (h : a + τ a = a * τ a * hform τ J d d) : cT τ (reflM τ J a d) * J * reflM τ J a d = J

theorem **reflM_mulVec** (τ : A →+* A) (J : Matrix n n A) (a : A) (d p : n → A) : (reflM τ J a d).mulVec p = p − (a * hform τ J d p) • d

theorem **refl_facts** (τ : A →+* A) (hτ : ∀ a, τ (τ a) = a) (J : Matrix n n A) (hJ : ∀ i j, τ (J j i) = J i j) (p y : n → A) (hyp : hform τ J y y = hform τ J p p) (ci : A) (hci : (hform τ J p p − hform τ J y p) * ci = 1) : reflM τ J ci (y − p) * reflM τ J (τ ci) (y − p) = 1 ∧ reflM τ J (τ ci) (y − p) * reflM τ J ci (y − p) = 1 ∧ cT τ (reflM τ J ci (y − p)) * J * reflM τ J ci (y − p) = J ∧ (reflM τ J ci (y − p)).mulVec p = y

theorem **selT_of_mem** {W G : Type*} (g₁ : G) (Z : Set W) (V : ℕ → Set W) (f : ℕ → W → G) (k : ℕ) {y : W} (h : y ∈ V k ∩ Z) : selT g₁ Z V f (k + 1) y = f k y

theorem **exists_section** {W G : Type*} [TopologicalSpace W] [LocallyCompactSpace W] [SecondCountableTopology W] [T2Space W] [MeasurableSpace W] [BorelSpace W] [TopologicalSpace G] [MeasurableSpace G] [BorelSpace G] (g₁ : G) (Z : Set W) (hZ : IsClosed Z) (P : G → W → Prop) (loc : ∀ y₀ ∈ Z, ∃ U : Set W, IsOpen U ∧ y₀ ∈ U ∧ ∃ f : W → G, ContinuousOn f (U ∩ Z) ∧ ∀ y ∈ U ∩ Z, P (f y) y) : ∃ s : W → G, Measurable s ∧ (∀ y ∈ Z, P (s y) y) ∧ ∀ C : Set W, IsCompact C → ∃ C' : Set G, IsCompact C' ∧ ∀ y ∈ C, y ∈ Z → s y ∈ C'

theorem **hform_neg_neg** (τ : K →+* K) (J : Matrix n n K) (u : n → K) : hform τ J (−u) (−u) = hform τ J u u

theorem **hform_neg_right** (τ : K →+* K) (J : Matrix n n K) (u v : n → K) : hform τ J u (−v) = −hform τ J u v

theorem **reflM_cont** [TopologicalSpace K] [IsTopologicalRing K] {X : Type*} [TopologicalSpace X] (τ : K →+* K) (hτ : Continuous τ) (J : Matrix n n K) {a : X → K} {d : X → n → K} (ha : Continuous a) (hd : Continuous d) : Continuous fun x => reflM τ J (a x) (d x)

theorem **continuousAt_ringInverse** [TopologicalSpace K] [IsTopologicalRing K] (hU : IsOpenEmbedding (Units.val : Kˣ → K)) {a : K} (ha : IsUnit a) : ContinuousAt (Ring.inverse : K → K) a

theorem **local_refl** [TopologicalSpace K] [IsTopologicalRing K] (τ : K →+* K) (hτc : Continuous τ) (hτ : ∀ a, τ (τ a) = a) (J : Matrix n n K) (hJ : ∀ i j, τ (J j i) = J i j) (hU : IsOpenEmbedding (Units.val : Kˣ → K)) (h2 : IsUnit (2 : K)) (u : n → K) (hu : IsUnit (hform τ J u u)) : ∃ U : Set (n → K), IsOpen U ∧ u ∈ U ∧ ∃ t : {v : n → K // v ∈ U ∧ hform τ J v v = hform τ J u u} → UJ τ J, Continuous t ∧ ∀ v, (((t v : UJ τ J) : GL n K) : Matrix n n K).mulVec (−u) = v.1

theorem **orbit_section** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R], IsLocR R → ∀ x : Fin 1 → Fin M → R ⊗[F] E, IsUnit (gramR R σ J x).det → ∃ s : (Fin 1 → Fin M → R ⊗[F] E) → HA R σ J, Measurable s ∧ (∀ y, gramR R σ J y = gramR R σ J x → actW R σ J (s y) x = y) ∧ ∀ C : Set (Fin 1 → Fin M → R ⊗[F] E), IsCompact C → ∃ C' : Set (HA R σ J), IsCompact C' ∧ ∀ y ∈ C, gramR R σ J y = gramR R σ J x → s y ∈ C'

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.RankOneGramMeasurableSectionStatement

theorem **_root_.GlobalTheory.T.rank_one_gram_measurable_section** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R], IsLocR R → ∀ x : Fin 1 → Fin M → R ⊗[F] E, IsUnit (gramR R σ J x).det → ∃ s : (Fin 1 → Fin M → R ⊗[F] E) → HA R σ J, Measurable s ∧ (∀ y, gramR R σ J y = gramR R σ J x → actW R σ J (s y) x = y) ∧ ∀ C : Set (Fin 1 → Fin M → R ⊗[F] E), IsCompact C → ∃ C' : Set (HA R σ J), IsCompact C' ∧ ∀ y ∈ C, gramR R σ J y = gramR R σ J x → s y ∈ C'

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.RankOneLocalDensitiesProductConverges

theorem **val_alg** (v : HeightOneSpectrum (𝒪 F)) (x : F) : Valued.v (algebraMap F (Fv F v) x) = v.valuation F x

theorem **val_lt_one_of_mem** (v : HeightOneSpectrum (𝒪 F)) {r : 𝒪 F} (h : r ∈ v.asIdeal) : Valued.v ((algebraMap F (Fv F v)) (algebraMap (𝒪 F) F r)) < 1

theorem **val_le_one** (v : HeightOneSpectrum (𝒪 F)) (r : 𝒪 F) : Valued.v ((algebraMap F (Fv F v)) (algebraMap (𝒪 F) F r)) ≤ 1

theorem **val_eq_one** (v : HeightOneSpectrum (𝒪 F)) {r : 𝒪 F} (h : r ∉ v.asIdeal) : Valued.v ((algebraMap F (Fv F v)) (algebraMap (𝒪 F) F r)) = 1

theorem **two_le_norm** (v : HeightOneSpectrum (𝒪 F)) : 2 ≤ Ideal.absNorm v.asIdeal

theorem **exists_nat_mul_mem** (v : HeightOneSpectrum (𝒪 F)) (x : Fv F v) : ∃ n : ℕ, 0 < n ∧ (n : Fv F v) * x ∈ v.adicCompletionIntegers F

theorem **exists_exponents** (a : ℕ) (ha : 0 < a) : ∃ e : HeightOneSpectrum (𝒪 F) → ℕ, (∀ v, Ideal.span {(a : 𝒪 F)} ≤ v.asIdeal ^ e v ∧ ¬ Ideal.span {(a : 𝒪 F)} ≤ v.asIdeal ^ (e v + 1)) ∧ Function.HasFiniteMulSupport (fun v : HeightOneSpectrum (𝒪 F) => ((Ideal.absNorm v.asIdeal : ℝ)) ^ e v) ∧ ∏ᶠ v : HeightOneSpectrum (𝒪 F), ((Ideal.absNorm v.asIdeal : ℝ)) ^ e v = (a : ℝ) ^ Module.finrank ℚ F

theorem **exists_den** (β : F) : ∃ D : ℕ, 0 < D ∧ ∃ x : 𝒪 F, (D : F) * β = algebraMap (𝒪 F) F x

theorem **repr_natCast_mul** (a : ℕ) (y : Fv F v ⊗[F] E) (k : Fin d) : (Algebra.TensorProduct.basis (Fv F v) b).repr ((a : Fv F v ⊗[F] E) * y) k = (a : Fv F v) * (Algebra.TensorProduct.basis (Fv F v) b).repr y k

theorem **mem_dilSet** {a : ℕ} {X : Fin 1 → Fin M → Fv F v ⊗[F] E} : X ∈ dilSet b v a ↔ ∀ p i k, (a : Fv F v) * (Algebra.TensorProduct.basis (Fv F v) b).repr (X p i) k ∈ v.adicCompletionIntegers F

theorem **dilSet_mono** (a t : ℕ) : (dilSet b v a : Set (Fin 1 → Fin M → Fv F v ⊗[F] E)) ⊆ dilSet b v (a * t)

theorem **exists_mem_dilSet** (X : Fin 1 → Fin M → Fv F v ⊗[F] E) : ∃ n : ℕ, 0 < n ∧ X ∈ dilSet b v n

theorem **dilSet_unit** {a : ℕ} (h : (a : 𝒪 F) ∉ v.asIdeal) : (dilSet b v a : Set (Fin 1 → Fin M → Fv F v ⊗[F] E)) = latV b v

theorem **measurable_latV** : MeasurableSet (latV b v : Set (Fin 1 → Fin M → Fv F v ⊗[F] E))

theorem **exists_pos_mass** (μ : Measure (Fin 1 → Fin M → Fv F v ⊗[F] E)) (h : μ ≠ 0) : ∃ n : ℕ, 0 < n ∧ μ (dilSet b v n) ≠ 0

theorem **eventually_half** (hσ : IsCMExt F E σ) (hJ : IsHermND σ J) (hM : 3 ≤ M) (hlam : ∀ v, IsStdV v (lam v)) (hm : ∀ v, IsGramFib (Fv F v) σ J 1 (coordW (Fv F v) b (lam v) 1 M) (coordHerm (Fv F v) σ b (lam v) 1) (m v)) : ∀ᶠ v : HeightOneSpectrum (𝒪 F) in Filter.cofinite, ∀ s : Fv F v, s ∈ v.adicCompletionIntegers F → s ≠ 0 → 1 / 2 ≤ (m v (hermScal (Fv F v) σ s) (latV b v)).toReal

theorem **scal_fin** (hσ : IsCMExt F E σ) (hJ : IsHermND σ J) (hM : 3 ≤ M) (hlam : ∀ v, IsStdV v (lam v)) (hm : ∀ v, IsGramFib (Fv F v) σ J 1 (coordW (Fv F v) b (lam v) 1 M) (coordHerm (Fv F v) σ b (lam v) 1) (m v)) {s : F} (hs : s ≠ 0) : (∀ v, mL σ b m s v < ⊤) ∧ Summable fun v => |(mL σ b m s v).toReal - 1|

theorem **dil_eq** (hσ : IsCMExt F E σ) (hJ : IsHermND σ J) (hM : 3 ≤ M) (hlam : ∀ v, IsStdV v (lam v)) (hm : ∀ v, IsGramFib (Fv F v) σ J 1 (coordW (Fv F v) b (lam v) 1 M) (coordHerm (Fv F v) σ b (lam v) 1) (m v)) (v : HeightOneSpectrum (𝒪 F)) (t : F) (a e : ℕ) (ha : 0 < a) (h1 : Ideal.span {(a : 𝒪 F)} ≤ v.asIdeal ^ e) (h2 : ¬ Ideal.span {(a : 𝒪 F)} ≤ v.asIdeal ^ (e + 1)) : m v (hermScal (Fv F v) σ (algebraMap F _ t)) (dilSet b v a) = (Ideal.absNorm v.asIdeal : ℝ≥0∞) ^ (e * (2 * M - 2)) * mL σ b m ((a : F) ^ 2 * t) v

theorem **growth_main** (hσ : IsCMExt F E σ) (hJ : IsHermND σ J) (hM : 3 ≤ M) (hlam : ∀ v, IsStdV v (lam v)) (hm : ∀ v, IsGramFib (Fv F v) σ J 1 (coordW (Fv F v) b (lam v) 1 M) (coordHerm (Fv F v) σ b (lam v) 1) (m v)) (β : F) (hβ : β ≠ 0) (hne : ∀ v, m v (hermScal (Fv F v) σ (algebraMap F _ β)) ≠ 0) : ∃ c : ℝ, 0 < c ∧ ∀ k : ℕ, ∃ a : ℕ, k < a ∧ c * (a : ℝ) ^ (Module.finrank ℚ F * (2 * M - 2)) ≤ ∏' v, (m v (hermScal (Fv F v) σ (algebraMap F _ β)) (dilSet b v a)).toReal

theorem **growth** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → 3 ≤ M → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (lam : ∀ v : HeightOneSpectrum (𝒪 F), Measure (Fv F v)), (∀ v, IsStdV v (lam v)) → ∀ (m : ∀ v : HeightOneSpectrum (𝒪 F), HermR (Fv F v) σ 1 → Measure (Fin 1 → Fin M → Fv F v ⊗[F] E)), (∀ v, IsGramFib (Fv F v) σ J 1 (coordW (Fv F v) b (lam v) 1 M) (coordHerm (Fv F v) σ b (lam v) 1) (m v)) → ∀ β : F, β ≠ 0 → (∀ v, m v (hermScal (Fv F v) σ (algebraMap F _ β)) ≠ 0) → ∃ c : ℝ, 0 < c ∧ ∀ k : ℕ, ∃ a : ℕ, k < a ∧ c * (a : ℝ) ^ (Module.finrank ℚ F * (2 * M - 2)) ≤ ∏' v, (m v (hermScal (Fv F v) σ (algebraMap F _ β)) {X : (Fin 1 → Fin M → Fv F v ⊗[F] E) | (fun p i => (a : Fv F v ⊗[F] E) * X p i) ∈ latV b v}).toReal

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.RankOneLocalDensitiesProductConvergesStatement

theorem **_root_.GlobalTheory.T.exists_frequently_mul_pow_le_tprod_rank_one_local_density** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → 3 ≤ M → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (lam : ∀ v : HeightOneSpectrum (𝒪 F), Measure (Fv F v)), (∀ v, IsStdV v (lam v)) → ∀ (m : ∀ v : HeightOneSpectrum (𝒪 F), HermR (Fv F v) σ 1 → Measure (Fin 1 → Fin M → Fv F v ⊗[F] E)), (∀ v, IsGramFib (Fv F v) σ J 1 (coordW (Fv F v) b (lam v) 1 M) (coordHerm (Fv F v) σ b (lam v) 1) (m v)) → ∀ β : F, β ≠ 0 → (∀ v, m v (hermScal (Fv F v) σ (algebraMap F _ β)) ≠ 0) → ∃ c : ℝ, 0 < c ∧ ∀ k : ℕ, ∃ a : ℕ, k < a ∧ c * (a : ℝ) ^ (Module.finrank ℚ F * (2 * M - 2)) ≤ ∏' v, (m v (hermScal (Fv F v) σ (algebraMap F _ β)) {X : (Fin 1 → Fin M → Fv F v ⊗[F] E) | (fun p i => (a : Fv F v ⊗[F] E) * X p i

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.RankOneLocalDensityAlmostAll

theorem **isOpen_ball_sub** {F : Type} [Field F] [NumberField F] (v : HeightOneSpectrum (𝒪 F)) (s : Fv F v) : IsOpen {z : Fv F v | Valued.v (z - s) < Valued.v s}

theorem **local_mass** {F E : Type} [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E) (hCM : IsCMExt F E σ) (M : ℕ) (J : Matrix (Fin M) (Fin M) E) (hJ : IsHermND σ J) (d : ℕ) (b : Module.Basis (Fin d) F E) (v : HeightOneSpectrum (𝒪 F)) (Bv : Matrix (Fin M × Fin d) (Fin M × Fin d) (Fv F v)) (hform : ∀ x y : Fin M → Fv F v ⊗[F] E, Algebra.trace (Fv F v) (Fv F v ⊗[F] E) (hR (Fv F v) σ J x y) = (fun p : Fin M × Fin d => (Algebra.TensorProduct.basis (Fv F v) b).repr (x p.1) p.2) ⬝ᵥ (Bv *ᵥ fun p : Fin M × Fin d => (Algebra.TensorProduct.basis (Fv F v) b).repr (y p.1) p.2)) (lam : Measure (Fv F v)) (hlam : IsStdV v lam) (m : HermR (Fv F v) σ 1 → Measure (Fin 1 → Fin M → Fv F v ⊗[F] E)) (hm : IsGramFib (Fv F v) σ J 1 (coordW (Fv F v) b lam 1 M) (coordHerm (Fv F v) σ b lam 1) m) (s : Fv F v) (hs0 : s ≠ 0) (D : ℝ≥0) (hD : ∀ A : Set (Fv F v), MeasurableSet A → A ⊆ {z | V

theorem **density_standard_lattice** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → 3 ≤ M → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E), ∀ᶠ v : HeightOneSpectrum (𝒪 F) in Filter.cofinite, ∀ (lam : Measure (Fv F v)), IsStdV v lam → ∀ m, IsGramFib (Fv F v) σ J 1 (coordW (Fv F v) b lam 1 M) (coordHerm (Fv F v) σ b lam 1) m → ∀ s : Fv F v, s ∈ v.adicCompletionIntegers F → s ≠ 0 → |(m (hermScal (Fv F v) σ s) (latV b v)).toReal - 1| ≤ 2 / (Ideal.absNorm v.asIdeal : ℝ) ^ (M - 1)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.RankOneLocalDensityAlmostAllStatement

theorem **_root_.GlobalTheory.T.rank_one_local_density_almost_all** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → 3 ≤ M → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E), ∀ᶠ v : HeightOneSpectrum (𝒪 F) in Filter.cofinite, ∀ (lam : Measure (Fv F v)), IsStdV v lam → ∀ m, IsGramFib (Fv F v) σ J 1 (coordW (Fv F v) b lam 1 M) (coordHerm (Fv F v) σ b lam 1) m → ∀ s : Fv F v, s ∈ v.adicCompletionIntegers F → s ≠ 0 → |(m (hermScal (Fv F v) σ s) (latV b v)).toReal - 1| ≤ 2 / (Ideal.absNorm v.asIdeal : ℝ) ^ (M - 1)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.RankZeroGramFiberMeasure

theorem **fibre_eq_dirac_of_unique** {V X : Type} [MeasurableSpace X] [SubsingletonX] (μ : Measure V) (ν : Measure X) (m : X → Measure V) (x₀ : V) (hμ : μ Set.univ = 1) (hν : ν Set.univ = 1) (h : ∀ φ : V → ℝ≥0∞, Measurable φ → ∫⁻ v, φ v ∂μ = ∫⁻ b, ∫⁻ v, φ v ∂(m b) ∂ν) (S : X) : m S = Measure.dirac x₀

theorem **GlobalTheory.CompanionTheory.SW.GramEmpty.fibre_gram_zero** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R], IsLocR R → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E), ∀ (lam : Measure R), lam.IsAddHaarMeasure → ∀ m, IsGramFib R σ J 0 (coordW R b lam 0 M) (coordHerm R σ b lam 0) m → ∀ S, m S = Measure.dirac (fun a => a.elim0)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.RankZeroGramFiberMeasureStatement

theorem **_root_.GlobalTheory.T.rank_zero_gram_fiber_measure** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R], IsLocR R → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E), ∀ (lam : Measure R), lam.IsAddHaarMeasure → ∀ m, IsGramFib R σ J 0 (coordW R b lam 0 M) (coordHerm R σ b lam 0) m → ∀ S, m S = Measure.dirac (fun a => a.elim0)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.RationalGramAdelicCompactFinite

theorem **sigma_sigma** (σ : E →ₐ[F] E) (h : IsCMExt F E σ) (x : E) : σ (σ x) = x

theorem **exists_moved** (σ : E →ₐ[F] E) (h : IsCMExt F E σ) : ∃ x : E, σ x ≠ x

theorem **fixed_mem** (σ : E →ₐ[F] E) (h : IsCMExt F E σ) (e : E) (he : σ e = e) : ∃ t : F, e = algebraMap F E t

theorem **hE_self_fixed** (σ : E →ₐ[F] E) (h : IsCMExt F E σ) {M : ℕ} (J : Matrix (Fin M) (Fin M) E) (hJ : IsHermND σ J) (x : Fin M → E) : σ (hE σ J x x) = hE σ J x x

theorem **gram_rational_finite** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → (∀ X : Fin 1 → Fin M → E, ∃ t : F, gramE σ J X = fun _ _ => algebraMap F E t) ∧ (∀ (n : ℕ) (X : Fin n → Fin M → E), gramR (AF F) σ J (ratTup (AF F) X) = (gramE σ J X).map (inclR (AF F))) ∧ ∀ Ω : Set (Fin 1 → Fin M → AF F ⊗[F] E), IsCompact Ω → {t : F | ∃ Y ∈ Ω, gramR (AF F) σ J Y = fun _ _ => inclR (AF F) (algebraMap F E t)}.Finite

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.RationalGramAdelicCompactFiniteStatement

theorem **_root_.GlobalTheory.T.rational_gram_adelic_compact_finite** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → (∀ X : Fin 1 → Fin M → E, ∃ t : F, gramE σ J X = fun _ _ => algebraMap F E t) ∧ (∀ (n : ℕ) (X : Fin n → Fin M → E), gramR (AF F) σ J (ratTup (AF F) X) = (gramE σ J X).map (inclR (AF F))) ∧ ∀ Ω : Set (Fin 1 → Fin M → AF F ⊗[F] E), IsCompact Ω → {t : F | ∃ Y ∈ Ω, gramR (AF F) σ J Y = fun _ _ => inclR (AF F) (algebraMap F E t)}.Finite

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.RationalTuplesDiscreteCocompactAdelic

theorem **repr_inclR** {F E : Type} [Field F] [Field E] [Algebra F E] (R : Type) [CommRing R] [Algebra F R] {d : ℕ} (b : Module.Basis (Fin d) F E) (e : E) (k : Fin d) : (Algebra.TensorProduct.basis R b).repr (inclR R e) k = algebraMap F R (b.repr e k)

theorem **scalar** (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E) (h : IsCMExt F E σ) : (∃ C : Set (AF F ⊗[F] E), IsCompact C ∧ ∀ y, ∃ e : E, y − inclR (AF F) e ∈ C) ∧ (∀ C : Set (AF F ⊗[F] E), IsCompact C → {e : E | inclR (AF F) e ∈ C}.Finite) ∧ NoncompactSpace (AF F ⊗[F] E)

theorem **main** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (n M : ℕ), (∃ C : Set (Fin n → Fin M → AF F ⊗[F] E), IsCompact C ∧ ∀ y, ∃ X : Fin n → Fin M → E, y − ratTup (AF F) X ∈ C) ∧ (∀ C : Set (Fin n → Fin M → AF F ⊗[F] E), IsCompact C → {X : Fin n → Fin M → E | ratTup (AF F) X ∈ C}.Finite) ∧ (0 < n → 0 < M → NoncompactSpace (Fin n → Fin M → AF F ⊗[F] E))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.RationalTuplesDiscreteCocompactAdelicStatement

theorem **_root_.GlobalTheory.T.rational_tuples_discrete_cocompact_adelic** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (n M : ℕ), (∃ C : Set (Fin n → Fin M → AF F ⊗[F] E), IsCompact C ∧ ∀ y, ∃ X : Fin n → Fin M → E, y − ratTup (AF F) X ∈ C) ∧ (∀ C : Set (Fin n → Fin M → AF F ⊗[F] E), IsCompact C → {X : Fin n → Fin M → E | ratTup (AF F) X ∈ C}.Finite) ∧ (0 < n → 0 < M → NoncompactSpace (Fin n → Fin M → AF F ⊗[F] E))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.RealGaussianFourierTransform

lemma **beta_re** (τ : ℂ) {e : ℝ} : (beta τ e).re = τ.im

lemma **beta_one** (τ : ℂ) : beta τ 1 = -Complex.I * τ

lemma **beta_neg_one** (τ : ℂ) : beta τ (-1) = Complex.I * (starRingEnd ℂ) τ

lemma **beta_inv** {τ : ℂ} (hτ : τ ≠ 0) {e : ℝ} (he : e = 1 ∨ e = -1) : beta (-τ⁻¹) e = (beta τ e)⁻¹

lemma **norm_beta** (τ : ℂ) {e : ℝ} (he : e = 1 ∨ e = -1) : ‖beta τ e‖ = ‖τ‖

lemma **norm_half** (τ : ℂ) {e : ℝ} (he : e = 1 ∨ e = -1) : ‖((beta τ e)⁻¹) ^ (1 / 2 : ℂ)‖ = (Real.sqrt ‖τ‖)⁻¹

lemma **half_mul** {b : ℂ} (hb : 0 < b.re) : (b⁻¹) ^ (1 / 2 : ℂ) * ((b⁻¹)⁻¹) ^ (1 / 2 : ℂ) = 1

lemma **subst** (g : Matrix ι ι ℝ) (hg : g.det ≠ 0) (F : (ι → ℝ) → ℂ) (hF : Continuous F) : (Integrable (fun u => F (g *ᵥ u)) → Integrable F) ∧ ∫ u, F (g *ᵥ u) = ((|g.det|⁻¹ : ℝ) : ℂ) * ∫ x, F x

lemma **bil** (g A : Matrix ι ι ℝ) (u w : ι → ℝ) : (g *ᵥ u) ⬝ᵥ (A *ᵥ (g *ᵥ w)) = u ⬝ᵥ ((gᵀ * A * g) *ᵥ w)

lemma **quad** {g S P : Matrix ι ι ℝ} {e : ι → ℝ} (hS : gᵀ * S * g = Matrix.diagonal e) (hP : gᵀ * P * g = 1) (τ : ℂ) (u : ι → ℝ) : (Real.pi : ℂ) * Complex.I * (τ.re : ℂ) * (((g *ᵥ u) ⬝ᵥ (S *ᵥ (g *ᵥ u))) : ℝ) : ℂ) - (Real.pi : ℂ) * (τ.im : ℂ) * (((g *ᵥ u) ⬝ᵥ (P *ᵥ (g *ᵥ u))) : ℝ) : ℂ) = -∑ i, ((Real.pi : ℂ) * beta τ (e i)) * (u i : ℂ) ^ 2

lemma **lin** {g S : Matrix ι ι ℝ} {e : ι → ℝ} (hS : gᵀ * S * g = Matrix.diagonal e) (u v : ι → ℝ) : 2 * (Real.pi : ℂ) * Complex.I * (((g *ᵥ u) ⬝ᵥ (S *ᵥ (g *ᵥ v))) : ℝ) : ℂ) = ∑ i, (2 * (Real.pi : ℂ) * Complex.I * (e i : ℂ) * (v i : ℂ)) * (u i : ℂ)

lemma **det_scale** {g S : Matrix ι ι ℝ} {e : ι → ℝ} (he : ∀ i, e i = 1 ∨ e i = -1) (hS : gᵀ * S * g = Matrix.diagonal e) : Real.sqrt |S.det| * |g.det| = 1

theorem **fourier_gaussian_real_place** : ∀ (ι : Type) [Fintype ι] [DecidableEq ι] (S P : Matrix ι ι ℝ), S.IsSymm → S.det ≠ 0 → P.PosDef → P * S⁻¹ * P = S → ∃ c : ℂ → ℂ, ∀ τ : ℂ, 0 < τ.im → ‖c τ‖ = (Real.sqrt ‖τ‖ ^ Fintype.card ι)⁻¹ ∧ c τ * c (-τ⁻¹) = 1 ∧ Integrable (fun x : ι → ℝ => Complex.exp ((Real.pi : ℂ) * Complex.I * (τ.re : ℂ) * ((x ⬝ᵥ (S *ᵥ x)) : ℝ) : ℂ) - (Real.pi : ℂ) * (τ.im : ℂ) * ((x ⬝ᵥ (P *ᵥ x)) : ℝ) : ℂ))) ∧ ∀ y : ι → ℝ, ((Real.sqrt |S.det| : ℝ) : ℂ) * ∫ x : ι → ℝ, Complex.exp ((Real.pi : ℂ) * Complex.I * (τ.re : ℂ) * ((x ⬝ᵥ (S *ᵥ x)) : ℝ) : ℂ) - (Real.pi : ℂ) * (τ.im : ℂ) * ((x ⬝ᵥ (P *ᵥ x)) : ℝ) : ℂ)) * Complex.exp (2 * (Real.pi : ℂ) * Complex.I * ((x ⬝ᵥ (S *ᵥ y)) : ℝ) : ℂ)) = c τ * Complex.exp ((Real.pi : ℂ) * Complex.I * ((-τ⁻¹).re : ℂ) * ((y ⬝ᵥ (S *ᵥ y)) : ℝ) : ℂ) - (Real.pi : ℂ) * ((-τ⁻¹).im : ℂ) * ((y ⬝ᵥ (P *ᵥ y)) : ℝ) : ℂ))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.RealGaussianFourierTransformStatement

theorem **_root_.GlobalTheory.T.real_gaussian_fourier_transform** : ∀ (ι : Type) [Fintype ι] [DecidableEq ι] (S P : Matrix ι ι ℝ), S.IsSymm → S.det ≠ 0 → P.PosDef → P * S⁻¹ * P = S → ∃ c : ℂ → ℂ, ∀ τ : ℂ, 0 < τ.im → ‖c τ‖ = (Real.sqrt ‖τ‖ ^ Fintype.card ι)⁻¹ ∧ c τ * c (-τ⁻¹) = 1 ∧ Integrable (fun x : ι → ℝ => Complex.exp ((Real.pi : ℂ) * Complex.I * (τ.re : ℂ) * ((x ⬝ᵥ (S *ᵥ x)) : ℝ) : ℂ) - (Real.pi : ℂ) * (τ.im : ℂ) * ((x ⬝ᵥ (P *ᵥ x)) : ℝ) : ℂ))) ∧ ∀ y : ι → ℝ, ((Real.sqrt |S.det| : ℝ) : ℂ) * ∫ x : ι → ℝ, Complex.exp ((Real.pi : ℂ) * Complex.I * (τ.re : ℂ) * ((x ⬝ᵥ (S *ᵥ x)) : ℝ) : ℂ) - (Real.pi : ℂ) * (τ.im : ℂ) * ((x ⬝ᵥ (P *ᵥ x)) : ℝ) : ℂ)) * Complex.exp (2 * (Real.pi : ℂ) * Complex.I * ((x ⬝ᵥ (S *ᵥ y)) : ℝ) : ℂ)) = c τ * Complex.exp ((Real.pi : ℂ) * Complex.I * ((-τ⁻¹).re : ℂ) * ((y ⬝ᵥ (S *ᵥ y)) : ℝ) : ℂ) - (Real.pi : ℂ) * ((-τ⁻¹).im : ℂ) * ((y ⬝ᵥ (P *ᵥ y)) : ℝ) : ℂ))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.RealQuadraticFormLemma

lemma **tsum_lint** {Pr : Measure Ω} {s : Z → Ω → ℂ} (hm : ∀ z, Measurable (s z)) : ∫⁻ ω, ∑' z, ‖s z ω‖ₑ ∂Pr = ∑' z, ∫⁻ ω, ‖s z ω‖ₑ ∂Pr

lemma **term_lint_ne_top** {Pr : Measure Ω} {s : Z → Ω → ℂ} (hm : ∀ z, Measurable (s z)) (hfin : ∫⁻ ω, ∑' z, ‖s z ω‖ₑ ∂Pr ≠ ∞) (z : Z) : ∫⁻ ω, ‖s z ω‖ₑ ∂Pr ≠ ∞

lemma **term_integrable** {Pr : Measure Ω} {s : Z → Ω → ℂ} (hm : ∀ z, Measurable (s z)) (hfin : ∫⁻ ω, ∑' z, ‖s z ω‖ₑ ∂Pr ≠ ∞) (z : Z) : Integrable (s z) Pr

lemma **summable_integral_norm** {Pr : Measure Ω} {s : Z → Ω → ℂ} (hm : ∀ z, Measurable (s z)) (hfin : ∫⁻ ω, ∑' z, ‖s z ω‖ₑ ∂Pr ≠ ∞) : Summable fun z => ∫ ω, ‖s z ω‖ ∂Pr

lemma **summable_norm_integral** {Pr : Measure Ω} {s : Z → Ω → ℂ} (hm : ∀ z, Measurable (s z)) (hfin : ∫⁻ ω, ∑' z, ‖s z ω‖ₑ ∂Pr ≠ ∞) : Summable fun z => ‖∫ ω, s z ω ∂Pr‖

lemma **integral_series** {Pr : Measure Ω} {s : Z → Ω → ℂ} (hm : ∀ z, Measurable (s z)) (hfin : ∫⁻ ω, ∑' z, ‖s z ω‖ₑ ∂Pr ≠ ∞) (c : Z → ℂ) (hc : ∀ z, ‖c z‖ = 1) : ∫ ω, ∑' z, c z * s z ω ∂Pr = ∑' z, c z * ∫ ω, s z ω ∂Pr

theorem **fourier_series** {Pr : Measure Ω} {s : Z → Ω → ℂ} (hm : ∀ z, Measurable (s z)) (hfin : ∫⁻ ω, ∑' z, ‖s z ω‖ₑ ∂Pr ≠ ∞) {μ : Measure V} [IsFiniteMeasure μ] {u : Z → V → ℂ} (hu : ∀ z, Continuous (u z)) (hun : ∀ z x, ‖u z x‖ = 1) {v : V → ℂ} (hv : Continuous v) (hvn : ∀ x, ‖v x‖ = 1) (Sel : Set Z) (Vol : ℂ) (h1 : ∀ z ∈ Sel, ∫ x, u z x * v x ∂μ = Vol) (h0 : ∀ z ∉ Sel, (∫ x, u z x * v x ∂μ = 0) ∨ ∀ ω, s z ω = 0) : Integrable (fun x => (∫ ω, ∑' z, u z x * s z ω ∂Pr) * v x) μ ∧ ∫ x, (∫ ω, ∑' z, u z x * s z ω ∂Pr) * v x ∂μ = Vol * ∑' z : Sel, ∫ ω, s z.1 ω ∂Pr

theorem **sum_selected** {Pr : Measure Ω} {s : Z → Ω → ℂ} (hm : ∀ z, Measurable (s z)) (hfin : ∫⁻ ω, ∑' z, ‖s z ω‖ₑ ∂Pr ≠ ∞) (hpos : ∀ z ω, s z ω = ((‖s z ω‖ : ℝ) : ℂ)) (Sel : Set Z) : ∑' z : Sel, ∫ ω, s z.1 ω ∂Pr = ((((∫⁻ ω, ∑' z : Sel, ‖s z.1 ω‖ₑ ∂Pr).toReal : ℝ) : ℂ)

lemma **quad_orth** {R : Type} [CommRing R] (g C : Matrix ι ι R) (v : ι → R) : (g *ᵥ v) ⬝ᵥ (C *ᵥ (g *ᵥ v)) = v ⬝ᵥ ((gᵀ * C * g) *ᵥ v)

lemma **dot_map** {R S : Type} [CommRing R] [CommRing S] (f : R →+* S) (B : Matrix ι ι R) (v : ι → R) : (fun j => f (v j)) ⬝ᵥ (B.map f *ᵥ fun j => f (v j)) = f (v ⬝ᵥ (B *ᵥ v))

lemma **ph_continuous** (ρ : InfinitePlace F → (F →+* ℝ)) (b : F) : Continuous (ph ρ b)

lemma **ph_mul** (ρ : InfinitePlace F → (F →+* ℝ)) (b c : F) (x : InfinitePlace F → ℝ) : ph ρ b x * ph ρ c x = ph ρ (b + c) x

lemma **ph_zero** (ρ : InfinitePlace F → (F →+* ℝ)) (x : InfinitePlace F → ℝ) : ph ρ 0 x = 1

lemma **ph_neg** (ρ : InfinitePlace F → (F →+* ℝ)) (b : F) (x : InfinitePlace F → ℝ) : Complex.exp (-(2 * (Real.pi : ℂ) * Complex.I) * ((∑ w, ρ w b * x w : ℝ) : ℂ)) = ph ρ (-b) x

lemma **ph_norm** (ρ : InfinitePlace F → (F →+* ℝ)) (b : F) (x : InfinitePlace F → ℝ) : ‖ph ρ b x‖ = 1

lemma **thTerm_shift** {B : Matrix ι ι F} {ρ : InfinitePlace F → (F →+* ℝ)} (P : InfinitePlace F → Matrix ι ι ℝ) {gf : Matrix ι ι (A F)} {gi : InfinitePlace F → Matrix ι ι ℝ} (hO : IsOrth B ρ gf gi) (φ : (ι → A F) → ℂ) (x : InfinitePlace F → ℝ) (z : ι → F) : thTerm B ρ P gf gi φ (fun w => (x w : ℂ) + Complex.I) z = ph ρ (qf (1 / 2 : F) B z) x * thTerm B ρ P gf gi φ (fun _ => Complex.I) z

lemma **thTerm_nonneg** (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (P : InfinitePlace F → Matrix ι ι ℝ) (gf : Matrix ι ι (A F)) (gi : InfinitePlace F → Matrix ι ι ℝ) (a : ℕ) (z : ι → F) : thTerm B ρ P gf gi (indL a) (fun _ => Complex.I) z = (((‖thTerm B ρ P gf gi (indL a) (fun _ => Complex.I) z‖ : ℝ) : ℂ)

lemma **thTerm_zero_le** (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (P : InfinitePlace F → Matrix ι ι ℝ) (gf : Matrix ι ι (A F)) (gi : InfinitePlace F → Matrix ι ι ℝ) (a : ℕ) : ‖thTerm B ρ P gf gi (indL a) (fun _ => Complex.I) 0‖ₑ ≤ 1

lemma **qf_adelic** {B : Matrix ι ι F} {gf : Matrix ι ι (A F)} (hO : gfᵀ * Bf B * gf = Bf B) (z : ι → F) : qf (hf F) (Bf B) (gf *ᵥ fun j => algebraMap F (A F) (z j)) = algebraMap F (A F) (qf (1 / 2 : F) B z)

lemma **integral_of_ne_zero** {B : Matrix ι ι F} {ρ : InfinitePlace F → (F →+* ℝ)} {P : InfinitePlace F → Matrix ι ι ℝ} {gf : Matrix ι ι (A F)} {gi : InfinitePlace F → Matrix ι ι ℝ} (hO : IsOrth B ρ gf gi) {a N : ℕ} (hint : ∀ y : ι → A F, (∀ i, (a : A F) * y i ∈ Oh F) → (N : A F) * qf (hf F) (Bf B) y ∈ Oh F) (hmem : ∀ b : F, algebraMap F (A F) b ∈ Oh F ↔ ∃ r : 𝒪 F, (r : F) = b) {τ : InfinitePlace F → ℂ} {z : ι → F} (h : thTerm B ρ P gf gi (indL a) τ z ≠ 0) : ∃ r : 𝒪 F, (r : F) = (N : F) * qf (1 / 2 : F) B z

lemma **sb_indL** (hT2 : T2Space (A F)) (hOc : IsCompact (Oh F)) (hOo : IsOpen (Oh F)) {a : ℕ} (ha : 0 < a) : SB (indL a : (ι → A F) → ℂ)

lemma **countable_field** : Countable F

theorem **fourier_coefficient** : ∀ (F : Type) [Field F] [NumberField F], ∀ [MeasurableSpace (A F)] [BorelSpace (A F)], ∀ (ι : Type) [Fintype ι] [DecidableEq ι] (M : ℕ) (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (P : InfinitePlace F → Matrix ι ι ℝ), QSetup M B ρ P → ∀ (Ω : Type) [MeasurableSpace Ω] (Pr : Measure Ω) (gf : Ω → Matrix ι ι (A F)) (gi : Ω → InfinitePlace F → Matrix ι ι ℝ), CountData M B ρ Pr gf gi → ∀ a : ℕ, 0 < a → ∀ β : F, β ≠ 0 → ∀ N : ℕ, 0 < N → (∃ r : 𝒪 F, (r : F) = (N : F) * β) → (∀ y : ι → A F, (∀ i, (a : A F) * y i ∈ Oh F) → (N : A F) * qf (hf F) (Bf B) y ∈ Oh F) → ∀ D : Set (InfinitePlace F → ℝ), IsAddFundamentalDomain (latR ρ N) D volume → ∃ I : ℝ≥0, ∫⁻ ω, ∑' x : {x : ι → F // qf (1 / 2 : F) B x = β}, ‖thTerm B ρ P (gf ω) (gi ω) (indL a) (fun x => Complex.I) x.1‖ₑ ∂Pr = (I : ℝ≥0∞) ∧ IntegrableOn (fun x : InfinitePlace F → ℝ => (∫ ω, theta B ρ P (gf ω) (gi ω) (

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.RealQuadraticFormLemmaStatement

theorem **_root_.GlobalTheory.T.real_quadratic_form_lemma** : ∀ (F : Type) [Field F] [NumberField F], ∀ [MeasurableSpace (A F)] [BorelSpace (A F)], ∀ (ι : Type) [Fintype ι] [DecidableEq ι] (M : ℕ) (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (P : InfinitePlace F → Matrix ι ι ℝ), QSetup M B ρ P → ∀ (Ω : Type) [MeasurableSpace Ω] (Pr : Measure Ω) (gf : Ω → Matrix ι ι (A F)) (gi : Ω → InfinitePlace F → Matrix ι ι ℝ), CountData M B ρ Pr gf gi → ∀ a : ℕ, 0 < a → ∀ β : F, β ≠ 0 → ∀ N : ℕ, 0 < N → (∃ r : 𝒪 F, (r : F) = (N : F) * β) → (∀ y : ι → A F, (∀ i, (a : A F) * y i ∈ Oh F) → (N : A F) * qf (hf F) (Bf B) y ∈ Oh F) → ∀ D : Set (InfinitePlace F → ℝ), IsAddFundamentalDomain (latR ρ N) D volume → ∃ I : ℝ≥0, ∫⁻ ω, ∑' x : {x : ι → F // qf (1 / 2 : F) B x = β}, ‖thTerm B ρ P (gf ω) (gi ω) (indL a) (fun _ => Complex.I) x.1‖ₑ ∂Pr = (I : ℝ≥0∞) ∧ IntegrableOn (fun x : InfinitePlace F → ℝ => (∫ ω,

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ReductionToSiegelDomain

theorem **prod_sq_embedding** {F : Type} [Field F] [NumberField F] (hF : IsTotallyReal F) (ρ : InfinitePlace F → (F →+* ℝ)) (hρ : ∀ w x, ((ρ w x : ℝ) : ℂ) = w.embedding x) (x : F) : ∏ w, (ρ w x) ^ 2 = ((Algebra.norm ℚ x : ℚ) : ℝ) ^ 2

theorem **balanced_lower** {ι : Type} [Fintype ι] (z : ι → ℝ) (hz : ∀ i, 0 < z i) (κ P : ℝ) (hκ : 0 < κ) (hbal : ∀ v w, κ * z w ≤ z v) (hprod : P ≤ ∏ i, z i) (w : ι) : κ * min 1 P ≤ z w

theorem **reduction** : ∀ (F : Type) [Field F] [NumberField F], IsTotallyReal F → ∀ ρ : InfinitePlace F → (F →+* ℝ), (∀ w x, ((ρ w x : ℝ) : ℂ) = w.embedding x) → ∃ (s : Finset SL(2, F)) (κ t : ℝ), 0 < κ ∧ 0 < t ∧ ∀ τ : InfinitePlace F → ℂ, (∀ w, 0 < (τ w).im) → ∃ r ∈ s, ∃ k : SL(2, F), (∀ i j, ∃ z : 𝒪 F, (z : F) = k i j) ∧ (∀ w, t ≤ (mob ρ (r * k) τ w).im) ∧ ∀ v w, κ * (mob ρ (r * k) τ w).im ≤ (mob ρ (r * k) τ v).im

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ReductionToSiegelDomainStatement

theorem **_root_.GlobalTheory.T.reduction_to_siegel_domain** : ∀ (F : Type) [Field F] [NumberField F], IsTotallyReal F → ∀ ρ : InfinitePlace F → (F →+* ℝ), (∀ w x, ((ρ w x : ℝ) : ℂ) = w.embedding x) → ∃ (s : Finset SL(2, F)) (κ t : ℝ), 0 < κ ∧ 0 < t ∧ ∀ τ : InfinitePlace F → ℂ, (∀ w, 0 < (τ w).im) → ∃ r ∈ s, ∃ k : SL(2, F), (∀ i j, ∃ z : 𝒪 F, (z : F) = k i j) ∧ (∀ w, t ≤ (mob ρ (r * k) τ w).im) ∧ ∀ v w, κ * (mob ρ (r * k) τ w).im ≤ (mob ρ (r * k) τ v).im

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.RestrFamily

lemma **ae_g_eq** (hι : Function.Injective ι) (hf : ∀ v, f v = ι (g v)) {U : Set V} {m : Measure V} {b : X} (h : m (f ⁻¹' {ι b} ∩ U)ᶜ = 0) : ∀ᵐ v ∂m, g v = b

lemma **piece** (hι : Function.Injective ι) (hg : Continuous g) (hf : ∀ v, f v = ι (g v)) {s D : Set X} (hD : MeasurableSet D) (hDs : D ⊆ s) {F : X → Measure V} (h : IsFibreFamily μ f ι (g ⁻¹' s) ν s F) {φ : V → ℝ≥0∞} (hφ : Measurable φ) : ∫⁻ v in g ⁻¹' D, φ v ∂μ = ∫⁻ b in D, ∫⁻ v, φ v ∂(F b) ∂ν

lemma **restr_family** (hι : Function.Injective ι) (hg : Continuous g) (hf : ∀ v, f v = ι (g v)) {B B₂ : Set X} (hB₂ : IsOpen B₂) (hsub : B₂ ⊆ B) {fam : X → Measure V} (h : IsFibreFamily μ f ι (g ⁻¹' B) ν B fam) : IsFibreFamily μ f ι (g ⁻¹' B₂) ν B₂ (restr B₂ fam)

theorem **glue** : ∀ (V X Y : Type) [TopologicalSpace V] [LocallyCompactSpace V] [SecondCountableTopology V] [T2Space V] [MeasurableSpace V] [BorelSpace V] [TopologicalSpace X] [LocallyCompactSpace X] [SecondCountableTopology X] [T2Space X] [MeasurableSpace X] [BorelSpace X] (μ : Measure V) (ν : Measure X) [IsLocallyFiniteMeasure μ] [IsLocallyFiniteMeasure ν] [ν.IsOpenPosMeasure] (f : V → Y) (g : V → X) (ι : X → Y), Function.Injective ι → Continuous g → (∀ v, f v = ι (g v)) → ∀ (B : Set X), IsOpen B → (∀ b ∈ B, ∃ B' : Set X, IsOpen B' ∧ b ∈ B' ∧ B' ⊆ B ∧ ∃ fib' : X → Measure V, Formalism.MainNotions.GlobalTheoryTheory.GenusMassVolumes.IsFibreFamily μ f ι (g ⁻¹' B') ν B' fib') → ∃ fib : X → Measure V, Formalism.MainNotions.GlobalTheoryTheory.GenusMassVolumes.IsFibreFamily μ f ι (g ⁻¹' B) ν B fib

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.RightMultiplicationEulerProduct

theorem **map_eq_of_conj** {W V : Type} [AddCommGroup W] [TopologicalSpace W] [IsTopologicalAddGroup W] [MeasurableSpace W] [BorelSpace W] [AddCommGroup V] [TopologicalSpace V] [IsTopologicalAddGroup V] [MeasurableSpace V] [BorelSpace V] (e : W ≃+ V) (he : Continuous e) (hs : Continuous e.symm) (T : W → W) (B : V → V) (hB : Continuous B) (hc : ∀ x, e (T x) = B (e x)) (c : ℝ≥0) (H : ∀ ν : Measure V, ν.IsAddHaarMeasure → ν.map B = c • ν) (μ : Measure W) (hμ : μ.IsAddHaarMeasure) : μ.map T = c • μ

theorem **repr_mul_incl** (y : R ⊗[F] E) (e : E) (k : Fin d) : (Algebra.TensorProduct.basis R b).repr (y * inclR R e) k = ∑ l, (Algebra.TensorProduct.basis R b).repr y l * algebraMap F R (b.repr (b l * e) k)

theorem **coE_rmul** {n M : ℕ} (X : Fin n → Fin M → R ⊗[F] E) (g : Matrix (Fin n) (Fin n) E) : coE b R n M (rmul R X (g.map (inclR R))) = (coMat b g M).map (algebraMap F R) *ᵥ coE b R n M X

theorem **rmul_rmul** {n M : ℕ} (X : Fin n → Fin M → R ⊗[F] E) (G G' : Matrix (Fin n) (Fin n) (R ⊗[F] E)) : rmul R (rmul R X G) G' = rmul R X (G * G')

theorem **rmul_one'** {n M : ℕ} (X : Fin n → Fin M → R ⊗[F] E) : rmul R X 1 = X

theorem **rmul_zero'** {n M : ℕ} (G : Matrix (Fin n) (Fin n) (R ⊗[F] E)) : rmul R (0 : Fin n → Fin M → R ⊗[F] E) G = 0

theorem **coMat_det** {n : ℕ} (g : Matrix (Fin n) (Fin n) E) (hg : g.det ≠ 0) (M : ℕ) : (coMat b g M).det ≠ 0

theorem **rmul_map_haar** (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [IsTopologicalRing R] [MeasurableSpace R] [BorelSpace R] [SecondCountableTopology R] [IsTopologicalRing (R ⊗[F] E)] {n M : ℕ} (hBor : BorelSpace (Fin n → Fin M → R ⊗[F] E)) (hh : IsHomeomorph fun (y : R ⊗[F] E) (k : Fin d) => (Algebra.TensorProduct.basis R b).repr y k) (g : Matrix (Fin n) (Fin n) E) (c : ℝ≥0) (H : ∀ ν : Measure (Fin n × Fin M × Fin d → R), ν.IsAddHaarMeasure → ν.map (fun x => (coMat b g M).map (algebraMap F R) *ᵥ x) = c • ν) (μ : Measure (Fin n → Fin M → R ⊗[F] E)) (hμ : μ.IsAddHaarMeasure) : μ.map (fun X => rmul R X (g.map (inclR R))) = c • μ

theorem **rmul_product_formula** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (n M : ℕ) (g : Matrix (Fin n) (Fin n) E), g.det ≠ 0 → ∃ (cinf : ℝ≥0) (c : HeightOneSpectrum (𝒪 F) → ℝ≥0), IsUnitProd cinf c ∧ (∀ μ : Measure (Fin n → Fin M → Ainf F ⊗[F] E), μ.IsAddHaarMeasure → μ.map (fun X => rmul (Ainf F) X (g.map (inclR (Ainf F)))) = cinf • μ) ∧ ∀ v : HeightOneSpectrum (𝒪 F), (∀ μ : Measure (Fin n → Fin M → Fv F v ⊗[F] E), μ.IsAddHaarMeasure → μ.map (fun X => rmul (Fv F v) X (g.map (inclR (Fv F v)))) = c v • μ)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.RightMultiplicationEulerProductStatement

theorem **_root_.GlobalTheory.T.right_multiplication_euler_product** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (n M : ℕ) (g : Matrix (Fin n) (Fin n) E), g.det ≠ 0 → ∃ (cinf : ℝ≥0) (c : HeightOneSpectrum (𝒪 F) → ℝ≥0), IsUnitProd cinf c ∧ (∀ μ : Measure (Fin n → Fin M → Ainf F ⊗[F] E), μ.IsAddHaarMeasure → μ.map (fun X => rmul (Ainf F) X (g.map (inclR (Ainf F)))) = cinf • μ) ∧ ∀ v : HeightOneSpectrum (𝒪 F), (∀ μ : Measure (Fin n → Fin M → Fv F v ⊗[F] E), μ.IsAddHaarMeasure → μ.map (fun X => rmul (Fv F v) X (g.map (inclR (Fv F v)))) = c v • μ)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.SL2BruhatDecompositionIdentities

private theorem **coe_n** {R : Type} [CommRing R] (b : R) : ((nMat b : SL(2, R)) : Matrix (Fin 2) (Fin 2) R) = !![1, b; 0, 1]

private theorem **coe_m** {R : Type} [CommRing R] (a : Rˣ) : ((mMat a : SL(2, R)) : Matrix (Fin 2) (Fin 2) R) = !![(a : R), 0; 0, ((a⁻¹ : Rˣ) : R)]

private theorem **coe_w** {R : Type} [CommRing R] : ((wMat : SL(2, R)) : Matrix (Fin 2) (Fin 2) R) = !![0, 1; -1, 0]

private theorem **SL2_eq_of_entries_eq_matrix** {R : Type} [CommRing R] {g : SL(2, R)} {p q r s : R} {h : SL(2, R)} (hh : (h : Matrix (Fin 2) (Fin 2) R) = !![p, q; r, s]) (h0 : g 0 0 = p) (h1 : g 0 1 = q) (h2 : g 1 0 = r) (h3 : g 1 1 = s) : g = h

theorem **decompositions** : ∀ (R : Type) [CommRing R] (g : SL(2, R)), (∀ c : Rˣ, g 1 0 = (c : R) → g = nMat (g 0 0 * ((c⁻¹ : Rˣ) : R)) * mMat (-c⁻¹) * wMat * nMat (g 1 1 * ((c⁻¹ : Rˣ) : R))) ∧ (∀ a : Rˣ, g 0 0 = (a : R) → g = wMat⁻¹ * nMat (-(g 1 0 * ((a⁻¹ : Rˣ) : R))) * wMat * mMat a * nMat (g 0 1 * ((a⁻¹ : Rˣ) : R))) ∧ (∀ a : Rˣ, g 0 0 = (a : R) → g 1 0 = 0 → g = nMat (g 0 1 * (a : R)) * mMat a) ∧ (wMat : SL(2, R))⁻¹ = wMat * wMat * wMat

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.SL2BruhatDecompositionIdentitiesStatement

theorem **_root_.GlobalTheory.T.SL2_bruhat_decompositions_and_w_inv** : ∀ (R : Type) [CommRing R] (g : SL(2, R)), (∀ c : Rˣ, g 1 0 = (c : R) → g = nMat (g 0 0 * ((c⁻¹ : Rˣ) : R)) * mMat (-c⁻¹) * wMat * nMat (g 1 1 * ((c⁻¹ : Rˣ) : R))) ∧ (∀ a : Rˣ, g 0 0 = (a : R) → g = wMat⁻¹ * nMat (-(g 1 0 * ((a⁻¹ : Rˣ) : R))) * wMat * mMat a * nMat (g 0 1 * ((a⁻¹ : Rˣ) : R))) ∧ (∀ a : Rˣ, g 0 0 = (a : R) → g 1 0 = 0 → g = nMat (g 0 1 * (a : R)) * mMat a) ∧ (wMat : SL(2, R))⁻¹ = wMat * wMat * wMat

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.SbEqSumCosetIndicators

theorem **sb_eq_sum_coset_indicators** : ∀ (V : Type) [AddCommGroup V] [TopologicalSpace V] [MeasurableSpace V] (χ : V → V → ℂ) (μ : Measure V), NADual V χ μ → ∀ φ : V → ℂ, SB φ → ∀ L' : AddSubgroup V, IsOpen (L' : Set V) → (∀ x, ∀ l ∈ L', φ (x + l) = φ x) → ∃ s : Finset V, ∀ x, φ x = ∑ a ∈ s, φ a * Set.indicator {y | y - a ∈ L'} (fun _ => (1 : ℂ)) x

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ScalarHermitianTraceAndUnit

theorem **hermScal_entry** {F E : Type} [Field F] [Field E] [Algebra F E] (σ : E →ₐ[F] E) (R : Type) [CommRing R] [Algebra F R] (s : R) : (hermScal R σ s : Matrix (Fin 1) (Fin 1) (R ⊗[F] E)) 0 0 = algebraMap R (R ⊗[F] E) s

theorem **trace_det** : ∀ (F E : Type) [Field F] [Field E] [Algebra F E], (1 / 2 : F) ≠ 0 → Module.finrank F E = 2 → ∀ (σ : E →ₐ[F] E) (R : Type) [CommRing R] [Algebra F R] (β : F), algebraMap F R (1 / 2) * Algebra.trace R (R ⊗[F] E) ((hermScal R σ (algebraMap F R β) : Matrix (Fin 1) (Fin 1) (R ⊗[F] E)) 0 0) = algebraMap F R β ∧ (β ≠ 0 → IsUnit ((hermScal R σ (algebraMap F R β) : Matrix (Fin 1) (Fin 1) (R ⊗[F] E))).det)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ScalarHermitianTraceAndUnitStatement

theorem **_root_.GlobalTheory.T.scalar_hermitian_trace_and_unit** : ∀ (F E : Type) [Field F] [Field E] [Algebra F E], (1 / 2 : F) ≠ 0 → Module.finrank F E = 2 → ∀ (σ : E →ₐ[F] E) (R : Type) [CommRing R] [Algebra F R] (β : F), algebraMap F R (1 / 2) * Algebra.trace R (R ⊗[F] E) ((hermScal R σ (algebraMap F R β) : Matrix (Fin 1) (Fin 1) (R ⊗[F] E)) 0 0) = algebraMap F R β ∧ (β ≠ 0 → IsUnit ((hermScal R σ (algebraMap F R β) : Matrix (Fin 1) (Fin 1) (R ⊗[F] E))).det)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ScalarMultiplicationEulerProduct

theorem **repr_inclR_mul** {d : ℕ} (b : Module.Basis (Fin d) F E) (t : E) (y : R ⊗[F] E) : (fun k => (Algebra.TensorProduct.basis R b).repr (inclR R t * y) k) = (Algebra.leftMulMatrix b t).map (algebraMap F R) *ᵥ (fun k => (Algebra.TensorProduct.basis R b).repr y k)

theorem **map_inclR_mul** [Module.Finite F E] {d : ℕ} (b : Module.Basis (Fin d) F E) (t : E) (c : ℝ≥0) (h : ∀ μ : Measure (Fin d → R), μ.IsAddHaarMeasure → μ.map (fun x => (Algebra.leftMulMatrix b t).map (algebraMap F R) *ᵥ x) = c • μ) (μ : Measure (R ⊗[F] E)) (hμ : μ.IsAddHaarMeasure) : μ.map (fun y => inclR R t * y) = c • μ

theorem **GlobalTheory.CompanionTheory.SW.map_inclR_prodFormula** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ t : E, t ≠ 0 → ∃ (cinf : ℝ≥0) (c : HeightOneSpectrum (𝒪 F) → ℝ≥0), IsUnitProd cinf c ∧ (∀ μ : Measure (Ainf F ⊗[F] E), μ.IsAddHaarMeasure → μ.map (fun y => inclR (Ainf F) t * y) = cinf • μ) ∧ ∀ v : HeightOneSpectrum (𝒪 F), (∀ μ : Measure (Fv F v ⊗[F] E), μ.IsAddHaarMeasure → μ.map (fun y => inclR (Fv F v) t * y) = c v • μ)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ScalarMultiplicationEulerProductStatement

theorem **_root_.GlobalTheory.T.scalar_multiplication_euler_product** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ t : E, t ≠ 0 → ∃ (cinf : ℝ≥0) (c : HeightOneSpectrum (𝒪 F) → ℝ≥0), IsUnitProd cinf c ∧ (∀ μ : Measure (Ainf F ⊗[F] E), μ.IsAddHaarMeasure → μ.map (fun y => inclR (Ainf F) t * y) = cinf • μ) ∧ ∀ v : HeightOneSpectrum (𝒪 F), (∀ μ : Measure (Fv F v ⊗[F] E), μ.IsAddHaarMeasure → μ.map (fun y => inclR (Fv F v) t * y) = c v • μ)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.SchurFunctionCompactSupportModuloUnipotent

lemma **phi_eq_charFun** (μ : Measure V) (x : V) : phi μ x = charFun μ ((2 * Real.pi) • x)

lemma **continuous_phi** (μ : Measure V) [IsFiniteMeasure μ] : Continuous (phi μ)

lemma **ker_nonneg** {b : ℝ} (hb : 0 < b) (w : V) : 0 ≤ ker b w

lemma **continuous_ker** (b : ℝ) : Continuous (ker b : V → ℝ)

lemma **ker_le** {b : ℝ} (hb : 0 < b) (w : V) : ker b w ≤ (Real.pi / b) ^ ((Module.finrank ℝ V : ℝ) / 2)

lemma **integrable_gauss** {b : ℝ} (hb : 0 < b) : Integrable (fun x : V => Real.exp (-b * ‖x‖ ^ 2)) volume

lemma **integrable_ker** {b : ℝ} (hb : 0 < b) : Integrable (ker b : V → ℝ) volume

lemma **integral_ker** {b : ℝ} (hb : 0 < b) : ∫ w : V, ker b w = 1

lemma **gauss_inner** {b : ℝ} (hb : 0 < b) (u : V) : ∫ x : V, Complex.exp (-(b : ℂ) * (‖x‖ : ℂ) ^ 2 + (2 * (Real.pi : ℂ) * Complex.I) * ((inner ℝ u x : ℝ) : ℂ)) = (ker b u : ℂ)

lemma **smoothed_eq** (μ : Measure V) [IsFiniteMeasure μ] {b : ℝ} (hb : 0 < b) (y : V) : ∫ x, phi μ x * (Complex.exp (-(b : ℂ) * (‖x‖ : ℂ) ^ 2) * Complex.exp (-(2 * (Real.pi : ℂ) * Complex.I) * ((inner ℝ y x : ℝ) : ℂ))) = ((∫ z, ker b (z - y) ∂μ : ℝ) : ℂ)

lemma **dens_eq_fourier** (μ : Measure V) (y : V) : dens μ y = 𝓕 (phi μ) y

lemma **bn_pos** (n : ℕ) : 0 < bn n

lemma **tendsto_smoothed** (μ : Measure V) [IsFiniteMeasure μ] (hφ : Integrable (phi μ) volume) (y : V) : Tendsto (fun n : ℕ => ((∫ z, ker (bn n) (z - y) ∂μ : ℝ) : ℂ)) atTop (𝓝 (dens μ y))

lemma **dens_im** (μ : Measure V) [IsFiniteMeasure μ] (hφ : Integrable (phi μ) volume) (y : V) : (dens μ y).im = 0

lemma **dens_re_nonneg** (μ : Measure V) [IsFiniteMeasure μ] (hφ : Integrable (phi μ) volume) (y : V) : 0 ≤ (dens μ y).re

lemma **integrable_ker_shift** (μ : Measure V) [IsFiniteMeasure μ] {b : ℝ} (hb : 0 < b) (y : V) : Integrable (fun z => ker b (z - y)) μ

lemma **lintegral_smoothed** (μ : Measure V) [IsFiniteMeasure μ] {b : ℝ} (hb : 0 < b) : ∫⁻ y, ENNReal.ofReal (∫ z, ker b (z - y) ∂μ) = μ Set.univ

lemma **lintegral_dens_le** (μ : Measure V) [IsFiniteMeasure μ] (hφ : Integrable (phi μ) volume) : ∫⁻ y, ENNReal.ofReal (dens μ y).re ≤ μ Set.univ

theorem **exists_density** (μ : Measure V) [IsFiniteMeasure μ] (hφ : Integrable (phi μ) volume) : ∃ D : V → ℝ, Continuous D ∧ (∀ y, 0 ≤ D y) ∧ (∀ y, (D y : ℂ) = dens μ y) ∧ μ = (volume : Measure V).withDensity (fun y => ENNReal.ofReal (D y))

theorem **density_of_integrable_charFun** : ∀ (κ : Type) [Fintype κ] (m : Measure (κ → ℝ)), IsFiniteMeasure m → Integrable (fun x : κ → ℝ => ∫ z, Complex.exp ((2 * (Real.pi : ℂ) * Complex.I) * ((∑ w, z w * x w : ℝ) : ℂ)) ∂m) → ∃ D : (κ → ℝ) → ℝ, Continuous D ∧ (∀ y, 0 ≤ D y) ∧ (∀ y, (D y : ℂ) = ∫ x : κ → ℝ, (∫ z, Complex.exp ((2 * (Real.pi : ℂ) * Complex.I) * ((∑ w, z w * x w : ℝ) : ℂ)) ∂m) * Complex.exp (-(2 * (Real.pi : ℂ) * Complex.I) * ((∑ w, y w * x w : ℝ) : ℂ))) ∧ ∀ U : Set (κ → ℝ), MeasurableSet U → m U = ∫⁻ y in U, ENNReal.ofReal (D y)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.SchurFunctionCompactSupportModuloUnipotentStatement

theorem **_root_.GlobalTheory.T.schur_function_compact_support_modulo_unipotent** : ∀ (κ : Type) [Fintype κ] (m : Measure (κ → ℝ)), IsFiniteMeasure m → Integrable (fun x : κ → ℝ => ∫ z, Complex.exp ((2 * (Real.pi : ℂ) * Complex.I) * ((∑ w, z w * x w : ℝ) : ℂ)) ∂m) → ∃ D : (κ → ℝ) → ℝ, Continuous D ∧ (∀ y, 0 ≤ D y) ∧ (∀ y, (D y : ℂ) = ∫ x : κ → ℝ, (∫ z, Complex.exp ((2 * (Real.pi : ℂ) * Complex.I) * ((∑ w, z w * x w : ℝ) : ℂ)) ∂m) * Complex.exp (-(2 * (Real.pi : ℂ) * Complex.I) * ((∑ w, y w * x w : ℝ) : ℂ))) ∧ ∀ U : Set (κ → ℝ), MeasurableSet U → m U = ∫⁻ y in U, ENNReal.ofReal (D y)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.SchurFunctionSmoothRightInvariant

theorem **tsum_integral_swap** : ∀ (K X : Type) [Countable K] [MeasurableSpace X] (μ : Measure X) [IsFiniteMeasure μ] (T : K → X → ℂ) (C : ℝ), (∀ b, AEStronglyMeasurable (T b) μ) → (∀ x, Summable fun b => ‖T b x‖) → (∀ x, ∑' b, ‖T b x‖ ≤ C) → Integrable (fun x => ∑' b, T b x) μ ∧ Summable (fun b => ∫ x, T b x ∂μ) ∧ ∫ x, ∑' b, T b x ∂μ = ∑' b, ∫ x, T b x ∂μ

theorem **tsum_cosets** : ∀ (K : Type) [AddCommGroup K] (H : AddSubgroup K) (f : K → ℂ), Summable f → ∑' b, f b = ∑' q : K ⧸ H, ∑' h : H, f (q.out + (h : K))

theorem **unfold_char** : ∀ (κ : Type) [Fintype κ] (Λ : AddSubgroup (κ → ℝ)) [Countable Λ] (D : Set (κ → ℝ)), IsAddFundamentalDomain Λ D volume → ∀ (c e : (κ → ℝ) → ℂ), Integrable c → Continuous e → (∀ x, ‖e x‖ = 1) → (∀ x y, e (x + y) = e x * e y) → (∀ l ∈ Λ, e l = 1) → ∀ s : κ → ℝ, ∑' l : Λ, ∫ x in D, c (x + (s + (l : κ → ℝ))) * e x = e (-s) * ∫ x, c x * e x

theorem **hasSum_integral_cosets** : ∀ (G Q : Type) [AddCommGroup G] [TopologicalSpace G] [IsTopologicalAddGroup G] [MeasurableSpace G] [BorelSpace G] [Countable Q] (lf : Measure G) [lf.IsAddLeftInvariant] (s : Q → G) (W : Set G), IsOpen W → (∀ y, ∃ q, y - s q ∈ W) → (∀ q q', (∃ w ∈ W, ∃ w' ∈ W, s q + w = s q' + w') → q = q') → ∀ g : G → ℂ, Integrable g lf → (∀ y, ∀ w ∈ W, g (y + w) = g y) → HasSum (fun q => ((lf W).toReal : ℂ) * g (s q)) (∫ y, g y ∂lf)

theorem **cosets_of_multiples** : ∀ (F : Type) [Field F] [NumberField F] (N : ℕ), 0 < N → ∀ H : AddSubgroup F, (∀ b, b ∈ H ↔ ∃ r : 𝒪 F, b = (N : F) * (r : F)) → (∀ y : A F, ∃ q : F ⧸ H, y - algebraMap F (A F) q.out ∈ {x : A F | ∃ z ∈ Oh F, x = (N : A F) * z}) ∧ ∀ q q' : F ⧸ H, (∃ w ∈ {x : A F | ∃ z ∈ Oh F, x = (N : A F) * z}, ∃ w' ∈ {x : A F | ∃ z ∈ Oh F, x = (N : A F) * z}, algebraMap F (A F) q.out + w = algebraMap F (A F) q'.out + w') → q = q'

theorem **mass_multiples** : ∀ (F : Type) [Field F] [NumberField F] [MeasurableSpace (A F)] [BorelSpace (A F)] (lf : Measure (A F)), lf.IsAddHaarMeasure → lf (Oh F) = 1 → ∀ N : ℕ, 0 < N → (lf {x : A F | ∃ z ∈ Oh F, x = (N : A F) * z}).toReal * (N : ℝ) ^ Module.finrank ℚ F = 1

theorem **sum_places_eq_trace** : ∀ (F : Type) [Field F] [NumberField F], IsTotallyReal F → ∀ ρ : InfinitePlace F → (F →+* ℝ), (∀ w x, ((ρ w x : ℝ) : ℂ) = w.embedding x) → ∀ z : F, ∑ w, ρ w z = ((Algebra.trace ℚ F z : ℚ) : ℝ)

theorem **Oh_mul_mem** : ∀ (F : Type) [Field F] [NumberField F] (x y : A F), x ∈ Oh F → y ∈ Oh F → x * y ∈ Oh F

theorem **fourier_coeff_eis** : ∀ (F : Type) [Field F] [NumberField F] [MeasurableSpace (A F)] [BorelSpace (A F)] (lf : Measure (A F)), lf.IsAddHaarMeasure → lf (Oh F) = 1 → ∀ ψ : AddChar (A F) ℂ, IsStdChar F ψ → ∀ (ι : Type) [Fintype ι] [DecidableEq ι] (M : ℕ) (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (P : InfinitePlace F → Matrix ι ι ℝ), QSetup M B ρ P → 3 ≤ M → ∀ φ : (ι → A F) → ℂ, SB φ → Integrable (fstar ψ (hf F) (Bf B) (muF B lf) φ) lf → Integrable (fun x : InfinitePlace F → ℝ => cG B ρ P (fun w =>

(x w : ℂ) + Complex.I)) → ∀ β : F, β ≠ 0 → ∀ N : ℕ, 0 < N → (∃ r : 𝒪 F, (r : F) = (N : F) * β) → (∀ x ∈ Oh F, ∀ y : A F, fstar ψ (hf F) (Bf B) (muF B lf) φ (y + (N : A F) * x) = fstar ψ (hf F) (Bf B) (muF B lf) φ y) → ∀ D : Set (InfinitePlace F → ℝ), IsAddFundamentalDomain (latR ρ N) D volume → IntegrableOn (fun x : InfinitePlace F → ℝ => eis ψ lf B ρ P φ (fun w => (x w : ℂ) + Com

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.SchurFunctionSmoothRightInvariantStatement

theorem **_root_.GlobalTheory.T.schur_function_smooth_right_invariant** : ∀ (F : Type) [Field F] [NumberField F] [MeasurableSpace (A F)] [BorelSpace (A F)] (lf : Measure (A F)), lf.IsAddHaarMeasure → lf (Oh F) = 1 → ∀ ψ : AddChar (A F) ℂ, IsStdChar F ψ → ∀ (ι : Type) [Fintype ι] [DecidableEq ι] (M : ℕ) (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (P : InfinitePlace F → Matrix ι ι ℝ), QSetup M B ρ P → 3 ≤ M → ∀ φ : (ι → A F) → ℂ, SB φ → Integrable (fstar ψ (hf F) (Bf B) (muF B lf) φ) lf → Integrable (fun x : InfinitePlace F → ℝ => cG B ρ P (fun w => (x w : ℂ) + Complex.I)) → ∀ β : F, β ≠ 0 → ∀ N : ℕ, 0 < N → (∃ r : 𝒪 F, (r : F) = (N : F) * β) → (∀ x ∈ Oh F, ∀ y : A F, fstar ψ (hf F) (Bf B) (muF B lf) φ (y + (N : A F) * x) = fstar ψ (hf F) (Bf B) (muF B lf) φ y) → ∀ D : Set (InfinitePlace F → ℝ), IsAddFundamentalDomain (latR ρ N) D volume → IntegrableOn (fun x : InfinitePlace F → ℝ =>

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.SchurFunctionWhittakerEquivariant

lemma **oh_add** {x y : A F} (hx : x ∈ Oh F) (hy : y ∈ Oh F) : x + y ∈ Oh F

lemma **oh_neg** {x : A F} (hx : x ∈ Oh F) : -x ∈ Oh F

lemma **oh_mul** {x y : A F} (hx : x ∈ Oh F) (hy : y ∈ Oh F) : x * y ∈ Oh F

lemma **oh_alg** (r : 𝒪 F) : algebraMap F (A F) (r : F) ∈ Oh F

lemma **nat_cast_eq** (n : ℕ) : (n : A F) = algebraMap F (A F) (n : F)

lemma **oh_nat** (n : ℕ) : (n : A F) ∈ Oh F

lemma **nat_inv_mul** {N : ℕ} (hN : 0 < N) : algebraMap F (A F) (1 / (N : F)) * (N : A F) = 1

lemma **setS_isOpen** (a : ℕ) : IsOpen {x : A F | (a : A F) * x ∈ Oh F}

lemma **setS_isCompact** {a : ℕ} (ha : 0 < a) : IsCompact {x : A F | (a : A F) * x ∈ Oh F}

lemma **setL_eq** (ι : Type) (a : ℕ) : {x : ι → A F | ∀ i, (a : A F) * x i ∈ Oh F} = Set.pi Set.univ fun _ => {x : A F | (a : A F) * x ∈ Oh F}

lemma **setL_isCompact** (ι : Type) {a : ℕ} (ha : 0 < a) : IsCompact {x : ι → A F | ∀ i, (a : A F) * x i ∈ Oh F}

lemma **setL_isOpen** (ι : Type) [Fintype ι] (a : ℕ) : IsOpen {x : ι → A F | ∀ i, (a : A F) * x i ∈ Oh F}

lemma **indL_SB** (ι : Type) [Fintype ι] {a : ℕ} (ha : 0 < a) : SB (indL a : (ι → A F) → ℂ)

lemma **hs_add** {N : ℕ} {x y : A F} (hx : x ∈ Hs F N) (hy : y ∈ Hs F N) : x + y ∈ Hs F N

lemma **hs_neg** {N : ℕ} {x : A F} (hx : x ∈ Hs F N) : -x ∈ Hs F N

lemma **hs_zero** (N : ℕ) : (0 : A F) ∈ Hs F N

lemma **hs_alg** (N : ℕ) (r : 𝒪 F) : algebraMap F (A F) ((N : F) * (r : F)) ∈ Hs F N

lemma **hs_sub_oh** {N : ℕ} {x : A F} (hx : x ∈ Hs F N) : x ∈ Oh F

lemma **hs_isOpen** {N : ℕ} (hN : 0 < N) : IsOpen (Hs F N)

lemma **hs_alg_iff** {N : ℕ} (hN : 0 < N) {c : F} (h : algebraMap F (A F) c ∈ Hs F N) : ∃ r : 𝒪 F, c = (N : F) * (r : F)

theorem **integrable_of_periodic** [MeasurableSpace (A F)] [BorelSpace (A F)] (lf : Measure (A F)) (hlf : lf.IsAddHaarMeasure) (h1 : lf (Oh F) = 1) (f : A F → ℂ) {N : ℕ} (hN : 0 < N) (hper : ∀ x ∈ Oh F, ∀ y, f (y + (N : A F) * x) = f y) {C : ℝ} {M : ℕ} (hM : 3 ≤ M) (hdec : ∀ b : F, ‖f (algebraMap F (A F) b)‖ ≤ C * ((den b : ℝ) ^ M)⁻¹) : Integrable f lf

theorem **density_main** : ∀ (F : Type) [Field F] [NumberField F] [MeasurableSpace (A F)] [BorelSpace (A F)] (lf : Measure (A F)), lf.IsAddHaarMeasure → lf (Oh F) = 1 → ∀ ψ : AddChar (A F) ℂ, IsStdChar F ψ → ∀ (ι : Type) [Fintype ι] [DecidableEq ι] (M : ℕ) (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (P : InfinitePlace F → Matrix ι ι ℝ), QSetup M B ρ P → 3 ≤ M → ∀ a : ℕ, 0 < a → ∃ Df : A F → ℝ, Continuous Df ∧ (∀ y, 0 ≤ Df y) ∧ (∀ U : Set (A F), MeasurableSet U → (Measure.pi fun _ : ι => lf) {x | (∀ i, (a : A F) * x i ∈ Oh F) ∧ qf (hf F) (Bf B) x ∈ U} = ∫⁻ y in U, ENNReal.ofReal (Df y) ∂lf) ∧ Integrable (fstar ψ (hf F) (Bf B) (muF B lf) (indL a)) lf ∧ ∀ z : A F, ∫ y, fstar ψ (hf F) (Bf B) (muF B lf) (indL a) y * ψ (-z * y) ∂lf = ((kap F ^ (2 * M - 2) * (Real.sqrt |((Algebra.norm ℚ B.det : ℚ) : ℝ)|)⁻¹ * Df z : ℝ) : ℂ)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.SchurFunctionWhittakerEquivariantStatement

theorem **_root_.GlobalTheory.T.schur_function_whittaker_equivariant** : ∀ (F : Type) [Field F] [NumberField F] [MeasurableSpace (A F)] [BorelSpace (A F)] (lf : Measure (A F)), lf.IsAddHaarMeasure → lf (Oh F) = 1 → ∀ ψ : AddChar (A F) ℂ, IsStdChar F ψ → ∀ (ι : Type) [Fintype ι] [DecidableEq ι] (M : ℕ) (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (P : InfinitePlace F → Matrix ι ι ℝ), QSetup M B ρ P → 3 ≤ M → ∀ a : ℕ, 0 < a → ∃ Df : A F → ℝ, Continuous Df ∧ (∀ y, 0 ≤ Df y) ∧ (∀ U : Set (A F), MeasurableSet U → (Measure.pi fun _ : ι => lf) {x | (∀ i, (a : A F) * x i ∈ Oh F) ∧ qf (hf F) (Bf B) x ∈ U} = ∫⁻ y in U, ENNReal.ofReal (Df y) ∂lf) ∧ Integrable (fstar ψ (hf F) (Bf B) (muF B lf) (indL a)) lf ∧ ∀ z : A F, ∫ y, fstar ψ (hf F) (Bf B) (muF B lf) (indL a) y * ψ (-z * y) ∂lf = ((kap F ^ (2 * M - 2) * (Real.sqrt |((Algebra.norm ℚ B.det : ℚ) : ℝ)|)⁻¹ * Df z : ℝ) : ℂ)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.SchwartzBruhatSpanIndicators

theorem **sb_eq_sum_rational_coset_indicators** : ∀ (F : Type) [Field F] [NumberField F] [MeasurableSpace (A F)] [BorelSpace (A F)] (ι : Type) [Fintype ι] [DecidableEq ι] (χ : (ι → A F) → (ι → A F) → ℂ) (μ : Measure (ι → A F)), NADual (ι → A F) χ μ → (∀ N : ℕ, 0 < N → IsOpen (latN F ι N : Set (ι → A F))) → ∀ φ : (ι → A F) → ℂ, SB φ → ∃ N : ℕ, 0 < N ∧ ∃ (s : Finset (ι → F)) (c : (ι → F) → ℂ), φ = ∑ a ∈ s, c a • indC N a

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.SchwartzBruhatSpanIndicatorsStatement

theorem **_root_.GlobalTheory.T.schwartz_bruhat_span_indicators** : ∀ (F : Type) [Field F] [NumberField F] [MeasurableSpace (A F)] [BorelSpace (A F)] (ι : Type) [Fintype ι] [DecidableEq ι] (χ : (ι → A F) → (ι → A F) → ℂ) (μ : Measure (ι → A F)), NADual (ι → A F) χ μ → (∀ N : ℕ, 0 < N → IsOpen (latN F ι N : Set (ι → A F))) → ∀ φ : (ι → A F) → ℂ, SB φ → ∃ N : ℕ, 0 < N ∧ ∃ (s : Finset (ι → F)) (c : (ι → F) → ℂ), φ = ∑ a ∈ s, c a • indC N a

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.SelfDualStandardMeasures

theorem **psiV_std** (v : HeightOneSpectrum (𝒪 F)) (y : Fv F v) : psiV F (Circle.coeHom.compAddChar (adelicChar F)) v y = ((localAdelicChar F v y : Circle) : ℂ)

theorem **coe_integers_eq** (v : HeightOneSpectrum (𝒪 F)) : ((ValuativeRel.valuation (Fv F v)).integer : Set (Fv F v)) = (v.adicCompletionIntegers F : Set (Fv F v))

theorem **smul_eq_selfDualMeasure** (v : HeightOneSpectrum (𝒪 F)) (lam : Measure (Fv F v)) (h : IsStdV v lam) : cV F v • lam = GroundTheory.LocalField.selfDualMeasure (localAdelicChar F v)

theorem **selfDual_std** : ∀ (F : Type) [Field F] [NumberField F], IsTotallyReal F → ∀ (laminf : Measure (Ainf F)) (lam : ∀ v : HeightOneSpectrum (𝒪 F), Measure (Fv F v)), IsStdInf laminf → (∀ v, IsStdV v (lam v)) → ∃ ψ, IsCharAF F ψ ∧ ∃ (cinf : ℝ≥0) (c : HeightOneSpectrum (𝒪 F) → ℝ≥0), IsUnitProd cinf c ∧ (SelfDual (cinf • laminf) (fun s t => psiInf F ψ (s * t))) ∧ ∀ v : HeightOneSpectrum (𝒪 F), (SelfDual (c v • lam v) (fun s t => psiV F ψ v (s * t)))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.SelfDualStandardMeasuresStatement

theorem **_root_.GlobalTheory.T.self_dual_standard_measures** : ∀ (F : Type) [Field F] [NumberField F], IsTotallyReal F → ∀ (laminf : Measure (Ainf F)) (lam : ∀ v : HeightOneSpectrum (𝒪 F), Measure (Fv F v)), IsStdInf laminf → (∀ v, IsStdV v (lam v)) → ∃ ψ, IsCharAF F ψ ∧ ∃ (cinf : ℝ≥0) (c : HeightOneSpectrum (𝒪 F) → ℝ≥0), IsUnitProd cinf c ∧ (SelfDual (cinf • laminf) (fun s t => psiInf F ψ (s * t))) ∧ ∀ v : HeightOneSpectrum (𝒪 F), (SelfDual (c v • lam v) (fun s t => psiV F ψ v (s * t)))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.SelfdualMeasureTransportEquiv

theorem **selfDual_map_of_comp** {V V' : Type} [AddCommGroup V] [TopologicalSpace V] [IsTopologicalAddGroup V] [MeasurableSpace V] [BorelSpace V] [AddCommGroup V'] [TopologicalSpace V'] [IsTopologicalAddGroup V'] [MeasurableSpace V'] [BorelSpace V'] (e : V ≃ₜ+ V') (μ : Measure V) (χ : V' → V' → ℂ) (h : SelfDual μ (fun x y => χ (e x) (e y))) : SelfDual (μ.map e) χ

theorem **selfDual_transport** (V V' : Type) [AddCommGroup V] [TopologicalSpace V] [IsTopologicalAddGroup V] [MeasurableSpace V] [BorelSpace V] [AddCommGroup V'] [TopologicalSpace V'] [IsTopologicalAddGroup V'] [MeasurableSpace V'] [BorelSpace V'] (e : V ≃ₜ+ V') (μ : Measure V) (χ : V' → V' → ℂ) : SelfDual μ (fun x y => χ (e x) (e y)) ↔ SelfDual (μ.map e) χ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.SelfdualMeasureTransportEquivStatement

theorem **_root_.GlobalTheory.T.selfdual_measure_transport_equiv** : ∀ (V V' : Type) [AddCommGroup V] [TopologicalSpace V] [IsTopologicalAddGroup V] [MeasurableSpace V] [BorelSpace V] [AddCommGroup V'] [TopologicalSpace V'] [IsTopologicalAddGroup V'] [MeasurableSpace V'] [BorelSpace V'] (e : V ≃ₜ+ V') (μ : Measure V) (χ : V' → V' → ℂ), SelfDual μ (fun x y => χ (e x) (e y)) ↔ SelfDual (μ.map e) χ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.SelfdualMeasureTwistBySymmetricAuto

theorem **selfDual_twist** : ∀ (V : Type) [AddCommGroup V] [TopologicalSpace V] [IsTopologicalAddGroup V] [LocallyCompactSpace V] [SecondCountableTopology V] [T2Space V] [MeasurableSpace V] [BorelSpace V] (μ : Measure V) (χ : V → V → ℂ), SelfDual μ χ → ∀ (B : V ≃ₜ+ V), (∀ x y, χ x (B y) = χ (B x) y) → ∀ c : ℝ≥0, μ.map B = c • μ → SelfDual ((NNReal.sqrt c)⁻¹ • μ) (fun x y => χ x (B y))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.SelfdualMeasureTwistBySymmetricAutoStatement

theorem **_root_.GlobalTheory.T.selfdual_measure_twist_by_symmetric_auto** : ∀ (V : Type) [AddCommGroup V] [TopologicalSpace V] [IsTopologicalAddGroup V] [LocallyCompactSpace V] [SecondCountableTopology V] [T2Space V] [MeasurableSpace V] [BorelSpace V] (μ : Measure V) (χ : V → V → ℂ), SelfDual μ χ → ∀ (B : V ≃ₜ+ V), (∀ x y, χ x (B y) = χ (B x) y) → ∀ c : ℝ≥0, μ.map B = c • μ → SelfDual ((NNReal.sqrt c)⁻¹ • μ) (fun x y => χ x (B y))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.SiegelDensitiesOfHermitianForm

theorem **rescale_tprod** {ι : Type} (a : ℝ≥0) (r : ι → ℝ≥0) (h1 : ∀ᶠ v in Filter.cofinite, r v = 1) (h2 : a ∗ ∏ᶠ v, r v = 1) (y : ℂ) (x : ι → ℂ) (hx : Multipliable x) : Multipliable (fun v => r v • x v) ∧ (a • y) ∗ ∏' v, r v • x v = y ∗ ∏' v, x v

theorem **siegelDensities** (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E) (hCM : IsCMExt F E σ) (M : ℕ) (J : Matrix (Fin M) (Fin M) E) (hJ : IsHermND σ J) : SiegelDensities F E σ M J

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.SiegelDensitiesOfHermitianFormStatement

theorem **_root_.GlobalTheory.T.siegel_densities_of_hermitian_form** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → SiegelDensities F E σ M J

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.SiegelMatrixSymmetricPosdefImag

theorem **dotProduct_blockDiagonal_mulVec** {R ι W : Type} [CommSemiring R] [Fintype ι] [DecidableEq ι] [Fintype W] [DecidableEq W] (f : W → Matrix ι ι R) (y : ι × W → R) : y ⬝ᵥ (blockDiagonal f *ᵥ y) = ∑ w, (fun i => y (i, w)) ⬝ᵥ (f w *ᵥ fun i => y (i, w))

theorem **dotProduct_block_ofReal** {ι : Type} [Fintype ι] (a b : ℝ) (C P : Matrix ι ι ℝ) (y : ι → ℝ) : (fun i => (y i : ℂ)) ⬝ᵥ ((Matrix.of fun i j => (a : ℂ) ∗ ((C i j : ℝ) : ℂ) + Complex.I ∗ (b : ℂ) ∗ ((P i j : ℝ) : ℂ)) *ᵥ fun i => (y i : ℂ)) = (a : ℂ) ∗ ((y ⬝ᵥ (C *ᵥ y) : ℝ) : ℂ) + Complex.I ∗ (b : ℂ) ∗ ((y ⬝ᵥ (P *ᵥ y) : ℝ) : ℂ)

theorem **ZM_eq_blockDiagonal** {F : Type} [Field F] [NumberField F] {ι : Type} [Fintype ι] [DecidableEq ι] [DecidableEq (InfinitePlace F)] (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (P : InfinitePlace F → Matrix ι ι ℝ) (τ : InfinitePlace F → ℂ) : ZM B ρ P τ = blockDiagonal (ZB B ρ P τ)

theorem **ZM_map_im** {F : Type} [Field F] [NumberField F] {ι : Type} [Fintype ι] [DecidableEq ι] [DecidableEq (InfinitePlace F)] (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (P : InfinitePlace F → Matrix ι ι ℝ) (τ : InfinitePlace F → ℂ) : (ZM B ρ P τ).map Complex.im = blockDiagonal fun w => (τ w).im • P w

theorem **ZM_symm_posDef_gauss** (F : Type) [Field F] [NumberField F] (ι : Type) [Fintype ι] [DecidableEq ι] [DecidableEq (InfinitePlace F)] (M : ℕ) (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (P : InfinitePlace F → Matrix ι ι ℝ) : QSetup M B ρ P → ∀ τ : InfinitePlace F → ℂ, (∀ w, 0 < (τ w).im) → (ZM B ρ P τ).IsSymm ∧ ((ZM B ρ P τ).map Complex.im).PosDef ∧ ∀ x : ι × InfinitePlace F → ℝ, Complex.exp ((Real.pi : ℂ) ∗ Complex.I ∗ ((fun i => (x i : ℂ)) ⬝ᵥ (ZM B ρ P τ *ᵥ fun i => (x i : ℂ)))) = gauss B ρ P τ (cur x)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.SiegelMatrixSymmetricPosdefImagStatement

theorem **_root_.GlobalTheory.T.siegel_matrix_symmetric_posdef_imag** : ∀ (F : Type) [Field F] [NumberField F] (ι : Type) [Fintype ι] [DecidableEq ι] [DecidableEq (InfinitePlace F)] (M : ℕ) (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (P : InfinitePlace F → Matrix ι ι ℝ), QSetup M B ρ P → ∀ τ : InfinitePlace F → ℂ, (∀ w, 0 < (τ w).im) → (ZM B ρ P τ).IsSymm ∧ ((ZM B ρ P τ).map Complex.im).PosDef ∧ ∀ x : ι × InfinitePlace F → ℝ, Complex.exp ((Real.pi : ℂ) ∗ Complex.I ∗ ((fun i => (x i : ℂ)) ⬝ᵥ (ZM B ρ P τ *ᵥ fun i => (x i : ℂ)))) = gauss B ρ P τ (cur x)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.SiegelMeanValueIntegrable

theorem **mean_thetaS_eq_integral** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (μ : Measure (HA (AF F) σ J)) (ℱ : Set (HA (AF F) σ J)), IsHaarH F E σ M J μ → IsFDH F E σ M J μ ℱ → ∀ (n : ℕ) (S : Set (Fin n → Fin M → E)) (Φ : (Fin n → Fin M → AF F ⊗[F] E) → ℂ), Measurable Φ → Integrable Φ (thetaMeas σ J μ ℱ S) → meanC μ ℱ (thetaS F E σ M J Φ S) = ∫ X, Φ X ∂(thetaMeas σ J μ ℱ S)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.SiegelMeanValueIntegrableStatement

theorem **_root_.GlobalTheory.T.siegel_mean_value_integrable** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (μ : Measure (HA (AF F) σ J)) (ℱ : Set (HA (AF F) σ J)), IsHaarH F E σ M J μ → IsFDH F E σ M J μ ℱ → ∀ (n : ℕ) (S : Set (Fin n → Fin M → E)) (Φ : (Fin n → Fin M → AF F ⊗[F] E) → ℂ), Measurable Φ → Integrable Φ (thetaMeas σ J μ ℱ S) → meanC μ ℱ (thetaS F E σ M J Φ S) = ∫ X, Φ X ∂(thetaMeas σ J μ ℱ S)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.SiegelMeanValueNonnegative

theorem **theta_lint_eq** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (μ : Measure (HA (AF F) σ J)) (ℱ : Set (HA (AF F) σ J)), IsHaarH F E σ M J μ → IsFDH F E σ M J μ ℱ → ∀ (n : ℕ) (S : Set (Fin n → Fin M → E)) (Φ : (Fin n → Fin M → AF F ⊗[F] E) → ℝ≥0∞), Measurable Φ → ∫⁻ X, Φ X ∂(thetaMeas σ J μ ℱ S) = (μ ℱ)⁻¹ ∗ ∫⁻ h in ℱ, ∑' X : S, Φ (actW (AF F) σ J h⁻¹ (ratTup (AF F) (X : Fin n → Fin M → E))) ∂μ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.SiegelMeanValueNonnegativeStatement

theorem **_root_.GlobalTheory.T.siegel_mean_value_nonnegative** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (μ : Measure (HA (AF F) σ J)) (ℱ : Set (HA (AF F) σ J)), IsHaarH F E σ M J μ → IsFDH F E σ M J μ ℱ → ∀ (n : ℕ) (S : Set (Fin n → Fin M → E)) (Φ : (Fin n → Fin M → AF F ⊗[F] E) → ℝ≥0∞), Measurable Φ → ∫⁻ X, Φ X ∂(thetaMeas σ J μ ℱ S) = (μ ℱ)⁻¹ ∗ ∫⁻ h in ℱ, ∑' X : S, Φ (actW (AF F) σ J h⁻¹ (ratTup (AF F) (X : Fin n → Fin M → E))) ∂μ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.SiegelWeilArchimedeanDensity

theorem **size_eq_sum** {F E : Type} [Field F] [NumberField F] [Field E] [Algebra F E] {M d : ℕ} (b : Module.Basis (Fin d) F E) (e : Ainf F ≃+* (InfinitePlace F → ℝ)) (he : ∀ y w, |e y w| = ‖y w‖) (X : Fin 1 → Fin M → Ainf F ⊗[F] E) : sizeTup b X = ∑ w, ∑ p, |xw e b X w p|

theorem **size_le** {F E : Type} [Field F] [NumberField F] [Field E] [Algebra F E] {M d : ℕ} (b : Module.Basis (Fin d) F E) (e : Ainf F ≃+* (InfinitePlace F → ℝ)) (he : ∀ y w, |e y w| = ‖y w‖) (X : Fin 1 → Fin M → Ainf F ⊗[F] E) {R : ℝ} (hR : 0 < R) (h : ∀ w, ‖xw e b X w‖ ≤ R) : sizeTup b X ≤ ((M ∗ d ∗ Fintype.card (InfinitePlace F) + 1 : ℕ) : ℝ) ∗ R

theorem **ofF_ne_zero_iff** {F E : Type} [Field F] [Field E] [Algebra F E] {M d : ℕ} (b : Module.Basis (Fin d) F E) (x : Fin M × Fin d → F) : ofF b x ≠ 0 ↔ x ≠ 0

theorem **real_bound** {C₀ c R : ℝ} (hc : 1 ≤ c) (hR : 0 < R) (a b : ℕ) : C₀ ∗ ((c ∗ R) ^ a + (c ∗ R) ^ b) ≤ (|C₀| ∗ (c ^ a + c ^ b)) ∗ (R ^ a + R ^ b)

theorem **main_bound** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → 3 ≤ M → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (ρ : InfinitePlace F → (F →+* ℝ)) (e : Ainf F ≃+* (InfinitePlace F → ℝ)), IsHomeomorph e → (∀ x w, e (algebraMap F (Ainf F) x) w = ρ w x) → (∀ y w, |e y w| = ‖y w‖) → ∀ (μ : Measure (HA (AF F) σ J)) (ℱ : Set (HA (AF F) σ J)), IsHaarH F E σ M J μ → IsFDH F E σ M J μ ℱ → ∀ N : ℕ, 0 < N → ∃ C : ℝ, ∀ R : ℝ, 0 < R → ∫⁻ ω, ∑' x : {x : Fin M × Fin d → F // x ≠ 0}, Set.indicator {x : Fin M × Fin d → F | (∀ i v, ((N : Af F) ∗ (gfin σ J b ω ∗ᵥ fun j => algebraMap F (Af F) (x j)) i) v ∈ v.adicCompletionIntegers F) ∧ ∀ w, ‖ginf e σ J b ω w ∗ᵥ fun j => ρ w (x j)‖ ≤ R} (fun _ => (1 : ℝ≥0∞)) x.1 ∂(prH μ ℱ) ≤ ENNReal.ofReal (C ∗ (R ^ (Module.finrank ℚ F ∗ M) + R ^ (2

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.SiegelWeilArchimedeanDensityStatement

theorem **_root_.GlobalTheory.T.average_rational_vector_count_ball_bound** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → 3 ≤ M → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (ρ : InfinitePlace F → (F →+* ℝ)) (e : Ainf F ≃+* (InfinitePlace F → ℝ)), IsHomeomorph e → (∀ x w, e (algebraMap F (Ainf F) x) w = ρ w x) → (∀ y w, |e y w| = ‖y w‖) → ∀ (μ : Measure (HA (AF F) σ J)) (ℱ : Set (HA (AF F) σ J)), IsHaarH F E σ M J μ → IsFDH F E σ M J μ ℱ → ∀ N : ℕ, 0 < N → ∃ C : ℝ, ∀ R : ℝ, 0 < R → ∫⁻ ω, ∑' x : {x : Fin M × Fin d → F // x ≠ 0}, Set.indicator {x : Fin M × Fin d → F | (∀ i v, ((N : Af F) ∗ (gfin σ J b ω ∗ᵥ fun j => algebraMap F (Af F) (x j)) i) v ∈ v.adicCompletionIntegers F) ∧ ∀ w, ‖ginf e σ J b ω w ∗ᵥ fun j => ρ w (x j)‖ ≤ R} (fun _ => (1 : ℝ≥0∞)) x.1 ∂(prH μ ℱ) ≤ ENNRea

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theorem **exists_test_set** {α : Type} [MeasurableSpace α] (μ : Measure α) [SigmaFinite μ] (h : μ ≠ 0) : ∃ s, MeasurableSet s ∧ μ s ≠ 0 ∧ μ s ≠ ⊤

theorem **fib_facts** {F E : Type} [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E) (hCM : IsCMExt F E σ) {M : ℕ} (J : Matrix (Fin M) (Fin M) E) (hJ : IsHermND σ J) (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R] (hR : IsLocR R) {n : ℕ} (T : Matrix (Fin n) (Fin n) E) (hT : ∀ a b, σ (T b a) = T a b) (hdet : T.det ≠ 0) {d : ℕ} (b : Module.Basis (Fin d) F E) (lam : Measure R) (hlam : lam.IsAddHaarMeasure) (mm : HermR R σ n → Measure (Fin n

→ Fin M → R ⊗[F] E)) (hm : IsGramFib R σ J n (coordW R b lam n M) (coordHerm R σ b lam n) mm) : SigmaFinite (mm (hermOf R σ T hT)) ∧ (mm (hermOf R σ T hT) = 0 → ∀ X, gramR R σ J X ≠ T.map (inclR R))

theorem **borel_tup** {F E : Type} [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E) (hCM : IsCMExt F E σ) (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] (hR : IsLocR R) (n M : ℕ) : BorelSpace (Fin n → Fin M → R ⊗[F] E)

theorem **pureL_insert** {F E : Type} [Field F] [NumberField F] [Field E] [Algebra F E] {d M n : ℕ} (b : Module.Basis (Fin d) F E) [DecidableEq (HeightOneSpectrum (𝒪 F))] (S : Finset (HeightOneSpectrum (𝒪 F))) (v : HeightOneSpectrum (𝒪 F)) (hv : v ∉ S) (φinf : (Fin n → Fin M → Ainf F ⊗[F] E) → ℝ≥0∞) (φ : ∀ v : HeightOneSpectrum (𝒪 F), (Fin n → Fin M → Fv F v ⊗[F] E) → ℝ≥0∞) (X : Fin n → Fin M → AF F ⊗[F] E) : pureL b (insert v S) φinf (Function.update φ v ((latV b v).indicator 1)) X = pureL b S φinf φ X

theorem **exists_isEulerProd** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (n : ℕ), n = 0 ∨ n + 2 ≤ M → ∀ (T : Matrix (Fin n) (Fin n) E) (hT : ∀ a b, σ (T b a) = T a b), T.det ≠ 0 → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (laminf : Measure (Ainf F)) (lam : ∀ v : HeightOneSpectrum (𝒪 F), Measure (Fv F v)), IsStdInf laminf → (∀ v, IsStdV v (lam v)) → ∀ (minf : HermR (Ainf F) σ n → Measure (Fin n → Fin M → Ainf F ⊗[F] E)), IsGramFib (Ainf F) σ J n (coordW (Ainf F) b laminf n M) (coordHerm (Ainf F) σ b laminf n) minf → ∀ (m : ∀ v : HeightOneSpectrum (𝒪 F), HermR (Fv F v) σ n → Measure (Fin n → Fin M → Fv F v ⊗[F] E)), (∀ v, IsGramFib (Fv F v) σ J n (coordW (Fv F v) b (lam v) n M) (coordHerm (Fv F v) σ b (lam v) n)) → ∀ I : Measure (Fin n → Fin M → AF F ⊗[F] E)

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theorem **_root_.GlobalTheory.T.exists_isEulerProd_of_invariant_of_fiber_supported** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (n : ℕ), n = 0 ∨ n + 2 ≤ M → ∀ (T : Matrix (Fin n) (Fin n) E) (hT : ∀ a b, σ (T b a) = T a b), T.det ≠ 0 → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (laminf : Measure (Ainf F)) (lam : ∀ v : HeightOneSpectrum (𝒪 F), Measure (Fv F v)), IsStdInf laminf → (∀ v, IsStdV v (lam v)) → ∀ (minf : HermR (Ainf F) σ n → Measure (Fin n → Fin M → Ainf F ⊗[F] E)), IsGramFib (Ainf F) σ J n (coordW (Ainf F) b laminf n M) (coordHerm (Ainf F) σ b laminf n) minf → ∀ (m : ∀ v : HeightOneSpectrum (𝒪 F), HermR (Fv F v) σ n → Measure (Fin n → Fin M → Fv F v ⊗[F] E)), (∀ v, IsGramFib (Fv F v) σ J n (coordW (Fv F v) b (lam v) n M) (coordHerm (Fv F v) σ b (lam v) n)

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theorem **thetaMeas_of_empty** (σ : E →ₐ[F] E) {M n : ℕ} (J : Matrix (Fin M) (Fin M) E) (μ : Measure (HA (AF F) σ J)) (𝒯 : Set (HA (AF F) σ J)) (S : Set (Fin n → Fin M → E)) (hS : S = ∅) : thetaMeas σ J μ 𝒯 S = 0

theorem **fibre_zero_of_one** (σ : E →ₐ[F] E) (hCM : IsCMExt F E σ) {M : ℕ} (J : Matrix (Fin M) (Fin M) E) (hJ : IsHermND σ J) (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R] (hLoc : IsLocR R) {d : ℕ} (b : Module.Basis (Fin d) F E) (lam : Measure R) (hlam : lam.IsAddHaarMeasure) (t : F) (ht : t ≠ 0) {n : ℕ} (T : Matrix (Fin n) (Fin n) E) (hT' : ∀ a b, σ (blockOne (algebraMap F E t) T b a) = blockOne (algebraMap F E t) T a b) (T1 : Matrix (Fin 1) (Fin 1) E) (hT1e : T1 = fun _ _ => algebraMap F E t) (hT1 : ∀ a b, σ (T1 b a) = T1 a b) (m1 : HermR R σ 1 → Measure (Fin 1 → Fin M → R ⊗[F] E)) (m : HermR R σ (n + 1) → Measure (Fin (n + 1) → Fin M → R ⊗[F] E)) (hm1 : IsGramFib R σ J 1 (coordW R b lam 1 M) (coordHerm R σ b lam 1) m1) (hm : IsGramFib R σ J (n + 1) (coordW R b lam (n + 1) M) (coordHerm R σ b lam (n + 1)) m) (h0 : m1 (hermOf R σ T1 hT1) = 0)

theorem **swMeas_of_not_represented** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ t : F, t ≠ 0 → ∀ n : ℕ, n + 3 ≤ M → ∀ (T : Matrix (Fin n) (Fin n) E) (hT : ∀ a b, σ (T b a) = T a b), T.det ≠ 0 → ∀ (hT' : ∀ a b, σ (blockOne (algebraMap F E t) T b a) = blockOne (algebraMap F E t) T a b), (∀ x : Fin M → E, hE σ J x x ≠ algebraMap F E t) → SwMeas σ J (blockOne (algebraMap F E t) T) hT'

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theorem **_root_.GlobalTheory.T.siegel_weil_measure_block_one_of_not_represented** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ t : F, t ≠ 0 → ∀ n : ℕ, n + 3 ≤ M → ∀ (T : Matrix (Fin n) (Fin n) E) (hT : ∀ a b, σ (T b a) = T a b), T.det ≠ 0 → ∀ (hT' : ∀ a b, σ (blockOne (algebraMap F E t) T b a) = blockOne (algebraMap F E t) T a b), (∀ x : Fin M → E, hE σ J x x ≠ algebraMap F E t) → SwMeas σ J (blockOne (algebraMap F E t) T) hT'

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theorem **of_cofinite** {α : Type} {p : α → Prop} (h : ∀ᶠ v in Filter.cofinite, p v) {S : Finset α} (hS : (Filter.eventually_cofinite.1 h).toFinset ⊆ S) {v : α} (hv : v ∉ S) : p v

theorem **lattice_measurable** {F E : Type} [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E) (hCM : IsCMExt F E σ) {d : ℕ} (b : Module.Basis (Fin d) F E) (n M : ℕ) (v : HeightOneSpectrum (𝒪 F)) : MeasurableSet (latV b v : Set (Fin n → Fin M → Fv F v ⊗[F] E))

theorem **theta_hypothesis** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ t : F, t ≠ 0 → ∀ (n d : ℕ) (b : Module.Basis (Fin d) F E) (I : Measure (Fin n → Fin M → AF F ⊗[F] E)) (minf : Measure (Fin n → Fin M → Ainf F ⊗[F] E)) (m : ∀ v : HeightOneSpectrum (𝒪 F), Measure (Fin n → Fin M → Fv F v ⊗[F] E)), IsEulerProd b I 1 minf m → ∃ S₂ : Finset (HeightOneSpectrum (𝒪 F)), ∀ S, S₂ ⊆ S → ∀ (φinf : (Fin (n + 1) → Fin (M + 1) → Ainf F ⊗[F] E) → ℝ≥0∞) (φ : ∀ v : HeightOneSpectrum (𝒪 F), (Fin (n + 1) → Fin (M + 1) → Fv F v ⊗[F] E) → ℝ≥0∞) (Ψinf : (Fin 1 → Fin (M + 1) → Ainf F ⊗[F] E) → ℝ≥0∞) (Ψ : ∀ v : HeightOneSpectrum (𝒪 F), (Fin 1 → Fin (M + 1) → Fv F v ⊗[F] E) → ℝ≥0∞), Measurable φinf → (∀ v, Measurable (φ v)) → (∀ g : HA (Ainf F) σ (blockOne (algebraMap F E t) J), Ψinf

theorem **loc_families** {F E : Type} [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E) (hCM : IsCMExt F E σ) {M : ℕ} (J : Matrix (Fin M) (Fin M) E) (hJ : IsHermND σ J) (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R] (hR : IsLocR R) {d : ℕ} (b : Module.Basis (Fin d) F E) (lam : Measure R) (hlam : lam.IsAddHaarMeasure) (n : ℕ) : ∃ m, IsGramFib R σ J n (coordW R b lam n M) (coordHerm R σ b lam n) m

theorem **blockOne_step** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ t : F, t ≠ 0 → ∀ n : ℕ, n + 2 ≤ M → ∀ (T : Matrix (Fin n) (Fin n) E) (hT : ∀ a b, σ (T b a) = T a b), T.det ≠ 0 → ∀ (hT' : ∀ a b, σ (blockOne (algebraMap F E t) T b a) = blockOne (algebraMap F E t) T a b), SWMeas σ J T hT → SWMeas σ (blockOne (algebraMap F E t) J) (blockOne (algebraMap F E t) T) hT'

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theorem **_root_.GlobalTheory.T.siegel_weil_measure_add_rank_one_block** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ t : F, t ≠ 0 → ∀ n : ℕ, n + 2 ≤ M → ∀ (T : Matrix (Fin n) (Fin n) E) (hT : ∀ a b, σ (T b a) = T a b), T.det ≠ 0 → ∀ (hT' : ∀ a b, σ (blockOne (algebraMap F E t) T b a) = blockOne (algebraMap F E t) T a b), SWMeas σ J T hT → SWMeas σ (blockOne (algebraMap F E t) J) (blockOne (algebraMap F E t) T) hT'

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theorem **hform_conj** {A : Type} [CommRing A] {M : ℕ} (s : A →+* A) (J G : Matrix (Fin M) (Fin M) A) (x y : Fin M → A) : hform s (cT s G * J * G) x y = hform s J (G.mulVec x) (G.mulVec y)

theorem **gramE_ratAct** (σ : E →ₐ[F] E) {M n : ℕ} (J g : Matrix (Fin M) (Fin M) E) (X : Fin n → Fin M → E) : gramE σ J (ratAct g X) = gramE σ (cT (σ : E →+* E) g * J * g) X

theorem **ratAct_mul** {M n : ℕ} (g h : Matrix (Fin M) (Fin M) E) (X : Fin n → Fin M → E) : ratAct g (ratAct h X) = ratAct (g * h) X

theorem **ratAct_one** {M n : ℕ} (X : Fin n → Fin M → E) : ratAct (1 : Matrix (Fin M) (Fin M) E) X = X

theorem **ratTup_ratAct** (R : Type) [CommRing R] [Algebra F R] {M n : ℕ} (g : GL (Fin M) E) (X : Fin n → Fin M → E) : ratTup (F

theorem **actTup_mul** (R : Type) [CommRing R] [Algebra F R] {M n : ℕ} (g h : GL (Fin M) (R ⊗[F] E)) (X : Fin n → Fin M → R ⊗[F] E) : actTup R (g * h) X = actTup R g (actTup R h X)

theorem **actTup_one** (R : Type) [CommRing R] [Algebra F R] {M n : ℕ} (X : Fin n → Fin M → R ⊗[F] E) : actTup R (1 : GL (Fin M) (R ⊗[F] E)) X = X

theorem **map_actTup** {R R' : Type} [CommRing R] [Algebra F R] [CommRing R'] [Algebra F R'] (φ : R →ₐ[F] R') {M n : ℕ} (g : GL (Fin M) E) (X : Fin n → Fin M → R ⊗[F] E) (a : Fin n) (i : Fin M) : Algebra.TensorProduct.map φ (AlgHom.id F E) (actTup R (bc (inclR R) g) X a i) = actTup R' (bc (inclR R') g) (fun a i => Algebra.TensorProduct.map φ (AlgHom.id F E) (X a i)) a i

theorem **sigma_sigma** [NumberField E] [NumberField F] (σ : E →ₐ[F] E) (hCM : IsCMExt F E σ) (x : E) : σ (σ x) = x

theorem **isHermND_conj** [NumberField E] [NumberField F] (σ : E →ₐ[F] E) (hCM : IsCMExt F E σ) {M : ℕ} (J : Matrix (Fin M) (Fin M) E) (hJ : IsHermND σ J) (g : GL (Fin M) E) : IsHermND σ (cT (σ : E →+* E) (g : Matrix (Fin M) (Fin M) E) * J * (g : Matrix (Fin M) (Fin M) E))

theorem **continuous_actTup** (σ : E →ₐ[F] E) (hCM : IsCMExt F E σ) (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [IsTopologicalRing R] [LocallyCompactSpace R] [SecondCountableTopology R] [T2Space R] {M n : ℕ} (g : GL (Fin M) (R ⊗[F] E)) : Continuous (actTup R g : (Fin n → Fin M → R ⊗[F] E) → _)

theorem **measurable_actTup** (σ : E →ₐ[F] E) (hCM : IsCMExt F E σ) (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [IsTopologicalRing R] [LocallyCompactSpace R] [SecondCountableTopology R] [T2Space R] {M n : ℕ} (g : GL (Fin M) (R ⊗[F] E)) : Measurable (actTup R g : (Fin n → Fin M → R ⊗[F] E) → _)

theorem **mat_inv** (R : Type) [CommRing R] [Algebra F R] (σ : E →ₐ[F] E) {M : ℕ} (J : Matrix (Fin M) (Fin M) E) (h : HA R σ J) : HA.mat R σ J h⁻¹ = (HA.mat R σ J h)⁻¹

theorem **act_conj** (R : Type) [CommRing R] [Algebra F R] (σ : E →ₐ[F] E) {M n : ℕ} {J J' : Matrix (Fin M) (Fin M) E} (G : GL (Fin M) (R ⊗[F] E)) (e : HA R σ J' → HA R σ J) (hmat : ∀ u, HA.mat R σ J (e u) = G * HA.mat R σ J' u * G⁻¹) (u : HA R σ J') (Y : Fin n → Fin M → R ⊗[F] E) : actTup R G (actW R σ J' u⁻¹ Y) = actW R σ J (e u)⁻¹ (actTup R G Y)

theorem **sum_reindex** {ι κ α : Type} [MeasurableSpace α] (e : ι ≃ κ) (m : κ → Measure α) : Measure.sum (fun i => m (e i)) = Measure.sum m

theorem **theta_map** (σ : E →ₐ[F] E) (hCM : IsCMExt F E σ) {M n : ℕ} (J : Matrix (Fin M) (Fin M) E) (hJ : IsHermND σ J) (g : GL (Fin M) E) [LocallyCompactSpace (AF F)] [SecondCountableTopology (AF F)] [T2Space (AF F)] (He : HA (AF F) σ (cT (σ : E →+* E) (g : Matrix (Fin M) (Fin M) E) ∗ J ∗ (g : Matrix (Fin M) (Fin M) E)) ≃ₙ HA (AF F) σ J) (hmat : ∀ u, HA.mat (AF F) σ J (He u) = bc (inclR (AF F)) g ∗ HA.mat (AF F) σ _ u ∗ (bc (inclR (AF F)) g)⁻¹) (μ : Measure (HA (AF F) σ J)) (ℱ : Set (HA (AF F) σ J)) (T : Matrix (Fin n) (Fin n) E) : (thetaMeas σ _ (μ.map He.symm) (He ⁻¹' ℱ) {X | gramE σ (cT (σ : E →+* E) (g : Matrix (Fin M) (Fin M) E) ∗ J ∗ (g : Matrix (Fin M) (Fin M) E)) X = T}).map (actTup (AF F) (bc (inclR (AF F)) g)) = thetaMeas σ J μ ℱ {X | gramE σ J X = T}

theorem **isFD_pull** (σ : E →ₐ[F] E) {M : ℕ} {J J' : Matrix (Fin M) (Fin M) E} (e : HA (AF F) σ J' ≃* HA (AF F) σ J) (hrat : ∀ u, HA.mat (AF F) σ J (e u) ∈ HFin (AF F) σ J ↔ HA.mat (AF F) σ J' u ∈ HFin (AF F) σ J') {μ : Measure (HA (AF F) σ J)} {μ' : Measure (HA (AF F) σ J')} (hq : Measure.QuasiMeasurePreserving e μ' μ) {ℱ : Set (HA (AF F) σ J)} (hℱ : IsFDH F E σ M J μ ℱ) : IsFDH F E σ M J' μ' (e ⁻¹' ℱ)

theorem **local_map** (σ : E →ₐ[F] E) (hCM : IsCMExt F E σ) {M : ℕ} (J : Matrix (Fin M) (Fin M) E) (hJ : IsHermND σ J) (g : GL (Fin M) E) (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R] (hR : IsLocR R) (n : ℕ) (μ : Measure (Fin n → Fin M → R ⊗[F] E)) (ν : Measure (HermR R σ n)) (hμ : μ.IsAddHaarMeasure) (hν : ν.IsAddHaarMeasure) (m m' : HermR R σ n → Measure (Fin n → Fin M → R ⊗[F] E)) (hm : IsGramFib R σ J n μ ν m) (hm' : IsGramFib R σ (cT (σ : E →+* E) (g : Matrix (Fin M) (Fin M) E) ∗ J ∗ (g : Matrix (Fin M) (Fin M) E)) n μ ν m') (a : ℝ≥0) (ha : μ.map (actTup R (bc (inclR R) g)) = a • μ) (S : HermR R σ n) : (m' S).map (actTup R (bc (inclR R) g)) = a • m S

theorem **haar_of_loc** (σ : E →ₐ[F] E) (hCM : IsCMExt F E σ) (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R] (hR : IsLocR R) {d : ℕ} (b : Module.Basis (Fin d) F E) (lam : Measure R) (hlam : lam.IsAddHaarMeasure) (n M : ℕ) : (coordW R b lam n M).IsAddHaarMeasure ∧ (coordHerm R σ b lam n).IsAddHaarMeasure

theorem **measurable_of_loc** (σ : E →ₐ[F] E) (hCM : IsCMExt F E σ) (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] (hR : IsLocR R) {M n : ℕ} (g : GL (Fin M) (R ⊗[F] E)) : Measurable (actTup R g : (Fin n → Fin M → R ⊗[F] E) → _)

theorem **swMeas_of_isometry** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (g : GL (Fin M) E) (J' : Matrix (Fin M) (Fin M) E), J' = cT (σ : E →+* E) (g : Matrix (Fin M) (Fin M) E) ∗ J ∗ g → ∀ n : ℕ, ∀ (T : Matrix (Fin n) (Fin n) E) (hT : ∀ a b, σ (T b a) = T a b), T.det ≠ 0 → SWMeas σ J' T hT → SWMeas σ J T hT

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.SiegelWeilMeasureOfCongruentFormStatement

theorem **_root_.GlobalTheory.T.siegel_weil_measure_of_congruent_form** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (g : GL (Fin M) E) (J' : Matrix (Fin M) (Fin M) E), J' = cT (σ : E →+* E) (g : Matrix (Fin M) (Fin M) E) ∗ J ∗ g → ∀ n : ℕ, ∀ (T : Matrix (Fin n) (Fin n) E) (hT : ∀ a b, σ (T b a) = T a b), T.det ≠ 0 → SWMeas σ J' T hT → SWMeas σ J T hT

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.SiegelWeilMeasureOfCongruentTarget

theorem **rE_eq** {M n : ℕ} (X : Fin n → Fin M → E) (G : Matrix (Fin n) (Fin n) E) : Matrix.of (rE X G) = Gᵀ ∗ Matrix.of X

theorem **rE_rE** {M n : ℕ} (X : Fin n → Fin M → E) (G G' : Matrix (Fin n) (Fin n) E) : rE (rE X G) G' = rE X (G ∗ G')

theorem **rE_one** {M n : ℕ} (X : Fin n → Fin M → E) : rE X (1 : Matrix (Fin n) (Fin n) E) = X

theorem **gramE_eq** (σ : E →ₐ[F] E) {M n : ℕ} (J : Matrix (Fin M) (Fin M) E) (X : Fin n → Fin M → E) : gramE σ J X = (Matrix.of X).map (σ : E →+* E) ∗ J ∗ (Matrix.of X)ᵀ

theorem **gramE_rE** (σ : E →ₐ[F] E) {M n : ℕ} (J : Matrix (Fin M) (Fin M) E) (X : Fin n → Fin M → E) (G : Matrix (Fin n) (Fin n) E) : gramE σ J (rE X G) = cT (σ : E →+* E) G ∗ gramE σ J X ∗ G

theorem **ratTup_rE** {M n : ℕ} (X : Fin n → Fin M → E) (G : Matrix (Fin n) (Fin n) E) : ratTup (F

theorem **cT_map_incl** (σ : E →ₐ[F] E) {n : ℕ} (G : Matrix (Fin n) (Fin n) E) : cT (sigR R σ) (G.map (inclR R)) = (cT (σ : E →+* E) G).map (inclR R)

theorem **tauH_hermOf** (σ : E →ₐ[F] E) (hinv : ∀ x, σ (σ x) = x) {n : ℕ} (H T T' : Matrix (Fin n) (Fin n) E) (hT : ∀ a b, σ (T b a) = T a b) (hT' : ∀ a b, σ (T' b a) = T' a b) (hrel : cT (σ : E →+* E) H ∗ T' ∗ H = T) : tauH R σ hinv (H.map (inclR R)) (hermOf R σ T' hT') = hermOf R σ T hT

theorem **measurable_rmul** (σ : E →ₐ[F] E) (hcm : IsCMExt F E σ) (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [IsTopologicalRing R] [LocallyCompactSpace R] [SecondCountableTopology R] [T2Space R] (n M : ℕ) (G : Matrix (Fin n) (Fin n) (R ⊗[F] E)) : Measurable (fun X : Fin n → Fin M → R ⊗[F] E => rmul R X G)

theorem **unitary_conjugate_local** (σ : E →ₐ[F] E) (hcm : IsCMExt F E σ) (hinv : ∀ x, σ (σ x) = x) {M : ℕ} (J : Matrix (Fin M) (Fin M) E) (hJ : IsHermND σ J) (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R] (hR : IsLocR R) {n d : ℕ} (b : Module.Basis (Fin d) F E) (lam : Measure R) (hlam : lam.IsAddHaarMeasure) (m : HermR R σ n → Measure (Fin n → Fin M → R ⊗[F] E)) (hm : IsGramFib R σ J n (coordW R b lam n M) (coordHerm R σ b lam n) m) (H T T' : Matrix (Fin n) (Fin n) E) (hH : H.det ≠ 0) (hT : ∀ a b, σ (T b a) = T a b) (hT' : ∀ a b, σ (T' b a) = T' a b) (hrel : cT (σ : E →+* E) H ∗ T' ∗ H = T) (a c : ℝ≥0) (hc : c ≠ 0) (ha : ∀ μ : Measure (Fin n → Fin M → R ⊗[F] E), μ.IsAddHaarMeasure → μ.map (fun X => rmul R X (H.map (inclR R))) = a • μ) (hc' : ∀ ν : Measure (HermR R σ n), ν.IsAddHaarMeasure → ∀ τ : HermR R σ n → HermR R σ n, (∀ S, (τ S : Matrix (

theorem **projInf_rmul** {M n : ℕ} (X : Fin n → Fin M → AF F ⊗[F] E) (G : Matrix (Fin n) (Fin n) E) : projInf (rmul (AF F) X (G.map (inclR (AF F)))) = rmul (Ainf F) (projInf X) (G.map (inclR (Ainf F)))

theorem **projV_rmul** (v : HeightOneSpectrum (𝒪 F)) {M n : ℕ} (X : Fin n → Fin M → AF F ⊗[F] E) (G : Matrix (Fin n) (Fin n) E) : projV v (projFin (rmul (AF F) X (G.map (inclR (AF F))))) = rmul (Fv F v) (projV v (projFin X)) (G.map (inclR (Fv F v)))

theorem **theta_map** (σ : E →ₐ[F] E) (hcm : IsCMExt F E σ) {M : ℕ} (J : Matrix (Fin M) (Fin M) E) (hJ : IsHermND σ J) (μ : Measure (HA (AF F) σ J)) (ℱ : Set (HA (AF F) σ J)) (hH : IsHaarH F E σ M J μ) (hF : IsFDH F E σ M J μ ℱ) {n : ℕ} (G H T T' : Matrix (Fin n) (Fin n) E) (hGH : G ∗ H = 1) (hHG : H ∗ G = 1) (h1 : cT (σ : E →+* E) H ∗ T' ∗ H = T) (h2 : cT (σ : E →+* E) G ∗ T ∗ G = T') (he : Measurable fun X : Fin n → Fin M → AF F ⊗[F] E => rmul (AF F) X (H.map (inclR (F

theorem **sw_scale** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ n : ℕ, ∀ (T : Matrix (Fin n) (Fin n) E) (hT : ∀ a b, σ (T b a) = T a b), T.det ≠ 0 → ∀ (g : GL (Fin n) E) (T' : Matrix (Fin n) (Fin n) E) (hT' : ∀ a b, σ (T' b a) = T' a b), T' = cT (σ : E →+* E) (g : Matrix (Fin n) (Fin n) E) ∗ T ∗ g → SWMeas σ J T' hT' → SWMeas σ J T hT

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.SiegelWeilMeasureOfCongruentTargetStatement

theorem **_root_.GlobalTheory.T.siegel_weil_measure_of_congruent_target** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ n : ℕ, ∀ (T : Matrix (Fin n) (Fin n) E) (hT : ∀ a b, σ (T b a) = T a b), T.det ≠ 0 → ∀ (g : GL (Fin n) E) (T' : Matrix (Fin n) (Fin n) E) (hT' : ∀ a b, σ (T' b a) = T' a b), T' = cT (σ : E →+* E) (g : Matrix (Fin n) (Fin n) E) ∗ T ∗ g → SWMeas σ J T' hT' → SWMeas σ J T hT

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.SiegelWeilMeasureOfRankSmallOrZero

theorem **swMeas_all** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (n M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → n = 0 ∨ n + 2 ≤ M → ∀ (T : Matrix (Fin n) (Fin n) E) (hT : ∀ a b, σ (T b a) = T a b), T.det ≠ 0 → SWMeas σ J T hT

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.SiegelWeilMeasureOfRankSmallOrZeroStatement

theorem **_root_.GlobalTheory.T.siegel_weil_measure_of_rank_small_or_zero** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (n M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → n = 0 ∨ n + 2 ≤ M → ∀ (T : Matrix (Fin n) (Fin n) E) (hT : ∀ a b, σ (T b a) = T a b), T.det ≠ 0 → SWMeas σ J T hT

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.SiegelWeilMeasureRankOne

theorem **siegel_one_vector** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → 3 ≤ M → ∀ (T : Matrix (Fin 1) (Fin 1) E) (hT : ∀ a b, σ (T b a) = T a b), T.det ≠ 0 → SWMeas σ J T hT

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.SiegelWeilMeasureRankOneStatement

theorem **_root_.GlobalTheory.T.siegel_weil_measure_rank_one** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → 3 ≤ M → ∀ (T : Matrix (Fin 1) (Fin 1) E) (hT : ∀ a b, σ (T b a) = T a b), T.det ≠ 0 → SWMeas σ J T hT

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.SiegelWeilMeasureRankZero

theorem **lintegral_unique** {V : Type} [MeasurableSpace V] [Unique V] (m : Measure V) (h : m Set.univ = 1) (φ : V → ℝ≥0∞) : ∫⁻ x, φ x ∂m = φ default

theorem **coordW_univ** {d : ℕ} (b : Module.Basis (Fin d) F E) (lam : Measure R) (M : ℕ) : coordW R b lam 0 M Set.univ = 1

theorem **coordHerm_univ** (σ : E →ₐ[F] E) {d : ℕ} (b : Module.Basis (Fin d) F E) (lam : Measure R) : coordHerm R σ b lam 0 Set.univ = 1

theorem **fib_univ** (σ : E →ₐ[F] E) {M : ℕ} (J : Matrix (Fin M) (Fin M) E) (μ : Measure (Fin 0 → Fin M → R ⊗[F] E)) (ν : Measure (HermR R σ 0)) (m : HermR R σ 0 → Measure (Fin 0 → Fin M → R ⊗[F] E)) (hμ : μ Set.univ = 1) (hν : ν Set.univ = 1) (h : IsGramFib R σ J 0 μ ν m) (S : HermR R σ 0) : m S Set.univ = 1

theorem **theta_univ** (σ : E →ₐ[F] E) {M : ℕ} (J : Matrix (Fin M) (Fin M) E) (μ : Measure (HA (AF F) σ J)) (ℱ : Set (HA (AF F) σ J)) (h0 : μ ℱ ≠ 0) (h1 : μ ℱ ≠ ⊤) (T : Matrix (Fin 0) (Fin 0) E) : thetaMeas σ J μ ℱ {X | gramE σ J X = T} Set.univ = 1

theorem **swMeas_zero** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (T : Matrix (Fin 0) (Fin 0) E) (hT : ∀ a b, σ (T b a) = T a b), T.det ≠ 0 → SWMeas σ J T hT

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.SiegelWeilMeasureRankZeroStatement

theorem **_root_.GlobalTheory.T.siegel_weil_measure_rank_zero** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (T : Matrix (Fin 0) (Fin 0) E) (hT : ∀ a b, σ (T b a) = T a b), T.det ≠ 0 → SWMeas σ J T hT

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.SiegelWeilOrbitalMeasure

theorem **prod_factor** (a₀ : G₀ → W₀ → W₀) (a : ∀ i, G i → W i → W i) (hc₀ : Continuous fun p : G₀ × W₀ => a₀ p.1 p.2) (h1₀ : ∀ x, a₀ 1 x = x) (hm₀ : ∀ g h x, a₀ (g * h) x = a₀ g (a₀ h x)) (hc : ∀ i, Continuous fun p : G i × W i => a i p.1 p.2) (h1 : ∀ i x, a i 1 x = x) (hm : ∀ i g h x, a i (g * h) x = a i g (a i h x)) (Z₀ : Set W₀) (Z : ∀ i, Set (W i)) (hZ₀ : IsClosed Z₀) (hZ : ∀ i, IsClosed (Z i)) (hs₀ : ∀ g, ∀ x ∈ Z₀, a₀ g x ∈ Z₀) (hs : ∀ i g, ∀ x ∈ Z i, a i g x ∈ Z i) (ht₀ : ∀ x ∈ Z₀, ∀ y ∈ Z₀, ∃ g, a₀ g x = y) (ht : ∀ i, ∀ x ∈ Z i, ∀ y ∈ Z i, ∃ g, a i g x = y) (m₀ : Measure W₀) (m : ∀ i, Measure (W i)) [IsLocallyFiniteMeasure m₀] [∀ i, IsLocallyFiniteMeasure (m i)] (hi₀ : ∀ g, m₀.map (a₀ g) = m₀) (hi : ∀ i g, (m i).map (a i g) = m i) (hz₀ : m₀ Z₀ᶜ = 0) (hz : ∀ i, m i (Z i)ᶜ = 0) (hn₀ : Z₀.Nonempty → m₀ ≠ 0) (hn : ∀ i, (Z i).Nonempty → m i ≠ 0) (P : Measure (W₀ × ∀ i, W i)) [IsLocallyFi

theorem **lat_natMul** {d M n : ℕ} (b : Module.Basis (Fin d) F E) (v : HeightOneSpectrum (𝒪 F)) (Y : Fin n → Fin M → Fv F v ⊗[F] E) (hY : Y ∈ latV b v) (N : ℕ) : (fun a i => (N : Fv F v ⊗[F] E) * Y a i) ∈ latV b v

theorem **zero_mem_lat** {d M n : ℕ} (b : Module.Basis (Fin d) F E) (v : HeightOneSpectrum (𝒪 F)) : (0 : Fin n → Fin M → Fv F v ⊗[F] E) ∈ latV b v

theorem **exists_scale** (σ : E →ₐ[F] E) (hσ : IsCMExt F E σ) {d : ℕ} (b : Module.Basis (Fin d) F E) (n M : ℕ) (v : HeightOneSpectrum (𝒪 F)) (K : Set (Fin n → Fin M → Fv F v ⊗[F] E)) (hK : IsCompact K) : ∃ N : ℕ, 0 < N ∧ ∀ Y ∈ K, (fun a i => (N : Fv F v ⊗[F] E) * Y a i) ∈ latV b v

theorem **preimage_lt_top** (σ : E →ₐ[F] E) (hσ : IsCMExt F E σ) {d : ℕ} (b : Module.Basis (Fin d) F E) {n M : ℕ} (S : Finset (HeightOneSpectrum (𝒪 F))) (I : Measure (Fin n → Fin M → AF F ⊗[F] E)) [IsLocallyFiniteMeasure I] (K : Set ((Fin n → Fin M → Ainf F ⊗[F] E) × ∀ v : S, Fin n → Fin M → Fv F v.1 ⊗[F] E)) (hK : IsCompact K) : I (pr S ⁻¹' K ∩ intOff b S) < ⊤

theorem **push_inv** (σ : E →ₐ[F] E) (hσ : IsCMExt F E σ) {M : ℕ} (J : Matrix (Fin M) (Fin M) E) (hJ : IsHermND σ J) {d : ℕ} (b : Module.Basis (Fin d) F E) {n : ℕ} (S : Finset (HeightOneSpectrum (𝒪 F))) (I : Measure (Fin n → Fin M → AF F ⊗[F] E)) (hI : ∀ h : HA (AF F) σ J, I.map (actW (AF F) σ J h) = I) (hone : ∀ (v : HeightOneSpectrum (𝒪 F)) (X : Fin n → Fin M → Fv F v ⊗[F] E), actW (Fv F v) σ J 1 X = X) (hpr : Measurable (pr S : (Fin n → Fin M → AF F ⊗[F] E) → _)) (hf : ∀ h : HA (AF F) σ J, Measurable (actW (AF F) σ J h : (Fin n → Fin M → AF F ⊗[F] E) → _)) (hint : MeasurableSet (intOff b S : Set (Fin n → Fin M → AF F ⊗[F] E))) (g₀ : HA (Ainf F) σ J) (g : ∀ v : S, HA (Fv F v.1) σ J) (hA : Measurable fun x : (Fin n → Fin M → Ainf F ⊗[F] E) × ∀ v : S, Fin n → Fin M → Fv F v.1 ⊗[F] E => (actW (Ainf F) σ J g₀ x.1, fun v => actW (Fv F v.1) σ J (g v) (x.2 v))) : ((I.restrict (intOff b S)).map (p

theorem **local_pack** (σ : E →ₐ[F] E) (hσ : IsCMExt F E σ) {M : ℕ} (J : Matrix (Fin M) (Fin M) E) (hJ : IsHermND σ J) (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R] (hR : IsLocR R) {n : ℕ} (T : Matrix (Fin n) (Fin n) E) (hT : ∀ a b, σ (T b a) = T a b) (hdet : T.det ≠ 0) {d : ℕ} (b : Module.Basis (Fin d) F E) (lam : Measure R) (hlam : lam.IsAddHaarMeasure) (m : HermR R σ n → Measure (Fin n → Fin M → R ⊗[F] E)) (hm : IsGramFib R σ J n (coordW R b lam n M) (coordHerm R σ b lam n) m) : IsLocallyFiniteMeasure (m (hermOf R σ T hT)) ∧ (∀ h : HA R σ J, (m (hermOf R σ T hT)).map (actW R σ J h) = m (hermOf R σ T hT)) ∧ m (hermOf R σ T hT) {Y | gramR R σ J Y = T.map (inclR R)}ᶜ = 0 ∧ ({Y : Fin n → Fin M → R ⊗[F] E | gramR R σ J Y = T.map (inclR R)}.Nonempty → m (hermOf R σ T hT) ≠ 0) ∧ (∀ X ∈ {Y : Fin n → Fin M → R ⊗[F] E | gramR R σ J Y = T.map (inclR R)

theorem **fact_S** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (n : ℕ), ∀ (T : Matrix (Fin n) (Fin n) E) (hT : ∀ a b, σ (T b a) = T a b), T.det ≠ 0 → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (laminf : Measure (Ainf F)) (lam : ∀ v : HeightOneSpectrum (𝒪 F), Measure (Fv F v)), IsStdInf laminf → (∀ v, IsStdV v (lam v)) → ∀ (minf : HermR (Ainf F) σ n → Measure (Fin n → Fin M → Ainf F ⊗[F] E)), IsGramFib (Ainf F) σ J n (coordW (Ainf F) b laminf n M) (coordHerm (Ainf F) σ b laminf n) minf → ∀ (m : ∀ v : HeightOneSpectrum (𝒪 F), HermR (Fv F v) σ n → Measure (Fin n → Fin M → Fv F v ⊗[F] E)), (∀ v, IsGramFib (Fv F v) σ J n (coordW (Fv F v) b (lam v) n M) (coordHerm (Fv F v) σ b (lam v) n) (m v)) → ∀ I : Measure (Fin n → Fin M → AF F ⊗[F] E), IsLocallyFiniteMeasure I → (∀

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.SiegelWeilOrbitalMeasureStatement

theorem **_root_.GlobalTheory.T.siegel_weil_orbital_measure** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (n : ℕ), ∀ (T : Matrix (Fin n) (Fin n) E) (hT : ∀ a b, σ (T b a) = T a b), T.det ≠ 0 → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (laminf : Measure (Ainf F)) (lam : ∀ v : HeightOneSpectrum (𝒪 F), Measure (Fv F v)), IsStdInf laminf → (∀ v, IsStdV v (lam v)) → ∀ (minf : HermR (Ainf F) σ n → Measure (Fin n → Fin M → Ainf F ⊗[F] E)), IsGramFib (Ainf F) σ J n (coordW (Ainf F) b laminf n M) (coordHerm (Ainf F) σ b laminf n) minf → ∀ (m : ∀ v : HeightOneSpectrum (𝒪 F), HermR (Fv F v) σ n → Measure (Fin n → Fin M → Fv F v ⊗[F] E)), (∀ v, IsGramFib (Fv F v) σ J n (coordW (Fv F v) b (lam v) n M) (coordHerm (Fv F v) σ b (lam v) n) (m v)) → ∀ I : Measure (Fin n → Fin M → A

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.SimultaneousNormalFormMajorant

theorem **exists_eq_transpose_mul_self** {ι : Type} [Fintype ι] [DecidableEq ι] {T : Matrix ι ι ℝ} (hT : T.PosSemidef) : ∃ B : Matrix ι ι ℝ, T = Bᵀ * B

theorem **simultaneous_normal_form_majorant** : ∀ (ι : Type) [Fintype ι] [DecidableEq ι] (S P : Matrix ι ι ℝ), S.IsSymm → S.det ≠ 0 → P.PosDef → P * S⁻¹ * P = S → ∃ (g : Matrix ι ι ℝ) (e : ι → ℝ), g.det ≠ 0 ∧ (∀ i, e i = 1 ∨ e i = -1) ∧ gᵀ * P * g = 1 ∧ gᵀ * S * g = Matrix.diagonal e

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.SizeFunctionRapidDecayIntegrable

theorem **continuous_compReal** (hF : IsTotallyReal F) (v : InfinitePlace F) : Continuous (compReal hF v)

theorem **continuous_compReal_symm** (hF : IsTotallyReal F) (v : InfinitePlace F) : Continuous (compReal hF v).symm

theorem **norm_compReal** (hF : IsTotallyReal F) (v : InfinitePlace F) (x : v.Completion) : ‖compReal hF v x‖ = ‖x‖

theorem **secondCountable_Ainf** (hF : IsTotallyReal F) : SecondCountableTopology (Ainf F)

theorem **continuous_ainfEquiv** : Continuous (ainfEquiv hF)

theorem **continuous_ainfEquiv_symm** : Continuous (ainfEquiv hF).symm

theorem **abs_coordEquiv** (y : Ainf F ⊗[F] E) (k : Fin d) (v : InfinitePlace F) : ‖coordEquiv hF b y k v‖ = ‖(Algebra.TensorProduct.basis (Ainf F) b).repr y k v‖

theorem **tupEquiv_apply** (n M : ℕ) (X : Fin n → Fin M → Ainf F ⊗[F] E) (a : Fin n) (i : Fin M) : tupEquiv hF b n M X a i = coordEquiv hF b (X a i)

theorem **norm_tupEquiv_le** (n M : ℕ) (X : Fin n → Fin M → Ainf F ⊗[F] E) : ‖tupEquiv hF b n M X‖ ≤ sizeTup b X

theorem **continuous_coordEquiv** (hc : IsHomeomorph fun (y : Ainf F ⊗[F] E) (k : Fin d) => (Algebra.TensorProduct.basis (Ainf F) b).repr y k) : Continuous (coordEquiv hF b)

theorem **continuous_coordEquiv_symm** (hc : IsHomeomorph fun (y : Ainf F ⊗[F] E) (k : Fin d) => (Algebra.TensorProduct.basis (Ainf F) b).repr y k) : Continuous (coordEquiv hF b).symm

theorem **size_neg_pow_integrable** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E), ∀ (n M : ℕ) (μ : Measure (Fin n → Fin M → Ainf F ⊗[F] E)), μ.IsAddHaarMeasure → Continuous (sizeTup b : (Fin n → Fin M → Ainf F ⊗[F] E) → ℝ) ∧ (∀ X : Fin n → Fin M → Ainf F ⊗[F] E, 0 ≤ sizeTup b X) ∧ ∃ N : ℕ, ∫⁻ X, ENNReal.ofReal (((1 + sizeTup b X) ^ N)⁻¹) ∂μ < ⊤

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.SizeFunctionRapidDecayIntegrableStatement

theorem **_root_.GlobalTheory.T.size_function_rapid_decay_integrable** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E), ∀ (n M : ℕ) (μ : Measure (Fin n → Fin M → Ainf F ⊗[F] E)), μ.IsAddHaarMeasure → Continuous (sizeTup b : (Fin n → Fin M → Ainf F ⊗[F] E) → ℝ) ∧ (∀ X : Fin n → Fin M → Ainf F ⊗[F] E, 0 ≤ sizeTup b X) ∧ ∃ N : ℕ, ∫⁻ X, ENNReal.ofReal (((1 + sizeTup b X) ^ N)⁻¹) ∂μ < ⊤

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.SmallBoxGramScalarZero

theorem **gramR_smul** {F E : Type} [Field F] [Field E] [Algebra F E] (R : Type) [CommRing R] [Algebra F R] (σ : E →ₐ[F] E) {M n : ℕ} (J : Matrix (Fin M) (Fin M) E) (s : R) (X : Fin n → Fin M → R ⊗[F] E) (a c : Fin n) : gramR R σ J ((s ⊗ₜ[F] (1 : E)) • X) a c = ((s ⊗ₜ[F] (1 : E)) * (s ⊗ₜ[F] (1 : E))) * gramR R σ J X a c

theorem **small_length** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (N : ℕ), 0 < N → ∃ r₀ : ℝ, 0 < r₀ ∧ ∀ (t : F) (Y : Fin 1 → Fin M → AF F ⊗[F] E), Y ∈ box b N r₀ → (gramR (AF F) σ J Y = fun _ _ => inclR (AF F) (algebraMap F E t)) → t = 0

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.SmallBoxGramScalarZeroStatement

theorem **_root_.GlobalTheory.T.small_box_gram_scalar_zero** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (N : ℕ), 0 < N → ∃ r₀ : ℝ, 0 < r₀ ∧ ∀ (t : F) (Y : Fin 1 → Fin M → AF F ⊗[F] E), Y ∈ box b N r₀ → (gramR (AF F) σ J Y = fun _ _ => inclR (AF F) (algebraMap F E t)) → t = 0

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.SplitAnisotropic

theorem **det_blockOne** {K : Type} [CommRing K] {M : ℕ} (t : K) (A : Matrix (Fin M) (Fin M) K) : (blockOne t A).det = t * A.det

theorem **split_anisotropic** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin (M + 1)) (Fin (M + 1)) E), IsHermND σ J → ∀ (t : F) (x : Fin (M + 1) → E), t ≠ 0 → hE σ J x x = algebraMap F E t → ∃ (g : GL (Fin (M + 1)) E) (J₁ : Matrix (Fin M) (Fin M) E), IsHermND σ J₁ ∧ blockOne (algebraMap F E t) J₁ = cT (σ : E →+* E) (g : Matrix (Fin (M + 1)) (Fin (M + 1)) E) * J * g ∧ ∀ i, (g : Matrix (Fin (M + 1)) (Fin (M + 1)) E) i 0 = x i

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.StageData

theorem **schur_coe** (σ : E →ₐ[F] E) {n : ℕ} (t : F) (a : R → Rˣ) (S : HermR R σ (n + 1)) (h : schurM R σ t a (S : Matrix (Fin (n + 1)) (Fin (n + 1)) (R ⊗[F] E)) ∈ HermR R σ n) : (schur R σ t a S : Matrix (Fin n) (Fin n) (R ⊗[F] E)) = schurM R σ t a S

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.StandardAdelicCharacter

lemma **fieldChar_add** (a b : F) : fieldChar F (a + b) = fieldChar F a * fieldChar F b

lemma **fieldChar_zero** : fieldChar F 0 = 1

lemma **norm_fieldChar** (b : F) : ‖fieldChar F b‖ = 1

lemma **trace_integer** (r : 𝒪 F) : ∃ n : ℤ, Algebra.trace ℚ F (r : F) = n

lemma **fieldChar_eq_one_iff** (b : F) : fieldChar F b = 1 ↔ ∃ n : ℤ, Algebra.trace ℚ F b = n

lemma **zero_mem_Oh** : (0 : A F) ∈ Oh F

lemma **add_mem_Oh** {x y : A F} (hx : x ∈ Oh F) (hy : y ∈ Oh F) : x + y ∈ Oh F

lemma **sub_mem_Oh** {x y : A F} (hx : x ∈ Oh F) (hy : y ∈ Oh F) : x - y ∈ Oh F

lemma **exists_rep** (x : A F) : ∃ b : F, x - algebraMap F (A F) b ∈ Oh F

lemma **charFun_eq** (x : A F) (b : F) (h : x - algebraMap F (A F) b ∈ Oh F) : charFun F x = fieldChar F b

lemma **stdChar_apply** (x : A F) : stdChar F x = charFun F x

lemma **continuous_stdChar** : Continuous (stdChar F)

lemma **Oh_subset_of_isClosed** {S : Set (A F)} (hS : IsClosed S) (h : ∀ r : 𝒪 F, algebraMap F (A F) (r : F) ∈ S) : Oh F ⊆ S

theorem **main** : ∀ (F : Type) [Field F] [NumberField F], (∃ ψ : AddChar (A F) ℂ, IsStdChar F ψ) ∧ ∀ ψ : AddChar (A F) ℂ, IsStdChar F ψ → (∀ x, ‖ψ x‖ = 1) ∧ (∀ x ∈ Oh F, ψ x = 1) ∧ ∀ b : F, (∀ x ∈ Oh F, ψ (algebraMap F (A F) b * x) = 1) ↔ ∀ r : 𝒪 F, ∃ n : ℤ, Algebra.trace ℚ F (b * (r : F)) = n

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.StandardAdelicCharacterStatement

theorem **_root_.GlobalTheory.T.standard_adelic_character** : ∀ (F : Type) [Field F] [NumberField F], (∃ ψ : AddChar (A F) ℂ, IsStdChar F ψ) ∧ ∀ ψ : AddChar (A F) ℂ, IsStdChar F ψ → (∀ x, ‖ψ x‖ = 1) ∧ (∀ x ∈ Oh F, ψ x = 1) ∧ ∀ b : F, (∀ x ∈ Oh F, ψ (algebraMap F (A F) b * x) = 1) ↔ ∀ r : 𝒪 F, ∃ n : ℤ, Algebra.trace ℚ F (b * (r : F)) = n

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.StandardLocalHaarProperties

theorem **isCompact_integers** : IsCompact (v.adicCompletionIntegers F : Set (Fv F v))

theorem **isOpen_integers** : IsOpen (v.adicCompletionIntegers F : Set (Fv F v))

theorem **mulAdd_continuous** (c : Fv F v) (hc : c ≠ 0) : Continuous (mulAdd v c hc)

theorem **mulAdd_symm_continuous** (c : Fv F v) (hc : c ≠ 0) : Continuous (mulAdd v c hc).symm

theorem **vol_lt_one** (lam : Measure (Fv F v)) (h : IsStdV v lam) : lam {t | Valued.v t < 1} = (Ideal.absNorm v.asIdeal : ℝ≥0∞)⁻¹

theorem **absNorm_ne_zero** : (Ideal.absNorm v.asIdeal : ℝ≥0∞) ≠ 0

theorem **map_unit** (lam : Measure (Fv F v)) (h : IsStdV v lam) (u : Fv F v) (hu : Valued.v u = 1) : lam.map (fun t => u * t) = lam

theorem **exp_neg_one_mul_lt_one_iff** (a : WithZero (Multiplicative ℤ)) : WithZero.exp (-1 : ℤ) * a < 1 ↔ a ≤ 1

theorem **map_unif** (lam : Measure (Fv F v)) (h : IsStdV v lam) (π : Fv F v) (hπ : Valued.v π = WithZero.exp (-1 : ℤ)) : lam.map (fun t => π * t) = (Ideal.absNorm v.asIdeal : ℝ≥0∞) • lam

theorem **volumes_in_completion** : ∀ (F : Type) [Field F] [NumberField F] (v : HeightOneSpectrum (𝒪 F)) (lam : Measure (Fv F v)), IsStdV v lam → lam {t | Valued.v t < 1} = (Ideal.absNorm v.asIdeal : ℝ≥0∞)⁻¹ ∧ (∀ π : Fv F v, Valued.v π = WithZero.exp (-1 : ℤ) → lam.map (fun t => π * t) = (Ideal.absNorm v.asIdeal : ℝ≥0∞) • lam) ∧ (∀ u : Fv F v, Valued.v u = 1 → lam.map (fun t => u * t) = lam) ∧ Nat.card (IsLocalRing.ResidueField (v.adicCompletionIntegers F)) = Ideal.absNorm v.asIdeal

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.StandardLocalHaarPropertiesStatement

theorem **_root_.GlobalTheory.T.standard_local_haar_properties** : ∀ (F : Type) [Field F] [NumberField F] (v : HeightOneSpectrum (𝒪 F)) (lam : Measure (Fv F v)), IsStdV v lam → lam {t | Valued.v t < 1} = (Ideal.absNorm v.asIdeal : ℝ≥0∞)⁻¹ ∧ (∀ π : Fv F v, Valued.v π = WithZero.exp (-1 : ℤ) → lam.map (fun t => π * t) = (Ideal.absNorm v.asIdeal : ℝ≥0∞) • lam) ∧ (∀ u : Fv F v, Valued.v u = 1 → lam.map (fun t => u * t) = lam) ∧ Nat.card (IsLocalRing.ResidueField (v.adicCompletionIntegers F)) = Ideal.absNorm v.asIdeal

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.SumRhoEqTrace

theorem **oh_add** {x y : A F} (hx : x ∈ Oh F) (hy : y ∈ Oh F) : x + y ∈ Oh F

theorem **oh_zero** : (0 : A F) ∈ Oh F

theorem **oh_neg** {x : A F} (hx : x ∈ Oh F) : -x ∈ Oh F

theorem **sum_rho_eq_trace** {ρ : InfinitePlace F → (F →+* ℝ)} (hT : IsTotallyReal F) (hρ : ∀ w x, ((ρ w x : ℝ) : ℂ) = w.embedding x) (c : F) : ∑ w, ρ w c = ((Algebra.trace ℚ F c : ℚ) : ℝ)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.SummableOneDivAbsNormSq

theorem **summable_one_div_absNorm_sq** : ∀ (F : Type) [Field F] [NumberField F], Summable fun v : HeightOneSpectrum (𝒪 F) => 1 / (Ideal.absNorm v.asIdeal : ℝ) ^ 2

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.SymmetricInvertibleExistsPosdefCongruence

theorem **exists_posDef_majorant_of_isSymm** : ∀ (ι : Type) [Fintype ι] [DecidableEq ι] (B : Matrix ι ι ℝ), B.IsSymm → B.det ≠ 0 → ∃ P : Matrix ι ι ℝ, P.PosDef ∧ P * B⁻¹ * P = B

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.SymmetricInvertibleExistsPosdefCongruenceStatement

theorem **_root_.GlobalTheory.T.symmetric_invertible_exists_posdef_congruence** : ∀ (ι : Type) [Fintype ι] [DecidableEq ι] (B : Matrix ι ι ℝ), B.IsSymm → B.det ≠ 0 → ∃ P : Matrix ι ι ℝ, P.PosDef ∧ P * B⁻¹ * P = B

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.TensorIntegerRingAlmostAllPlaces

theorem **exists_good_nat** {k : Type} [Field k] (h2 : (2 : k) ≠ 0) (h3 : (3 : k) ≠ 0) (D A B C : k) (hne : A ≠ 0 ∨ B ≠ 0 ∨ C ≠ 0) : ∃ a : ℕ, 1 - D * (a : k) ^ 2 ≠ 0 ∧ A + B * (a : k) + C * (a : k) ^ 2 ≠ 0

theorem **finite_maxSpec** [IsLocalRing R] (hsp : ∀ y : O, ∃ p q : R, y = ι p + ι q * δ) : Finite (MaximalSpectrum O)

theorem **norm_eq** (hδ : δ * δ = ι D) (hτι : ∀ r, τ (ι r) = ι r) (hτδ : τ δ = -δ) (p q : R) : (ι p + ι q * δ) * τ (ι p + ι q * δ) = ι (p ^ 2 - D * q ^ 2)

theorem **fixed_unit** (hτι : ∀ r, τ (ι r) = ι r) (hτδ : τ δ = -δ) (hsp : ∀ y : O, ∃ p q : R, y = ι p + ι q * δ) (hinj : ∀ p q : R, ι p + ι q * δ = 0 → p = 0 ∧ q = 0) (h2 : IsUnit (2 : R)) (β : O) (hβ : IsUnit β) (hτβ : τ β = β) : ∃ b : R, IsUnit b ∧ β = ι b

theorem **exists_eps** [IsLocalRing R] (hδ : δ * δ = ι D) (hτι : ∀ r, τ (ι r) = ι r) (hτδ : τ δ = -δ) (hsp : ∀ y : O, ∃ p q : R, y = ι p + ι q * δ) (hinj : ∀ p q : R, ι p + ι q * δ = 0 → p = 0 ∧ q = 0) (hD : IsUnit D) (h2 : IsUnit (2 : R)) (h3 : IsUnit (3 : R)) (s β : O) (hβ : IsUnit β) (hτβ : τ β = β) : ∃ ε z : O, ε * τ ε = 1 ∧ (β - τ ε * s) * z = 1

theorem **coord_mem** {ι₁ ι₂ : Type} [Fintype ι₁] (P : Module.Basis ι₁ K A) (Q : Module.Basis ι₂ K A) (hPQ : ∀ i l, Q.repr (P i) l ∈ R) (y : A) (hy : ∀ i, P.repr y i ∈ R) (l : ι₂) : Q.repr y l ∈ R

theorem **coord_two** (B : Module.Basis (Fin 2) K A) (hB0 : B 0 = 1) (p q : K) : B.repr (algebraMap K A p + algebraMap K A q * B 1) 0 = p ∧ B.repr (algebraMap K A p + algebraMap K A q * B 1) 1 = q

theorem **eq_coord** (B : Module.Basis (Fin 2) K A) (hB0 : B 0 = 1) (y : A) : y = algebraMap K A (B.repr y 0) + algebraMap K A (B.repr y 1) * B 1

theorem **exists_subring** {d : ℕ} (B : Module.Basis (Fin 2) K A) (C : Module.Basis (Fin d) K A) (D : K) (hB0 : B 0 = 1) (hD : B 1 * B 1 = algebraMap K A D) (hDR : D ∈ R) (hBC : ∀ k j, B.repr (C k) j ∈ R) (hCB : ∀ j k, C.repr (B j) k ∈ R) : ∃ O : Subring A, (∀ y, y ∈ O ↔ ∀ k, C.repr y k ∈ R) ∧ (∀ y, y ∈ O ↔ ∀ j, B.repr y j ∈ R)

theorem **sigR_algebraMap** (σ : E →ₐ[F] E) (v : HeightOneSpectrum (𝒪 F)) (c : Fv F v) : sigR (Fv F v) σ (algebraMap (Fv F v) (Fv F v ⊗[F] E) c) = algebraMap (Fv F v) (Fv F v ⊗[F] E) c

theorem **good_at** (σ : E →ₐ[F] E) {d : ℕ} (b : Module.Basis (Fin d) F E) (b₂ : Module.Basis (Fin 2) F E) (D : F) (hb0 : b₂ 0 = 1) (hσ1 : σ (b₂ 1) = -b₂ 1) (hD : b₂ 1 * b₂ 1 = algebraMap F E D) (v : HeightOneSpectrum (𝒪 F)) (hD1 : Valued.v (algebraMap F (Fv F v) D) = 1) (h21 : Valued.v (algebraMap F (Fv F v) 2) = 1) (h31 : Valued.v (algebraMap F (Fv F v) 3) = 1) (hBC : ∀ k j, (Algebra.TensorProduct.basis (Fv F v) b₂).repr (inclR (Fv F v) (b k)) j ∈ v.adicCompletionIntegers F) (hCB : ∀ j k, (Algebra.TensorProduct.basis (Fv F v) b).repr (inclR (Fv F v) (b₂ j)) k ∈ v.adicCompletionIntegers F) : ∃ O : Subring (Fv F v ⊗[F] E), (∀ y, y ∈ O ↔ ∀ k, (Algebra.TensorProduct.basis (Fv F v) b).repr y k ∈ v.adicCompletionIntegers F) ∧ (∀ y ∈ O, sigR (Fv F v) σ y ∈ O) ∧ Finite (MaximalSpectrum O) ∧ IsUnit (2 : O) ∧ (∃ δ : O, IsUnit δ ∧ sigR (Fv F v) σ δ = -δ) ∧ (∀ s β : O, IsUnit β → sigR (Fv F v) σ β = β

theorem **good_ring** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E), ∀ᶠ v : HeightOneSpectrum (𝒪 F) in Filter.cofinite, ∃ O : Subring (Fv F v ⊗[F] E), (∀ y, y ∈ O ↔ ∀ k, (Algebra.TensorProduct.basis (Fv F v) b).repr y k ∈ v.adicCompletionIntegers F) ∧ (∀ y ∈ O, sigR (Fv F v) σ y ∈ O) ∧ Finite (MaximalSpectrum O) ∧ IsUnit (2 : O) ∧ (∃ δ : O, IsUnit δ ∧ sigR (Fv F v) σ δ = -δ) ∧ (∀ s β : O, IsUnit β → sigR (Fv F v) σ β = β → ∃ ε z : O, (ε : Fv F v ⊗[F] E) * sigR (Fv F v) σ ε = 1 ∧ ((β : Fv F v ⊗[F] E) - sigR (Fv F v) σ ε * s) * z = 1) ∧ ∀ β : O, IsUnit β → sigR (Fv F v) σ β = β → ∃ a : O, (a : Fv F v ⊗[F] E) * sigR (Fv F v) σ a = β

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.TensorIntegerRingAlmostAllPlacesStatement

theorem **_root_.GlobalTheory.T.tensor_integer_ring_almost_all_places** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E), ∀ᶠ v : HeightOneSpectrum (𝒪 F) in Filter.cofinite, ∃ O : Subring (Fv F v ⊗[F] E), (∀ y, y ∈ O ↔ ∀ k, (Algebra.TensorProduct.basis (Fv F v) b).repr y k ∈ v.adicCompletionIntegers F) ∧ (∀ y ∈ O, sigR (Fv F v) σ y ∈ O) ∧ Finite (MaximalSpectrum O) ∧ IsUnit (2 : O) ∧ (∃ δ : O, IsUnit δ ∧ sigR (Fv F v) σ δ = -δ) ∧ (∀ s β : O, IsUnit β → sigR (Fv F v) σ β = β → ∃ ε z : O, (ε : Fv F v ⊗[F] E) * sigR (Fv F v) σ ε = 1 ∧ ((β : Fv F v ⊗[F] E) - sigR (Fv F v) σ ε * s) * z = 1) ∧ ∀ β : O, IsUnit β → sigR (Fv F v) σ β = β → ∃ a : O, (a : Fv F v ⊗[F] E) * sigR (Fv F v) σ a = β

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.TensorInvolutionAlgebraProperties

theorem **exists_basis_one_delta** (σ : E →ₐ[F] E) {δ : E} (hδ : σ δ = -δ) (h0 : δ ≠ 0) (hfr : Module.finrank F E = 2) (h2 : (2 : F) ≠ 0) : ∃ b : Module.Basis (Fin 2) F E, b 0 = 1 ∧ b 1 = δ

theorem **exists_coord** (b : Module.Basis (Fin 2) F E) {δ : E} (hb0 : b 0 = 1) (hb1 : b 1 = δ) (y : R ⊗[F] E) : ∃ a c : R, y = a ⊗ₜ[F] (1 : E) + c ⊗ₜ[F] δ

theorem **coord_inj** (b : Module.Basis (Fin 2) F E) {δ : E} (hb0 : b 0 = 1) (hb1 : b 1 = δ) {a c a' c' : R} (h : a ⊗ₜ[F] (1 : E) + c ⊗ₜ[F] δ = a' ⊗ₜ[F] (1 : E) + c' ⊗ₜ[F] δ) : a = a' ∧ c = c'

theorem **tmul_one_injective** (b : Module.Basis (Fin 2) F E) {δ : E} (hb0 : b 0 = 1) (hb1 : b 1 = δ) : Function.Injective (Algebra.TensorProduct.includeLeft : R →ₐ[F] R ⊗[F] E)

theorem **algebra_tensor_invol** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (R : Type) [CommRing R] [Algebra F R] [IsArtinianRing R] [IsReduced R], IsArtinianRing (R ⊗[F] E) ∧ IsReduced (R ⊗[F] E) ∧ IsUnit (2 : R ⊗[F] E) ∧ (∀ y, sigR R σ (sigR R σ y) = y) ∧ (∃ δ : R ⊗[F] E, IsUnit δ ∧ sigR R σ δ = -δ) ∧ (∀ y : R ⊗[F] E, sigR R σ y = y → ∃ r : R, y = r ⊗ₜ[F] (1 : E)) ∧ ∀ r : R, IsUnit (r ⊗ₜ[F] (1 : E) : R ⊗[F] E) ↔ IsUnit r

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.TensorInvolutionAlgebraPropertiesStatement

theorem **_root_.GlobalTheory.T.tensor_involution_algebra_properties** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (R : Type) [CommRing R] [Algebra F R] [IsArtinianRing R] [IsReduced R], IsArtinianRing (R ⊗[F] E) ∧ IsReduced (R ⊗[F] E) ∧ IsUnit (2 : R ⊗[F] E) ∧ (∀ y, sigR R σ (sigR R σ y) = y) ∧ (∃ δ : R ⊗[F] E, IsUnit δ ∧ sigR R σ δ = -δ) ∧ (∀ y : R ⊗[F] E, sigR R σ y = y → ∃ r : R, y = r ⊗ₜ[F] (1 : E)) ∧ ∀ r : R, IsUnit (r ⊗ₜ[F] (1 : E) : R ⊗[F] E) ↔ IsUnit r

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.TensorLocallyCompactRing

theorem **isModTop** : IsModuleTopology R (R ⊗[F] E)

theorem **isHomeomorph_coord** {d : ℕ} (b : Module.Basis (Fin d) F E) : IsHomeomorph fun (y : R ⊗[F] E) (k : Fin d) => (Algebra.TensorProduct.basis R b).repr y k

theorem **isTopRing** [Module.Finite F E] : IsTopologicalRing (R ⊗[F] E)

theorem **continuous_sigR** (σ : E →ₐ[F] E) : Continuous (sigR R σ)

theorem **continuous_trace** [Module.Finite F E] : Continuous (Algebra.trace R (R ⊗[F] E))

theorem **topTens_props** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [IsTopologicalRing R] [LocallyCompactSpace R] [SecondCountableTopology R] [T2Space R], IsTopologicalRing (R ⊗[F] E) ∧ LocallyCompactSpace (R ⊗[F] E) ∧ SecondCountableTopology (R ⊗[F] E) ∧ T2Space (R ⊗[F] E) ∧ Continuous (sigR R σ) ∧ Continuous (Algebra.trace R (R ⊗[F] E)) ∧ (∀ (d : ℕ) (b : Module.Basis (Fin d) F E), IsHomeomorph fun (y : R ⊗[F] E) (k : Fin d) => (Algebra.TensorProduct.basis R b).repr y k) ∧ (∀ n M : ℕ, BorelSpace (Fin n → Fin M → R ⊗[F] E)) ∧ (∀ n : ℕ, IsClosed (HermR R σ n : Set (Matrix (Fin n) (Fin n) (R ⊗[F] E)))) ∧ (∀ (n M : ℕ) (J : Matrix (Fin M) (Fin M) E), Continuous (gramR R σ J : (Fin n → Fin M → R ⊗[F] E) → _))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.TensorLocallyCompactRingStatement

theorem **_root_.GlobalTheory.T.tensor_locally_compact_ring** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [IsTopologicalRing R] [LocallyCompactSpace R] [SecondCountableTopology R] [T2Space R], IsTopologicalRing (R ⊗[F] E) ∧ LocallyCompactSpace (R ⊗[F] E) ∧ SecondCountableTopology (R ⊗[F] E) ∧ T2Space (R ⊗[F] E) ∧ Continuous (sigR R σ) ∧ Continuous (Algebra.trace R (R ⊗[F] E)) ∧ (∀ (d : ℕ) (b : Module.Basis (Fin d) F E), IsHomeomorph fun (y : R ⊗[F] E) (k : Fin d) => (Algebra.TensorProduct.basis R b).repr y k) ∧ (∀ n M : ℕ, BorelSpace (Fin n → Fin M → R ⊗[F] E)) ∧ (∀ n : ℕ, IsClosed (HermR R σ n : Set (Matrix (Fin n) (Fin n) (R ⊗[F] E)))) ∧ (∀ (n M : ℕ) (J : Matrix (Fin M) (Fin M) E), Continuous (gramR R σ J : (Fin n → Fin M → R ⊗[F] E) → _))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ThetaAverageExpectedCount

theorem **countable_field** : Countable F

theorem **card_places** (h : IsTotallyReal F) : Fintype.card (InfinitePlace F) = Module.finrank ℚ F

theorem **posDef_lower** {P : Matrix ι ι ℝ} (hP : P.PosDef) : ∃ c : ℝ, 0 < c ∧ ∀ y : ι → ℝ, c * ‖y‖ ^ 2 ≤ y ⬝ᵥ (P *ᵥ y)

theorem **posDef_lower_family** {W : Type} [Finite W] [Nonempty W] (P : W → Matrix ι ι ℝ) (hP : ∀ w, (P w).PosDef) : ∃ c : ℝ, 0 < c ∧ ∀ w, ∀ y : ι → ℝ, c * ‖y‖ ^ 2 ≤ y ⬝ᵥ (P w *ᵥ y)

theorem **norm_gauss** (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (P : InfinitePlace F → Matrix ι ι ℝ) (τ : InfinitePlace F → ℂ) (x : InfinitePlace F → ι → ℝ) : ‖gauss B ρ P τ x‖ = Real.exp (-(Real.pi * ∑ w, (τ w).im * (x w ⬝ᵥ (P w *ᵥ x w))))

theorem **sq_norm_le_sum** {W : Type} [Fintype W] (x : W → ι → ℝ) : ‖x‖ ^ 2 ≤ ∑ w, ‖x w‖ ^ 2

theorem **gauss_bound** (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (P : InfinitePlace F → Matrix ι ι ℝ) (τ : InfinitePlace F → ℂ) {c t' : ℝ} (hc : 0 < c) (ht' : 0 < t') (hlow : ∀ w, ∀ y : ι → ℝ, c * ‖y‖ ^ 2 ≤ y ⬝ᵥ (P w *ᵥ y)) (hτ : ∀ w, t' ≤ (τ w).im) (x : InfinitePlace F → ι → ℝ) : ‖gauss B ρ P τ x‖ ≤ Real.exp (-(Real.pi * c * t' * ‖x‖ ^ 2))

theorem **wt_bound** (M : ℕ) (τ : InfinitePlace F → ℂ) {T : ℝ} (h0 : ∀ w, 0 ≤ (τ w).im) (hT : ∀ w, (τ w).im ≤ T) : wt M τ ≤ Real.sqrt T ^ (Fintype.card (InfinitePlace F) * M)

theorem **phi_bound** {φ : (ι → A F) → ℂ} (hφ : SB φ) : ∃ C₀ : ℝ, 0 ≤ C₀ ∧ ∃ N : ℕ, 0 < N ∧ (∀ y, ‖φ y‖ ≤ C₀) ∧ ∀ y, φ y ≠ 0 → ∀ i, (N : A F) * y i ∈ Oh F

theorem **measurable_fin** {Ω : Type} [MeasurableSpace Ω] [MeasurableSpace (A F)] [BorelSpace (A F)] {gf : Ω → Matrix ι ι (A F)} (h : ∀ i j, Measurable fun ω => gf ω i j) (x : ι → F) : Measurable fun ω => gf ω *ᵥ fun j => algebraMap F (A F) (x j)

theorem **measurable_inf** {Ω : Type} [MeasurableSpace Ω] {gi : Ω → InfinitePlace F → Matrix ι ι ℝ} (h : ∀ w i j, Measurable fun ω => gi ω w i j) (ρ : InfinitePlace F → (F →+* ℝ)) (x : ι → F) : Measurable fun ω => (fun w => gi ω w *ᵥ fun j => ρ w (x j) : InfinitePlace F → ι → ℝ)

theorem **continuous_gauss** (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (P : InfinitePlace F → Matrix ι ι ℝ) (τ : InfinitePlace F → ℂ) : Continuous (gauss B ρ P τ)

theorem **measurable_term** {Ω : Type} [MeasurableSpace Ω] [MeasurableSpace (A F)] [BorelSpace (A F)] (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (P : InfinitePlace F → Matrix ι ι ℝ) {gf : Ω → Matrix ι ι (A F)} {gi : Ω → InfinitePlace F → Matrix ι ι ℝ} (hf : ∀ i j, Measurable fun ω => gf ω i j) (hi : ∀ w i j, Measurable fun ω => gi ω w i j) {φ : (ι → A F) → ℂ} (hφ : SB φ) (τ : InfinitePlace F → ℂ) (x : ι → F) : Measurable fun ω => thTerm B ρ P (gf ω) (gi ω) φ τ x

theorem **term_le** (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (P : InfinitePlace F → Matrix ι ι ℝ) (gf : Matrix ι ι (A F)) (gi : InfinitePlace F → Matrix ι ι ℝ) {φ : (ι → A F) → ℂ} {C₀ : ℝ} {N : ℕ} (hC₀ : 0 ≤ C₀) (hb : ∀ y, ‖φ y‖ ≤ C₀) (hs : ∀ y, φ y ≠ 0 → ∀ i, (N : A F) * y i ∈ Oh F) (τ : InfinitePlace F → ℂ) {c t' : ℝ} (hc : 0 < c) (ht' : 0 < t') (hlow : ∀ w, ∀ y : ι → ℝ, c * ‖y‖ ^ 2 ≤ y ⬝ᵥ (P w *ᵥ y)) (hτ : ∀ w, t' ≤ (τ w).im) (x : {x : ι → F // x ≠ 0}) : ‖thTerm B ρ P gf gi φ τ x.1‖ₑ ≤ ENNReal.ofReal C₀ * Set.indicator {x : {x : ι → F // x ≠ 0} | ∀ i, (N : A F) * (gf *ᵥ fun j => algebraMap F (A F) (x.1 j)) i ∈ Oh F} (fun x => ENNReal.ofReal (Real.exp (-(Real.pi * c * t' * ‖(fun w => gi w *ᵥ fun j => ρ w (x.1 j) : InfinitePlace F → ι → ℝ)‖ ^ 2)))) x

theorem **add_le_mul_pow_of_sq_le_of_le_mul** {w T κ C₀ C' : ℝ} {n : ℕ} (hT : 0 < T) (hκ : 0 < κ) (hC₀ : 0 ≤ C₀) (hw0 : 0 ≤ w) (hw : w ≤ Real.sqrt (T / κ) ^ n) : w * (C₀ * (C' * (Real.sqrt T ^ n)⁻¹)) ≤ C₀ * |C'| * Real.sqrt κ⁻¹ ^ n

theorem **theta_integral_bound** : ∀ (F : Type) [Field F] [NumberField F], ∀ [MeasurableSpace (A F)] [BorelSpace (A F)], ∀ (ι : Type) [Fintype ι] [DecidableEq ι] (M : ℕ) (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (P : InfinitePlace F → Matrix ι ι ℝ), QSetup M B ρ P → ∀ (Ω : Type) [MeasurableSpace Ω] (Pr : Measure Ω) (gf : Ω → Matrix ι ι (A F)) (gi : Ω → InfinitePlace F → Matrix ι ι ℝ), CountData M B ρ Pr gf gi → ∀ φ : (ι → A F) → ℂ, SB φ → ∀ κ t : ℝ, 0 < κ → 0 < t → ∃ C : ℝ, ∀ τ : InfinitePlace F → ℂ, (∀ w, t ≤ (τ w).im) → (∀ v w, κ * (τ w).im ≤ (τ v).im) → (∀ x, Measurable fun ω => thTerm B ρ P (gf ω) (gi ω) φ τ x) ∧ ENNReal.ofReal (wt M τ) * ∫⁻ ω, ∑' x : {x : ι → F // x ≠ 0}, ‖thTerm B ρ P (gf ω) (gi ω) φ τ x.1‖ₑ ∂Pr ≤ ENNReal.ofReal C

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ThetaAverageExpectedCountStatement

theorem **_root_.GlobalTheory.T.weighted_theta_average_le_of_countData** : ∀ (F : Type) [Field F] [NumberField F], ∀ [MeasurableSpace (A F)] [BorelSpace (A F)], ∀ (ι : Type) [Fintype ι] [DecidableEq ι] (M : ℕ) (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (P : InfinitePlace F → Matrix ι ι ℝ), QSetup M B ρ P → ∀ (Ω : Type) [MeasurableSpace Ω] (Pr : Measure Ω) (gf : Ω → Matrix ι ι (A F)) (gi : Ω → InfinitePlace F → Matrix ι ι ℝ), CountData M B ρ Pr gf gi → ∀ φ : (ι → A F) → ℂ, SB φ → ∀ κ t : ℝ, 0 < κ → 0 < t → ∃ C : ℝ, ∀ τ : InfinitePlace F → ℂ, (∀ w, t ≤ (τ w).im) → (∀ v w, κ * (τ w).im ≤ (τ v).im) → (∀ x, Measurable fun ω => thTerm B ρ P (gf ω) (gi ω) φ τ x) ∧ ENNReal.ofReal (wt M τ) * ∫⁻ ω, ∑' x : {x : ι → F // x ≠ 0}, ‖thTerm B ρ P (gf ω) (gi ω) φ τ x.1‖ₑ ∂Pr ≤ ENNReal.ofReal C

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ThetaAverageIntegralFormula

theorem **thTerm_zero** {F : Type} [Field F] [NumberField F] {ι : Type} [Fintype ι] [DecidableEq ι] (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (P : InfinitePlace F → Matrix ι ι ℝ) (gf : Matrix ι ι (A F)) (gi : InfinitePlace F → Matrix ι ι ℝ) (φ : (ι → A F) → ℂ) (τ : InfinitePlace F → ℂ) : thTerm B ρ P gf gi φ τ 0 = φ 0

theorem **main** : ∀ (F : Type) [Field F] [NumberField F] [MeasurableSpace (A F)] [BorelSpace (A F)] (lf : Measure (A F)), lf.IsAddHaarMeasure → lf (Oh F) = 1 → ∀ ψ : AddChar (A F) ℂ, IsStdChar F ψ → ∀ (ι : Type) [Fintype ι] [DecidableEq ι] (M : ℕ) (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (P : InfinitePlace F → Matrix ι ι ℝ), QSetup M B ρ P → ∀ (Ω : Type) [MeasurableSpace Ω] (Pr : Measure Ω) (gf : Ω → Matrix ι ι (A F)) (gi : Ω → InfinitePlace F → Matrix ι ι ℝ), CountData M B ρ Pr gf gi → ∀ φ τ, SB φ → (∀ w, 0 < (τ w).im) → Integrable (fun ω => theta B ρ P (gf ω) (gi ω) φ τ) Pr ∧ ENNReal.ofReal ‖(∫ ω, theta B ρ P (gf ω) (gi ω) φ τ ∂Pr) − φ 0‖ ≤ ∫⁻ ω, ∑' x : {x : ι → F // x ≠ 0}, ‖thTerm B ρ P (gf ω) (gi ω) φ τ x.1‖ₑ ∂Pr

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ThetaAverageIntegralFormulaStatement

theorem **_root_.GlobalTheory.T.theta_average_integral_formula** : ∀ (F : Type) [Field F] [NumberField F] [MeasurableSpace (A F)] [BorelSpace (A F)] (lf : Measure (A F)), lf.IsAddHaarMeasure → lf (Oh F) = 1 → ∀ ψ : AddChar (A F) ℂ, IsStdChar F ψ → ∀ (ι : Type) [Fintype ι] [DecidableEq ι] (M : ℕ) (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (P : InfinitePlace F → Matrix ι ι ℝ), QSetup M B ρ P → ∀ (Ω : Type) [MeasurableSpace Ω] (Pr : Measure Ω) (gf : Ω → Matrix ι ι (A F)) (gi : Ω → InfinitePlace F → Matrix ι ι ℝ), CountData M B ρ Pr gf gi → ∀ φ τ, SB φ → (∀ w, 0 < (τ w).im) → Integrable (fun ω => theta B ρ P (gf ω) (gi ω) φ τ) Pr ∧ ENNReal.ofReal ‖(∫ ω, theta B ρ P (gf ω) (gi ω) φ τ ∂Pr) − φ 0‖ ≤ ∫⁻ ω, ∑' x : {x : ι → F // x ≠ 0}, ‖thTerm B ρ P (gf ω) (gi ω) φ τ x.1‖ₑ ∂Pr

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ThetaCountCompactFinite

theorem **cntI_mono_fun** {S : Set (Fin n → Fin M → E)} {Φ Ψ : (Fin n → Fin M → AF F ⊗[F] E) → ℝ≥0∞} (h : ∀ Y, Φ Y ≤ Ψ Y) : cntI σ J μ 𝓕 S Φ ≤ cntI σ J μ 𝓕 S Ψ

theorem **cntI_le_of_contrib** {S S' : Set (Fin n → Fin M → E)} {Φ : (Fin n → Fin M → AF F ⊗[F] E) → ℝ≥0∞} (h : ∀ (g : HA (AF F) σ J) (X : Fin n → Fin M → E), X ∈ S → Φ (actW (AF F) σ J g⁻¹ (ratTup (AF F) X)) ≠ 0 → X ∈ S') : cntI σ J μ 𝓕 S Φ ≤ cntI σ J μ 𝓕 S' Φ

theorem **cntI_union_le** [Countable E] {S S' : Set (Fin n → Fin M → E)} {Φ : (Fin n → Fin M → AF F ⊗[F] E) → ℝ≥0∞} (hm : ∀ X : Fin n → Fin M → E, Measurable fun g : HA (AF F) σ J => Φ (actW (AF F) σ J g⁻¹ (ratTup (AF F) X))) : cntI σ J μ 𝓕 (S ∪ S') Φ ≤ cntI σ J μ 𝓕 S Φ + cntI σ J μ 𝓕 S' Φ

theorem **cntI_nonzero_lt_top** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → 3 ≤ M → ∀ (μ : Measure (HA (AF F) σ J)) (𝓕 : Set (HA (AF F) σ J)), IsHaarH F E σ M J μ → IsFDH F E σ M J μ 𝓕 → ∀ Ω : Set (Fin 1 → Fin M → AF F ⊗[F] E), IsCompact Ω → cntI σ J μ 𝓕 {X | X ≠ 0} (Set.indicator Ω (fun _ => (1 : ℝ≥0∞))) < ⊤

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ThetaCountCompactFiniteStatement

theorem **_root_.GlobalTheory.T.theta_count_compact_finite** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → 3 ≤ M → ∀ (μ : Measure (HA (AF F) σ J)) (𝓕 : Set (HA (AF F) σ J)), IsHaarH F E σ M J μ → IsFDH F E σ M J μ 𝓕 → ∀ Ω : Set (Fin 1 → Fin M → AF F ⊗[F] E), IsCompact Ω → cntI σ J μ 𝓕 {X | X ≠ 0} (Set.indicator Ω (fun _ => (1 : ℝ≥0∞))) < ⊤

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ThetaFunctionAutomorphy

theorem **sb_add** {V : Type} [TopologicalSpace V] {φ φ' : V → ℂ} (h : SB φ) (h' : SB φ') : SB (φ + φ')

theorem **sb_smul** {V : Type} [TopologicalSpace V] (c : ℂ) {φ : V → ℂ} (h : SB φ) : SB (c • φ)

theorem **sb_integrable** {V : Type} [AddCommGroup V] [TopologicalSpace V] [MeasurableSpace V] {χ : V → V → ℂ} {μ : Measure V} (hV : NADual V χ μ) {φ : V → ℂ} (h : SB φ) (y : V) : Integrable (fun x => φ x * χ x y) μ

theorem **fou_add** {V : Type} [AddCommGroup V] [TopologicalSpace V] [MeasurableSpace V] {χ : V → V → ℂ} {μ : Measure V} (hV : NADual V χ μ) {φ φ' : V → ℂ} (h : SB φ) (h' : SB φ') : fou χ μ (φ + φ') = fou χ μ φ + fou χ μ φ'

theorem **fou_smul** {V : Type} [MeasurableSpace V] (χ : V → V → ℂ) (μ : Measure V) (c : ℂ) (φ : V → ℂ) : fou χ μ (c • φ) = c • fou χ μ φ

theorem **thTerm_one** (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (P : InfinitePlace F → Matrix ι ι ℝ) (φ : (ι → A F) → ℂ) (τ : InfinitePlace F → ℂ) (x : ι → F) : thTerm B ρ P 1 (fun _ => 1) φ τ x = φ (toA x) * gauss B ρ P τ (cur (emb ρ x))

theorem **theta_add** {B : Matrix ι ι F} {ρ : InfinitePlace F → (F →+* ℝ)} {P : InfinitePlace F → Matrix ι ι ℝ} {τ : InfinitePlace F → ℂ} {φ φ' : (ι → A F) → ℂ} (h : Summable fun x => ‖thTerm B ρ P 1 (fun _ => 1) φ τ x‖) (h' : Summable fun x => ‖thTerm B ρ P 1 (fun _ => 1) φ' τ x‖) : theta B ρ P 1 (fun _ => 1) (φ + φ') τ = theta B ρ P 1 (fun _ => 1) φ τ + theta B ρ P 1 (fun _ => 1) φ' τ

theorem **theta_smul** (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (P : InfinitePlace F → Matrix ι ι ℝ) (τ : InfinitePlace F → ℂ) (c : ℂ) (φ : (ι → A F) → ℂ) : theta B ρ P 1 (fun _ => 1) (c • φ) τ = c * theta B ρ P 1 (fun _ => 1) φ τ

theorem **orth_one** (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) : IsOrth B ρ 1 (fun _ => 1)

theorem **poisson_form** [DecidableEq (InfinitePlace F)] {M : ℕ} {B : Matrix ι ι F} {ρ : InfinitePlace F → (F →+* ℝ)} {P : InfinitePlace F → Matrix ι ι ℝ} (hQ : QSetup M B ρ P) {τ : InfinitePlace F → ℂ} (hτ : ∀ w, 0 < (τ w).im) (g : Matrix (ι × InfinitePlace F) (ι × InfinitePlace F) ℝ) (hg : g.det ≠ 0) (a : ι × InfinitePlace F → ℝ) (u : (ι × InfinitePlace F → ℤ) → ι × InfinitePlace F → ℝ) (hu : ∀ n, DM B ρ *ᵥ u n = (g⁻¹)ᵀ *ᵥ fun i => (n i : ℝ)) : ∑' n : ι × InfinitePlace F → ℤ, gauss B ρ P τ (cur (a + g *ᵥ fun i => (n i : ℝ))) = ((|g.det|⁻¹ * (Real.sqrt |((Algebra.norm ℚ B.det : ℚ) : ℝ)|)⁻¹ : ℝ) : ℂ) * cG B ρ P τ * ∑' n : ι × InfinitePlace F → ℤ, gauss B ρ P (fun w => -(τ w)⁻¹) (cur (u n)) * Complex.exp (-(2 * (Real.pi : ℂ) * Complex.I) * ((a ⬝ᵥ (DM B ρ *ᵥ u n) : ℝ) : ℂ))

theorem **theta_indC** [DecidableEq (InfinitePlace F)] (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (P : InfinitePlace F → Matrix ι ι ℝ) (τ : InfinitePlace F → ℂ) (a : ι → F) (N : ℕ) (g : Matrix (ι × InfinitePlace F) (ι × InfinitePlace F) ℝ) (e : (ι × InfinitePlace F → ℤ) → ι → F) (hi : Function.Injective e) (hr : ∀ x : ι → F, toA x - toA a ∈ latN F ι N ↔ x ∈ Set.range e) (he : ∀ n, emb ρ (e n) = emb ρ a + g *ᵥ fun p => (n p : ℝ)) : theta B ρ P 1 (fun _ => 1) (indC N a) τ = ∑' n : ι × InfinitePlace F → ℤ, gauss B ρ P τ (cur (emb ρ a + g *ᵥ fun i => (n i : ℝ)))

theorem **base** [MeasurableSpace (A F)] [BorelSpace (A F)] {lf : Measure (A F)} (hl : lf.IsAddHaarMeasure) (h1 : lf (Oh F) = 1) {ψ : AddChar (A F) ℂ} (hψ : IsStdChar F ψ) {M : ℕ} {B : Matrix ι ι F} {ρ : InfinitePlace F → (F →+* ℝ)} {P : InfinitePlace F → Matrix ι ι ℝ} (hQ : QSetup M B ρ P) (hV : NADual (ι → A F) (chiB ψ (Bf B)) (muF B lf)) (a : ι → F) {N : ℕ} (hN : 0 < N) {τ : InfinitePlace F → ℂ} (hτ : ∀ w, 0 < (τ w).im) : cG B ρ P τ * theta B ρ P 1 (fun _ => 1) (fou (chiB ψ (Bf B)) (muF B lf) (indC N a)) (fun w => -(τ w)⁻¹) = theta B ρ P 1 (fun _ => 1) (indC N a) τ

theorem **core** [MeasurableSpace (A F)] [BorelSpace (A F)] {lf : Measure (A F)} (hl : lf.IsAddHaarMeasure) (h1 : lf (Oh F) = 1) {ψ : AddChar (A F) ℂ} (hψ : IsStdChar F ψ) {M : ℕ} {B : Matrix ι ι F} {ρ : InfinitePlace F → (F →+* ℝ)} {P : InfinitePlace F → Matrix ι ι ℝ} (hQ : QSetup M B ρ P) (hV : NADual (ι → A F) (chiB ψ (Bf B)) (muF B lf)) {φ : (ι → A F) → ℂ} (hφ : SB φ) {τ : InfinitePlace F → ℂ} (hτ : ∀ w, 0 < (τ w).im) : cG B ρ P τ * theta B ρ P 1 (fou (chiB ψ (Bf B)) (muF B lf) φ) (fun w => -(τ w)⁻¹) = theta B ρ P 1 (fun _ => 1) φ τ

theorem **thetaInvariance** : ∀ (F : Type) [Field F] [NumberField F] [MeasurableSpace (A F)] [BorelSpace (A F)] (lf : Measure (A F)), lf.IsAddHaarMeasure → lf (Oh F) = 1 → ∀ ψ : AddChar (A F) ℂ, IsStdChar F ψ → ∀ (ι : Type) [Fintype ι] [DecidableEq ι] (M : ℕ) (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (P : InfinitePlace F → Matrix ι ι ℝ), QSetup M B ρ P → ∀ (gf : Matrix ι ι (A F)) (gi : InfinitePlace F → Matrix ι ι ℝ), IsOrth B ρ gf gi → ∀ φ : (ι → A F) → ℂ, SB φ → ∀ τ : InfinitePlace F → ℂ, (∀ w, 0 < (τ w).im) → cG B ρ P τ * theta B ρ P gf gi (fou (chiB ψ (Bf B)) (muF B lf) φ) (fun w => -(τ w)⁻¹) = theta B ρ P gf gi φ τ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ThetaFunctionAutomorphyStatement

theorem **_root_.GlobalTheory.T.theta_function_automorphy** : ∀ (F : Type) [Field F] [NumberField F] [MeasurableSpace (A F)] [BorelSpace (A F)] (lf : Measure (A F)), lf.IsAddHaarMeasure → lf (Oh F) = 1 → ∀ ψ : AddChar (A F) ℂ, IsStdChar F ψ → ∀ (ι : Type) [Fintype ι] [DecidableEq ι] (M : ℕ) (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (P : InfinitePlace F → Matrix ι ι ℝ), QSetup M B ρ P → ∀ (gf : Matrix ι ι (A F)) (gi : InfinitePlace F → Matrix ι ι ℝ), IsOrth B ρ gf gi → ∀ φ : (ι → A F) → ℂ, SB φ → ∀ τ : InfinitePlace F → ℂ, (∀ w, 0 < (τ w).im) → cG B ρ P τ * theta B ρ P gf gi (fou (chiB ψ (Bf B)) (muF B lf) φ) (fun w => -(τ w)⁻¹) = theta B ρ P gf gi φ τ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ThetaFunctionCuspExpansion

theorem **ei_inv** : ∀ (F : Type) [Field F] [NumberField F] [MeasurableSpace (A F)] [BorelSpace (A F)] (lf : Measure (A F)), lf.IsAddHaarMeasure → lf (Oh F) = 1 → ∀ ψ : AddChar (A F) ℂ, IsStdChar F ψ → ∀ (ι : Type) [Fintype ι] [DecidableEq ι] (M : ℕ) (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (P : InfinitePlace F → Matrix ι ι ℝ), QSetup

M B ρ P → 3 ≤ M → ∀ φ : (ι → A F) → ℂ, SB φ → ∀ τ : InfinitePlace F → ℂ, (∀ w, 0 < (τ w).im) → (∀ b : F, eis ψ lf B ρ P (opU ψ (hf F) (Bf B) (algebraMap F (A F) b) φ) (fun w => τ w + (ρ w b : ℝ)) = eis ψ lf B ρ P φ τ) ∧ (∀ a : F, a ≠ 0 → eis ψ lf B ρ P (opD 1 (algebraMap F (A F) a) φ) (fun w => ((ρ w a : ℝ) : ℂ) ^ 2 * τ w) = eis ψ lf B ρ P φ τ) ∧ (cG B ρ P τ * eis ψ lf B ρ P (fou (chiB ψ (Bf B)) (muF B lf) φ) (fun w => -(τ w)⁻¹) = eis ψ lf B ρ P φ τ) ∧ ∀ c : ℂ, eis ψ lf B ρ P (c • φ) τ = c * eis ψ lf B ρ P φ τ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ThetaFunctionCuspExpansionStatement

theorem **_root_.GlobalTheory.T.theta_function_cusp_expansion** : ∀ (F : Type) [Field F] [NumberField F] [MeasurableSpace (A F)] [BorelSpace (A F)] (lf : Measure (A F)), lf.IsAddHaarMeasure → lf (Oh F) = 1 → ∀ ψ : AddChar (A F) ℂ, IsStdChar F ψ → ∀ (ι : Type) [Fintype ι] [DecidableEq ι] (M : ℕ) (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (P : InfinitePlace F → Matrix ι ι ℝ), QSetup M B ρ P → 3 ≤ M → ∀ φ : (ι → A F) → ℂ, SB φ → ∀ τ : InfinitePlace F → ℂ, (∀ w, 0 < (τ w).im) → (∀ b : F, eis ψ lf B ρ P (opU ψ (hf F) (Bf B) (algebraMap F (A F) b) φ) (fun w => τ w + (ρ w b : ℝ)) = eis ψ lf B ρ P φ τ) ∧ (∀ a : F, a ≠ 0 → eis ψ lf B ρ P (opD 1 (algebraMap F (A F) a) φ) (fun w => ((ρ w a : ℝ) : ℂ) ^ 2 * τ w) = eis ψ lf B ρ P φ τ) ∧ (cG B ρ P τ * eis ψ lf B ρ P (fou (chiB ψ (Bf B)) (muF B lf) φ) (fun w => -(τ w)⁻¹) = eis ψ lf B ρ P φ τ) ∧ ∀ c : ℂ, eis ψ lf B ρ P (c • φ) τ = c * eis ψ lf B ρ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ThetaFunctionGrowthBound

theorem **prod_sum_bound** {S R : Type} (h : S × R → ℝ) (a : S → ℝ) (K : ℝ) (h0 : ∀ p, 0 ≤ h p) (ha : Summable a) (hin : ∀ s, Summable (fun r => h (s, r)) ∧ ∑' r, h (s, r) ≤ a s * K) : Summable h ∧ ∑' p, h p ≤ (∑' s, a s) * K

theorem **exists_decomp** (F : Type) [Field F] [NumberField F] : ∃ (Q : Type) (out : Q → F), Function.Injective out ∧ (∀ q q', (∃ r : 𝒪 F, out q - out q' = (r : F)) → q = q') ∧ Function.Bijective (fun p : Q × 𝒪 F => out p.1 + (p.2 : F))

theorem **place_bound** (M : ℕ) (hM : 2 ≤ M) (t y : ℝ) (ht : 0 < t) (hy : t ≤ y) : Real.sqrt y ^ M * (y * (y ^ M)⁻¹) ≤ ((Real.sqrt t) ^ (M - 2))⁻¹

theorem **ei_bound** : ∀ (F : Type) [Field F] [NumberField F] [MeasurableSpace (A F)] [BorelSpace (A F)] (lf : Measure (A F)), lf.IsAddHaarMeasure → lf (Oh F) = 1 → ∀ ψ : AddChar (A F) ℂ, IsStdChar F ψ → ∀ (ι : Type) [Fintype ι] [DecidableEq ι] (M : ℕ) (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (P : InfinitePlace F → Matrix ι ι ℝ), QSetup M B ρ P → 3 ≤ M → ∀ φ : (ι → A F) → ℂ, SB φ → ∀ t : ℝ, 0 < t → ∃ C : ℝ, ∀ τ : InfinitePlace F → ℂ, (∀ w, t ≤ (τ w).im) → Summable (fun b => ‖eiTerm ψ lf B ρ P φ τ b‖) ∧ wt M τ * ∑' b, ‖eiTerm ψ lf B ρ P φ τ b‖ ≤ C

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ThetaFunctionGrowthBoundStatement

theorem **_root_.GlobalTheory.T.summable_norm_eiTerm_and_wt_mul_tsum_le** : ∀ (F : Type) [Field F] [NumberField F] [MeasurableSpace (A F)] [BorelSpace (A F)] (lf : Measure (A F)), lf.IsAddHaarMeasure → lf (Oh F) = 1 → ∀ ψ : AddChar (A F) ℂ, IsStdChar F ψ → ∀ (ι : Type) [Fintype ι] [DecidableEq ι] (M : ℕ) (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (P : InfinitePlace F → Matrix ι ι ℝ), QSetup M B ρ P → 3 ≤ M → ∀ φ : (ι → A F) → ℂ, SB φ → ∀ t : ℝ, 0 < t → ∃ C : ℝ, ∀ τ : InfinitePlace F → ℂ, (∀ w, t ≤ (τ w).im) → Summable (fun b => ‖eiTerm ψ lf B ρ P φ τ b‖) ∧ wt M τ * ∑' b, ‖eiTerm ψ lf B ρ P φ τ b‖ ≤ C

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ThetaFunctionModularity

theorem **form_mulVec** {R ι : Type} [CommRing R] [Fintype ι] (G Q : Matrix ι ι R) (h : Gᵀ * Q * G = Q) (v : ι → R) : (G *ᵥ v) ⬝ᵥ (Q *ᵥ (G *ᵥ v)) = v ⬝ᵥ (Q *ᵥ v)

theorem **form_map** {R S ι : Type} [CommRing R] [CommRing S] [Fintype ι] (f : R →+* S) (Q : Matrix ι ι R) (x : ι → R) : (fun j => f (x j)) ⬝ᵥ (Q.map f *ᵥ fun j => f (x j)) = f (x ⬝ᵥ (Q *ᵥ x))

theorem **trace_eq_sum** (hT : IsTotallyReal F) (ρ : InfinitePlace F → (F →+* ℝ)) (hp : ∀ w x, ((ρ w x : ℝ) : ℂ) = w.embedding x) (y : F) : ((Algebra.trace ℚ F y : ℚ) : ℂ) = ∑ w, ((ρ w y : ℝ) : ℂ)

theorem **gauss_add** (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (P : InfinitePlace F → Matrix ι ι ℝ) (τ : InfinitePlace F → ℂ) (c : InfinitePlace F → ℝ) (y : InfinitePlace F → ι → ℝ) : gauss B ρ P (fun w => τ w + (c w : ℝ)) y = gauss B ρ P τ y * Complex.exp ((Real.pi : ℂ) * Complex.I * ∑ w, ((c w : ℂ) * ((y w ⬝ᵥ (B.map (ρ w) *ᵥ y w) : ℝ) : ℂ)))

theorem **gauss_sq** (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (P : InfinitePlace F → Matrix ι ι ℝ) (τ : InfinitePlace F → ℂ) (c : InfinitePlace F → ℝ) (y : InfinitePlace F → ι → ℝ) : gauss B ρ P (fun w => ((c w : ℝ) : ℂ) ^ 2 * τ w) y = gauss B ρ P τ (fun w => c w • y w)

theorem **term_U** (ψ : AddChar (A F) ℂ) (hψ : IsStdChar F ψ) (M : ℕ) (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (P : InfinitePlace F → Matrix ι ι ℝ) (hQ : QSetup M B ρ P) (gf : Matrix ι ι (A F)) (gi : InfinitePlace F → Matrix ι ι ℝ) (hO : IsOrth B ρ gf gi) (φ : (ι → A F) → ℂ) (τ : InfinitePlace F → ℂ) (b : F) (x : ι → F) : thTerm B ρ P gf gi (opU ψ (hf F) (Bf B) (algebraMap F (A F) b) φ) (fun w => τ w + (ρ w b : ℝ)) x = thTerm B ρ P gf gi φ τ x

theorem **term_D** (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (P : InfinitePlace F → Matrix ι ι ℝ) (gf : Matrix ι ι (A F)) (gi : InfinitePlace F → Matrix ι ι ℝ) (φ : (ι → A F) → ℂ) (τ : InfinitePlace F → ℂ) (a : F) (x : ι → F) : thTerm B ρ P gf gi (opD 1 (algebraMap F (A F) a) φ) (fun w => ((ρ w a : ℝ) : ℂ) ^ 2 * τ w) x = thTerm B ρ P gf gi φ τ (a • x)

theorem **main** : ∀ (F : Type) [Field F] [NumberField F], ∀ ψ : AddChar (A F) ℂ, IsStdChar F ψ → ∀ (ι : Type) [Fintype ι] [DecidableEq ι] (M : ℕ) (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (P : InfinitePlace F → Matrix ι ι ℝ), QSetup M B ρ P → ∀ (gf : Matrix ι ι (A F)) (gi : InfinitePlace F → Matrix ι ι ℝ), IsOrth B ρ gf gi → ∀ (φ : (ι → A F) → ℂ) (τ : InfinitePlace F → ℂ), (∀ b : F, theta B ρ P gf gi (opU ψ (hf F) (Bf B) (algebraMap F (A F) b) φ) (fun w => τ w + (ρ w b : ℝ)) = theta B ρ P gf gi φ τ) ∧ (∀ a : F, a ≠ 0 → theta B ρ P gf gi (opD 1 (algebraMap F (A F) a) φ) (fun w => ((ρ w a : ℝ) : ℂ) ^ 2 * τ w) = theta B ρ P gf gi φ τ) ∧ ∀ c : ℂ, theta B ρ P gf gi (c • φ) τ = c * theta B ρ P gf gi φ τ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ThetaFunctionModularityStatement

theorem **_root_.GlobalTheory.T.theta_function_modularity** : ∀ (F : Type) [Field F] [NumberField F], ∀ ψ : AddChar (A F) ℂ, IsStdChar F ψ → ∀ (ι : Type) [Fintype ι] [DecidableEq ι] (M : ℕ) (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (P : InfinitePlace F → Matrix ι ι ℝ), QSetup M B ρ P → ∀ (gf : Matrix ι ι (A F)) (gi : InfinitePlace F → Matrix ι ι ℝ), IsOrth B ρ gf gi → ∀ (φ : (ι → A F) → ℂ) (τ : InfinitePlace F → ℂ), (∀ b : F, theta B ρ P gf gi (opU ψ (hf F) (Bf B) (algebraMap F (A F) b) φ) (fun w => τ w + (ρ w b : ℝ)) = theta B ρ P gf gi φ τ) ∧ (∀ a : F, a ≠ 0 → theta B ρ P gf gi (opD 1 (algebraMap F (A F) a) φ) (fun w => ((ρ w a : ℝ) : ℂ) ^ 2 * τ w) = theta B ρ P gf gi φ τ) ∧ ∀ c : ℂ, theta B ρ P gf gi (c • φ) τ = c * theta B ρ P gf gi φ τ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ThetaIntegralOverCoordinates

theorem **lintegral_thetaMeas_coord** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (μ : Measure (HA (AF F) σ J)) (ℱ : Set (HA (AF F) σ J)), IsHaarH F E σ M J μ → IsFDH F E σ M J μ ℱ → ∀ (S : Set (Fin M × Fin d → F)) (T : Set (Fin 1 → Fin M → E)), (∀ x, ofF b x ∈ T ↔ x ∈ S) → ∀ Φ : (Fin 1 → Fin M → AF F ⊗[F] E) → ℝ≥0∞, Measurable Φ → ∫⁻ X, Φ X ∂(thetaMeas σ J μ ℱ T) = ∫⁻ ω, ∑' x : S, Φ (actW (AF F) σ J ω⁻¹ (ratTup (AF F) (ofF b x.1))) ∂(prH μ ℱ)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ThetaIntegralOverCoordinatesStatement

theorem **_root_.GlobalTheory.T.theta_integral_over_coordinates** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (μ : Measure (HA (AF F) σ J)) (ℱ : Set (HA (AF F) σ J)), IsHaarH F E σ M J μ → IsFDH F E σ M J μ ℱ → ∀ (S : Set (Fin M × Fin d → F)) (T : Set (Fin 1 → Fin M → E)), (∀ x, ofF b x ∈ T ↔ x ∈ S) → ∀ Φ : (Fin 1 → Fin M → AF F ⊗[F] E) → ℝ≥0∞, Measurable Φ → ∫⁻ X, Φ X ∂(thetaMeas σ J μ ℱ T) = ∫⁻ ω, ∑' x : S, Φ (actW (AF F) σ J ω⁻¹ (ratTup (AF F) (ofF b x.1))) ∂(prH μ ℱ)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ThetaMeasureIntegralFormula

theorem **lintegral_thetaMeas** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (μ : Measure (HA (AF F) σ J)) (ℱ : Set (HA (AF F) σ J)), IsHaarH F E σ M J μ → IsFDH F E σ M J μ ℱ → ∀ (n : ℕ) (S : Set (Fin n → Fin M → E)) (Φ : (Fin n → Fin M → AF F ⊗[F] E) → ℝ≥0∞), Measurable Φ → ∫⁻ Y, Φ Y ∂(thetaMeas σ J μ ℱ S) = (μ ℱ)⁻¹ * cntI σ J μ ℱ S Φ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ThetaMeasureIntegralFormulaStatement

theorem **_root_.GlobalTheory.T.theta_measure_integral_formula** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (μ : Measure (HA (AF F) σ J)) (ℱ : Set (HA (AF F) σ J)), IsHaarH F E σ M J μ → IsFDH F E σ M J μ ℱ → ∀ (n : ℕ) (S : Set (Fin n → Fin M → E)) (Φ : (Fin n → Fin M → AF F ⊗[F] E) → ℝ≥0∞), Measurable Φ → ∫⁻ Y, Φ Y ∂(thetaMeas σ J μ ℱ S) = (μ ℱ)⁻¹ * cntI σ J μ ℱ S Φ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ThetaMeasureNonzeroVectorsBound

theorem **box_mono** {F E : Type} [Field F] [NumberField F] [Field E] [Algebra F E] {d M n : ℕ} (b : Module.Basis (Fin d) F E) (N : ℕ) {r r' : ℝ} (h : r ≤ r') : (box b N r : Set (Fin n → Fin M → AF F ⊗[F] E)) ⊆ box b N r'

theorem **real_bound** {C₁ C₂ x y z : ℝ} (hx : 0 ≤ x) (hy : 0 ≤ y) (hz : 0 ≤ z) (hxy : x ≤ y) : C₁ * x ≤ (|C₁| + |C₂|) * (y + z)

theorem **mean_count_box** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → 3 ≤ M → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (μ : Measure (HA (AF F) σ J)) (ℱ : Set (HA (AF F) σ J)), IsHaarH F E σ M J μ → IsFDH F E σ M J μ ℱ → ∀ N : ℕ, 0 < N → ∃ C : ℝ, ∀ r : ℝ, 0 < r → thetaMeas σ J μ ℱ {X : Fin 1 → Fin M → E | X ≠ 0} {X | (∀ v, (fun a i => (N : Fv F v ⊗[F] E) * projV v (projFin X) a i) ∈ latV b v) ∧ sizeTup b (projInf X) ≤ r} ≤ ENNReal.ofReal (C * (r ^ (Module.finrank ℚ F * M) + r ^ (2 * Module.finrank ℚ F * M)))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ThetaMeasureNonzeroVectorsBoundStatement

theorem **_root_.GlobalTheory.T.theta_measure_nonzero_vectors_bound** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → 3 ≤ M → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (μ : Measure (HA (AF F) σ J)) (ℱ : Set (HA (AF F) σ J)), IsHaarH F E σ M J μ → IsFDH F E σ M J μ ℱ → ∀ N : ℕ, 0 < N → ∃ C : ℝ, ∀ r : ℝ, 0 < r → thetaMeas σ J μ ℱ {X : Fin 1 → Fin M → E | X ≠ 0} {X | (∀ v, (fun a i => (N : Fv F v ⊗[F] E) ∗ projV v (projFin X) a i) ∈ latV b v) ∧ sizeTup b (projInf X) ≤ r} ≤ ENNReal.ofReal (C ∗ (r ^ (Module.finrank ℚ F ∗ M) + r ^ (2 ∗ Module.finrank ℚ F ∗ M)))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ThetaSeriesSummable

theorem **norm_mixed_sq_le** (F : Type) [Field F] [NumberField F] (y : F) : ‖mixedEmbedding F y‖ ^ 2 ≤ ∑ w : InfinitePlace F, (w y) ^ 2

theorem **summable_integers** (F : Type) [Field F] [NumberField F] {a : ℝ} (ha : 0 < a) : Summable fun r : 𝒪 F => Real.exp (-a ∗ ∑ w : InfinitePlace F, (w (r : F)) ^ 2)

theorem **summable_pi_integers** (F : Type) [Field F] [NumberField F] (ι : Type) [Fintype ι] {a : ℝ} (ha : 0 < a) : Summable fun r : ι → 𝒪 F => Real.exp (-a ∗ ∑ j, ∑ w : InfinitePlace F, (w (r j : F)) ^ 2)

theorem **exists_quad_lower** {ι : Type} [Fintype ι] [DecidableEq ι] (P g : Matrix ι ι ℝ) (hP : P.PosDef) (hg : IsUnit g.det) : ∃ c : ℝ, 0 < c ∧ ∀ v : ι → ℝ, c ∗ ‖v‖ ^ 2 ≤ (g *ᵥ v) ⬝ᵥ (P *ᵥ (g *ᵥ v))

theorem **isUnit_det_of_orth** {R ι : Type} [CommRing R] [Fintype ι] [DecidableEq ι] (G C : Matrix ι ι R) (h : Gᵀ ∗ C ∗ G = C) (hC : IsUnit C.det) : IsUnit G.det

theorem **norm_gauss** (B : Matrix ι ι F) (ρ : InfinitePlace F → Matrix ι ι ℝ) (P : InfinitePlace F → Matrix ι ι ℝ) (τ : InfinitePlace F → ℂ) (x : InfinitePlace F → ι → ℝ) : ‖gauss B ρ P τ x‖ = Real.exp (-∑ w, Real.pi ∗ (τ w).im ∗ (x w ⬝ᵥ (P w *ᵥ x w)))

theorem **isUnit_det_Bf** (B : Matrix ι ι F) (h : B.det ≠ 0) : IsUnit (Bf B).det

theorem **gauss_bound** {M : ℕ} {B : Matrix ι ι F} {ρ : InfinitePlace F → (F →+* ℝ)} {P : InfinitePlace F → Matrix ι ι ℝ} (hQ : QSetup M B ρ P) {gi : InfinitePlace F → Matrix ι ι ℝ} (hgi : ∀ w, (gi w)ᵀ ∗ B.map (ρ w) ∗ gi w = B.map (ρ w)) {τ : InfinitePlace F → ℂ} (hτ : ∀ w, 0 < (τ w).im) : ∃ a : ℝ, 0 < a ∧ ∀ x : ι → F, ‖gauss B ρ P τ (fun w => gi w *ᵥ fun j => ρ w (x j))‖ ≤ Real.exp (-a ∗ ∑ j, ∑ w : InfinitePlace F, (w (x j)) ^ 2)

theorem **support_denominator** {gf : Matrix ι ι (A F)} (hu : IsUnit gf.det) {φ : (ι → A F) → ℂ} (hφ : SB φ) : ∃ N : ℕ, 0 < N ∧ ∀ x : ι → F, φ (gf *ᵥ fun j => algebraMap F (A F) (x j)) ≠ 0 → ∀ j, ∃ r : 𝒪 F, (r : F) = (N : F) ∗ x j

theorem **summable_thTerm** {M : ℕ} {B : Matrix ι ι F} {ρ : InfinitePlace F → (F →+* ℝ)} {P : InfinitePlace F → Matrix ι ι ℝ} (hQ : QSetup M B ρ P) {gf : Matrix ι ι (A F)} {gi : InfinitePlace F → Matrix ι ι ℝ} (hO : IsOrth B ρ gf gi) {τ : InfinitePlace F → ℂ} (hτ : ∀ w, 0 < (τ w).im) {φ : (ι → A F) → ℂ} (hφ : SB φ) : Summable fun x => ‖thTerm B ρ P gf gi φ τ x‖

theorem **sb_indN** {gf : Matrix ι ι (A F)} (hu : IsUnit gf.det) {N : ℕ} (hN : 0 < N) : SB (indN gf N)

theorem **natCast_mul_mem_Oh** (n : ℕ) {z : A F} (hz : z ∈ Oh F) : (n : A F) ∗ z ∈ Oh F

theorem **vecN_subset** (gf : Matrix ι ι (A F)) (n : ℕ) : vecN gf n ⊆ vecN gf 1

theorem **exists_not_multiple** {y : F} (hy : y ≠ 0) : ∃ n₀ : ℕ, ∀ n ≥ n₀, ∀ r : 𝒪 F, y ≠ ((n + 1 : ℕ) : F) ∗ (r : F)

theorem **indN_eventually_zero** {gf : Matrix ι ι (A F)} (hu : IsUnit gf.det) {x : ι → F} (hx : x ≠ 0) : ∀ᶠ n : ℕ in Filter.atTop, indN gf (n + 1) (gf *ᵥ fun j => algebraMap F (A F) (x j)) = 0

theorem **indN_zero** (gf : Matrix ι ι (A F)) (n : ℕ) (h0 : (0 : A F) ∈ Oh F) : indN gf n 0 = 1

theorem **exists_theta_ne_zero** {M : ℕ} {B : Matrix ι ι F} {ρ : InfinitePlace F → (F →+* ℝ)} {P : InfinitePlace F → Matrix ι ι ℝ} (hQ : QSetup M B ρ P) {gf : Matrix ι ι (A F)} {gi : InfinitePlace F → Matrix ι ι ℝ} (hO : IsOrth B ρ gf gi) {τ : InfinitePlace F → ℂ} (hτ : ∀ w, 0 < (τ w).im) : ∃ φ : (ι → A F) → ℂ, SB φ ∧ theta B ρ P gf gi φ τ ≠ 0

theorem **theta_summable_and_ne_zero** : ∀ (F : Type) [Field F] [NumberField F], ∀ (ι : Type) [Fintype ι] [DecidableEq ι] (M : ℕ) (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (P : InfinitePlace F → Matrix ι ι ℝ), QSetup M B ρ P → ∀ (gf : Matrix ι ι (A F)) (gi : InfinitePlace F → Matrix ι ι ℝ), IsOrth B ρ gf gi → ∀ τ : InfinitePlace F → ℂ, (∀ w, 0 < (τ w).im) → (∀ φ : (ι → A F) → ℂ, SB φ → Summable fun x => ‖thTerm B ρ P gf gi φ τ x‖) ∧ ∃ φ : (ι → A F) → ℂ, SB φ ∧ theta B ρ P gf gi φ τ ≠ 0

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.ThetaSeriesSummableStatement

theorem **_root_.GlobalTheory.T.theta_series_summable** : ∀ (F : Type) [Field F] [NumberField F], ∀ (ι : Type) [Fintype ι] [DecidableEq ι] (M : ℕ) (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (P : InfinitePlace F → Matrix ι ι ℝ), QSetup M B ρ P → ∀ (gf : Matrix ι ι (A F)) (gi : InfinitePlace F → Matrix ι ι ℝ), IsOrth B ρ gf gi → ∀ τ : InfinitePlace F → ℂ, (∀ w, 0 < (τ w).im) → (∀ φ : (ι → A F) → ℂ, SB φ → Summable fun x => ‖thTerm B ρ P gf gi φ τ x‖) ∧ ∃ φ : (ι → A F) → ℂ, SB φ ∧ theta B ρ P gf gi φ τ ≠ 0

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.TotallyRealContinuousEmbeddingInfiniteAdeles

theorem **norm_eq_abs_ringEquiv** {F : Type} [Field F] {w : InfinitePlace F} (hw : w.IsReal) (x : w.Completion) : ‖x‖ = |InfinitePlace.Completion.ringEquivRealOfIsReal hw x|

theorem **exists_continuous_real_embedding** : ∀ (F : Type) [Field F] [NumberField F], IsTotallyReal F → (∃ θ : ℝ →+* InfiniteAdeleRing F, Continuous θ) ∧ ∀ θ : ℝ →+* InfiniteAdeleRing F, Continuous θ → ∀ (t : ℝ) (w : InfinitePlace F), ‖θ t w‖ = |t|

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.TotallyRealContinuousEmbeddingInfiniteAdelesStatement

theorem **_root_.GlobalTheory.T.totally_real_continuous_embedding_infinite_adeles** : ∀ (F : Type) [Field F] [NumberField F], IsTotallyReal F → (∃ θ : ℝ →+* InfiniteAdeleRing F, Continuous θ) ∧ ∀ θ : ℝ →+* InfiniteAdeleRing F, Continuous θ → ∀ (t : ℝ) (w : InfinitePlace F), ‖θ t w‖ = |t|

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.TotallyRealIntegralBasisExists

theorem **exists_integralBasis_infinitePlace** (F : Type) [Field F] [NumberField F] (hF : IsTotallyReal F) : Nonempty (Module.Basis (InfinitePlace F) ℤ (𝒪 F))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.TotallyRealIntegralBasisExistsStatement

theorem **_root_.GlobalTheory.T.totally_real_integral_basis_exists** : ∀ (F : Type) [Field F] [NumberField F], IsTotallyReal F → Nonempty (Module.Basis (InfinitePlace F) ℤ (𝒪 F))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.TprodComplementSplitFinset

theorem **tprod_tail_cut** : ∀ (ι : Type) [DecidableEq ι] (a : ι → ℝ) (S S' : Finset ι), Summable (fun v => |a v - 1|) → S ⊆ S' → ∏' v : {v // v ∉ S}, a v.1 = (∏ v ∈ S' \ S, a v) ∗ ∏' v : {v // v ∉ S'}, a v.1

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.TprodComplementSplitFinsetStatement

theorem **_root_.GlobalTheory.T.tprod_complement_split_finset** : ∀ (ι : Type) [DecidableEq ι] (a : ι → ℝ) (S S' : Finset ι), Summable (fun v => |a v - 1|) → S ⊆ S' → ∏' v : {v // v ∉ S}, a v.1 = (∏ v ∈ S' \ S, a v) ∗ ∏' v : {v // v ∉ S'}, a v.1

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.TraceFormInvarianceUnderUnitary

theorem **isometry_map_ringHom** {ι R S K : Type} [Fintype ι] [CommRing R] [CommRing S] [CommRing K] (φ : R →+* S) (α : K →+* R) (ψ : K →+* S) (hφ : ∀ x, φ (α x) = ψ x) (A : Matrix ι ι R) (B : Matrix ι ι K) (h : Aᵀ ∗ B.map α ∗ A = B.map α) : (A.map φ)ᵀ ∗ B.map ψ ∗ A.map φ = B.map ψ

theorem **ginf_gfin_isometry** (F E : Type) [Field F] [NumberField F] [Field E] [Algebra F E] (σ : E →ₐ[F] E) (M : ℕ) (J : Matrix (Fin M) (Fin M) E) (d : ℕ) (b : Module.Basis (Fin d) F E) (B : Matrix (Fin M × Fin d) (Fin M × Fin d) F) (ρ : InfinitePlace F → (F →+* ℝ)) (e : Ainf F ≃+* (InfinitePlace F → ℝ)) (he : ∀ x w, e (algebraMap F (Ainf F) x) w = ρ w x) (hB : ∀ h : HA (AF F) σ J, (gmat (AF F) σ J b h)ᵀ ∗ B.map (algebraMap F (AF F)) ∗ gmat (AF F) σ J b h = B.map (algebraMap F (AF F))) : (∀ ω w, (ginf e σ J b ω w)ᵀ ∗ B.map (ρ w) ∗ ginf e σ J b ω w = B.map (ρ w)) ∧ ∀ ω, (gfin σ J b ω)ᵀ ∗ B.map (algebraMap F (Af F)) ∗ gfin σ J b ω = B.map (algebraMap F (Af F))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.TraceFormInvarianceUnderUnitaryStatement

theorem **_root_.GlobalTheory.T.trace_form_invariance_under_unitary** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [Algebra F E] (σ : E →ₐ[F] E) (M : ℕ) (J : Matrix (Fin M) (Fin M) E) (d : ℕ) (b : Module.Basis (Fin d) F E) (B : Matrix (Fin M × Fin d) (Fin M × Fin d) F) (ρ : InfinitePlace F → (F →+* ℝ)) (e : Ainf F ≃+* (InfinitePlace F → ℝ)), (∀ x w, e (algebraMap F (Ainf F) x) w = ρ w x) → (∀ h : HA (AF F) σ J, (gmat (AF F) σ J b h)ᵀ ∗ B.map (algebraMap F (AF F)) ∗ gmat (AF F) σ J b h = B.map (algebraMap F (AF F))) → (∀ ω w, (ginf e σ J b ω w)ᵀ ∗ B.map (ρ w) ∗ ginf e σ J b ω w = B.map (ρ w)) ∧ ∀ ω, (gfin σ J b ω)ᵀ ∗ B.map (algebraMap F (Af F)) ∗ gfin σ J b ω = B.map (algebraMap F (Af F))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.TraceFormRationalThetaIdentity

theorem **continuous_gaussQ** {F E : Type} [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E) (hσ : IsCMExt F E σ) {M d : ℕ} (b : Module.Basis (Fin d) F E) (e : Ainf F ≃+* (InfinitePlace F → ℝ)) (hom : IsHomeomorph e) (Pm : InfinitePlace F → Matrix (Fin M × Fin d) (Fin M × Fin d) ℝ) : Continuous (gaussQ e b Pm)

theorem **lintegral_thetaMeas_gauss** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (B : Matrix (Fin M × Fin d) (Fin M × Fin d) F), (∀ x : Fin M → E, Algebra.trace F E (hE σ J x x) = (fun p : Fin M × Fin d => b.repr (x p.1) p.2) ⬝ᵥ (B *ᵥ fun p : Fin M × Fin d => b.repr (x p.1) p.2)) → ∀ (ρ : InfinitePlace F → (F →+* ℝ)) (e : Ainf F ≃+* (InfinitePlace

F → ℝ)), IsHomeomorph e → (∀ x w, e (algebraMap F (Ainf F) x) w = ρ w x) → ∀ (Pm : InfinitePlace F → Matrix (Fin M × Fin d) (Fin M × Fin d) ℝ) (μ : Measure (HA (AF F) σ J)) (ℱ : Set (HA (AF F) σ J)), IsHaarH F E σ M J μ → IsFDH F E σ M J μ ℱ → ∀ (a : ℕ), 0 < a → ∀ β : F, ∫⁻ X in {X | ∀ v, projV v (projFin X) ∈ {X : (Fin 1 → Fin M → Fv F v ⊗[F] E) | (fun p i => (a : Fv F v ⊗[F] E)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.TraceFormRationalThetaIdentityStatement

theorem **_root_.GlobalTheory.T.trace_form_rational_theta_identity** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (B : Matrix (Fin M × Fin d) (Fin M × Fin d) F), (∀ x : Fin M → E, Algebra.trace F E (hE σ J x x) = (fun p : Fin M × Fin d => b.repr (x p.1) p.2) ⬝ᵥ (B *ᵥ fun p : Fin M × Fin d => b.repr (x p.1) p.2)) → ∀ (ρ : InfinitePlace F → (F →+* ℝ)) (e : Ainf F ≃+* (InfinitePlace F → ℝ)), IsHomeomorph e → (∀ x w, e (algebraMap F (Ainf F) x) w = ρ w x) → ∀ (Pm : InfinitePlace F → Matrix (Fin M × Fin d) (Fin M × Fin d) ℝ) (μ : Measure (HA (AF F) σ J)) (ℱ : Set (HA (AF F) σ J)), IsHaarH F E σ M J μ → IsFDH F E σ M J μ ℱ → ∀ (a : ℕ), 0 < a → ∀ β : F, ∫⁻ X in {X | ∀ v, projV v (projFin X) ∈ {X : (Fin 1 → Fin M → Fv F v ⊗[F] E) | (

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.TraceGramDetNeZero

theorem **trace_one_tmul** [FiniteDimensional F E] (R : Type) [CommRing R] [Algebra F R] (e : E) : Algebra.trace R (R ⊗[F] E) ((1 : R) ⊗ₜ[F] e) = algebraMap F R (Algebra.trace F E e)

theorem **coordL_apply** (R : Type) [CommRing R] [Algebra F R] (M : ℕ) {d : ℕ} (b : Module.Basis (Fin d) F E) (x : Fin M → R ⊗[F] E) (p : Fin M × Fin d) : coordL R M b x p = (Algebra.TensorProduct.basis R b).repr (x p.1) p.2

theorem **trace_expand** [FiniteDimensional F E] (R : Type) [CommRing R] [Algebra F R] (σ : E →ₐ[F] E) {d : ℕ} (b : Module.Basis (Fin d) F E) (u v : R ⊗[F] E) (e : E) : Algebra.trace R (R ⊗[F] E) (sigR R σ u * inclR R e * v) = ∑ k, ∑ l, (Algebra.TensorProduct.basis R b).repr u k * (algebraMap F R (Algebra.trace F E (σ (b k) * e * b l)) * (Algebra.TensorProduct.basis R b).repr v l)

theorem **trace_hR** [FiniteDimensional F E] (R : Type) [CommRing R] [Algebra F R] (σ : E →ₐ[F] E) {M d : ℕ} (J : Matrix (Fin M) (Fin M) E) (b : Module.Basis (Fin d) F E) (x y : Fin M → R ⊗[F] E) : Algebra.trace R (R ⊗[F] E) (hR R σ J x y) = coordL R M b x ⬝ᵥ (traceGram σ J b).map (algebraMap F R) *ᵥ coordL R M b y

theorem **hform_mulVec** {A : Type} [CommRing A] (s : A →+* A) {M : ℕ} (J g : Matrix (Fin M) (Fin M) A) (x y : Fin M → A) : hform s J (g *ᵥ x) (g *ᵥ y) = hform s (cT s g * J * g) x y

theorem **matrix_ext_dot** {ι A : Type} [Fintype ι] [DecidableEq ι] [CommRing A] (P Q : Matrix ι ι A) (h : ∀ u v : ι → A, u ⬝ᵥ P *ᵥ v = u ⬝ᵥ Q *ᵥ v) : P = Q

theorem **dot_conj** {ι A : Type} [Fintype ι] [CommRing A] (g P : Matrix ι ι A) (u v : ι → A) : u ⬝ᵥ (gᵀ * P * g) *ᵥ v = (g *ᵥ u) ⬝ᵥ P *ᵥ (g *ᵥ v)

theorem **traceGram_isSymm** [FiniteDimensional F E] (σ : E →ₐ[F] E) (hσ : ∀ x, σ (σ x) = x) {M d : ℕ} (J : Matrix (Fin M) (Fin M) E) (hJ : ∀ i j, σ (J j i) = J i j) (b : Module.Basis (Fin d) F E) : (traceGram σ J b).IsSymm

theorem **traceGram_det_ne_zero** [FiniteDimensional F E] [Algebra.IsSeparable F E] (σ : E →ₐ[F] E) (hσ : ∀ x, σ (σ x) = x) {M d : ℕ} (J : Matrix (Fin M) (Fin M) E) (hJ : J.det ≠ 0) (b : Module.Basis (Fin d) F E) : (traceGram σ J b).det ≠ 0

theorem **hR_mat** (R : Type) [CommRing R] [Algebra F R] (σ : E →ₐ[F] E) {M : ℕ} (J : Matrix (Fin M) (Fin M) E) (h : HA R σ J) (x y : Fin M → R ⊗[F] E) : hR R σ J ((HA.mat R σ J h : Matrix (Fin M) (Fin M) (R ⊗[F] E)) *ᵥ x) ((HA.mat R σ J h : Matrix (Fin M) (Fin M) (R ⊗[F] E)) *ᵥ y) = hR R σ J x y

theorem **coord_orthogonal** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E), ∃ B : Matrix (Fin M × Fin d) (Fin M × Fin d) F, B.IsSymm ∧ B.det ≠ 0 ∧ ∀ (R : Type) [CommRing R] [Algebra F R] (h : HA R σ J), ∃ g : Matrix (Fin M × Fin d) (Fin M × Fin d) R, gᵀ * B.map (algebraMap F R) * g = B.map (algebraMap F R) ∧ ∀ X : Fin 1 → Fin M → R ⊗[F] E, (fun p : Fin M × Fin d => (Algebra.TensorProduct.basis R b).repr (actW R σ J h X 0 p.1) p.2) = g *ᵥ fun p : Fin M × Fin d => (Algebra.TensorProduct.basis R b).repr (X 0 p.1) p.2

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.UnitBalancingImaginaryParts

theorem **exists_bound_log_centered** (E : Type) [Field E] [NumberField E] : ∃ R : ℝ, ∀ t : InfinitePlace E → ℝ, ∃ (u : (𝓞 E)ˣ) (s : ℝ), ∀ w : InfinitePlace E, |t w + Real.log (w (u : E)) − s| ≤ R

theorem **balance_by_units** : ∀ (F : Type) [Field F] [NumberField F], IsTotallyReal F → ∀ ρ : InfinitePlace F → (F →+* ℝ), (∀ w x, ((ρ w x : ℝ) : ℂ) = w.embedding x) → ∃ κ : ℝ, 0 < κ ∧ ∀ y : InfinitePlace F → ℝ, (∀ w, 0 < y w) → ∃ u : (𝓞 F)ˣ, ∀ v w, κ * (ρ w ((u : 𝓞 F) : F) ^ 2 * y w) ≤ ρ v ((u : 𝓞 F) : F) ^ 2 * y v

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.UnitBalancingImaginaryPartsStatement

theorem **_root_.GlobalTheory.T.unit_balancing_imaginary_parts** : ∀ (F : Type) [Field F] [NumberField F], IsTotallyReal F → ∀ ρ : InfinitePlace F → (F →+* ℝ), (∀ w x, ((ρ w x : ℝ) : ℂ) = w.embedding x) → ∃ κ : ℝ, 0 < κ ∧ ∀ y : InfinitePlace F → ℝ, (∀ w, 0 < y w) → ∃ u : (𝓞 F)ˣ, ∀ v w, κ * (ρ w ((u : 𝓞 F) : F) ^ 2 * y w) ≤ ρ v ((u : 𝓞 F) : F) ^ 2 * y v

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.UnitChoiceOfIsArtinianRingIsReduced

theorem **unitChoice_of_isArtinianRing_isReduced** : ∀ (R : Type) [CommRing R] [IsArtinianRing R] [IsReduced R], PIW.AdelicOrbitU.UnitChoice R

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.UnitProductClosureProperties

theorem **mulSupport_finite** {a : ℝ≥0} {c : HeightOneSpectrum (𝓞 F) → ℝ≥0} (h : IsUnitProd a c) : (Function.mulSupport c).Finite

theorem **map_hom** (f : ℝ≥0 →* ℝ≥0) {a : ℝ≥0} {c : HeightOneSpectrum (𝓞 F) → ℝ≥0} (h : IsUnitProd a c) : IsUnitProd (f a) (fun v => f (c v))

theorem **mul** {a b : ℝ≥0} {c e : HeightOneSpectrum (𝓞 F) → ℝ≥0} (h : IsUnitProd a c) (k : IsUnitProd b e) : IsUnitProd (a * b) (fun v => c v * e v)

theorem **ne_zero** {a : ℝ≥0} {c : HeightOneSpectrum (𝓞 F) → ℝ≥0} (h : IsUnitProd a c) : a ≠ 0

theorem **family_ne_zero** {a : ℝ≥0} {c : HeightOneSpectrum (𝓞 F) → ℝ≥0} (h : IsUnitProd a c) (v : HeightOneSpectrum (𝓞 F)) : c v ≠ 0

theorem **inv** {a : ℝ≥0} {c : HeightOneSpectrum (𝓞 F) → ℝ≥0} (h : IsUnitProd a c) : IsUnitProd a⁻¹ (fun v => (c v)⁻¹)

theorem **main** : ∀ (F : Type) [Field F] [NumberField F] (a b : ℝ≥0) (c e : HeightOneSpectrum (𝓞 F) → ℝ≥0), IsUnitProd a c → IsUnitProd b e → a ≠ 0 ∧ (∀ v, c v ≠ 0) ∧ IsUnitProd (a * b) (fun v => c v * e v) ∧ IsUnitProd a⁻¹ (fun v => (c v)⁻¹) ∧ IsUnitProd (a / b) (fun v => c v / e v) ∧ IsUnitProd (NNReal.sqrt a) (fun v => NNReal.sqrt (c v)) ∧ (∀ n : ℕ, IsUnitProd (a ^ n) (fun v => c v ^ n)) ∧ ∃ S₀ : Finset (HeightOneSpectrum (𝓞 F)), ∀ S, S₀ ⊆ S → (∀ v, v ∉ S → c v = 1) ∧ a * ∏ v ∈ S, c v = 1

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.UnitProductClosurePropertiesStatement

theorem **_root_.GlobalTheory.T.unit_product_closure_properties** : ∀ (F : Type) [Field F] [NumberField F] (a b : ℝ≥0) (c e : HeightOneSpectrum (𝓞 F) → ℝ≥0), IsUnitProd a c → IsUnitProd b e → a ≠ 0 ∧ (∀ v, c v ≠ 0) ∧ IsUnitProd (a * b) (fun v => c v * e v) ∧ IsUnitProd a⁻¹ (fun v => (c v)⁻¹) ∧ IsUnitProd (a / b) (fun v => c v / e v) ∧ IsUnitProd (NNReal.sqrt a) (fun v => NNReal.sqrt (c v)) ∧ (∀ n : ℕ, IsUnitProd (a ^ n) (fun v => c v ^ n)) ∧ ∃ S₀ : Finset (HeightOneSpectrum (𝓞 F)), ∀ S, S₀ ⊆ S → (∀ v, v ∉ S → c v = 1) ∧ a * ∏ v ∈ S, c v = 1

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.UnitaryActionConsTupleMeasurable

theorem **continuous_consTup** {K : Type} [Zero K] [One K] [TopologicalSpace K] (n M : ℕ) : Continuous (fun X : Fin n → Fin M → K => (consTup X : Fin (n + 1) → Fin (M + 1) → K))

theorem **measurable_actW_consTup** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R], IsLocR R → ∀ (n M : ℕ) (J : Matrix (Fin (M + 1)) (Fin (M + 1)) E) (g : HA R σ J), Measurable (fun X : Fin n → Fin M → R ⊗[F] E => actW R σ J g (consTup X))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.UnitaryActionConsTupleMeasurableStatement

theorem **_root_.GlobalTheory.T.unitary_action_cons_tuple_measurable** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R], IsLocR R → ∀ (n M : ℕ) (J : Matrix (Fin (M + 1)) (Fin (M + 1)) E) (g : HA R σ J), Measurable (fun X : Fin n → Fin M → R ⊗[F] E => actW R σ J g (consTup X))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.UnitaryActionEulerProduct

theorem **repr_incl_mul** {d : ℕ} (b : Module.Basis (Fin d) F E) (e : E) (y : R ⊗[F] E) (k : Fin d) : (Algebra.TensorProduct.basis R b).repr (inclR R e * y) k = ∑ l, algebraMap F R (Algebra.leftMulMatrix b e k l) * (Algebra.TensorProduct.basis R b).repr y l

theorem **coTup_mulVec** {n M d : ℕ} (b : Module.Basis (Fin d) F E) (G : Matrix (Fin M) (Fin M) E) (X : Fin n → Fin M → R ⊗[F] E) : coTup R b (fun a => G.map (inclR R) *ᵥ X a) = (actMat n b G).map (algebraMap F R) *ᵥ coTup R b X

theorem **coTup_act** {n M d : ℕ} (b : Module.Basis (Fin d) F E) (g : GL (Fin M) E) (X : Fin n → Fin M → R ⊗[F] E) : coTup R b (actTup R (bc (inclR R) g) X) = (actMat n b (g : Matrix (Fin M) (Fin M) E)).map (algebraMap F R) *ᵥ coTup R b X

theorem **det_actMat_ne_zero** (n : ℕ) {M d : ℕ} (b : Module.Basis (Fin d) F E) (g : GL (Fin M) E) : (actMat n b (g : Matrix (Fin M) (Fin M) E)).det ≠ 0

theorem **map_actTup** {n M d : ℕ} (b : Module.Basis (Fin d) F E) (g : GL (Fin M) E) (c : ℝ≥0) (h : ∀ ν : Measure (Fin n × Fin M × Fin d → R), ν.IsAddHaarMeasure → ν.map (fun x => (actMat n b (g : Matrix (Fin M) (Fin M) E)).map (algebraMap F R) *ᵥ x) = c • ν) (μ : Measure (Fin n → Fin M → R ⊗[F] E)) (hμ : μ.IsAddHaarMeasure) : μ.map (actTup R (bc (inclR R) g)) = c • μ

theorem **product_formula_actTup** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (n M : ℕ) (g : GL (Fin M) E), ∃ (cinf : ℝ≥0) (c : HeightOneSpectrum (𝒪 F) → ℝ≥0), IsUnitProd cinf c ∧ (∀ μ : Measure (Fin n → Fin M → Ainf F ⊗[F] E), μ.IsAddHaarMeasure → μ.map (actTup (Ainf F) (bc (inclR (Ainf F)) g)) = cinf • μ) ∧ ∀ v : HeightOneSpectrum (𝒪 F), (∀ μ : Measure (Fin n → Fin M → Fv F v ⊗[F] E), μ.IsAddHaarMeasure → μ.map (actTup (Fv F v) (bc (inclR (Fv F v)) g)) = c v • μ)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.UnitaryActionEulerProductStatement

theorem **_root_.GlobalTheory.T.haar_modulus_gl_action_product_formula** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (n M : ℕ) (g : GL (Fin M) E), ∃ (cinf : ℝ≥0) (c : HeightOneSpectrum (𝒪 F) → ℝ≥0), IsUnitProd cinf c ∧ (∀ μ : Measure (Fin n → Fin M → Ainf F ⊗[F] E), μ.IsAddHaarMeasure → μ.map (actTup (Ainf F) (bc (inclR (Ainf F)) g)) = cinf • μ) ∧ ∀ v : HeightOneSpectrum (𝒪 F), (∀ μ : Measure (Fin n → Fin M → Fv F v ⊗[F] E), μ.IsAddHaarMeasure → μ.map (actTup (Fv F v) (bc (inclR (Fv F v)) g)) = c v • μ)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.UnitaryActionPreservesHaarLocal

theorem **det_mul_conj_eq_one** {A : Type} [CommRing A] {m : Type} [Fintype m] [DecidableEq m] (τ : A →+* A) (g J : Matrix m m A) (hg : cT τ g * J * g = J) (hJ : IsUnit J.det) : g.det * τ g.det = 1

theorem **map_tuple_eq** {K : Type} [CommRing K] [TopologicalSpace K] [IsTopologicalRing K] [LocallyCompactSpace K] [SecondCountableTopology K] [T2Space K] [MeasurableSpace K] [BorelSpace K] [IsArtinianRing K] [IsReduced K] (τ : K →+* K) (hc : Continuous τ) (hi : ∀ y, τ (τ y) = y) {M n : ℕ} (A : Matrix (Fin M) (Fin M) K) (hA : A.det * τ A.det = 1) (μ : Measure (Fin n → Fin M → K)) [μ.IsAddHaarMeasure] : μ.map (fun X a => A *ᵥ X a) = μ

theorem **map_actW_eq** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R], IsLocR R → ∀ (n : ℕ) (μ : Measure (Fin n → Fin M → R ⊗[F] E)), μ.IsAddHaarMeasure → ∀ h : HA R σ J, μ.map (actW R σ J h) = μ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.UnitaryActionPreservesHaarLocalStatement

theorem **_root_.GlobalTheory.T.unitary_action_preserves_haar_local** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R], IsLocR R → ∀ (n : ℕ) (μ : Measure (Fin n → Fin M → R ⊗[F] E)), μ.IsAddHaarMeasure → ∀ h : HA R σ J, μ.map (actW R σ J h) = μ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.UnitaryGroupFiniteVolume

theorem **finVol_unitaryGroup** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → FinVol F E σ M J

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.UnitaryGroupFiniteVolumeStatement

theorem **_root_.GlobalTheory.T.unitary_group_finite_volume** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → FinVol F E σ M J

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.UnitaryIsomOfIsometry

theorem **conj_mem_UJ** (s : A →+* A) (J : Matrix n n A) (G u : GL n A) (hu : u ∈ UJ s (cT s (G : Matrix n n A) * J * (G : Matrix n n A))) : G * u * G⁻¹ ∈ UJ s J

theorem **conj_inv_mem_UJ** (s : A →+* A) (J : Matrix n n A) (G u : GL n A) (hu : u ∈ UJ s J) : G⁻¹ * u * G ∈ UJ s (cT s (G : Matrix n n A) * J * (G : Matrix n n A))

theorem **cT_inclR** (σ : E →ₐ[F] E) {M : ℕ} (g : Matrix (Fin M) (Fin M) E) : cT (sigR R σ) (g.map (inclR R)) = (cT (σ : E →+* E) g).map (inclR R)

theorem **JR_conj** (σ : E →ₐ[F] E) {M : ℕ} (J : Matrix (Fin M) (Fin M) E) (g : GL (Fin M) E) : JR R (cT (σ : E →+* E) (g : Matrix (Fin M) (Fin M) E) * J * (g : Matrix (Fin M) (Fin M) E)) = cT (sigR R σ) ((bc (inclR R) g : GL (Fin M) (R ⊗[F] E)) : Matrix (Fin M) (Fin M) (R ⊗[F] E)) * JR R J * ((bc (inclR R) g : GL (Fin M) (R ⊗[F] E)) : Matrix (Fin M) (Fin M) (R ⊗[F] E))

theorem **isTopologicalRing_tens** (F E : Type) [Field F] [Field E] [Algebra F E] [Module.Finite F E] (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [IsTopologicalRing R] : IsTopologicalRing (R ⊗[F] E)

theorem **GlobalTheory.CompanionTheory.SW.HIsom.unitaryIsom_of_isometry** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (g : GL (Fin M) E) (J' : Matrix (Fin M) (Fin M) E), J' = cT (σ : E →+* E) (g : Matrix (Fin M) (Fin M) E) * J * g → ∀ (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [IsTopologicalRing R], ∃ e : HA R σ J' ≃ₗ* HA R σ J, (∀ u, HA.mat R σ J (e u) = bc (inclR R) g * HA.mat R σ J' u * (bc (inclR R) g)⁻¹) ∧ ∀ u, HA.mat R σ J (e u) ∈ HFin R σ J ↔ HA.mat R σ J' u ∈ HFin R σ J'

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.UnitaryTransitiveOnIsotropicVectors

theorem **hE_eq_sum** (x y : Fin M → E) : hE σ J x y = ∑ i, ∑ j, σ (x i) * J i j * y j

theorem **hE_add_right** (x y z : Fin M → E) : hE σ J x (y + z) = hE σ J x y + hE σ J x z

theorem **hE_smul_right** (c : E) (x y : Fin M → E) : hE σ J x (c • y) = c * hE σ J x y

theorem **hE_add_left** (x y z : Fin M → E) : hE σ J (x + y) z = hE σ J x z + hE σ J y z

theorem **hE_smul_left** (c : E) (x y : Fin M → E) : hE σ J (c • x) y = σ c * hE σ J x y

theorem **hE_conj** (hσ : ∀ a, σ (σ a) = a) (hJ : ∀ i j, σ (J j i) = J i j) (x y : Fin M → E) : σ (hE σ J x y) = hE σ J y x

theorem **exists_hE_ne_zero** (hdet : J.det ≠ 0) {x : Fin M → E} (hx : x ≠ 0) : ∃ w, hE σ J x w ≠ 0

theorem **exists_hyperbolic_partner** [CharZero E] (hσ : ∀ a, σ (σ a) = a) (hJ : ∀ i j, σ (J j i) = J i j) (hdet : J.det ≠ 0) {x : Fin M → E} (hx : x ≠ 0) (hxx : hE σ J x x = 0) : ∃ x', hE σ J x x' = 1 ∧ hE σ J x' x = 1 ∧ hE σ J x' x' = 0

theorem **transitive_isotropic** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ x y : Fin M → E, x ≠ 0 → y ≠ 0 → hE σ J x x = 0 → hE σ J y y = 0 → ∃ g ∈ UJ (σ : E →+* E) J, (g : Matrix (Fin M) (Fin M) E) *ᵥ x = y

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.UnitaryTransitiveOnIsotropicVectorsStatement

theorem **_root_.GlobalTheory.T.unitary_transitive_on_isotropic_vectors** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ x y : Fin M → E, x ≠ 0 → y ≠ 0 → hE σ J x x = 0 → hE σ J y y = 0 → ∃ g ∈ UJ (σ : E →+* E) J, (g : Matrix (Fin M) (Fin M) E) *ᵥ x = y

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.UnitsOpenEmbeddingLocalTensor

theorem **continuousOn_ringInverse** {R : Type} [CommRing R] [TopologicalSpace R] [IsTopologicalRing R] (h : IsOpenEmbedding (Units.val : Rˣ → R)) : ContinuousOn (Ring.inverse : R → R) {r | IsUnit r}

theorem **units_val_isOpenEmbedding** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R], IsLocR R → IsOpenEmbedding (Units.val : (R ⊗[F] E)ˣ → R ⊗[F] E)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.UnitsOpenEmbeddingLocalTensorStatement

theorem **_root_.GlobalTheory.T.units_open_embedding_local_tensor** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R], IsLocR R → IsOpenEmbedding (Units.val : (R ⊗[F] E)ˣ → R ⊗[F] E)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.WeilOperatorIntegralInvariance

theorem **oh_mul** {F : Type} [Field F] [NumberField F] {x y : A F} (hx : x ∈ Oh F) (hy : y ∈ Oh F) : x * y ∈ Oh F

theorem **continuous_qf** {F : Type} [Field F] [NumberField F] {ι : Type} [Fintype ι] (h : A F) (B : Matrix ι ι (A F)) : Continuous (qf h B)

theorem **fstar_periodic** : ∀ (F : Type) [Field F] [NumberField F] [MeasurableSpace (A F)] [BorelSpace (A F)] (lf : Measure (A F)), lf.IsAddHaarMeasure → lf (Oh F) = 1 → ∀ ψ : AddChar (A F) ℂ, IsStdChar F ψ → ∀ (ι : Type) [Fintype ι] [DecidableEq ι] (M : ℕ) (B : Matrix ι ι F) (ρ : InfinitePlace F → (F →+* ℝ)) (P : InfinitePlace F → Matrix ι ι ℝ), QSetup M B ρ P → ∀ φ : (ι → A F) → ℂ, SB φ → ∃ N : ℕ, 0 < N ∧ ∀ x ∈ Oh F, opU ψ (hf F) (Bf B) ((N : A F) * x) φ = φ ∧ ∀ y : A F, fstar ψ (hf F) (Bf B) (muF B lf) φ (y + (N : A F) * x) = fstar ψ (hf F) (Bf B) (muF B lf) φ y

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.WeilOperatorIntegralInvarianceStatement

theorem **_root_.GlobalTheory.T.weil_operator_integral_invariance** : ∀ (F : Type) [Field F] [NumberField F] [MeasurableSpace (A F)] [BorelSpace (A F)] (lf : Measure (A F)), lf.IsAddHaarMeasure → lf (Oh F) = 1 → ∀ ψ : AddChar (A F) ℂ, IsStdChar F ψ → ∀ (ι : Type) [Fintype ι] [DecidableEq ι] (M : ℕ) (B : Matrix ι ι F) (ρ : InfinitePlace F → (F

→+* ℝ)) (P : InfinitePlace F → Matrix ι ι ℝ), QSetup M B ρ P → ∀ φ : (ι → A F) → ℂ, SB φ → ∃ N : ℕ, 0 < N ∧ ∀ x ∈ Oh F, opU ψ (hf F) (Bf B) ((N : A F) * x) φ = φ ∧ ∀ y : A F, fstar ψ (hf F) (Bf B) (muF B lf) φ (y + (N : A F) * x) = fstar ψ (hf F) (Bf B) (muF B lf) φ y

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.WeilOrthogonalGroupInvariance

theorem **chiB_mulVec_mulVec** {R : Type} [CommRing R] {ι : Type} [Fintype ι] (ψ : AddChar R ℂ) (B g : Matrix ι ι R) (hg : gᵀ * B * g = B) (x y : ι → R) : chiB ψ B (g *ᵥ x) (g *ᵥ y) = chiB ψ B x y

theorem **perp_image** {V : Type} (χ : V → V → ℂ) (e : V ≃ V) (hχ : ∀ x y, χ (e x) (e y) = χ x y) (L : Set V) : perp χ (e '' L) = e '' perp χ L

theorem **map_eq_self** {V : Type} [AddCommGroup V] [TopologicalSpace V] [MeasurableSpace V] (χ : V → V → ℂ) (μ : Measure V) (hD : NADual V χ μ) (e : V ≃+ V) (he : Continuous e) (hes : Continuous e.symm) (hχ : ∀ x y, χ (e x) (e y) = χ x y) : Measure.map e μ = μ

theorem **orth_meas** : ∀ (R : Type) [CommRing R] [TopologicalSpace R] [MeasurableSpace R] (ι : Type) [Fintype ι] [DecidableEq ι] (ψ : AddChar R ℂ) (h : R) (B : Matrix ι ι R) (μ : Measure (ι → R)), NAQuad ψ h B μ → ∀ g : Matrix ι ι R, IsUnit g.det → gᵀ * B * g = B → Measure.map (fun x => g *ᵥ x) μ = μ ∧ ∀ φ : (ι → R) → ℂ, SB φ → SB (fun x => φ (g *ᵥ x)) ∧ ∀ y, fou (chiB ψ B) μ (fun x => φ (g *ᵥ x)) y = fou (chiB ψ B) μ φ (g *ᵥ y)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.WeilOrthogonalGroupInvarianceStatement

theorem **_root_.GlobalTheory.T.weil_orthogonal_group_invariance** : ∀ (R : Type) [CommRing R] [TopologicalSpace R] [MeasurableSpace R] (ι : Type) [Fintype ι] [DecidableEq ι] (ψ : AddChar R ℂ) (h : R) (B : Matrix ι ι R) (μ : Measure (ι → R)), NAQuad ψ h B μ → ∀ g : Matrix ι ι R, IsUnit g.det → gᵀ * B * g = B → Measure.map (fun x => g *ᵥ x) μ = μ ∧ ∀ φ : (ι → R) → ℂ, SB φ → SB (fun x => φ (g *ᵥ x)) ∧ ∀ y, fou (chiB ψ B) μ (fun x => φ (g *ᵥ x)) y = fou (chiB ψ B) μ φ (g *ᵥ y)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.WeilRepresentationQuadraticDatum

lemma **Bf_mul** (g h : Matrix ι ι F) : Bf (g * h) = Bf g * Bf h

lemma **Bf_one** : Bf (1 : Matrix ι ι F) = 1

lemma **Bf_transpose** (g : Matrix ι ι F) : Bf gᵀ = (Bf g)ᵀ

lemma **Bf_inv_mul** {g : Matrix ι ι F} (hg : g.det ≠ 0) : Bf g⁻¹ * Bf g = 1

lemma **Bf_mul_inv** {g : Matrix ι ι F} (hg : g.det ≠ 0) : Bf g * Bf g⁻¹ = 1

lemma **Bf_smul_one** (c : F) : Bf (c • (1 : Matrix ι ι F)) = algebraMap F (A F) c • 1

lemma **mem_Gs_iff** {g : Matrix ι ι F} (hg : g.det ≠ 0) {S : Set (A F)} {y : ι → A F} : y ∈ Gs g S ↔ ∀ i, (Bf g⁻¹ *ᵥ y) i ∈ S

lemma **continuous_mulVec'** (M : Matrix ι ι (A F)) : Continuous fun y : ι → A F => M *ᵥ y

lemma **isOpen_Gs** {g : Matrix ι ι F} (hg : g.det ≠ 0) {S : Set (A F)} (hS : IsOpen S) : IsOpen (Gs g S)

lemma **isCompact_Gs** (g : Matrix ι ι F) {S : Set (A F)} (hS : IsCompact S) : IsCompact (Gs g S)

lemma **Gs_mono** (g : Matrix ι ι F) {S T : Set (A F)} (h : S ⊆ T) : Gs g S ⊆ Gs g T

lemma **det_inv_ne** {g : Matrix ι ι F} (hg : g.det ≠ 0) : (g⁻¹).det ≠ 0

lemma **psi_sum** {R : Type} [CommRing R] (ψ : AddChar R ℂ) (s : Finset ι) (f : ι → R) : ψ (∑ i ∈ s, f i) = ∏ i ∈ s, ψ (f i)

lemma **perp_Gs** (ψ : AddChar (A F) ℂ) (B g : Matrix ι ι F) (hg : (gᵀ * B).det ≠ 0) {S : Set (A F)} (h0 : (0 : A F) ∈ S) : perp (chiB ψ (Bf B)) (Gs g S) = Gs (gᵀ * B)⁻¹ (perp (fun x y => ψ (x * y)) S)

lemma **norm_inv'** (x : F) (hx : x ≠ 0) : Algebra.norm ℚ x⁻¹ = (Algebra.norm ℚ x)⁻¹

lemma **coe_LL** {c : F} (hc : c ≠ 0) : (LL ι c : Set (ι → A F)) = Gs (c • (1 : Matrix ι ι F)) (Oh F)

lemma **pi_Gs** (lf : Measure (A F)) (hlf : lf.IsAddHaarMeasure) (h1 : lf (Oh F) = 1) {g : Matrix ι ι F} (hg : g.det ≠ 0) {S : Set (A F)} (hS : MeasurableSet S) : (Measure.pi fun _ : ι => lf) (Gs g S) = ENNReal.ofReal (|((Algebra.norm ℚ g.det : ℚ) : ℝ)|⁻¹) * lf S ^ Fintype.card ι

lemma **exists_LL_subset** {U : Set (ι → A F)} (hU : U ∈ 𝓝 (0 : ι → A F)) : ∃ N : ℕ, 0 < N ∧ (LL ι (N : F) : Set (ι → A F)) ⊆ U

lemma **exists_subset_LL** {K : Set (ι → A F)} (hK : IsCompact K) : ∃ N : ℕ, 0 < N ∧ K ⊆ (LL ι ((N : F)⁻¹) : Set (ι → A F))

lemma **real_id** (k s d : ℝ) (n : ℕ) (hk : k ≠ 0) (hs : s ≠ 0) (hd : d ≠ 0) : k ^ n * s⁻¹ * d⁻¹ * (k ^ n * s⁻¹ * (d * (s * s) * ((k * k)⁻¹) ^ n)) = 1

lemma **mass_one** (lf : Measure (A F)) (hlf : lf.IsAddHaarMeasure) (h1 : lf (Oh F) = 1) {B g : Matrix ι ι F} (hB : B.det ≠ 0) (hg : g.det ≠ 0) {P : Set (A F)} (hPm : MeasurableSet P) (hP : ENNReal.ofReal (kap F) * (ENNReal.ofReal (kap F) * lf P) = 1) : muF B lf (Gs g (Oh F)) * muF B lf (Gs (gᵀ * B)⁻¹ P) = 1

lemma **LL_good** (lf : Measure (A F)) (hlf : lf.IsAddHaarMeasure) (h1 : lf (Oh F) = 1) (ψ : AddChar (A F) ℂ) {B : Matrix ι ι F} (hS : B.IsSymm) (hB : B.det ≠ 0) (hOc : IsCompact (Oh F)) (hOo : IsOpen (Oh F)) (hPc : IsCompact (perp (fun x y => ψ (x * y)) (Oh F))) (hPo : IsOpen (perp (fun x y => ψ (x * y)) (Oh F))) (hPP : perp (fun x y => ψ (x * y)) (perp (fun x y => ψ (x * y)) (Oh F)) ⊆ Oh F) (hP : ENNReal.ofReal (kap F) * (ENNReal.ofReal (kap F) * lf (perp (fun x y => ψ (x * y)) (Oh F))) = 1) {c : F} (hc : c ≠ 0) : IsCompact (LL ι c : Set (ι → A F)) ∧ IsOpen (LL ι c : Set (ι → A F)) ∧ IsCompact (perp (chiB ψ (Bf B)) (LL ι c : Set (ι → A F))) ∧ IsOpen (perp (chiB ψ (Bf B)) (LL ι c : Set (ι → A F))) ∧ perp (chiB ψ (Bf B)) (perp (chiB ψ (Bf B)) (LL ι c : Set (ι → A F))) ⊆ LL ι c ∧ muF B lf (LL ι c : Set (ι → A F)) * muF B lf (perp (chiB ψ (Bf B)) (LL ι c : Set (ι → A F))) = 1

theorem **naQuad** : ∀ (F : Type) [Field F] [NumberField F] [MeasurableSpace (A F)] [BorelSpace (A F)] (lf : Measure (A F)), lf.IsAddHaarMeasure → lf (Oh F) = 1 → ∀ ψ : AddChar (A F) ℂ, IsStdChar F ψ → ∀ (ι : Type) [Fintype ι] [DecidableEq ι] (B : Matrix ι ι F), B.IsSymm → B.det ≠ 0 → NAQuad ψ (hf F) (Bf B) (muF B lf)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.WeilRepresentationQuadraticDatumStatement

theorem **_root_.GlobalTheory.T.weil_representation_quadratic_datum** : ∀ (F : Type) [Field F] [NumberField F] [MeasurableSpace (A F)] [BorelSpace (A F)] (lf : Measure (A F)), lf.IsAddHaarMeasure → lf (Oh F) = 1 → ∀ ψ : AddChar (A F) ℂ, IsStdChar F ψ → ∀ (ι : Type) [Fintype ι] [DecidableEq ι] (B : Matrix ι ι F), B.IsSymm → B.det ≠ 0 → NAQuad ψ (hf F) (Bf B) (muF B lf)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.WittExtensionArtinianRing

theorem **exists_actW_eq_of_gramR_eq** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (R : Type) [CommRing R] [Algebra F R] [IsArtinianRing R] [IsReduced R] (n : ℕ) (X Y : Fin n → Fin M → R ⊗[F] E), gramR R σ J X = gramR R σ J Y → IsUnit (gramR R σ J X).det → ∃ h : HA R σ J, actW R σ J h X = Y

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.WittExtensionArtinianRingStatement

theorem **_root_.GlobalTheory.T.witt_extension_artinian_ring** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (R : Type) [CommRing R] [Algebra F R] [IsArtinianRing R] [IsReduced R] (n : ℕ) (X Y : Fin n → Fin M → R ⊗[F] E), gramR R σ J X = gramR R σ J Y → IsUnit (gramR R σ J X).det → ∃ h : HA R σ J, actW R σ J h X = Y

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.WittExtensionDiagonalGramTuples

theorem **trans_diag** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (R : Type) [CommRing R] [Algebra F R], PIW.AdelicOrbitU.UnitChoice R → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (n : ℕ) (D : Fin n → Rˣ) (X Y : Fin n → Fin M → R ⊗[F] E), gramR R σ J X = Matrix.diagonal (fun i => algebraMap R (R ⊗[F] E) (D i : R)) → gramR R σ J Y = gramR R σ J X → ∃ h : HA R σ J, actW R σ J h X = Y

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.WittExtensionDiagonalGramTuplesStatement

theorem **_root_.GlobalTheory.T.witt_extension_diagonal_gram_tuples** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (R : Type) [CommRing R] [Algebra F R], PIW.AdelicOrbitU.UnitChoice R → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (n : ℕ) (D : Fin n → Rˣ) (X Y : Fin n → Fin M → R ⊗[F] E), gramR R σ J X = Matrix.diagonal (fun i => algebraMap R (R ⊗[F] E) (D i : R)) → gramR R σ J Y = gramR R σ J X → ∃ h : HA R σ J, actW R σ J h X = Y

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.WittExtensionGlobal

theorem **IsIsom.comp** {B : V →ₛₗ[σ] V →ₗ[E] E} {γ δ : V →ₗ[E] V} (hγ : IsIsom B γ) (hδ : IsIsom B δ) : IsIsom B (γ ∘ₗ δ)

theorem **exists_quasiRefl** (B : V →ₛₗ[σ] V →ₗ[E] E) (hσ : ∀ x, σ (σ x) = x) (hB : ∀ x y, σ (B x y) = B y x) (x y : V) (hxy : B x x = B y y) (h0 : B (x − y) x ≠ 0) : ∃ γ : V →ₗ[E] V, IsIsom B γ ∧ γ x = y ∧ ∀ z, B (x − y) z = 0 → γ z = z

theorem **exists_isom_of_eq** (B : V →ₛₗ[σ] V →ₗ[E] E) (hσ : ∀ x, σ (σ x) = x) (hB : ∀ x y, σ (B x y) = B y x) (h2 : (2 : E) ≠ 0) (x y : V) (hxy : B x x = B y y) (hq : B x x ≠ 0) : ∃ γ : V →ₗ[E] V, IsIsom B γ ∧ γ x = y ∧ ∀ z, B x z = 0 → B y z = 0 → γ z = z

theorem **form_lin_left** (B : V →ₛₗ[σ] V →ₗ[E] E) {n : ℕ} (X : Fin n → V) (c : Fin n → E) (z : V) : B (lin X c) z = ∑ a, σ (c a) * B (X a) z

theorem **form_lin_right** (B : V →ₛₗ[σ] V →ₗ[E] E) {n : ℕ} (X : Fin n → V) (c : Fin n → E) (z : V) : B z (lin X c) = ∑ a, c a * B z (X a)

theorem **lin_single** {n : ℕ} (X : Fin n → V) (a : Fin n) : lin X (Pi.single a (1 : E)) = X a

theorem **lin_add** {n : ℕ} (X : Fin n → V) (c d : Fin n → E) : lin X (c + d) = lin X c + lin X d

theorem **lin_smul** {n : ℕ} (X : Fin n → V) (t : E) (c : Fin n → E) : lin X (t • c) = t • lin X c

theorem **lin_sub** {n : ℕ} (X : Fin n → V) (c d : Fin n → E) : lin X (c - d) = lin X c - lin X d

theorem **lin_mem_span** {n : ℕ} (X : Fin n → V) (c : Fin n → E) : lin X c ∈ Submodule.span E (Set.range X)

theorem **exists_norm_ne_zero** (B : V →ₗ[σ] V →ₗ[E] E) (hB : ∀ x y, σ (B x y) = B y x) (h2 : (2 : E) ≠ 0) {n : ℕ} (X : Fin n → V) (hX : ¬ ∀ a b, B (X a) (X b) = 0) : ∃ c : Fin n → E, B (lin X c) (lin X c) ≠ 0

theorem **exists_isom_tuple** (B : V →ₗ[σ] V →ₗ[E] E) (hσ : ∀ x, σ (σ x) = x) (hB : ∀ x y, σ (B x y) = B y x) (h2 : (2 : E) ≠ 0) {n : ℕ} (k : ℕ) : ∀ X Y : Fin n → V, Module.finrank E (Submodule.span E (Set.range X)) = k → (∀ a b, B (X a) (X b) = B (Y a) (Y b)) → (∀ c : Fin n → E, (∀ a, B (X a) (lin X c) = 0) → lin X c = 0 ∧ lin Y c = 0) → ∃ γ : V →ₗ[E] V, IsIsom B γ ∧ ∀ a, γ (X a) = Y a

theorem **formB_apply** (s : E →+* E) {M : ℕ} (J : Matrix (Fin M) (Fin M) E) (x y : Fin M → E) : formB s J x y = hform s J x y

theorem **formB_conj** (s : E →+* E) (hs : ∀ x, s (s x) = x) {M : ℕ} (J : Matrix (Fin M) (Fin M) E) (hJ : ∀ i j, s (J j i) = J i j) (x y : Fin M → E) : s (formB s J x y) = formB s J y x

theorem **cT_mul_mul_apply** (s : E →+* E) {M : ℕ} (J g : Matrix (Fin M) (Fin M) E) (i j : Fin M) : (cT s g * J * g) i j = hform s J (fun k => g k i) (fun k => g k j)

theorem **exists_mem_UJ** (s : E →+* E) {M : ℕ} (J : Matrix (Fin M) (Fin M) E) (hJ : J.det ≠ 0) (γ : (Fin M → E) →ₗ[E] (Fin M → E)) (hγ : IsIsom (formB s J) γ) : ∃ g ∈ UJ s J, ∀ z, (g : Matrix (Fin M) (Fin M) E) *ᵥ z = γ z

theorem **witt_tuple** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (n : ℕ) (X Y : Fin n → Fin M → E), gramE σ J X = gramE σ J Y → (gramE σ J X).det ≠ 0 → ∃ g ∈ UJ (σ : E →+* E) J, ∀ a, (g : Matrix (Fin M) (Fin M) E) *ᵥ X a = Y a

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.WittExtensionGlobalStatement

theorem **_root_.GlobalTheory.T.witt_extension_global** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (n : ℕ) (X Y : Fin n → Fin M → E), gramE σ J X = gramE σ J Y → (gramE σ J X).det ≠ 0 → ∃ g ∈ UJ (σ : E →+* E) J, ∀ a, (g : Matrix (Fin M) (Fin M) E) *ᵥ X a = Y a

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.WittExtensionRationalToAdelic

theorem **trans_adelic** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (n : ℕ) (X : Fin n → Fin M → E) (Y : Fin n → Fin M → AF F ⊗[F] E), (gramE σ J X).det ≠ 0 → gramR (AF F) σ J Y = (gramE σ J X).map (inclR (AF F)) → ∃ h : HA (AF F) σ J, actW (AF F) σ J h (ratTup (AF F) X) = Y

CompanionTheory.SpectralGeometry.LocalGlobalProofs.AdelicAnalysis.WittExtensionRationalToAdelicStatement

theorem **_root_.GlobalTheory.T.witt_extension_rational_to_adelic** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (n : ℕ) (X : Fin n → Fin M → E) (Y : Fin n → Fin M → AF F ⊗[F] E), (gramE σ J X).det ≠ 0 → gramR (AF F) σ J Y = (gramE σ J X).map (inclR (AF F)) → ∃ h : HA (AF F) σ J, actW (AF F) σ J h (ratTup (AF F) X) = Y

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.BalancedFormDecompositionBound

theorem **transport** {φ χ : InvFun O} {b : ℕ} (e : X ≃ₗ[O] Y) {β : LinearMap.BilinForm ℤ Y} (h : ∃ (n : ℕ) (u : Fin n → LinearMap.BilinForm ℤ X), (∀ i, u i ∈ balForms bar X) ∧ (∀ i y, 0 ≤ u i y y) ∧ φ.LinearOn u ∧ χ.LinearOn u ∧ (∀ i, φ.evalZ (u i) ≤ b) ∧ (∀ i, χ.evalZ (u i) ≤ 1) ∧ InRCone u (pull e.toLinearMap β)) : ∃ (n : ℕ) (u : Fin n → LinearMap.BilinForm ℤ Y), (∀ i, u i ∈ balForms bar Y) ∧ (∀ i y, 0 ≤ u i y y) ∧ φ.LinearOn u ∧ χ.LinearOn u ∧ (∀ i, φ.evalZ (u i) ≤ b) ∧ (∀ i, χ.evalZ (u i) ≤ 1) ∧ InRCone u β

theorem **linearOn_starDepth** {ψ w : InvFun O} {n : ℕ} {u : Fin n → LinearMap.BilinForm ℤ X} (h : (starFun bar ψ w 0 1).LinearOn u) : (starDepth bar ψ w).LinearOn u

theorem **mul_le_of_sub_mem_rcone** {m : ℕ} {γ : Fin m → LinearMap.BilinForm ℤ X} {h : LinearMap.BilinForm ℤ X →ₗ[ℤ] ℤ} (hγ : ∀ i, 0 ≤ h (γ i)) {β v : LinearMap.BilinForm ℤ X} {t : ℝ} (hm : toReal β - t • toReal v ∈ rcone γ) : t * (h v : ℝ) ≤ h β

theorem **exists_nat_bound** {α : Type} {S : Set α} (hS : S.Finite) (f : α → ℝ) : ∃ b : ℕ, ∀ x ∈ S, f x ≤ b

theorem **centres_finite** (hw : IsCentreSet bar q ψ w) (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X] (hX : Module.finrank O X ≤ q) {S : Set (LinearMap.BilinForm ℤ X)} (hS : S.Finite) : {v | v ∈ balForms bar X ∧ (∀ y, 0 ≤ v y y) ∧ w.evalZ v = 1 ∧ ∃ (m : ℕ) (γ : Fin m → LinearMap.BilinForm ℤ X), IsCell bar ψ γ ∧ (∀ i, γ i ∈ S) ∧ InRCone γ v}.Finite

theorem **of_cell** (hcm : CMOrd O bar) (hw : IsCentreSet bar q ψ w) (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X] (hX : Module.finrank O X ≤ q) {b m : ℕ} {γ : Fin m → LinearMap.BilinForm ℤ X} (hγ : IsCell bar ψ γ) (hb : ∀ i, ψ.evalZ (γ i) ≤ b) (hb' : ∀ v ∈ balForms bar X, (∀ y, 0 ≤ v y y) → w.evalZ v = 1 → InRCone γ v → ψ.evalZ v ≤ b) {β : LinearMap.BilinForm ℤ X} (hβ : InRCone γ β) : ∃ (n : ℕ) (u : Fin n → LinearMap.BilinForm ℤ X), (∀ i, u i ∈ balForms bar X) ∧ (∀ i y, 0 ≤ u i y y) ∧ ψ.LinearOn u ∧ (starDepth bar ψ w).LinearOn u ∧ (∀ i, ψ.evalZ (u i) ≤ b) ∧ (∀ i, (starDepth bar ψ w).evalZ (u i) ≤ 1) ∧ InRCone u β

theorem **per_module** (hcm : CMOrd O bar) (hψ : IsPolyFan bar q ψ) (hw : IsCentreSet bar q ψ w) (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X] (hX : Module.finrank O X ≤ q) : ∃ b : ℕ, ∀ β ∈ balForms bar X, (∀ y, 0 ≤ β y y) → ∃ (n : ℕ) (u : Fin n → LinearMap.BilinForm ℤ X), (∀ i, u i ∈ balForms bar X) ∧ (∀ i y, 0 ≤ u i y y) ∧ ψ.LinearOn u ∧ (starDepth bar ψ w).LinearOn u ∧ (∀ i, ψ.evalZ (u i) ≤ b) ∧ (∀ i, (starDepth bar ψ w).evalZ (u i) ≤ 1) ∧ InRCone u β

theorem **exists_bounded_gens** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O), CMOrd O bar → ∀ (q : ℕ) (ψ w : InvFun O), IsPolyFan bar q ψ → IsCentreSet bar q ψ w → ∃ b : ℕ, ∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Module.finrank O X ≤ q → ∀ β ∈ balForms bar X, (∀ y, 0 ≤ β y y) → ∃ (n : ℕ) (u : Fin n → LinearMap.BilinForm ℤ X), (∀ i, u i ∈ balForms bar X) ∧ (∀ i y, 0 ≤ u i y y) ∧ ψ.LinearOn u ∧ (starDepth bar ψ w).LinearOn u ∧ (∀ i, ψ.evalZ (u i) ≤ b) ∧ (∀ i, (starDepth bar ψ w).evalZ (u i) ≤ 1) ∧ InRCone u β

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.BalancedFormDecompositionBoundStatement

theorem **_root_.GlobalTheory.T.balanced_form_decomposition_bound** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O), CMOrd O bar → ∀ (q : ℕ) (ψ w : InvFun O), IsPolyFan bar q ψ → IsCentreSet bar q ψ w → ∃ b : ℕ, ∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Module.finrank O X ≤ q → ∀ β ∈ balForms bar X, (∀ y, 0 ≤ β y y) → ∃ (n : ℕ) (u : Fin n → LinearMap.BilinForm ℤ X), (∀ i, u i ∈ balForms bar X) ∧ (∀ i y, 0 ≤ u i y y) ∧ ψ.LinearOn u ∧ (starDepth bar ψ w).LinearOn u ∧ (∀ i, ψ.evalZ (u i) ≤ b) ∧ (∀ i, (starDepth bar ψ w).evalZ (u i) ≤ 1) ∧ InRCone u β

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.BalancedNonnegFormInPsdCone

theorem **toReal_mem_psdCone_of_nonneg** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O), CMOrd O bar → ∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X] (β : LinearMap.BilinForm ℤ X), β ∈ balForms bar X → (∀ y, 0 ≤ β y y) → toReal β ∈ psdCone bar X

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.BalancedNonnegFormInPsdConeStatement

theorem **_root_.GlobalTheory.T.balanced_nonneg_form_in_psd_cone** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O), CMOrd O bar → ∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X] (β : LinearMap.BilinForm ℤ X), β ∈ balForms bar X → (∀ y, 0 ≤ β y y) → toReal β ∈ psdCone bar X

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.BarycentricStageExtension

theorem **baryStep** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O) (w0 : O), CMOrd O bar → ∀ (q : ℕ) (ψ lab φ : InvFun O) (i : ℕ), (∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Dominates bar X) → IsPolyFan bar q ψ → IsTwG w0 ψ → IsTwG w0 lab → IsCarrierDim bar q ψ lab → IsTwG w0 φ → IsBaryStage bar q ψ lab φ (i + 1) → ∃ φ' : InvFun O, IsTwG w0 φ' ∧ IsBaryStage bar q ψ lab φ' i

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.BarycentricStageExtensionStatement

theorem **_root_.GlobalTheory.T.barycentric_stage_extension** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O) (w0 : O), CMOrd O bar → ∀ (q : ℕ) (ψ lab φ : InvFun O) (i : ℕ), (∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Dominates bar X) → IsPolyFan bar q ψ → IsTwG w0 ψ → IsTwG w0 lab → IsCarrierDim bar q ψ lab → IsTwG w0 φ → IsBaryStage bar q ψ lab φ (i + 1) → ∃ φ' : InvFun O, IsTwG w0 φ' ∧ IsBaryStage bar q ψ lab φ' i

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.CarrierDimFanCellExists

theorem **inFace_iff** (hb : CMOrd O bar) (hψ : IsPolyFan bar q ψ) (hl : IsCarrierDim bar q ψ lab) (hX : Module.finrank O X ≤ q) (hD : Dominates bar X) {m : ℕ} {τ : Fin m → LinearMap.BilinForm ℤ X} (hτ : IsCell bar ψ τ) {β : LinearMap.BilinForm ℤ X} (hβ : InRCone τ β) (hr : (genRank τ : ℝ) ≤ lab.evalZ β) (x : LinearMap.BilinForm ℤ X) : InFace bar ψ lab β x ↔ InRCone τ x

theorem **isIndec_iff** (hb : CMOrd O bar) (hψ : IsPolyFan bar q ψ) (hl : IsCarrierDim bar q ψ lab) (hX : Module.finrank O X ≤ q) (hD : Dominates bar X) {m : ℕ} {τ : Fin m → LinearMap.BilinForm ℤ X} (hτ : IsCell bar ψ τ) {β : LinearMap.BilinForm ℤ X} (hβ : InRCone τ β) (hr : (genRank τ : ℝ) ≤ lab.evalZ β) (x : LinearMap.BilinForm ℤ X) : IsIndec bar ψ lab β x ↔ InRCone τ x ∧ x ≠ 0 ∧ ∀ y z, InRCone τ y → InRCone τ z → y ≠ 0 → z ≠ 0 → y + z ≠ x

theorem **exists_isMarked_inRCone** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O), CMOrd O bar → ∀ (q : ℕ) (ψ lab : InvFun O) (i : ℕ), (∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Dominates bar X) → IsPolyFan bar q ψ → IsCarrierDim bar q ψ lab → ∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Module.finrank O X ≤ q → ∀ (m : ℕ) (v : Fin m → LinearMap.BilinForm ℤ X), IsCell bar ψ v → ∀ β ∈ balForms bar X, (∀ y, 0 ≤ β y y) → InRCone v β → lab.evalZ β = ((i + 1 : ℕ) : ℝ) → ∃ β₀ : LinearMap.BilinForm ℤ X, IsMarked bar ψ lab i X β₀ ∧ InRCone v β₀

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.CarrierDimFanCellExistsStatement

theorem **_root_.GlobalTheory.T.exists_isMarked_inRCone_of_isCell_of_evalZ_eq** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O), CMOrd O bar → ∀ (q : ℕ) (ψ lab : InvFun O) (i : ℕ), (∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Dominates bar X) → IsPolyFan bar q ψ → IsCarrierDim bar q ψ lab → ∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Module.finrank O X ≤ q → ∀ (m : ℕ) (v : Fin m → LinearMap.BilinForm ℤ X), IsCell bar ψ v → ∀ β ∈ balForms bar X, (∀ y, 0 ≤ β y y) → InRCone v β → lab.evalZ β = ((i + 1 : ℕ) : ℝ) → ∃ β₀ : LinearMap.BilinForm ℤ X, IsMarked bar ψ lab i X β₀ ∧ InRCone v β₀

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.CarrierDimensionLabelBounded

private theorem **exists_int_ne_zero_eq_mul_of_ne_zero** (O : Type) [CommRing O] [IsDomain O] [Module.Finite ℤ O] (s : O) (hs : s ≠ 0) : ∃ n : ℤ, n ≠ 0 ∧ ∃ o : O, o ∗ s = (n : O)

private theorem **isLocalizedModule_int_restrictScalars_mkLinearMap** (O : Type) [CommRing O] [IsDomain O] [CharZero O] [Module.Finite ℤ O] (X : Type) [AddCommGroup X] [Module O X] : IsLocalizedModule (nonZeroDivisors ℤ) ((LocalizedModule.mkLinearMap (nonZeroDivisors O) X).restrictScalars ℤ)

private theorem **finrank_int_eq_mul_finrank** (O : Type) [CommRing O] [IsDomain O] [CharZero O] [Module.Finite ℤ O] (X : Type) [AddCommGroup X] [Module O X] : Module.finrank ℤ X = Module.finrank ℤ O ∗ Module.finrank O X

theorem **finrank_int_eq_fraction_mul** (O : Type) [CommRing O] [IsDomain O] [CharZero O] [Module.Finite ℤ O] (K : Type) [Field K] [CharZero K] [Algebra O K] [IsFractionRing O K] (X : Type) [AddCommGroup X] [Module O X] : Module.finrank ℤ X = Module.finrank ℚ K ∗ Module.finrank O X

theorem **finrank_int_eq_finrank_mul** (O : Type) [CommRing O] [IsDomain O] [CharZero O] [Module.Finite ℤ O] (X : Type) [AddCommGroup X] [Module O X] : Module.finrank ℤ X = Module.finrank ℤ O ∗ Module.finrank O X

private theorem **isTorsionFree_int_of_isTorsionFree** (O : Type) [CommRing O] [IsDomain O] [CharZero O] (Y : Type) [AddCommGroup Y] [Module O Y] [Module.IsTorsionFree O Y] : Module.IsTorsionFree ℤ Y

theorem **free_int_of_projective** (O : Type) [CommRing O] [IsDomain O] [CharZero O] [Module.Finite ℤ O] {Y : Type} [AddCommGroup Y] [Module O Y] [Module.Finite O Y] [Module.Projective O Y] : Module.Free ℤ Y

theorem **genRank_le_sq** (O : Type) [CommRing O] [IsDomain O] [CharZero O] [Module.Finite ℤ O] (q : ℕ) (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X] (hX : Module.finrank O X ≤ q) {m : ℕ} (γ : Fin m → LinearMap.BilinForm ℤ X) : genRank γ ≤ (q ∗ Module.finrank ℤ O) ^ 2

private theorem **nat_sInf_le_of_forall_le** {s : Set ℕ} {N : ℕ} (h : ∀ k ∈ s, k ≤ N) : sInf s ≤ N

theorem **carrierDim_bounded** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O), CMOrd O bar → ∀ (q : ℕ) (ψ lab : InvFun O), IsCarrierDim bar q ψ lab → ∃ N : ℕ, ∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Module.finrank O X ≤ q → ∀ β ∈ balForms bar X, (∀ y, 0 ≤ β y y) → lab.evalZ β ≤ N

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.CarrierDimensionLabelBoundedStatement

theorem **_root_.GlobalTheory.T.carrier_dimension_label_bounded** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O), CMOrd O bar → ∀ (q : ℕ) (ψ lab : InvFun O), IsCarrierDim bar q ψ lab → ∃ N : ℕ, ∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Module.finrank O X ≤ q → ∀ β ∈ balForms bar X, (∀ y, 0 ≤ β y y) → lab.evalZ β ≤ N

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.CarrierFunctionTwistedGalois

theorem **twEval_of_isTwG** {O : Type} [CommRing O] {w0 : O} {ψ : InvFun O} (htw : IsTwG w0 ψ) {σ : O ≃+* O} (hσ : σ w0 = w0) : Tw.TwEval σ ψ

theorem **isTwG_carFun** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O) (w0 : O), CMOrd O bar → ∀ (ψ : InvFun O), IsTwG w0 ψ → IsTwG w0 (carFun bar ψ)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.CarrierFunctionTwistedGaloisStatement

theorem **_root_.GlobalTheory.T.carrier_function_twisted_galois** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O) (w0 : O), CMOrd O bar → ∀ (ψ : InvFun O), IsTwG w0 ψ → IsTwG w0 (carFun bar ψ)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.CarrierLabelMarkedFaceBound

theorem **genRank_le_of_label_le** {O : Type} [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O) (q : ℕ) (ψ lab : InvFun O) (i : ℕ) (hlab : IsCarrierDim bar q ψ lab) (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X] (hX : Module.finrank O X ≤ q) {m : ℕ} (v : Fin m → LinearMap.BilinForm ℤ X) (hv : IsCell bar ψ v) (hb : ∀ β ∈ balForms bar X, (∀ y, 0 ≤ β y y) → InRCone v β → lab.evalZ β ≤ ((i + 1 : ℕ) : ℝ)) : genRank v ≤ i + 1

theorem **inFace_iff_of_isMarked** {O : Type} [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O) (hO : CMOrd O bar) (q : ℕ) (ψ lab : InvFun O) (i : ℕ) (hψ : IsPolyFan bar q ψ) (hlab : IsCarrierDim bar q ψ lab) (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X] (hX : Module.finrank O X ≤ q) (hDom : Dominates bar X) {m : ℕ} (v : Fin m → LinearMap.BilinForm ℤ X) (hv : IsCell bar ψ v) (hb : ∀ β ∈ balForms bar X, (∀ y, 0 ≤ β y y) → InRCone v β → lab.evalZ β ≤ ((i + 1 : ℕ) : ℝ)) {β : LinearMap.BilinForm ℤ X} (hβ : IsMarked bar ψ lab i X β) (hβv : InRCone v β) (x : LinearMap.BilinForm ℤ X) : InFace bar ψ lab β x ↔ x ∈ balForms bar X ∧ (∀ y, 0 ≤ x y y) ∧ InRCone v x

theorem **eq_of_isMarked_of_inRCone** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O), CMOrd O bar → ∀ (q : ℕ) (ψ lab : InvFun O) (i : ℕ), (∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Dominates bar X) → IsPolyFan bar q ψ → IsCarrierDim bar q ψ lab → ∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Module.finrank O X ≤ q → ∀ (m : ℕ) (v : Fin m → LinearMap.BilinForm ℤ X), IsCell bar ψ v → (∀ β ∈ balForms bar X, (∀ y, 0 ≤ β y y) → InRCone v β → lab.evalZ β ≤ ((i + 1 : ℕ) : ℝ)) → ∀ β β' : LinearMap.BilinForm ℤ X, IsMarked bar ψ lab i X β → IsMarked bar ψ lab i X β' → InRCone v β → InRCone v β' → β = β'

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.CarrierLabelMarkedFaceBoundStatement

theorem **_root_.GlobalTheory.T.isMarked_unique_of_inRCone** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O), CMOrd O bar → ∀ (q : ℕ) (ψ lab : InvFun O) (i : ℕ), (∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Dominates bar X) → IsPolyFan bar q ψ → IsCarrierDim bar q ψ lab → ∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Module.finrank O X ≤ q → ∀ (m : ℕ) (v : Fin m → LinearMap.BilinForm ℤ X), IsCell bar ψ v → (∀ β ∈ balForms bar X, (∀ y, 0 ≤ β y y) → InRCone v β → lab.evalZ β ≤ ((i + 1 : ℕ) : ℝ)) → ∀ β β' : LinearMap.BilinForm ℤ X, IsMarked bar ψ lab i X β → IsMarked bar ψ lab i X β' → InRCone v β → InRCone v β' → β = β'

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.CellFaceOfFaceCut

theorem **apply_eq_sum_of_toReal** {m : ℕ} {β : LinearMap.BilinForm ℤ X} {u : Fin m → LinearMap.BilinForm ℤ X} {c : Fin m → ℝ} (hβ : toReal β = ∑ i, c i • toReal (u i)) (F : LinearMap.BilinForm ℤ X →ₗ[ℤ] ℤ) : (F β : ℝ) = ∑ i, c i ∗ (F (u i) : ℝ)

theorem **est_at** {m : ℕ} {γ : Fin m → LinearMap.BilinForm ℤ X} {N : ℕ} {ℓ h : LinearMap.BilinForm ℤ X →ₗ[ℤ] ℤ} (hstr : ∀ β ∈ balForms bar X, (∀ y, 0 ≤ β y y) → N ∗ ψ.evalZ β ≤ ℓ β → ∃ (k : ℕ) (c : Fin m → ℕ), 0 < k ∧ k • β = ∑ i, c i • γ i) (hle : ∀ β ∈ balForms bar X, (∀ y, 0 ≤ β y y) → N ∗ ψ.evalZ β ≤ ℓ β) (hγ : ∀ j, 0 ≤ h (γ j)) {C : ℕ} (hC : ∀ β ∈ balForms bar X, (∀ y, 0 ≤ β y y) → ¬(h β : ℝ) ≤ C ∗ ℓ β) {u : LinearMap.BilinForm ℤ X} (hu : u ∈ balForms bar X) (hp : ∀ y, 0 ≤ u y y) (n : ℤ) (hn : ψ.evalZ u = n) {B0 L : ℝ} (hb : N ∗ ψ.evalZ u ≤ B0) (hB0 : 0 ≤ B0) (hL : C ∗ (B0 + 1) + C ≤ L) : L ∗ (N ∗ ψ.evalZ u) ≤ L ∗ ℓ u + h u

theorem **isCell_comp** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O), CMOrd O bar → ∀ (ψ : InvFun O) (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X], IsPolyFanOn bar ψ X → Dominates bar X → ∀ (m m' : ℕ) (γ : Fin m → LinearMap.BilinForm ℤ X) (s : Fin m' ↪ Fin m) (h : LinearMap.BilinForm ℤ X →ₗ[ℤ] ℤ), IsCell bar ψ γ → IsFaceCut γ s h → IsCell bar ψ (fun i => γ (s i))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.CellFaceOfFaceCutStatement

theorem **_root_.GlobalTheory.T.cell_face_of_face_cut** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O), CMOrd O bar → ∀ (ψ : InvFun O) (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X], IsPolyFanOn bar ψ X → Dominates bar X → ∀ (m m' : ℕ) (γ : Fin m → LinearMap.BilinForm ℤ X) (s : Fin m' ↪ Fin m) (h : LinearMap.BilinForm ℤ X →ₗ[ℤ] ℤ), IsCell bar ψ γ → IsFaceCut γ s h → IsCell bar ψ (fun i => γ (s i))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.CellFaceRestriction

theorem **isCell_face_of_isFaceCut** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O), CMOrd O bar → ∀ (q : ℕ) (ψ : InvFun O), IsPolyFan bar q ψ → ∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Module.finrank O X ≤ q → Dominates bar X → ∀ (m : ℕ) (v : Fin m → LinearMap.BilinForm ℤ X), IsCell bar ψ v → ∀ (m' : ℕ) (s : Fin m' ↪ Fin m) (h : LinearMap.BilinForm ℤ X →ₗ[ℤ] ℤ), IsFaceCut v s h → IsCell bar ψ (fun j => v (s j))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.CellFaceRestrictionStatement

theorem **_root_.GlobalTheory.T.cell_face_restriction** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O), CMOrd O bar → ∀ (q : ℕ) (ψ : InvFun O), IsPolyFan bar q ψ → ∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Module.finrank O X ≤ q → Dominates bar X → ∀ (m : ℕ) (v : Fin m → LinearMap.BilinForm ℤ X), IsCell bar ψ v → ∀ (m' : ℕ) (s : Fin m' ↪ Fin m) (h : LinearMap.BilinForm ℤ X →ₗ[ℤ] ℤ), IsFaceCut v s h → IsCell bar ψ (fun j => v (s j))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.CellOfQuotientCell

theorem **projective_quot_orad** (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] (β : LinearMap.BilinForm ℤ X) : Module.Projective O (X ⧸ (orad β : Submodule O X))

theorem **cell_of_quot_cell** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O), CMOrd O bar → ∀ (ψ : InvFun O) (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], IsPolyFanOn bar ψ X → (∀ (m m' : ℕ) (γ : Fin m → LinearMap.BilinForm ℤ X) (s : Fin m' ↪ Fin m) (h : LinearMap.BilinForm ℤ X →ₗ[ℤ] ℤ), IsCell bar ψ γ → IsFaceCut γ s h → IsCell bar ψ (fun i => γ (s i))) → ∀ β ∈ balForms bar X, (∀ y, 0 ≤ β y y) → ∀ (m' : ℕ) (δ : Fin m' → LinearMap.BilinForm ℤ (X ⧸ (orad β : Submodule O X))), IsCell bar ψ δ → InRCone (fun i => pull (orad β : Submodule O X).mkQ (δ i)) β → ∃ (m : ℕ) (γ : Fin m → LinearMap.BilinForm ℤ X), IsCell bar ψ γ ∧ InRCone γ β ∧ genRank γ ≤ genRank δ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.CellOfQuotientCellStatement

theorem **_root_.GlobalTheory.T.cell_of_quotient_cell** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O), CMOrd O bar → ∀ (ψ : InvFun O) (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], IsPolyFanOn bar ψ X → (∀ (m m' : ℕ) (γ : Fin m → LinearMap.BilinForm ℤ X) (s : Fin m' ↪ Fin m) (h : LinearMap.BilinForm ℤ X →ₗ[ℤ] ℤ), IsCell bar ψ γ → IsFaceCut γ s h → IsCell bar ψ (fun i => γ (s i))) → ∀ β ∈ balForms bar X, (∀ y, 0 ≤ β y y) → ∀ (m' : ℕ) (δ : Fin m' → LinearMap.BilinForm ℤ (X ⧸ (orad β : Submodule O X))), IsCell bar ψ δ → InRCone (fun i => pull (orad β : Submodule O X).mkQ (δ i)) β → ∃ (m : ℕ) (γ : Fin m → LinearMap.BilinForm ℤ X), IsCell bar ψ γ ∧ InRCone γ β ∧ genRank γ ≤ genRank δ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.CellOfQuotientRadicalCone

theorem **exists_cell_quot_orad_of_inRCone** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O), CMOrd O bar → ∀ (ψ : InvFun O) (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X], (∀ (m m' : ℕ) (γ : Fin m → LinearMap.BilinForm ℤ X) (s : Fin m' ↪ Fin m) (h : LinearMap.BilinForm ℤ X →ₗ[ℤ] ℤ), IsCell bar ψ γ → IsFaceCut γ s h → IsCell bar ψ (fun i => γ (s i))) → ∀ β ∈ balForms bar X, (∀ y, 0 ≤ β y y) → ∀ (m : ℕ) (γ : Fin m → LinearMap.BilinForm ℤ X), IsCell bar ψ γ → InRCone γ β → ∃ (m' : ℕ) (δ : Fin m' → LinearMap.BilinForm ℤ (X ⧸ (orad β : Submodule O X))), IsCell bar ψ δ ∧ InRCone (fun i => pull (orad β : Submodule O X).mkQ (δ i)) β ∧ genRank δ ≤ genRank γ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.CellOfQuotientRadicalConeStatement

theorem **_root_.GlobalTheory.T.cell_of_quotient_radical_cone** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O), CMOrd O bar → ∀ (ψ : InvFun O) (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X], (∀ (m m' : ℕ) (γ : Fin m → LinearMap.BilinForm ℤ X) (s : Fin m' ↪ Fin m) (h : LinearMap.BilinForm ℤ X →ₗ[ℤ] ℤ), IsCell bar ψ γ → IsFaceCut γ s h → IsCell bar ψ (fun i => γ (s i))) → ∀ β ∈ balForms bar X, (∀ y, 0 ≤ β y y) → ∀ (m : ℕ) (γ : Fin m → LinearMap.BilinForm ℤ X), IsCell bar ψ γ → InRCone γ β → ∃ (m' : ℕ) (δ : Fin m' → LinearMap.BilinForm ℤ (X ⧸ (orad β : Submodule O X))), IsCell bar ψ δ ∧ InRCone (fun i => pull (orad β : Submodule O X).mkQ (δ i)) β ∧ genRank δ ≤ genRank γ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.CentreFunctionOneOnCell

theorem **starDepth_on_cell** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O), CMOrd O bar → ∀ (q : ℕ) (ψ w : InvFun O), IsPolyFan bar q ψ → IsCentreSet bar q ψ w → ∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X], Module.finrank O X ≤ q → ∀ (m : ℕ) (γ : Fin m → LinearMap.BilinForm ℤ X), IsCell bar ψ γ → ∀ β₀ ∈ balForms bar X, (∀ y, 0 ≤ β₀ y y) → w.evalZ β₀ = 1 → InRCone γ β₀ → ∀ (m' : ℕ) (s : Fin m' ↪ Fin m) (h : LinearMap.BilinForm ℤ X →ₗ[ℤ] ℤ), IsFaceCut γ s h → 0 < h β₀ → (∀ β ∈ balForms bar X, (∀ y, 0 ≤ β y y) → InRCone γ β → (h β₀ : ℝ) * (starDepth bar ψ w).evalZ β ≤ h β) ∧ (starDepth bar ψ w).evalZ β₀ = 1 ∧ (∀ j, (starDepth bar ψ w).evalZ (γ (s j)) = 0) ∧ ∀ β ∈ balForms bar X, (∀ y, 0 ≤ β y y) → InRCone γ β → (h β₀ : ℝ) * (starDepth bar ψ w).evalZ β = h β → ∃ (k : ℕ) (c : Fin (m' + 1) → ℕ), 0 < k ∧ k

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.CentreFunctionOneOnCellStatement

theorem **_root_.GlobalTheory.T.mul_starDepth_le_faceCut_on_cell** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O), CMOrd O bar → ∀ (q : ℕ) (ψ w : InvFun O), IsPolyFan bar q ψ → IsCentreSet bar q ψ w → ∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Module.finrank O X ≤ q → ∀ (m : ℕ) (γ : Fin m → LinearMap.BilinForm ℤ X), IsCell bar ψ γ → ∀ β₀ ∈ balForms bar X, (∀ y, 0 ≤ β₀ y y) → w.evalZ β₀ = 1 → InRCone γ β₀ → ∀ (m' : ℕ) (s : Fin m' ↪ Fin m) (h : LinearMap.BilinForm ℤ X →ₗ[ℤ] ℤ), IsFaceCut γ s h → 0 < h β₀ → (∀ β ∈ balForms bar X, (∀ y, 0 ≤ β y y) → InRCone γ β → (h β₀ : ℝ) * (starDepth bar ψ w).evalZ β ≤ h β) ∧ (starDepth bar ψ w).evalZ β₀ = 1 ∧ (∀ j, (starDepth bar ψ w).evalZ (γ (s j)) = 0) ∧ ∀ β ∈ balForms bar X, (∀ y, 0 ≤ β y y) → InRCone γ β → (h β₀ : ℝ) * (starDepth bar ψ w).evalZ β = h β → ∃ (k :

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.CmOrdIntId

theorem **cmOrd_int_id** : CMOrd ℤ id

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.ContactSetSpannedOfCover

theorem **isMin2_pullR_iff** {r : X →ₗ[O] X₁} (hr : Function.Surjective r) (B : X₁ →ₗ[ℤ] X₁ →ₗ[ℤ] ℝ) (y : X) : IsMin2 (pullR r B) y ↔ IsMin2 B (r y)

theorem **pullR_mem_contact** {r : X →ₗ[O] X₁} (hr : Function.Surjective r) {B : X₁ →ₗ[ℤ] X₁ →ₗ[ℤ] ℝ} {ι : Type} {a : ι → X} (h : B ∈ contact bar fun i => r (a i)) : pullR r B ∈ contact bar a

theorem **mem_contact_of_pullR** {r : X →ₗ[O] X₁} (hr : Function.Surjective r) {B : X₁ →ₗ[ℤ] X₁ →ₗ[ℤ] ℝ} {ι : Type} {a : ι → X} (hB : B ∈ psdCone bar X₁) (h : pullR r B ∈ contact bar a) : B ∈ contact bar fun i => r (a i)

theorem **isSpanned_contact** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O), (∀ (Y : Type) [AddCommGroup Y] [Module O Y] [Module.Finite O Y] [Module.Projective O Y], HasRedCover bar Y) → ∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X] (ι : Type) [Finite ι] (a : ι → X), Covers2 a → IsSpanned bar (contact bar a)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.ContactSetSpannedOfCoverStatement

theorem **_root_.GlobalTheory.T.contact_set_spanned_of_cover** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O), (∀ (Y : Type) [AddCommGroup Y] [Module O Y] [Module.Finite O Y] [Module.Projective O Y], HasRedCover bar Y) → ∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X] (ι : Type) [Finite ι] (a : ι → X), Covers2 a → IsSpanned bar (contact bar a)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.ContactSetSpansOfPosDef

theorem **IsMin2.add** {B B' : X →ₗ[ℤ] X →ₗ[ℤ] ℝ} {y : X} (h : IsMin2 B y) (h' : IsMin2 B' y) : IsMin2 (B + B') y

theorem **IsMin2.smul** {B : X →ₗ[ℤ] X →ₗ[ℤ] ℝ} {y : X} (h : IsMin2 B y) {t : ℝ} (ht : 0 ≤ t) : IsMin2 (t • B) y

theorem **isPosDef_smul_of_pos** {B : X →ₗ[ℤ] X →ₗ[ℤ] ℝ} (h : IsPosDef B) {t : ℝ} (ht : 0 < t) : IsPosDef (t • B)

theorem **rcone_subset_sigma** {k : ℕ} {s : Fin k → ℕ} (γ : ∀ j, Fin (s j) → LinearMap.BilinForm ℤ X) {N : ℕ} (e : (Σ j, Fin (s j)) ≃ Fin N) (j : Fin k) : rcone (γ j) ⊆ rcone (fun i => γ (e.symm i).1 (e.symm i).2)

theorem **isSpanned_contact_of_posDef** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O) (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X] (ι : Type) [Finite ι] (a : ι → X), HasRedCover bar X → Covers2 a → (∃ B₀ ∈ contact bar a, IsPosDef B₀) → IsSpanned bar (contact bar a)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.ContactSetSpansOfPosDefStatement

theorem **_root_.GlobalTheory.T.contact_set_spans_of_pos_def** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O) (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X] (ι : Type) [Finite ι] (a : ι → X), HasRedCover bar X → Covers2 a → (∃ B₀ ∈ contact bar a, IsPosDef B₀) → IsSpanned bar (contact bar a)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.EqOfOrbitMarkInCellCone

theorem **eq_of_orbMark_of_inRCone_cell** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O) (w0 : O), CMOrd O bar → ∀ (q : ℕ) (φ lab : InvFun O), IsTwG w0 φ → IsTwG w0 lab → SimpLab bar q φ lab → ∀ {X₀ : Type} [AddCommGroup X₀] [Module O X₀] [Module.Finite O X₀] {m₀ : ℕ} (δ₀ : Fin m₀ → LinearMap.BilinForm ℤ X₀) (h₀ : IsCell bar φ δ₀), LinearIndependent ℤ δ₀ → (∀ i, IsPrim bar ((δ₀ i, h₀.mem i) : balForms bar X₀)) → ∀ (v : LinearMap.BilinForm ℤ X₀) (t : Fin m₀ → ℝ), v ≠ 0 → (∀ i, 0 ≤ t i) → toReal v = ∑ i, t i • toReal (δ₀ i) → ∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Module.finrank O X ≤ q → ∀ (m : ℕ) (γ : Fin m → LinearMap.BilinForm ℤ X), IsCell bar φ γ → ∀ β ∈ balForms bar X, (∀ y, 0 ≤ β y y) → ∀ β' ∈ balForms bar X, (∀ y, 0 ≤ β' y y) → (OrbMark.orbFun O w0 v).evalZ β = 1 → (OrbMark.orbFun O w0 v).

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.EqOfOrbitMarkInCellConeStatement

theorem **_root_.GlobalTheory.T.eq_of_orbit_mark_in_cell_cone** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O) (w0 : O), CMOrd O bar → ∀ (q : ℕ) (φ lab : InvFun O), IsTwG w0 φ → IsTwG w0 lab → SimpLab bar q φ lab → ∀ {X₀ : Type} [AddCommGroup X₀] [Module O X₀] [Module.Finite O X₀] {m₀ : ℕ} (δ₀ : Fin m₀ → LinearMap.BilinForm ℤ X₀) (h₀ : IsCell bar φ δ₀), LinearIndependent ℤ δ₀ → (∀ i, IsPrim bar ((δ₀ i, h₀.mem i) : balForms bar X₀)) → ∀ (v : LinearMap.BilinForm ℤ X₀) (t : Fin m₀ → ℝ), v ≠ 0 → (∀ i, 0 ≤ t i) → toReal v = ∑ i, t i • toReal (δ₀ i) → ∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Module.finrank O X ≤ q → ∀ (m : ℕ) (γ : Fin m → LinearMap.BilinForm ℤ X), IsCell bar φ γ → ∀ β ∈ balForms bar X, (∀ y, 0 ≤ β y y) → ∀ β' ∈ balForms bar X, (∀ y, 0 ≤ β' y y) → (OrbMark.orbFun O w0 v).evalZ β = 1 → (O

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.EvalPullbackSemilinearSurjective

theorem **eval_pull_semilinear_of_isTwG** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O) (w0 : O), CMOrd O bar → ∀ (φ : InvFun O), IsTwG w0 φ → ∀ {X Y : Type} [AddCommGroup X] [Module O X] [Module.Finite O X] [AddCommGroup Y] [Module O Y] [Module.Finite O Y] (τ : O ≃+* O), τ w0 = w0 → ∀ (f : X →+ Y), Function.Surjective f → (∀ (a : O) (x : X), f (a • x) = τ a • f x) → ∀ B : Y → Y → ℝ, φ.eval X (fun x y => B (f x) (f y)) = φ.eval Y B

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.EvalPullbackSemilinearSurjectiveStatement

theorem **_root_.GlobalTheory.T.eval_pullback_semilinear_surjective** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O) (w0 : O), CMOrd O bar → ∀ (φ : InvFun O), IsTwG w0 φ → ∀ {X Y : Type} [AddCommGroup X] [Module O X] [Module.Finite O X] [AddCommGroup Y] [Module O Y] [Module.Finite O Y] (τ : O ≃+* O), τ w0 = w0 → ∀ (f : X →+ Y), Function.Surjective f → (∀ (a : O) (x : X), f (a • x) = τ a • f x) → ∀ B : Y → Y → ℝ, φ.eval X (fun x y => B (f x) (f y)) = φ.eval Y B

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.ExistsBalancedFormsDominating

theorem **contact_inter_rcone_spanned** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O) (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X] (ι : Type) [Finite ι] (a : ι → X) (m : ℕ) (u : Fin m → LinearMap.BilinForm ℤ X), (∀ i y z, u i y z = u i z y) → (∀ i y, 0 ≤ u i y y) → ∃ (s : ℕ) (γ : Fin s → LinearMap.BilinForm ℤ X), (∀ i, γ i ∈ balForms bar X) ∧ (∀ i y, 0 ≤ γ i y y) ∧ (∀ i, toReal (γ i) ∈ contact bar a) ∧ ∀ B ∈ contact bar a, B ∈ rcone u → B ∈ rcone γ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.ExistsBalancedFormsDominatingStatement

theorem **_root_.GlobalTheory.T.exists_balanced_forms_dominating** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O) (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X] (ι : Type) [Finite ι] (a : ι → X) (m : ℕ) (u : Fin m → LinearMap.BilinForm ℤ X), (∀ i y z, u i y z = u i z y) → (∀ i y, 0 ≤ u i y y) → ∃ (s : ℕ) (γ : Fin s → LinearMap.BilinForm ℤ X), (∀ i, γ i ∈ balForms bar X) ∧ (∀ i y, 0 ≤ γ i y y) ∧ (∀ i, toReal (γ i) ∈ contact bar a) ∧ ∀ B ∈ contact bar a, B ∈ rcone u → B ∈ rcone γ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.ExistsCarrierDimensionLabel

theorem **exists_cell** {O : Type} [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] {bar : O → O} {ψ : InvFun O} {X : Type} [AddCommGroup X] [Module O X] [Module.Finite O X] (hfan : IsPolyFanOn bar ψ X) {β : LinearMap.BilinForm ℤ X} (hβ : β ∈ balForms bar X) (hp : ∀ y, 0 ≤ β y y) : ∃ (m : ℕ) (γ : Fin m → LinearMap.BilinForm ℤ X), IsCell bar ψ γ ∧ InRCone γ β

theorem **exists_isCarrierDim** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O) (w0 : O), CMOrd O bar → ∀ (q : ℕ) (ψ : InvFun O), (∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Dominates bar X) → IsPolyFan bar q ψ → IsTwG w0 ψ → ∃ lab : InvFun O, IsTwG w0 lab ∧ IsCarrierDim bar q ψ lab

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.ExistsCarrierDimensionLabelStatement

theorem **_root_.GlobalTheory.T.exists_carrier_dimension_label** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O) (w0 : O), CMOrd O bar → ∀ (q : ℕ) (ψ : InvFun O), (∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Dominates bar X) → IsPolyFan bar q ψ → IsTwG w0 ψ → ∃ lab : InvFun O, IsTwG w0 lab ∧ IsCarrierDim bar q ψ lab

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.ExistsDescendingRaysPullback

theorem **exists_desc_rays_pullAdd** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O) (w0 : O), CMOrd O bar → ∀ {X₀ : Type} [AddCommGroup X₀] [Module O X₀] [Module.Finite O X₀] {m₀ : ℕ} (δ₀ : Fin m₀ → LinearMap.BilinForm ℤ X₀) (hm : ∀ i, δ₀ i ∈ balForms bar X₀), (∀ i y, 0 ≤ δ₀ i y y) → LinearIndependent ℤ δ₀ → (∀ i, IsPrim bar ((δ₀ i, hm i) : balForms bar X₀)) → ∀ (v : LinearMap.BilinForm ℤ X₀) (t : Fin m₀ → ℝ), (∀ i, 0 ≤ t i) → toReal v = ∑ i, t i • toReal (δ₀ i) → ∀ {X : Type} [AddCommGroup X] [Module O X] [Module.Finite O X] (β : LinearMap.BilinForm ℤ X) (τ : O ≃+* O) (f : X →+ X₀ ⧸ (CarDim.orad v : Submodule O X₀)), τ w0 = w0 → Function.Surjective f → (∀ (a : O) (x : X), f (a • x) = τ a • f x) → (∀ (x y : X) (x₀ y₀ : X₀), Submodule.Quotient.mk x₀ = f x → Submodule.Quotient.mk y₀ = f y → β x y = v x₀ y₀) → ∃ u : Fin m₀ → LinearMap.BilinForm ℤ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.ExistsDescendingRaysPullbackStatement

theorem **_root_.GlobalTheory.T.exists_descending_rays_pullback** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O) (w0 : O), CMOrd O bar → ∀ {X₀ : Type} [AddCommGroup X₀] [Module O X₀] [Module.Finite O X₀] {m₀ : ℕ} (δ₀ : Fin m₀ → LinearMap.BilinForm ℤ X₀) (hm : ∀ i, δ₀ i ∈ balForms bar X₀), (∀ i y, 0 ≤ δ₀ i y y) → LinearIndependent ℤ δ₀ → (∀ i, IsPrim bar ((δ₀ i, hm i) : balForms bar X₀)) → ∀ (v : LinearMap.BilinForm ℤ X₀) (t : Fin m₀ → ℝ), (∀ i, 0 ≤ t i) → toReal v = ∑ i, t i • toReal (δ₀ i) → ∀ {X : Type} [AddCommGroup X] [Module O X] [Module.Finite O X] (β : LinearMap.BilinForm ℤ X) (τ : O ≃+* O) (f : X →+ X₀ ⧸ (CarDim.orad v : Submodule O X₀)), τ w0 = w0 → Function.Surjective f → (∀ (a : O) (x : X), f (a • x) = τ a • f x) → (∀ (x y : X) (x₀ y₀ : X₀), Submodule.Quotient.mk x₀ = f x → Submodule.Quotient.mk y₀ = f y → β x y = v x₀ y₀) → ∃ u : F

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.ExistsFan

theorem **exists_fan** (E : Type) [Field E] [NumberField E] [IsCMField E] (H : Host E) (q : ℕ) : ∃ φ : InvFun (𝒪 E), IsPolFan H.bar q φ ∧ NoSelfInt H.bar q φ ∧ GlobalTheory.TorRef.IsPosFn H φ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.ExistsFanLowerMaxMultiplicity

theorem **exists_round_lt_maxMult** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O) (w0 : O), CMOrd O bar → ∀ (q : ℕ) (φ lab : InvFun O), (∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Dominates bar X) → IsPolyFan bar q φ → IsTwG w0 φ → IsTwG w0 lab → SimpLab bar q φ lab → maxMult bar q φ < ⊤ → 2 ≤ maxMult bar q φ → ∃ φ' lab' : InvFun O, IsPolyFan bar q φ' ∧ IsTwG w0 φ' ∧ IsTwG w0 lab' ∧ SimpLab bar q φ' lab' ∧ Subordinate bar q φ' φ ∧ maxMult bar q φ' < maxMult bar q φ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.ExistsFanLowerMaxMultiplicityStatement

theorem **_root_.GlobalTheory.T.exists_fan_lower_max_multiplicity** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O) (w0 : O), CMOrd O bar → ∀ (q : ℕ) (φ lab : InvFun O), (∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Dominates bar X) → IsPolyFan bar q φ → IsTwG w0 φ → IsTwG w0 lab → SimpLab bar q φ lab → maxMult bar q φ < ⊤ → 2 ≤ maxMult bar q φ → ∃ φ' lab' : InvFun O, IsPolyFan bar q φ' ∧ IsTwG w0 φ' ∧ IsTwG w0 lab' ∧ SimpLab bar q φ' lab' ∧ Subordinate bar q φ' φ ∧ maxMult bar q φ' < maxMult bar q φ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.ExistsFiniteBelowModTwo

theorem **exists_finite_below_mod_two** {X : Type} [AddCommGroup X] [Module.Finite ℤ X] [Module.Free ℤ X] {m : ℕ} (f : X → Fin m → ℕ) : ∃ G : Set X, G.Finite ∧ ∀ y, ∃ y' ∈ G, (∃ x : X, y' = y + (x + x)) ∧ f y' ≤ f y

theorem **finite_isMin2** : ∀ (X : Type) [AddCommGroup X] [Module.Finite ℤ X] [Module.Free ℤ X] (m : ℕ) (P : Fin m → LinearMap.BilinForm ℤ X), (∀ i y z, P i y z = P i z y) → (∀ i y, 0 ≤ P i y y) → ∃ F : Set X, F.Finite ∧ ∀ B ∈ rcone P, IsPosDef B → ∀ y, IsMin2 B y → y ∈ F

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.ExistsIsPosDefOfBasis

theorem **exists_isPosDef_of_basis** {X : Type} [AddCommGroup X] {n : ℕ} (b : Module.Basis (Fin n) ℤ X) : ∃ B₀ : X →ₗ[ℤ] X →ₗ[ℤ] ℝ, (∀ y z, B₀ y z = B₀ z y) ∧ IsPosDef B₀

theorem **hasRedCover_int** : ∀ (X : Type) [AddCommGroup X] [Module.Finite ℤ X] [Module.Free ℤ X], HasRedCover (id : ℤ → ℤ) X

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.ExistsMarkingCentreFunction

theorem **exists_markFun_props_gen** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O) (w0 : O), CMOrd O bar → ∀ (q : ℕ) (ψ lab : InvFun O) (i : ℕ), (∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Dominates bar X) → IsPolyFan bar q ψ → IsTwG w0 ψ → IsTwG w0 lab → IsCarrierDim bar q ψ lab → ∃ w : InvFun O, IsTwG w0 w ∧ ∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Module.finrank O X ≤ q → (∀ β ∈ balForms bar X, (∀ y, 0 ≤ β y y) → w.evalZ β = 0 ∨ w.evalZ β = 1) ∧ (∀ β ∈ balForms bar X, (∀ y, 0 ≤ β y y) → w.evalZ β = 1 → ∀ (k : ℕ) (β' : LinearMap.BilinForm ℤ X), (k : ℤ) • β' = β → k = 1) ∧ (∀ β ∈ balForms bar X, (∀ y, 0 ≤ β y y) → w.evalZ β = 1 → lab.evalZ β = ((i + 1 : ℕ) : ℝ)) ∧ (∀ (m : ℕ) (v : Fin m → LinearMap.BilinForm ℤ X), IsCell bar ψ v → (∀ β ∈ balForms bar X

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.ExistsMarkingCentreFunctionStatement

theorem **_root_.GlobalTheory.T.exists_marking_centre_function** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O) (w0 : O), CMOrd O bar → ∀ (q : ℕ) (ψ lab : InvFun O) (i : ℕ), (∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Dominates bar X) → IsPolyFan bar q ψ → IsTwG w0 ψ → IsTwG w0 lab → IsCarrierDim bar q ψ lab → ∃ w : InvFun O, IsTwG w0 w ∧ ∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Module.finrank O X ≤ q → (∀ β ∈ balForms bar X, (∀ y, 0 ≤ β y y) → w.evalZ β = 0 ∨ w.evalZ β = 1) ∧ (∀ β ∈ balForms bar X, (∀ y, 0 ≤ β y y) → w.evalZ β = 1 → ∀ (k : ℕ) (β' : LinearMap.BilinForm ℤ X), (k : ℤ) • β' = β → k = 1) ∧ (∀ β ∈ balForms bar X, (∀ y, 0 ≤ β y y) → w.evalZ β = 1 → lab.evalZ β = ((i + 1 : ℕ) : ℝ)) ∧ (∀ (m : ℕ) (v : Fin m → LinearMap.BilinForm ℤ X), IsCell bar

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.ExistsMatchingPiecesOfCell

theorem **exists_pieces_match** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O) (w0 : O), CMOrd O bar → ∀ (φ : InvFun O), IsTwG w0 φ → ∀ {X₀ : Type} [AddCommGroup X₀] [Module O X₀] [Module.Finite O X₀] {m₀ : ℕ} (δ₀ : Fin m₀ → LinearMap.BilinForm ℤ X₀) (h₀ : IsCell bar φ δ₀), LinearIndependent ℤ δ₀ → (∀ i, IsPrim bar ((δ₀ i, h₀.mem i) : balForms bar X₀)) → ∀ (v : LinearMap.BilinForm ℤ X₀) (t : Fin m₀ → ℝ), v ≠ 0 → (∀ i, 0 ≤ t i) → toReal v = ∑ i, t i • toReal (δ₀ i) → ∀ {X : Type} [AddCommGroup X] [Module O X] [Module.Finite O X] (β : LinearMap.BilinForm ℤ X), (OrbMark.orbFun O w0 v).evalZ β = 1 → ∃ p : Fin m₀ → LinearMap.BilinForm ℤ X, toReal β = ∑ i, t i • toReal (p i) ∧ (∀ i, 0 < t i → p i ∈ balForms bar X ∧ (∀ y, 0 ≤ p i y y) ∧ (∀ (k : ℕ) (β' : LinearMap.BilinForm ℤ X), (k : ℤ) • β' = p i → k = 1) ∧ ∀ lab : InvFun O, IsTwG w0 lab → lab.evalZ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.ExistsMatchingPiecesOfCellStatement

theorem **_root_.GlobalTheory.T.exists_matching_pieces_of_cell** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O) (w0 : O), CMOrd O bar → ∀ (φ : InvFun O), IsTwG w0 φ → ∀ {X₀ : Type} [AddCommGroup X₀] [Module O X₀] [Module.Finite O X₀] {m₀ : ℕ} (δ₀ : Fin m₀ → LinearMap.BilinForm ℤ X₀) (h₀ : IsCell bar φ δ₀), LinearIndependent ℤ δ₀ → (∀ i, IsPrim bar ((δ₀ i, h₀.mem i) : balForms bar X₀)) → ∀ (v : LinearMap.BilinForm ℤ X₀) (t : Fin m₀ → ℝ), v ≠ 0 → (∀ i, 0 ≤ t i) → toReal v = ∑ i, t i • toReal (δ₀ i) → ∀ {X : Type} [AddCommGroup X] [Module O X] [Module.Finite O X] (β : LinearMap.BilinForm ℤ X), (OrbMark.orbFun O w0 v).evalZ β = 1 → ∃ p : Fin m₀ → LinearMap.BilinForm ℤ X, toReal β = ∑ i, t i • toReal (p i) ∧ (∀ i, 0 < t i → p i ∈ balForms bar X ∧ (∀ y, 0 ≤ p i y y) ∧ (∀ (k : ℕ) (β' : LinearMap.BilinForm ℤ X), (k : ℤ) • β' = p i → k = 1) ∧ ∀ lab : I

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.ExistsNatCombOfToRealMemRcone

theorem **exists_nat_comb_of_toReal_mem_rcone** (X : Type) [AddCommGroup X] (m : ℕ) (γ : Fin m → LinearMap.BilinForm ℤ X) (β : LinearMap.BilinForm ℤ X) (hβ : toReal β ∈ rcone γ) : ∃ (k : ℕ) (c : Fin m → ℕ), 0 < k ∧ k • β = ∑ i, c i • γ i

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.ExistsPsdFormsCoverContactSet

theorem **finite_free_int_of_projective** (O : Type) [CommRing O] [IsDomain O] [CharZero O] [Module.Finite ℤ O] (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X] : Module.Finite ℤ X ∧ Module.Free ℤ X

theorem **exists_fin_reindex** {T : Type} [Finite T] {X : Type} [AddCommGroup X] (m' : T → ℕ) (u' : ∀ t, Fin (m' t) → LinearMap.BilinForm ℤ X) (hsym : ∀ t i y z, u' t i y z = u' t i z y) (hnn : ∀ t i y, 0 ≤ u' t i y y) : ∃ (k : ℕ) (m : Fin k → ℕ) (u : ∀ j, Fin (m j) → LinearMap.BilinForm ℤ X), (∀ j i y z, u j i y z = u j i z y) ∧ (∀ j i y, 0 ≤ u j i y y) ∧ ∀ B : X →ₗ[ℤ] X →ₗ[ℤ] ℝ, (∃ t, B ∈ rcone (u' t)) → ∃ j, B ∈ rcone (u j)

theorem **isPosDef_pullR** (e : X ≃ₗ[0] X) {B : X →ₗ[ℤ] X →ₗ[ℤ] ℝ} (hB : IsPosDef B) : IsPosDef (pullR e.toLinearMap B)

theorem **isMin2_pullR** (e : X ≃ₗ[0] X) {B : X →ₗ[ℤ] X →ₗ[ℤ] ℝ} {y : X} (hy : IsMin2 B y) : IsMin2 (pullR e.toLinearMap B) (e.symm y)

theorem **exists_rcone_cover_contact** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O) (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X] (ι : Type) [Finite ι] (a : ι → X), HasRedCover bar X → Covers2 a → ∃ (k : ℕ) (m : Fin k → ℕ) (u : ∀ j, Fin (m j) → LinearMap.BilinForm ℤ X), (∀ j i y z, u j i y z = u j i z y) ∧ (∀ j i y, 0 ≤ u j i y y) ∧ ∀ B ∈ contact bar a, IsPosDef B → ∃ j, B ∈ rcone (u j)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.ExistsPsdFormsCoverContactSetStatement

theorem **_root_.GlobalTheory.T.exists_psd_forms_cover_contact_set** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O) (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X] (ι : Type) [Finite ι] (a : ι → X), HasRedCover bar X → Covers2 a → ∃ (k : ℕ) (m : Fin k → ℕ) (u : ∀ j, Fin (m j) → LinearMap.BilinForm ℤ X), (∀ j i y z, u j i y z = u j i z y) ∧ (∀ j i y, 0 ≤ u j i y y) ∧ ∀ B ∈ contact bar a, IsPosDef B → ∃ j, B ∈ rcone (u j)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.ExistsPullbackCoordinateOfPos

theorem **mem_balForms_of_descend** {O : Type} [CommRing O] {bar : O → O} (hb : CMOrd O bar) {X Y : Type} [AddCommGroup X] [Module O X] [AddCommGroup Y] [Module O Y] (τ : O ≃+* O) {f : X →+ Y} (hf : Function.Surjective f) (hsm : ∀ (a : O) (x : X), f (a • x) = τ a • f x) {δ : LinearMap.BilinForm ℤ X} (hδ : δ ∈ balForms bar X) {ε : LinearMap.BilinForm ℤ Y} (hε : ∀ x y, ε (f x) (f y) = δ x y) : ε ∈ balForms bar Y

theorem **exists_coord_pullAdd_of_pos** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O) (w0 : O), CMOrd O bar → ∀ (φ : InvFun O), IsTwG w0 φ → ∀ {X₀ : Type} [AddCommGroup X₀] [Module O X₀] [Module.Finite O X₀] {m₀ : ℕ} (δ₀ : Fin m₀ → LinearMap.BilinForm ℤ X₀), IsCell bar φ δ₀ → LinearIndependent ℤ δ₀ → ∀ (v : LinearMap.BilinForm ℤ X₀) (t : Fin m₀ → ℝ), (∀ i, 0 ≤ t i) → toReal v = ∑ i, t i • toReal (δ₀ i) → ∀ {X Y : Type} [AddCommGroup X] [Module O X] [Module.Finite O X] [AddCommGroup Y] [Module O Y] [Module.Finite O Y] (π : X₀ →ₗ[O] Y), Function.Surjective π → ∀ (τ : O ≃+* O) (f : X →+ Y), τ w0 = w0 → Function.Surjective f → (∀ (a : O) (x : X), f (a • x) = τ a • f x) → ∀ u : Fin m₀ → LinearMap.BilinForm ℤ Y, (∀ i, 0 < t i → pull π (u i) = δ₀ i) → ∀ β : LinearMap.BilinForm ℤ X, (∀ (x y : X) (x₀ y₀ : X₀), π x₀ = f x → π y₀ = f y → β x y = v x₀ y₀

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.ExistsPullbackCoordinateOfPosStatement

theorem **_root_.GlobalTheory.T.exists_pullback_coordinate_of_pos** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O) (w0 : O), CMOrd O bar → ∀ (φ : InvFun O), IsTwG w0 φ → ∀ {X₀ : Type} [AddCommGroup X₀] [Module O X₀] [Module.Finite O X₀] {m₀ : ℕ} (δ₀ : Fin m₀ → LinearMap.BilinForm ℤ X₀), IsCell bar φ δ₀ → LinearIndependent ℤ δ₀ → ∀ (v : LinearMap.BilinForm ℤ X₀) (t : Fin m₀ → ℝ), (∀ i, 0 ≤ t i) → toReal v = ∑ i, t i • toReal (δ₀ i) → ∀ {X Y : Type} [AddCommGroup X] [Module O X] [Module.Finite O X] [AddCommGroup Y] [Module O Y] [Module.Finite O Y] (π : X₀ →ₗ[O] Y), Function.Surjective π → ∀ (τ : O ≃+* O) (f : X →+ Y), τ w0 = w0 → Function.Surjective f → (∀ (a : O) (x : X), f (a • x) = τ a • f x) → ∀ u : Fin m₀ → LinearMap.BilinForm ℤ Y, (∀ i, 0 < t i → pull π (u i) = δ₀ i) → ∀ β : LinearMap.BilinForm ℤ X, (∀ (x y : X) (x₀ y₀ : X₀), π x₀ = f x →

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.ExistsSimplicialLabelledRefinement

theorem **exists_simpLab_refinement** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O) (w0 : O), CMOrd O bar → ∀ (q : ℕ) (ψ : InvFun O), (∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Dominates bar X) → IsPolyFan bar q ψ → IsTwG w0 ψ → ∃ φ lab : InvFun O, IsPolyFan bar q φ ∧ IsTwG w0 φ ∧ IsTwG w0 lab ∧ SimpLab bar q φ lab ∧ Subordinate bar q φ ψ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.ExistsSimplicialLabelledRefinementStatement

theorem **_root_.GlobalTheory.T.exists_simplicial_labelled_refinement** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O) (w0 : O), CMOrd O bar → ∀ (q : ℕ) (ψ : InvFun O), (∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Dominates bar X) → IsPolyFan bar q ψ → IsTwG w0 ψ → ∃ φ lab : InvFun O, IsPolyFan bar q φ ∧ IsTwG w0 φ ∧ IsTwG w0 lab ∧ SimpLab bar q φ lab ∧ Subordinate bar q φ ψ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.ExistsSmoothFanInt

theorem **hasRedCover_int_proj** (Y : Type) [AddCommGroup Y] [m : Module ℤ Y] [Module.Finite ℤ Y] [Module.Projective ℤ Y] : HasRedCover (id : ℤ → ℤ) Y

theorem **dominates_int_proj** (Y : Type) [AddCommGroup Y] [m : Module ℤ Y] [Module.Finite ℤ Y] [Module.Projective ℤ Y] : Dominates (id : ℤ → ℤ) Y

theorem **exists_smooth_fan_int** : ∀ g : ℕ, ∃ φ : InvFun ℤ, IsPolFan id g φ ∧ NoSelfInt id g φ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.ExistsSmoothPolyhedralFanRefinement

theorem **exists_smooth_refinement** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O) (w0 : O), CMOrd O bar → ∀ (q : ℕ) (ψ : InvFun O), (∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Dominates bar X) → IsPolyFan bar q ψ → IsTwG w0 ψ → ∃ φ' : InvFun O, IsPolFan bar q φ' ∧ NoSelfInt bar q φ' ∧ IsTwG w0 φ' ∧ ∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Module.finrank O X ≤ q → ∀ (m : ℕ) (γ : Fin m → LinearMap.BilinForm ℤ X), IsCone bar φ' γ → ψ.LinearOn γ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.ExistsSmoothPolyhedralFanRefinementStatement

theorem **_root_.GlobalTheory.T.exists_smooth_polyhedral_fan_refinement** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O) (w0 : O), CMOrd O bar → ∀ (q : ℕ) (ψ : InvFun O), (∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Dominates bar X) → IsPolyFan bar q ψ → IsTwG w0 ψ → ∃ φ' : InvFun O, IsPolFan bar q φ' ∧ NoSelfInt bar q φ' ∧ IsTwG w0 φ' ∧ ∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Module.finrank O X ≤ q → ∀ (m : ℕ) (γ : Fin m → LinearMap.BilinForm ℤ X), IsCone bar φ' γ → ψ.LinearOn γ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.ExistsSmoothSubordinateFan

theorem **resolution** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O) (w0 : O), CMOrd O bar → ∀ (q : ℕ) (φ lab : InvFun O), (∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Dominates bar X) → IsPolyFan bar q φ → IsTwG w0 φ → IsTwG w0 lab → SimpLab bar q φ lab → ∃ φ' : InvFun O, IsPolFan bar q φ' ∧ NoSelfInt bar q φ' ∧ IsTwG w0 φ' ∧ Subordinate bar q φ' φ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.ExistsSmoothSubordinateFanStatement

theorem **_root_.GlobalTheory.T.exists_smooth_subordinate_fan** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O) (w0 : O), CMOrd O bar → ∀ (q : ℕ) (φ lab : InvFun O), (∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Dominates bar X) → IsPolyFan bar q φ → IsTwG w0 φ → IsTwG w0 lab → SimpLab bar q φ lab → ∃ φ' : InvFun O, IsPolFan bar q φ' ∧ NoSelfInt bar q φ' ∧ IsTwG w0 φ' ∧ Subordinate bar q φ' φ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.ExistsStarSubdivision

theorem **isTwG_starFun** {O : Type} [CommRing O] [Module.Finite ℤ O] {bar : O → O} {w0 : O} {ψ w : InvFun O} (hψ : IsTwG w0 ψ) (hd : IsTwG w0 (starDepth bar ψ w)) (K D : ℕ) : IsTwG w0 (starFun bar ψ w K D)

theorem **exists_starSubdivision** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O) (w0 : O), CMOrd O bar → ∀ (q : ℕ) (ψ w : InvFun O), (∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Dominates bar X) → IsPolyFan bar q ψ → IsTwG w0 ψ → IsTwG w0 w → IsCentreSet bar q ψ w → ∃ ψ' : InvFun O, IsPolyFan bar q ψ' ∧ IsTwG w0 ψ' ∧ Subordinate bar q ψ' ψ ∧ IsStarOf bar q ψ w ψ'

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.ExistsStarSubdivisionStatement

theorem **_root_.GlobalTheory.T.exists_star_subdivision** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O) (w0 : O), CMOrd O bar → ∀ (q : ℕ) (ψ w : InvFun O), (∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Dominates bar X) → IsPolyFan bar q ψ → IsTwG w0 ψ → IsTwG w0 w → IsCentreSet bar q ψ w → ∃ ψ' : InvFun O, IsPolyFan bar q ψ' ∧ IsTwG w0 ψ' ∧ Subordinate bar q ψ' ψ ∧ IsStarOf bar q ψ w ψ'

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.ExistsTwistedCentreSet

theorem **isTwG_orbFun** {O : Type} [CommRing O] {X₀ : Type} [AddCommGroup X₀] [Module O X₀] (w0 : O) (v : LinearMap.BilinForm ℤ X₀) : IsTwG w0 (OrbMark.orbFun O w0 v)

theorem **centres_exists** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O) (w0 : O), CMOrd O bar → ∀ (q : ℕ) (φ lab : InvFun O), (∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Dominates bar X) → IsPolyFan bar q φ → IsTwG w0 φ → IsTwG w0 lab → SimpLab bar q φ lab → ∀ M : ℕ, 2 ≤ M → maxMult bar q φ ≤ M → ∀ c : FormFamily O, ∃ w : InvFun O, IsTwG w0 w ∧ IsCentreSet bar q φ w ∧ (∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Module.finrank O X ≤ q → ∀ (m : ℕ) (δ : Fin m → LinearMap.BilinForm ℤ X), IsMultCell bar φ M δ → c.Carries δ → ∃ β ∈ balForms bar X, (∀ y, 0 ≤ β y y) ∧ w.evalZ β = 1 ∧ InRCone δ β) ∧ (∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Module.finrank O X ≤ q → ∀ (m : ℕ) (δ : Fin m → LinearMap.BilinForm ℤ X) (h

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.ExistsTwistedCentreSetStatement

theorem **_root_.GlobalTheory.T.exists_twisted_centre_set** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O) (w0 : O), CMOrd O bar → ∀ (q : ℕ) (φ lab : InvFun O), (∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Dominates bar X) → IsPolyFan bar q φ → IsTwG w0 φ → IsTwG w0 lab → SimpLab bar q φ lab → ∀ M : ℕ, 2 ≤ M → maxMult bar q φ ≤ M → ∀ c : FormFamily O, ∃ w : InvFun O, IsTwG w0 w ∧ IsCentreSet bar q φ w ∧ (∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Module.finrank O X ≤ q → ∀ (m : ℕ) (δ : Fin m → LinearMap.BilinForm ℤ X), IsMultCell bar φ M δ → c.Carries δ → ∃ β ∈ balForms bar X, (∀ y, 0 ≤ β y y) ∧ w.evalZ β = 1 ∧ InRCone δ β) ∧ (∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Module.finrank O X ≤ q → ∀ (m : ℕ) (δ : Fin

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.ExistsTwistedSimplicialStarLabel

theorem **isTwG_starLab** {O : Type} [CommRing O] (w0 : O) {lab w : InvFun O} (hl : IsTwG w0 lab) (hw : IsTwG w0 w) : IsTwG w0 (starLab lab w)

theorem **exists_isTwG_simpLab_star** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O) (w0 : O), CMOrd O bar → ∀ (q : ℕ) (φ lab w ψ' : InvFun O), (∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Dominates bar X) → IsPolyFan bar q φ → IsTwG w0 lab → SimpLab bar q φ lab → IsTwG w0 w → IsCentreSet bar q φ w → IsPolyFan bar q ψ' → Subordinate bar q ψ' φ → IsStarOf bar q φ w ψ' → ∃ lab' : InvFun O, IsTwG w0 lab' ∧ SimpLab bar q ψ' lab'

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.ExistsTwistedSimplicialStarLabelStatement

theorem **_root_.GlobalTheory.T.exists_twisted_simplicial_star_label** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O) (w0 : O), CMOrd O bar → ∀ (q : ℕ) (φ lab w ψ' : InvFun O), (∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Dominates bar X) → IsPolyFan bar q φ → IsTwG w0 lab → SimpLab bar q φ lab → IsTwG w0 w → IsCentreSet bar q φ w → IsPolyFan bar q ψ' → Subordinate bar q ψ' φ → IsStarOf bar q φ w ψ' → ∃ lab' : InvFun O, IsTwG w0 lab' ∧ SimpLab bar q ψ' lab'

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.FinitePosDefFormSublevel

theorem **posDef_coercive** {n : ℕ} (A : Matrix (Fin n) (Fin n) ℝ) (hA : A.PosDef) : ∃ ε : ℝ, 0 < ε ∧ ∀ x : Fin n → ℝ, ε * ‖x‖ ^ 2 ≤ x ⬝ᵥ (A *ᵥ x)

theorem **sublevel_finite** : ∀ (X : Type) [AddCommGroup X] [Module.Finite ℤ X] [Module.Free ℤ X] (β : LinearMap.BilinForm ℤ X), (∀ y z, β y z = β z y) → (∀ y, y ≠ 0 → 0 < β y y) → ∀ c : ℤ, {y : X | β y y ≤ c}.Finite

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.FinitePosDefFormSublevelStatement

theorem **_root_.GlobalTheory.T.finite_pos_def_form_sublevel** : ∀ (X : Type) [AddCommGroup X] [Module.Finite ℤ X] [Module.Free ℤ X] (β : LinearMap.BilinForm ℤ X), (∀ y z, β y z = β z y) → (∀ y, y ≠ 0 → 0 < β y y) → ∀ c : ℤ, {y : X | β y y ≤ c}.Finite

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.IntEqZeroOfForallTwoPowDvd

theorem **int_eq_zero_of_forall_two_pow_dvd** (n : ℤ) (h : ∀ k : ℕ, (2 : ℤ) ^ k ∣ n) : n = 0

theorem **exists_two_pow_smul_of_covers2** {X : Type} [AddCommGroup X] {ι : Type} {a : ι → X} (ha : Covers2 a) (d : X →+ X) (hd : ∀ i, d (a i) = 0) (k : ℕ) : ∀ y : X, ∃ z : X, d y = ((2 : ℤ) ^ k) • z

theorem **addHom_eq_zero_of_covers2** {X : Type} [AddCommGroup X] [Module.Finite ℤ X] [Module.Free ℤ X] {ι : Type} {a : ι → X} (ha : Covers2 a) (d : X →+ X) (hd : ∀ i, d (a i) = 0) : d = 0

theorem **addHom_ext_of_covers2** {X : Type} [AddCommGroup X] [Module.Finite ℤ X] [Module.Free ℤ X] {ι : Type} {a : ι → X} (ha : Covers2 a) {f g : X →+ X} (h : ∀ i, f (a i) = g (a i)) : f = g

theorem **finite_addHom_of_covers2** : ∀ (X : Type) [AddCommGroup X] [Module.Finite ℤ X] [Module.Free ℤ X] (ι : Type) [Finite ι] (a : ι → X), Covers2 a → ∀ F : Set X, F.Finite → {f : X →+ X | ∀ i, f (a i) ∈ F}.Finite

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.IntegerLatticeDominates

theorem **quadOf_eq** {n : ℕ} (A : Matrix (Fin n) (Fin n) ℝ) (x : Fin n → ℝ) : quadOf A x = star x ⬝ᵥ (A *ᵥ x)

theorem **continuous_quadOf** {n : ℕ} (A : Matrix (Fin n) (Fin n) ℝ) : Continuous (quadOf A)

theorem **quadOf_smul** {n : ℕ} (A : Matrix (Fin n) (Fin n) ℝ) (t : ℝ) (x : Fin n → ℝ) : quadOf A (t • x) = t ^ 2 * quadOf A x

theorem **exists_nat_quadOf_le** {n : ℕ} {P : Matrix (Fin n) (Fin n) ℝ} (hP : P.PosDef) (R : Matrix (Fin n) (Fin n) ℝ) : ∃ C : ℕ, ∀ x, quadOf R x ≤ C * quadOf P x

theorem **sum_mul_nonneg** {n : ℕ} {A G : Matrix (Fin n) (Fin n) ℝ} (hA : A.PosSemidef) (hG : G.PosSemidef) : 0 ≤ ∑ i, ∑ j, A i j * G i j

theorem **posSemidef_of_int_nonneg** {n : ℕ} {G : Matrix (Fin n) (Fin n) ℝ} (hs : G.IsHermitian) (h : ∀ v : Fin n → ℤ, 0 ≤ quadOf G fun i => (v i : ℝ)) : G.PosSemidef

theorem **elemForm_comm** (i j : Fin n) : elemForm b i j = elemForm b j i

theorem **two_smul_eq** {β : LinearMap.BilinForm ℤ X} (hβ : ∀ y z, β y z = β z y) : (2 : ℤ) • β = ∑ i, ∑ j, β (b i) (b j) • elemForm b i j

theorem **two_mul_map** (L : LinearMap.BilinForm ℤ X →ₗ[ℤ] ℤ) {β : LinearMap.BilinForm ℤ X} (hβ : ∀ y z, β y z = β z y) : 2 * L β = ∑ i, ∑ j, L (elemForm b i j) * β (b i) (b j)

theorem **dominates_of_basis** {X : Type} [AddCommGroup X] {n : ℕ} (b : Module.Basis (Fin n) ℤ X) : GlobalTheory.PolFan.Dominates (id : ℤ → ℤ) X

theorem **dominates_int** : ∀ (X : Type) [AddCommGroup X] [Module.Finite ℤ X] [Module.Free ℤ X], GlobalTheory.PolFan.Dominates (id : ℤ → ℤ) X

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.IntegerLatticeDominatesStatement

theorem **_root_.GlobalTheory.T.integer_lattice_dominates** : ∀ (X : Type) [AddCommGroup X] [Module.Finite ℤ X] [Module.Free ℤ X], Dominates (id : ℤ → ℤ) X

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.IsMin2IffFinite

theorem **isMin2_iff_finite** : ∀ (X : Type) [AddCommGroup X] (m : ℕ) (u : Fin m → LinearMap.BilinForm ℤ X) (y : X), (∀ i x, 0 ≤ u i x x) → ∃ F : Finset X, ∀ B ∈ rcone u, (IsMin2 B y ↔ ∀ x ∈ F, B y y ≤ B (y + (x + x)) (y + (x + x)))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.IsPosDefAddAndMemPsdCone

theorem **free_int_of_projective** (O : Type) [CommRing O] [IsDomain O] [CharZero O] [Module.Finite ℤ O] (Y : Type) [AddCommGroup Y] [Module O Y] [Module.Finite O Y] [Module.Projective O Y] : Module.Finite ℤ Y ∧ Module.Free ℤ Y

theorem **sum_nonneg_of_isPosDef** {Y ι : Type} [AddCommGroup Y] [Fintype ι] (b : Module.Basis ι ℤ Y) {B : Y →ₗ[ℤ] Y →ₗ[ℤ] ℝ} (h : IsPosDef B) {n : ℕ} (y : Fin n → Y) (c : Fin n → ℝ) : 0 ≤ ∑ i, ∑ j, c i * c j * B (y i) (y j)

theorem **sum_nonneg_of_mem_psdCone** (O : Type) [CommRing O] [IsDomain O] [CharZero O] [Module.Finite ℤ O] {bar : O → O} {X : Type} [AddCommGroup X] [Module O X] [Module.Finite O X] {B : X →ₗ[ℤ] X →ₗ[ℤ] ℝ} (hB : B ∈ psdCone bar X) {n : ℕ} (x : Fin n → X) (c : Fin n → ℝ) : 0 ≤ ∑ i, ∑ j, c i * c j * B (x i) (x j)

theorem **isPosDef_add_and_mem_psdCone** (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O) (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X] (B B₀ : X →ₗ[ℤ] X →ₗ[ℤ] ℝ) (hB : B ∈ psdCone bar X) (hB₀ : B₀ ∈ realForms bar X) (h₀ : IsPosDef B₀) : IsPosDef (B + B₀) ∧ B + B₀ ∈ psdCone bar X

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.IsStarOfClauses

theorem **isStarOf_of_clauses** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O), CMOrd O bar → ∀ (q : ℕ) (ψ w ψ' : InvFun O), IsPolyFan bar q ψ → IsCentreSet bar q ψ w → (∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Module.finrank O X ≤ q → ∀ (m : ℕ) (γ : Fin m → LinearMap.BilinForm ℤ X), IsCell bar ψ γ → (∀ β ∈ balForms bar X, (∀ y, 0 ≤ β y y) → InRCone γ β → w.evalZ β = 0) → IsCell bar ψ' γ) → (∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Module.finrank O X ≤ q → ∀ (m : ℕ) (γ : Fin m → LinearMap.BilinForm ℤ X), IsCell bar ψ γ → ∀ β₀ ∈ balForms bar X, (∀ y, 0 ≤ β₀ y y) → w.evalZ β₀ = 1 → InRCone γ β₀ → ∀ (m' : ℕ) (s : Fin m' ↪ Fin m) (h : LinearMap.BilinForm ℤ X →ₗ[ℤ] ℤ), IsFaceCut γ s h → 0 < h β₀ → IsCell bar ψ' (Fin.cons β₀ fun j => γ (s j))) → IsStarOf ba

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.IsStarOfClausesStatement

theorem **_root_.GlobalTheory.T.is_star_of_clauses** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O), CMOrd O bar → ∀ (q : ℕ) (ψ w ψ' : InvFun O), IsPolyFan bar q ψ → IsCentreSet bar q ψ w → (∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Module.finrank O X ≤ q → ∀ (m : ℕ) (γ : Fin m → LinearMap.BilinForm ℤ X), IsCell bar ψ γ → (∀ β ∈ balForms bar X, (∀ y, 0 ≤ β y y) → InRCone γ β → w.evalZ β = 0) → IsCell bar ψ' γ) → (∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Module.finrank O X ≤ q → ∀ (m : ℕ) (γ : Fin m → LinearMap.BilinForm ℤ X), IsCell bar ψ γ → ∀ β₀ ∈ balForms bar X, (∀ y, 0 ≤ β₀ y y) → w.evalZ β₀ = 1 → InRCone γ β₀ → ∀ (m' : ℕ) (s : Fin m' ↪ Fin m) (h : LinearMap.BilinForm ℤ X →ₗ[ℤ] ℤ), IsFaceCut γ s h → 0 < h β₀ → IsCell bar ψ' (Fin.cons β₀ fun j => γ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.IsStarOfFiniteCover

private theorem **inRCone_mono_of_range_subset** {X : Type} [AddCommGroup X] {m n : ℕ} (γ : Fin m → LinearMap.BilinForm ℤ X) (γ' : Fin n → LinearMap.BilinForm ℤ X) (hsub : Set.range γ' ⊆ Set.range γ) {β : LinearMap.BilinForm ℤ X} (h : GlobalTheory.PolFan.InRCone γ' β) : GlobalTheory.PolFan.InRCone γ β

theorem **isStarOf_finite_cover** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O), CMOrd O bar → ∀ (q : ℕ) (ψ w ψ' : InvFun O), IsPolyFan bar q ψ → IsCentreSet bar q ψ w → IsStarOf bar q ψ w ψ' → ∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Module.finrank O X ≤ q → ∃ S : Set (LinearMap.BilinForm ℤ X), S.Finite ∧ ∀ B ∈ psdCone bar X, ∃ (e : X ≃ₗ[O] X) (m : ℕ) (γ : Fin m → LinearMap.BilinForm ℤ X) (c : Fin m → ℝ), IsCell bar ψ' γ ∧ (∀ i, γ i ∈ S) ∧ (∀ i, 0 ≤ c i) ∧ pullR e.toLinearMap B = ∑ i, c i • toReal (γ i)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.MarkingValueGaloisInvariant

@[simp] theorem **twPi_apply** (σ : O ≃+* O) (n : ℕ) (a : Fin n → O) : twPi σ n a = fun j => σ (a j)

@[simp] theorem **twForm_apply** (σ : O ≃+* O) (n : ℕ) (β : LinearMap.BilinForm ℤ (Fin n → O)) (a b : Fin n → O) : twForm σ n β a b = β (fun j => σ (a j)) (fun j => σ (b j))

theorem **twForm_mem_balForms** {bar : O → O} (hb : CMOrd O bar) (σ : O ≃+* O) {n : ℕ} {β : LinearMap.BilinForm ℤ (Fin n → O)} (h : β ∈ balForms bar (Fin n → O)) : twForm σ n β ∈ balForms bar (Fin n → O)

theorem **twForm_symm_twForm** (σ : O ≃+* O) {n : ℕ} (β : LinearMap.BilinForm ℤ (Fin n → O)) : twForm σ.symm n (twForm σ n β) = β

theorem **twForm_mem_balForms_iff** {bar : O → O} (hb : CMOrd O bar) (σ : O ≃+* O) {n : ℕ} (β : LinearMap.BilinForm ℤ (Fin n → O)) : twForm σ n β ∈ balForms bar (Fin n → O) ↔ β ∈ balForms bar (Fin n → O)

theorem **evalZ_twForm** {w0 : O} {φ : InvFun O} (hφ : IsTwG w0 φ) {σ : O ≃+* O} (hσ : σ w0 = w0) {n : ℕ} (β : LinearMap.BilinForm ℤ (Fin n → O)) : φ.evalZ (twForm σ n β) = φ.evalZ β

theorem **markVal_twist** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O) (w0 : O), CMOrd O bar → ∀ (ψ lab : InvFun O) (i : ℕ), IsTwG w0 ψ → IsTwG w0 lab → ∀ σ : O ≃+* O, σ w0 = w0 → ∀ (n : ℕ) (B : (Fin n → O) → (Fin n → O) → ℝ), markVal bar ψ lab i n (fun a b => B (fun j => σ (a j)) (fun j => σ (b j))) = markVal bar ψ lab i n B

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.MarkingValueGaloisInvariantStatement

theorem **_root_.GlobalTheory.T.marking_value_galois_invariant** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O) (w0 : O), CMOrd O bar → ∀ (ψ lab : InvFun O) (i : ℕ), IsTwG w0 ψ → IsTwG w0 lab → ∀ σ : O ≃+* O, σ w0 = w0 → ∀ (n : ℕ) (B : (Fin n → O) → (Fin n → O) → ℝ), markVal bar ψ lab i n (fun a b => B (fun j => σ (a j)) (fun j => σ (b j))) = markVal bar ψ lab i n B

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.MaxMultiplicityLtTop

theorem **maxMult_lt_top** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O), CMOrd O bar → ∀ (q : ℕ) (φ : InvFun O), IsPolyFan bar q φ → maxMult bar q φ < ⊤

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.MaxMultiplicityLtTopStatement

theorem **_root_.GlobalTheory.T.max_multiplicity_lt_top** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O), CMOrd O bar → ∀ (q : ℕ) (φ : InvFun O), IsPolyFan bar q φ → maxMult bar q φ < ⊤

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.MemConeOfModTwoLift

theorem **isMin2_sum_smul** {X : Type} [AddCommGroup X] {m : ℕ} (B : Fin m → X →ₗ[ℤ] X →ₗ[ℤ] ℝ) (c : Fin m → ℝ) (y : X) (hc : ∀ i, 0 ≤ c i) (hB : ∀ i, IsMin2 (B i) y) : IsMin2 (∑ i, c i • B i) y

theorem **isCell_vorFun_of_contact** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O) (g : ℕ) (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X] (n : ℕ) (r : (Fin n → ℤ) →ₗ[ℤ] X) (a : (Fin n → ZMod (1 + 1)) → Fin n → ℤ), Function.Surjective r → (∀ y, redV 1 (a y) = y) → ∀ (m : ℕ) (γ : Fin m → LinearMap.BilinForm ℤ X), (∀ i, γ i ∈ balForms bar X) → (∀ i y, 0 ≤ γ i y y) → (∀ i, toReal (γ i) ∈ contact bar fun y => r (a y)) → (contact bar fun y => r (a y)) ⊆ rcone γ → IsCell bar (vorFun O g) γ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.MemConeOfModTwoLiftStatement

theorem **_root_.GlobalTheory.T.isCell_vorFun_of_contact_subset_rcone** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O) (g : ℕ) (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X] (n : ℕ) (r : (Fin n → ℤ) →ₗ[ℤ] X) (a : (Fin n → ZMod (1 + 1)) → Fin n → ℤ), Function.Surjective r → (∀ y, redV 1 (a y) = y) → ∀ (m : ℕ) (γ : Fin m → LinearMap.BilinForm ℤ X), (∀ i, γ i ∈ balForms bar X) → (∀ i y, 0 ≤ γ i y y) → (∀ i, toReal (γ i) ∈ contact bar fun y => r (a y)) → (contact bar fun y => r (a y)) ⊆ rcone γ → IsCell bar (vorFun O g) γ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.MemRationalConeOfPerturbation

theorem **exists_indep_rep** (v : ι → V) (n : ℕ) : ∀ (t : ι → ℝ), (∀ i, 0 ≤ t i) → (by classical exact (Finset.univ.filter (fun i => t i ≠ 0)).card) = n → ∃ (S : Finset ι) (s : ι → ℝ), LinearIndepOn ℝ v (S : Set ι) ∧ (∀ i, 0 ≤ s i) ∧ (∀ i, i ∉ S → s i = 0) ∧ ∑ i, s i • v i = ∑ i, t i • v i

theorem **seg_mem_cone** (v : ι → V) (B B₀ : V) (h : ∀ ε : ℝ, 0 < ε → ∃ t : ι → ℝ, (∀ i, 0 ≤ t i) ∧ B + ε • B₀ = ∑ i, t i • v i) : ∃ t : ι → ℝ, (∀ i, 0 ≤ t i) ∧ B = ∑ i, t i • v i

theorem **rcone_closed_seg** (X : Type) [AddCommGroup X] [Module.Finite ℤ X] (m : ℕ) (γ : Fin m → LinearMap.BilinForm ℤ X) (B B₀ : X →ₗ[ℤ] X →ₗ[ℤ] ℝ) (h : ∀ ε : ℝ, 0 < ε → B + ε • B₀ ∈ rcone γ) : B ∈ rcone γ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.MemRationalConeOfPerturbationStatement

theorem **_root_.GlobalTheory.T.mem_rational_cone_of_perturbation** : ∀ (X : Type) [AddCommGroup X] [Module.Finite ℤ X] (m : ℕ) (γ : Fin m → LinearMap.BilinForm ℤ X) (B B₀ : X →ₗ[ℤ] X →ₗ[ℤ] ℝ), (∀ ε : ℝ, 0 < ε → B + ε • B₀ ∈ rcone γ) → B ∈ rcone γ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.MultipleCellsAmongPolyhedralFan

theorem **perModule** {O : Type} [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O) (φ : InvFun O) (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] (hX : IsPolyFanOn bar φ X) : ∃ T : Set (LinearMap.BilinForm ℤ X), T.Finite ∧ ∀ (m : ℕ) (δ : Fin m → LinearMap.BilinForm ℤ X) (h : IsCell bar φ δ), LinearIndependent ℤ δ → (∀ i, IsPrim bar ((δ i, h.mem i) : balForms bar X)) → ∃ e : X ≃ₗ[O] X, ∀ i, pull e.toLinearMap (δ i) ∈ T

theorem **multCellsAmong_of_isPolyFan** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O), CMOrd O bar → ∀ (q : ℕ) (φ : InvFun O), IsPolyFan bar q φ → ∀ M : ℕ, ∃ n : ℕ, MultCellsAmong bar q φ M n

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.MultipleCellsAmongPolyhedralFanStatement

theorem **_root_.GlobalTheory.T.multiple_cells_among_polyhedral_fan** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O), CMOrd O bar → ∀ (q : ℕ) (φ : InvFun O), IsPolyFan bar q φ → ∀ M : ℕ, ∃ n : ℕ, MultCellsAmong bar q φ M n

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.NewCellMultLtAfterStar

theorem **cellMult_lt_of_star_new** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O), CMOrd O bar → ∀ (q : ℕ) (φ lab w ψ' : InvFun O), SimpLab bar q φ lab → IsCentreSet bar q φ w → (∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Module.finrank O X ≤ q → ∀ (m : ℕ) (δ : Fin m → LinearMap.BilinForm ℤ X) (h : IsCell bar φ δ), LinearIndependent ℤ δ → (∀ i, IsPrim bar ((δ i, h.mem i) : balForms bar X)) → ∀ β ∈ balForms bar X, (∀ y, 0 ≤ β y y) → w.evalZ β = 1 → ∀ c : Fin m → ℝ, (∀ i, 0 ≤ c i) → toReal β = ∑ i, c i • toReal (δ i) → ∀ i, c i < 1) → ∀ M : ℕ, maxMult bar q φ ≤ M → ∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Module.finrank O X ≤ q → ∀ (n : ℕ) (γ' : Fin n → LinearMap.BilinForm ℤ X) (hγ' : IsCell bar ψ' γ'), LinearIndependent ℤ γ' → (∀ i, IsPrim bar ((γ' i, h

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.NewCellMultLtAfterStarStatement

theorem **_root_.GlobalTheory.T.new_cell_mult_lt_after_star** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O), CMOrd O bar → ∀ (q : ℕ) (φ lab w ψ' : InvFun O), SimpLab bar q φ lab → IsCentreSet bar q φ w → (∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Module.finrank O X ≤ q → ∀ (m : ℕ) (δ : Fin m → LinearMap.BilinForm ℤ X) (h : IsCell bar φ δ), LinearIndependent ℤ δ → (∀ i, IsPrim bar ((δ i, h.mem i) : balForms bar X)) → ∀ β ∈ balForms bar X, (∀ y, 0 ≤ β y y) → w.evalZ β = 1 → ∀ c : Fin m → ℝ, (∀ i, 0 ≤ c i) → toReal β = ∑ i, c i • toReal (δ i) → ∀ i, c i < 1) → ∀ M : ℕ, maxMult bar q φ ≤ M → ∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Module.finrank O X ≤ q → ∀ (n : ℕ) (γ' : Fin n → LinearMap.BilinForm ℤ X) (hγ' : IsCell bar ψ' γ'), LinearIndependent ℤ γ' →

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.OldCellMultLeAfterStar

theorem **starOld_cell_mult_le** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O), CMOrd O bar → ∀ (q : ℕ) (φ lab w ψ' : InvFun O), (∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Dominates bar X) → IsPolyFan bar q φ → SimpLab bar q φ lab → ∀ M : ℕ, maxMult bar q φ ≤ M → ∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Module.finrank O X ≤ q → ∀ (n : ℕ) (γ' : Fin n → LinearMap.BilinForm ℤ X) (hγ' : IsCell bar ψ' γ'), LinearIndependent ℤ γ' → (∀ i, IsPrim bar ((γ' i, hγ'.mem i) : balForms bar X)) → ∀ (m : ℕ) (γ : Fin m → LinearMap.BilinForm ℤ X), IsCell bar φ γ → IsCell bar ψ' γ → (∀ β ∈ balForms bar X, (∀ y, 0 ≤ β y y) → InRCone γ β → w.evalZ β = 0) → (∀ j, InRCone γ (γ' j)) → ∃ hφ : IsCell bar φ γ', cellMult bar (fun i => ((γ' i, hφ.mem i) : balForms bar X)) ≤ M ∧ ∀

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.OldCellMultLeAfterStarStatement

theorem **_root_.GlobalTheory.T.old_cell_mult_le_after_star** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O), CMOrd O bar → ∀ (q : ℕ) (φ lab w ψ' : InvFun O), (∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Dominates bar X) → IsPolyFan bar q φ → SimpLab bar q φ lab → ∀ M : ℕ, maxMult bar q φ ≤ M → ∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Module.finrank O X ≤ q → ∀ (n : ℕ) (γ' : Fin n → LinearMap.BilinForm ℤ X) (hγ' : IsCell bar ψ' γ'), LinearIndependent ℤ γ' → (∀ i, IsPrim bar ((γ' i, hγ'.mem i) : balForms bar X)) → ∀ (m : ℕ) (γ : Fin m → LinearMap.BilinForm ℤ X), IsCell bar φ γ → IsCell bar ψ' γ → (∀ β ∈ balForms bar X, (∀ y, 0 ≤ β y y) → InRCone γ β → w.evalZ β = 0) → (∀ j, InRCone γ (γ' j)) → ∃ hφ : IsCell bar φ γ', cellMult bar (fun i => ((γ' i, hφ.mem

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.OrbitMarkCoordinateLtOne

theorem **orbMark_coord_lt_one** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O) (w0 : O), CMOrd O bar → ∀ (φ : InvFun O), IsTwG w0 φ → ∀ {X₀ : Type} [AddCommGroup X₀] [Module O X₀] [Module.Finite O X₀] {m₀ : ℕ} (δ₀ : Fin m₀ → LinearMap.BilinForm ℤ X₀) (h₀ : IsCell bar φ δ₀), LinearIndependent ℤ δ₀ → (∀ i, IsPrim bar ((δ₀ i, h₀.mem i) : balForms bar X₀)) → ∀ (v : LinearMap.BilinForm ℤ X₀) (t : Fin m₀ → ℝ), v ≠ 0 → (∀ i, 0 ≤ t i ∧ t i < 1) → toReal v = ∑ i, t i • toReal (δ₀ i) → ∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] (m : ℕ) (δ : Fin m → LinearMap.BilinForm ℤ X) (h : IsCell bar φ δ), LinearIndependent ℤ δ → (∀ i, IsPrim bar ((δ i, h.mem i) : balForms bar X)) → ∀ β ∈ balForms bar X, (∀ y, 0 ≤ β y y) → (OrbMark.orbFun O w0 v).evalZ β = 1 → ∀ c : Fin m → ℝ, (∀ i, 0 ≤ c i) → toReal β = ∑ i, c i • toReal (δ i) → ∀ i, c i < 1

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.OrbitMarkCoordinateLtOneStatement

theorem **_root_.GlobalTheory.T.orbit_mark_coordinate_lt_one** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O) (w0 : O), CMOrd O bar → ∀ (φ : InvFun O), IsTwG w0 φ → ∀ {X₀ : Type} [AddCommGroup X₀] [Module O X₀] [Module.Finite O X₀] {m₀ : ℕ} (δ₀ : Fin m₀ → LinearMap.BilinForm ℤ X₀) (h₀ : IsCell bar φ δ₀), LinearIndependent ℤ δ₀ → (∀ i, IsPrim bar ((δ₀ i, h₀.mem i) : balForms bar X₀)) → ∀ (v : LinearMap.BilinForm ℤ X₀) (t : Fin m₀ → ℝ), v ≠ 0 → (∀ i, 0 ≤ t i ∧ t i < 1) → toReal v = ∑ i, t i • toReal (δ₀ i) → ∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] (m : ℕ) (δ : Fin m → LinearMap.BilinForm ℤ X) (h : IsCell bar φ δ), LinearIndependent ℤ δ → (∀ i, IsPrim bar ((δ i, h.mem i) : balForms bar X)) → ∀ β ∈ balForms bar X, (∀ y, 0 ≤ β y y) → (OrbMark.orbFun O w0 v).evalZ β = 1 → ∀ c : Fin m → ℝ, (∀ i, 0 ≤ c i) → toReal β = ∑ i, c i •

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.PsdBalancedFormMemCellCone

theorem **inRCone_of_add** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O), CMOrd O bar → ∀ (q : ℕ) (ψ lab : InvFun O), IsPolyFan bar q ψ → IsCarrierDim bar q ψ lab → ∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Module.finrank O X ≤ q → Dominates bar X → ∀ (m : ℕ) (v : Fin m → LinearMap.BilinForm ℤ X), IsCell bar ψ v → ∀ β ∈ balForms bar X, (∀ y, 0 ≤ β y y) → InRCone v β → ∀ x ∈ balForms bar X, (∀ y, 0 ≤ x y y) → ψ.evalZ (β + x) = ψ.evalZ β + ψ.evalZ x → lab.evalZ (β + x) = lab.evalZ β → InRCone v x

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.PsdBalancedFormMemCellConeStatement

theorem **_root_.GlobalTheory.T.psd_balanced_form_mem_cell_cone** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O), CMOrd O bar → ∀ (q : ℕ) (ψ lab : InvFun O), IsPolyFan bar q ψ → IsCarrierDim bar q ψ lab → ∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Module.finrank O X ≤ q → Dominates bar X → ∀ (m : ℕ) (v : Fin m → LinearMap.BilinForm ℤ X), IsCell bar ψ v → ∀ β ∈ balForms bar X, (∀ y, 0 ≤ β y y) → InRCone v β → ∀ x ∈ balForms bar X, (∀ y, 0 ≤ x y y) → ψ.evalZ (β + x) = ψ.evalZ β + ψ.evalZ x → lab.evalZ (β + x) = lab.evalZ β → InRCone v x

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.PsdBalancedFormVanishesLowRank

theorem **vanish_of_inRCone_of_genRank_le** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O), CMOrd O bar → ∀ (q : ℕ) (ψ lab : InvFun O), IsPolyFan bar q ψ → IsCarrierDim bar q ψ lab → ∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Module.finrank O X ≤ q → Dominates bar X → ∀ (m : ℕ) (τ : Fin m → LinearMap.BilinForm ℤ X), IsCell bar ψ τ → ∀ β ∈ balForms bar X, (∀ y, 0 ≤ β y y) → InRCone τ β → (genRank τ : ℝ) ≤ lab.evalZ β → ∀ j y, β y y = 0 → τ j y y = 0

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.PsdBalancedFormVanishesLowRankStatement

theorem **_root_.GlobalTheory.T.psd_balanced_form_vanishes_low_rank** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O), CMOrd O bar → ∀ (q : ℕ) (ψ lab : InvFun O), IsPolyFan bar q ψ → IsCarrierDim bar q ψ lab → ∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Module.finrank O X ≤ q → Dominates bar X → ∀ (m : ℕ) (τ : Fin m → LinearMap.BilinForm ℤ X), IsCell bar ψ τ → ∀ β ∈ balForms bar X, (∀ y, 0 ≤ β y y) → InRCone τ β → (genRank τ : ℝ) ≤ lab.evalZ β → ∀ j y, β y y = 0 → τ j y y = 0

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.PsdConeAddMem

@[simp] theorem **orthSum_apply** (B₁ : X₁ →ₗ[ℤ] X₁ →ₗ[ℤ] ℝ) (B₂ : X₂ →ₗ[ℤ] X₂ →ₗ[ℤ] ℝ) (u v : X₁ × X₂) : orthSum B₁ B₂ u v = B₁ u.1 v.1 + B₂ u.2 v.2

theorem **sum_basis_nonneg** {K : Type} [Fintype K] (e : Module.Basis K ℤ X₁) {B : X₁ →ₗ[ℤ] X₁ →ₗ[ℤ] ℝ} (h : IsPosDef B) (c : K → ℝ) : 0 ≤ ∑ k, ∑ l, c k * c l * B (e k) (e l)

theorem **isPosDef_orthSum** {K₁ K₂ : Type} [Fintype K₁] [Fintype K₂] (e₁ : Module.Basis K₁ ℤ X₁) (e₂ : Module.Basis K₂ ℤ X₂) {B₁ : X₁ →ₗ[ℤ] X₁ →ₗ[ℤ] ℝ} {B₂ : X₂ →ₗ[ℤ] X₂ →ₗ[ℤ] ℝ} (h₁ : IsPosDef B₁) (h₂ : IsPosDef B₂) : IsPosDef (orthSum B₁ B₂)

theorem **isPosDef_compl_injective** {Y : Type} [AddCommGroup Y] {C : X₁ →ₗ[ℤ] X₁ →ₗ[ℤ] ℝ} (h : IsPosDef C) (j : Y →ₗ[ℤ] X₁) (hj : Function.Injective j) : IsPosDef (C.compl₁₂ j j)

theorem **free_int_of_torsion_eq_bot** (M : Type) [AddCommGroup M] [Module O M] [Module.Finite O M] (ht : Submodule.torsion O M = ⊥) : Module.Free ℤ M

theorem **add_mem_psdCone** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O) (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] (B B' : X →ₗ[ℤ] X →ₗ[ℤ] ℝ), B ∈ psdCone bar X → B' ∈ psdCone bar X → B + B' ∈ psdCone bar X

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.PsdConeAddMemStatement

theorem **_root_.GlobalTheory.T.psd_cone_add_mem** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O) (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] (B B' : X →ₗ[ℤ] X →ₗ[ℤ] ℝ), B ∈ psdCone bar X → B' ∈ psdCone bar X → B + B' ∈ psdCone bar X

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.PsdConeDescendsAlongSurjection

theorem **eq_zero_of_zsmul_eq_zero_of_projective** {O : Type} [CommRing O] [IsDomain O] [CharZero O] {P : Type} [AddCommGroup P] [Module O P] [Module.Projective O P] {n : ℤ} (hn : n ≠ 0) {z : P} (h : n • z = 0) : z = 0

theorem **IsPosDef.eq_zero_of_apply_self** {O : Type} [CommRing O] [IsDomain O] [CharZero O] {P : Type} [AddCommGroup P] [Module O P] [Module.Projective O P] {B : P →ₗ[ℤ] P →ₗ[ℤ] ℝ} (h : IsPosDef B) {z : P} (hz : B z z = 0) : z = 0

theorem **exists_psdCone_descend** (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O) (X X₁ : Type) [AddCommGroup X] [Module O X] [AddCommGroup X₁] [Module O X₁] (r : X →ₗ[O] X₁) (B : X →ₗ[ℤ] X →ₗ[ℤ] ℝ) (hr : Function.Surjective r) (hB : B ∈ psdCone bar X) (hk : ∀ y, r y = 0 → B y y = 0) : ∃ B₁ ∈ psdCone bar X₁, B = pullR r B₁

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.PsdConeDescendsAlongSurjectionStatement

theorem **_root_.GlobalTheory.T.psd_cone_descends_along_surjection** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O) (X X₁ : Type) [AddCommGroup X] [Module O X] [AddCommGroup X₁] [Module O X₁] (r : X →ₗ[O] X₁) (B : X →ₗ[ℤ] X →ₗ[ℤ] ℝ), Function.Surjective r → B ∈ psdCone bar X → (∀ y, r y = 0 → B y y = 0) → ∃ B₁ ∈ psdCone bar X₁, B = pullR r B₁

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.PsdConeExistsMaximalSupport

theorem **exists_smul_mem_of_le_of_finrank_le** {M : Type} [AddCommGroup M] [Module.Finite ℤ M] {P Q : Submodule ℤ M} (h : P ≤ Q) (hr : Module.finrank ℤ Q ≤ Module.finrank ℤ P) {y : M} (hy : y ∈ Q) : ∃ n : ℤ, n ≠ 0 ∧ n • y ∈ P

theorem **diag_eq_zero_iff_of_isPosDef** {X₁ : Type} [AddCommGroup X₁] {B : X₁ →ₗ[ℤ] X₁ →ₗ[ℤ] ℝ} (h : IsPosDef B) (z : X₁) : B z z = 0 ↔ z ∈ Submodule.torsion ℤ X₁

theorem **exists_radical_of_mem_psdCone** {O : Type} [CommRing O] {bar : O → O} {X : Type} [AddCommGroup X] [Module O X] {B : X →ₗ[ℤ] X →ₗ[ℤ] ℝ} (hB : B ∈ psdCone bar X) : ∃ N : Submodule ℤ X, ∀ y, y ∈ N ↔ B y y = 0

theorem **exists_least_radical** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O) (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] (K : Set (X →ₗ[ℤ] X →ₗ[ℤ] ℝ)), K.Nonempty → K ⊆ psdCone bar X → (∀ B ∈ K, ∀ B' ∈ K, B + B' ∈ K) → ∃ B₁ ∈ K, ∀ B ∈ K, ∀ y, B₁ y y = 0 → B y y = 0

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.PsdConeExistsMaximalSupportStatement

theorem **_root_.GlobalTheory.T.psd_cone_exists_maximal_support** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O) (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] (K : Set (X →ₗ[ℤ] X →ₗ[ℤ] ℝ)), K.Nonempty → K ⊆ psdCone bar X → (∀ B ∈ K, ∀ B' ∈ K, B + B' ∈ K) → ∃ B₁ ∈ K, ∀ B ∈ K, ∀ y, B₁ y y = 0 → B y y = 0

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.PsdConeFinitelyManyOrbits

theorem **exists_vorFun_cells** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O) (g : ℕ), (∀ (Y : Type) [AddCommGroup Y] [Module O Y] [Module.Finite O Y] [Module.Projective O Y], HasRedCover bar Y) → ∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], ∃ S : Set (LinearMap.BilinForm ℤ X), S.Finite ∧ ∀ B ∈ psdCone bar X, ∃ (e : X ≃ₗ[O] X) (m : ℕ) (γ : Fin m → LinearMap.BilinForm ℤ X) (c : Fin m → ℝ), IsCell bar (vorFun O g) γ ∧ (∀ i, γ i ∈ S) ∧ (∀ i, 0 ≤ c i) ∧ pullR e.toLinearMap B = ∑ i, c i • toReal (γ i)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.PsdConeFinitelyManyOrbitsStatement

theorem **_root_.GlobalTheory.T.psd_cone_finitely_many_orbits** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O) (g : ℕ), (∀ (Y : Type) [AddCommGroup Y] [Module O Y] [Module.Finite O Y] [Module.Projective O Y], HasRedCover bar Y) → ∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], ∃ S : Set (LinearMap.BilinForm ℤ X), S.Finite ∧ ∀ B ∈ psdCone bar X, ∃ (e : X ≃ₗ[O] X) (m : ℕ) (γ : Fin m → LinearMap.BilinForm ℤ X) (c : Fin m → ℝ), IsCell bar (vorFun O g) γ ∧ (∀ i, γ i ∈ S) ∧ (∀ i, 0 ≤ c i) ∧ pullR e.toLinearMap B = ∑ i, c i • toReal (γ i)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.PullbackMemCellCone

theorem **inRCone_pullAdd_of_isCell** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O) (w0 : O), CMOrd O bar → ∀ (φ : InvFun O), IsTwG w0 φ → ∀ {X₀ : Type} [AddCommGroup X₀] [Module O X₀] [Module.Finite O X₀] {m₀ : ℕ} (δ₀ : Fin m₀ → LinearMap.BilinForm ℤ X₀), IsCell bar φ δ₀ → ∀ t : Fin m₀ → ℝ, (∀ i, 0 ≤ t i) → ∀ {X Y : Type} [AddCommGroup X] [Module O X] [Module.Finite O X] [AddCommGroup Y] [Module O Y] [Module.Finite O Y] (π : X₀ →ₗ[O] Y), Function.Surjective π → ∀ (τ : O ≃+* O) (f : X →+ Y), τ w0 = w0 → Function.Surjective f → (∀ (a : O) (x : X), f (a • x) = τ a • f x) → ∀ u : Fin m₀ → LinearMap.BilinForm ℤ Y, (∀ i, 0 < t i → pull π (u i) = δ₀ i ∧ pullAdd f (u i) ∈ balForms bar X ∧ ∀ y, 0 ≤ pullAdd f (u i) y y) → ∀ β : LinearMap.BilinForm ℤ X, toReal β = ∑ i, t i • toReal (pullAdd f (u i)) → ∀ (m : ℕ) (δ : Fin m → LinearMap.BilinForm ℤ X), Is

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.PullbackMemCellConeStatement

theorem **_root_.GlobalTheory.T.pullback_mem_cell_cone** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O) (w0 : O), CMOrd O bar → ∀ (φ : InvFun O), IsTwG w0 φ → ∀ {X₀ : Type} [AddCommGroup X₀] [Module O X₀] [Module.Finite O X₀] {m₀ : ℕ} (δ₀ : Fin m₀ → LinearMap.BilinForm ℤ X₀), IsCell bar φ δ₀ → ∀ t : Fin m₀ → ℝ, (∀ i, 0 ≤ t i) → ∀ {X Y : Type} [AddCommGroup X] [Module O X] [Module.Finite O X] [AddCommGroup Y] [Module O Y] [Module.Finite O Y] (π : X₀ →ₗ[O] Y), Function.Surjective π → ∀ (τ : O ≃+* O) (f : X →+ Y), τ w0 = w0 → Function.Surjective f → (∀ (a : O) (x : X), f (a • x) = τ a • f x) → ∀ u : Fin m₀ → LinearMap.BilinForm ℤ Y, (∀ i, 0 < t i → pull π (u i) = δ₀ i ∧ pullAdd f (u i) ∈ balForms bar X ∧ ∀ y, 0 ≤ pullAdd f (u i) y y) → ∀ β : LinearMap.BilinForm ℤ X, toReal β = ∑ i, t i • toReal (pullAdd f (u i)) → ∀ (m : ℕ) (δ : Fin m → LinearMap

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.RatFormPosOfIntPos

theorem **ratForm_pos_of_int_pos** {n : ℕ} (Q : Matrix (Fin n) (Fin n) ℚ) (hQ : ∀ v : Fin n → ℤ, v ≠ 0 → 0 < ∑ i, ∑ j, Q i j * v i * v j) (w : Fin n → ℚ) (hw : w ≠ 0) : 0 < ∑ i, ∑ j, Q i j * w i * w j

theorem **posDef_of_int_pos** (n : ℕ) (Q : Matrix (Fin n) (Fin n) ℚ) (hs : Q.IsSymm) (hQ : ∀ v : Fin n → ℤ, v ≠ 0 → 0 < ∑ i, ∑ j, Q i j * v i * v j) : (Q.map (Rat.cast : ℚ → ℝ)).PosDef

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.SmoothOfMaxMultiplicityLeOne

theorem **smoothEnd** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O), CMOrd O bar → ∀ (q : ℕ) (φ lab : InvFun O), (∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Dominates bar X) → IsPolyFan bar q φ → SimpLab bar q φ lab → maxMult bar q φ ≤ 1 → IsPolFan bar q φ ∧ NoSelfInt bar q φ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.SmoothOfMaxMultiplicityLeOneStatement

theorem **_root_.GlobalTheory.T.smooth_of_max_multiplicity_le_one** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O), CMOrd O bar → ∀ (q : ℕ) (φ lab : InvFun O), (∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Dominates bar X) → IsPolyFan bar q φ → SimpLab bar q φ lab → maxMult bar q φ ≤ 1 → IsPolFan bar q φ ∧ NoSelfInt bar q φ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.StarDepthBoundedDenominator

theorem **starDepth_den** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O), CMOrd O bar → ∀ (q : ℕ) (ψ w : InvFun O), IsPolyFan bar q ψ → IsCentreSet bar q ψ w → ∃ D₀ : ℕ, 0 < D₀ ∧ ∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Module.finrank O X ≤ q → ∀ β ∈ balForms bar X, (∀ y, 0 ≤ β y y) → ∃ n : ℤ, (D₀ : ℝ) * (starDepth bar ψ w).evalZ β = n

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.StarDepthBoundedDenominatorStatement

theorem **_root_.GlobalTheory.T.star_depth_bounded_denominator** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O), CMOrd O bar → ∀ (q : ℕ) (ψ w : InvFun O), IsPolyFan bar q ψ → IsCentreSet bar q ψ w → ∃ D₀ : ℕ, 0 < D₀ ∧ ∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Module.finrank O X ≤ q → ∀ β ∈ balForms bar X, (∀ y, 0 ≤ β y y) → ∃ n : ℤ, (D₀ : ℝ) * (starDepth bar ψ w).evalZ β = n

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.StarDepthGapOnCell

theorem **starDepth_le_gap** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O), CMOrd O bar → ∀ (q : ℕ) (ψ w : InvFun O), IsPolyFan bar q ψ → IsCentreSet bar q ψ w → ∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Module.finrank O X ≤ q → ∀ (m : ℕ) (γ : Fin m → LinearMap.BilinForm ℤ X) (N : ℕ) (ℓ : LinearMap.BilinForm ℤ X →ₗ[ℤ] ℤ), IsCell bar ψ γ → (∀ β ∈ balForms bar X, (∀ y, 0 ≤ β y y) → InRCone γ β → w.evalZ β = 0) → 0 < N → StrictWith bar ψ N ℓ γ → ∀ β ∈ balForms bar X, (∀ y, 0 ≤ β y y) → (starDepth bar ψ w).evalZ β ≤ ℓ β − N * ψ.evalZ β

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.StarDepthGapOnCellStatement

theorem **_root_.GlobalTheory.T.star_depth_gap_on_cell** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O), CMOrd O bar → ∀ (q : ℕ) (ψ w : InvFun O), IsPolyFan bar q ψ → IsCentreSet bar q ψ w → ∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Module.finrank O X ≤ q → ∀ (m : ℕ) (γ : Fin m → LinearMap.BilinForm ℤ X) (N : ℕ) (ℓ : LinearMap.BilinForm ℤ X →ₗ[ℤ] ℤ), IsCell bar ψ γ → (∀ β ∈ balForms bar X, (∀ y, 0 ≤ β y y) → InRCone γ β → w.evalZ β = 0) → 0 < N → StrictWith bar ψ N ℓ γ → ∀ β ∈ balForms bar X, (∀ y, 0 ≤ β y y) → (starDepth bar ψ w).evalZ β ≤ ℓ β − N * ψ.evalZ β

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.StarDepthTwistedGalois

theorem **mem_balForms_twist** (hb : CMOrd O bar) (τ : O ≃+* O) (β : LinearMap.BilinForm ℤ X) : β ∈ balForms bar X ↔ β ∈ (letI

theorem **eval_twist_of_isTwG** {φ : InvFun O} (hφ : IsTwG w0 φ) (τ : O ≃+* O) (hτ : τ w0 = w0) [Module.Finite O X] (hf : letI

theorem **depthSet_twist_subset** (hb : CMOrd O bar) {ψ w : InvFun O} (hψ : IsTwG w0 ψ) (hw : IsTwG w0 w) (σ : O ≃+* O) (hσ : σ w0 = w0) (n : ℕ) (B : (Fin n → O) → (Fin n → O) → ℝ) : depthSet bar ψ w n (fun a b => B (fun i => σ (a i)) (fun i => σ (b i))) ⊆ depthSet bar ψ w n B

theorem **isTwG_starDepth** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O) (w0 : O), CMOrd O bar → ∀ (ψ w : InvFun O), IsTwG w0 ψ → IsTwG w0 w → IsTwG w0 (starDepth bar ψ w)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.StarDepthTwistedGaloisStatement

theorem **_root_.GlobalTheory.T.star_depth_twisted_galois** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O) (w0 : O), CMOrd O bar → ∀ (ψ w : InvFun O), IsTwG w0 ψ → IsTwG w0 w → IsTwG w0 (starDepth bar ψ w)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.StarFunctionLinearOnCell

theorem **starFun_linearOn** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O), CMOrd O bar → ∀ (q : ℕ) (ψ w : InvFun O) (K D : ℕ), IsCentreSet bar q ψ w → (∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Module.finrank O X ≤ q → ∀ (m : ℕ) (γ : Fin m → LinearMap.BilinForm ℤ X), IsCell bar ψ γ → (∀ β ∈ balForms bar X, (∀ y, 0 ≤ β y y) → InRCone γ β → w.evalZ β = 0) → (starFun bar ψ w K D).LinearOn γ) ∧ (∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Module.finrank O X ≤ q → ∀ (m : ℕ) (γ : Fin m → LinearMap.BilinForm ℤ X), IsCell bar ψ γ → ∀ β₀ ∈ balForms bar X, (∀ y, 0 ≤ β₀ y y) → w.evalZ β₀ = 1 → InRCone γ β₀ → ∀ (m' : ℕ) (s : Fin m' ↪ Fin m) (h : LinearMap.BilinForm ℤ X →ₗ[ℤ] ℤ), IsFaceCut γ s h → 0 < h β₀ → (starFun bar ψ w K D).LinearOn (Fin.cons β₀ fun j => γ (s

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.StarFunctionLinearOnCellStatement

theorem **_root_.GlobalTheory.T.star_function_linear_on_cell** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O), CMOrd O bar → ∀ (q : ℕ) (ψ w : InvFun O) (K D : ℕ), IsCentreSet bar q ψ w → (∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Module.finrank O X ≤ q → ∀ (m : ℕ) (γ : Fin m → LinearMap.BilinForm ℤ X), IsCell bar ψ γ → (∀ β ∈ balForms bar X, (∀ y, 0 ≤ β y y) → InRCone γ β → w.evalZ β = 0) → (starFun bar ψ w K D).LinearOn γ) ∧ (∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Module.finrank O X ≤ q → ∀ (m : ℕ) (γ : Fin m → LinearMap.BilinFormℤ X), IsCell bar ψ γ → ∀ β₀ ∈ balForms bar X, (∀ y, 0 ≤ β₀ y y) → w.evalZ β₀ = 1 → InRCone γ β₀ → ∀ (m' : ℕ) (s : Fin m' ↪ Fin m) (h : LinearMap.BilinForm ℤ X →ₗ[ℤ] ℤ), IsFaceCut γ s h → 0 < h β₀ → (starFun bar ψ w K D).Li

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.StarFunctionOffCell

theorem **off_cell** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O), CMOrd O bar → ∀ (q : ℕ) (ψ w : InvFun O) (b : ℕ), IsPolyFan bar q ψ → IsCentreSet bar q ψ w → ∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Module.finrank O X ≤ q → (∀ β ∈ balForms bar X, (∀ y, 0 ≤ β y y) → ∃ (n : ℕ) (u : Fin n → LinearMap.BilinForm ℤ X), (∀ i, u i ∈ balForms bar X) ∧ (∀ i y, 0 ≤ u i y y) ∧ ψ.LinearOn u ∧ (starDepth bar ψ w).LinearOn u ∧ (∀ i, ψ.evalZ (u i) ≤ b) ∧ (∀ i, (starDepth bar ψ w).evalZ (u i) ≤ 1) ∧ InRCone u β) → ∀ (m : ℕ) (γ : Fin m → LinearMap.BilinForm ℤ X) (N : ℕ) (ℓ : LinearMap.BilinForm ℤ X →ₗ[ℤ] ℤ), IsCell bar ψ γ → 0 < N → StrictWith bar ψ N ℓ γ → ∀ β₀ ∈ balForms bar X, (∀ y, 0 ≤ β₀ y y) → w.evalZ β₀ = 1 → InRCone γ β₀ → ∀ (m' : ℕ) (s : Fin m' ↪ Fin m) (h : LinearMap.BilinForm ℤ X →ₗ[ℤ] ℤ), IsFaceCut

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.StarFunctionOffCellStatement

theorem **_root_.GlobalTheory.T.star_function_off_cell** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O), CMOrd O bar → ∀ (q : ℕ) (ψ w : InvFun O) (b : ℕ), IsPolyFan bar q ψ → IsCentreSet bar q ψ w → ∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Module.finrank O X ≤ q → (∀ β ∈ balForms bar X, (∀ y, 0 ≤ β y y) → ∃ (n : ℕ) (u : Fin n → LinearMap.BilinForm ℤ X), (∀ i, u i ∈ balForms bar X) ∧ (∀ i y, 0 ≤ u i y y) ∧ ψ.LinearOn u ∧ (starDepth bar ψ w).LinearOn u ∧ (∀ i, ψ.evalZ (u i) ≤ b) ∧ (∀ i, (starDepth bar ψ w).evalZ (u i) ≤ 1) ∧ InRCone u β) → ∀ (m : ℕ) (γ : Fin m → LinearMap.BilinForm ℤ X) (N : ℕ) (ℓ : LinearMap.BilinForm ℤ X →ₗ[ℤ] ℤ), IsCell bar ψ γ → 0 < N → StrictWith bar ψ N ℓ γ → ∀ β₀ ∈ balForms bar X, (∀ y, 0 ≤ β₀ y y) → w.evalZ β₀ = 1 → InRCone γ β₀ → ∀ (m' : ℕ) (s : Fin m' ↪ Fin m) (h : LinearM

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.StarIntegralValuesPolyhedral

theorem **isPolyFan_star** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O), CMOrd O bar → ∀ (q : ℕ) (ψ w ψ' : InvFun O), IsPolyFan bar q ψ → IsCentreSet bar q ψ w → IsStarOf bar q ψ w ψ' → (∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Module.finrank O X ≤ q → ∀ β ∈ balForms bar X, (∀ y, 0 ≤ β y y) → ∃ n : ℤ, ψ'.evalZ β = n) → (∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Module.finrank O X ≤ q → ∀ β ∈ balForms bar X, (∀ y, 0 ≤ β y y) → β ≠ 0 → 0 < ψ'.evalZ β) → IsPolyFan bar q ψ'

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.StarIntegralValuesPolyhedralStatement

theorem **_root_.GlobalTheory.T.star_integral_values_polyhedral** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O), CMOrd O bar → ∀ (q : ℕ) (ψ w ψ' : InvFun O), IsPolyFan bar q ψ → IsCentreSet bar q ψ w → IsStarOf bar q ψ w ψ' → (∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Module.finrank O X ≤ q → ∀ β ∈ balForms bar X, (∀ y, 0 ≤ β y y) → ∃ n : ℤ, ψ'.evalZ β = n) → (∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Module.finrank O X ≤ q → ∀ β ∈ balForms bar X, (∀ y, 0 ≤ β y y) → β ≠ 0 → 0 < ψ'.evalZ β) → IsPolyFan bar q ψ'

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.StarSubdivisionDropsMultCells

theorem **exists_star_drop_multCells** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O) (w0 : O), CMOrd O bar → ∀ (q : ℕ) (φ lab : InvFun O), (∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Dominates bar X) → IsPolyFan bar q φ → IsTwG w0 φ → IsTwG w0 lab → SimpLab bar q φ lab → ∀ M n : ℕ, 2 ≤ M → maxMult bar q φ ≤ M → MultCellsAmong bar q φ M (n + 1) → ∃ φ' lab' : InvFun O, IsPolyFan bar q φ' ∧ IsTwG w0 φ' ∧ IsTwG w0 lab' ∧ SimpLab bar q φ' lab' ∧ Subordinate bar q φ' φ ∧ maxMult bar q φ' ≤ M ∧ MultCellsAmong bar q φ' M n

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.StarSubdivisionDropsMultCellsStatement

theorem **_root_.GlobalTheory.T.star_subdivision_drops_mult_cells** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O) (w0 : O), CMOrd O bar → ∀ (q : ℕ) (φ lab : InvFun O), (∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Dominates bar X) → IsPolyFan bar q φ → IsTwG w0 φ → IsTwG w0 lab → SimpLab bar q φ lab → ∀ M n : ℕ, 2 ≤ M → maxMult bar q φ ≤ M → MultCellsAmong bar q φ M (n + 1) → ∃ φ' lab' : InvFun O, IsPolyFan bar q φ' ∧ IsTwG w0 φ' ∧ IsTwG w0 lab' ∧ SimpLab bar q φ' lab' ∧ Subordinate bar q φ' φ ∧ maxMult bar q φ' ≤ M ∧ MultCellsAmong bar q φ' M n

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.StarSubdivisionMultLe

theorem **starMult** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O), CMOrd O bar → ∀ (q : ℕ) (φ lab w ψ' : InvFun O), (∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Dominates bar X) → IsPolyFan bar q φ → SimpLab bar q φ lab → IsCentreSet bar q φ w → (∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Module.finrank O X ≤ q → ∀ (m : ℕ) (δ : Fin m → LinearMap.BilinForm ℤ X) (h : IsCell bar φ δ), LinearIndependent ℤ δ → (∀ i, IsPrim bar ((δ i, h.mem i) : balForms bar X)) → ∀ β ∈ balForms bar X, (∀ y, 0 ≤ β y y) → w.evalZ β = 1 → ∀ c : Fin m → ℝ, (∀ i, 0 ≤ c i) → toReal β = ∑ i, c i • toReal (δ i) → ∀ i, c i < 1) → IsPolyFan bar q ψ' → Subordinate bar q ψ' φ → IsStarOf bar q φ w ψ' → ∀ M : ℕ, maxMult bar q φ ≤ M → maxMult bar q ψ' ≤ M ∧ ∀ (X : Type) [AddCommGroup X] [Mo

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.StarSubdivisionMultLeStatement

theorem **_root_.GlobalTheory.T.star_subdivision_mult_le** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O), CMOrd O bar → ∀ (q : ℕ) (φ lab w ψ' : InvFun O), (∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Dominates bar X) → IsPolyFan bar q φ → SimpLab bar q φ lab → IsCentreSet bar q φ w → (∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Module.finrank O X ≤ q → ∀ (m : ℕ) (δ : Fin m → LinearMap.BilinForm ℤ X) (h : IsCell bar φ δ), LinearIndependent ℤ δ → (∀ i, IsPrim bar ((δ i, h.mem i) : balForms bar X)) → ∀ β ∈ balForms bar X, (∀ y, 0 ≤ β y y) → w.evalZ β = 1 → ∀ c : Fin m → ℝ, (∀ i, 0 ≤ c i) → toReal β = ∑ i, c i • toReal (δ i) → ∀ i, c i < 1) → IsPolyFan bar q ψ' → Subordinate bar q ψ' φ → IsStarOf bar q φ w ψ' → ∀ M : ℕ, maxMult bar q φ ≤ M → maxMult bar q ψ' ≤

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.StrictStarFunctionBounds

theorem **exists_starFun_strict** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O), CMOrd O bar → ∀ (q : ℕ) (ψ w : InvFun O), (∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Dominates bar X) → IsPolyFan bar q ψ → IsCentreSet bar q ψ w → ∃ K D : ℕ, 0 < K ∧ 0 < D ∧ (∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Module.finrank O X ≤ q → ∀ β ∈ balForms bar X, (∀ y, 0 ≤ β y y) → ∃ n : ℤ,

(starFun bar ψ w K D).evalZ β = n) ∧ (∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Module.finrank O X ≤ q → ∀ (m : ℕ) (γ : Fin m → LinearMap.BilinForm ℤ X), IsCell bar ψ γ → (∀ β ∈ balForms bar X, (∀ y, 0 ≤ β y y) → InRCone γ β → w.evalZ β = 0) → IsStrict bar (starFun bar ψ w K D) γ) ∧ (∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.StrictStarFunctionBoundsStatement

theorem **_root_.GlobalTheory.T.strict_star_function_bounds** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O), CMOrd O bar → ∀ (q : ℕ) (ψ w : InvFun O), (∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Dominates bar X) → IsPolyFan bar q ψ → IsCentreSet bar q ψ w → ∃ K D : ℕ, 0 < K ∧ 0 < D ∧ (∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Module.finrank O X ≤ q → ∀ β ∈ balForms bar X, (∀ y, 0 ≤ β y y) → ∃ n : ℤ, (starFun bar ψ w K D).evalZ β = n) ∧ (∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Module.finrank O X ≤ q → ∀ (m : ℕ) (γ : Fin m → LinearMap.BilinForm ℤ X), IsCell bar ψ γ → (∀ β ∈ balForms bar X, (∀ y, 0 ≤ β y y) → InRCone γ β → w.evalZ β = 0) → IsStrict bar (starFun bar ψ w K D) γ) ∧ (∀ (X : Type) [AddCommGroup X]

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.UniformCellBoundPolyhedralFan

theorem **uniform_bound** (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O) (hb : CMOrd O bar) (q : ℕ) (ψ : InvFun O) (hψ : IsPolyFan bar q ψ) : ∃ N₀ : ℕ, ∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Module.finrank O X ≤ q → ∀ (m : ℕ) (γ : Fin m → LinearMap.BilinForm ℤ X), IsCell bar ψ γ → ∃ (N : ℕ) (ℓ : LinearMap.BilinForm ℤ X →ₗ[ℤ] ℤ), 0 < N ∧ N ≤ N₀ ∧ StrictWith bar ψ N ℓ γ

theorem **exists_strict_data** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O), CMOrd O bar → ∀ (q : ℕ) (ψ : InvFun O), (∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Dominates bar X) → IsPolyFan bar q ψ → ∃ N₀ C₀ : ℕ, ∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Module.finrank O X ≤ q → ∀ (m : ℕ) (γ : Fin m → LinearMap.BilinForm ℤ X), IsCell bar ψ γ → ∃ (N : ℕ) (ℓ : LinearMap.BilinForm ℤ X →ₗ[ℤ] ℤ), 0 < N ∧ N ≤ N₀ ∧ StrictWith bar ψ N ℓ γ ∧ ∀ (m' : ℕ) (s : Fin m' ↪ Fin m) (h : LinearMap.BilinForm ℤ X →ₗ[ℤ] ℤ), IsFaceCut γ s h → ∃ h' : LinearMap.BilinForm ℤ X →ₗ[ℤ] ℤ, IsFaceCut γ s h' ∧ (∀ β, InRCone γ β → 0 < h β → 0 < h' β) ∧ ∀ β ∈ balForms bar X, (∀ y, 0 ≤ β y y) → -(h' β) ≤ C₀ * ℓ β

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.UniformCellBoundPolyhedralFanStatement

theorem **_root_.GlobalTheory.T.uniform_cell_bound_polyhedral_fan** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O), CMOrd O bar → ∀ (q : ℕ) (ψ : InvFun O), (∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Dominates bar X) → IsPolyFan bar q ψ → ∃ N₀ C₀ : ℕ, ∀ (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Module.finrank O X ≤ q → ∀ (m : ℕ) (γ : Fin m → LinearMap.BilinForm ℤ X), IsCell bar ψ γ → ∃ (N : ℕ) (ℓ : LinearMap.BilinForm ℤ X →ₗ[ℤ] ℤ), 0 < N ∧ N ≤ N₀ ∧ StrictWith bar ψ N ℓ γ ∧ ∀ (m' : ℕ) (s : Fin m' ↪ Fin m) (h : LinearMap.BilinForm ℤ X →ₗ[ℤ] ℤ), IsFaceCut γ s h → ∃ h' : LinearMap.BilinForm ℤ X →ₗ[ℤ] ℤ, IsFaceCut γ s h' ∧ (∀ β, InRCone γ β → 0 < h β → 0 < h' β) ∧ ∀ β ∈ balForms bar X, (∀ y, 0 ≤ β y y) → -(h' β) ≤ C₀ * ℓ β

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.VorFunHomogSuperadd

theorem **evalR_vorFun_eq** (O : Type) [CommRing O] [Module.Finite ℤ O] (g : ℕ) {X : Type} [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Finite ℤ X] {d : ℕ} {s : (Fin d → ℤ) →ₗ[ℤ] X} (hs : Function.Surjective s) (B : X →ₗ[ℤ] X →ₗ[ℤ] ℝ) : evalR (vorFun O g) B = (fineDen g {1} : ℝ) * ((((1 : ℕ) : ℝ) + 1) ^ d)⁻¹ * ∑ y, classMin 1 (fun a b => B (s a) (s b)) y

theorem **vorFun_homog_superadd** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O) (g : ℕ) (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X], (∀ B ∈ psdCone bar X, ∀ t : ℝ, 0 ≤ t → evalR (vorFun O g) (t • B) = t * evalR (vorFun O g) B) ∧ ∀ B ∈ psdCone bar X, ∀ B' ∈ psdCone bar X, evalR (vorFun O g) B + evalR (vorFun O g) B' ≤ evalR (vorFun O g) (B + B')

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.VorFunValueLe

theorem **vorFun_value_le** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O) (g : ℕ) (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] (n : ℕ) (r : (Fin n → ℤ) →ₗ[ℤ] X) (a : (Fin n → ZMod (1 + 1)) → Fin n → ℤ), Function.Surjective r → (∀ y, redV 1 (a y) = y) → Covers2 (fun y => r (a y)) ∧ ∀ B : X →ₗ[ℤ] X →ₗ[ℤ] ℝ, (∀ y, 0 ≤ B y y) → (2 : ℝ) ^ n * evalR (vorFun O g) B ≤ 2 ^ g * ∑ y, B (r (a y)) (r (a y)) ∧ ((2 : ℝ) ^ n * evalR (vorFun O g) B = 2 ^ g * ∑ y, B (r (a y)) (r (a y)) ↔ ∀ y, IsMin2 B (r (a y)))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.VoronoiFanIsPolyhedral

theorem **vorFun_isPolyFan** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O) (g q : ℕ), (∀ (Y : Type) [AddCommGroup Y] [Module O Y] [Module.Finite O Y] [Module.Projective O Y], HasRedCover bar Y) → Module.finrank ℤ O * q ≤ g → IsPolyFan bar q (vorFun O g)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.VoronoiFanIsPolyhedralStatement

theorem **_root_.GlobalTheory.T.voronoi_fan_is_polyhedral** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O) (g q : ℕ), (∀ (Y : Type) [AddCommGroup Y] [Module O Y] [Module.Finite O Y] [Module.Projective O Y], HasRedCover bar Y) → Module.finrank ℤ O * q ≤ g → IsPolyFan bar q (vorFun O g)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.VoronoiFunctionIntegralPositive

theorem **exists_least_points** {X : Type} [AddCommGroup X] {n : ℕ} (r : (Fin n → ℤ) →ₗ[ℤ] X) (hr : Function.Surjective r) (β : LinearMap.BilinForm ℤ X) (hβ : ∀ y, 0 ≤ β y y) : ∃ a : (Fin n → ZMod (1 + 1)) → Fin n → ℤ, (∀ y, redV 1 (a y) = y) ∧ ∀ y, IsMin2 (toReal β) (r (a y))

theorem **apply_eq_zero_of_self** {X : Type} [AddCommGroup X] (β : LinearMap.BilinForm ℤ X) (hs : ∀ y z, β y z = β z y) (hβ : ∀ y, 0 ≤ β y y) {z : X} (hz : β z z = 0) (w : X) : β z w = 0

theorem **eq_zero_of_covers** {X : Type} [AddCommGroup X] (β : LinearMap.BilinForm ℤ X) (hs : ∀ y z, β y z = β z y) (hβ : ∀ y, 0 ≤ β y y) {ι : Type} {a : ι → X} (hc : Covers2 a) (h0 : ∀ i, β (a i) (a i) = 0) : β = 0

theorem **vorFun_int_pos** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O) (g q : ℕ) (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Module.finrank ℤ O * q ≤ g → Module.finrank O X ≤ q → (∀ β : LinearMap.BilinForm ℤ X, (∀ y, 0 ≤ β y y) → ∃ n : ℤ, (vorFun O g).evalZ β = n) ∧ ∀ β : LinearMap.BilinForm ℤ X, (∀ y z, β y z = β z y) → (∀ y, 0 ≤ β y y) → β ≠ 0 → 0 < (vorFun O g).evalZ β

CompanionTheory.SpectralGeometry.LocalGlobalProofs.ConvexCones.VoronoiFunctionIntegralPositiveStatement

theorem **_root_.GlobalTheory.T.voronoi_function_integral_positive** : ∀ (O : Type) [CommRing O] [IsDedekindDomain O] [CharZero O] [Module.Finite ℤ O] (bar : O → O) (g q : ℕ) (X : Type) [AddCommGroup X] [Module O X] [Module.Finite O X] [Module.Projective O X], Module.finrank ℤ O * q ≤ g → Module.finrank O X ≤ q → (∀ β : LinearMap.BilinForm ℤ X, (∀ y, 0 ≤ β y y) → ∃ n : ℤ, (vorFun O g).evalZ β = n) ∧ ∀ β : LinearMap.BilinForm ℤ X, (∀ y z, β y z = β z y) → (∀ y, 0 ≤ β y y) → β ≠ 0 → 0 < (vorFun O g).evalZ β

CompanionTheory.SpectralGeometry.LocalGlobalProofs.GlobalTheory.ExistsExpUnitary

theorem **map_exp_matrix** {m n : Type} [Fintype m] [DecidableEq m] [Fintype n] [DecidableEq n] (f : Matrix m m ℂ →ₐ[ℂ] Matrix n n ℂ) (hf : Continuous f) (S : Matrix m m ℂ) : f (NormedSpace.exp S) = NormedSpace.exp (f S)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.HermitianCharacters.CharacteristicIdentity

theorem **eval_mul_scalar** (P Q : B[X]) (u : ℂ) : (P * Q).eval (algebraMap ℂ B u) = P.eval (algebraMap ℂ B u) * Q.eval (algebraMap ℂ B u)

theorem **IsPolyFun.const** (b : B) : IsPolyFun (fun _ => b)

theorem **IsPolyFun.scalar** : IsPolyFun (fun u => algebraMap ℂ B u)

theorem **IsPolyFun.add** {f g : ℂ → B} (hf : IsPolyFun f) (hg : IsPolyFun g) : IsPolyFun (fun u => f u + g u)

theorem **IsPolyFun.mul** {f g : ℂ → B} (hf : IsPolyFun f) (hg : IsPolyFun g) : IsPolyFun (fun u => f u * g u)

theorem **IsPolyFun.sum** {ι : Type*} (s : Finset ι) (f : ι → ℂ → B) (h : ∀ i, IsPolyFun (f i)) : IsPolyFun (fun u => ∑ i ∈ s, f i u)

theorem **IsPolyFun.prod_ofFn** : ∀ (t : ℕ) (f : Fin t → ℂ → B), (∀ i, IsPolyFun (f i)) → IsPolyFun (fun u => (List.ofFn fun k => f k u).prod) | 0, f, _ => by simpa using IsPolyFun.const (1 : B) | t + 1, f, h => by simpa [List.ofFn_succ] using (h 0).mul (IsPolyFun.prod_ofFn t (fun k => f k.succ) fun k => h _) theorem IsPolyFun.map {B' : Type*} [Ring B'] [Algebra ℂ B'] (g : B →ₐ[ℂ] B') {f : ℂ → B} (hf : IsPolyFun f) : IsPolyFun (fun u => g (f u))

theorem **isPoly_Tm** {n : ℕ} (E : Fin n → Fin n → B) (c : ℂ) (a b : Fin n) : IsPolyFun (fun u => Tm E (u - c) a b)

theorem **isPoly_qminor** {n t : ℕ} (E : Fin n → Fin n → B) (a b : Fin t → Fin n) (c : ℂ) : IsPolyFun (fun u => qminor E a b (u + c))

theorem **eq_zero_of_eval** (P : B[X]) (h : ∀ u : ℂ, P.eval (algebraMap ℂ B u) = 0) : P = 0

CompanionTheory.SpectralGeometry.LocalGlobalProofs.HermitianCharacters.SelfDualLatticeFiltrationCharacter

theorem **exists_eq_exp** (g : ℝ → ℂ) (hc : Continuous g) (hn : ∀ t, ‖g t‖ = 1) (h0 : g 0 = 1) (hadd : ∀ s t, g (s + t) = g s * g t) : ∃ c : ℝ, ∀ t : ℝ, g t = Complex.exp (2 * Real.pi * Complex.I * ((c * t : ℝ) : ℂ))

theorem **pairA_add** (x x' : Ainf F) (y y' : Af F) : pairA F (x + x') (y + y') = pairA F x y + pairA F x' y'

theorem **algebraMap_eq_pairA** (x : F) : algebraMap F (AF F) x = pairA F (algebraMap F _ x) 0 + pairA F 0 (algebraMap F _ x)

theorem **denseRange_fin** : DenseRange (algebraMap F (Af F))

theorem **ext_of_inf** {ψ ψ' : AF F → ℂ} (hc : Continuous ψ) (hc' : Continuous ψ') (ha : ∀ x y, ψ (x + y) = ψ x * ψ y) (ha' : ∀ x y, ψ' (x + y) = ψ' x * ψ' y) (hr : ∀ a : F, ψ (algebraMap F _ a) = 1) (hr' : ∀ a : F, ψ' (algebraMap F _ a) = 1) (h : ∀ a : F, ψ (pairA F (algebraMap F _ a) 0) = ψ' (pairA F (algebraMap F _ a) 0)) (x : AF F) : ψ x = ψ' x

theorem **ker_mem_nhds** {ψ : AddChar (AF F) ℂ} (hc : Continuous ψ) (hn : ∀ x, ‖ψ x‖ = 1) : {a : Af F | ψ (pairA F 0 a) = 1} ∈ 𝒩 (0 : Af F)

theorem **exists_nsmul_inf** {ψ : AddChar (AF F) ℂ} (hψ : IsCharAF F ψ) : ∃ N : ℕ, 0 < N ∧ ∀ r : 𝒪 F, ψ (pairA F (algebraMap F _ ((N : F) * r)) 0) = 1

theorem **trace_mul_ne_zero** {ψ : AddChar (AF F) ℂ} (hψ : IsCharAF F ψ) {a : F} (h : ∀ x : F, ψ (pairA F (algebraMap F _ x) 0) = Complex.exp (2 * Real.pi * Complex.I * ((Algebra.trace ℚ F (a * x) : ℚ) : ℂ))) : a ≠ 0

theorem **exists_realHom** : ∃ ρ : ℝ →+* Ainf F, Continuous ρ

theorem **exists_trace** {ψ : AddChar (AF F) ℂ} (hψ : IsCharAF F ψ) : ∃ a : F, ∀ x : F, ψ (pairA F (algebraMap F _ x) 0) = Complex.exp (2 * Real.pi * Complex.I * ((Algebra.trace ℚ F (a * x) : ℚ) : ℂ))

theorem **trace_eq_sum** (a : F) : ((Algebra.trace ℚ F a : ℚ) : ℝ) = ∑ w : InfinitePlace F, w.mult * (w.embedding a).re

theorem **exists_inf** {ψ : AddChar (AF F) ℂ} (hψ : IsCharAF F ψ) (w : InfinitePlace F) : ∃ x : Ainf F, (∀ w', w' ≠ w → x w' = 0) ∧ psiInf F ψ x ≠ 1

theorem **continuous_singleV** (v : HeightOneSpectrum (𝒪 F)) : Continuous (singleV F v)

theorem **exists_mem_only** (v : HeightOneSpectrum (𝒪 F)) : ∃ t : 𝒪 F, t ≠ 0 ∧ t ∈ v.asIdeal ∧ ∀ w : HeightOneSpectrum (𝒪 F), w ≠ v → t ∉ w.asIdeal

theorem **exists_fin** {ψ : AddChar (AF F) ℂ} (hψ : IsCharAF F ψ) (v : HeightOneSpectrum (𝒪 F)) : ∃ y, psiV F ψ v y ≠ 1

theorem **char_nontrivial_at_every_place** : ∀ (F : Type) [Field F] [NumberField F] (ψ : AddChar (AF F) ℂ), IsCharAF F ψ → (∀ v : HeightOneSpectrum (𝒪 F), Continuous (psiV F ψ v) ∧ ∃ y, psiV F ψ v y ≠ 1) ∧ Continuous (psiInf F ψ) ∧ ∀ (w : InfinitePlace F), ∃ x : Ainf F, (∀ w', w' ≠ w → x w' = 0) ∧ psiInf F ψ x ≠ 1

CompanionTheory.SpectralGeometry.LocalGlobalProofs.LocalMassSeries.HermitianRankOneLocalVolume

theorem **mass_fibre_summable_one** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (n : ℕ), n = 1 → n + 2 ≤ M → ∀ (T : Matrix (Fin n) (Fin n) E) (hT : ∀ a b, σ (T b a) = T a b), T.det ≠ 0 → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (lam : ∀ v : HeightOneSpectrum (𝒪 F), Measure (Fv F v)), (∀ v, IsStdV v (lam v)) → ∀ (m : ∀ v : HeightOneSpectrum (𝒪 F), HermR (Fv F v) σ n → Measure (Fin n → Fin M → Fv F v ⊗[F] E)), (∀ v, IsGramFib (Fv F v) σ J n (coordW (Fv F v) b (lam v) n M) (coordHerm (Fv F v) σ b (lam v) n) (m v)) → (∀ v, m v (hermOf (Fv F v) σ T hT) (latV b v) < ⊤) ∧ Summable fun v => |(m v (hermOf (Fv F v) σ T hT) (latV b v)).toReal − 1|

CompanionTheory.SpectralGeometry.LocalGlobalProofs.LocalMassSeries.HermitianRankOneLocalVolumeStatement

theorem **_root_.GlobalTheory.T.hermitian_rank_one_local_volume** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (n : ℕ), n = 1 → n + 2 ≤ M → ∀ (T : Matrix (Fin n) (Fin n) E) (hT : ∀ a b, σ (T b a) = T a b), T.det ≠ 0 → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (lam : ∀ v : HeightOneSpectrum (𝒪 F), Measure (Fv F v)), (∀ v, IsStdV v (lam v)) → ∀ (m : ∀ v : HeightOneSpectrum (𝒪 F), HermR (Fv F v) σ n → Measure (Fin n → Fin M → Fv F v ⊗[F] E)), (∀ v, IsGramFib (Fv F v) σ J n (coordW (Fv F v) b (lam v) n M) (coordHerm (Fv F v) σ b (lam v) n) (m v)) → (∀ v, m v (hermOf (Fv F v) σ T hT) (latV b v) < ⊤) ∧ Summable fun v => |(m v (hermOf (Fv F v) σ T hT) (latV b v)).toReal − 1|

CompanionTheory.SpectralGeometry.LocalGlobalProofs.NefEffectiveCones.EventuallyEvalNegOfCoeffNeg

theorem **eventually_eval_neg_of_coeff_neg** (q : Polynomial ℝ) (D : ℕ) (hdeg : q.natDegree ≤ D) (hc : q.coeff D < 0) : ∀ᶠ m : ℕ in atTop, q.eval (m : ℝ) < 0

theorem **interNum_top_pos_of_isAmpleLB** : ∀ (k : Type) [Field k] (X : Scheme.{0}) [IsIntegral X] [Algebra k Γ(X, ⊤)] (hp : IsProjOver (strHom k X)) (n : ℕ) (hd : topologicalKrullDim X = (n : WithBot ℕ∞)) (A : LB X) (hA : IsAmpleLB A), 0 < interNum k (GlobalTheory.structureModule X) (fun _ : Fin n => A)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.NefEffectiveCones.ExistsNsmulMulVecMemHat

theorem **exists_nat_mul_integer** (K : Type) [Field K] [NumberField K] (x : K) : ∃ m : ℕ, 0 < m ∧ ∃ r : 𝒪 K, (r : K) = m * x

theorem **exists_nsmul_mem_lattice** [NumberField E] (L : Submodule (𝒪 E) (Fin n → E)) (hL : Submodule.span E (L : Set (Fin n → E)) = ⊤) (x : Fin n → E) : ∃ m : ℕ, 0 < m ∧ m • x ∈ L

CompanionTheory.SpectralGeometry.LocalGlobalProofs.NefEffectiveCones.NefPsEffUpTo

theorem **NefPsEffUpTo.mono** {k : Type} [Field k] {d e : ℕ} (h : NefPsEffUpTo k d) (he : e ≤ d) : NefPsEffUpTo k e

CompanionTheory.SpectralGeometry.LocalGlobalProofs.SmoothMorphisms.AbelianSchemeSpreadsOut

theorem **isCommMonObj_of_iso** {C : Type*} [Category C] [MonoidalCategory C] [BraidedCategory C] {M N : C} [MonObj M] [MonObj N] [IsCommMonObj N] (e : M ≅ N) [IsMonHom e.hom] : IsCommMonObj M where

theorem **isAbelianScheme_of_isGrpPullback** {S S' : Scheme.{0}} {f : S' ⟶ S} {G : Over S} [GrpObj G] [GrpObj (Over.mk (pullback.snd G.hom f))] (φ : pullback G.hom f ⟶ G.left) (h : IsGrpPullback (Over.mk (pullback.snd G.hom f)) G f φ) (hG : IsAbelianScheme G) : IsAbelianScheme (Over.mk (pullback.snd G.hom f))

theorem **exists_model_relDim** : ∀ (R : Type) [CommRing R] (g : ℕ) (A : Over (Spec R)) [GrpObj A] (F : Finset R), IsAbelianScheme A → SmoothOfRelativeDimension g A.hom → ∃ (S : Subring R) (B : Over (Spec S)) (_ : GrpObj B) (φ : A.left ⟶ B.left), (F : Set R) ⊆ S ∧ Algebra.FiniteType ℤ S ∧ IsAbelianScheme B ∧ SmoothOfRelativeDimension g B.hom ∧ IsGrpPullback A B (spMap S.subtype) φ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.SmoothMorphisms.AbelianSchemeSpreadsOutStatement

theorem **_root_.GlobalTheory.T.abelian_scheme_spreads_out** : ∀ (R : Type) [CommRing R] (g : ℕ) (A : Over (Spec R)) [GrpObj A] (F : Finset R), IsAbelianScheme A → SmoothOfRelativeDimension g A.hom → ∃ (S : Subring R) (B : Over (Spec S)) (_ : GrpObj B) (φ : A.left ⟶ B.left), (F : Set R) ⊆ S ∧ Algebra.FiniteType ℤ S ∧ IsAbelianScheme B ∧ SmoothOfRelativeDimension g B.hom ∧ IsGrpPullback A B (spMap S.subtype) φ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.SmoothMorphisms.ExistsFgSmoothOfRelativeDimension

theorem **smoothOfRelativeDimension_snd** {X S T : Scheme.{0}} (p : X ⟶ S) (b : T ⟶ S) (g : ℕ) (h : pullback.fst p b ⁻¹ᵘ dimLocus p g = ⊤) : SmoothOfRelativeDimension g (pullback.snd p b)

theorem **preimage_dimLocus_eq_top** {X S T : Scheme.{0}} (p : X ⟶ S) [Smooth p] (b : T ⟶ S) (g : ℕ) (h : SmoothOfRelativeDimension g (pullback.snd p b)) : pullback.fst p b ⁻¹ᵘ dimLocus p g = ⊤

theorem **exists_fg_smoothOfRelativeDimension** : ∀ (k V : Type) [CommRing k] [IsNoetherianRing k] [CommRing V] [Algebra k V] (X : Scheme.{0}) (p : X ⟶ Sp k) (g : ℕ), Smooth p → QuasiCompact p → SmoothOfRelativeDimension g (pullback.snd p (spMap (algebraMap k V))) → ∃ R : Subalgebra k V, R.FG ∧ SmoothOfRelativeDimension g (pullback.snd p (spMap (algebraMap k R)))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.SmoothMorphisms.ExistsModelAbelianLb

theorem **exists_model_abelian_lb** : ∀ (R : Type) [CommRing R] (A : Over (Spec R)) [GrpObj A], IsAbelianScheme A → ∀ L : LB A.left, L.IsLB → ∃ (S : Subring R) (B : Over (Spec S)) (_ : GrpObj B) (φ : A.left ⟶ B.left) (M : LB B.left), Algebra.FiniteType ℤ S ∧ IsAbelianScheme B ∧ IsGrpPullback A B (spMap S.subtype) φ ∧ M.IsLB ∧ LBIso L (M.pull φ)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.SmoothMorphisms.LineBundleSpreadsOut

theorem **lb_descends_fg** : ∀ (k V : Type) [CommRing k] [CommRing V] [Algebra k V] (X₀ X : Scheme.{0}) (p₀ : X₀ ⟶ Sp k) (p : X ⟶ Sp V) (φ : X ⟶ X₀), QuasiCompact p₀ → IsSeparated p₀ → IsPullback φ p p₀ (spMap (algebraMap k V)) → ∀ L : LB X, L.IsLB → ∃ R : Subalgebra k V, R.FG ∧ ∃ M : LB (pullback p₀ (spMap (algebraMap k R))), M.IsLB ∧ ∀ σ : X ⟶ pullback p₀ (spMap (algebraMap k R)), σ ≫ pullback.fst _ _ = φ → σ ≫ pullback.snd _ _ = p ≫ spMap R.val.toRingHom → LBIso L (M.pull σ)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.SmoothMorphisms.LineBundleSpreadsOutStatement

theorem **_root_.GlobalTheory.T.line_bundle_spreads_out** : ∀ (k V : Type) [CommRing k] [CommRing V] [Algebra k V] (X₀ X : Scheme.{0}) (p₀ : X₀ ⟶ Sp k) (p : X ⟶ Sp V) (φ : X ⟶ X₀), QuasiCompact p₀ → IsSeparated p₀ → IsPullback φ p p₀ (spMap (algebraMap k V)) → ∀ L : LB X, L.IsLB → ∃ R : Subalgebra k V, R.FG ∧ ∃ M : LB (pullback p₀ (spMap (algebraMap k R))), M.IsLB ∧ ∀ σ : X ⟶ pullback p₀ (spMap (algebraMap k R)), σ ≫ pullback.fst _ _ = φ → σ ≫ pullback.snd _ _ = p ≫ spMap R.val.toRingHom → LBIso L (M.pull σ)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.AdaptExists

theorem **herm_add_left** (H : Matrix ι ι ℂ) (x y w : ι → ℂ) : herm H (x + y) w = herm H x w + herm H y w

theorem **herm_sub_left** (H : Matrix ι ι ℂ) (x y w : ι → ℂ) : herm H (x − y) w = herm H x w − herm H y w

theorem **herm_smul_left** (H : Matrix ι ι ℂ) (a : ℂ) (x w : ι → ℂ) : herm H (a • x) w = a * herm H x w

theorem **herm_add_right** (H : Matrix ι ι ℂ) (x y w : ι → ℂ) : herm H w (x + y) = herm H w x + herm H w y

theorem **herm_sub_right** (H : Matrix ι ι ℂ) (x y w : ι → ℂ) : herm H w (x − y) = herm H w x − herm H w y

theorem **herm_smul_right** (H : Matrix ι ι ℂ) (a : ℂ) (x w : ι → ℂ) : herm H w (a • x) = star a * herm H w x

theorem **herm_neg_left** (H : Matrix ι ι ℂ) (x w : ι → ℂ) : herm H (−x) w = − herm H x w

theorem **herm_neg_right** (H : Matrix ι ι ℂ) (x w : ι → ℂ) : herm H w (−x) = − herm H w x

theorem **star_herm** {H : Matrix ι ι ℂ} (hH : Hᴴ = H) (x y : ι → ℂ) : star (herm H x y) = herm H y x

theorem **herm_mulVec** (H g : Matrix ι ι ℂ) (x y : ι → ℂ) : herm H (g *ᵥ x) (g *ᵥ y) = herm (gᴴ * H * g) x y

theorem **herm_single** (H : Matrix ι ι ℂ) (i j : ι) : herm H (Pi.single j 1) (Pi.single i 1) = H i j

theorem **mem_UH_of_herm** {H g : Matrix ι ι ℂ} (h : ∀ x y, herm H (g *ᵥ x) (g *ᵥ y) = herm H x y) : gᴴ * H * g = H

theorem **refl_iso** {H : Matrix ι ι ℂ} (hH : Hᴴ = H) (v : ι → ℂ) (c : ℂ) (hc : c ≠ 0) (hvv : herm H v v = c + star c) (x y : ι → ℂ) : herm H (x − (c⁻¹ * herm H x v) • v) (y − (c⁻¹ * herm H y v) • v) = herm H x y

theorem **exists_refl** {H : Matrix ι ι ℂ} (hH : Hᴴ = H) (u z : ι → ℂ) (huz : herm H u u = herm H z z) (hc : herm H u u − herm H u z ≠ 0) : ∃ γ : Matrix ι ι ℂ, γᴴ * H * γ = H ∧ γ *ᵥ u = z ∧ ∀ x, herm H x u = 0 → herm H x z = 0 → γ *ᵥ x = x

theorem **exists_move** {H : Matrix ι ι ℂ} (hH : Hᴴ = H) (u z : ι → ℂ) (huz : herm H u u = herm H z z) (h0 : herm H u u ≠ 0) : ∃ γ : Matrix ι ι ℂ, γᴴ * H * γ = H ∧ γ *ᵥ u = z ∧ ∀ x, herm H x u = 0 → herm H x z = 0 → γ *ᵥ x = x

theorem **star_inv_sqrt_two_bis** : star s2 = s2

theorem **inv_sqrt_two_mul_self_bis** : s2 * s2 = 1 / 2

theorem **unitary_adapted_basis_apply_basis** (k r : ℕ) (j : Fin (2 * r + k + 1)) : G0 k r *ᵥ bvec j = col k r j

theorem **herm_bvec** {m : ℕ} (H : Matrix (Fin m) (Fin m) ℂ) (i j : Fin m) : herm H (bvec j) (bvec i) = H i j

theorem **unitary_adapted_basis_entry** (k r : ℕ) (H : Mat (2 * r + k + 1)) (i j : Fin (2 * r + k + 1)) : ((G0 k r)ᴴ * H * G0 k r) i j = herm H (col k r j) (col k r i)

theorem **unitary_adapted_basis_is_unitary** (k r : ℕ) (hk : 1 ≤ k) : (G0 k r)ᴴ * G0 k r = 1

theorem **Hpk_val** (i j : Fin (2 * r + k + 1)) : Hpk k r i j = if i.val < r ∧ i.val + j.val = 2 * r + k then 1 else if j.val < r ∧ i.val + j.val = 2 * r + k then 1 else if r ≤ i.val ∧ i.val ≤ r + k ∧ i.val = j.val then (if i.val = r + k then −1 else 1) else 0

theorem **Hq_val** (i j : Fin (2 * r + k + 1)) : Hq k r i j = if i.val ≤ r ∨ r + k ≤ i.val then (if i.val + j.val = 2 * r + k then 1 else 0) else if i.val = j.val then 1 else 0

theorem **A_pp** (hk : 1 ≤ k) (hi : i.val = r) (hj : j.val = r) : Hpk k r i j = 1

theorem **A_qq** (hk : 1 ≤ k) (hi : i.val = r + k) (hj : j.val = r + k) : Hpk k r i j = −1

theorem **A_zp** (hk : 1 ≤ k) (hi : i.val ≠ r) (hj : j.val = r) : Hpk k r i j = 0

theorem **A_pz** (hk : 1 ≤ k) (hi : i.val = r) (hj : j.val ≠ r) : Hpk k r i j = 0

theorem **A_zq** (hk : 1 ≤ k) (hi : i.val ≠ r + k) (hj : j.val = r + k) : Hpk k r i j = 0

theorem **A_qz** (hk : 1 ≤ k) (hi : i.val = r + k) (hj : j.val ≠ r + k) : Hpk k r i j = 0

theorem **Q_pq** (hi : i.val = r) (hj : j.val = r + k) : Hq k r i j = 1

theorem **Q_qp** (hi : i.val = r + k) (hj : j.val = r) : Hq k r i j = 1

theorem **Q_pz** (hi : i.val = r) (hj : j.val ≠ r + k) : Hq k r i j = 0

theorem **Q_qz** (hi : i.val = r + k) (hj : j.val ≠ r) : Hq k r i j = 0

theorem **Q_zp** (hi : i.val ≠ r + k) (hi' : i.val ≠ r) (hj : j.val = r) : Hq k r i j = 0

theorem **Q_zq** (hi : i.val ≠ r) (hi' : i.val ≠ r + k) (hj : j.val = r + k) : Hq k r i j = 0

theorem **Q_eq_A** (hi : i.val ≠ r) (hi' : i.val ≠ r + k) (hj : j.val ≠ r) (hj' : j.val ≠ r + k) : Hq k r i j = Hpk k r i j

theorem **col_p** (hj : j.val = r) : col k r j = s2 • (bvec (wI k r 0) + bvec (wI k r (Fin.last k)))

theorem **col_q** (hk : 1 ≤ k) (hj : j.val = r + k) : col k r j = s2 • (bvec (wI k r 0) − bvec (wI k r (Fin.last k)))

theorem **col_o** (hj : j.val ≠ r) (hj' : j.val ≠ r + k) : col k r j = bvec j

theorem **unitary_adapted_basis_transforms_form** (k r : ℕ) (hk : 1 ≤ k) : (G0 k r)ᴴ * Hpk k r * G0 k r = Hq k r

theorem **Hpk_herm** (k r : ℕ) : (Hpk k r)ᴴ = Hpk k r

theorem **herm_outer** (k r : ℕ) (i : Fin (2 * r + k + 1)) (hi : i.val < r ∨ r + k < i.val) (z : Fin (2 * r + k + 1) → ℂ) (hz : inWb k r z) : herm (Hpk k r) (bvec i) z = 0

theorem **adaptExists** : AdaptExists

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.AdaptExistsStatement

theorem **_root_.GlobalTheory.T.AdaptedBasisExistsAlt** : AdaptExists

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.AffineCoverOfProductFibres

theorem **over_affineOpen_of_isPullback** : ∀ {k : Type} [Field k] {X T Z : Scheme.{0}} (a : X ⟶ Sp k) (b : T ⟶ Sp k) (p : Z ⟶ X) (q : Z ⟶ T) (H : IsPullback p q a b) {V : T.Opens} (hV : IsAffineOpen V) {κ : Type} [LinearOrder κ] [Fintype κ] (D : κ → X.Opens) (hD : ∀ i, IsAffineOpen (D i)) (hc : ⨆ i, D i = ⊤) (G : LB Z) (hG : G.IsLB), (∀ i, IsAffineOpen (p ⁻¹ᵁ D i ⊓ q ⁻¹ᵁ V)) ∧ ∃ (Y : Scheme.{0}) (π : Y ⟶ Z) (f : Y ⟶ Spec Γ(T, V)) (E : κ → Y.Opens), IsPullback π f q hV.fromSpec ∧ (∀ i, IsAffineOpen (E i)) ∧ ⨆ i, E i = ⊤ ∧ Nonempty (cechLB (f ≫ hV.fromSpec ≫ b) (G.pull π) E ≅ cechLB (p ≫ a) G fun i => p ⁻¹ᵁ D i ⊓ q ⁻¹ᵁ V)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.AffineCoverOfProductFibresStatement

theorem **_root_.GlobalTheory.T.affine_cover_of_product_fibres** : ∀ {k : Type} [Field k] {X T Z : Scheme.{0}} (a : X ⟶ Sp k) (b : T ⟶ Sp k) (p : Z ⟶ X) (q : Z ⟶ T) (H : IsPullback p q a b) {V : T.Opens} (hV : IsAffineOpen V) {κ : Type} [LinearOrder κ] [Fintype κ] (D : κ → X.Opens) (hD : ∀ i, IsAffineOpen (D i)) (hc : ⨆ i, D i = ⊤) (G : LB Z) (hG : G.IsLB), (∀ i, IsAffineOpen (p ⁻¹ᵁ D i ⊓ q ⁻¹ᵁ V)) ∧ ∃ (Y : Scheme.{0}) (π : Y ⟶ Z) (f : Y ⟶ Spec Γ(T, V)) (E : κ → Y.Opens), IsPullback π f q hV.fromSpec ∧ (∀ i, IsAffineOpen (E i)) ∧ ⨆ i, E i = ⊤ ∧ Nonempty (cechLB (f ≫ hV.fromSpec ≫ b) (G.pull π) E ≅ cechLB (p ≫ a) G fun i => p ⁻¹ᵁ D i ⊓ q ⁻¹ᵁ V)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.AffineCoverSeparatingFiniteClosedPoints

theorem **exists_affine_cover_finite_closed_points** : ∀ (X : Scheme.{0}) [CompactSpace X] (F : Set X) (hF : F.Finite) (hc : ∀ x ∈ F, IsClosed ({x} : Set X)), ∃ (r : ℕ) (V : Fin (r + 1) → X.Opens), (∀ j, IsAffineOpen (V j)) ∧ ⨆ j, V j = ⊤ ∧ ∀ x ∈ F, ∀ i j, x ∈ V i → x ∈ V j → i = j

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.AffineCoverSeparatingFiniteClosedPointsStatement

theorem **_root_.GlobalTheory.T.affine_cover_separating_finite_closed_points** : ∀ (X : Scheme.{0}) [CompactSpace X] (F : Set X) (hF : F.Finite) (hc : ∀ x ∈ F, IsClosed ({x} : Set X)), ∃ (r : ℕ) (V : Fin (r + 1) → X.Opens), (∀ j, IsAffineOpen (V j)) ∧ ⨆ j, V j = ⊤ ∧ ∀ x ∈ F, ∀ i j, x ∈ V i → x ∈ V j → i = j

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.AlgCoreSubOfAlgCore

theorem **ofFn_prod_intertwine** (φ : V' →ₗ[ℂ] V) : ∀ {t : ℕ} (f' : Fin t → Module.End ℂ V') (f : Fin t → Module.End ℂ V), (∀ k x, φ (f' k x) = f k (φ x)) → ∀ x, φ ((List.ofFn f').prod x) = (List.ofFn f).prod (φ x) | 0, _, _, _, x => by simp | t + 1, f', f, h, x => by rw [List.ofFn_succ, List.ofFn_succ, List.prod_cons, List.prod_cons, Module.End.mul_apply, Module.End.mul_apply, h, ofFn_prod_intertwine φ _ (fun k => f k.succ) (fun k => h k.succ)] theorem qdet_intertwine (φ : V' →ₗ[ℂ] V) (E' : Fin n → Fin n → Module.End ℂ V') (E : Fin n → Fin n → Module.End ℂ V) (h : ∀ a b x, φ (E' a b x) = E a b (φ x)) (u : ℂ) (x : V') : φ (qdet E' u x) = qdet E u (φ x)

theorem **algCoreSub_of_algCore** : AlgCore → AlgCoreSub

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.AnisotropicOfHenselAndSandwich

lemma **add_left** (hh : IsHerm s h) (x y z : V) : h (x + y) z = h x z + h y z

lemma **smul_left** (hh : IsHerm s h) (a : L) (x y : V) : h (a • x) y = s a * h x y

lemma **q_add** (hh : IsHerm s h) (x y : V) : h (x + y) (x + y) = h x x + h y y + (h x y + h y x)

lemma **norm_q_smul** (hs : IsInv s) (hh : IsHerm s h) (a : L) (x : V) : ‖h (a • x) (a • x)‖ = ‖a‖ * ‖a‖ * ‖h x x‖

lemma **pair_le** [IsUltrametricDist L] [CompleteSpace L] (hH : StHensel) (hs : IsInv s) (hh : IsHerm s h) (hani : ∀ x, h x x = 0 → x = 0) {x y : V} (hx : ‖h x x‖ ≤ 1) (hy : ‖h y y‖ ≤ 1) : ‖h x y + h y x‖ ≤ 1

lemma **exists_ball** [IsUltrametricDist L] (hs : IsInv s) (hh : IsHerm s h) (hp : ∀ x y : V, ‖h x x‖ ≤ 1 → ‖h y y‖ ≤ 1 → ‖h x y + h y x‖ ≤ 1) : ∃ V₀ : AddSubgroup V, ∀ x, x ∈ V₀ ↔ ‖h x x‖ ≤ 1

lemma **coord_comb** [FiniteDimensional L V] (hh : IsHerm s h) {m : ℕ} (b : Module.Basis (Fin m) L V) : ∃ M : Fin m → Fin m → L, ∀ x i, b.repr x i = ∑ j, M i j * h (b j) x

theorem **aniso** : StHensel → StSandwich → StAniso

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.AnisotropicOfHenselAndSandwichStatement

theorem **_root_.GlobalTheory.T.anisotropic_of_hensel_and_sandwich** : StHensel → StSandwich → StAniso

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.AntiInvolutionSwapsUpperLower

theorem **piece_mul_mul** {P : ι → E} (hP : IsPieces P) (x y : E) (i j : ι) : P i * (x * y) * P j = ∑ k, (P i * x * P k) * (P k * y * P j)

theorem **upper_mul** {wt : ι → ℕ} {P : ι → E} (hP : IsPieces P) {x y : E} (hx : IsUpper wt P x) (hy : IsUpper wt P y) : IsUpper wt P (x * y)

theorem **lower_mul** {wt : ι → ℕ} {P : ι → E} (hP : IsPieces P) {x y : E} (hx : IsLower wt P x) (hy : IsLower wt P y) : IsLower wt P (x * y)

theorem **diag_mul** {wt : ι → ℕ} {P : ι → E} (hP : IsPieces P) (hw : Function.Injective wt) {x y : E} (hx : IsLower wt P x) (hy : IsLower wt P y) (i : ι) : P i * (x * y) * P i = (P i * x * P i) * (P i * y * P i)

theorem **lowerUni_mul** {wt : ι → ℕ} {P : ι → E} (hP : IsPieces P) (hw : Function.Injective wt) {x y : E} (hx : IsLowerUni wt P x) (hy : IsLowerUni wt P y) : IsLowerUni wt P (x * y)

theorem **lowerUni_of_inv** {wt : ι → ℕ} {P : ι → E} (hP : IsPieces P) (hw : Function.Injective wt) {b b' : E} (hb : IsLowerUni wt P b) (hb' : IsLower wt P b') (h : b * b' = 1) : IsLowerUni wt P b'

theorem **eq_one_of_upper_lowerUni** {wt : ι → ℕ} {P : ι → E} (hP : IsPieces P) (hw : Function.Injective wt) {c : E} (hu : IsUpper wt P c) (hl : IsLowerUni wt P c) : c = 1

theorem **st_block** (hrev : ∀ i, rev (rev i) = i) (hmul : ∀ a b, st (a * b) = st b * st a) (hst : ∀ i, st (P i) = P (rev i)) (x : E) (i j : ι) : P i * st x * P j = st (P (rev j) * x * P (rev i))

theorem **st_upper** (hrev : ∀ i, rev (rev i) = i) (hord : ∀ i j, wt i < wt j → wt (rev j) < wt (rev i)) (hmul : ∀ a b, st (a * b) = st b * st a) (h0 : st 0 = 0) (hst : ∀ i, st (P i) = P (rev i)) {x : E} (hx : IsUpper wt P x) : IsUpper wt P (st x)

theorem **st_lower** (hrev : ∀ i, rev (rev i) = i) (hord : ∀ i j, wt i < wt j → wt (rev j) < wt (rev i)) (hmul : ∀ a b, st (a * b) = st b * st a) (h0 : st 0 = 0) (hst : ∀ i, st (P i) = P (rev i)) {x : E} (hx : IsLower wt P x) : IsLower wt P (st x)

theorem **st_lowerUni** (hrev : ∀ i, rev (rev i) = i) (hord : ∀ i j, wt i < wt j → wt (rev j) < wt (rev i)) (hmul : ∀ a b, st (a * b) = st b * st a) (h0 : st 0 = 0) (hst : ∀ i, st (P i) = P (rev i)) {x : E} (hx : IsLowerUni wt P x) : IsLowerUni wt P (st x)

theorem **st_mul_self_of_factor** : ∀ {E ι : Type} [Ring E] [Fintype ι] (wt : ι → ℕ) (P : ι → E), GlobalTheory.CompanionTheory.Iwahori.IsPieces P → ∀ (rev : ι → ι), (∀ i, rev (rev i) = i) → (∀ i j, wt i < wt j → wt (rev j) < wt (rev i)) → Function.Injective wt → ∀ (st : E → E), (∀ a b, st (a * b) = st b * st a) → st 1 = 1 → st 0 = 0 → (∀ i, st (P i) = P (rev i)) → (∀ a a' : E, a * a' = 1 → a' * a = 1 → GlobalTheory.CompanionTheory.Iwahori.IsLower wt P a → GlobalTheory.CompanionTheory.Iwahori.IsUpper wt P a' → ∀ b b' : E, b * b' = 1 → b' * b = 1 → GlobalTheory.CompanionTheory.Iwahori.IsLower wt P b → GlobalTheory.CompanionTheory.Iwahori.IsLower wt P b') → ∀ a a' b b' : E, a * a' = 1 → a' * a = 1 → b * b' = 1 → b' * b = 1 → GlobalTheory.CompanionTheory.Iwahori.IsUpper wt P a → GlobalTheory.CompanionTheory.Iwahori.IsUpper wt P b → GlobalTheory.CompanionTheory.Iwahori.IsLowerUni wt P b → st (a * b) * (a * b) = 1 → (a * b) * st (a * b) = 1 → st a

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.AntiInvolutionSwapsUpperLowerStatement

theorem **_root_.GlobalTheory.T.unitary_factors_of_unitary_upper_mul_lowerUni** : ∀ {E ι : Type} [Ring E] [Fintype ι] (wt : ι → ℕ) (P : ι → E), GlobalTheory.CompanionTheory.Iwahori.IsPieces P → ∀ (rev : ι → ι), (∀ i, rev (rev i) = i) → (∀ i j, wt i < wt j → wt (rev j) < wt (rev i)) → Function.Injective wt → ∀ (st : E → E), (∀ a b, st (a * b) = st b * st a) → st 1 = 1 → st 0 = 0 → (∀ i, st (P i) = P (rev i)) → (∀ a a' : E, a * a' = 1 → a' * a = 1 → GlobalTheory.CompanionTheory.Iwahori.IsUpper wt P a → GlobalTheory.CompanionTheory.Iwahori.IsUpper wt P a') → (∀ b b' : E, b * b' = 1 → b' * b = 1 → GlobalTheory.CompanionTheory.Iwahori.IsLower wt P b → GlobalTheory.CompanionTheory.Iwahori.IsLower wt P b') → ∀ a a' : E, a * a' = 1 → a' * a = 1 → b * b' = 1 → b' * b = 1 → GlobalTheory.CompanionTheory.Iwahori.IsUpper wt P a → GlobalTheory.CompanionTheory.Iwahori.IsLowerUni wt P b → st (a * b) *

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.AntitonePowersInDominantMonoid

theorem **exists_tExp_eq_indicator** (b : Fin n → ℤ) (hb : Antitone b) : ∃ i : Fin (n + 2), tExp n i = fun j => if 0 < b j then 1 else 0

theorem **dPow_mem_closure_of_nonneg** (p : Lˣ) (N : ℕ) : ∀ b : Fin n → ℤ, Antitone b → (∀ j, 0 ≤ b j) → (∀ j, b j ≤ N) → dPow p b ∈ Submonoid.closure (Set.range (tGen p : Fin (n + 2) → GL (Fin n) L))

theorem **dPow_mem_closure_of_ge** (p : Lˣ) (C : ℕ) : ∀ a : Fin n → ℤ, Antitone a → (∀ j, -(C : ℤ) ≤ a j) → dPow p a ∈ Submonoid.closure (Set.range (tGen p : Fin (n + 2) → GL (Fin n) L))

theorem **dPow_antitone_mem_closure** (L : Type) [NontriviallyNormedField L] [IsUltrametricDist L] (n : ℕ) (p : Lˣ) (a : Fin n → ℤ) (ha : Antitone a) : dPow p a ∈ Submonoid.closure (Set.range (tGen p : Fin (n + 2) → GL (Fin n) L))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.AntitonePowersInDominantMonoidStatement

theorem **_root_.GlobalTheory.T.antitone_powers_in_dominant_monoid** : ∀ (L : Type) [NontriviallyNormedField L] [IsUltrametricDist L] (n : ℕ) (p : Lˣ) (a : Fin n → ℤ), Antitone a → GlobalTheory.CompanionTheory.CartGL.dPow p a ∈ Submonoid.closure (Set.range (GlobalTheory.CompanionTheory.CartGL.tGen p : Fin (n + 2) → GL (Fin n) L))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.AppTopCompApply

lemma **comp_toLine** {T T' : Scheme.{0}} (g : T' ⟶ T) (s : Γ(T, ⊤)) : g ≫ toLine T s = toLine T' (g.appTop s)

lemma **toLine_appTop** (T : Scheme.{0}) (s : Γ(T, ⊤)) : (toLine T s).appTop ((Scheme.ΓSpecIso (.of (Polynomial ℤ))).inv Polynomial.X) = s

lemma **appTop_injective** {W Z : Scheme.{0}} (q : W ⟶ Z) [Epi q] (s s' : Γ(Z, ⊤)) (h : q.appTop s = q.appTop s') : s = s'

lemma **exists_descent** {W Z : Scheme.{0}} (q : W ⟶ Z) [EffectiveEpi q] (v : Γ(W, ⊤)) (h : ∀ (T : Scheme.{0}) (g₁ g₂ : T ⟶ W), g₁ ≫ q = g₂ ≫ q → g₁.appTop v = g₂.appTop v) : ∃ t : Γ(Z, ⊤), q.appTop t = v

lemma **exists_invariant** [Module.Flat A B] (u w : B ⊗[A] B) (huw : u * w = 1) (hc : ∀ (C : Type) [CommRing C] [Algebra A C] (α β γ : B →ₐ[A] C), pr α γ u = pr α β u * pr β γ u) (q : Ideal B) (hq : q ≠ ⊤) : ∃ n : B, n ∉ q ∧ n ⊗ₜ[A] (1 : B) = u * ((1 : B) ⊗ₜ[A] n)

lemma **appTop_comp_apply** {T₁ T₂ T₃ : Scheme.{0}} (g : T₁ ⟶ T₂) (h : T₂ ⟶ T₃) (s : Γ(T₃, ⊤)) : (g ≫ h).appTop s = g.appTop (h.appTop s)

lemma **lift_appTop_comp** {W W' : Scheme.{0}} (s : W' ⟶ W) (a b : W ⟶ X) (h : a ≫ f = b ≫ f) (h' : (s ≫ a) ≫ f = (s ≫ b) ≫ f) : (pullback.lift (s ≫ a) (s ≫ b) h').appTop u = s.appTop ((pullback.lift a b h).appTop u)

lemma **lift_congr** {W : Scheme.{0}} {a a' b b' : W ⟶ X} (ha : a = a') (hb : b = b') (h : a ≫ f = b ≫ f) (h' : a' ≫ f = b' ≫ f) : (pullback.lift a b h).appTop u = (pullback.lift a' b' h').appTop u

lemma **effectiveEpi_fst** {V T : Scheme.{0}} (G : V ⟶ Y) [Flat G] [LocallyOfFinitePresentation G] (c : T ⟶ Y) (h : ∀ w : T, c w ∈ Set.range G) : EffectiveEpi (pullback.fst c G)

theorem **extend** {Z V : Scheme.{0}} (ι : Z ⟶ X) (e : V ⟶ X) [Flat (e ≫ f)] [LocallyOfFinitePresentation (e ≫ f)] (hZ : ∀ z : Z, f (ι z) ∈ Set.range (e ≫ f)) (hu : IsUnit u) (hc : ∀ (W : Scheme.{0}) (a b c : W ⟶ X) (hab : a ≫ f = b ≫ f) (hbc : b ≫ f = c ≫ f), (pullback.lift a c (hab.trans hbc)).appTop u = (pullback.lift a b hab).appTop u * (pullback.lift b c hbc).appTop u) (t₀ : Γ(V, ⊤)) (ht₀ : IsUnit t₀) (P : ∀ (W : Scheme.{0}) (a b : W ⟶ V) (h : (a ≫ e) ≫ f = (b ≫ e) ≫ f), a.appTop t₀ = (pullback.lift (a ≫ e) (b ≫ e) h).appTop u * b.appTop t₀) : ∃ t : Γ(Z, ⊤), IsUnit t ∧ ∀ (W : Scheme.{0}) (a b : W ⟶ Z) (hab : (a ≫ ι) ≫ f = (b ≫ ι) ≫ f), a.appTop t = (pullback.lift (a ≫ ι) (b ≫ ι) hab).appTop u * b.appTop t

lemma **spec_appTop** {R S : Type} [CommRing R] [CommRing S] (ψ : R →+* S) (r : R) : (Spec.map (CommRingCat.ofHom ψ)).appTop ((Scheme.ΓSpecIso (.of R)).inv r) = (Scheme.ΓSpecIso (.of S)).inv (ψ r)

theorem **local_unit** {A B : Type} [CommRing A] [CommRing B] [Algebra A B] [Module.Flat A B] (i : Spec (.of B) ⟶ X) (j : Spec (.of A) ⟶ Y) [Mono j] [IsOpenImmersion i] (hij : Spec.map (CommRingCat.ofHom (algebraMap A B)) ≫ j = i ≫ f) (hu : IsUnit u) (hc : ∀ (W : Scheme.{0}) (a b c : W ⟶ X) (hab : a ≫ f = b ≫ f) (hbc : b ≫ f = c ≫ f), (pullback.lift a c (hab.trans hbc)).appTop u = (pullback.lift a b hab).appTop u * (pullback.lift b c hbc).appTop u) (x : PrimeSpectrum B) : ∃ (V : Scheme.{0}) (e : V ⟶ X) (_ : IsOpenImmersion e) (t₀ : Γ(V, ⊤)), i x ∈ Set.range e ∧ IsUnit t₀ ∧ ∀ (W : Scheme.{0}) (a b : W ⟶ V) (h : (a ≫ e) ≫ f = (b ≫ e) ≫ f), a.appTop t₀ = (pullback.lift (a ≫ e) (b ≫ e) h).appTop u * b.appTop t₀

theorem **h90** : CompanionTheory.ObstructionTrivialLocus.H90

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.ArchimedeanPrincipalSeries

theorem **inner_rep** {σ : K →* (H →L[ℂ] H)} (hiso : ∀ g v, ‖σ g v‖ = ‖v‖) (k : K) (a b : H) : ⟪σ k a, σ k b⟫_ℂ = ⟪a, b⟫_ℂ

theorem **rep_mul_apply** (σ : K →* (H →L[ℂ] H)) (a b : K) (x : H) : σ (a * b) x = σ a (σ b x)

theorem **rep_inv_apply** (σ : K →* (H →L[ℂ] H)) (k : K) (x : H) : σ k (σ k⁻¹ x) = x

theorem **inner_rep_inv** {σ : K →* (H →L[ℂ] H)} (hiso : ∀ g v, ‖σ g v‖ = ‖v‖) (k : K) (a b : H) : ⟪σ k⁻¹ a, b⟫_ℂ = ⟪a, σ k b⟫_ℂ

theorem **schur_scalar** (σ : K →* (H →L[ℂ] H)) (S : Submodule ℂ H) [FiniteDimensional ℂ S] (hS0 : S ≠ ⊥) (hS : ∀ k, ∀ x ∈ S, σ k x ∈ S) (hirr : ∀ T : Submodule ℂ H, T ≤ S → (∀ k, ∀ x ∈ T, σ k x ∈ T) → T = ⊥ ∨ T = S) (A : H →ₗ[ℂ] H) (hA : ∀ x ∈ S, A x ∈ S) (hc : ∀ k, ∀ x ∈ S, A (σ k x) = σ k (A x)) : ∃ c : ℂ, ∀ x ∈ S, A x = c • x

theorem **exists_minimal_stable** (σ : K →* (H →L[ℂ] H)) (S₀ : Submodule ℂ H) [FiniteDimensional ℂ S₀] (h0 : S₀ ≠ ⊥) (hS₀ : ∀ k, ∀ x ∈ S₀, σ k x ∈ S₀) : ∃ S : Submodule ℂ H, FiniteDimensional ℂ S ∧ S ≠ ⊥ ∧ (∀ k, ∀ x ∈ S, σ k x ∈ S) ∧ ∀ T : Submodule ℂ H, T ≤ S → (∀ k, ∀ x ∈ T, σ k x ∈ T) → T = ⊥ ∨ T = S

theorem **exists_rankOneAvg** (hiso : ∀ g v, ‖σ g v‖ = ‖v‖) (hc : ∀ v, Continuous fun x => σ x v) (s u : H) : ∃ B : H →L[ℂ] H, ∀ x, B x = ∫ k, ⟪σ k s, x⟫_ℂ • σ k u ∂μ

theorem **rankOneAvg_comm** (hiso : ∀ g v, ‖σ g v‖ = ‖v‖) (hc : ∀ v, Continuous fun x => σ x v) {s u : H} {B : H →L[ℂ] H} (hB : ∀ x, B x = ∫ k, ⟪σ k s, x⟫_ℂ • σ k u ∂μ) (g : K) (x : H) : B (σ g x) = σ g (B x)

theorem **schur_avg** (hiso : ∀ g v, ‖σ g v‖ = ‖v‖) (hc : ∀ v, Continuous fun x => σ x v) (S : Submodule ℂ H) [FiniteDimensional ℂ S] (hS0 : S ≠ ⊥) (hS : ∀ k, ∀ x ∈ S, σ k x ∈ S) (hirr : ∀ T : Submodule ℂ H, T ≤ S → (∀ k, ∀ x ∈ T, σ k x ∈ T) → T = ⊥ ∨ T = S) {s t : H} (hs : s ∈ S) (ht : t ∈ S) : (Module.finrank ℂ S : ℂ) • ∫ k, ⟪σ k s, s⟫_ℂ • σ k t ∂μ = ⟪s, t⟫_ℂ • s

theorem **avg_idem** (hiso : ∀ g v, ‖σ g v‖ = ‖v‖) (hc : ∀ v, Continuous fun x => σ x v) {s : H} {d : ℂ} {E : H →L[ℂ] H} (hE : ∀ u, E u = ∫ k, (d * ⟪σ k s, s⟫_ℂ) • σ k u ∂μ) (hs : E s = s) : E * E = E

theorem **avg_symm** [μ.IsInvInvariant] (hiso : ∀ g v, ‖σ g v‖ = ‖v‖) (hc : ∀ v, Continuous fun x => σ x v) {s : H} {d : ℝ} {E : H →L[ℂ] H} (hE : ∀ u, E u = ∫ k, ((d : ℂ) * ⟪σ k s, s⟫_ℂ) • σ k u ∂μ) (u w : H) : ⟪E u, w⟫_ℂ = ⟪u, E w⟫_ℂ

theorem **rFin_coef** {K : Type} [Group K] [TopologicalSpace K] {H : Type} [NormedAddCommGroup H] [InnerProductSpace ℂ H] {σ : K →* (H →L[ℂ] H)} (hc : ∀ v, Continuous fun x => σ x v) (S : Submodule ℂ H) [FiniteDimensional ℂ S] (hS : ∀ k, ∀ x ∈ S, σ k x ∈ S) {s : H} (hs : s ∈ S) (d : ℂ) : RFin (fun k => d * ⟪σ k s, s⟫_ℂ) (Module.finrank ℂ S)

theorem **exists_projAvg** (K : Type) [Group K] [TopologicalSpace K] [IsTopologicalGroup K] [CompactSpace K] [T2Space K] [SecondCountableTopology K] [MeasurableSpace K] [BorelSpace K] (μ : Measure K) [IsProbabilityMeasure μ] [μ.IsMulLeftInvariant] [μ.IsInvInvariant] (H : Type) [NormedAddCommGroup H] [InnerProductSpace ℂ H] [CompleteSpace H] [Nontrivial H] (σ : K →* (H →L[ℂ] H)) (hiso : ∀ g v, ‖σ g v‖ = ‖v‖) (hc : ∀ v, Continuous fun x => σ x v) : ∃ (e : K → ℂ) (d : ℕ) (E : H →L[ℂ] H), Continuous e ∧ RFin e d ∧ (∀ u, E u = ∫ k, e k • σ k u ∂μ) ∧ E ≠ 0 ∧ E * E = E ∧ ∀ u w, ⟪E u, w⟫_ℂ = ⟪u, E w⟫_ℂ

theorem **isPIProj_of_equiv** {G G' H : Type} [Group G] [Group G'] [NormedAddCommGroup H] [InnerProductSpace ℂ H] (φ : G ≃* G') (ρ : G →* (H →L[ℂ] H)) {E : H →L[ℂ] H} (h : IsPIProj (ρ.comp φ.symm.toMonoidHom) E) : IsPIProj ρ E

theorem **exists_isPIProj_UJ** {n : ℕ} (ε : Fin n → Bool) (H : Type) [NormedAddCommGroup H] [InnerProductSpace ℂ H] [CompleteSpace H] [Nontrivial H] (ρ : UJ ε →* (H →L[ℂ] H)) (hiso : ∀ g v, ‖ρ g v‖ = ‖v‖) (hc : ∀ v, Continuous fun g => ρ g v) : ∃ P, IsPIProj ρ P

theorem **archPI** : StArchPI

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.ArchimedeanPrincipalSeriesStatement

theorem **_root_.GlobalTheory.T.ArchimedeanPrincipalSeries** : StArchPI

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.BaseFamilyAlt

theorem **baseFam** : CompanionTheory.RgExh.Dec.J.BaseFam

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.Bdy

theorem **map_qdet** {A B : Type} [Ring A] [Algebra ℂ A] [Ring B] [Algebra ℂ B] (ψ : A →ₐ[ℂ] B) {n : ℕ} (X : Fin n → Fin n → A) (u : ℂ) : ψ (qdet X u) = qdet (fun p q => ψ (X p q)) u

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.BicommutantRestrictionPrime

theorem **bicommutant_restriction_prime** : ∀ (G G₁ H : Type) [Group G] [Group G₁] [NormedAddCommGroup H] [InnerProductSpace ℂ H] (π : G →* (H →L[ℂ] H)), IsTopIrreducible π → ∀ (j : G₁ →* G) (p : G →* G₁), (∀ x, p (j x) = x) → (∀ x g, p g = 1 → Commute (j x) g) → ∀ x y : H →L[ℂ] H, (∀ T : H →L[ℂ] H, (∀ g, T * π (j g) = π (j g) * T) → T * y = y * T) → y ≠ 0 → (∀ g, x * π (j g) * y = 0) → x = 0

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.BlkBijective

theorem **qminor_equiv** {A : Type} [Ring A] [Algebra ℂ A] {m n : ℕ} (E : Fin n → Fin n → A) (hE : GlRel E) (τ : Fin m ≃ Fin n) (u : ℂ) : qminor E τ τ u = qdet E u

theorem **qminor_sum** {A : Type} [Ring A] [Algebra ℂ A] {n q p : ℕ} (E : Fin n → Fin n → A) (r : Fin q ⊕ Fin p → Fin n) (u : ℂ) : qminor E (r ∘ finSumFinEquiv.symm) (r ∘ finSumFinEquiv.symm) u = ∑ σ : Perm (Fin q ⊕ Fin p), ((Perm.sign σ : ℤ) : A) * ((List.ofFn fun k : Fin q => Tm E (u − (k : ℂ)) (r (σ (Sum.inl k))) (r (Sum.inl k))).prod * (List.ofFn fun k : Fin p => Tm E (u − (q : ℂ) − (k : ℂ)) (r (σ (Sum.inr k))) (r (Sum.inr k))).prod)

theorem **prod_mem** {V : Type} [AddCommGroup V] [Module ℂ V] (P : Submodule ℂ V) : ∀ l : List (Module.End ℂ V), (∀ T ∈ l, ∀ v ∈ P, T v ∈ P) → (∃ T ∈ l, ∀ v, T v ∈ P) → ∀ v, l.prod v ∈ P | [], _, h, _ => by simp at h | T :: l, h1, h2, v => by rw [List.prod_cons, Module.End.mul_apply] obtain ⟨S, hS, hS'⟩

theorem **mem_pplus** (ε : Fin n → Bool) (E : Fin n → Fin n → Module.End ℂ V) {a b : Fin n} (ha : ε a = true) (hb : ε b = false) (v : V) : E a b v ∈ pplus ε E

theorem **stab_pplus** (ε : Fin n → Bool) (E : Fin n → Fin n → Module.End ℂ V) (hE : GlRel E) {c d : Fin n} (hc : ε c = false) (hd : ε d = false) : ∀ v ∈ pplus ε E, E c d v ∈ pplus ε E

theorem **Tm_mem** (ε : Fin n → Bool) (E : Fin n → Fin n → Module.End ℂ V) {a b : Fin n} (ha : ε a = true) (hb : ε b = false) (u : ℂ) (v : V) : Tm E u a b v ∈ pplus ε E

theorem **Tm_stab** (ε : Fin n → Bool) (E : Fin n → Fin n → Module.End ℂ V) (hE : GlRel E) (a : Fin n) {b : Fin n} (hb : ε b = false) (u : ℂ) : ∀ v ∈ pplus ε E, Tm E u a b v ∈ pplus ε E

theorem **block_mem** (ε : Fin n → Bool) (E : Fin n → Fin n → Module.End ℂ V) (hE : GlRel E) {q p : ℕ} (r : Fin q ⊕ Fin p → Fin n) (hl : ∀ k, ε (r (Sum.inl k)) = false) (hr : ∀ k, ε (r (Sum.inr k)) = true) (u : ℂ) (v : V) : qminor E (r ∘ finSumFinEquiv.symm) (r ∘ finSumFinEquiv.symm) u v − (qminor E (r ∘ Sum.inl) (r ∘ Sum.inl) u * qminor E (r ∘ Sum.inr) (r ∘ Sum.inr) (u − (q : ℂ))) v ∈ pplus ε E

theorem **blk_val** (ε : Fin n → Bool) (s : Bool) (k : Fin (nb ε s)) : ε (blk ε s k) = s

theorem **blk_bijective** (ε : Fin n → Bool) : Function.Bijective (Sum.elim (blk ε false) (blk ε true))

theorem **co0** : CompanionTheory.D.DSIC.CapelliDeterminantBlockProduct

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.BlkStrictMono

theorem **blk_strictMono** (ε : Fin n → Bool) (s : Bool) : StrictMono (blk ε s)

theorem **blk_mem** (ε : Fin n → Bool) (s : Bool) (i : Fin (nb ε s)) : ε (blk ε s i) = s

theorem **blk_surj** (ε : Fin n → Bool) (s : Bool) (a : Fin n) (h : ε a = s) : ∃ i, blk ε s i = a

theorem **prod_blk** (ε : Fin n → Bool) (s : Bool) (f : Fin n → ℂ) : ∏ i, f (blk ε s i) = ∏ a ∈ univ.filter (fun a => ε a = s), f a

theorem **card_below** (ε : Fin n → Bool) (s : Bool) (i : Fin (nb ε s)) : (univ.filter fun b => b < blk ε s i ∧ ε b = ε (blk ε s i)).card = i

theorem **card_total** (ε : Fin n → Bool) (c : Fin n) : ((univ.filter fun b => c < b ∧ ε b = ε c).card : ℝ) + ((univ.filter fun b => b < c ∧ ε b = ε c).card : ℝ) + ((univ.filter fun b => c < b ∧ ε b ≠ ε c).card : ℝ) + ((univ.filter fun b => b < c ∧ ε b ≠ ε c).card : ℝ) + 1 = n

theorem **card_other** (ε : Fin n → Bool) (c : Fin n) (hc : ε c = true) : ((univ.filter fun b => c < b ∧ ε b ≠ ε c).card : ℝ) + ((univ.filter fun b => b < c ∧ ε b ≠ ε c).card : ℝ) = nb ε false

theorem **wt_false** (ε : Fin n → Bool) (c : Fin n) (hc : ε c = false) : wtA (A1 ε) c = −((univ.filter fun b => c < b ∧ ε b ≠ ε c).card : ℝ)

theorem **wt_true** (ε : Fin n → Bool) (c : Fin n) (hc : ε c = true) : wtA (A1 ε) c = ((univ.filter fun b => b < c ∧ ε b ≠ ε c).card : ℝ)

theorem **gam_false** (ε : Fin n → Bool) (lam : Fin n → ℝ) (i : Fin (nb ε false)) : gam ε lam (blk ε false i) = lam (blk ε false i) − ((n : ℝ) − 1) / 2 + (i : ℕ)

theorem **gam_true** (ε : Fin n → Bool) (lam : Fin n → ℝ) (i : Fin (nb ε true)) : gam ε lam (blk ε true i) = lam (blk ε true i) − ((n : ℝ) − 1) / 2 + (nb ε false : ℝ) + (i : ℕ)

theorem **block_eigen** (ε : Fin n → Bool) (s : Bool) (μ : Fin n → ℝ) (V : Type) [AddCommGroup V] [Module ℂ V] (F : Fin n → Fin n → Module.End ℂ V) (x : V) (hF : KRel ε F) (hx : KHighest ε F μ x) (u : ℂ) : qminor F (blk ε s) (blk ε s) u x = (∏ i : Fin (nb ε s), (u − (i : ℂ) + (μ (blk ε s i) : ℂ))) • x

theorem **zetaHW** : CompanionTheory.D.DSIC.ZetaHW

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.BlockTriangularMemMinimalParabolic

theorem **cT_add** {m p : Type} (M N : Matrix m p K) : cT c (M + N) = cT c M + cT c N

theorem **cT_sub** {m p : Type} (M N : Matrix m p K) : cT c (M − N) = cT c M − cT c N

theorem **cT_cT** (hc : ∀ x, c (c x) = x) {m p : Type} (M : Matrix m p K) : cT c (cT c M) = M

theorem **cT_diagonal** {m : Type} [DecidableEq m] (d : m → K) : cT c (diagonal d) = diagonal (fun i => c (d i))

theorem **levi_core** (hc : ∀ x, c (c x) = x) {n r : ℕ} {Jh : Matrix (Fin n) (Fin n) K} {Xa Ya : Matrix (Fin n) (Fin r) K} {Xd Yd : Matrix (Fin r) (Fin n) K} (hJ : cT c Jh = Jh) (hXd : Xd = cT c Xa * Jh) (hYd : Yd = cT c Ya * Jh) (hxx : Xd * Xa = 0) (hyy : Yd * Ya = 0) (hxy : Xd * Ya = 1) (g : GL (Fin n) K) (hg : cT c (g : Matrix (Fin n) (Fin

n) K) * Jh * (g : Matrix (Fin n) (Fin n) K) = Jh) (B : Matrix (Fin r) (Fin r) K) (hBt : B.BlockTriangular id) (hB : (g : Matrix (Fin n) (Fin n) K) * Xa = Xa * B) (L : Matrix (Fin n) (Fin n) K) (hL : L = Xa * diagonal (fun i => (Yd * (g : Matrix (Fin n) (Fin n) K) * Xa) i i) * Yd + Ya * diagonal (fun i => (Xd * (g : Matrix (Fin n) (Fin n) K) * Ya) i i) * Xd + (1 - Xa * Yd - Ya * Xd) * (g : Matrix (Fin n) (Fin n) K) * (1 - Xa * Yd - Ya * Xd)) : ∃ u : GL (Fin n) K, (u : Matrix (Fin n) (Fin n) K) = L ∧ cT c L * Jh * L = Jh ∧ (∃ d, L * Xa = Xa * diagonal d)

theorem **sigma_invol** {F E : Type} [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] {σ : E →ₐ[F] E} (h : IsCMExt F E σ) (x : E) : σ (σ x) = x

theorem **sigR_invol** {F E : Type} [Field F] [Field E] [Algebra F E] (A : Type) [CommRing A] [Algebra F A] {σ : E →ₐ[F] E} (h : ∀ x, σ (σ x) = x) (x : A ⊗[F] E) : sigR A σ (sigR A σ x) = x

theorem **sigR_inclR** {F E : Type} [Field F] [Field E] [Algebra F E] (A : Type) [CommRing A] [Algebra F A] (σ : E →ₐ[F] E) (e : E) : sigR A σ (inclR A e) = inclR A (σ e)

theorem **cT_map_inclR** {F E : Type} [Field F] [Field E] [Algebra F E] (A : Type) [CommRing A] [Algebra F A] (σ : E →ₐ[F] E) {a b : ℕ} (N : Matrix (Fin a) (Fin b) E) : cT (sigR A σ) (N.map (inclR (F

theorem **exists_levi** : ∀ {F E : Type} [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] {σ : E →ₐ[F] E} {n : ℕ} {J : Matrix (Fin n) (Fin n) E} {r : ℕ} {X Y : Matrix (Fin n) (Fin r) E}, IsCMExt F E σ → IsHermND σ J → IsWittFrame σ J X Y → ∀ (A : Type) [CommRing A] [Algebra F A] [TopologicalSpace A] (p : HA A σ J) (B : Matrix (Fin r) (Fin r) (A ⊗[F] E)), B.BlockTriangular id → (matHom A σ J p).1 * X.map (inclR (F

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.BoundaryCreationAnticommutator

theorem **anticommutator_triple_product** {R : Type*} [Ring R] (K Aj Ai C d₁ d₂ : R) (h₁ : Ai * C + C * Ai = d₁) (h₂ : Aj * C + C * Aj = d₂) (h₃ : K * C + C * K = 0) : K * Aj * Ai * C + C * (K * Aj * Ai) = K * Aj * d₁ - K * d₂ * Ai

theorem **tri_anti** (i j k e : Pos n) : (cre k * ann j * ann i * cre e + cre e * (cre k * ann j * ann i) : Module.End ℂ (Cn n V)) = (if i = e then cre k * ann j else 0) - (if j = e then cre k * ann i else 0)

theorem **cre_mul_lift** (φ : Module.End ℂ V) (e : Pos n) : (cre e * (lift n φ : Module.End ℂ (Cn n V)) : Module.End ℂ (Cn n V)) = (lift n φ : Module.End ℂ (Cn n V)) * cre e

theorem **part_one** (E : Fin n → Fin n → Module.End ℂ V) (e : Pos n) : ((∑ i : Pos n, if cpt (allT n) i then (lift n (E i.1.1 i.1.2) : Module.End ℂ (Cn n V)) * ann i else 0) * cre e + cre e * ∑ i : Pos n, if cpt (allT n) i then (lift n (E i.1.1 i.1.2) : Module.End ℂ (Cn n V)) * ann i else 0 : Module.End ℂ (Cn n V)) = lift n (E e.1.1 e.1.2)

theorem **sum_anti** {R : Type*} [Ring R] (T P Q : Pos n → Pos n → R) (c : R) (h : ∀ i j k, T i j k * c + c * T i j k = P i j k - Q i j k) : (∑ i, ∑ j, ∑ k, T i j k) * c + c * ∑ i, ∑ j, ∑ k, T i j k = ∑ i, ∑ j, ∑ k, P i j k - ∑ i, ∑ j, ∑ k, Q i j k

theorem **part_two** (e : Pos n) : ((∑ i : Pos n, ∑ j : Pos n, ∑ k : Pos n, if cpt (allT n) i ∧ cpt (allT n) j ∧ i.1.2 = j.1.1 ∧ k.1.1 = i.1.1 ∧ k.1.2 = j.1.2 then cre k * ann j * ann i else 0) * cre e + cre e * ∑ i : Pos n, ∑ j : Pos n, ∑ k : Pos n, if cpt (allT n) i ∧ cpt (allT n) j ∧ i.1.2 = j.1.1 ∧ k.1.1 = i.1.1 ∧ k.1.2 = j.1.2 then cre k * ann j * ann i else 0 : Module.End ℂ (Cn n V)) = adL e.1.1 e.1.2

theorem **cartan** (n : ℕ) (V : Type) [AddCommGroup V] [Module ℂ V] (E : Fin n → Fin n → Module.End ℂ V) (e : Pos n) : (bd (allT n) E * cre e + cre e * bd (allT n) E : Module.End ℂ (Cn n V)) = lF E e.1.1 e.1.2

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.BoundaryCreationAnticommutatorStatement

theorem **_root_.GlobalTheory.T.boundary_creation_anticommutator** : ∀ (n : ℕ) (V : Type) [AddCommGroup V] [Module ℂ V] (E : Fin n → Fin n → Module.End ℂ V) (e : Pos n), (bd (allT n) E * cre e + cre e * bd (allT n) E : Module.End ℂ (Cn n V)) = lF E e.1.1 e.1.2

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.CanonicalAnticommutationRelationsConsequence

theorem **comm_prod** (D C X : R) : (D * C) * X - X * (D * C) = D * (C * X + X * C) - (D * X + X * D) * C

theorem **ad_mul** (A x y : R) : A * (x * y) - (x * y) * A = (A * x - x * A) * y + x * (A * y - y * A)

theorem **ad_sub** (x y z : R) : (x - y) * z - z * (x - y) = (x * z - z * x) - (y * z - z * y)

theorem **comm_sum** {ι : Type} [Fintype ι] (u v : ι → R) (x : R) : (∑ r, u r * v r) * x - x * (∑ r, u r * v r) = ∑ r, (u r * (v r * x + x * v r) - (u r * x + x * u r) * v r)

theorem **ad_d** (hd : ∀ a b a' b', d a b * d a' b' + d a' b' * d a b = 0) (hcd : ∀ a b a' b', c a b * d a' b' + d a' b' * c a b = if a = a' ∧ b = b' ∧ τ a b then 1 else 0) (hdτ : ∀ a b, ¬τ a b → d a b = 0) (h1 : ∀ b, τ p b ↔ τ q b) (h2 : ∀ a, τ a p ↔ τ a q) (a e : Fin n) : (∑ r, d p r * c q r - ∑ r, d r q * c r p) * d a e - d a e * (∑ r, d p r * c q r - ∑ r, d r q * c r p) = (if q = a then d p e else 0) - (if p = e then d a q else 0)

theorem **ad_c** (hc : ∀ a b a' b', c a b * c a' b' + c a' b' * c a b = 0) (hcd : ∀ a b a' b', c a b * d a' b' + d a' b' * c a b = if a = a' ∧ b = b' ∧ τ a b then 1 else 0) (hcτ : ∀ a b, ¬τ a b → c a b = 0) (h1 : ∀ b, τ p b ↔ τ q b) (h2 : ∀ a, τ a p ↔ τ a q) (a b : Fin n) : (∑ r, d p r * c q r - ∑ r, d r q * c r p) * c a b - c a b * (∑ r, d p r * c q r - ∑ r, d r q * c r p) = (if q = b then c a p else 0) - (if p = a then c q b else 0)

theorem **ad_triple** (A : R) (hAd : ∀ a e, A * d a e - d a e * A = (if q = a then d p e else 0) - (if p = e then d a q else 0)) (hAc : ∀ a b, A * c a b - c a b * A = (if q = b then c a p else 0) - (if p = a then c q b else 0)) : A * (∑ a, ∑ b, ∑ e, d a e * c b e * c a b) - (∑ a, ∑ b, ∑ e, d a e * c b e * c a b) * A = 0

theorem **ad_first** (A : R) (hAL : ∀ a b, A * L a b = L a b * A) (hAc : ∀ a b, A * c a b - c a b * A = (if q = b then c a p else 0) - (if p = a then c q b else 0)) : A * (∑ a, ∑ b, L a b * c a b) - (∑ a, ∑ b, L a b * c a b) * A = ∑ a, L a q * c a p - ∑ b, L p b * c q b

theorem **ad_L_first** (hLc : ∀ a b a' b', Commute (L a b) (c a' b')) (hLL : ∀ a b, L p q * L a b - L a b * L p q = (if q = a then L p b else 0) - (if b = p then L a q else 0)) : L p q * (∑ a, ∑ b, L a b * c a b) - (∑ a, ∑ b, L a b * c a b) * L p q = ∑ b, L p b * c q b - ∑ a, L a q * c a p

theorem **commute_shifted_dirac** : ∀ (n : ℕ) (R : Type) [Ring R] (τ : Fin n → Fin n → Prop) [DecidableRel τ] (c d L : Fin n → Fin n → R) (p q : Fin n), (∀ a b a' b', c a b * c a' b' + c a' b' * c a b = 0) → (∀ a b a' b', d a b * d a' b' + d a' b' * d a b = 0) → (∀ a b a' b', c a b * d a' b' + d a' b' * c a b = if a = a' ∧ b = b' ∧ τ a b then 1 else 0) → (∀ a b, ¬τ a b → c a b = 0) → (∀ a b, ¬τ a b → d a b = 0) → (∀ b, τ p b ↔ τ q b) → (∀ a, τ a p ↔ τ a q) → (∀ a b a' b', Commute (L a b) (c a' b')) → (∀ a b a' b', Commute (L a b) (d a' b')) → (∀ a b, L p q * L a b - L a b * L p q = (if q = a then L p b else 0) - (if b = p then L a q else 0)) → Commute (L p q + (∑ r, d p r * c q r - ∑ r, d r q * c r p)) ((∑ a, ∑ b, L a b * c a b) + ∑ a, ∑ b, ∑ e, d a e * c b e * c a b)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.CanonicalAnticommutationRelationsConsequenceStatement

theorem **_root_.GlobalTheory.T.canonical_anticommutation_relations_consequence** : ∀ (n : ℕ) (R : Type) [Ring R] (τ : Fin n → Fin n → Prop) [DecidableRel τ] (c d L : Fin n → Fin n → R) (p q : Fin n), (∀ a b a' b', c a b * c a' b' + c a' b' * c a b = 0) → (∀ a b a' b', d a b * d a' b' + d a' b' * d a b = 0) → (∀ a b a' b', c a b * d a' b' + d a' b' * c a b = if a = a' ∧ b = b' ∧ τ a b then 1 else 0) → (∀ a b, ¬τ a b → c a b = 0) → (∀ a b, ¬τ a b → d a b = 0) → (∀ b, τ p b ↔ τ q b) → (∀ a, τ a p ↔ τ a q) → (∀ a b a' b', Commute (L a b) (c a' b')) → (∀ a b a' b', Commute (L a b) (d a' b')) → (∀ a b, L p q * L a b - L a b * L p q = (if q = a then L p b else 0) - (if b = p then L a q else 0)) → Commute (L p q + (∑ r, d p r * c q r - ∑ r, d r q * c r p)) ((∑ a, ∑ b, L a b * c a b) + ∑ a, ∑ b, ∑ e, d a e * c b e * c a b)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.CapelliPolynomialPermutationMatch

theorem **exists_linear_factors_indexed** {ι : Type} [DecidableEq ι] (F : Finset ι) : ∀ P : ℂ[X], P.Monic → P.natDegree = F.card → ∃ r : ι → ℂ, P = ∏ a ∈ F, (X - C (r a))

theorem **exists_perm_of_monic_factors** (n M : ℕ) (lev : Fin n → ℕ) (χ : Fin n → ℂ) (P : Fin M → ℂ[X]) (s : Fin M → ℂ) (hlev : ∀ a, lev a < M) (hP : ∀ i, (P i).Monic ∧ (P i).natDegree = (univ.filter fun a => lev a = (i : ℕ)).card) (hcap : ∀ Y : ℂ, ∏ i, (P i).eval (Y + s i) = Formalism.QuantumDeterminants.capP χ Y) : ∃ σ : Equiv.Perm (Fin n), ∀ i : Fin M, ∑ a ∈ univ.filter (fun a => lev a = (i : ℕ)), χ (σ a) = (P i).nextCoeff + ((univ.filter fun a => lev a = (i : ℕ)).card : ℂ) * s i

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.CapelliPolynomialPermutationMatchStatement

theorem **_root_.GlobalTheory.T.capelli_polynomial_permutation_match** : ∀ (n M : ℕ) (lev : Fin n → ℕ) (χ : Fin n → ℂ) (P : Fin M → ℂ[X]) (s : Fin M → ℂ), (∀ a, lev a < M) → (∀ i, (P i).Monic ∧ (P i).natDegree = (univ.filter fun a => lev a = (i : ℕ)).card) → (∀ Y : ℂ, ∏ i, (P i).eval (Y + s i) = Formalism.QuantumDeterminants.capP χ Y) → ∃ σ : Equiv.Perm (Fin n), ∀ i : Fin M, ∑ a ∈ univ.filter (fun a => lev a = (i : ℕ)), χ (σ a) = (P i).nextCoeff + ((univ.filter fun a => lev a = (i : ℕ)).card : ℂ) * s i

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.CarImpFrel

theorem **pair_comm** {ι A : Type} [DecidableEq ι] [Ring A] (c a : ι → A) (haa : ∀ i j, a i * a j + a j * a i = 0) (hcc : ∀ i j, c i * c j + c j * c i = 0) (hac : ∀ i j, a i * c j + c j * a i = if i = j then 1 else 0) (i j k l : ι) : (c i * a j) * (c k * a l) - (c k * a l) * (c i * a j) = (if j = k then c i * a l else 0) - (if l = i then c k * a j else 0)

theorem **bil_comm** {ι A : Type} [Fintype ι] [DecidableEq ι] [Ring A] [Algebra ℂ A] (c a : ι → A) (haa : ∀ i j, a i * a j + a j * a i = 0) (hcc : ∀ i j, c i * c j + c j * c i = 0) (hac : ∀ i j, a i * c j + c j * a i = if i = j then 1 else 0) (m m' : ι → ι → ℂ) : bil c a m * bil c a m' - bil c a m' * bil c a m = bil c a (fun i l => ∑ k, m i k * m' k l - ∑ k, m' i k * m k l)

theorem **dl_comm** (x y : Fin n) : dl x y = dl y x

theorem **gop_eq** (ε : Fin n → Bool) (V : Type) [AddCommGroup V] [Module ℂ V] (c d : Fin n) : (Gop ε c d : Module.End ℂ (Ch ε V)) = bil cre ann (fun i j : NC ε => mm c d i.1 j.1)

theorem **sum_pt** (ε : Fin n → Bool) (x y : Fin n) (hx : ε x = true) (hy : ε y = false) (f : Fin n × Fin n → ℂ) : ∑ k : NC ε, dl k.1.1 x * dl k.1.2 y * f k.1 = f (x, y)

theorem **sum_mm** (ε : Fin n → Bool) (a b : Fin n) (hab : ε a = ε b) (i : NC ε) (f : Fin n × Fin n → ℂ) : ∑ k : NC ε, mm a b i.1 k.1 * f k.1 = dl i.1.1 a * f (b, i.1.2) - dl i.1.2 b * f (i.1.1, a)

theorem **gop_rel** (h : CAR) (ε : Fin n → Bool) (V : Type) [AddCommGroup V] [Module ℂ V] : KRel ε (Gop ε : Fin n → Fin n → Module.End ℂ (Ch ε V))

theorem **lift_gop** (T : Module.End ℂ V) (c d : Fin n) : Commute (lift ε T : Module.End ℂ (Ch ε V)) (Gop ε c d)

theorem **lift_rel** (E : Fin n → Fin n → Module.End ℂ V) (hE : KRel ε E) : KRel ε (fun c d => (lift ε (E c d) : Module.End ℂ (Ch ε V)))

theorem **add_rel** {A : Type} [Ring A] (ε : Fin n → Bool) (L G : Fin n → Fin n → A) (hL : KRel ε L) (hG : KRel ε G) (hc : ∀ a b c d, Commute (L a b) (G c d)) : KRel ε (fun c d => L c d + G c d)

theorem **neg_rel** {A : Type} [Ring A] (ε : Fin n → Bool) (G : Fin n → Fin n → A) (hG : KRel ε G) : KRel ε (fun c d => -G d c)

theorem **frel_of_car** (h : CAR) : FRel

theorem **car_imp_frel** : CompanionTheory.D.DSIC.CAR → CompanionTheory.D.DSIC.FRel

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.CardOther

theorem **card_four** (ε : Fin n → Bool) (a : Fin n) : ((univ.filter fun b => a < b ∧ ε b = ε a).card : ℝ) + (univ.filter fun b => b < a ∧ ε b = ε a).card + (univ.filter fun b => a < b ∧ ε b ≠ ε a).card + (univ.filter fun b => b < a ∧ ε b ≠ ε a).card + 1 = n

theorem **card_fst** (ε : Fin n → Bool) (a : Fin n) : ((A1 ε).filter fun i => i.1.1 = a).card = if ε a = true then (univ.filter fun b => b < a ∧ ε b ≠ ε a).card else 0

theorem **card_snd** (ε : Fin n → Bool) (a : Fin n) : ((A1 ε).filter fun i => i.1.2 = a).card = if ε a = true then 0 else (univ.filter fun b => a < b ∧ ε b ≠ ε a).card

theorem **card_other** (ε : Fin n → Bool) (a : Fin n) (h : ε a = true) : (univ.filter fun b => a < b ∧ ε b ≠ ε a).card + (univ.filter fun b => b < a ∧ ε b ≠ ε a).card = nb ε false

theorem **gam_eq** (ε : Fin n → Bool) (lam : Fin n → ℝ) (a : Fin n) : gam ε lam a = lam a - ((n : ℝ) - 1) / 2 + (univ.filter fun b => b < a ∧ ε b = ε a).card + (if ε a = true then (nb ε false : ℝ) else 0)

theorem **pos_le** (ε : Fin n → Bool) (m : Fin n → ℤ) (hm : StrictAnti m) (c d : Fin n) (hcd : c < d) (e : ε c = ε d) : m d + (univ.filter fun b => b < d ∧ ε b = ε d).card ≤ m c + (univ.filter fun b => b < c ∧ ε b = ε c).card

theorem **gamDom** : CompanionTheory.D.DSIC.GamDom

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.CartanDecomposition

theorem **exists_refl** (H : Matrix (Fin n) (Fin n) ℂ) (hH : Hᴴ = H) (ε : ℂ) (hε : star ε = ε) (a b : Fin n → ℂ) (ha : H *ᵥ a = ε • a) (hb : H *ᵥ b = ε • b) (hab : star a ⬝ᵥ a = star b ⬝ᵥ b) : ∃ U : Matrix (Fin n) (Fin n) ℂ, Uᴴ * U = 1 ∧ H * U = U * H ∧ U *ᵥ a = b ∧ ∀ w, star a ⬝ᵥ w = 0 → star b ⬝ᵥ w = 0 → U *ᵥ w = w

theorem **dot_herm** (H : Matrix (Fin n) (Fin n) ℂ) (hH : Hᴴ = H) (x w : Fin n → ℂ) : star (H *ᵥ x) ⬝ᵥ w = star x ⬝ᵥ H *ᵥ w

theorem **exists_unitary** (H : Matrix (Fin n) (Fin n) ℂ) (hH : Hᴴ = H) (hHH : H * H = 1) (x y : Fin n → ℂ) (h1 : star x ⬝ᵥ x = star y ⬝ᵥ y) (h2 : star x ⬝ᵥ H *ᵥ x = star y ⬝ᵥ H *ᵥ y) : ∃ K : Matrix (Fin n) (Fin n) ℂ, Kᴴ * K = 1 ∧ H * K = K * H ∧ K *ᵥ x = y ∧ ∀ w, star x ⬝ᵥ w = 0 → star y ⬝ᵥ w = 0 → star x ⬝ᵥ H *ᵥ w = 0 → star y ⬝ᵥ H *ᵥ w = 0 → K *ᵥ w = w

theorem **two_le** (H : Matrix (Fin n) (Fin n) ℂ) (hH : Hᴴ = H) (hHH : H * H = 1) (y : Fin n → ℂ) (h : star y ⬝ᵥ H *ᵥ y = -2) : ∃ N : ℝ, 2 ≤ N ∧ star y ⬝ᵥ y = N

theorem **exists_u** (N : ℝ) (hN : 2 ≤ N) : ∃ u : ℝ, 0 < u ∧ u ≤ 1 ∧ u⁻¹ ^ 2 + u ^ 2 = N

theorem **Hq_herm** (k r : ℕ) : (Hq k r)ᴴ = Hq k r

theorem **Hq_outer** (k r : ℕ) (i j : Fin (2 * r + k + 1)) (hi : outer k r i) (hij : i.val + j.val = 2 * r + k) : Hq k r *ᵥ bvec j = bvec i

theorem **Hq_inner** (k r : ℕ) (j : Fin (2 * r + k + 1)) (hj : ¬ outer k r j) : Hq k r *ᵥ bvec j = bvec j

theorem **Hq_sq** (k r : ℕ) : Hq k r * Hq k r = 1

theorem **dg_bvec** {m : ℕ} (d : Fin m → ℝ) (i : Fin m) : dg d *ᵥ bvec i = (d i : ℂ) • bvec i

theorem **dg_mul** (k r : ℕ) (u : ℝ) (hu : u ≠ 0) : dg (tor k r u⁻¹) * dg (tor k r u) = 1

theorem **tor_pair** (k r : ℕ) (hk : 1 ≤ k) (u : ℝ) (hu : u ≠ 0) (i j : Fin (2 * r + k + 1)) (hij : i.val + j.val = 2 * r + k) : tor k r u i * tor k r u j = 1

theorem **tor_inner** (k r : ℕ) (u : ℝ) (i : Fin (2 * r + k + 1)) (hi : ¬ outer k r i) : tor k r u i = 1

theorem **dg_mem** (k r : ℕ) (hk : 1 ≤ k) (u : ℝ) (hu : u ≠ 0) : dg (tor k r u) ∈ UH (Hq k r)

theorem **UH_mul** {m : ℕ} {H g h : Matrix (Fin m) (Fin m) ℂ} (hg : g ∈ UH H) (hh : h ∈ UH H) : g * h ∈ UH H

theorem **bvec_dot** {m : ℕ} (i j : Fin m) : star (bvec i) ⬝ᵥ bvec j = if i = j then (1 : ℂ) else 0

theorem **cartanQ** : CartanQ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.CartanDecompositionGL

theorem **stCartanGL** : StCartanGL

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.CartanDecompositionGLStatement

theorem **_root_.GlobalTheory.T.CartanDecompositionGL** : StCartanGL

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.CartanDecompositionGlNonarchimedean

theorem **mem_ball** {x : L} : x ∈ ball L ↔ ‖x‖ ≤ 1

theorem **isUnit_of_norm_eq_one** {x : ball L} (h : ‖(x : L)‖ = 1) : IsUnit x

theorem **norm_eq_one_of_isUnit** {x : ball L} (h : IsUnit x) : ‖(x : L)‖ = 1

theorem **norm_unif_pos** (p : Lˣ) : 0 < ‖(p : L)‖

theorem **exists_nat_norm** (hp : IsUnif p) {x : L} (hx : x ≠ 0) (h1 : ‖x‖ ≤ 1) : ∃ m : ℕ, ‖x‖ = ‖(p : L)‖ ^ m

theorem **unif_coe** (hp : IsUnif p) : ((unif hp : ball L) : L) = p

theorem **irreducible_unif** (hp : IsUnif p) : Irreducible (unif hp)

theorem **isDVR** (hp : IsUnif p) : IsDiscreteValuationRing (ball L)

theorem **map_mem_congK** (n : ℕ) (U : GL (Fin n) (ball L)) : Matrix.GeneralLinearGroup.map (algebraMap (ball L) L) U ∈ congK L n 1 zero_le_one le_rfl

theorem **cartan_unitBall** (L : Type) [NontriviallyNormedField L] [IsUltrametricDist L] [ProperSpace L] (n : ℕ) (p : Lˣ) (hp : IsUnif p) (g : GL (Fin n) L) : ∃ a : Fin n → ℤ, Antitone a ∧ ∃ k₁ ∈ congK L n 1 zero_le_one le_rfl, ∃ k₂ ∈ congK L n 1 zero_le_one le_rfl, g = k₁ * dPow p a * k₂

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.CartanDecompositionGlNonarchimedeanStatement

theorem **_root_.GlobalTheory.T.cartan_decomposition_gl_nonarchimedean** : ∀ (L : Type) [NontriviallyNormedField L] [IsUltrametricDist L] [ProperSpace L] (n : ℕ) (p : Lˣ), GlobalTheory.CompanionTheory.CartGL.IsUnif p → ∀ g : GL (Fin n) L, ∃ a : Fin n → ℤ, Antitone a ∧ ∃ k₁ ∈ GlobalTheory.CompanionTheory.CartGL.congK L n 1 zero_le_one le_rfl, ∃ k₂ ∈ GlobalTheory.CompanionTheory.CartGL.congK L n 1 zero_le_one le_rfl, g = k₁ * GlobalTheory.CompanionTheory.CartGL.dPow p a * k₂

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.CartanDecompositionStatement

theorem **_root_.GlobalTheory.T.CartanDecomposition** : CartanQ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.CartanExists

theorem **cartan_exists** : ∀ (R K : Type) [CommRing R] [IsDomain R] [IsDiscreteValuationRing R] [Field K] [Algebra R K] [IsFractionRing R K] (n : ℕ) (ϖ : R), Irreducible ϖ → ∀ g : Matrix (Fin n) (Fin n) K, IsUnit g.det → ∃ (u v : Matrix (Fin n) (Fin n) R) (a : Fin n → ℤ), IsUnit u.det ∧ IsUnit v.det ∧ Antitone a ∧ g = u.map (algebraMap R K) * Matrix.diagonal (fun i => (algebraMap R K ϖ) ^ (a i)) * v.map (algebraMap R K)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.CasApplyOfScalChar

theorem **etr_two_eq_cas** {A : Type*} [Ring A] {n : ℕ} (E : Fin n → Fin n → A) : etr E 2 = Cas E

theorem **cas_apply_of_scalChar** {n : ℕ} (V : Type) [AddCommGroup V] [Module ℂ V] (E : Fin n → Fin n → Module.End ℂ V) (hE : GlRel E) (χ : Fin n → ℂ) (hS : CompanionTheory.RgExh.ScalChar (V, E, hE) χ) (u : V) : Cas E u = (∑ a, χ a * χ a - ∑ a, ((Formalism.QuantumDeterminants.rhoStd n a : ℝ) : ℂ) * ((Formalism.QuantumDeterminants.rhoStd n a : ℝ) : ℂ)) • u

theorem **re_sum_mul_self** {n : ℕ} (χ : Fin n → ℂ) : (∑ a, χ a * χ a).re = ∑ a, ((χ a).re ^ 2 - (χ a).im ^ 2)

theorem **unitIneq** : CompanionTheory.RgExh.U.UnitIneq

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.CasimirDiagonalSum

theorem **qdet_smooth** (hC : PI.DSInfCharCap.DSInfCharCap) (k r : ℕ) (ε : Fin (2 * r + k + 1) → Bool) (mu : Fin (2 * r + k + 1) → ℝ) (R : Rep k r) (hR : R.IsDS ε mu) (Q : Mx k r) (E : Fin (2 * r + k + 1) → Fin (2 * r + k + 1) → Module.End ℂ R.smSub) (hQ : IsUnit Q) (hE : R.IsE Q E) (Y : ℂ) (w : R.smSub) : Formalism.REAL.qdet E (Y + ((((2 * r + k + 1) : ℕ) : ℂ) − 1) / 2) w = (∏ s, (Y + (mu s : ℂ))) • w

theorem **dC_coe** (U : URep k r) (Y : Mx k r) (v : U.R.smSub) : (U.dC Y v : U.R.H) = U.R.dC Y v

theorem **isE_one** (U : URep k r) : U.R.IsE 1 (fun a b => U.dC (Matrix.single a b 1))

theorem **cas_smooth** (hC : PI.DSInfCharCap.DSInfCharCap) {ε : Fin (2 * r + k + 1) → Bool} {μ : Fin (2 * r + k + 1) → ℝ} (U : URep k r) (hk : 1 ≤ k) (hU : U.R.IsDS ε μ) (w : U.R.smSub) : (∑ a, ∑ b, U.dC (Matrix.single a b 1) * U.dC (Matrix.single b a 1)) w = (casG μ : ℂ) • w

theorem **triv_dC** (Y : Mx k r) : (triv k r).dC Y = 0

theorem **L_one** (U : URep k r) (Y : Mx k r) (x : M (one U)) (c : Unit) : L (one U) Y x c = U.dC Y (x c)

theorem **casDS** : CasDS

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.CasimirDiagonalSumStatement

theorem **_root_.GlobalTheory.T.CasimirDiagonalSum** : CasDS

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.CasimirSplitting

theorem **hpk_herm** (k r : ℕ) : (Formalism.QuantumDeterminants.Hpk k r)ᴴ = Formalism.QuantumDeterminants.Hpk k r

theorem **hpk_row** (k r : ℕ) (i : Fin (2 * r + k + 1)) : ∃ (j₀ : Fin (2 * r + k + 1)) (s : ℂ), s * s = 1 ∧ (∀ j, Formalism.QuantumDeterminants.Hpk k r i j = if j = j₀ then s else 0) ∧ ∀ j, Formalism.QuantumDeterminants.Hpk k r j₀ j = if j = i then s else 0

theorem **hpk_sq** (k r : ℕ) : Formalism.QuantumDeterminants.Hpk k r * Formalism.QuantumDeterminants.Hpk k r = 1

theorem **flip_facts** {n : ℕ} (Hm Y : Matrix (Fin n) (Fin n) ℂ) (h1 : Hmᴴ = Hm) (h2 : Hm * Hm = 1) : (Hm * Yᴴ * Hm)ᴴ * Hm = Hm * Y ∧ Hm * (Hm * Yᴴ * Hm) = Yᴴ * Hm

theorem **reH_lie** (Y : Mx k r) : InLieHpk k r (reH k r Y)

theorem **imH_lie** (Y : Mx k r) : InLieHpk k r (imH k r Y)

theorem **diff_entry** {F : FRep k r} (hF : F.OK) {X : Mx k r} (hX : InLieHpk k r X) (c d : F.ι) : HasDerivAt (fun t : ℝ => F.ρ (NormedSpace.exp ((t : ℂ) • X)) c d) (F.dR X c d) 0

theorem **dR_tens** {F F' : FRep k r} (hF : F.OK) (hF' : F'.OK) {X : Mx k r} (hX : InLieHpk k r X) : (F.tens F').dR X = F.dR X ⊗ₖ (1 : Matrix F'.ι F'.ι ℂ) + (1 : Matrix F.ι F.ι ℂ) ⊗ₖ F'.dR X

theorem **dC_tens** {F F' : FRep k r} (hF : F.OK) (hF' : F'.OK) (Y : Mx k r) : (F.tens F').dC Y = F.dC Y ⊗ₖ (1 : Matrix F'.ι F'.ι ℂ) + (1 : Matrix F.ι F.ι ℂ) ⊗ₖ F'.dC Y

theorem **mAct_apply** (A : Matrix κ ι ℂ) (x : ι → V) (c : κ) : mAct A x c = ∑ d, A c d • x d

theorem **mAct_add** (A B : Matrix ι ι ℂ) : (mAct (A + B) : Module.End ℂ (ι → V)) = mAct A + mAct B

theorem **mAct_sum** {α : Type} (s : Finset α) (A : α → Matrix ι ι ℂ) : (mAct (∑ a ∈ s, A a) : Module.End ℂ (ι → V)) = ∑ a ∈ s, mAct (A a)

theorem **mAct_mul** (A B : Matrix ι ι ℂ) : (mAct A * mAct B : Module.End ℂ (ι → V)) = mAct (A * B)

theorem **dOp_mul** (S T : Module.End ℂ V) : (dOp ι (S * T) : Module.End ℂ (ι → V)) = dOp ι S * dOp ι T

theorem **dOp_sum** {α : Type} (s : Finset α) (T : α → Module.End ℂ V) : (dOp ι (∑ a ∈ s, T a) : Module.End ℂ (ι → V)) = ∑ a ∈ s, dOp ι (T a)

theorem **left_factor_action_mul** (f g : Module.End ℂ (ι → V)) : (on1 (f * g) : Module.End ℂ (ι × κ → V)) = on1 f * on1 g

theorem **right_factor_action_mul** (f g : Module.End ℂ (κ → V)) : (on2 (f * g) : Module.End ℂ (ι × κ → V)) = on2 f * on2 g

theorem **left_factor_action_add** (f g : Module.End ℂ (ι → V)) : (on1 (f + g) : Module.End ℂ (ι × κ → V)) = on1 f + on1 g

theorem **right_factor_action_add** (f g : Module.End ℂ (κ → V)) : (on2 (f + g) : Module.End ℂ (ι × κ → V)) = on2 f + on2 g

theorem **left_factor_action_sum** {α : Type} (s : Finset α) (T : α → Module.End ℂ (ι → V)) : (on1 (∑ a ∈ s, T a) : Module.End ℂ (ι × κ → V)) = ∑ a ∈ s, on1 (T a)

theorem **right_factor_action_sum** {α : Type} (s : Finset α) (T : α → Module.End ℂ (κ → V)) : (on2 (∑ a ∈ s, T a) : Module.End ℂ (ι × κ → V)) = ∑ a ∈ s, on2 (T a)

theorem **left_factor_action_diag_op** (T : Module.End ℂ V) : (on1 (dOp ι T) : Module.End ℂ (ι × κ → V)) = dOp (ι × κ) T

theorem **right_factor_action_diag_op** (T : Module.End ℂ V) : (on2 (dOp κ T) : Module.End ℂ (ι × κ → V)) = dOp (ι × κ) T

theorem **left_factor_action_kronecker** (A : Matrix ι ι ℂ) : (on1 (mAct A) : Module.End ℂ (ι × κ → V)) = mAct (A ⊗ₖ (1 : Matrix κ κ ℂ))

theorem **right_factor_action_kronecker** (A : Matrix κ κ ℂ) : (on2 (mAct A) : Module.End ℂ (ι × κ → V)) = mAct ((1 : Matrix ι ι ℂ) ⊗ₖ A)

theorem **expand** {R α : Type} [Ring R] [Fintype α] (E A B : α → α → R) (Q : R) (h1 : ∑ a, ∑ b, A a b * B b a = Q) (h2 : ∑ a, ∑ b, B a b * A b a = Q) : ∑ a, ∑ b, (E a b + (A a b + B a b)) * (E b a + (A b a + B b a)) = ∑ a, ∑ b, (E a b + A a b) * (E b a + A b a) + ∑ a, ∑ b, (E a b + B a b) * (E b a + B b a) − ∑ a, ∑ b, E a b * E b a + (Q + Q)

theorem **casSplit** : CasSplit

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.CasimirSplittingStatement

theorem **_root_.GlobalTheory.T.CasimirSplitting** : CasSplit

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.CayleyHamiltonStatement

theorem **eval_mul_of_commute** (p q : A[X]) (t : A) (h : ∀ k, Commute (q.coeff k) t) : (p * q).eval t = p.eval t * q.eval t

theorem **eval_pow_of_commute** (q : A[X]) (t : A) (h : ∀ k, Commute (q.coeff k) t) (k : ℕ) : (q ^ k).eval t = q.eval t ^ k

theorem **commute_alg** (a : A) (y : ℂ) : Commute a (algebraMap ℂ A y)

theorem **eval_mul_alg** (p q : A[X]) (y : ℂ) : (p * q).eval (algebraMap ℂ A y) = p.eval (algebraMap ℂ A y) * q.eval (algebraMap ℂ A y)

theorem **commute_coeff_X_add_C** (a t : A) (h : Commute a t) (k : ℕ) : Commute ((X + C a : A[X]).coeff k) t

theorem **eq_zero_of_eval** (R : A[X]) (h : ∀ y : ℂ, R.eval (algebraMap ℂ A y) = 0) : R = 0

theorem **polF_const** (a : A) : PolF fun _ : ℂ => a

theorem **polF_alg** (c : ℂ) : PolF fun y : ℂ => algebraMap ℂ A (y + c)

theorem **PolF.add** {f g : ℂ → A} (hf : PolF f) (hg : PolF g) : PolF fun y => f y + g y

theorem **PolF.mul** {f g : ℂ → A} (hf : PolF f) (hg : PolF g) : PolF fun y => f y * g y

theorem **polF_sum** {ι : Type*} (s : Finset ι) (f : ι → ℂ → A) (h : ∀ i, PolF (f i)) : PolF fun y => ∑ i ∈ s, f i y

theorem **polF_ofFn** {m : ℕ} (f : Fin m → ℂ → A) (h : ∀ i, PolF (f i)) : PolF fun y => (List.ofFn fun i => f i y).prod

theorem **polF_Tm** {n : ℕ} (E : Fin n → Fin n → A) (c : ℂ) (a b : Fin n) : PolF fun y : ℂ => Tm E (y + c) a b

theorem **polF_qdet** {n : ℕ} (E : Fin n → Fin n → A) (c : ℂ) : PolF fun y : ℂ => qdet E (y + c)

theorem **eval_map_hom** {B : Type*} [Ring B] (f : A →+* B) (p : A[X]) (x : A) : (p.map f).eval (f x) = f (p.eval x)

theorem **prod_smul_ofFn** {m : ℕ} (s : ℂ) (f : Fin m → A) : (List.ofFn fun i => s • f i).prod = s ^ m • (List.ofFn f).prod

theorem **commute_qdet_om** (M : GMod n) (u : ℂ) : Commute ((qdet M.E u).compLeft (Fin n)) (om M)

theorem **commute_coeff_om** (M : GMod n) (p : (Module.End ℂ (Fin n → M.V))[X]) (h : ∀ y : ℂ, Commute (p.eval (algebraMap ℂ _ y)) (om M)) (k : ℕ) : Commute (p.coeff k) (om M)

theorem **root** (H : QdetTens) (M : GMod n) (c : ℂ) (pC : (Module.End ℂ (Fin n → M.V))[X]) (hC : ∀ y : ℂ, pC.eval (algebraMap ℂ _ y) = (qdet M.E (y + c)).compLeft (Fin n)) : pC.eval (−om M − algebraMap ℂ _ c) = 0

theorem **neg_sub_algebra_map_smul_identity** {A : Type*} [Ring A] [Algebra ℂ A] (w : A) (x c : ℂ) : −w − algebraMap ℂ A c + algebraMap ℂ A x = (−1 : ℂ) • (w − (x − c) • (1 : A))

theorem **cayleyHam_of** (H : QdetTens) : CayleyHam

theorem **cayleyHam** : CayleyHam

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.CayleyHamiltonStatementStatement

theorem **_root_.GlobalTheory.T.CayleyHamiltonHk** : CompanionTheory.RgExh.Hk.CayleyHam

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.CechAcyclicityOverAffineBase

theorem **cech_fibre_acyclic** : ∀ {k : Type} [Field k] {X T Z : Scheme.{0}} (a : X ⟶ Sp k) (b : T ⟶ Sp k) (p : Z ⟶ X) (q : Z ⟶ T) (H : IsPullback p q a b) [IsSeparated a] [Flat a] (G : LB Z) (hG : G.IsLB) (R : Type) [CommRing R] (t : Spec (CommRingCat.of R) ⟶ T) (Y : Scheme.{0}) (π : Y ⟶ Z) (f : Y ⟶ Spec (CommRingCat.of R)) (Hy : IsPullback π f q t) {κ : Type} [LinearOrder κ] [Fintype κ] (D : κ → Y.Opens) (hD : ∀ i, IsAffineOpen (D i)) (hc : ⊔ i, D i = ⊤) (hac : (cechLB f (G.pull π) D).Acyclic) (s : Sp k ⟶ Spec (CommRingCat.of R)) (σ : X ⟶ Z) (hs : s ≫ t ≫ b = 𝟙 _) (h₁ : σ ≫ p = 𝟙 X) (h₂ : σ ≫ q = a ≫ s ≫ t), ∃ D' : κ → X.Opens, (∀ i, IsAffineOpen (D' i)) ∧ ⊔ i, D' i = ⊤ ∧ (cechLB a (G.pull σ) D').Acyclic

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.CechAcyclicityOverAffineBaseStatement

theorem **_root_.GlobalTheory.T.exists_affine_cover_cechLB_fibre_acyclic** : ∀ {k : Type} [Field k] {X T Z : Scheme.{0}} (a : X ⟶ Sp k) (b : T ⟶ Sp k) (p : Z ⟶ X) (q : Z ⟶ T) (H : IsPullback p q a b) [IsSeparated a] [Flat a] (G : LB Z) (hG : G.IsLB) (R : Type) [CommRing R] (t : Spec (CommRingCat.of R) ⟶ T) (Y : Scheme.{0}) (π : Y ⟶ Z) (f : Y ⟶ Spec (CommRingCat.of R)) (Hy : IsPullback π f q t) {κ : Type} [LinearOrder κ] [Fintype κ] (D : κ → Y.Opens) (hD : ∀ i, IsAffineOpen (D i)) (hc : ⊔ i, D i = ⊤) (hac : (cechLB f (G.pull π) D).Acyclic) (s : Sp k ⟶ Spec (CommRingCat.of R)) (σ : X ⟶ Z) (hs : s ≫ t ≫ b = 𝟙 _) (h₁ : σ ≫ p = 𝟙 X) (h₂ : σ ≫ q = a ≫ s ≫ t), ∃ D' : κ → X.Opens, (∀ i, IsAffineOpen (D' i)) ∧ ⊔ i, D' i = ⊤ ∧ (cechLB a (G.pull σ) D').Acyclic

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.CechComplexTrivialLineBundleNotAcyclic

theorem **not_acyclic_cechLB_of_triv** {R : Type} [CommRing R] {X : Scheme.{0}} [Nonempty X] (f : X ⟶ Spec (CommRingCat.of R)) (N : LB X) (hN : N.IsLB) (hT : Triv N) (κ : Type) [LinearOrder κ] [Fintype κ] (D : κ → X.Opens) (hc : ⊔ i, D i = ⊤) : ¬ (cechLB f N D).Acyclic

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.CechComplexTrivialLineBundleNotAcyclicStatement

theorem **_root_.GlobalTheory.T.cech_complex_trivial_line_bundle_not_acyclic** : ∀ {R : Type} [CommRing R] {X : Scheme.{0}} [Nonempty X] (f : X ⟶ Spec (CommRingCat.of R)) (N : LB X) (hN : N.IsLB) (hT : Triv N) (κ : Type) [LinearOrder κ] [Fintype κ] (D : κ → X.Opens) (hc : ⊔ i, D i = ⊤), ¬ (cechLB f N D).Acyclic

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.CechFlatComplex

theorem **isRegular_fibre** (k E : Type) [Field k] [CommRing E] [Algebra k E] (g : ℕ) [Algebra.IsStandardSmoothOfRelativeDimension g k E] (f : Fin g → E) (hA : IsArtinianRing (E ⧸ Ideal.span (Set.range f))) (Q : Ideal E) [Q.IsPrime] (hQ : Ideal.span (Set.range f) ≤ Q) : IsRegular (Localization.AtPrime Q) (List.ofFn fun i => algebraMap E (Localization.AtPrime Q) (f i))

theorem **residueField_exists** (R : Type) [CommRing R] (p : Ideal R) [p.IsPrime] (x : p.ResidueField) : ∃ a : R, ∃ b : R, b ∉ p ∧ b • x = algebraMap R p.ResidueField a

theorem **fibre_exists** (R C : Type) [CommRing R] [CommRing C] [Algebra R C] (p : Ideal R) [p.IsPrime] (e : p.Fiber C) : ∃ b : R, b ∉ p ∧ ∃ c : C, b • e = (1 : p.ResidueField) ⊗ₜ[R] c

theorem **exists_localization_fibre** (R C : Type) [CommRing R] [CommRing C] [Algebra R C] (q : Ideal C) [q.IsPrime] : ∃ (Q : Ideal ((q.under R).Fiber C)) (_ : Q.IsPrime) (e : Localization.AtPrime Q ≃+* (Localization.AtPrime q ⧸ (q.under R).map (algebraMap R (Localization.AtPrime q)))), (∀ c ∈ q, (1 : (q.under R).ResidueField) ⊗ₜ[R] c ∈ Q) ∧ ∀ c : C, e (algebraMap _ _ ((1 : (q.under R).ResidueField) ⊗ₜ[R] c)) = Ideal.Quotient.mk _ (algebraMap C (Localization.AtPrime q) c)

theorem **isArtinianRing_fibre_quotient** (R C : Type) [CommRing R] [CommRing C] [Algebra R C] (p : Ideal R) [p.IsPrime] {g : ℕ} (f : Fin g → C) [Algebra.QuasiFinite R (C ⧸ Ideal.span (Set.range f))] : IsArtinianRing (p.Fiber C ⧸ Ideal.span (Set.range fun i => (1 : p.ResidueField) ⊗ₜ[R] f i))

theorem **flat_localization** (R C : Type) [CommRing R] [IsNoetherianRing R] [CommRing C] [Algebra R C] (g : ℕ) [Algebra.IsStandardSmoothOfRelativeDimension g R C] (f : Fin g → C) [Algebra.QuasiFinite R (C ⧸ Ideal.span (Set.range f))] (q : Ideal C) [q.IsPrime] : Module.Flat R (Localization.AtPrime q ⧸ (Ideal.span (Set.range f)).map (algebraMap C (Localization.AtPrime q)))

theorem **ciFlat** : CompanionTheory.ObstructionTrivialLocus.CIFlat

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.CechFlatComplexStatement

theorem **_root_.GlobalTheory.T.CechFlatComplex** : CIFlat

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.CechSliceAcyclicOfTruncation

theorem **isZero_homology_supp_singleton** {ν κ : Type} [LinearOrder ν] [LinearOrder κ] {R : Type} [CommRing R] (A : GlobalTheory.Cube.{0} (ν ⊕ₗ κ) R) {Q : Set (Finset ν)} (hQ : Q.OrdConnected) (hT₁ : Set.OrdConnected {t : Finset κ | t.card = 1}) (j : κ) (n : ℕ) (h : Limits.IsZero ((GlobalTheory.cubeComplex (supp A (partsIn (inlEmb ν κ) (inrEmb ν κ) Q {t : Finset κ | t.card = 1}) (ordConnected_partsIn _ _ hQ hT₁))).homology n)) : Limits.IsZero ((GlobalTheory.cubeComplex (supp A (partsIn (inlEmb ν κ) (inrEmb ν κ) Q {{j}})) (ordConnected_partsIn _ _ hQ Set.ordConnected_singleton))).homology n)

theorem **cechComplex_sliceFst_singleton_acyclic_of_trunc₂** : ∀ {ν κ : Type} [LinearOrder ν] [LinearOrder κ] [Finite ν] [Finite κ] {R : Type} [CommRing R] (A : GlobalTheory.Cube.{0} (ν ⊕ₗ κ) R) (hA : ∀ t : Finset κ, 2 ≤ t.card → (GlobalTheory.cechComplex (GlobalTheory.Cube.sliceFst A t)).Acyclic) (hT : ∀ n, Limits.IsZero ((GlobalTheory.cubeComplex (GlobalTheory.Cube.trunc₂ A)).homology (n + 2))) (j : κ), (GlobalTheory.cechComplex (GlobalTheory.Cube.sliceFst A {j})).Acyclic

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.CechSliceAcyclicOfTruncationStatement

theorem **_root_.GlobalTheory.T.cech_slice_acyclic_of_truncation** : ∀ {ν κ : Type} [LinearOrder ν] [LinearOrder κ] [Finite ν] [Finite κ] {R : Type} [CommRing R] (A : GlobalTheory.Cube.{0} (ν ⊕ₗ κ) R) (hA : ∀ t : Finset κ, 2 ≤ t.card → (GlobalTheory.cechComplex (GlobalTheory.Cube.sliceFst A t)).Acyclic) (hT : ∀ n, Limits.IsZero ((GlobalTheory.cubeComplex (GlobalTheory.Cube.trunc₂ A)).homology (n + 2))) (j : κ), (GlobalTheory.cechComplex (GlobalTheory.Cube.sliceFst A {j})).Acyclic

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.CenterActionCommutesLeviDifference

theorem **ev_fa** {n : ℕ} {V : Type} [AddCommGroup V] [Module ℂ V] (E : Fin n → Fin n → Module.End ℂ V) (Y : ℂ) : ev E (fa n Y) = qdet E (Y + ((n : ℂ) − 1) / 2)

theorem **evL_fb** {n M : ℕ} (lev : Fin n → ℕ) (d : Fin M → ℕ) (h : ∑ i, d i = n) (hlev : ∀ (i : Fin M) (p : Fin (d i)), lev (stdBlocks d h ⟨i, p⟩) = (i : ℕ)) {V : Type} [AddCommGroup V] [Module ℂ V] (E : Fin n → Fin n → Module.End ℂ V) (Y : ℂ) : evL lev E (fb d h Y) = (List.ofFn fun i : Fin M => qdet (fun p q : Fin (d i) => E (stdBlocks d h ⟨i, p⟩) (stdBlocks d h ⟨i, q⟩))) (Y + ((d i : ℂ) − 1) / 2 − (modExp (fun i => Fin (d i)) i : ℂ) / 2)).prod

theorem **span_apply_sum** {n : ℕ} (lev : Fin n → ℕ) (hm : Monotone lev) {V : Type} [AddCommGroup V] [Module ℂ V] (E : Fin n → Fin n → Module.End ℂ V) (r : Fin n → Fin n → Prop) (hr : ∀ p q, r p q → lev p < lev q) {y : Module.End ℂ V} (hy : y ∈ Submodule.span ℂ {y : Module.End ℂ V | ∃ (p q : Fin n) (w : Module.End ℂ V), r p q ∧ y = E p q ∗ w}) (x : V) : ∃ w : Pos n → V, y x = ∑ i : Pos n, if inU lev i then E i.1.1 i.1.2 (w i) else 0

theorem **co_hyp** : ∀ (n M : ℕ) (lev : Fin n → ℕ) (d : Fin M → ℕ) (h : ∑ i, d i = n) (Y : ℂ), Monotone lev → (∀ (i : Fin M) (p : Fin (d i)), lev (stdBlocks d h ⟨i, p⟩) = (i : ℕ)) → ∀ (V : Type) [AddCommGroup V] [Module ℂ V] (E : Fin n → Fin n → Module.End ℂ V), GlRel E → (∀ i : Pos n, inU lev i → Commute (E i.1.1 i.1.2) (ev E (fa n Y))) ∧ ∀ x : V, ∃ w : Pos n → V, ev E (fa n Y) x − evL lev E (fb d h Y) x = ∑ i : Pos n, if inU lev i then E i.1.1 i.1.2 (w i) else 0

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.CenterActionCommutesLeviDifferenceStatement

theorem **_root_.GlobalTheory.T.center_action_commutes_levi_difference** : ∀ (n M : ℕ) (lev : Fin n → ℕ) (d : Fin M → ℕ) (h : ∑ i, d i = n) (Y : ℂ), Monotone lev → (∀ (i : Fin M) (p : Fin (d i)), lev (stdBlocks d h ⟨i, p⟩) = (i : ℕ)) → ∀ (V : Type) [AddCommGroup V] [Module ℂ V] (E : Fin n → Fin n → Module.End ℂ V), GlRel E → (∀ i : Pos n, inU lev i → Commute (E i.1.1 i.1.2) (ev E (fa n Y))) ∧ ∀ x : V, ∃ w : Pos n → V, ev E (fa n Y) x − evL lev E (fb d h Y) x = ∑ i : Pos n, if inU lev i then E i.1.1 i.1.2 (w i) else 0

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.ChartExistence

theorem **smooth_solve** {P F V : Type*} [NormedAddCommGroup P] [NormedSpace ℝ P] [NormedAddCommGroup F] [NormedSpace ℝ F] [NormedAddCommGroup V] [NormedSpace ℝ V] [FiniteDimensional ℝ V] (L : Submodule ℝ V) (M : P → F →L[ℝ] V) (hM : ContDiff ℝ ∞ M) (hL : ∀ t c, M t c ∈ L) (h0 : ∀ y ∈ L, ∃ c, M 0 c = y) : ∃ U : Set P, IsOpen U ∧ (0 : P) ∈ U ∧ ∀ y : P → V, ContDiff ℝ ∞ y → (∀ t, y t ∈ L) → ∃ c : P → F, ContDiffOn ℝ ∞ c U ∧ ∀ t ∈ U, M t (c t) = y t

theorem **smooth_coeffs** {P V : Type*} [NormedAddCommGroup P] [NormedSpace ℝ P] [NormedAddCommGroup V] [NormedSpace ℝ V] [FiniteDimensional ℝ V] {d e : ℕ} (L : Submodule ℝ V) (Z : Fin d → P → V) (Q : Fin e → V) (h3 : ∀ j, ContDiff ℝ ∞ (Z j)) (h1 : ∀ j t, Z j t ∈ L) (hQ : ∀ i, Q i ∈ L) (h0 : ∀ y ∈ L, ∃ (a : Fin d → ℝ) (b : Fin e → ℝ), y = ∑ j, a j • Z j 0 + ∑ i, b i • Q i) : ∃ U : Set P, IsOpen U ∧ (0 : P) ∈ U ∧ ∀ y : P → V, ContDiff ℝ ∞ y → (∀ t, y t ∈ L) → ∃ (a : Fin d → P → ℝ) (b : Fin e → P → ℝ), (∀ j, ContDiffOn ℝ ∞ (a j) U) ∧ (∀ i, ContDiffOn ℝ ∞ (b i) U) ∧ ∀ t ∈ U, y t = ∑ j, a j t • Z j t + ∑ i, b i t • Q i

theorem **contDiff_ex** (X : Mat n) : ContDiff ℝ ∞ (ex X)

theorem **contDiff_conjT** : ContDiff ℝ ∞ (fun A : Mat n => Aᴴ)

theorem **left_inv_of_mem** {Hm g : Mat n} (h : Hm ∗ Hm = 1) (hg : g ∈ UH Hm) : Hm ∗ gᴴ ∗ Hm ∗ g = 1

theorem **inv_of_mem** {Hm g : Mat n} (h : Hm ∗ Hm = 1) (hg : g ∈ UH Hm) : g⁻¹ = Hm ∗ gᴴ ∗ Hm

theorem **isUnit_of_mem** {Hm g : Mat n} (h : Hm ∗ Hm = 1) (hg : g ∈ UH Hm) : IsUnit g

theorem **lie_conj** {Hm g Y : Mat n} (h : Hm ∗ Hm = 1) (hg : g ∈ UH Hm) (hY : Lie Hm Y) : Lie Hm (g⁻¹ ∗ Y ∗ g)

theorem **ex_conj** {g : Mat n} (hg : IsUnit g) (Y : Mat n) (τ : ℝ) : ex (g⁻¹ ∗ Y ∗ g) τ = g⁻¹ ∗ ex Y τ ∗ g

theorem **contDiff_conj** {P : Type*} [NormedAddCommGroup P] [NormedSpace ℝ P] {Hm : Mat n} (h : Hm * Hm = 1) {s : P → Mat n} (hs : ContDiff ℝ ∞ s) (hm : ∀ t, s t ∈ UH Hm) (Y : Mat n) : ContDiff ℝ ∞ (fun t => (s t)⁻¹ * Y * s t)

theorem **prodEx_succ** {d : ℕ} (X : Fin (d + 1) → Mat n) (t : Fin (d + 1) → ℝ) : prodEx X t = ex (X 0) (t 0) * prodEx (fun j => X j.succ) (fun j => t j.succ)

theorem **prodEx_zero** {d : ℕ} (X : Fin d → Mat n) : prodEx X 0 = 1

theorem **prodEx_mem** {Hm : Mat n} : ∀ {d : ℕ} (X : Fin d → Mat n), (∀ j, Lie Hm (X j)) → ∀ t, prodEx X t ∈ UH Hm | 0, X, _, t => by simp [prodEx, UH] | d + 1, X, hX, t => by rw [prodEx_succ] exact CCRuleCW.UH_mul Hm _ _ (ex_mem_of_lie (hX 0) _) (prodEx_mem _ (fun j => hX j.succ) _) theorem contDiff_tail {d : ℕ} : ContDiff ℝ ∞ (fun (t : Fin (d + 1) → ℝ) (j : Fin d) => t j.succ)

theorem **contDiff_prodEx** : ∀ {d : ℕ} (X : Fin d → Mat n), ContDiff ℝ ∞ (prodEx X) | 0, X => by have : prodEx X = fun _ => 1

theorem **exists_fields** {Hm : Mat n} (h : Hm * Hm = 1) : ∀ {d : ℕ} (X : Fin d → Mat n), (∀ j, Lie Hm (X j)) → ∃ Z : Fin d → (Fin d → ℝ) → Mat n, (∀ j t, Lie Hm (Z j t)) ∧ (∀ j t (τ : ℝ), prodEx X (t + τ • (Pi.single j 1 : Fin d → ℝ)) = prodEx X t * ex (Z j t) τ) ∧ (∀ j, ContDiff ℝ ∞ (Z j)) ∧ ∀ j, Z j 0 = X j | 0, X, _ => (fun j => j.elim0, fun j => j.elim0, fun j => j.elim0, fun j => j.elim0) | d + 1, X, hX => by obtain (Z, h1, h2, h3, h4)

theorem **chartExists** : ChartExists

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.ChartExistenceStatement

theorem **_root_.GlobalTheory.T.ChartExistence** : ChartExists

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.CocycleDefinesDescendedLineBundle

theorem **gm_comp_val** {W' W X : Scheme.{0}} (p : W' ⟶ W) (y : W') (f : W ⟶ X) (V : X.Opens) (hy : f (p y) ∈ V) (x : Γ(X, V)ˣ) : (gm (p ≫ f) y V x).val = (p.stalkMap y).hom (gm f (p y) V x).val

theorem **gm_top_val** {W' X : Scheme.{0}} (V : X.Opens) (g : W' ⟶ V.toScheme) (w : W') (τ : Γ(X, V)ˣ) : (gm (g ≫ V.ι) w V τ).val = (W'.presheaf.germ ⊤ w trivial).hom (g.appTop.hom ((GlobalTheory.FunDescent.res V).hom τ.val))

theorem **gm_global_val** {W' X : Scheme.{0}} (k : W' ⟶ X) (w : W') (u : Γ(X, ⊤)ˣ) : (gm k w ⊤ u).val = (W'.presheaf.germ ⊤ w trivial).hom (k.appTop.hom u.val)

theorem **exists_local_unit** (H : H90) {Z Q : Scheme.{0}} (h : Z ⟶ Q) (hf : Flat h) (hl : LocallyOfFinitePresentation h) (hs : Surjective h) (u : Γ(pullback h h, ⊤)) (hu : IsUnit u) (hc : ∀ (W : Scheme.{0}) (a b c : W ⟶ Z) (hab : a ≫ h = b ≫ h) (hbc : b ≫ h = c ≫ h), (pullback.lift a c (hab.trans hbc)).appTop u = (pullback.lift a b hab).appTop u * (pullback.lift b c hbc).appTop u) (y : Q) : ∃ U : Q.Opens, y ∈ U ∧ ∃ τ : Γ(Z, h ⁻¹ᵁ U)ˣ, ∀ (W : Scheme.{0}) (a b : W ⟶ Z) (hab : a ≫ h = b ≫ h) (w : W), h (a w) ∈ U → gm a w (h ⁻¹ᵁ U) τ = gm (pullback.lift a b hab) w ⊤ hu.unit * gm b w (h ⁻¹ᵁ U) τ

theorem **exists_lb** : H90 → ∀ {Z Q : Scheme.{0}} (h : Z ⟶ Q), Flat h → LocallyOfFinitePresentation h → Surjective h → ∀ (u : Γ(pullback h h, ⊤)) (hu : IsUnit u), (∀ (W : Scheme.{0}) (a b c : W ⟶ Z) (hab : a ≫ h = b ≫ h) (hbc : b ≫ h = c ≫ h), (pullback.lift a c (hab.trans hbc)).appTop u = (pullback.lift a b hab).appTop u * (pullback.lift b c hbc).appTop u) → ∃ N : LB Q, N.IsLB ∧ ∃ T : LB.Gen (N.pull h), T.IsGen ∧ ∀ (W : Scheme.{0}) (a b : W ⟶ Z) (hab : a ≫ h = b ≫ h) (w : W) (i : N.ι), h (a w) ∈ N.U i → gm a w ((N.pull h).U fun _ => i) (T.s fun _ => i) = gm (pullback.lift a b hab) w ⊤ hu.unit * gm b w ((N.pull h).U fun _ => i) (T.s fun _ => i)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.CocycleDefinesDescendedLineBundleStatement

theorem **_root_.GlobalTheory.T.cocycle_defines_descended_line_bundle** : H90 → ∀ {Z Q : Scheme.{0}} (h : Z ⟶ Q), Flat h → LocallyOfFinitePresentation h → Surjective h → ∀ (u : Γ(pullback h h, ⊤)) (hu : IsUnit u), (∀ (W : Scheme.{0}) (a b c : W ⟶ Z) (hab : a ≫ h = b ≫ h) (hbc : b ≫ h = c ≫ h), (pullback.lift a c (hab.trans hbc)).appTop u = (pullback.lift a b hab).appTop u * (pullback.lift b c hbc).appTop u) → ∃ N : LB Q, N.IsLB ∧ ∃ T : LB.Gen (N.pull h), T.IsGen ∧ ∀ (W : Scheme.{0}) (a b : W ⟶ Z) (hab : a ≫ h = b ≫ h) (w : W) (i : N.ι), h (a w) ∈ N.U i → gm a w ((N.pull h).U fun _ => i) (T.s fun _ => i) = gm (pullback.lift a b hab) w ⊤ hu.unit * gm b w ((N.pull h).U fun _ => i) (T.s fun _ => i)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.CommuteOfMemAdjoin

theorem **commute_of_mem_adjoin** {V : Type} [AddCommGroup V] [Module ℂ V] {l : ℕ} (a : Fin (l + 1) → Module.End ℂ V) (hc : ∀ i j, Commute (a i) (a j)) (i : Fin (l + 1)) {y : Module.End ℂ V} (hy : y ∈ Algebra.adjoin ℂ (Set.range a)) : Commute y (a i)

theorem **adjoin_le_sup** {V : Type} [AddCommGroup V] [Module ℂ V] {l : ℕ} (a : Fin (l + 1) → Module.End ℂ V) (hc : ∀ i j, Commute (a i) (a j)) : Subalgebra.toSubmodule (Algebra.adjoin ℂ (Set.range a)) ≤ Subalgebra.toSubmodule (Algebra.adjoin ℂ (Set.range fun i : Fin l => a i.succ)) ⊔ (Subalgebra.toSubmodule (Algebra.adjoin ℂ (Set.range a))).map (LinearMap.mulRight ℂ (a 0))

theorem **finrank_map_pow_le** {V : Type} [AddCommGroup V] [Module ℂ V] [FiniteDimensional ℂ V] {l : ℕ} (a : Fin (l + 1) → Module.End ℂ V) (hc : ∀ i j, Commute (a i) (a j)) (k : ℕ) : finrank ℂ ((Subalgebra.toSubmodule (Algebra.adjoin ℂ (Set.range a))).map (LinearMap.mulRight ℂ (a 0 ^ k))) ≤ finrank ℂ (Algebra.adjoin ℂ (Set.range fun i : Fin l => a i.succ)) + finrank ℂ ((Subalgebra.toSubmodule (Algebra.adjoin ℂ (Set.range a))).map (LinearMap.mulRight ℂ (a 0 ^ (k + 1))))

theorem **comm_step** : ∀ (V : Type) [AddCommGroup V] [Module ℂ V] [FiniteDimensional ℂ V] (l : ℕ) (a : Fin (l + 1) → Module.End ℂ V), (∀ i j, Commute (a i) (a j)) → ∀ x : ℕ, Module.finrank ℂ (Algebra.adjoin ℂ (Set.range a)) ≤ x * Module.finrank ℂ (Algebra.adjoin ℂ (Set.range fun i : Fin l => a i.succ)) + Module.finrank ℂ ((Subalgebra.toSubmodule (Algebra.adjoin ℂ (Set.range a))).map (LinearMap.mulRight ℂ (a 0 ^ x)))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.CommuteOfUnitaryConjugate

theorem **sum_pick** {R : Type} [AddCommGroup R] [Module ℂ R] (f : Fin n → Fin n → R) (a b : Fin n) (P : Prop) [Decidable P] (hP : P → ε a = true ∧ ε b = false) (Q : NC ε → Prop) [DecidablePred Q] (hQ : ∀ k, Q k ↔ P ∧ k.1.1 = a ∧ k.1.2 = b) : ∑ k : NC ε, (if Q k then (1 : ℂ) else 0) • f k.1.1 k.1.2 = if P then f a b else 0

theorem **sq_zero** (L A : ι → R) (hLL : ∀ i j, Commute (L i) (L j)) (hLA : ∀ i j, Commute (L i) (A j)) (hAA : ∀ i j, A i * A j + A j * A i = 0) : (∑ i, L i * A i) * (∑ i, L i * A i) = 0

theorem **commute_of_unitary_conjugate** (T G : R) (L A : ι → R) (hTA : ∀ k, Commute T (A k)) (hGL : ∀ k, Commute G (L k)) (h : ∑ k, (T * L k - L k * T) * A k + ∑ k, L k * (G * A k - A k * G) = 0) : Commute (T + G) (∑ k, L k * A k)

theorem **comm_ann** (A C : ι → R) (q : ι → ι → ℂ) (hAA : ∀ i j, A i * A j + A j * A i = 0) (hAC : ∀ i j, A i * C j + C j * A i = if i = j then 1 else 0) (k : ι) : (∑ i, ∑ j, q i j • (C i * A j)) * A k - A k * (∑ i, ∑ j, q i j • (C i * A j)) = -∑ j, q k j • A j

theorem **comm_cre** (A C : ι → R) (q : ι → ι → ℂ) (hCC : ∀ i j, C i * C j + C j * C i = 0) (hAC : ∀ i j, A i * C j + C j * A i = if i = j then 1 else 0) (k : ι) : (∑ i, ∑ j, q i j • (C i * A j)) * C k - C k * (∑ i, ∑ j, q i j • (C i * A j)) = ∑ i, q i k • C i

theorem **lift_ann** (T : Module.End ℂ V) (i : NC ε) : Commute (lift ε T : Module.End ℂ (Ch ε V)) (ann i)

theorem **lift_cre** (T : Module.End ℂ V) (i : NC ε) : Commute (lift ε T : Module.End ℂ (Ch ε V)) (cre i)

theorem **lift_Gop** (T : Module.End ℂ V) (c d : Fin n) : Commute (Gop ε c d) (lift ε T : Module.End ℂ (Ch ε V))

theorem **glRel_lift** (E : Fin n → Fin n → Module.End ℂ V) (hE : GlRel E) : GlRel (fun a b => (lift ε (E a b) : Module.End ℂ (Ch ε V)))

theorem **bdFacts** : CAR → BdFacts

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.CommutingLocallyNilpotentCommonKernel

theorem **exists_joint_kernel_finset** {V : Type} [AddCommGroup V] [Module ℂ V] (I : Submodule ℂ V) (hI : I ≠ ⊥) (D : ℂ → Module.End ℂ V) (hcomm : ∀ Y Y', Commute (D Y) (D Y')) (hstab : ∀ Y, ∀ x ∈ I, D Y x ∈ I) (hnil : ∀ Y, ∀ x ∈ I, ∃ N : ℕ, (D Y ^ N) x = 0) (F : Finset ℂ) : ∃ w ∈ I, w ≠ 0 ∧ ∀ Y ∈ F, D Y w = 0

theorem **exists_joint_kernel_polynomial_family** : ∀ {V : Type} [AddCommGroup V] [Module ℂ V] (I : Submodule ℂ V), I ≠ ⊥ → ∀ (D : ℂ → Module.End ℂ V), (∀ Y Y', Commute (D Y) (D Y')) → (∀ Y, ∀ x ∈ I, D Y x ∈ I) → (∀ Y, ∀ x ∈ I, ∃ N : ℕ, (D Y ^ N) x = 0) → ∀ (l : ℕ) (c : ℕ → Module.End ℂ V), (∀ Y, D Y = ∑ j ∈ Finset.range l, Y ^ j • c j) → ∃ w ∈ I, w ≠ 0 ∧ ∀ Y, D Y w = 0

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.CommutingLocallyNilpotentCommonKernelStatement

theorem **_root_.GlobalTheory.T.commuting_locally_nilpotent_common_kernel** : ∀ {V : Type} [AddCommGroup V] [Module ℂ V] (I : Submodule ℂ V), I ≠ ⊥ → ∀ (D : ℂ → Module.End ℂ V), (∀ Y Y', Commute (D Y) (D Y')) → (∀ Y, ∀ x ∈ I, D Y x ∈ I) → (∀ Y, ∀ x ∈ I, ∃ N : ℕ, (D Y ^ N) x = 0) → ∀ (l : ℕ) (c : ℕ → Module.End ℂ V), (∀ Y, D Y = ∑ j ∈ Finset.range l, Y ^ j • c j) → ∃ w ∈ I, w ≠ 0 ∧ ∀ Y, D Y w = 0

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.CompactFiniteAdelesAwayPBoundedDenominators

theorem **kHighest_iff** {n : ℕ} (ε : Fin n → Bool) {V W : Type} [AddCommGroup V] [Module ℂ V] [AddCommGroup W] [Module ℂ W] (ι : V →ₗ[ℂ] W) (hι : Function.Injective ι) (E' : Fin n → Fin n → Module.End ℂ V) (E : Fin n → Fin n → Module.End ℂ W) (h : ∀ a b v, ι (E' a b v) = E a b (ι v)) (μ : Fin n → ℝ) (v : V) : KHighest ε E' μ v ↔ KHighest ε E μ (ι v)

theorem **glRel_of_inj** {n : ℕ} {V W : Type} [AddCommGroup V] [Module ℂ V] [AddCommGroup W] [Module ℂ W] (ι : V →ₗ[ℂ] W) (hι : Function.Injective ι) (E' : Fin n → Fin n → Module.End ℂ V) (E : Fin n → Fin n → Module.End ℂ W) (h : ∀ a b v, ι (E' a b v) = E a b (ι v)) (hE : GlRel E) : GlRel E'

theorem **kModExists** (h1 : SmMod) (h2 : KFinSm) (h3 : KFinStab) (h4 : StabLink) (h5 : HWofType) (h6 : TypeOfHW) : CompanionTheory.D.LK.KModExists

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.CompactSubgroupFiniteOpenCosets

theorem **exists_finite_coset_cover** : ∀ (G : Type) [Group G] [TopologicalSpace G] [IsTopologicalGroup G] (K₀ K : Subgroup G), IsCompact (K₀ : Set G) → IsOpen (K : Set G) → ∃ (d : ℕ) (c : Fin d → G), (∀ i, c i ∈ K₀) ∧ ∀ g ∈ K₀, ∃ i, ∃ κ ∈ K, g = κ * c i

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.CompactSubgroupFiniteOpenCosetsStatement

theorem **_root_.GlobalTheory.T.compact_subgroup_finite_open_cosets** : ∀ (G : Type) [Group G] [TopologicalSpace G] [IsTopologicalGroup G] (K₀ K : Subgroup G), IsCompact (K₀ : Set G) → IsOpen (K : Set G) → ∃ (d : ℕ) (c : Fin d → G), (∀ i, c i ∈ K₀) ∧ ∀ g ∈ K₀, ∃ i, ∃ κ ∈ K, g = κ * c i

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.CongruenceEndomorphismInverseExists

theorem **norm_one_add_of_lt** {t : L} (ht : ‖t‖ < 1) : ‖1 + t‖ = 1

theorem **exists_coord** {m : ℕ} (b : Module.Basis (Fin m) L V) {Λ : AddSubgroup V} (hb : ∀ x, x ∈ Λ ↔ ∀ i, ‖b.repr x i‖ ≤ 1) {π : L} (hπ : ‖π‖ < 1) {g : Module.End L V} (hg : g ∈ congJ Λ π) {x : V} (hx : x ≠ 0) : ∃ i, (∀ j, ‖b.repr x j‖ ≤ ‖b.repr x i‖) ∧ b.repr x i ≠ 0 ∧ ‖b.repr ((1 + g) x) i‖ = ‖b.repr x i‖

theorem **eq_zero_of_apply** {m : ℕ} (b : Module.Basis (Fin m) L V) {Λ : AddSubgroup V} (hb : ∀ x, x ∈ Λ ↔ ∀ i, ‖b.repr x i‖ ≤ 1) {π : L} (hπ : ‖π‖ < 1) {g : Module.End L V} (hg : g ∈ congJ Λ π) {x : V} (h0 : (1 + g) x = 0) : x = 0

theorem **mem_of_apply_mem** {m : ℕ} (b : Module.Basis (Fin m) L V) {Λ : AddSubgroup V} (hb : ∀ x, x ∈ Λ ↔ ∀ i, ‖b.repr x i‖ ≤ 1) {π : L} (hπ : ‖π‖ < 1) {g : Module.End L V} (hg : g ∈ congJ Λ π) {x : V} (hG : (1 + g) x ∈ Λ) : x ∈ Λ

theorem **main** : ∀ {L V : Type} [NontriviallyNormedField L] [IsUltrametricDist L] [AddCommGroup V] [Module L V] [FiniteDimensional L V] (Λ : AddSubgroup V), IsLat L Λ → ∀ π : L, ‖π‖ < 1 → ∀ g ∈ GlobalTheory.CompanionTheory.Iwahori.congJ Λ π, ∃ g' ∈ GlobalTheory.CompanionTheory.Iwahori.congJ Λ π, (1 + g) * (1 + g') = 1 ∧ (1 + g') * (1 + g) = 1

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.CongruenceEndomorphismInverseExistsStatement

theorem **_root_.GlobalTheory.T.congruence_endomorphism_inverse_exists** : ∀ {L V : Type} [NontriviallyNormedField L] [IsUltrametricDist L] [AddCommGroup V] [Module L V] [FiniteDimensional L V] (Λ : AddSubgroup V), IsLat L Λ → ∀ π : L, ‖π‖ < 1 → ∀ g ∈ GlobalTheory.CompanionTheory.Iwahori.congJ Λ π, ∃ g' ∈ GlobalTheory.CompanionTheory.Iwahori.congJ Λ π, (1 + g) * (1 + g') = 1 ∧ (1 + g') * (1 + g) = 1

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.CongruenceSubgroupIsCompact

theorem **isCompact_entryBall** (L : Type) [NontriviallyNormedField L] [ProperSpace L] (n : ℕ) (r : ℝ) : IsCompact {M : Matrix (Fin n) (Fin n) L | ∀ i j, ‖(M - 1) i j‖ ≤ r}

theorem **isCompact_congK** (L : Type) [NontriviallyNormedField L] [IsUltrametricDist L] [ProperSpace L] (n : ℕ) (r : ℝ) (h0 : 0 ≤ r) (h1 : r ≤ 1) : IsCompact (congK L n r h0 h1 : Set (GL (Fin n) L))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.CongruenceSubgroupIsCompactStatement

theorem **_root_.GlobalTheory.T.congruence_subgroup_is_compact** : ∀ (L : Type) [NontriviallyNormedField L] [IsUltrametricDist L] [ProperSpace L] (n : ℕ) (r : ℝ) (h0 : 0 ≤ r) (h1 : r ≤ 1), IsCompact (GlobalTheory.CompanionTheory.CartGL.congK L n r h0 h1 : Set (GL (Fin n) L))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.CongruenceSubgroupIsOpen

theorem **isOpen_congK** : ∀ (L : Type) [NontriviallyNormedField L] [IsUltrametricDist L] (n : ℕ) (r : ℝ) (h0 : 0 ≤ r) (h1 : r ≤ 1), 0 < r → IsOpen (GlobalTheory.CompanionTheory.CartGL.congK L n r h0 h1 : Set (GL (Fin n) L))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.CongruenceSubgroupIsOpenStatement

theorem **_root_.GlobalTheory.T.congruence_subgroup_is_open** : ∀ (L : Type) [NontriviallyNormedField L] [IsUltrametricDist L] (n : ℕ) (r : ℝ) (h0 : 0 ≤ r) (h1 : r ≤ 1), 0 < r → IsOpen (GlobalTheory.CompanionTheory.CartGL.congK L n r h0 h1 : Set (GL (Fin n) L))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.CongruenceSubgroupNormalInMaximalCompact

theorem **entry_le_one** {A : Matrix (Fin n) (Fin n) L} (hA : ∀ i j, ‖(A - 1) i j‖ ≤ 1) (i j : Fin n) : ‖A i j‖ ≤ 1

theorem **entry_conj** {C D X : Matrix (Fin n) (Fin n) L} {r : ℝ} (h0 : 0 ≤ r) (hC : ∀ i j, ‖C i j‖ ≤ 1) (hD : ∀ i j, ‖D i j‖ ≤ 1) (hCD : C * D = 1) (hX : ∀ i j, ‖(X - 1) i j‖ ≤ r) (i j : Fin n) : ‖(C * X * D - 1) i j‖ ≤ r

theorem **conj_mem_congK** (L : Type) [NontriviallyNormedField L] [IsUltrametricDist L] (n : ℕ) (r : ℝ) (h0 : 0 ≤ r) (h1 : r ≤ 1) (c x : GL (Fin n) L) (hc : c ∈ GlobalTheory.CompanionTheory.CartGL.congK L n 1 zero_le_one le_rfl) (hx : x ∈ GlobalTheory.CompanionTheory.CartGL.congK L n r h0 h1) : c * x * c⁻¹ ∈ GlobalTheory.CompanionTheory.CartGL.congK L n r h0 h1

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.CongruenceSubgroupNormalInMaximalCompactStatement

theorem **_root_.GlobalTheory.T.congruence_subgroup_normal_in_maximal_compact** : ∀ (L : Type) [NontriviallyNormedField L] [IsUltrametricDist L] (n : ℕ) (r : ℝ) (h0 : 0 ≤ r) (h1 : r ≤ 1) (c x : GL (Fin n) L), c ∈ GlobalTheory.CompanionTheory.CartGL.congK L n 1 zero_le_one le_rfl → x ∈ GlobalTheory.CompanionTheory.CartGL.congK L n r h0 h1 → c * x * c⁻¹ ∈ GlobalTheory.CompanionTheory.CartGL.congK L n r h0 h1

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.CongruenceSubgroupsNeighborhoodBasis

theorem **exists_pi_entry_box** {L : Type} [NontriviallyNormedField L] {n : ℕ} {A : Set (Fin n → Fin n → L)} {x : Fin n → Fin n → L} (hA : A ∈ 𝒩 x) : ∃ ε : ℝ, 0 < ε ∧ ∀ y : Fin n → Fin n → L, (∀ i j, ‖y i j - x i j‖ ≤ ε) → y ∈ A

theorem **exists_entry_box** {L : Type} [NontriviallyNormedField L] {n : ℕ} {A : Set (Matrix (Fin n) (Fin n) L)} (hA : A ∈ 𝒩 (1 : Matrix (Fin n) (Fin n) L)) : ∃ ε : ℝ, 0 < ε ∧ ∀ M : Matrix (Fin n) (Fin n) L, (∀ i j, ‖(M - 1) i j‖ ≤ ε) → M ∈ A

theorem **congK_cofinal** (L : Type) [NontriviallyNormedField L] [IsUltrametricDist L] (n : ℕ) (U : Set (GL (Fin n) L)) (hU : IsOpen U) (h1U : (1 : GL (Fin n) L) ∈ U) : ∃ ε : ℝ, 0 < ε ∧ ∀ (r : ℝ) (h0 : 0 ≤ r) (h1 : r ≤ 1), r ≤ ε → (GlobalTheory.CompanionTheory.CartGL.congK L n r h0 h1 : Set (GL (Fin n) L)) ⊆ U

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.CongruenceSubgroupsNeighborhoodBasisStatement

theorem **_root_.GlobalTheory.T.congruence_subgroups_neighborhood_basis** : ∀ (L : Type) [NontriviallyNormedField L] [IsUltrametricDist L] (n : ℕ) (U : Set (GL (Fin n) L)), IsOpen U → (1 : GL (Fin n) L) ∈ U → ∃ ε : ℝ, 0 < ε ∧ ∀ (r : ℝ) (h0 : 0 ≤ r) (h1 : r ≤ 1), r ≤ ε → (GlobalTheory.CompanionTheory.CartGL.congK L n r h0 h1 : Set (GL (Fin n) L)) ⊆ U

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.ConjQdet

theorem **conj_single_apply** {n : ℕ} (U Ui : Matrix (Fin n) (Fin n) ℂ) (p q a b : Fin n) : (U * Matrix.single p q 1 * Ui : Matrix (Fin n) (Fin n) ℂ) a b = U a p * Ui q b

theorem **conjQdet** : CompanionTheory.RgExh.Dec.J.ConjQdet

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.ContinuousDiag

theorem **null_of_lines** {G : Type} [AddCommGroup G] [MeasurableSpace G] [MeasurableAdd₂ G] (μ : Measure G) [μ.IsAddLeftInvariant] [SFinite μ] (φ : ℝ → G) (hφ : Measurable φ) {S : Set G} (hS : MeasurableSet S) (h : ∀ x, (volume : Measure ℝ) {t | φ t + x ∈ S} = 0) : μ S = 0

theorem **q_line** (hσ : Formalism.MainNotions.GlobalTheoryTheory.GenusMassVolumes.IsCMExt F E σ) {M : ℕ} (J : Matrix (Fin M) (Fin M) E) (y0 : Fin M → E) (a : F) (hy0 : Formalism.MainNotions.GlobalTheoryTheory.GenusMassVolumes.hE σ J y0 y0 = algebraMap F E a) (R : Type) [CommRing R] [Nontrivial R] [Algebra F R] (x : Fin M → R ⊗[F] E) : ∃ c : R, ∀ t : R, Formalism.MainNotions.GlobalTheoryTheory.GenusMassVolumes.qR R σ J (x + t • fun i => Formalism.MainNotions.GlobalTheoryTheory.GenusMassVolumes.inclR R (y0 i)) = algebraMap F R a * t ^ 2 + c * t + Formalism.MainNotions.GlobalTheoryTheory.GenusMassVolumes.qR R σ J x

theorem **continuous_diag** : Continuous (diag F)

theorem **coord_diag** (t : ℝ) (w : InfinitePlace F) : coord F (diag F t) w = t

theorem **continuous_coord** (w : InfinitePlace F) : Continuous fun c : Ainf F => coord F c w

theorem **null_nonunit** {G : Type} [AddCommGroup G] [MeasurableSpace G] [MeasurableAdd₂ G] (μ : Measure G) [μ.IsAddLeftInvariant] [SFinite μ] (f : G → Ainf F) (hf : ∀ w, Measurable fun X => coord F (f X) w) (φ : ℝ → G) (hφ : Measurable φ) (h : ∀ x w, {t : ℝ | coord F (f (φ t + x)) w = 0}.Finite) : μ {X | ¬ IsUnit (f X)} = 0

theorem **null_sets** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → 1 ≤ M → ∀ (μ : Measure (Fin 1 → Fin M → Ainf F ⊗[F] E)) (ν : Measure (Ainf F)), μ.IsAddHaarMeasure → ν.IsAddHaarMeasure → ν {c | ¬ IsUnit c} = 0 ∧ μ {X | ¬ IsUnit ((fun X : Fin 1 → Fin M → Ainf F ⊗[F] E => Formalism.MainNotions.GlobalTheoryTheory.GenusMassVolumes.qR (Ainf F) σ J (X 0)) X)} = 0

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.ContinuousUnitsRenSymm

lemma **ren_mul** (r : Equiv.Perm (Fin n)) (g h : Matrix (Fin n) (Fin n) ℂ) : ren r (g * h) = ren r g * ren r h

lemma **ren_one** (r : Equiv.Perm (Fin n)) : ren r (1 : Matrix (Fin n) (Fin n) ℂ) = 1

lemma **ren_ct** (r : Equiv.Perm (Fin n)) (g : Matrix (Fin n) (Fin n) ℂ) : (ren r g)ᴴ = ren r gᴴ

lemma **ren_ren_symm** (r : Equiv.Perm (Fin n)) (g : Matrix (Fin n) (Fin n) ℂ) : ren r (ren r.symm g) = g

lemma **ren_Jm** (ε : Fin n → Bool) (r : Equiv.Perm (Fin n)) : Jm (ε ∘ r) = ren r (Jm ε)

lemma **ren_diag** (r : Equiv.Perm (Fin n)) (d : Fin n → ℂ) : ren r (diagonal d) = diagonal (d ∘ r)

lemma **mem_K** {ε : Fin n → Bool} (r : Equiv.Perm (Fin n)) {g : Matrix (Fin n) (Fin n) ℂ} (hg : g ∈ Kset ε) : ren r g ∈ Kset (ε ∘ r)

lemma **mem_UJ** {ε : Fin n → Bool} (r : Equiv.Perm (Fin n)) {g : Matrix (Fin n) (Fin n) ℂ} (hg : g ∈ UJset ε) : ren r g ∈ UJset (ε ∘ r)

lemma **mem_inG** {ε : Fin n → Bool} (r : Equiv.Perm (Fin n)) {X : Matrix (Fin n) (Fin n) ℂ} (hX : inG ε X) : inG (ε ∘ r) (ren r X)

lemma **continuous_ren** (r : Equiv.Perm (Fin n)) : Continuous (ren r)

lemma **exp_ren** (r : Equiv.Perm (Fin n)) (X : Matrix (Fin n) (Fin n) ℂ) : NormedSpace.exp (ren r X) = ren r (NormedSpace.exp X)

lemma **Ord.symm** {ε ε₁ : Fin n → Bool} {r : Equiv.Perm (Fin n)} (hε : ε₁ = ε ∘ r) (h : Ord ε₁ r) : Ord ε r.symm

lemma **hε_symm** (hε : ε₁ = ε ∘ r) : ε = ε₁ ∘ r.symm

lemma **hτ_symm** (hτ : ∀ g, τ' g = τ (ren r g)) : ∀ g, τ g = τ' (ren r.symm g)

lemma **stab_ren** (hε : ε₁ = ε ∘ r) (hτ : ∀ g, τ' g = τ (ren r g)) {S : Submodule ℂ H} (h : IsKStableOn ε₁ τ S) : IsKStableOn ε τ' S

lemma **kfin_ren** (hε : ε₁ = ε ∘ r) (hτ : ∀ g, τ' g = τ (ren r g)) {v : H} (h : IsKFin ε₁ τ v) : IsKFin ε τ' v

lemma **wt_ren** (hε : ε₁ = ε ∘ r) (hτ : ∀ g, τ' g = τ (ren r g)) {μ : Fin n → ℝ} {v : H} (h : IsWtOn (Kset ε₁) τ μ v) : IsWtOn (Kset ε) τ' (μ ∘ r.symm) v

lemma **root_ren** (r : Equiv.Perm (Fin n)) (a b x : Fin n) : root (r a) (r b) x = root a b (r.symm x)

lemma **cdom_ren** (hε : ε₁ = ε ∘ r) (ho : Ord ε₁ r) {μ Λ : Fin n → ℝ} (h : CDom ε₁ μ Λ) : CDom ε (μ ∘ r.symm) (Λ ∘ r.symm)

lemma **kType_ren** (hε : ε₁ = ε ∘ r) (ho : Ord ε₁ r) (hτ : ∀ g, τ' g = τ (ren r g)) {Λ : Fin n → ℝ} {S : Submodule ℂ H} (h : IsKTypeOf ε₁ τ Λ S) : IsKTypeOf ε τ' (Λ ∘ r.symm) S

lemma **kHighest_fwd** {V : Type} [AddCommGroup V] [Module ℂ V] (hε : ε₁ = ε ∘ r) (ho : Ord ε₁ r) {E : Fin n → Fin n → Module.End ℂ V} {μ : Fin n → ℝ} {x : V} (h : KHighest ε₁ E μ x) : KHighest ε (fun a b => E (r.symm a) (r.symm b)) (μ ∘ r.symm) x

lemma **kHighest_bwd** {V : Type} [AddCommGroup V] [Module ℂ V] (hε : ε₁ = ε ∘ r) (ho : Ord ε₁ r) {E : Fin n → Fin n → Module.End ℂ V} {μ : Fin n → ℝ} {x : V} (h : KHighest ε (fun a b => E (r.symm a) (r.symm b)) μ x) : KHighest ε₁ E (μ ∘ r) x

lemma **kMod_ren** (hε : ε₁ = ε ∘ r) (ho : Ord ε₁ r) (hτ : ∀ g, τ' g = τ (ren r g)) {D : Submodule ℂ H} {E : Fin n → Fin n → Module.End ℂ D} (hK : IsKMod ε₁ τ D E) : IsKMod ε τ' D (fun a b => E (r.symm a) (r.symm b))

lemma **unitsRen_val** (r : Equiv.Perm (Fin n)) (g : (Matrix (Fin n) (Fin n) ℂ)ˣ) : ((unitsRen r g : (Matrix (Fin n) (Fin n) ℂ)ˣ) : Matrix (Fin n) (Fin n) ℂ) = ren r g

lemma **unitsRen_symm_val** (r : Equiv.Perm (Fin n)) (g : (Matrix (Fin n) (Fin n) ℂ)ˣ) : (((unitsRen r).symm g : (Matrix (Fin n) (Fin n) ℂ)ˣ) : Matrix (Fin n) (Fin n) ℂ) = ren r.symm g

lemma **continuous_unitsRen** (r : Equiv.Perm (Fin n)) : Continuous (unitsRen r)

lemma **continuous_unitsRen_symm** (r : Equiv.Perm (Fin n)) : Continuous (unitsRen r).symm

lemma **continuous_ujRen** (ε : Fin n → Bool) (r : Equiv.Perm (Fin n)) : Continuous (ujRen ε r)

lemma **continuous_ujRen_symm** (ε : Fin n → Bool) (r : Equiv.Perm (Fin n)) : Continuous (ujRen ε r).symm

lemma **memLp_ren** (ε : Fin n → Bool) (r : Equiv.Perm (Fin n)) {f : Formalism.REAL.UJ (ε ∘ r) → ℂ} (h : MemLp f 2 (Formalism.REAL.muG (ε ∘ r))) : MemLp (f ∘ ujRen ε r) 2 (Formalism.REAL.muG ε)

lemma **sqIrr_ren** (hε : ε₁ = ε ∘ r) (hτ : ∀ g, τ' g = τ (ren r g)) (h : GlobalTheory.Dec.IsSqIntIrr ε₁ τ) : GlobalTheory.Dec.IsSqIntIrr ε τ'

theorem **dsOfArr** : CompanionTheory.RgExh.Fin.DSOfArr

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.CountingBound

theorem **casG_polar** {n : ℕ} (μ : Fin n → ℝ) (J : Finset (Fin n)) (s : Fin n) : casG (addSet μ J) + casG (μ + Pi.single s 1) + (2 − casG μ) − casG (addSet μ J + Pi.single s 1) = if s ∈ J then 0 else 2

theorem **addSet_insert** {n : ℕ} (μ : Fin n → ℝ) (J : Finset (Fin n)) (s : Fin n) (hs : s ∉ J) : addSet μ J + Pi.single s 1 = addSet μ (insert s J)

theorem **addSet_eq_iff** {n : ℕ} (μ : Fin n → ℝ) (J I : Finset (Fin n)) (s : Fin n) : addSet μ J + Pi.single s 1 = addSet μ I ↔ s ∉ J ∧ I = insert s J

theorem **casG_polar_complex** {n : ℕ} (μ : Fin n → ℝ) (J : Finset (Fin n)) (s : Fin n) : (casG (addSet μ J) : ℂ) + (casG (μ + Pi.single s 1) : ℂ) + (2 − (casG μ : ℂ)) − (casG (addSet μ J + Pi.single s 1) : ℂ) = if s ∈ J then 0 else 2

theorem **count** : Count

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.CountingBoundStatement

theorem **_root_.GlobalTheory.T.CountingBound** : Count

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.CreAnnApply

lemma **sg_eq** (i : NC ε) (A : Finset (NC ε)) : sg i A = (sgR i A : ℂ)

lemma **sgR_erase** (i : NC ε) (A : Finset (NC ε)) : sgR i (A.erase i) = sgR i A

lemma **sgR_mul_self** (i : NC ε) (A : Finset (NC ε)) : sgR i A * sgR i A = 1

lemma **hh_symm** (i j : NC ε) (A B : Finset (NC ε)) : hh i j A B = hh j i B A

lemma **coef_symm** (c d : Fin n) (i j : NC ε) : coef c d i j = coef d c j i

lemma **cre_ann_apply** {V : Type} [AddCommGroup V] [Module ℂ V] (i j : NC ε) (x : Ch ε V) (A : Finset (NC ε)) : (cre i * ann j : Module.End ℂ (Ch ε V)) x A = ∑ B, (hh i j A B : ℂ) • x B

lemma **Gop_apply** {V : Type} [AddCommGroup V] [Module ℂ V] (c d : Fin n) (x : Ch ε V) (A : Finset (NC ε)) : Gop ε c d x A = ∑ B, (gg c d A B : ℂ) • x B

lemma **wtA_erase** (i : NC ε) (A : Finset (NC ε)) (hi : i ∈ A) : wtA A = wtA (A.erase i) + Pi.single i.1.1 1 − Pi.single i.1.2 1

lemma **exists_of_ne** {c d : Fin n} {A B : Finset (NC ε)} (h : gg c d A B ≠ 0) : ∃ i j : NC ε, ((i.1.1 = c ∧ j.1.1 = d ∧ i.1.2 = j.1.2) ∨ (i.1.2 = d ∧ j.1.2 = c ∧ i.1.1 = j.1.1)) ∧ i ∈ A ∧ j ∈ B ∧ A.erase i = B.erase j

lemma **gg_diag** (c : Fin n) (A B : Finset (NC ε)) : gg c c A B = if A = B then wtA A c else 0

theorem **gFacts** : GFacts

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.CreApply

theorem **key_inj** {i j : NC ε} (h : key i = key j) : i = j

theorem **tw_self** (i : NC ε) : tw i i = 1

theorem **tw_mul_self** (i j : NC ε) : tw i j * tw i j = 1

theorem **tw_swap** {i j : NC ε} (h : i ≠ j) : tw j i = −tw i j

theorem **sg_mul_self** (i : NC ε) (A : Finset (NC ε)) : sg i A * sg i A = 1

theorem **sg_insert** (i : NC ε) {j : NC ε} {A : Finset (NC ε)} (h : j ∉ A) : sg i (insert j A) = tw i j * sg i A

theorem **sg_erase** (i : NC ε) {j : NC ε} {A : Finset (NC ε)} (h : j ∈ A) : sg i (A.erase j) = tw i j * sg i A

theorem **ann_apply** (i : NC ε) (x : Ch ε V) (A : Finset (NC ε)) : ann i x A = if i ∈ A then 0 else sg i A • x (insert i A)

theorem **cre_apply** (i : NC ε) (x : Ch ε V) (A : Finset (NC ε)) : cre i x A = if i ∈ A then sg i A • x (A.erase i) else 0

theorem **ann_ann** (i j : NC ε) : (ann i * ann j + ann j * ann i : Module.End ℂ (Ch ε V)) = 0

theorem **cre_cre** (i j : NC ε) : (cre i * cre j + cre j * cre i : Module.End ℂ (Ch ε V)) = 0

theorem **ann_cre** (i j : NC ε) : (ann i * cre j + cre j * ann i : Module.End ℂ (Ch ε V)) = if i = j then 1 else 0

theorem **car** : CompanionTheory.D.DSIC.CAR

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.CreationProductSingleBasis

theorem **cre_single** {n : ℕ} {V : Type} [AddCommGroup V] [Module ℂ V] (f : Pos n) (S : Finset (Pos n)) (w : V) (h : f ∉ S) : (cre f : Module.End ℂ (Cn n V)) (Pi.single S w : Cn n V) = CompanionTheory.D.LK.sg f (insert f S) • (Pi.single (insert f S) w : Cn n V)

theorem **sg_ne_zero** {n : ℕ} (f : Pos n) (A : Finset (Pos n)) : CompanionTheory.D.LK.sg f A ≠ 0

theorem **prod_cre_single** : ∀ (n : ℕ) (V : Type) [AddCommGroup V] [Module ℂ V] (l : List (Pos n)) (S : Finset (Pos n)) (w : V), l.Nodup → (∀ i ∈ l, i ∉ S) → ∃ c : ℂ, c ≠ 0 ∧ ((l.map cre).prod : Module.End ℂ (Cn n V)) (Pi.single S w : Cn n V) = c • (Pi.single (S ∪ l.toFinset) w : Cn n V)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.CreationProductSingleBasisStatement

theorem **_root_.GlobalTheory.T.creation_product_single_basis** : ∀ (n : ℕ) (V : Type) [AddCommGroup V] [Module ℂ V] (l : List (Pos n)) (S : Finset (Pos n)) (w : V), l.Nodup → (∀ i ∈ l, i ∉ S) → ∃ c : ℂ, c ≠ 0 ∧ ((l.map cre).prod : Module.End ℂ (Cn n V)) (Pi.single S w : Cn n V) = c • (Pi.single (S ∪ l.toFinset) w : Cn n V)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.CreationProductSupportDegree

theorem **suppIn_deg_cre_prod** : ∀ (n : ℕ) (V : Type) [AddCommGroup V] [Module ℂ V] (P : Pos n → Prop) (l : List (Pos n)) (x : Cn n V) (j : ℕ), (∀ i ∈ l, P i) → SuppIn P x → Deg j x → SuppIn P (((l.map cre).prod : Module.End ℂ (Cn n V)) x) ∧ Deg (j + l.length) (((l.map cre).prod : Module.End ℂ (Cn n V)) x)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.CreationProductSupportDegreeStatement

theorem **_root_.GlobalTheory.T.creation_product_support_degree** : ∀ (n : ℕ) (V : Type) [AddCommGroup V] [Module ℂ V] (P : Pos n → Prop) (l : List (Pos n)) (x : Cn n V) (j : ℕ), (∀ i ∈ l, P i) → SuppIn P x → Deg j x → SuppIn P (((l.map cre).prod : Module.End ℂ (Cn n V)) x) ∧ Deg (j + l.length) (((l.map cre).prod : Module.End ℂ (Cn n V)) x)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.DecompositionOfFrameWittTorus

theorem **mem_pull** {k : V ≃ₗ[L] V} {Λ : AddSubgroup V} {x : V} : x ∈ pull k Λ ↔ k x ∈ Λ

theorem **isIso_mul** {h : V → V → L} {a b : V ≃ₗ[L] V} (ha : IsIso h a) (hb : IsIso h b) : IsIso h (a * b)

theorem **isIso_inv** {h : V → V → L} {a : V ≃ₗ[L] V} (ha : IsIso h a) : IsIso h a⁻¹

theorem **adapted_unique** {h : V → V → L} {δ : L} {l : ℕ} {e f : Fin l → V} {W : Submodule L V} {u v : Fin l → L} {Λ₁ Λ₂ : AddSubgroup V} (hF : IsFrame h δ e f W) (h1 : IsAdapted h e f W u Λ₁) (h2 : IsAdapted h e f W v Λ₂) (huv : ∀ i, ‖u i‖ = ‖v i‖) (x : V) : x ∈ Λ₁ ↔ x ∈ Λ₂

theorem **adapted_pull** {h : V → V → L} {l : ℕ} {e f e' f' : Fin l → V} {W W' : Submodule L V} {u : Fin l → L} {Λ : AddSubgroup V} {k : V ≃ₗ[L] V} (hk : IsIso h k) (he : ∀ i, k (e i) = e' i) (hf : ∀ i, k (f i) = f' i) (hW : ∀ w ∈ W, k w ∈ W') (hA : IsAdapted h e' f' W' u Λ) : IsAdapted h e f W u (pull k Λ)

theorem **adapted_reindex** {h : V → V → L} {l l' : ℕ} {e f : Fin l' → V} {W : Submodule L V} {u : Fin l' → L} {Λ : AddSubgroup V} (hA : IsAdapted h e f W u Λ) (τ : Fin l ≃ Fin l') : IsAdapted h (e ∘ τ) (f ∘ τ) W (u ∘ τ) Λ

theorem **adapted_tor** {s : L →+* L} (hs : IsInv s) {h : V → V → L} {l : ℕ} {e f : Fin l → V} {W : Submodule L V} {u : Fin l → L} {Λ : AddSubgroup V} {t : V ≃ₗ[L] V} (hu : ∀ i, u i ≠ 0) (ht : IsTor s e f W u t) (hA : IsAdapted h e f W u Λ) : IsAdapted h e f W (fun _ => 1) (pull t Λ)

theorem **norm_discrete** {p : L} (hp0 : p ≠ 0) (hp1 : ‖p‖ < 1) (hpu : ∀ a : L, ‖a‖ < 1 → ‖a‖ ≤ ‖p‖) {x : L} (hx : x ≠ 0) : ∃ n : ℤ, ‖x‖ = ‖p‖ ^ n

theorem **exists_antitone_perm** {n : ℕ} (m : Fin n → ℕ) : ∃ σ : Equiv.Perm (Fin n), Antitone (m ∘ σ)

theorem **isLat_pull** (k : V ≃ₗ[L] V) {Λ : AddSubgroup V} (hΛ : IsLat L Λ) : IsLat L (pull k Λ)

theorem **isMax_pull** {h : V → V → L} {Λ : AddSubgroup V} (hM : IsMax h Λ) {g : V ≃ₗ[L] V} (hg : IsIso h g) : IsMax h (pull g⁻¹ Λ)

theorem **hl_apply** (hh : IsHerm s h) (x y : V) : hl hh x y = h x y

theorem **h_fe** (hh : IsHerm s h) (hF : IsFrame h δ e f W) (i j : Fin l) : h (f i) (e j) = if i = j then s δ else 0

theorem **form_e** (hh : IsHerm s h) (hF : IsFrame h δ e f W) (a b : Fin l → L) {w : V} (hw : w ∈ W) (i : Fin l) : h (e i) (∑ j, a j • e j + ∑ j, b j • f j + w) = b i * δ

theorem **form_f** (hh : IsHerm s h) (hF : IsFrame h δ e f W) (a b : Fin l → L) {w : V} (hw : w ∈ W) (i : Fin l) : h (f i) (∑ j, a j • e j + ∑ j, b j • f j + w) = a i * s δ

theorem **form_w** (hh : IsHerm s h) (hF : IsFrame h δ e f W) (a b : Fin l → L) {w v : V} (hw : w ∈ W) (hv : v ∈ W) : h v (∑ j, a j • e j + ∑ j, b j • f j + w) = h v w

theorem **frame_form** (hh : IsHerm s h) (hF : IsFrame h δ e f W) (a b a' b' : Fin l → L) {w w' : V} (hw : w ∈ W) (hw' : w' ∈ W) : h (∑ j, a' j • e j + ∑ j, b' j • f j + w') (∑ j, a j • e j + ∑ j, b j • f j + w) = ∑ j, a j * s (b' j * δ) + ∑ j, b j * s (a' j * s δ) + s (h w w')

theorem **permMap_apply** (hδ : δ ≠ 0) (hh : IsHerm s h) (hF : IsFrame h δ e f W) (τ : Equiv.Perm (Fin l)) (a b : Fin l → L) {w : V} (hw : w ∈ W) : permMap hh δ e f τ (∑ j, a j • e j + ∑ j, b j • f j + w) = ∑ j, a (τ.symm j) • e j + ∑ j, b (τ.symm j) • f j + w

theorem **frame_perm** {δ : L} (hδ : δ ≠ 0) {h : V → V → L} (hh : IsHerm s h) {l : ℕ} {e f : Fin l → V} {W : Submodule L V} (hF : IsFrame h δ e f W) (σ : Equiv.Perm (Fin l)) : ∃ P : V ≃ₗ[L] V, IsIso h P ∧ (∀ i, P (e i) = e (σ i)) ∧ (∀ i, P (f i) = f (σ i)) ∧ ∀ w ∈ W, P w = w

theorem **flip_sum** (F : Fin l → Prop) [DecidablePred F] (c : L) (a b : Fin l → L) : ∑ i, a i • (if F i then f i else e i) + ∑ i, b i • (if F i then c • e i else f i) = ∑ i, (if F i then c * b i else a i) • e i + ∑ i, (if F i then a i else b i) • f i

theorem **adapted_flip** (F : Fin l → Prop) [DecidablePred F] {c : L} (hc : ‖c‖ = 1) {u v : Fin l → L} {Λ : AddSubgroup V} (hv : ∀ i, ‖v i‖ = if F i then ‖u i‖⁻¹ else ‖u i‖) (hA : IsAdapted h e f W u Λ) : IsAdapted h (fun i => if F i then f i else e i) (fun i => if F i then c • e i else f i) W v Λ

theorem **hermitian_frame_partial_flip** (hs : IsInv s) (hδ : δ ≠ 0) (hh : IsHerm s h) (hF : IsFrame h δ e f W) (F : Fin l → Prop) [DecidablePred F] : IsFrame h δ (fun i => if F i then f i else e i) (fun i => if F i then (δ / s δ) • e i else f i) W

theorem **frame_flip** (hs : IsInv s) (hδ : δ ≠ 0) (hh : IsHerm s h) {u : Fin l → L} {Λ Λ' : AddSubgroup V} (hF : IsFrame h δ e f W) (h1 : IsAdapted h e f W (fun _ => 1) Λ) (h2 : IsAdapted h e f W u Λ') (p : L) (n : Fin l → ℤ) (hn : ∀ i, ‖u i‖ = ‖p‖ ^ n i) (hp0 : p ≠ 0) : ∃ (e' f' : Fin l → V) (m : Fin l → ℕ), IsFrame h δ e' f' W ∧ IsAdapted h e' f' W (fun _ => 1) Λ ∧ IsAdapted h e' f' W (fun i => p ^ m i) Λ'

theorem **stDecomp_of_frame_witt_tor** : StFrame → StWitt → StTor → StDecomp

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.DecompositionOfFrameWittTorusStatement

theorem **_root_.GlobalTheory.T.decomposition_of_frame_witt_torus** : StFrame → StWitt → StTor → StDecomp

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.DecompositionOfIwasawaLeviCartan

theorem **iso_mul** {A B C P Q : Matrix (Fin n) (Fin n) ℂ} (hP : Pᴴ * A * P = B) (hQ : Qᴴ * B * Q = C) : (P * Q)ᴴ * A * (P * Q) = C

theorem **iso_inv** {A B P Q : Matrix (Fin n) (Fin n) ℂ} (hP : Pᴴ * A * P = B) (h : P * Q = 1) : Qᴴ * B * Q = A

theorem **iso_exists_inv** {A B P : Matrix (Fin n) (Fin n) ℂ} (hP : Pᴴ * A * P = B) (hB : B * B = 1) : ∃ Q, P * Q = 1 ∧ Q * P = 1

theorem **uni_mul** {P Q : Matrix (Fin n) (Fin n) ℂ} (hP : Pᴴ * P = 1) (hQ : Qᴴ * Q = 1) : (P * Q)ᴴ * (P * Q) = 1

theorem **fix_inv** {P Q : Matrix (Fin n) (Fin n) ℂ} (h : Q * P = 1) {v : Fin n → ℂ} (hv : P *ᵥ v = v) : Q *ᵥ v = v

theorem **fix_mul** {P Q : Matrix (Fin n) (Fin n) ℂ} {v : Fin n → ℂ} (hv : P *ᵥ v = v) (hw : Q *ᵥ v = v) : (P * Q) *ᵥ v = v

theorem **fix_adj** {A B P : Matrix (Fin n) (Fin n) ℂ} (hP : Pᴴ * A * P = B) {v w : Fin n → ℂ} (hv : P *ᵥ v = v) (hA : A *ᵥ v = w) (hB : B *ᵥ v = w) : Pᴴ *ᵥ w = w

theorem **col_of_fix** {P : Matrix (Fin n) (Fin n) ℂ} {i : Fin n} (h : P *ᵥ bvec i = bvec i) (j : Fin n) : P j i = if j = i then 1 else 0

theorem **row_of_fix** {P : Matrix (Fin n) (Fin n) ℂ} {i : Fin n} (h : Pᴴ *ᵥ bvec i = bvec i) (j : Fin n) : P i j = if j = i then 1 else 0

theorem **hq_sq** : Hq k r * Hq k r = 1

theorem **hq_e** (a : Fin r) : Hq k r *ᵥ bvec (eI k r a) = bvec (eI k r a)

theorem **hq_es** (a : Fin r) : Hq k r *ᵥ bvec (esI k r a) = bvec (eI k r a)

theorem **hpk_e** (a : Fin r) : Hpk k r *ᵥ bvec (eI k r a) = bvec (esI k r a)

theorem **hpk_es** (a : Fin r) : Hpk k r *ᵥ bvec (esI k r a) = bvec (eI k r a)

theorem **block_of_fix** {A B P : Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ} (hP : Pᴴ * A * P = B) (hA : ∀ a : Fin r, A *ᵥ bvec (eI k r a) = bvec (esI k r a) ∧ A *ᵥ bvec (esI k r a) = bvec (eI k r a)) (hB : ∀ a : Fin r, B *ᵥ bvec (eI k r a) = bvec (esI k r a) ∧ B *ᵥ bvec (esI k r a) = bvec (eI k r a)) (hf : ∀ a : Fin r, P *ᵥ bvec (eI k r a) = bvec (eI k r a) ∧ P *ᵥ bvec (esI k r a) = bvec (esI k r a)) (i : Fin (2 * r + k + 1)) (hi : i.val < r ∨ r + k < i.val) (j : Fin (2 * r + k + 1)) : P j i = (if j = i then 1 else 0) ∧ P i j = (if j = i then 1 else 0)

theorem **conj_mem_N** {P Q m : Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ} (hP : Pᴴ * Hpk k r * P = Hq k r) (hPQ : P * Q = 1) (hQP : Q * P = 1) (hf : ∀ a : Fin r, P *ᵥ bvec (eI k r a) = bvec (eI k r a) ∧ P *ᵥ bvec (esI k r a) = bvec (esI k r a)) (hm : m ∈ Nq k r) : P * m * Q ∈ Nset k r

theorem **dg_mem** {d : Fin (2 * r + k + 1) → ℝ} (hd : IsTor k r d) : dg d ∈ UH (Hq k r)

theorem **dg_mul** (t t' : Fin (2 * r + k + 1) → ℝ) : dg t * dg t' = dg (fun i => t i * t' i)

theorem **isTor_mul** {t t' : Fin (2 * r + k + 1) → ℝ} (h : IsTor k r t) (h' : IsTor k r t') : IsTor k r (fun i => t i * t' i)

theorem **dg_comm** {t : Fin (2 * r + k + 1) → ℝ} (ht : IsTor k r t) (ht0 : t (wI k r 0) = 1) {h : Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ} (hh : h ∈ Wbq k r) : dg t * h = h * dg t

theorem **decomp** : AdaptExists → IwaQ → LeviQ → CartanQ → Decomp

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.DecompositionOfIwasawaLeviCartanStatement

theorem **_root_.GlobalTheory.T.decomposition_of_iwasawa_levi_cartan** : AdaptExists → IwaQ → LeviQ → CartanQ → Decomp

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.DecompositionOfTensorFromHomLaws

theorem **dec_comp** {Hom : ∀ a b, (N a →ₗ[ℂ] N b) → Prop} (hc : IsLinCat N Hom) {A B P : Type} [Fintype A] [Fintype B] [Fintype P] (e : P ≃ A × B) {X : O} {Z : A → O} {S : A → B → O} {j : ∀ a, N (Z a) →ₗ[ℂ] N X} {q : ∀ a, N X →ₗ[ℂ] N (Z a)} (h : IsDec Hom X Z j q) {io : ∀ a b, N (S a b) →ₗ[ℂ] N (Z a)} {pr : ∀ a b, N (Z a) →ₗ[ℂ] N (S a b)} (h2 : ∀ a, IsDec Hom (Z a) (S a) (io a) (pr a)) : IsDec Hom X (fun p => S (e p).1 (e p).2) (fun p => j (e p).1 ∘ₗ io (e p).1 (e p).2) (fun p => pr (e p).1 (e p).2 ∘ₗ q (e p).1) where

theorem **mAct_reindex** {ι ι' V : Type} [Fintype ι] [DecidableEq ι] [AddCommGroup V] [Module ℂ V] (e : ι' → ι) : (mAct ((1 : Matrix ι ι ℂ).submatrix e id) : (ι → V) →ₗ[ℂ] (ι' → V)) = LinearMap.funLeft ℂ V e

theorem **hom_reindex** (hM : MatHom) (F F' : FRep k r) (U : URep k r) (e : F'.ι ≃ F.ι) (h : ∀ g, F'.ρ g = (F.ρ g).submatrix e e) : IsHom (F, U) (F', U) (LinearMap.funLeft ℂ U.R.smSub e)

theorem **hom_cas** (hL : HomLie) {X Y : Obj k r} {f : M X →ₗ[ℂ] M Y} (hX : X.1.OK) (hY : Y.1.OK) (hf : IsHom X Y f) : cas Y ∘ₗ f = f ∘ₗ cas X

theorem **dec1** (hC : HomCat) (hM : MatHom) (hT : TensHom) (F F' : FRep k r) (U : URep k r) {A : Type} [Fintype A] (T : A → URep k r) {io : ∀ a, M (one (T a)) →ₗ[ℂ] M (F, U)} {pr : ∀ a, M (F, U) →ₗ[ℂ] M (one (T a))} (h : IsDec (N

theorem **dec2** (hC : HomCat) (hM : MatHom) (hT : TensHom) (F F' : FRep k r) (U : URep k r) {A : Type} [Fintype A] (T : A → URep k r) {io : ∀ a, M (one (T a)) →ₗ[ℂ] M (F', U)} {pr : ∀ a, M (F', U) →ₗ[ℂ] M (one (T a))} (h : IsDec (N

theorem **decTens** : HomCat → MatHom → TensHom → HomLie → OKTriv → DecTens

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.DecompositionOfTensorFromHomLawsStatement

theorem **_root_.GlobalTheory.T.decomposition_of_tensor_from_hom_laws** : HomCat → MatHom → TensHom → HomLie → OKTriv → DecTens

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.DecompositionOfTrace

theorem **exists_iso_of_eq** (hc : IsLinCat N Hom) {a b : O} (h : a = b) : ∃ (θ : N a →ₗ[ℂ] N b) (θ' : N b →ₗ[ℂ] N a), Hom a b θ ∧ Hom b a θ' ∧ θ ∘ₗ θ' = LinearMap.id ∧ θ' ∘ₗ θ = LinearMap.id

theorem **exists_basis** (hc : IsLinCat N Hom) {P : Type} [DecidableEq P] {T : P → O} (hT : IsSimpleFam Hom T) {κ : Type} [Fintype κ] {par : κ → P} {X : O} {io : ∀ k, N (T (par k)) →ₗ[ℂ] N X} {pr : ∀ k, N X →ₗ[ℂ] N (T (par k))} (hd : IsDec Hom X (fun k => T (par k)) io pr) (ν : P) : ∃ b : Module.Basis {k // par k = ν} ℂ (homSub hc (T ν) X), ∀ k, ∃ θ : N (T ν) →ₗ[ℂ] N (T (par k.1)), (b k).1 = io k.1 ∘ₗ θ

theorem **exists_trace** (hc : IsLinCat N Hom) {P : Type} [DecidableEq P] {T : P → O} (hT : IsSimpleFam Hom T) {κ : Type} [Fintype κ] {par : κ → P} {X : O} {io : ∀ k, N (T (par k)) →ₗ[ℂ] N X} {pr : ∀ k, N X →ₗ[ℂ] N (T (par k))} (hd : IsDec Hom X (fun k => T (par k)) io pr) (ν : P) (Φ : Module.End ℂ (N X)) (a : κ → ℂ) (h : ∀ k, Φ ∘ₗ io k = a k • io k) : ∃ F : Module.End ℂ (homSub hc (T ν) X), (∀ w, (F w).1 = Φ ∘ₗ w.1) ∧ LinearMap.trace ℂ _ F = ∑ k ∈ univ.filter (fun k => par k = ν), a k

theorem **dec_of_gen** (hc : IsLinCat N Hom) {P : Type} {T : P → O} (hT : IsSimpleFam Hom T) {κ : Type} [Fintype κ] {par : κ → P} {X Z : O} {io : ∀ k, N (T (par k)) →ₗ[ℂ] N X} {pr : ∀ k, N X →ₗ[ℂ] N (T (par k))} (hd : IsDec Hom X (fun k => T (par k)) io pr) {j : N Z →ₗ[ℂ] N X} {q : N X →ₗ[ℂ] N Z} (hj : Hom Z X j) (hq : Hom X Z q) (hqj : q ∘ₗ j = LinearMap.id) {I : Type} [Fintype I] {e : I → P} (he : Function.Injective e) (g : ∀ i, homSub hc (T (e i)) Z) (hg0 : ∀ i, g i ≠ 0) (hg : ∀ i w, ∃ c : ℂ, c • g i = w) (h0 : ∀ ν, (∀ i, e i ≠ ν) → ∀ w : homSub hc (T ν) Z, w = 0) : ∃ io pr, IsDec Hom Z (fun i => T (e i)) io pr

theorem **decOfTrace** : DecOfTrace

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.DecompositionOfTraceStatement

theorem **_root_.GlobalTheory.T.DecompositionOfTrace** : DecOfTrace

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.DefinitenessStatement

theorem **cT_congr** {R : Type} [CommRing R] (s : R →+* R) {n : ℕ} (Q Qi D J J' Δ : Matrix (Fin n) (Fin n) R) (hQ : Q * Qi = 1) (h1 : Formalism.UG.cT s Q * J * Q = Δ) (h2 : Formalism.UG.cT s D * J' * D = Δ) : Formalism.UG.cT s (D * Qi) * J' * (D * Qi) = J

theorem **exists_large_sq** {F : Type} [Field F] [NumberField F] (v : HeightOneSpectrum (𝓞 F)) {n : ℕ} (a : Fin n → F) (ha : ∀ i, a i ≠ 0) (B : ℝ) : ∃ N : ℕ, B < N ∧ ∀ i, ∃ c : v.adicCompletion F, c * c * algebraMap F (v.adicCompletion F) (a i) = algebraMap F (v.adicCompletion F) (a i + N)

theorem **isHermND_diagonal** (e : Fin n → F) (he : ∀ i, e i ≠ 0) : IsHermND σ (diagonal fun i => algebraMap F E (e i))

theorem **isPosDef_diagonal** (hσ : ∀ (τ : E →+* ℂ) (x : E), τ (σ x) = star (τ x)) (e : Fin n → F) (he : ∀ (τ : E →+* ℂ) i, 0 < (τ (algebraMap F E (e i))).re) : IsPosDef σ (diagonal fun i => algebraMap F E (e i))

theorem **sigR_includeLeft** (x : R) : sigR R σ (Algebra.TensorProduct.includeLeft (S

theorem **sigR_inclR** (x : E) : sigR R σ (inclR R x) = inclR R (σ x)

theorem **inclR_algebraMap** (f : F) : inclR R (algebraMap F E f) = Algebra.TensorProduct.includeLeft (S

theorem **locIso** {F E : Type} [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E) (hinv : ∀ x, σ (σ x) = x) {n : ℕ} (J P : Matrix (Fin n) (Fin n) E) (hP : P.det ≠ 0) (a e : Fin n → F) (hPd : Pᵀ * J * P.map σ = diagonal fun i => algebraMap F E (a i)) (v : HeightOneSpectrum (𝓞 F)) (c : Fin n → Fv F v) (hc0 : ∀ i, c i ≠ 0) (hc : ∀ i, c i * c i * algebraMap F (Fv F v) (a i) = algebraMap F (Fv F v) (e i)) : Nonempty (GlobalTheory.AdelicU.GLoc σ J v ≃ₜ* GlobalTheory.AdelicU.GLoc σ (diagonal fun i => algebraMap F E (e i)) v)

theorem **stDefinite** : StDefinite

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.DefinitenessStatementStatement

theorem **_root_.GlobalTheory.T.DefinitenessStatement** : StDefinite

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.DeltaStatement

theorem **exists_isDelta** (L : Type) [NontriviallyNormedField L] [IsUltrametricDist L] [ProperSpace L] [CharZero L] (s : L →+* L) (hs : IsInv s) : ∃ δ, IsDelta s δ

theorem **stDelta** : StDelta

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.DeltaStatementStatement

theorem **_root_.GlobalTheory.T.DeltaStatement** : StDelta

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.DerivedSeriesAdjoint

theorem **ex_mul_eq** {y : Mat n} (hy : IsUnit y) (X : Mat n) (t : ℝ) : ex X t * y = y * ex (y⁻¹ * X * y) t

theorem **lie_conj** {Hm y X : Mat n} (hy : IsUnit y) (hyH : yᴴ * Hm * y = Hm) (hX : Lie Hm X) : Lie Hm (y⁻¹ * X * y)

theorem **dsAd** : DsAd

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.DerivedSeriesAdjointStatement

theorem **_root_.GlobalTheory.T.DerivedSeriesAdjoint** : DsAd

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.DerivedSeriesDerivation

theorem **contDiff_dL** {a : E → E} (ha : ContDiff ℝ ∞ a) {Φ : E × E' → V} (hΦ : ContDiff ℝ ∞ Φ) : ContDiff ℝ ∞ (dL a Φ)

theorem **contDiff_dR** {b : E' → E'} (hb : ContDiff ℝ ∞ b) {Φ : E × E' → V} (hΦ : ContDiff ℝ ∞ Φ) : ContDiff ℝ ∞ (dR b Φ)

theorem **dR_dL** {a : E → E} (ha : ContDiff ℝ ∞ a) {b : E' → E'} (hb : ContDiff ℝ ∞ b) {Φ : E × E' → V} (hΦ : ContDiff ℝ ∞ Φ) : dR b (dL a Φ) = dL a (dR b Φ)

theorem **contDiff_mulR** (Y : Mat n) : ContDiff ℝ ∞ fun g : Mat n => g * Y

theorem **contDiff_dW** {Φ : Mat n × Mat n → H} (hΦ : ContDiff ℝ ∞ Φ) : ∀ W, ContDiff ℝ ∞ (dW W Φ)

theorem **dR_dW** {b : Mat n → Mat n} (hb : ContDiff ℝ ∞ b) {Φ : Mat n × Mat n → H} (hΦ : ContDiff ℝ ∞ Φ) : ∀ W, dR b (dW W Φ) = dW W (dR b Φ)

theorem **hasDerivAt_dL** {Φ : Mat n × Mat n → H} (hΦ : ContDiff ℝ ∞ Φ) (Y g y : Mat n) : HasDerivAt (fun t => Φ (g * ex Y t, y)) (dL (fun g => g * Y) Φ (g, y)) 0

theorem **hasDerivAt_dR** {Φ : Mat n × Mat n → H} (hΦ : ContDiff ℝ ∞ Φ) (X g y : Mat n) : HasDerivAt (fun t => Φ (g, y * ex X t)) (dR (fun y => y * X) Φ (g, y)) 0

private theorem **dW_eq_smul_dsW_of_eq_on_open** (h : RepOn G σ) {u : H} (hu : IsSm G σ u) {Φ : Mat n × Mat n → H} (hΦ : ContDiff ℝ ∞ Φ) {y : Mat n} {O : Set (Mat n)} (hO : IsOpen O) (hΦu : ∀ g ∈ G, g ∈ O → Φ (g, y) = σ g u) : ∀ {W : List (Mat n)}, LieW G W → ∀ g ∈ G, g ∈ O → dW W Φ (g, y) = σ g (dsW σ W u)

private theorem **hasDerivAt_dsW_smul_mul_ex** (h : RepOn G σ) {x : H} (hx : IsSm G σ x) {F : Mat n → H} (hF : ContDiff ℝ ∞ F) {O : Set (Mat n)} (hO : IsOpen O) (hFx : ∀ g ∈ G, g ∈ O → σ g x = F g) {W : List (Mat n)} (hW : LieW G W) {p : Mat n} (hp : p ∈ G) (hpO : p ∈ O) {X : Mat n} (hX : ∀ t, ex X t ∈ G) : HasDerivAt (fun τ : ℝ => dsW σ W (σ (p * ex X τ) x)) (dsW σ W (σ p (ds σ X x))) 0

private theorem **exists_contDiff_eq_on_nhds_of_contDiffOn** {F : Mat n → H} {U : Set (Mat n)} {p : Mat n} (hU : IsOpen U) (hpU : p ∈ U) (hF : ContDiffOn ℝ ∞ F U) : ∃ F' : Mat n → H, ContDiff ℝ ∞ F' ∧ ∃ O : Set (Mat n), IsOpen O ∧ p ∈ O ∧ O ⊆ U ∧ ∀ g ∈ O, F' g = F g

theorem **dsDer** : DsDer

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.DerivedSeriesDerivationStatement

theorem **_root_.GlobalTheory.T.DerivedSeriesDerivation** : DsDer

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.DerivedSeriesLinear

theorem **dsLin** : DsLin

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.DerivedSeriesLinearStatement

theorem **_root_.GlobalTheory.T.DerivedSeriesLinear** : DsLin

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.DictionaryStatement

theorem **exists_model_field** {K A : Type} [NontriviallyNormedField K] [IsUltrametricDist K] [ProperSpace K] [Field A] [Algebra K A] [FiniteDimensional K A] [TopologicalSpace A] [IsModuleTopology K A] : ∃ (L : Type) (_ : NontriviallyNormedField L) (_ : IsUltrametricDist L) (_ : ProperSpace L) (e : A ≃+* L), Continuous e ∧ Continuous e.symm ∧ ∀ (c : A ≃ₐ[K] A) (x : A), ‖e (c x)‖ = ‖e x‖

theorem **isField_tensor_of_no_algHom** {F K E : Type} [Field F] [Field K] [Field E] [Algebra F K] [Algebra F E] (h2 : Module.finrank F E = 2) (hn : IsEmpty (E →ₐ[F] K)) : IsField (K ⊗[F] E)

theorem **exists_model** {K A : Type} [NontriviallyNormedField K] [IsUltrametricDist K] [ProperSpace K] [CommRing A] [Algebra K A] [Module.Finite K A] [TopologicalSpace A] [IsModuleTopology K A] (hA : IsField A) : ∃ (L : Type) (_ : NontriviallyNormedField L) (_ : IsUltrametricDist L) (_ : ProperSpace L) (e : A ≃+* L), Continuous e ∧ Continuous e.symm ∧ ∀ (c : A ≃ₐ[K] A) (x : A), ‖e (c x)‖ = ‖e x‖

theorem **sigEquiv_apply** (K : Type) [Field K] [Algebra F K] (h : ∀ x, σ (σ x) = x) (y : K ⊗[F] E) : sigEquiv K h y = sigR K σ y

theorem **sigR_sigR** (K : Type) [Field K] [Algebra F K] (h : ∀ x, σ (σ x) = x) (y : K ⊗[F] E) : sigR K σ (sigR K σ y) = y

theorem **properSpace_Fv** (v : HeightOneSpectrum (𝒪 F)) : ProperSpace (Fv F v)

theorem **charZero_Fv** : CharZero (Fv F v)

theorem **exists_equiv_UG** {L : Type} [Field L] [TopologicalSpace L] (e : (Fv F v ⊗[F] E) ≃+* L) (he : Continuous e) (he' : Continuous e.symm) : Nonempty (GLoc σ J v ≃ₜ* CompanionTheory.Rg.Fl.UG (e.toRingHom.comp ((sigR (Fv F v) σ).comp e.symm.toRingHom)) ((JR (Fv F v) J).map e))

theorem **stDict** : CompanionTheory.Rg.Fl.StDict

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.DictionaryStatementStatement

theorem **_root_.GlobalTheory.T.DictionaryStatement** : StDict

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.DominantConjugationPreservesLowerCongruence

theorem **conj_entry** {L : Type} [NontriviallyNormedField L] [IsUltrametricDist L] {n : ℕ} (p : Lˣ) (a : Fin n → ℤ) (w : GL (Fin n) L) (i j : Fin n) : (((GlobalTheory.CompanionTheory.CartGL.dPow p a)⁻¹ * w * GlobalTheory.CompanionTheory.CartGL.dPow p a : GL (Fin n) L) : Matrix (Fin n) (Fin n) L) i j = ((p ^ (-a i) : Lˣ) : L) * (w : Matrix (Fin n) (Fin n) L) i j * ((p ^ a j : Lˣ) : L)

theorem **norm_conj_le** {L : Type} [NontriviallyNormedField L] (p : Lˣ) (hp : ‖(p : L)‖ < 1) {b c : ℤ} (hbc : b ≤ c) (x : L) : ‖((p ^ (-b) : Lˣ) : L) * x * ((p ^ c : Lˣ) : L)‖ ≤ ‖x‖

theorem **conj_lower_contract** : ∀ (L : Type) [NontriviallyNormedField L] [IsUltrametricDist L] (n : ℕ) (p : Lˣ), ‖(p : L)‖ < 1 → ∀ (a : Fin n → ℤ), Antitone a → ∀ (r : ℝ) (w : GL (Fin n) L), (∀ i j, i < j → (w : Matrix (Fin n) (Fin n) L) i j = 0) → (∀ i j, ‖((w : Matrix (Fin n) (Fin n) L) - 1) i j‖ ≤ r) → ∀ i j, ‖(((((GlobalTheory.CompanionTheory.CartGL.dPow p a)⁻¹ * w * GlobalTheory.CompanionTheory.CartGL.dPow p a : GL (Fin n) L) : Matrix (Fin n) (Fin n) L) - 1) i j‖ ≤ r

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.DominantConjugationPreservesLowerCongruenceStatement

theorem **_root_.GlobalTheory.T.dominant_conjugation_preserves_lower_congruence** : ∀ (L : Type) [NontriviallyNormedField L] [IsUltrametricDist L] (n : ℕ) (p : Lˣ), ‖(p : L)‖ < 1 → ∀ (a : Fin n → ℤ), Antitone a → ∀ (r : ℝ) (w : GL (Fin n) L), (∀ i j, i < j → (w : Matrix (Fin n) (Fin n) L) i j = 0) → (∀ i j, ‖((w : Matrix (Fin n) (Fin n) L) - 1) i j‖ ≤ r) → ∀ i j, ‖((((GlobalTheory.CompanionTheory.CartGL.dPow p a)⁻¹ * w * GlobalTheory.CompanionTheory.CartGL.dPow p a : GL (Fin n) L) : Matrix (Fin n) (Fin n) L) - 1) i j‖ ≤ r

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.DominantConjugationPreservesUpperCongruence

theorem **conj_upper_contract** : ∀ (L : Type) [NontriviallyNormedField L] [IsUltrametricDist L] (n : ℕ) (p : Lˣ), ‖(p : L)‖ < 1 → ∀ (a : Fin n → ℤ), Antitone a → ∀ (r : ℝ) (u : GL (Fin n) L), (∀ i j, j < i → (u : Matrix (Fin n) (Fin n) L) i j = 0) → (∀ i j, ‖((u : Matrix (Fin n) (Fin n) L) - 1) i j‖ ≤ r) → ∀ i j, ‖(((GlobalTheory.CompanionTheory.CartGL.dPow p a * u * (GlobalTheory.CompanionTheory.CartGL.dPow p a)⁻¹ : GL (Fin n) L) : Matrix (Fin n) (Fin n) L) - 1) i j‖ ≤ r

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.DominantConjugationPreservesUpperCongruenceStatement

theorem **_root_.GlobalTheory.T.dominant_conjugation_preserves_upper_congruence** : ∀ (L : Type) [NontriviallyNormedField L] [IsUltrametricDist L] (n : ℕ) (p : Lˣ), ‖(p : L)‖ < 1 → ∀ (a : Fin n → ℤ), Antitone a → ∀ (r : ℝ) (u : GL (Fin n) L), (∀ i j, j < i → (u : Matrix (Fin n) (Fin n) L) i j = 0) → (∀ i j, ‖((u : Matrix (Fin n) (Fin n) L) - 1) i j‖ ≤ r) → ∀ i j, ‖(((GlobalTheory.CompanionTheory.CartGL.dPow p a * u * (GlobalTheory.CompanionTheory.CartGL.dPow p a)⁻¹ : GL (Fin n) L) : Matrix (Fin n) (Fin n) L) - 1) i j‖ ≤ r

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.DominantMonoidElementsAreAntitonePowers

theorem **tExp_antitone** (n : ℕ) (i : Fin (n + 2)) : Antitone (tExp n i)

theorem **closure_tGen_antitone** (L : Type) [NontriviallyNormedField L] [IsUltrametricDist L] (n : ℕ) (p : Lˣ) (s : GL (Fin n) L) (hs : s ∈ Submonoid.closure (Set.range (tGen p : Fin (n + 2) → GL (Fin n) L))) : ∃ a : Fin n → ℤ, Antitone a ∧ s = dPow p a

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.DominantMonoidElementsAreAntitonePowersStatement

theorem **_root_.GlobalTheory.T.dominant_monoid_elements_are_antitone_powers** : ∀ (L : Type) [NontriviallyNormedField L] [IsUltrametricDist L] (n : ℕ) (p : Lˣ) (s : GL (Fin n) L), s ∈ Submonoid.closure (Set.range (GlobalTheory.CompanionTheory.CartGL.tGen p : Fin (n + 2) → GL (Fin n) L)) → ∃ a : Fin n → ℤ, Antitone a ∧ s = GlobalTheory.CompanionTheory.CartGL.dPow p a

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.DominantOfKHighest

lemma **kRel_of_glRel** {V : Type} [AddCommGroup V] [Module ℂ V] {ε : Fin n → Bool} {E : Fin n → Fin n → Module.End ℂ V} (hE : GlRel E) : CompanionTheory.D.DSIC.KRel ε E

lemma **kUnit_of_isUnitary** {V : Type} [NormedAddCommGroup V] [InnerProductSpace ℂ V] {ε : Fin n → Bool} {E : Fin n → Fin n → Module.End ℂ V} (hU : IsUnitary ε E) : CompanionTheory.D.DSIC.KUnit ε E

lemma **dominant_of_kHighest** {V : Type} [NormedAddCommGroup V] [InnerProductSpace ℂ V] {ε : Fin n → Bool} {E : Fin n → Fin n → Module.End ℂ V} (hF : CompanionTheory.D.DSIC.KRel ε E) (hU : CompanionTheory.D.DSIC.KUnit ε E) {μ : Fin n → ℝ} {u : V} (hu : u ≠ 0) (hh : KHighest ε E μ u) {a b : Fin n} (hab : a < b) (he : ε a = ε b) : μ b ≤ μ a

lemma **two_rhoC_int** (ε : Fin n → Bool) (a : Fin n) : ∃ m : ℤ, ((2 : ℝ) • rhoC ε) a = m

lemma **vNormSq_nat** (ε : Fin n → Bool) {μ : Fin n → ℝ} (hμ : ∀ a, ∃ m : ℤ, μ a = m) : ∃ k : ℕ, vNormSq ε μ = k

theorem **lktExists** : LKTExists

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.EigenvalueGapExists

theorem **exists_gap_of_mem_Icc_top** : ∀ (n : ℕ) (l : Fin n → ℝ) (L : ℝ), 0 < L → (∀ i, 0 ≤ l i ∧ l i ≤ L) → (∃ i, l i = L) → ∃ r : ℕ, r < n ∧ ∀ i, l i < r * (L / (n + 1)) ∨ (r + 1) * (L / (n + 1)) ≤ l i

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.EigenvalueGapExistsStatement

theorem **_root_.GlobalTheory.T.exists_gap_interval_avoiding_finite_family** : ∀ (n : ℕ) (l : Fin n → ℝ) (L : ℝ), 0 < L → (∀ i, 0 ≤ l i ∧ l i ≤ L) → (∃ i, l i = L) → ∃ r : ℕ, r < n ∧ ∀ i, l i < r * (L / (n + 1)) ∨ (r + 1) * (L / (n + 1)) ≤ l i

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.EntryMulSubOne

theorem **entry_mul** {A B : Matrix (Fin n) (Fin n) L} {a b : ℝ} (ha : 0 ≤ a) (hb : 0 ≤ b) (hA : ∀ i j, ‖A i j‖ ≤ a) (hB : ∀ i j, ‖B i j‖ ≤ b) (i j : Fin n) : ‖(A * B) i j‖ ≤ a * b

theorem **entry_mul_sub_one** {A B : Matrix (Fin n) (Fin n) L} {r : ℝ} (h0 : 0 ≤ r) (h1 : r ≤ 1) (hA : ∀ i j, ‖(A - 1) i j‖ ≤ r) (hB : ∀ i j, ‖(B - 1) i j‖ ≤ r) (i j : Fin n) : ‖(A * B - 1) i j‖ ≤ r

theorem **mem_congK** {r : ℝ} {h0 : 0 ≤ r} {h1 : r ≤ 1} {k : GL (Fin n) L} : k ∈ congK L n r h0 h1 ↔ ∀ i j, ‖((k : Matrix (Fin n) (Fin n) L) - 1) i j‖ ≤ r ∧ ‖(((k⁻¹ : GL (Fin n) L) : Matrix (Fin n) (Fin n) L) - 1) i j‖ ≤ r

theorem **dPow_val** (p : Lˣ) (a : Fin n → ℤ) : ((dPow p a : GL (Fin n) L) : Matrix (Fin n) (Fin n) L) = diagonal fun i => ((p ^ a i : Lˣ) : L)

theorem **dPow_add** (p : Lˣ) (a b : Fin n → ℤ) : (dPow p (a + b) : GL (Fin n) L) = dPow p a * dPow p b

theorem **dPow_zero** (p : Lˣ) : (dPow p (0 : Fin n → ℤ) : GL (Fin n) L) = 1

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.EvaluationLiftDegreeZero

theorem **deg_of** (j : ℕ) {x : Cn n V} {y : Cn n W} (hx : CompanionTheory.RgExh.U.Deg j x) (h : ∀ A, x A = 0 → y A = 0) : CompanionTheory.RgExh.U.Deg j y

theorem **supp_of** {P : Pos n → Prop} {x : Cn n V} {y : Cn n W} (hx : SuppIn P x) (h : ∀ A, x A = 0 → y A = 0) : SuppIn P y

theorem **deg_add** {j : ℕ} {x y : Cn n V} (hx : CompanionTheory.RgExh.U.Deg j x) (hy : CompanionTheory.RgExh.U.Deg j y) : CompanionTheory.RgExh.U.Deg j (x + y)

theorem **supp_add** {P : Pos n → Prop} {x y : Cn n V} (hx : SuppIn P x) (hy : SuppIn P y) : SuppIn P (x + y)

theorem **deg_sub** {j : ℕ} {x y : Cn n V} (hx : CompanionTheory.RgExh.U.Deg j x) (hy : CompanionTheory.RgExh.U.Deg j y) : CompanionTheory.RgExh.U.Deg j (x - y)

theorem **supp_sub** {P : Pos n → Prop} {x y : Cn n V} (hx : SuppIn P x) (hy : SuppIn P y) : SuppIn P (x - y)

theorem **deg_iff** (j : ℕ) (x : Cn n V) : CompanionTheory.RgExh.U.Deg j x ↔ CompanionTheory.D.LK.Sup (allT n) ∅ j x

theorem **bd_deg** (E : Fin n → Fin n → Module.End ℂ V) {j : ℕ} {x : Cn n V} (h : CompanionTheory.RgExh.U.Deg (j + 1) x) : CompanionTheory.RgExh.U.Deg j (bd (allT n) E x)

theorem **bd_supp** (lev : Fin n → ℕ) (E : Fin n → Fin n → Module.End ℂ V) {x : Cn n V} (h : SuppIn (inU lev) x) : SuppIn (inU lev) (bd (allT n) E x)

theorem **lift_adL** (φ : V →ₗ[ℂ] W) (p q : Fin n) (x : Cn n V) : lift n φ (adL p q x) = adL p q (lift n φ x)

theorem **lift_lF** (φ : V →ₗ[ℂ] W) (E : Fin n → Fin n → Module.End ℂ V) (F : Fin n → Fin n → Module.End ℂ W) (h : ∀ a b x, φ (E a b x) = F a b (φ x)) (p q : Fin n) (x : Cn n V) : lift n φ (lF E p q x) = lF F p q (lift n φ x)

theorem **lift_evLc** (lev : Fin n → ℕ) (φ : V →ₗ[ℂ] W) (E : Fin n → Fin n → Module.End ℂ V) (F : Fin n → Fin n → Module.End ℂ W) (h : ∀ a b x, φ (E a b x) = F a b (φ x)) (b : FreeAlgebra ℂ (Fin n × Fin n)) (x : Cn n V) : lift n φ (evLc lev E b x) = evLc lev F b (lift n φ x)

theorem **lift_ev** (φ : V →ₗ[ℂ] W) (E : Fin n → Fin n → Module.End ℂ V) (F : Fin n → Fin n → Module.End ℂ W) (h : ∀ a b x, φ (E a b x) = F a b (φ x)) (a : FreeAlgebra ℂ (Fin n × Fin n)) (z : Cn n V) : lift n φ (lift n (ev E a) z) = lift n (ev F a) (lift n φ z)

theorem **bd_lift_comm** (lev : Fin n → ℕ) (E : Fin n → Fin n → Module.End ℂ V) (T : Module.End ℂ V) (hT : ∀ i : Pos n, inU lev i → Commute (E i.1.1 i.1.2) T) {x : Cn n V} (hx : SuppIn (inU lev) x) : bd (allT n) E (lift n T x) = lift n T (bd (allT n) E x)

theorem **adC_U** {lev : Fin n → ℕ} {p q : Fin n} (hpq : lev p = lev q) {i k : Pos n} (hi : inU lev i) (h : adC p q i k ≠ 0) : inU lev k

theorem **adL_apply** (p q : Fin n) (x : Cn n V) (A : Finset (Pos n)) : adL p q x A = ∑ i : Pos n, ∑ k : Pos n, adC p q i k • cre k (ann i x) A

theorem **adL_ne** {p q : Fin n} {x : Cn n V} {A : Finset (Pos n)} (h : adL p q x A ≠ 0) : ∃ i k, adC p q i k ≠ 0 ∧ k ∈ A ∧ i ∉ A.erase k ∧ x (insert i (A.erase k)) ≠ 0

theorem **lF_supp** {lev : Fin n → ℕ} (E : Fin n → Fin n → Module.End ℂ V) {p q : Fin n} (hpq : lev p = lev q) {x : Cn n V} (hx : SuppIn (inU lev) x) : SuppIn (inU lev) (lF E p q x)

theorem **lF_deg** (E : Fin n → Fin n → Module.End ℂ V) (p q : Fin n) {j : ℕ} {x : Cn n V} (hx : CompanionTheory.RgExh.U.Deg j x) : CompanionTheory.RgExh.U.Deg j (lF E p q x)

theorem **adjoint_left_degree_zero** (p q : Fin n) {x : Cn n V} (hx : CompanionTheory.RgExh.U.Deg 0 x) : adL p q x = 0

theorem **evaluation_lift_degree_zero** (lev : Fin n → ℕ) (E : Fin n → Fin n → Module.End ℂ V) (b : FreeAlgebra ℂ (Fin n × Fin n)) : ∀ x : Cn n V, CompanionTheory.RgExh.U.Deg 0 x → CompanionTheory.RgExh.U.Deg 0 (evLc lev E b x) ∧ evLc lev E b x = lift n (evL lev E b) x

theorem **acycU** (lev : Fin n → ℕ) (E : Fin n → Fin n → Module.End ℂ Z) (a d : Pos n → Module.End ℂ Z) (Fil : ℕ → Submodule ℂ Z) (hE : GlRel E) (hF : Filt (inU lev) E a d Fil) (j : ℕ) (x : Cn n Z) (hxs : SuppIn (inU lev) x) (hxd : CompanionTheory.RgExh.U.Deg (j + 1) x) (hcyc : bd (allT n) E x = 0) : ∃ t : Cn n Z, SuppIn (inU lev) t ∧ CompanionTheory.RgExh.U.Deg (j + 2) t ∧ bd (allT n) E t = x

theorem **coverU** (lev : Fin n → ℕ) (E : Fin n → Fin n → Module.End ℂ V) (hE : GlRel E) : ∃ (Y : Type) (_ : AddCommGroup Y) (_ : Module ℂ Y) (EY : Fin n → Fin n → Module.End ℂ Y) (π : Y →ₗ[ℂ] V), GlRel EY ∧ Function.Surjective π ∧ (∀ a b y, π (EY a b y) = E a b (π y)) ∧ ∀ (j : ℕ) (x : Cn n Y), SuppIn (inU lev) x → CompanionTheory.RgExh.U.Deg (j + 1) x → bd (allT n) EY x = 0 → ∃ t : Cn n Y, SuppIn (inU lev) t ∧ CompanionTheory.RgExh.U.Deg (j + 2) t ∧ bd (allT n) EY t = x

theorem **quad_ann** (a b : ι → R) (haa : ∀ i j, a i * a j + a j * a i = 0) (hab : ∀ i j, a i * b j + b j * a i = if i = j then 1 else 0) (c : ι → ι → ℂ) (m : ι) : (∑ i, ∑ k, c i k • (b k * a i)) * a m - a m * (∑ i, ∑ k, c i k • (b k * a i)) = - ∑ l, c l m • a l

theorem **quad_cre** (a b : ι → R) (hbb : ∀ i j, b i * b j + b j * b i = 0) (hab : ∀ i j, a i * b j + b j * a i = if i = j then 1 else 0) (c : ι → ι → ℂ) (m : ι) : (∑ i, ∑ k, c i k • (b k * a i)) * b m - b m * (∑ i, ∑ k, c i k • (b k * a i)) = ∑ k, c m k • b k

theorem **triple_comm** {R : Type} [Ring R] (A x y z Sx Sy Sz : R) (hx : A * x = x * A + Sx) (hy : A * y = y * A - Sy) (hz : A * z = z * A - Sz) : A * (x * y * z) - x * y * z * A = Sx * y * z - x * Sy * z - x * y * Sz

theorem **cubic_comm** (a b : ι → R) (haa : ∀ i j, a i * a j + a j * a i = 0) (hbb : ∀ i j, b i * b j + b j * b i = 0) (hab : ∀ i j, a i * b j + b j * a i = if i = j then 1 else 0) (c : ι → ι → ι → ℂ) (t : ι → ι → ι → ι → ℂ) (h : ∀ I J κ, ∑ k, t I J k * c k κ = ∑ j, t I j κ * c J j + ∑ i, t i J κ * c I i) : Commute (∑ i, ∑ k, c i k • (b k * a i)) (∑ i, ∑ j, ∑ k, t i j k • (b k * a j * a i))

theorem **sum_root** {M : Type} [AddCommMonoid M] (f : Pos n → M) (P : Prop) [Decidable P] (x y : Fin n) (h : P → x < y) : ∑ j : Pos n, (if P ∧ j.1 = (x, y) then f j else 0) = if hP : P then f ((x, y), h hP) else 0

theorem **sum_cU** {M : Type} [AddCommGroup M] [Module ℂ M] {lev : Fin n → ℕ} (hlev : Monotone lev) {p q : Fin n} (hpq : lev p = lev q) (g : Pos n → M) (l : Pos n) (hl : inU lev l) : ∑ j, cU lev p q l j • g j = (if h : q = l.1.1 then g ((p, l.1.2), hlev.reflect_lt (by rw [hpq, h]; exact hl)) else 0) - (if h : l.1.2 = p then g ((l.1.1, q), hlev.reflect_lt (by rw [← hpq, ← h]; exact hl)) else 0)

theorem **struct_id** {lev : Fin n → ℕ} (hlev : Monotone lev) {p q : Fin n} (hpq : lev p = lev q) (I J κ : Pos n) : ∑ k, tU lev I J k * cU lev p q k κ = ∑ j, tU lev I j κ * cU lev p q J j + ∑ i, tU lev i J κ * cU lev p q I i

theorem **ann_zero_of** {lev : Fin n → ℕ} {x : Cn n V} (hx : SuppIn (inU lev) x) {i : Pos n} (hi : ¬inU lev i) : ann i x = 0

theorem **adL_eq** {lev : Fin n → ℕ} {x : Cn n V} (hx : SuppIn (inU lev) x) (p q : Fin n) : adL p q x = AU V lev p q x

theorem **bd_eq** {lev : Fin n → ℕ} (E : Fin n → Fin n → Module.End ℂ V) {x : Cn n V} (hx : SuppIn (inU lev) x) : bd (allT n) E x = (LU lev E + TU V lev) x

theorem **lift_comm_ann** (T : Module.End ℂ V) (i : Pos n) : Commute (lift n T : Module.End ℂ (Cn n V)) (ann i)

theorem **lift_comm_cre** (T : Module.End ℂ V) (i : Pos n) : Commute (lift n T : Module.End ℂ (Cn n V)) (cre i)

theorem **lift_mul** (S T : Module.End ℂ V) : (lift n (S * T) : Module.End ℂ (Cn n V)) = lift n S * lift n T

theorem **lift_sub** (S T : Module.End ℂ V) : (lift n (S - T) : Module.End ℂ (Cn n V)) = lift n S - lift n T

theorem **lift_comm_AU** (T : Module.End ℂ V) (lev : Fin n → ℕ) (p q : Fin n) : Commute (lift n T : Module.End ℂ (Cn n V)) (AU V lev p q)

theorem **lift_comm_TU** (T : Module.End ℂ V) (lev : Fin n → ℕ) : Commute (lift n T : Module.End ℂ (Cn n V)) (TU V lev)

theorem **AU_comm_TU** {lev : Fin n → ℕ} (hlev : Monotone lev) {p q : Fin n} (hpq : lev p = lev q) : Commute (AU V lev p q) (TU V lev)

theorem **gl_ad** {lev : Fin n → ℕ} (hlev : Monotone lev) {p q : Fin n} (hpq : lev p = lev q) {E : Fin n → Fin n → Module.End ℂ V} (hE : GlRel E) (l : Pos n) (hl : inU lev l) : E p q * E l.1.1 l.1.2 - E l.1.1 l.1.2 * E p q = ∑ k, cU lev p q l k • E k.1.1 k.1.2

theorem **cU_uI** {lev : Fin n → ℕ} {p q : Fin n} (hpq : lev p = lev q) (l k : Pos n) : uI lev k * cU lev p q l k = cU lev p q l k

theorem **comm_U** {lev : Fin n → ℕ} (hlev : Monotone lev) {p q : Fin n} (hpq : lev p = lev q) {E : Fin n → Fin n → Module.End ℂ V} (hE : GlRel E) : Commute ((lift n (E p q) : Module.End ℂ (Cn n V)) + AU V lev p q) (LU lev E + TU V lev)

theorem **lF_eq** {lev : Fin n → ℕ} (E : Fin n → Fin n → Module.End ℂ V) {x : Cn n V} (hx : SuppIn (inU lev) x) (p q : Fin n) : lF E p q x = ((lift n (E p q) : Module.End ℂ (Cn n V)) + AU V lev p q) x

theorem **lF_comm** {lev : Fin n → ℕ} (hlev : Monotone lev) {p q : Fin n} (hpq : lev p = lev q) {E : Fin n → Fin n → Module.End ℂ V} (hE : GlRel E) {x : Cn n V} (hx : SuppIn (inU lev) x) : lF E p q (bd (allT n) E x) = bd (allT n) E (lF E p q x)

theorem **evLc_facts** {lev : Fin n → ℕ} (hlev : Monotone lev) {E : Fin n → Fin n → Module.End ℂ V} (hE : GlRel E) (b : FreeAlgebra ℂ (Fin n × Fin n)) : ∀ x : Cn n V, SuppIn (inU lev) x → SuppIn (inU lev) (evLc lev E b x) ∧ (∀ j, CompanionTheory.RgExh.U.Deg j x → CompanionTheory.RgExh.U.Deg j (evLc lev E b x)) ∧ bd (allT n) E (evLc lev E b x) = evLc lev E b (bd (allT n) E x)

theorem **genCOU** : GenCOU

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.ExistenceOfSandwichStatement

theorem **herm_add_left** {s : L →+* L} {h : V → V → L} (hh : IsHerm s h) (x y z : V) : h (x + y) z = h x z + h y z

theorem **herm_eq_sum** {s : L →+* L} {h : V → V → L} (hh : IsHerm s h) {ι : Type} [Fintype ι] (b : Module.Basis ι L V) (z y : V) : h z y = ∑ j, b.repr y j * h z (b j)

theorem **norm_sub_le_max_of_ultrametric_field** (a b : L) : ‖a − b‖ ≤ max ‖a‖ ‖b‖

theorem **norm_herm_le_delta** {s : L →+* L} {δ : L} {h : V → V → L} (hδ : IsDelta s δ) (hh : IsHerm s h) {M : AddSubgroup V} (hO : ∀ a : L, ‖a‖ ≤ 1 → ∀ x ∈ M, a • x ∈ M) (hM : ∀ x ∈ M, ‖h x x‖ ≤ 1) {x y : V} (hx : x ∈ M) (hy : y ∈ M) : ‖h x y‖ ≤ ‖δ‖

theorem **exists_basis_herm_le_one** [FiniteDimensional L V] {s : L →+* L} {h : V → V → L} (hs : IsInv s) (hh : IsHerm s h) : ∃ (m : ℕ) (b : Module.Basis (Fin m) L V), ∀ i j, ‖h (b i) (b j)‖ ≤ 1

theorem **basis_mem_ball** {ι : Type} [DecidableEq ι] (b : Module.Basis ι L V) (j : ι) : b j ∈ ball b

theorem **IsLat.smul_mem** {Λ : AddSubgroup V} (hΛ : IsLat L Λ) {a : L} (ha : ‖a‖ ≤ 1) {x : V} (hx : x ∈ Λ) : a • x ∈ Λ

theorem **ball_absorbing** {m : ℕ} (b : Module.Basis (Fin m) L V) (x : V) : ∃ a : L, a ≠ 0 ∧ a • x ∈ ball b

theorem **norm_herm_ball_le_one** {s : L →+* L} {h : V → V → L} (hs : IsInv s) (hh : IsHerm s h) {m : ℕ} (b : Module.Basis (Fin m) L V) (hb : ∀ i j, ‖h (b i) (b j)‖ ≤ 1) {x : V} (hx : x ∈ ball b) : ‖h x x‖ ≤ 1

theorem **exists_scale** {s : L →+* L} {h : V → V → L} (hh : IsHerm s h) (B : ℝ) (hB : 0 ≤ B) {m : ℕ} (b : Module.Basis (Fin m) L V) : ∃ c : L, c ≠ 0 ∧ ∀ x, (∀ j, ‖h (b j) x‖ ≤ B) → c • x ∈ ball b

theorem **stExists_of_stSandwich** : StSandwich → StExists

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.ExistenceOfSandwichStatementStatement

theorem **_root_.GlobalTheory.T.existence_of_sandwich_statement** : StSandwich → StExists

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.ExistsAdFam

theorem **exists_adFam** (a : FreeAlgebra ℂ (Fin n × Fin n)) : ∃ (ι : Type) (_ : Fintype ι) (w : ι → FreeAlgebra ℂ (Fin n × Fin n)) (M : Fin n → Fin n → Matrix ι ι ℂ) (t : ι → ℂ), AdFam w M ∧ a = ∑ i, t i • w i

theorem **AdFam.comm** {ι : Type} [Fintype ι] {w : ι → FreeAlgebra ℂ (Fin n × Fin n)} {M : Fin n → Fin n → Matrix ι ι ℂ} (h : AdFam w M) {V : Type} [AddCommGroup V] [Module ℂ V] {E : Fin n → Fin n → Module.End ℂ V} (hE : GlRel E) (c d : Fin n) (t : ι → ℂ) : E c d ∗ ev E (∑ i, t i • w i) − ev E (∑ i, t i • w i) ∗ E c d = ev E (∑ j, (M c d ∗ᵥ t) j • w j)

theorem **invLift** : InvLift

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.ExistsCocycleOfMapToDegreeOneTerm

theorem **kRel_map** {n : ℕ} {A B : Type} [Ring A] [Ring B] (ε : Fin n → Bool) (f : A →+* B) (F : Fin n → Fin n → A) (h : KRel ε F) : KRel ε (fun a b => f (F a b))

theorem **fdual_single** {n : ℕ} {ε : Fin n → Bool} {U : Type} [AddCommGroup U] [Module ℂ U] (EU : Fin n → Fin n → Module.End ℂ U) (g : Fin n → Fin n → Finset (NC ε) → Finset (NC ε) → ℝ) (hg : ∀ (c d : Fin n) (x : Ch ε U) (A : Finset (NC ε)), Gop ε c d x A = ∑ B, (g c d A B : ℂ) • x B) (a b : Fin n) (A B : Finset (NC ε)) (u : U) : Fdual ε EU a b (Pi.single A u) B = (Pi.single A (EU a b u) : Ch ε U) B − (g b a B A : ℂ) • u

theorem **auxₐ** : FRel → GFacts → GamDom → KHWExists → HWMap → PRV

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.ExistsFactor

theorem **exists_factor** {S E V : Type*} [AddCommGroup S] [Module ℂ S] [NormedAddCommGroup E] [NormedSpace ℂ E] [NormedAddCommGroup V] [NormedSpace ℂ V] [FiniteDimensional ℂ V] (J : S →ₗ[ℂ] E) (B : S →ₗ[ℂ] V) (C : ℝ) (hb : ∀ v, ‖B v‖ ≤ C ∗ ‖J v‖) : ∃ T : E →L[ℂ] V, ∀ v, B v = T (J v)

theorem **factor** : Factor

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.ExistsFactorStatement

theorem **_root_.GlobalTheory.T.factorization'** : Factor

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.ExistsFixedOfFinset

theorem **ext_of_hom** (hi : IsIsotypic δ ρ) {C D : H →L[ℂ] H} (h : ∀ T ∈ homIn δ ρ ⊤, ∀ x, C (T x) = D (T x)) : C = D

theorem **eq_zero_of_hom** [CompleteSpace H] (hi : IsIsotypic δ ρ) {v : H} (h : ∀ T ∈ homIn δ ρ ⊤, ∀ x, ⟪T x, v⟫_ℂ = 0) : v = 0

theorem **IsOpFam.commute** (hA : IsOpFam δ ρ A) (hi : IsIsotypic δ ρ) {C : H →L[ℂ] H} (hC : ∀ g, Commute C (ρ g)) (y ξ : X) : Commute (A y ξ) C

theorem **IsOpFam.rep_mul** (hA : IsOpFam δ ρ A) (hi : IsIsotypic δ ρ) (g : G) (y ξ : X) : ρ g ∗ A y ξ = A (δ g y) ξ

theorem **IsOpFam.mul** (hA : IsOpFam δ ρ A) (hi : IsIsotypic δ ρ) (y ξ y' ξ' : X) : A y ξ ∗ A y' ξ' = ⟪ξ, y'⟫_ℂ • A y ξ'

theorem **IsOpFam.adjoint_eq** [CompleteSpace H] (hA : IsOpFam δ ρ A) (hi : IsIsotypic δ ρ) (hδ : IsIrrUnitary δ) (hρ : IsUnitaryRep ρ) (y ξ : X) : ContinuousLinearMap.adjoint (A y ξ) = A ξ y

theorem **adjoint_intertwine** [CompleteSpace H] (hδ : IsIrrUnitary δ) (hρ : IsUnitaryRep ρ) {T : X →L[ℂ] H} (hT : T ∈ homIn δ ρ ⊤) (g : G) (v : H) : have

theorem **IsOpFam.exists_ne_zero** [CompleteSpace H] (hA : IsOpFam δ ρ A) (hi : IsIsotypic δ ρ) (hδ : IsIrrUnitary δ) (hρ : IsUnitaryRep ρ) {w : H} (hw : w ≠ 0) : ∃ ξ : X, ‖ξ‖ = 1 ∧ A ξ ξ w ≠ 0

theorem **exists_fixed_ne_zero** [CompleteSpace H] (hi : IsIsotypic δ ρ) (hδ : IsIrrUnitary δ) (hρ : IsUnitaryRep ρ) (K : Subgroup G) {h : H} (h0 : h ≠ 0) (hK : ∀ k ∈ K, ρ k h = h) : ∃ x : X, x ≠ 0 ∧ ∀ k ∈ K, δ k x = x

theorem **IsOpFam.apply_fixed** [CompleteSpace H] (hA : IsOpFam δ ρ A) (hi : IsIsotypic δ ρ) (hδ : IsIrrUnitary δ) (hρ : IsUnitaryRep ρ) (K : Subgroup G) {ξ : X} (hξ : ‖ξ‖ = 1) (hl : ∀ x : X, (∀ k ∈ K, δ k x = x) → ∃ c : ℂ, x = c • ξ) {h : H} (hK : ∀ k ∈ K, ρ k h = h) : A ξ ξ h = h

@[simp] theorem **pr_empty** (y : ∀ i, Hs i) : pr hc ∅ y = 1

theorem **pr_insert** [DecidableEq ι] {s : Finset ι} {i : ι} (hi : i ∉ s) (y : ∀ i, Hs i) : pr hc (insert i s) y = a i (y i) ∗ pr hc s y

theorem **commute_pr** {C : H →L[ℂ] H} {s : Finset ι} {y : ∀ i, Hs i} (h : ∀ i ∈ s, Commute C (a i (y i))) : Commute C (pr hc s y)

theorem **pr_congr** {s : Finset ι} {y y' : ∀ i, Hs i} (h : ∀ i ∈ s, y i = y' i) : pr hc s y = pr hc s y'

theorem **pr_insert_apply** [DecidableEq ι] {s : Finset ι} {i : ι} (hi : i ∉ s) {y : ∀ i, Hs i} (hy : y i = ξ i) (hw : a i (ξ i) w = w) : pr hc (insert i s) y w = pr hc s y w

theorem **pr_union_apply** [DecidableEq ι] (hw : ∀ i, a i (ξ i) w = w) (t u : Finset ι) {y : ∀ i, Hs i} (hy : ∀ i ∈ u, i ∉ t → y i = ξ i) : pr hc (t ∪ u) y w = pr hc t y w

theorem **inner_pr** [CompleteSpace H] [DecidableEq ι] (hw : ∀ i, a i (ξ i) w = w) (hadj : ∀ i y y', ContinuousLinearMap.adjoint (a i y') ∗ a i y = ⟪y', y⟫_ℂ • a i (ξ i)) (s : Finset ι) (y y' : ∀ i, Hs i) : ⟪pr hc s y' w, pr hc s y w⟫_ℂ = (∏ i ∈ s, ⟪y' i, y i⟫_ℂ) ∗ ⟪w, w⟫_ℂ

theorem **pureVec_eq** [DecidableEq ι] (hw : ∀ i, a i (ξ i) w = w) {y : ∀ i, Hs i} (hy : Formalism.MainNotions.KudlaSiegelWeil.Restr ξ y) {s : Finset ι} (hs : ∀ i ∉ s, y i = ξ i) : pureVec hc ξ w y = pr hc s y w

theorem **exists_off** [DecidableEq ι] (hpe : ∀ i x, p i (e i x) = x) (hpe' : ∀ i j, i ≠ j → ∀ x, p j (e i x) = 1) (hcomm : ∀ v g, p v g = 1 → ∀ y, Commute (π g) (a v y)) (hrep : ∀ i x y, π (e i x) ∗ a i y = a i (δ i x y)) (s : Finset ι) (g : G) : ∃ k, (∀ i ∈ s, p i k = 1) ∧ (∀ i ∉ s, p i k = p i g) ∧ ∀ y, π g ∗ pr hc s y = pr hc s (fun i => δ i (p i g) (y i)) ∗ π k

theorem **exists_isRestrTensor** [CompleteSpace H] (hπ : IsTopIrreducible π) (hξ : ∀ i, ‖ξ i‖ = 1) (hw1 : ‖w‖ = 1) (hw : ∀ i, a i (ξ i) w = w) (hadj : ∀ i y y', ContinuousLinearMap.adjoint (a i y') ∗ a i y = ⟪y', y⟫_ℂ • a i (ξ i)) (hpe : ∀ i x, p i (e i x) = x) (hpe' : ∀ i j, i ≠ j → ∀ x, p j (e i x) = 1) (hcomm : ∀ v g, p v g = 1 → ∀ y, Commute (π g) (a v y)) (hrep : ∀ i x y, π (e i x) ∗ a i y = a i (δ i x y)) (hk : ∀ g, ∃ s₀ : Finset ι, (∀ i ∉ s₀, δ i (p i g) (ξ i) = ξ i) ∧ ∀ s, s₀ ⊆ s → ∀ k, (∀ i ∈ s, p i k = 1) → (∀ i ∉ s, p i k = p i g) → π k w = w) : ∃ B, IsRestrTensor p δ ξ π B

theorem **exists_fixed_of_finset** [DecidableEq ι] {A : ∀ i, X i → X i → (H →L[ℂ] H)} (hc : ∀ i j, i ≠ j → ∀ y ξ y' ξ', Commute (A i y ξ) (A j y' ξ')) (hsel : ∀ i (w : H), w ≠ 0 → ∃ ξ, ‖ξ‖ = 1 ∧ A i ξ ξ w ≠ 0) (hproj : ∀ i ξ, ‖ξ‖ = 1 → A i ξ ξ ∗ A i ξ ξ = A i ξ ξ) (Inv : H → Prop) (s : Finset ι) (hstep : ∀ i ∈ s, ∀ ξ w, Inv w → Inv (A i ξ ξ w)) {w₀ : H} (h₀ : Inv w₀) (hw₀ : w₀ ≠ 0) : ∃ (ξ : ∀ i, X i) (w : H), w ≠ 0 ∧ Inv w ∧ ∀ i ∈ s, ‖ξ i‖ = 1 ∧ A i (ξ i) (ξ i) w = w

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.ExistsGradedMatrixEigenBound

theorem **aeval_intertwine** {R : Type*} [Ring R] [Algebra ℝ R] {X A B : R} (h : X * A = B * X) (p : Polynomial ℝ) : X * Polynomial.aeval A p = Polynomial.aeval B p * X

theorem **cj_mul** (h1 : Uᴴ * U = 1) (d e : Fin n → ℝ) : cj U d * cj U e = cj U (fun i => d i * e i)

theorem **cj_conj** (d : Fin n → ℝ) : (cj U d)ᴴ = cj U d

theorem **cj_one** (h2 : U * Uᴴ = 1) : cj U (fun _ => 1) = 1

theorem **cj_add** (d e : Fin n → ℝ) : cj U (fun i => d i + e i) = cj U d + cj U e

theorem **cj_smul** (c : ℝ) (d : Fin n → ℝ) : cj U (fun i => c * d i) = (c : ℂ) • cj U d

theorem **cj_neg** (d : Fin n → ℝ) : cj U (fun i => - d i) = - cj U d

theorem **algebraMap_eq** (a : ℝ) : algebraMap ℝ (Matrix (Fin n) (Fin n) ℂ) a = (a : ℂ) • 1

theorem **cj_aeval** (h1 : Uᴴ * U = 1) (h2 : U * Uᴴ = 1) (d : Fin n → ℝ) (p : Polynomial ℝ) : Polynomial.aeval (cj U d) p = cj U (fun i => p.eval (d i))

theorem **cj_eq_zero** (h1 : Uᴴ * U = 1) {d : Fin n → ℝ} (h : cj U d = 0) (i : Fin n) : d i = 0

theorem **nsq_eq_dot** (v : Fin n → ℂ) : ((nsq v : ℝ) : ℂ) = star v ⬝ᵥ v

theorem **nsq_eq_of_gram** {M N : Matrix (Fin n) (Fin n) ℂ} (h : Mᴴ * M = Nᴴ * N) (y : Fin n → ℂ) : nsq (M *ᵥ y) = nsq (N *ᵥ y)

theorem **nsq_cj** (h1 : Uᴴ * U = 1) (d : Fin n → ℝ) (y : Fin n → ℂ) : nsq (cj U d *ᵥ y) = ∑ i, d i ^ 2 * ‖(Uᴴ *ᵥ y) i‖ ^ 2

theorem **core** {J G H : Matrix (Fin n) (Fin n) ℂ} (hJ : J * J = 1) (hHG : H * G = 1) (hGH : G * H = 1) (hJc : Jᴴ = J) (hHJ : H = J * Gᴴ * J) (hT : (Hᴴ * H).IsHermitian) (f : ℝ → ℝ) (α β : ℝ) (hf : ∀ l : ℝ, 0 < l → (f l = -1 ∨ f l = 0 ∨ f l = 1) ∧ f l⁻¹ = - f l) (hα : ∀ i, f (hT.eigenvalues i) = -1 → α ≤ hT.eigenvalues i) (hβ : ∀ i, f (hT.eigenvalues i) = 0 → β⁻¹ ≤ hT.eigenvalues i) (hne : ∃ i, f (hT.eigenvalues i) ≠ 0) : ∃ A : Matrix (Fin n) (Fin n) ℂ, Aᴴ = A ∧ A * A * A = A ∧ A ≠ 0 ∧ J * A = -A * J ∧ ∀ y : Fin n → ℂ, α * nsq (Pm A *ᵥ y) + β⁻¹ * nsq (Pz A *ᵥ y) ≤ nsq (H *ᵥ y)

theorem **GlobalTheory.CompanionTheory.UGeo.SpecGrad.grading_of_spectral** : ∀ (n : ℕ) (ε : Fin n → Bool) (g : S7.UJ ε) (hT : ((S7.toM g⁻¹)ᴴ * S7.toM g⁻¹).IsHermitian) (f : ℝ → ℝ) (α β : ℝ), (∀ l : ℝ, 0 < l → (f l = -1 ∨ f l = 0 ∨ f l = 1) ∧ f l⁻¹ = - f l) → (∀ i, f (hT.eigenvalues i) = -1 → α ≤ hT.eigenvalues i) → (∀ i, f (hT.eigenvalues i) = 0 → β⁻¹ ≤ hT.eigenvalues i) → (∃ i, f (hT.eigenvalues i) ≠ 0) → ∃ A' : Matrix (Fin n) (Fin n) ℂ, IsGrad ε A' ∧ ∀ y : Fin n → ℂ, α * nsq (Pm A' *ᵥ y) + β⁻¹ * nsq (Pz A' *ᵥ y) ≤ nsq (S7.toM g⁻¹ *ᵥ y)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.ExistsGradedMatrixEigenBoundStatement

theorem **_root_.GlobalTheory.T.exists_graded_matrix_eigen_bound** : ∀ (n : ℕ) (ε : Fin n → Bool) (g : S7.UJ ε) (hT : ((S7.toM g⁻¹)ᴴ * S7.toM g⁻¹).IsHermitian) (f : ℝ → ℝ) (α β : ℝ), (∀ l : ℝ, 0 < l → (f l = -1 ∨ f l = 0 ∨ f l = 1) ∧ f l⁻¹ = - f l) → (∀ i, f (hT.eigenvalues i) = -1 → α ≤ hT.eigenvalues i) → (∀ i, f (hT.eigenvalues i) = 0 → β⁻¹ ≤ hT.eigenvalues i) → (∃ i, f (hT.eigenvalues i) ≠ 0) → ∃ A' : Matrix (Fin n) (Fin n) ℂ, IsGrad ε A' ∧ ∀ y : Fin n → ℂ, α * nsq (Pm A' *ᵥ y) + β⁻¹ * nsq (Pz A' *ᵥ y) ≤ nsq (S7.toM g⁻¹ *ᵥ y)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.ExistsIsoOfHighest

@[simp] theorem **fstL_pair** (x : V) (y : V') : fstL V V' (pair x y) = x

@[simp] theorem **sndL_pair** (x : V) (y : V') : sndL V V' (pair x y) = y

theorem **ext2** {x y : WithLp 2 (V × V')} (h₁ : fstL V V' x = fstL V V' y) (h₂ : sndL V V' x = sndL V V' y) : x = y

theorem **inner_eq** (x y : WithLp 2 (V × V')) : ⟪x, y⟫_ℂ = ⟪fstL V V' x, fstL V V' y⟫_ℂ + ⟪sndL V V' x, sndL V V' y⟫_ℂ

@[simp] theorem **fstL_sumE** (a b : Fin n) (x : WithLp 2 (V × V')) : fstL V V' (sumE E E' a b x) = E a b (fstL V V' x)

@[simp] theorem **sndL_sumE** (a b : Fin n) (x : WithLp 2 (V × V')) : sndL V V' (sumE E E' a b x) = E' a b (sndL V V' x)

theorem **glRel_sumE** (hE : GlRel E) (hE' : GlRel E') : GlRel (sumE E E')

theorem **kUnit_sumE** {ε : Fin n → Bool} (hU : KUnit ε E) (hU' : KUnit ε E') : KUnit ε (sumE E E')

theorem **locFin_sumE** {ε : Fin n → Bool} (hF : LocFin ε E) (hF' : LocFin ε E') : LocFin ε (sumE E E')

theorem **kHighest_fst** {ε : Fin n → Bool} {μ : Fin n → ℝ} {x : WithLp 2 (V × V')} (h : KHighest ε (sumE E E') μ x) : KHighest ε E μ (fstL V V' x)

theorem **kHighest_snd** {ε : Fin n → Bool} {μ : Fin n → ℝ} {x : WithLp 2 (V × V')} (h : KHighest ε (sumE E E') μ x) : KHighest ε E' μ (sndL V V' x)

theorem **kHighest_pair** {ε : Fin n → Bool} {μ : Fin n → ℝ} {x : V} {y : V'} (h : KHighest ε E μ x) (h' : KHighest ε E' μ y) : KHighest ε (sumE E E') μ (pair x y)

theorem **isMin_sumE** {ε : Fin n → Bool} {lam : Fin n → ℝ} (hM : IsMin ε E lam) (hM' : IsMin ε E' lam) : IsMin ε (sumE E E') lam

theorem **exists_iso_of_highest** {ε : Fin n → Bool} {lam : Fin n → ℝ} (hlam : StrictAnti lam) (hE : GlRel E) (hE' : GlRel E') (hU : KUnit ε E) (hU' : KUnit ε E') (hF : LocFin ε E) (hF' : LocFin ε E') (hM : IsMin ε E lam) (hM' : IsMin ε E' lam) (hS : ∀ S : Submodule ℂ V, (∀ a b, ∀ x ∈ S, E a b x ∈ S) → S = ⊥ ∨ S = ⊤) (hS' : ∀ S : Submodule ℂ V', (∀ a b, ∀ x ∈ S, E' a b x ∈ S) → S = ⊥ ∨ S = ⊤) {v₁ : V} {v₂ : V'} (h₁ : v₁ ≠ 0) (h₂ : v₂ ≠ 0) (k₁ : KHighest ε E (lowK ε lam) v₁) (k₂ : KHighest ε E' (lowK ε lam) v₂) : ∃ e : V ≃ₗ[ℂ] V', ∀ a b x, e (E a b x) = E' a b (e x)

theorem **algUniq** : CompanionTheory.RgExh.Fin.AlgUniq

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.ExistsMinimalOfOrthogonalBound

theorem **exists_minimal_of_no_family_below** {H : Type} [NormedAddCommGroup H] [InnerProductSpace ℂ H] [CompleteSpace H] (L : Set (Submodule ℂ H)) (hcl : ∀ S ∈ L, IsClosed (S : Set H)) (hinf : ∀ S ∈ L, ∀ T ∈ L, S ⊓ T¹ ∈ L) (n : ℕ) : ∀ S ∈ L, S ≠ ⊥ → (∀ f : Fin n → Submodule ℂ H, (∀ i, f i ∈ L) → (∀ i, f i ≠ ⊥) → (∀ i, f i ≤ S) → ¬ Pairwise fun i k => f i ⊥ f k) → ∃ Q ∈ L, Q ≠ ⊥ ∧ ∀ T ∈ L, T ≤ Q → T = ⊥ ∨ T = Q

theorem **exists_minimal_of_orthogonal_bound** : ∀ (H : Type) [NormedAddCommGroup H] [InnerProductSpace ℂ H] [CompleteSpace H] (L : Set (Submodule ℂ H)) (m : ℕ) (S₀ : Submodule ℂ H), (∀ S ∈ L, IsClosed (S : Set H)) → (∀ S ∈ L, ∀ T ∈ L, S ⊓ T¹ ∈ L) → S₀ ∈ L → S₀ ≠ ⊥ → (∀ f : Fin (m + 1) → Submodule ℂ H, (∀ i, f i ∈ L) → (∀ i, f i ≠ ⊥) → ¬ Pairwise fun i k => f i ⊥ f k) → ∃ Q ∈ L, Q ≠ ⊥ ∧ ∀ T ∈ L, T ≤ Q → T = ⊥ ∨ T = Q

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.ExistsNoethModel

theorem **exists_good_notMem** {R S : Type} [CommRing R] [CommRing S] [IsNoetherianRing S] (σ : S →+* R) (A : Over (Sp R)) (B : Over (Sp S)) [IsProper B.hom] [Flat B.hom] {M : LB B.left} (hM : M.IsLB) {L : LB A.left} (hL : L.IsLB) {φ : A.left ⟶ B.left} (h : IsPullback φ A.hom B.hom (spMap σ)) (e : LBIso L (M.pull φ)) (hA : Out.RelAmple A L) (p : Ideal R) [p.IsPrime] : ∃ r ∈ good B M, σ r ∉ p

theorem **span_good** {R S : Type} [CommRing R] [CommRing S] [IsNoetherianRing S] (σ : S →+* R) (A : Over (Sp R)) (B : Over (Sp S)) [IsProper B.hom] [Flat B.hom] {M : LB B.left} (hM : M.IsLB) {L : LB A.left} (hL : L.IsLB) {φ : A.left ⟶ B.left} (h : IsPullback φ A.hom B.hom (spMap σ)) (e : LBIso L (M.pull φ)) (hA : Out.RelAmple A L) : Ideal.span (σ '' good B M) = ⊤

theorem **exists_fg_span** {R S : Type} [CommRing R] [CommRing S] [Algebra S R] (T : Set S) (hT : Ideal.span (algebraMap S R '' T) = ⊤) : ∃ C : Subalgebra S R, C.FG ∧ Ideal.span (algebraMap S C '' T) = ⊤

theorem **relAmple_of_span** {S S' : Type} [CommRing S] [CommRing S'] (τ : S →+* S') (B : Over (Sp S)) [IsProper B.hom] {M : LB B.left} (hM : M.IsLB) (T : Set S) (hT : T ⊆ good B M) (h : Ideal.span (τ '' T) = ⊤) : Out.RelAmple (Over.mk (pullback.snd B.hom (spMap τ))) (M.pull (pullback.fst B.hom (spMap τ)))

theorem **exists_noeth_model** (R : Type) [CommRing R] (A : Over (Sp R)) [GrpObj A] (g : ℕ) (hA : IsAbelianScheme A) (hg : SmoothOfRelativeDimension g A.hom) (L : LB A.left) (hL : L.IsLB) : ∃ (S : Type) (_ : CommRing S) (σ : S →+* R) (B : Over (Sp S)) (_ : GrpObj B) (φ : A.left ⟶ B.left) (M : LB B.left), IsNoetherianRing S ∧ IsAbelianScheme B ∧ SmoothOfRelativeDimension g B.hom ∧ IsGrpPullback A B (spMap σ) φ ∧ M.IsLB ∧ LBIso L (M.pull φ)

theorem **polModel** : PolModel

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.ExistsOrderedCreationList

theorem **exists_sorted_list** {α : Type} (S : Finset α) (h : α → ℕ) : ∃ l : List α, l.Nodup ∧ (∀ i, i ∈ l ↔ i ∈ S) ∧ l.Pairwise (fun a b => h a ≤ h b)

theorem **adC_ne_zero_cases** {n : ℕ} {p q : Fin n} {i k : Pos n} (h : adC p q i k ≠ 0) : (q = i.1.1 ∧ k.1 = (p, i.1.2)) ∨ (i.1.2 = p ∧ k.1 = (i.1.1, q))

theorem **mem_inside_iff** {n : ℕ} {ε : Fin n → Bool} {lev : Fin n → ℕ} (hlev : Monotone lev) (i : Pos n) : i ∈ om ε ∧ i ∉ omU ε lev ↔ ε i.1.1 = ε i.1.2 ∧ lev i.1.1 = lev i.1.2

theorem **exists_inside_order** : ∀ (n : ℕ) (ε : Fin n → Bool) (lev : Fin n → ℕ), Monotone lev → ∃ l : List (Pos n), l.Nodup ∧ (∀ i, i ∈ l ↔ i ∈ om ε ∧ i ∉ omU ε lev) ∧ ∀ (pre : List (Pos n)) (f : Pos n) (post : List (Pos n)), l = pre ++ f :: post → ∀ e ∈ l, ∀ k : Pos n, adC e.1.1 e.1.2 f k ≠ 0 → k ∈ post

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.ExistsOrderedCreationListStatement

theorem **_root_.GlobalTheory.T.exists_ordered_creation_list** : ∀ (n : ℕ) (ε : Fin n → Bool) (lev : Fin n → ℕ), Monotone lev → ∃ l : List (Pos n), l.Nodup ∧ (∀ i, i ∈ l ↔ i ∈ om ε ∧ i ∉ omU ε lev) ∧ ∀ (pre : List (Pos n)) (f : Pos n) (post : List (Pos n)), l = pre ++ f :: post → ∀ e ∈ l, ∀ k : Pos n, adC e.1.1 e.1.2 f k ≠ 0 → k ∈ post

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.ExistsPermOfCapPEq

theorem **roots_prod_X_sub_C** {n : ℕ} (k : Fin n → ℂ) : (∏ a, (Polynomial.X − Polynomial.C (k a))).roots = Multiset.map k Finset.univ.val

theorem **exists_perm_of_capelliPolynomial_eq** {n : ℕ} (f g : Fin n → ℂ) (h : ∀ Y, Formalism.QuantumDeterminants.capP f Y = Formalism.QuantumDeterminants.capP g Y) : ∃ τ : Equiv.Perm (Fin n), ∀ a, f a = g (τ a)

theorem **charOfAlgDS** : CompanionTheory.RgExh.Fin.CharOfAlgDS

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.ExistsPointOfIsAffine

theorem **point_ext** {P : LB (A ⊗ B).left} (hP : P.IsLB) (a₂ : IsAdditive (swap P)) (r : RightZero A B P) {x x' : T ⟶ B} (e : evc hP (fst A T) (snd A T ≫ x) = evc hP (fst A T) (snd A T ≫ x')) : x = x'

theorem **exists_level** [IsNoetherianRing R] (hA : IsAbelianScheme A) {C : Type} [CommRing C] (t : Sp C ⟶ Sp R) {M : LB (A ⊗ Over.mk t).left} (hM : M.IsLB) : ∃ (D : Type) (_ : CommRing D) (i : D →+* C) (tD : Sp D ⟶ Sp R) (N : LB (A ⊗ Over.mk tD).left) (hN : N.IsLB) (w : spMap i ≫ tD = t), IsNoetherianRing D ∧ cls M hM = evc hN (fst A _) (snd A _ ≫ (Over.homMk (spMap i) w : Over.mk t ⟶ Over.mk tD))

theorem **lbIso_uni_iff** {N : LB (A ⊗ T₀).left} (hN : N.IsLB) (y : Y ⟶ A ⊗ (A ⊗ T₀)) : LBIso ((uniE N).pull y.left) ((uniF N).pull y.left) ↔ evc hN ((y ≫ fst A _) ∗ (y ≫ snd A _ ≫ fst A T₀)) (y ≫ snd A _ ≫ snd A T₀) = evc hN (y ≫ fst A _) (y ≫ snd A _ ≫ snd A T₀) + evc hN (y ≫ snd A _ ≫ fst A T₀) (y ≫ snd A _ ≫ snd A T₀)

theorem **exists_additive_level** (hA : IsAbelianScheme A) {C D : Type} [CommRing C] [CommRing D] [IsNoetherianRing D] (i : D →+* C) (tD : Sp D ⟶ Sp R) {M : LB (A ⊗ Over.mk (spMap i ≫ tD)).left} (hM : M.IsLB) (hadd : IsAdditive M) {N : LB (A ⊗ Over.mk tD).left} (hN : N.IsLB) (e : cls M hM = evc hN (fst A _) (snd A _ ≫ (Over.homMk (spMap i) rfl : Over.mk (spMap i ≫ tD) ⟶ Over.mk tD))) : ∃ (D₁ : Type) (_ : CommRing D₁) (j : D →+* D₁) (i₁ : D₁ →+* C), IsNoetherianRing D₁ ∧ i₁.comp j = i ∧ IsAdditive (ev N (fst A (Over.mk (spMap j ≫ tD))) (snd A _ ≫ (Over.homMk (spMap j) rfl : Over.mk (spMap j ≫ tD) ⟶ Over.mk tD)))

theorem **exists_point_spec** (hA : IsAbelianScheme A) {P : LB (A ⊗ B).left} (hP : P.IsLB) (U : UnivNoeth A B P) {C : Type} [CommRing C] (t : Sp C ⟶ Sp R) {M : LB (A ⊗ Over.mk t).left} (hM : M.IsLB) (hadd : IsAdditive M) : ∃ x : Over.mk t ⟶ B, cls M hM = evc hP (fst A _) (snd A _ ≫ x)

theorem **cls_eq_evc** {M : LB (A ⊗ T).left} (hM : M.IsLB) : cls M hM = evc hM (fst A T) (snd A T)

theorem **isAdditive_ev** {M : LB (A ⊗ T).left} (hM : M.IsLB) (h : IsAdditive M) (v : T' ⟶ T) : IsAdditive (ev M (fst A T') (snd A T' ≫ v))

theorem **surjective_snd** (hA : IsAbelianScheme A) (V : T.left.Opens) : Function.Surjective ((snd A T).left ⁻¹_ V).appTop.hom

theorem **exists_point_of_isAffine** (hA : IsAbelianScheme A) {P : LB (A ⊗ B).left} (hP : P.IsLB) (U : UnivNoeth A B P) {T : Over (Sp R)} [IsAffine T.left] {M : LB (A ⊗ T).left} (hM : M.IsLB) (hadd : IsAdditive M) : ∃ x : T ⟶ B, cls M hM = evc hP (fst A T) (snd A T ≫ x)

theorem **exists_point** (hA : IsAbelianScheme A) {P : LB (A ⊗ B).left} (hP : P.IsLB) (a₁ : IsAdditive P) (a₂ : IsAdditive (swap P)) (r : RightZero A B P) (U : UnivNoeth A B P) {T : Over (Sp R)} {M : LB (A ⊗ T).left} (hM : M.IsLB) (hadd : IsAdditive M) : ∃ x : T ⟶ B, cls M hM = evc hP (fst A T) (snd A T ≫ x)

theorem **finDual_of_univNoeth** : (∀ (R : Type) [CommRing R] [IsNoetherianRing R] (A B : Over (Sp R)) [GrpObj A] [GrpObj B] (g : ℕ), IsAbelianScheme A → IsAbelianScheme B → SmoothOfRelativeDimension g A.hom → SmoothOfRelativeDimension g B.hom → ∀ P : LB (A ⊗ B).left, PreDual A B P → UnivNoeth A B P) → FinDual

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.ExistsPowEqZero

lemma **isWeight_pow** (hF : KRel ε E) {a b : Fin n} (he : ε a = ε b) {μ : Fin n → ℝ} {z : V} (hz : IsWeight E μ z) (k : ℕ) : IsWeight E (shift μ a b k) ((E a b ^ k) z)

lemma **norm_identity** (hF : KRel ε E) (hU : KUnit ε E) {a b : Fin n} (he : ε a = ε b) {μ : Fin n → ℝ} {w : V} (hw : IsWeight E μ w) : ‖E a b w‖ ^ 2 = ‖E b a w‖ ^ 2 + (μ b − μ a) ∗ ‖w‖ ^ 2

lemma **string_bound** (hF : KRel ε E) (hU : KUnit ε E) {a b : Fin n} (hab : a ≠ b) (he : ε a = ε b) {μ : Fin n → ℝ} {z : V} (hz : IsWeight E μ z) (k : ℕ) : (k + 1) ∗ (μ b − μ a − k) ∗ ‖(E a b ^ k) z‖ ^ 2 ≤ ‖(E a b ^ (k + 1)) z‖ ^ 2

lemma **exists_pow_eq_zero** (hF : KRel ε E) (hU : KUnit ε E) {L : Submodule ℂ V} [FiniteDimensional ℂ L] (hL : KStab ε E L) {a b : Fin n} (hab : a ≠ b) (he : ε a = ε b) {μ : Fin n → ℝ} {z : V} (hzL : z ∈ L) (hz : IsWeight E μ z) : (∀ k : ℕ, (E a b ^ k) z ∈ L) ∧ ∃ k : ℕ, (E a b ^ k) z = 0

lemma **swap_weight_of_le** (hF : KRel ε E) (hU : KUnit ε E) {L : Submodule ℂ V} [FiniteDimensional ℂ L] (hL : KStab ε E L) {a b : Fin n} (hab : a ≠ b) (he : ε a = ε b) {μ : Fin n → ℝ} {z : V} (hzL : z ∈ L) (hz0 : z ≠ 0) (hz : IsWeight E μ z) (hle : μ a ≤ μ b) : ∃ z' ∈ L, z' ≠ 0 ∧ IsWeight E (μ ∘ Equiv.swap a b) z'

lemma **swap_weight** (hF : KRel ε E) (hU : KUnit ε E) {L : Submodule ℂ V} [FiniteDimensional ℂ L] (hL : KStab ε E L) {a b : Fin n} (hab : a ≠ b) (he : ε a = ε b) {μ : Fin n → ℝ} {z : V} (hzL : z ∈ L) (hz0 : z ≠ 0) (hz : IsWeight E μ z) : ∃ z' ∈ L, z' ≠ 0 ∧ IsWeight E (μ ∘ Equiv.swap a b) z'

lemma **perm_weight** (hF : KRel ε E) (hU : KUnit ε E) {L : Submodule ℂ V} [FiniteDimensional ℂ L] (hL : KStab ε E L) : ∀ (m : ℕ) (w : Equiv.Perm (Fin n)), w.support.card = m → (∀ c, ε (w c) = ε c) → ∀ {μ : Fin n → ℝ} {z : V}, z ∈ L → z ≠ 0 → IsWeight E μ z → ∃ z' ∈ L, z' ≠ 0 ∧ IsWeight E (μ ∘ w) z'

private lemma **casK_mul_comm_of_eq_sign** {A : Type} [Ring A] {F : Fin n → Fin n → A} (hF : KRel ε F) {c d : Fin n} (hcd : ε c = ε d) : CasK ε F ∗ F c d = F c d ∗ CasK ε F

private lemma **casK_apply_eq_of_kHighest** (hF : KRel ε E) {v : Fin n → ℝ} {y : V} (hy : KHighest ε E v y) : CasK ε E y = ((PI.UpqRep.nsq (v + PI.UpqRep.rhoC ε) − PI.UpqRep.nsq (PI.UpqRep.rhoC ε) : ℝ) : ℂ) • y

theorem **wtMajor_of** (hlow : CompanionTheory.D.DSIC.LowSpan) : CompanionTheory.D.LK.WtMajor

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.ExistsUniformizerNonarchimedean

theorem **exists_isUnif** : ∀ (L : Type) [NontriviallyNormedField L] [IsUltrametricDist L] [ProperSpace L], ∃ p : Lˣ, GlobalTheory.CompanionTheory.CartGL.IsUnif p

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.ExistsUniformizerNonarchimedeanStatement

theorem **_root_.GlobalTheory.T.exists_uniformizer_nonarchimedean** : ∀ (L : Type) [NontriviallyNormedField L] [IsUltrametricDist L] [ProperSpace L], ∃ p : Lˣ, GlobalTheory.CompanionTheory.CartGL.IsUnif p

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.ExistsUniqueParam

theorem **exists_unique_param** {n : ℕ} (χ : Fin n → ℂ) (hχ : Function.Injective χ) (h : ∀ a, (χ a).im = 0 ∧ ∃ m : ℤ, (χ a).re − ((n : ℝ) − 1) / 2 = m) : ∃ μ : Fin n → ℝ, ∀ m : Fin n → ℝ, (Formalism.QuantumDeterminants.IsHCParam m ∧ ∃ τ : Equiv.Perm (Fin n), ∀ a, ((m a : ℝ) : ℂ) = χ (τ a)) ↔ m = μ

theorem **arrCount** : CompanionTheory.RgExh.Fin.ArrCount

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.ExistsUnitaryImage

theorem **injective_of_simple** {M : Type∗} [AddCommGroup M] [Module ℂ M] {E : Fin n → Fin n → Module.End ℂ X} {F : Fin n → Fin n → Module.End ℂ M} (hS : ∀ W : Submodule ℂ X, (∀ a b, ∀ x ∈ W, E a b x ∈ W) → W = ⊥ ∨ W = ⊤) (c : X →ₗ[ℂ] M) (hc : ∀ a b x, c (E a b x) = F a b (c x)) (h0 : c ≠ 0) : Function.Injective c

theorem **adjSign_eq_sgn** (ε : Fin n → Bool) (a b : Fin n) : adjSign ε a b = ((Formalism.REAL.sgn ε a b : ℝ) : ℂ)

theorem **exists_unitary_image** {ε : Fin n → Bool} {σ : Matrix (Fin n) (Fin n) ℂ → (H →L[ℂ] H)} (hσ : IsURepOn (UJset ε) σ) {E : Fin n → Fin n → Module.End ℂ X} (hS : ∀ W : Submodule ℂ X, (∀ a b, ∀ x ∈ W, E a b x ∈ W) → W = ⊥ ∨ W = ⊤) (c : X →ₗ[ℂ] smD ε σ) (hc : ∀ a b x, c (E a b x) = smE ε σ a b (c x)) (h0 : c ≠ 0) : ∃ (V : Submodule ℂ H) (E' : Fin n → Fin n → Module.End ℂ V) (e : X ≃ₗ[ℂ] V), (∀ x, ((e x : V) : H) = ((c x : smD ε σ) : H)) ∧ Formalism.REAL.GlRel E' ∧ Formalism.REAL.IsUnitary ε E' ∧ ∀ a b x, e (E a b x) = E' a b (e x)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.ExpMemUJ

theorem **flow_mem** {H : Type} [NormedAddCommGroup H] [InnerProductSpace ℂ H] (W : Submodule ℂ H) [FiniteDimensional ℂ W] (T : ℝ → (H →L[ℂ] H)) (h0 : ∀ v, T 0 v = v) (hadd : ∀ s t v, T (s + t) v = T s (T t v)) (hiso : ∀ t v, ‖T t v‖ = ‖v‖) (hder : ∀ w ∈ W, ∃ z ∈ W, HasDerivAt (fun t => T t w) z 0) (u : H) (hu : u ∈ W) : T 1 u ∈ W

theorem **deriv_mem** {H : Type} [NormedAddCommGroup H] [InnerProductSpace ℂ H] (W : Submodule ℂ H) [FiniteDimensional ℂ W] (c : ℝ → H) (z : H) (hc : ∀ t, c t ∈ W) (hz : HasDerivAt c z 0) : z ∈ W

theorem **inK_iff** (ε : Fin n → Bool) (X : Matrix (Fin n) (Fin n) ℂ) : inK ε X ↔ (∀ c d, ε c ≠ ε d → X c d = 0) ∧ ∀ c d, star (X d c) = −X c d

theorem **inK_smul** (ε : Fin n → Bool) {X : Matrix (Fin n) (Fin n) ℂ} (hX : inK ε X) (t : ℝ) : inK ε ((t : ℂ) • X)

theorem **exp_mem_K** (ε : Fin n → Bool) {X : Matrix (Fin n) (Fin n) ℂ} (hX : inK ε X) (t : ℝ) : NormedSpace.exp ((t : ℂ) • X) ∈ Kset ε

theorem **exp_mem_UJ** (ε : Fin n → Bool) {X : Matrix (Fin n) (Fin n) ℂ} (hX : inG ε X) (t : ℝ) : NormedSpace.exp ((t : ℂ) • X) ∈ UJset ε

theorem **stabLink** : KExp → StabLink

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.ExponentialMembership

theorem **lin_exp_eq** {n : ℕ} (A : Matrix (Fin n) (Fin n) ℂ) (L : Matrix (Fin n) (Fin n) ℂ →ₗ[ℂ] ℂ) (h : ∀ m : ℕ, L (A ^ (m + 1)) = 0) : L (NormedSpace.exp A) = L 1

theorem **ex_mulVec** {n : ℕ} (X : Mat n) (t : ℝ) (v : Fin n → ℂ) (hv : X ∗ᵥ v = 0) : ex X t ∗ᵥ v = v

theorem **ex_entry** {n : ℕ} (X : Mat n) (t : ℝ) (a : ℕ) (j : Fin n) (h1 : ∀ i l : Fin n, a ≤ i.val → l.val < a → X i l = 0) (h2 : ∀ i : Fin n, a ≤ i.val → X i j = 0) (i : Fin n) (hi : a ≤ i.val) : ex X t i j = if i = j then 1 else 0

private theorem **exp_mul_eq_mul_exp_of_semiconj** {𝔸 : Type*} [NormedRing 𝔸] [NormedAlgebra ℂ 𝔸] [CompleteSpace 𝔸] (B C H : 𝔸) (h : B * H = H * C) : NormedSpace.exp B * H = H * NormedSpace.exp C

theorem **expMem** : ExpMem

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.ExponentialMembershipStatement

theorem **_root_.GlobalTheory.T.ExponentialMembership** : ExpMem

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.ExteriorDifferentialMatrixUnitFormula

theorem **hasDerivAt_det** {m : ℕ} (A : ℝ → Matrix (Fin m) (Fin m) ℂ) (A' : Matrix (Fin m) (Fin m) ℂ) (h : ∀ i j, HasDerivAt (fun t => A t i j) (A' i j) 0) : HasDerivAt (fun t => (A t).det) (∑ j, ∑ i, adjugate (A 0) j i * A' i j) 0

theorem **det_one_submatrix** {m n : ℕ} (f g : Fin m → Fin n) (hf : StrictMono f) (hg : StrictMono g) : ((1 : Matrix (Fin n) (Fin n) ℂ).submatrix f g).det = if univ.image f = univ.image g then 1 else 0

theorem **card_filter_lt** {m n : ℕ} (f : Fin m → Fin n) (hf : StrictMono f) (i : Fin m) : (((univ.image f).erase (f i)).filter fun c => c < f i).card = i

theorem **image_succAbove** {m n : ℕ} (f : Fin (m + 1) → Fin n) (hf : StrictMono f) (i : Fin (m + 1)) : univ.image (f ∘ i.succAbove) = (univ.image f).erase (f i)

theorem **sum_adj** {m n : ℕ} (f g : Fin m → Fin n) (hf : StrictMono f) (hg : StrictMono g) (a b : Fin n) : ∑ j, ∑ i, adjugate ((1 : Matrix (Fin n) (Fin n) ℂ).submatrix f g) j i * (Matrix.single a b (1 : ℂ)) (f i) (g j) = if b ∈ univ.image g ∧ a ∉ (univ.image g).erase b ∧ univ.image f = insert a ((univ.image g).erase b) then (-1) ^ (((univ.image g).erase b).filter fun c => c < a).card * (-1) ^ (((univ.image g).erase b).filter fun c => c < b).card else 0

theorem **dR_wedge** (k r m : ℕ) (X : Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ) (J J' : Sub (2 * r + k + 1) m) : (wedge k r m).dR X J J' = ∑ j, ∑ i, adjugate ((1 : Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ).submatrix (J.1.orderEmbOfFin J.2) (J'.1.orderEmbOfFin J'.2)) j i * X (J.1.orderEmbOfFin J.2 i) (J'.1.orderEmbOfFin J'.2 j)

theorem **reH_add_imH** (k r : ℕ) (Y : Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ) : reH k r Y + Complex.I • imH k r Y = Y

theorem **dC_wedge_single** : ∀ (k r m : ℕ) (a b : Fin (2 * r + k + 1)) (J J' : Sub (2 * r + k + 1) m), (wedge k r m).dC (Matrix.single a b 1) J J' = if b ∈ J'.1 ∧ a ∉ J'.1.erase b ∧ J.1 = insert a (J'.1.erase b) then (-1) ^ ((J'.1.erase b).filter fun c => c < a).card * (-1) ^ ((J'.1.erase b).filter fun c => c < b).card else 0

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.ExteriorDifferentialMatrixUnitFormulaStatement

theorem **_root_.GlobalTheory.T.exterior_differential_matrix_unit_formula** : ∀ (k r m : ℕ) (a b : Fin (2 * r + k + 1)) (J J' : Sub (2 * r + k + 1) m), (wedge k r m).dC (Matrix.single a b 1) J J' = if b ∈ J'.1 ∧ a ∉ J'.1.erase b ∧ J.1 = insert a (J'.1.erase b) then (-1) ^ ((J'.1.erase b).filter fun c => c < a).card * (-1) ^ ((J'.1.erase b).filter fun c => c < b).card else 0

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.ExteriorDifferentialSignRuleQuadratic

theorem **jW_mul_transpose** (n m : ℕ) (J J' : Sub n m) (s s' : Fin n) : (jW n m * (jW n m)ᵀ) (J, s) (J', s') = if s ∉ J.1 ∧ s' ∉ J'.1 ∧ insert s J.1 = insert s' J'.1 then (-1) ^ (J.1.filter fun a => a < s).card * (-1) ^ (J'.1.filter fun a => a < s').card else 0

theorem **anticomm** {n : ℕ} (J J' : Finset (Fin n)) (s s' : Fin n) : (if s ∈ J' ∧ s' ∉ J'.erase s ∧ J = insert s' (J'.erase s) then (-1 : ℂ) ^ ((J'.erase s).filter fun c => c < s').card * (-1) ^ ((J'.erase s).filter fun c => c < s).card else 0) = (if J = J' ∧ s = s' then 1 else 0) - if s ∉ J ∧ s' ∉ J' ∧ insert s J = insert s' J' then (-1) ^ (J.filter fun a => a < s).card * (-1) ^ (J'.filter fun a => a < s').card else 0

theorem **wedgeQ** : StdD → (∀ (k r m : ℕ) (a b : Fin (2 * r + k + 1)) (J J' : Sub (2 * r + k + 1) m), (wedge k r m).dC (Matrix.single a b 1) J J' = if b ∈ J'.1 ∧ a ∉ J'.1.erase b ∧ J.1 = insert a (J'.1.erase b) then (-1) ^ ((J'.1.erase b).filter fun c => c < a).card * (-1) ^ ((J'.1.erase b).filter fun c => c < b).card else 0) → WedgeQ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.ExteriorDifferentialSignRuleQuadraticStatement

theorem **_root_.GlobalTheory.T.exterior_differential_sign_rule_quadratic** : StdD → (∀ (k r m : ℕ) (a b : Fin (2 * r + k + 1)) (J J' : Sub (2 * r + k + 1) m), (wedge k r m).dC (Matrix.single a b 1) J J' = if b ∈ J'.1 ∧ a ∉ J'.1.erase b ∧ J.1 = insert a (J'.1.erase b) then (-1) ^ ((J'.1.erase b).filter fun c => c < a).card * (-1) ^ ((J'.1.erase b).filter fun c => c < b).card else 0) → WedgeQ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.ExteriorIntegrality

theorem **er_emb** (I : BranchingSupport.Sub n (m + 1)) (j : Fin (m + 1)) : ⇑((er I j).1.orderEmbOfFin (er I j).2) = fun a => I.1.orderEmbOfFin I.2 (j.succAbove a)

theorem **exists_emb** (I : BranchingSupport.Sub n (m + 1)) {t : Fin n} (ht : t ∈ I.1) : ∃ j, I.1.orderEmbOfFin I.2 j = t

theorem **er_card** (I : BranchingSupport.Sub n (m + 1)) (j : Fin (m + 1)) : ((er I j).1.filter fun a => a < I.1.orderEmbOfFin I.2 j).card = j

theorem **jW_er** (I I' : BranchingSupport.Sub n (m + 1)) (j : Fin (m + 1)) : jW n m (er I j, I.1.orderEmbOfFin I.2 j) I' = if I' = I then (-1) ^ (j : ℕ) else 0

theorem **jW_ne** (p : BranchingSupport.Sub n m × Fin n) (I : BranchingSupport.Sub n (m + 1)) (h : jW n m p I ≠ 0) : ∃ j, (er I j, I.1.orderEmbOfFin I.2 j) = p

theorem **sum_jW** (F : BranchingSupport.Sub n m × Fin n → ℂ) (I : BranchingSupport.Sub n (m + 1)) : ∑ p, F p * jW n m p I = ∑ j : Fin (m + 1), (-1) ^ (j : ℕ) * F (er I j, I.1.orderEmbOfFin I.2 j)

theorem **laplace** (g : Matrix (Fin n) (Fin n) ℂ) (r : Fin (m + 1) → Fin n) (i : Fin (m + 1)) (J : BranchingSupport.Sub n m) (hJ : ⇑(J.1.orderEmbOfFin J.2) = fun a => r (i.succAbove a)) (I : BranchingSupport.Sub n (m + 1)) : ((cmp m g ⊗ₖ g) * jW n m) (J, r i) I = (-1) ^ (i : ℕ) * (g.submatrix r (I.1.orderEmbOfFin I.2)).det

theorem **first** (g : Matrix (Fin n) (Fin n) ℂ) : jW n m * cmp (m + 1) g = (cmp m g ⊗ₖ g) * jW n m

theorem **cmp_tr** (l : ℕ) (g : Matrix (Fin n) (Fin n) ℂ) : BranchingSupport.cmp l gᵀ = (BranchingSupport.cmp l g)ᵀ

theorem **third** : (jW n m)ᵀ * jW n m = ((m : ℂ) + 1) • (1 : Matrix (BranchingSupport.Sub n (m + 1)) (BranchingSupport.Sub n (m + 1)) ℂ)

theorem **wedgeInt** : WedgeInt

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.ExteriorIntegralityStatement

theorem **_root_.GlobalTheory.T.ExteriorIntegrality** : WedgeInt

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.ExteriorPowerIntegral

theorem **contDiff_minor** {n l : ℕ} (a b : Fin l → Fin n) : ContDiff ℝ ∞ fun g : Matrix (Fin n) (Fin n) ℂ => (g.submatrix a b).det

theorem **okWedge** : OKWedge

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.ExteriorPowerIntegralStatement

theorem **_root_.GlobalTheory.T.ExteriorPowerIntegral** : OKWedge

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.FactorizationStatement

theorem **proj_comm_of_star_closed** {H : Type} [NormedAddCommGroup H] [InnerProductSpace ℂ H] [CompleteSpace H] {N : Set (H →L[ℂ] H)} (hN : ∀ T ∈ N, star T ∈ N) (S : Submodule ℂ H) [S.HasOrthogonalProjection] (hS : ∀ T ∈ N, ∀ v ∈ S, T v ∈ S) (T : H →L[ℂ] H) (hT : T ∈ N) : T * S.starProjection = S.starProjection * T

theorem **stFactor_from** (h1 : ∀ (G H : Type) [Group G] [NormedAddCommGroup H] [InnerProductSpace ℂ H] [CompleteSpace H] (σ : G →* (H →L[ℂ] H)), (∀ g v, ‖σ g v‖ = ‖v‖) → ∀ (E : H →L[ℂ] H) (m : ℕ), (∀ g : Fin m → G, stdPoly E (fun i => σ (g i)) = 0) → ∀ a : Fin m → (H →L[ℂ] H), (∀ i, ∀ T : H →L[ℂ] H, (∀ g, T * σ g = σ g * T) → T * a i = a i * T) → stdPoly E a = 0) (h2 : ∀ (G G₁ H : Type) [Group G] [Group G₁] [NormedAddCommGroup H] [InnerProductSpace ℂ H] (π : G →* (H →L[ℂ] H)), IsTopIrreducible π → ∀ (j : G₁ →* G) (p : G →* G₁), (∀ x, p (j x) = x) → (∀ x g, p g = 1 → Commute (j x) g) → ∀ x y : H →L[ℂ] H, (∀ T : H →L[ℂ] H, (∀ g, T * π (j g) = π (j g) * T) → T * y = y * T) → y ≠ 0 → (∀ g, x * π (j g) * y = 0) → x = 0) (h3 : ∀ (R : Type) [Ring R] (A S : Set R) (E : R) (m : ℕ) (q : Fin (m + 1) → R), (∀ a ∈ A, ∀ b ∈ A, a * b ∈ A) → S ⊆ A → (∀ x ∈ A, ∀ y ∈ A, y ≠ 0 → (∀ s ∈ S, x * s * y = 0) → x =

theorem **stFactor** : StFactor

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.FactorizationStatementStatement

theorem **_root_.GlobalTheory.T.FactorizationStatement** : StFactor

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.FamilyFiltration

theorem **wInt_anti** {s s' : ℝ} (h : s ≤ s') (f : ℝ → ℂ) : wInt f s' ≤ wInt f s

theorem **enorm_add_sq_le** (a b : ℂ) : ‖a + b‖ₑ ^ 2 ≤ 2 * ‖a‖ₑ ^ 2 + 2 * ‖b‖ₑ ^ 2

theorem **measurable_integrand** {f : ℝ → ℂ} (hf : Continuous f) (s : ℝ) : Measurable fun t : ℝ => ‖f t‖ₑ ^ 2 * ENNReal.ofReal (Real.exp (-2 * s * t))

theorem **inUJ_mul** {ε : Fin n → Bool} {g h : Matrix (Fin n) (Fin n) ℂ} (hg : Formalism.REAL.inUJ ε g) (hh : Formalism.REAL.inUJ ε h) : Formalism.REAL.inUJ ε (g * h)

theorem **line_hasDerivAt** (hc : IsCoefFam ε X c) (hA : Formalism.REAL.inG ε A) {g : Matrix (Fin n) (Fin n) ℂ} (hg : Formalism.REAL.inUJ ε g) (i : ι) (u : X.V) (t : ℝ) : HasDerivAt (line c A g i u) (line c A g i (actM X.E A u) t) t

theorem **line_continuous** (hc : IsCoefFam ε X c) (hA : Formalism.REAL.inG ε A) {g : Matrix (Fin n) (Fin n) ℂ} (hg : Formalism.REAL.inUJ ε g) (i : ι) (u : X.V) : Continuous (line c A g i u)

theorem **mem_NSet_iff** (hc : IsCoefFam ε X c) (hA : Formalism.REAL.inG ε A) (μ : Measure (S7.UJ ε)) (s : ℝ) (u : X.V) : u ∈ NSet ε c A μ s ↔ ∀ i, ∀ᵐ x ∂μ, line c A (S7.toM x) i u ∈ weightedL2Continuous s

theorem **exp_conj_eq** {A Y : Matrix (Fin n) (Fin n) ℂ} {k : ℤ} (hY : A * Y - Y * A = (k : ℂ) • Y) (t : ℝ) : NormedSpace.exp ((t : ℂ) • A) * Y = ((Real.exp (k * t) : ℝ) : ℂ) • (Y * NormedSpace.exp ((t : ℂ) • A))

theorem **left_expand** {i : ι} {l : Fin n → Fin n → ι →₀ ℂ} (hl : ∀ (a b : Fin n) (u : X.V) (g : Matrix (Fin n) (Fin n) ℂ), Formalism.REAL.inUJ ε g → c i (X.E a b u) g = ∑ c', ∑ d, (g * Matrix.single a b (1 : ℂ) * g⁻¹ : Matrix (Fin n) (Fin n) ℂ) c' d * (l c' d).sum fun i' r => r * c i' u g) (Y : Matrix (Fin n) (Fin n) ℂ) (u : X.V) {g : Matrix (Fin n) (Fin n) ℂ} (hg : Formalism.REAL.inUJ ε g) : c i (actM X.E Y u) g = ∑ c', ∑ d, (g * Y * g⁻¹ : Matrix (Fin n) (Fin n) ℂ) c' d * (l c' d).sum fun i' r => r * c i' u g

theorem **wInt_shift** {f h : ℝ → ℂ} {k s : ℝ} (hp : ∀ t, f t = ((Real.exp (k * t) : ℝ) : ℂ) * h t) : wInt f (s + k) = wInt h s

theorem **shift_mem** (hc : IsCoefFam ε X c) (hA : Formalism.REAL.inG ε A) (μ : Measure (S7.UJ ε)) (Y : Matrix (Fin n) (Fin n) ℂ) (k : ℤ) (hY : A * Y - Y * A = (k : ℂ) • Y) (s : ℝ) (u : X.V) (hu : u ∈ NSet ε c A μ s) : actM X.E Y u ∈ NSet ε c A μ (s + k)

theorem **eig_bounds** (hc : IsCoefFam ε X c) (hA : Formalism.REAL.inG ε A) (μ : Measure (S7.UJ ε)) {s s' : ℝ} (hlt : s' < s) (htop : ∀ u, u ∈ NSet ε c A μ s) (z : ℂ) (u : X.V) (hu : u ∉ NSet ε c A μ s') (hv : actM X.E A u - z • u ∈ NSet ε c A μ s') : s' ≤ z.re ∧ z.re < s

theorem **famFilt** : CompanionTheory.RgExh.Dec.J.FamFilt

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.FamilyInduction

theorem **mul_inUJ** {a b : Matrix (Fin n) (Fin n) ℂ} (ha : Formalism.REAL.inUJ ε a) (hb : Formalism.REAL.inUJ ε b) : Formalism.REAL.inUJ ε (a * b)

theorem **inv_mul_of_inUJ** {g : Matrix (Fin n) (Fin n) ℂ} (hg : Formalism.REAL.inUJ ε g) : g⁻¹ * g = 1

theorem **exp_hasSum** (X : Matrix (Fin n) (Fin n) ℂ) : HasSum (fun k : ℕ => ((k.factorial : ℂ)⁻¹) • X ^ k) (NormedSpace.exp X)

theorem **exp_mul_eig** (A P : Matrix (Fin n) (Fin n) ℂ) (lam τ : ℂ) (h : A * P = lam • P) : NormedSpace.exp (τ • A) * P = Complex.exp (τ * lam) • P

theorem **exp_three** {A : Matrix (Fin n) (Fin n) ℂ} (hA : A * A * A = A) (τ : ℂ) : NormedSpace.exp (τ • A) = Complex.exp τ • ((1 / 2 : ℂ) • (A * A + A)) + Complex.exp (-τ) • ((1 / 2 : ℂ) • (A * A - A)) + (1 - A * A)

theorem **exp_entry_bound** {A : Matrix (Fin n) (Fin n) ℂ} (hA : A * A * A = A) : ∃ C : ℝ, 0 ≤ C ∧ ∀ t : ℝ, 0 ≤ t → ∀ i j, ‖NormedSpace.exp ((t : ℂ) • A) i j‖ ≤ C * Real.exp t ∧ ‖NormedSpace.exp (-((t : ℂ) • A)) i j‖ ≤ C * Real.exp t

theorem **mul_entry_le** (x y : Matrix (Fin n) (Fin n) ℂ) {B : ℝ} (hy : ∀ i j, ‖y i j‖ ≤ B) (j k : Fin n) : ‖(x * y) j k‖ ≤ (∑ a, ∑ b, ‖x a b‖) * B

theorem **mul_entry_le'** (x y : Matrix (Fin n) (Fin n) ℂ) {B : ℝ} (hy : ∀ i j, ‖y i j‖ ≤ B) (j k : Fin n) : ‖(y * x) j k‖ ≤ (∑ a, ∑ b, ‖x a b‖) * B

theorem **lint_lt_top** {f : ℝ → ℂ} {h w : Fin n → ℝ → ℂ} {C s : ℝ} (hf : ∀ t, 0 < t → f t = ∑ k, w k t * h k t) (hw : ∀ k t, 0 < t → ‖w k t‖ ≤ C * Real.exp t) (hh : ∀ k, Continuous (h k)) (hfin : ∀ k, ∫⁻ t in Set.Ioi (0 : ℝ), ‖h k t‖ₑ ^ 2 ∂ENNReal.ofReal (Real.exp (-2 * (s - 1) * t)) < ⊤) : ∫⁻ t in Set.Ioi (0 : ℝ), ‖f t‖ₑ ^ 2 ∂ENNReal.ofReal (Real.exp (-2 * s * t)) < ⊤

theorem **line_hasDeriv** (hc : IsCoefFam ε X c) (i : ι) (u : X.V) {Y g : Matrix (Fin n) (Fin n) ℂ} (hY : Formalism.REAL.inG ε Y) (hg : Formalism.REAL.inUJ ε g) (s : ℝ) : HasDerivAt (fun t : ℝ => c i u (g * NormedSpace.exp ((t : ℂ) • Y))) (c i (actM X.E Y u) (g * NormedSpace.exp ((s : ℂ) • Y))) s

theorem **line_cont** (hc : IsCoefFam ε X c) (i : ι) (u : X.V) {Y g : Matrix (Fin n) (Fin n) ℂ} (hY : Formalism.REAL.inG ε Y) (hg : Formalism.REAL.inUJ ε g) : Continuous (fun t : ℝ => c i u (g * NormedSpace.exp ((t : ℂ) • Y)))

theorem **stepCoef_apply** (X : GMod n) (z : ℂ) (c : X.V →ₗ[ℂ] (Matrix (Fin n) (Fin n) ℂ → ℂ)) (j : Fin n) (u : (step X z).V) (g : Matrix (Fin n) (Fin n) ℂ) : stepCoef X z c j u g = ∑ k, g j k * c (incl X z u k) g

theorem **incl_E** (X : GMod n) (z : ℂ) (a b : Fin n) (u : (step X z).V) : incl X z ((step X z).E a b u) = tensE X a b (incl X z u)

theorem **incl_actM** (X : GMod n) (z : ℂ) (Y : Matrix (Fin n) (Fin n) ℂ) (u : (step X z).V) : incl X z (actM (step X z).E Y u) = actM (tensE X) Y (incl X z u)

theorem **actM_tens_apply** (X : GMod n) (Y : Matrix (Fin n) (Fin n) ℂ) (v : Fin n → X.V) (k : Fin n) : actM (tensE X) Y v k = actM X.E Y (v k) + ∑ b, Y k b • v b

theorem **step_right** (hc : IsCoefFam ε X c) (z : ℂ) (i : ι) (j : Fin n) (u : (step X z).V) {Y g : Matrix (Fin n) (Fin n) ℂ} (hY : Formalism.REAL.inG ε Y) (hg : Formalism.REAL.inUJ ε g) : HasDerivAt (fun t : ℝ => stepCoef X z (c i) j u (g * NormedSpace.exp ((t : ℂ) • Y))) (stepCoef X z (c i) j (actM (step X z).E Y u) g) 0

theorem **finsupp_sum_comm** (L : ι →₀ ℂ) (w : Fin n → ℂ) (φ : ι → Fin n → ℂ) : (L.sum fun i r => r * ∑ k, w k * φ i k) = ∑ k, w k * L.sum fun i r => r * φ i k

theorem **step_left** (hc : IsCoefFam ε X c) (z : ℂ) (i : ι) (j : Fin n) : ∃ l : Fin n → Fin n →₀ ℂ, ∀ (a b : Fin n) (u : (step X z).V) (g : Matrix (Fin n) (Fin n) ℂ), Formalism.REAL.inUJ ε g → stepCoef X z (c i) j ((step X z).E a b u) g = ∑ c', ∑ d, (g * Matrix.single a b (1 : ℂ) * g⁻¹ : Matrix (Fin n) (Fin n) ℂ) c' d * (l c' d).sum fun q r => r * stepCoef X z (c q.1) q.2 u g

theorem **inv_combine** (W g : Matrix (Fin n) (Fin n) ℂ) (hW : W * g = 1) (φ : Fin n → ℂ) (k : Fin n) : φ k = ∑ j, W k j * ∑ l, g j l * φ l

theorem **stepFam** : StepFam

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.FarRegOfLktUnitIneqFarNum

theorem **rho_sum** (ε : Fin n → Bool) (a : Fin n) : rhoN ε a + rhoC ε a = ((n : ℝ) - 1) / 2 - ((a : ℕ) : ℝ)

theorem **abs_rhoC_le** (ε : Fin n → Bool) (a : Fin n) : |rhoC ε a| ≤ n

theorem **two_rhoC_int** (ε : Fin n → Bool) (a : Fin n) : ∃ m : ℤ, 2 * rhoC ε a = m

theorem **rhoC_lt** (ε : Fin n → Bool) {a b : Fin n} (hab : a < b) (he : ε a = ε b) : rhoC ε b < rhoC ε a

theorem **levS_mono** (y : ℕ → ℝ) (L : ℝ) : Monotone (levS y L)

theorem **levS_gap** {y : ℕ → ℝ} (hy : Antitone y) {L : ℝ} {a b : ℕ} (h : levS y L a < levS y L b) : L ≤ y a - y b

theorem **levS_flat** (y : ℕ → ℝ) (L : ℝ) (a m : ℕ) (h : levS y L a = levS y L (a + m)) : y a - y (a + m) ≤ m * L

theorem **exists_lev** (x : Fin n → ℝ) (hx : Antitone x) {L : ℝ} (hL : 0 ≤ L) : ∃ lev : Fin n → ℕ, Monotone lev ∧ (∀ a b, lev a < lev b → L ≤ x a - x b) ∧ ∀ a b, lev a = lev b → |x a - x b| ≤ n * L

theorem **farReg_of_lkt_unitIneq_farNum** : LKTExists → Renumber → UnitIneq → COBlock → FarNum → FarReg

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.FewEquationsStatementStatement

theorem **_root_.GlobalTheory.T.FewEquationsStatement** : FewEqSt

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.FibreAcyclicityOfSections

theorem **acyclic_over_affineOpen** {k : Type} [Field k] [IsAlgClosed k] {X T Z : Scheme.{0}} (a : X ⟶ Sp k) (b : T ⟶ Sp k) (p : Z ⟶ X) (q : Z ⟶ T) (H : IsPullback p q a b) (ha : IsProjBy a) [IsProper a] [Flat a] [IsProper b] (G : LB Z) (hG : G.IsLB) (hfib : ∀ (y : Sp k ⟶ T) (σ : X ⟶ Z), y ≫ b = 𝟙 _ → σ ≫ p = 𝟙 X → σ ≫ q = a ≫ y → FibAc a (G.pull σ)) {κ : Type} [LinearOrder κ] [Fintype κ] (B : κ → X.Opens) (hB : ∀ i, IsAffineOpen (B i)) (hc : ⨆ i, B i = ⊤) {V : T.Opens} (hV : IsAffineOpen V) : (cechLB (p ≫ a) G fun i => p ⁻¹ᵁ B i ⊓ q ⁻¹ᵁ V).Acyclic

theorem **fibAc_comp_of_fibres** : ∀ {k : Type} [Field k] [IsAlgClosed k] {X T Z : Scheme.{0}} (a : X ⟶ Sp k) (b : T ⟶ Sp k) (p : Z ⟶ X) (q : Z ⟶ T) (H : IsPullback p q a b) (ha : IsProjBy a) [IsProper a] [Flat a] [IsProper b] (G : LB Z) (hG : G.IsLB) (hfib : ∀ (y : Sp k ⟶ T) (σ : X ⟶ Z), y ≫ b = 𝟙 _ → σ ≫ p = 𝟙 X → σ ≫ q = a ≫ y → FibAc a (G.pull σ)), FibAc (p ≫ a) G

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.FibreAcyclicityOfSectionsStatement

theorem **_root_.GlobalTheory.T.fibre_acyclicity_of_sections** : ∀ {k : Type} [Field k] [IsAlgClosed k] {X T Z : Scheme.{0}} (a : X ⟶ Sp k) (b : T ⟶ Sp k) (p : Z ⟶ X) (q : Z ⟶ T) (H : IsPullback p q a b) (ha : IsProjBy a) [IsProper a] [Flat a] [IsProper b] (G : LB Z) (hG : G.IsLB) (hfib : ∀ (y : Sp k ⟶ T) (σ : X ⟶ Z), y ≫ b = 𝟙 _ → σ ≫ p = 𝟙 X → σ ≫ q = a ≫ y → FibAc a (G.pull σ)), FibAc (p ≫ a) G

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.FibreAcyclicityOutsideFiniteSet

theorem **exists_acyclic_cechLB_fibre** : ∀ {k : Type} [Field k] [IsAlgClosed k] {X T Z : Scheme.{0}} (a : X ⟶ Sp k) (b : T ⟶ Sp k) (p : Z ⟶ X) (q : Z ⟶ T) (H : IsPullback p q a b) (ha : IsProjBy a) [IsProper a] [Flat a] [IsProper b] (G : LB Z) (hG : G.IsLB) (F : Set (Sp k ⟶ T)) (hF : F.Finite) (hfib : ∀ (y : Sp k ⟶ T) (σ : X ⟶ Z), y ≫ b = 𝟙 _ → σ ≫ p = 𝟙 X → σ ≫ q = a ≫ y → y ∉ F → FibAc a (G.pull σ)) (htot : FibAc (p ≫ a) G) (y₀ : Sp k ⟶ T) (σ₀ : X ⟶ Z) (hy : y₀ ≫ b = 𝟙 _) (h₁ : σ₀ ≫ p = 𝟙 X) (h₂ : σ₀ ≫ q = a ≫ y₀), ∃ (m : ℕ) (D : Fin m → X.Opens), (∀ i, IsAffineOpen (D i)) ∧ ⨆ i, D i = ⊤ ∧ (cechLB a (G.pull σ₀) D).Acyclic

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.FibreAcyclicityOutsideFiniteSetStatement

theorem **_root_.GlobalTheory.T.exists_affine_cover_cech_acyclic_of_fibAc_off_finite** : ∀ {k : Type} [Field k] [IsAlgClosed k] {X T Z : Scheme.{0}} (a : X ⟶ Sp k) (b : T ⟶ Sp k) (p : Z ⟶ X) (q : Z ⟶ T) (H : IsPullback p q a b) (ha : IsProjBy a) [IsProper a] [Flat a] [IsProper b] (G : LB Z) (hG : G.IsLB) (F : Set (Sp k ⟶ T)) (hF : F.Finite) (hfib : ∀ (y : Sp k ⟶ T) (σ : X ⟶ Z), y ≫ b = 𝟙 _ → σ ≫ p = 𝟙 X → σ ≫ q = a ≫ y → y ∉ F → FibAc a (G.pull σ)) (htot : FibAc (p ≫ a) G) (y₀ : Sp k ⟶ T) (σ₀ : X ⟶ Z) (hy : y₀ ≫ b = 𝟙 _) (h₁ : σ₀ ≫ p = 𝟙 X) (h₂ : σ₀ ≫ q = a ≫ y₀), ∃ (m : ℕ) (D : Fin m → X.Opens), (∀ i, IsAffineOpen (D i)) ∧ ⨆ i, D i = ⊤ ∧ (cechLB a (G.pull σ₀) D).Acyclic

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.FibrewiseAcyclicityBaseChange

theorem **fibAc_of_closed_fibres** : ∀ {k : Type} [Field k] [IsAlgClosed k] {X T Z : Scheme.{0}} (a : X ⟶ Sp k) (b : T ⟶ Sp k) (p : Z ⟶ X) (q : Z ⟶ T) (H : IsPullback p q a b) (ha : IsProjBy a) [IsSeparated a] [Flat a] (G : LB Z) (hG : G.IsLB) (R : Type) [CommRing R] (t : Spec (CommRingCat.of R) ⟶ T) [LocallyOfFiniteType (t ≫ b)] (Y : Scheme.{0}) (π : Y ⟶ Z) (f : Y ⟶ Spec (CommRingCat.of R)) (Hy : IsPullback π f q t) (hfib : ∀ (s : Sp k ⟶ Spec (CommRingCat.of R)) (σ : X ⟶ Z), s ≫ t ≫ b = 𝟙 _ → σ ≫ p = 𝟙 X → σ ≫ q = a ≫ s ≫ t → FibAc a (G.pull σ)), FibAc f (G.pull π)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.FibrewiseAcyclicityBaseChangeStatement

theorem **_root_.GlobalTheory.T.fibrewise_acyclicity_base_change** : ∀ {k : Type} [Field k] [IsAlgClosed k] {X T Z : Scheme.{0}} (a : X ⟶ Sp k) (b : T ⟶ Sp k) (p : Z ⟶ X) (q : Z ⟶ T) (H : IsPullback p q a b) (ha : IsProjBy a) [IsSeparated a] [Flat a] (G : LB Z) (hG : G.IsLB) (R : Type) [CommRing R] (t : Spec (CommRingCat.of R) ⟶ T) [LocallyOfFiniteType (t ≫ b)] (Y : Scheme.{0}) (π : Y ⟶ Z) (f : Y ⟶ Spec (CommRingCat.of R)) (Hy : IsPullback π f q t) (hfib : ∀ (s : Sp k ⟶ Spec (CommRingCat.of R)) (σ : X ⟶ Z), s ≫ t ≫ b = 𝟙 _ → σ ≫ p = 𝟙 X → σ ≫ q = a ≫ s ≫ t → FibAc a (G.pull σ)), FibAc f (G.pull π)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.FiniteCommutativeGroupDualCharacter

theorem **pair_counit** [IsCocomm k D] (β : Du k D →ₐ[k] R) : pair (fun d => algebraMap k R (counit (R

theorem **pair_mul** [IsCocomm k D] (a a' : D →ₗ[k] R) (β : Du k D →ₐ[k] R) (ψ : D ⊗[k] D →+ R) (hψ : ∀ x y, ψ (x ⊗ₜ y) = a x * a' y) (c : D → R) (hc : ∀ d, c d = ψ (comul (R

theorem **fnR_comp** {X Y : Scheme.{0}} {A : Type} [CommRing A] (f : Y ⟶ X) (x : X ⟶ Spec (.of A)) (a : A) : fnR (f ≫ x) a = f.appTop (fnR x a)

theorem **fnR_specMap** {X : Scheme.{0}} {A B : Type} [CommRing A] [CommRing B] (φ : A →+* B) (x : X ⟶ Spec (.of B)) (a : A) : fnR (x ≫ Spec.map (CommRingCat.ofHom φ)) a = fnR x (φ a)

theorem **fnR_algebraMap** {W : Over (Sp k)} {A : Type} [CommRing A] [Algebra k A] (x : W ⟶ (Spec (.of A)).asOver (Spec (.of k))) (c : k) : fnR x.left (algebraMap k A c) = fnR W.hom c

theorem **fn_mul** {W : Over (Sp k)} (x y : W ⟶ (Spec (.of D)).asOver (Spec (.of k))) : ∃ ψ : D ⊗[k] D →+ Γ(W.left, ⊤), (∀ u v, ψ (u ⊗ₜ v) = fnR x.left u * fnR y.left v) ∧ ∀ d, fnR (x * y).left d = ψ (comul (R

theorem **fn_one** {W : Over (Sp k)} (d : D) : fnR (1 : W ⟶ (Spec (.of D)).asOver (Spec (.of k))).left d = fnR W.hom (counit (R

theorem **twoPow_mul_twoPow** {W : Over (Sp k)} (x y : W ⟶ GD k D) (t : W ⟶ TE k D) : P (x * y) t = P x t * P y t

theorem **P_one** {W : Over (Sp k)} (t : W ⟶ TE k D) : P (1 : W ⟶ GD k D) t = 1

theorem **pullback_comp₂** {W W' : Over (Sp k)} (f : W' ⟶ W) (x : W ⟶ GD k D) (t : W ⟶ TE k D) : f.left.appTop (P x t) = P (f ≫ x) (f ≫ t)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.FiniteDimInvariantSubspaceAdjointPair

theorem **highest_preimage_inner** {E : Type*} [NormedAddCommGroup E] [InnerProductSpace ℂ E] {ι : Type*} (T : Submodule ℂ E) (a b : ι → Module.End ℂ E) (D : Module.End ℂ E) (y : E) (hT : FiniteDimensional ℂ T) (hadj : ∀ i x z, ⟪a i x, z⟫_ℂ = ⟪x, b i z⟫_ℂ) (haT : ∀ i, ∀ t ∈ T, a i t ∈ T) (hbT : ∀ i, ∀ t ∈ T, b i t ∈ T) (haD : ∀ i, ∀ t ∈ T, a i (D t) = D (a i t)) (hbD : ∀ i, ∀ t ∈ T, b i (D t) = D (b i t)) (hy : y ∈ T) (hDy : ∀ i, a i (D y) = 0) : ∃ y' ∈ T, (∀ i, a i y' = 0) ∧ D y' = D y

theorem **highest_preimage_of_stable** : ∀ (n : ℕ) (V : Type) [NormedAddCommGroup V] [InnerProductSpace ℂ V] (ι : Type) (T : Submodule ℂ (Cn n V)) (a b : ι → Module.End ℂ (Cn n V)) (D : Module.End ℂ (Cn n V)) (y : Cn n V), FiniteDimensional ℂ T → (∀ i x z, ∑ A, ⟪a i x A, z A⟫_ℂ = ∑ A, ⟪x A, b i z A⟫_ℂ) → (∀ i, ∀ t ∈ T, a i t ∈ T) → (∀ i, ∀ t ∈ T, b i t ∈ T) → (∀ i, ∀ t ∈ T, a i (D t) = D (a i t)) → (∀ i, ∀ t ∈ T, b i (D t) = D (b i t)) → y ∈ T → (∀ i, a i (D y) = 0) → ∃ y' ∈ T, (∀ i, a i y' = 0) ∧ D y' = D y

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.FiniteDimInvariantSubspaceAdjointPairStatement

theorem **_root_.GlobalTheory.T.finite_dim_invariant_subspace_adjoint_pair** : ∀ (n : ℕ) (V : Type) [NormedAddCommGroup V] [InnerProductSpace ℂ V] (ι : Type) (T : Submodule ℂ (Cn n V)) (a b : ι → Module.End ℂ (Cn n V)) (D : Module.End ℂ (Cn n V)) (y : Cn n V), FiniteDimensional ℂ T → (∀ i x z, ∑ A, ⟪a i x A, z A⟫_ℂ = ∑ A, ⟪x A, b i z A⟫_ℂ) → (∀ i, ∀ t ∈ T, a i t ∈ T) → (∀ i, ∀ t ∈ T, b i t ∈ T) → (∀ i, ∀ t ∈ T, a i (D t) = D (a i t)) → (∀ i, ∀ t ∈ T, b i (D t) = D (b i t)) → y ∈ T → (∀ i, a i (D y) = 0) → ∃ y' ∈ T, (∀ i, a i y' = 0) ∧ D y' = D y

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.FiniteDimensionalComapSubtype

theorem **locFin_tens** (ε : Fin n → Bool) (X : GMod n) (h : LocFin ε X.E) : LocFin ε (tens X).E

theorem **finiteDimensional_comap_subtype** {V : Type} [AddCommGroup V] [Module ℂ V] (S T : Submodule ℂ V) [FiniteDimensional ℂ T] : FiniteDimensional ℂ (T.comap S.subtype)

theorem **locFin_sub** (ε : Fin n → Bool) (X : GMod n) (S : Submodule ℂ X.V) (h : ∀ a b, ∀ x ∈ S, X.E a b x ∈ S) (hX : LocFin ε X.E) : LocFin ε (X.sub S h).E

theorem **intWt_tens** (X : GMod n) (h : IntWt X) : IntWt (tens X)

theorem **intWt_sub** (X : GMod n) (S : Submodule ℂ X.V) (h : ∀ a b, ∀ x ∈ S, X.E a b x ∈ S) (hX : IntWt X) : IntWt (X.sub S h)

theorem **locFin_step** (ε : Fin n → Bool) (X : GMod n) (z : ℂ) (h : LocFin ε X.E) : LocFin ε (step X z).E

theorem **intWt_step** (X : GMod n) (z : ℂ) (h : IntWt X) : IntWt (step X z)

theorem **pathFin** : CompanionTheory.RgExh.Fin.PathFin

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.FiniteGroupHomKernelPullback

theorem **mono_of_isKernel** (hp : IsPullback i (toUnit H) q η[A']) : Mono i

theorem **kerLift_comp** (hp : IsPullback i (toUnit H) q η[A']) {W : C} (f : W ⟶ A) (hf : f ≫ q = 1) : kerLift hp f hf ≫ i = f

theorem **comp_kills** (hp : IsPullback i (toUnit H) q η[A']) {W : C} (x : W ⟶ H) : (x ≫ i) ≫ q = 1

theorem **isMonHom_of_isKernel** (hp : IsPullback i (toUnit H) q η[A']) : letI

theorem **isCommMonObj_of_isKernel** [BraidedCategory C] [IsCommMonObj A] (hp : IsPullback i (toUnit H) q η[A']) : letI

theorem **exists_finite_kernel** : ∀ (k : Type) [Field k] (A A' : Over (Sp k)) [GrpObj A] [GrpObj A'], IsAbelianScheme A → ∀ (q : A ⟶ A'), IsMonHom q → IsFinite q.left → ∃ (H : Over (Sp k)) (_ : GrpObj H) (_ : IsCommMonObj H) (i : H ⟶ A), IsMonHom i ∧ IsFinite H.hom ∧ IsPullback i (toUnit _) q η[A']

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.FiniteNetOfGradedProjections

theorem **norm_entry_sq_le_hs** (M : Matrix (Fin n) (Fin n) ℂ) (i j : Fin n) : ‖M i j‖ ^ 2 ≤ Formalism.REAL.hs M

theorem **norm_entry_le** {M : Matrix (Fin n) (Fin n) ℂ} (h : Formalism.REAL.hs M ≤ n) (i j : Fin n) : ‖M i j‖ ≤ (n : ℝ) + 1

theorem **hs_le_quarter** {M : Matrix (Fin n) (Fin n) ℂ} (h : ∀ i j, ‖M i j‖ < 1 / (2 * ((n : ℝ) + 1))) : Formalism.REAL.hs M ≤ 1 / 4

theorem **exists_net** (n : ℕ) (ε : Fin n → Bool) : ∃ (m : ℕ) (A : Fin m → Matrix (Fin n) (Fin n) ℂ), (∀ j, IsGrad ε (A j)) ∧ ∀ A' : Matrix (Fin n) (Fin n) ℂ, IsGrad ε A' → ∃ j, Formalism.REAL.hs (Pm (A j) − Pm A') ≤ 1 / 4 ∧ Formalism.REAL.hs (Pp (A j) − Pp A') ≤ 1 / 4

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.FiniteNetOfGradedProjectionsStatement

theorem **_root_.GlobalTheory.T.finite_net_of_graded_projections** : ∀ (n : ℕ) (ε : Fin n → Bool), ∃ (m : ℕ) (A : Fin m → Matrix (Fin n) (Fin n) ℂ), (∀ j, IsGrad ε (A j)) ∧ ∀ A' : Matrix (Fin n) (Fin n) ℂ, IsGrad ε A' → ∃ j, Formalism.REAL.hs (Pm (A j) − Pm A') ≤ 1 / 4 ∧ Formalism.REAL.hs (Pp (A j) − Pp A') ≤ 1 / 4

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.FiniteTypeAlgebraCommonEigencharacter

theorem **exists_character_of_smul_ne_zero** : ∀ (R : Type) [CommRing R] [Algebra ℂ R] [Algebra.FiniteType ℂ R] (W : Type) [AddCommGroup W] [Module ℂ W] [Module R W] [IsScalarTower ℂ R W] (t : W), t ≠ 0 → ∃ φ : R →ₐ[ℂ] ℂ, ∀ (r : R) (c : ℂ), r • t = c • t → φ r = c

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.FiniteTypeAlgebraCommonEigencharacterStatement

theorem **_root_.GlobalTheory.T.finite_type_algebra_common_eigencharacter** : ∀ (R : Type) [CommRing R] [Algebra ℂ R] [Algebra.FiniteType ℂ R] (W : Type) [AddCommGroup W] [Module ℂ W] [Module R W] [IsScalarTower ℂ R W] (t : W), t ≠ 0 → ∃ φ : R →ₐ[ℂ] ℂ, ∀ (r : R) (c : ℂ), r • t = c • t → φ r = c

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.FinprodEqIndicator

theorem **ofReal_re_prod** {ι : Type*} (s : Finset ι) (z : ι → ℂ) (h : ∀ i, 0 ≤ (z i).re ∧ (z i).im = 0) : ENNReal.ofReal (∏ i ∈ s, z i).re = ∏ i ∈ s, ENNReal.ofReal (z i).re

theorem **hasProd_ofReal** {ι : Type*} {a : ι → ℝ} {p : ℝ} (hp : HasProd a p) (h0 : ∀ i, 0 ≤ a i) : HasProd (fun i => ENNReal.ofReal (a i)) (ENNReal.ofReal p)

theorem **hasProd_of_tail** {ι : Type} (L : ι → ℝ≥0∞) (a : ι → ℝ) (S : Finset ι) (hL : ∀ v, L v ≠ ⊤) (ha0 : ∀ v, 0 ≤ a v) (hoff : ∀ v, v ∉ S → L v = ENNReal.ofReal (a v)) (hsum : Summable fun v => |a v - 1|) : HasProd L ((∏ v ∈ S, L v) * ENNReal.ofReal (∏' v : {v // v ∉ S}, a v.1))

theorem **finprod_eq_indicator** {F E : Type} [Field F] [NumberField F] [Field E] [Algebra F E] {d n M : ℕ} (b : Module.Basis (Fin d) F E) (Φv : ∀ v : HeightOneSpectrum (𝒪 F), (Fin n → Fin M → Fv F v ⊗[F] E) → ℂ) (S : Finset (HeightOneSpectrum (𝒪 F))) (hS : ∀ v, v ∉ S → Φv v = (latV b v).indicator 1) (X : Fin n → Fin M → AF F ⊗[F] E) : ∏ᶠ v, Φv v (projV v (projFin X)) = (intOff b S).indicator (fun Y => ∏ v ∈ S, Φv v (projV v (projFin Y))) X

theorem **GlobalTheory.CompanionTheory.SW.EulerPos.euler_pos** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (d : ℕ) (b : Module.Basis (Fin d) F E) (n M : ℕ) (I : Measure (Fin n → Fin M → AF F ⊗[F] E)) (minf : Measure (Fin n → Fin M → Ainf F ⊗[F] E)) (m : ∀ v : HeightOneSpectrum (𝒪 F), Measure (Fin n → Fin M → Fv F v ⊗[F] E)), IsEulerProd b I 1 minf m → (∀ v, IsLocallyFiniteMeasure (m v)) → ∀ (cinf : ℝ≥0) (c : HeightOneSpectrum (𝒪 F) → ℝ≥0), IsUnitProd cinf c → ∀ (Φ : (Fin n → Fin M → AF F ⊗[F] E) → ℝ≥0∞) (Φi : (Fin n → Fin M → Ainf F ⊗[F] E) → ℝ≥0∞) (Φf : (Fin n → Fin M → Af F ⊗[F] E) → ℂ) (Φv : ∀ v : HeightOneSpectrum (𝒪 F), (Fin n → Fin M → Fv F v ⊗[F] E) → ℂ), Measurable Φi → IsPureFin b Φf Φv → (∀ v Y, 0 ≤ (Φv v Y).re ∧ (Φv v Y).im = 0) → (∀ X, Φ X = Φi (projInf X) * ENNReal.ofReal (Φf (projFin X)).re) → Measurable

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.FinrankAdjoinLeAdd

lemma **compr_of_inv** (W W' : Submodule ℂ V) (hc : IsCompl W W') (b : End ℂ V) (hb : ∀ w ∈ W, b w ∈ W) (w : W) : compr W W' hc b w = (b w, hb w w.2)

lemma **compr_eq_restrict** (W W' : Submodule ℂ V) (hc : IsCompl W W') (b : End ℂ V) (hb : ∀ w ∈ W, b w ∈ W) : compr W W' hc b = b.restrict hb

lemma **compr_mul** (W W' : Submodule ℂ V) (hc : IsCompl W W') (b c : End ℂ V) (hcW : ∀ w ∈ W, c w ∈ W) : compr W W' hc (b * c) = compr W W' hc b * compr W W' hc c

lemma **compr_algebraMap** (W W' : Submodule ℂ V) (hc : IsCompl W W') (t : ℂ) : compr W W' hc (algebraMap ℂ (End ℂ V) t) = algebraMap ℂ (End ℂ W) t

lemma **adjoin_compr** {l : ℕ} (a : Fin l → End ℂ V) (W W' : Submodule ℂ V) (hc : IsCompl W W') (h : ∀ i, ∀ w ∈ W, a i w ∈ W) (b : End ℂ V) (hb : b ∈ Algebra.adjoin ℂ (Set.range a)) : (∀ w ∈ W, b w ∈ W) ∧ compr W W' hc b ∈ Algebra.adjoin ℂ (Set.range fun i => ((a i).restrict (h i) : End ℂ W))

theorem **finrank_adjoin_le_add** : ∀ (V : Type) [AddCommGroup V] [Module ℂ V] [FiniteDimensional ℂ V] (l : ℕ) (a : Fin l → Module.End ℂ V) (W W' : Submodule ℂ V), IsCompl W W' → ∀ (h : ∀ i, ∀ w ∈ W, a i w ∈ W) (h' : ∀ i, ∀ w ∈ W', a i w ∈ W'), Module.finrank ℂ (Algebra.adjoin ℂ (Set.range a)) ≤ Module.finrank ℂ (Algebra.adjoin ℂ (Set.range fun i => ((a i).restrict (h i) : Module.End ℂ W))) + Module.finrank ℂ (Algebra.adjoin ℂ (Set.range fun i => ((a i).restrict (h' i) : Module.End ℂ W')))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.FinrankRangePowConvex

private theorem **finrank_map_add_finrank_inf_ker'** {E F : Type} [AddCommGroup E] [Module ℂ E] [FiniteDimensional ℂ E] [AddCommGroup F] [Module ℂ F] (f : E →ₗ[ℂ] F) (A : Submodule ℂ E) : finrank ℂ (A.map f) + finrank ℂ (A ⊓ LinearMap.ker f : Submodule ℂ E) = finrank ℂ A

theorem **exists_rel_compl** [FiniteDimensional ℂ V] (W' W : Submodule ℂ V) (h : W' ≤ W) : ∃ Y : Submodule ℂ V, Y ⊔ W' = W ∧ finrank ℂ Y + finrank ℂ W' = finrank ℂ W

theorem **range_pow_succ** (a : Module.End ℂ V) (i : ℕ) : LinearMap.range (a ^ (i + 1)) = (LinearMap.range (a ^ i)).map a

theorem **range_pow_succ_le** (a : Module.End ℂ V) (i : ℕ) : LinearMap.range (a ^ (i + 1)) ≤ LinearMap.range (a ^ i)

theorem **finrank_range_pow_convex** [FiniteDimensional ℂ V] (a : Module.End ℂ V) (i : ℕ) : 2 * finrank ℂ (LinearMap.range (a ^ (i + 1))) ≤ finrank ℂ (LinearMap.range (a ^ i)) + finrank ℂ (LinearMap.range (a ^ (i + 2)))

theorem **succ_mul_drop_le** [FiniteDimensional ℂ V] (a : Module.End ℂ V) (x : ℕ) : (x + 1) * finrank ℂ (LinearMap.range (a ^ x)) + finrank ℂ (LinearMap.range (a ^ (x + 1))) ≤ (x + 1) * finrank ℂ (LinearMap.range (a ^ (x + 1))) + finrank ℂ (LinearMap.range (a ^ 0))

theorem **range_le_ker_of_commute** (a b : Module.End ℂ V) (hab : Commute b a) (ha : IsNilpotent a) (x : ℕ) (Y : Submodule ℂ V) (hY : Y ⊔ LinearMap.range (a ^ (x + 1)) = LinearMap.range (a ^ x)) (hb : Y ≤ LinearMap.ker b) : LinearMap.range (a ^ x) ≤ LinearMap.ker b

theorem **finrank_map_le_of_ker** {E F G : Type} [AddCommGroup E] [Module ℂ E] [FiniteDimensional ℂ E] [AddCommGroup F] [Module ℂ F] [AddCommGroup G] [Module ℂ G] (f : E →ₗ[ℂ] F) (T : E →ₗ[ℂ] G) (A : Submodule ℂ E) (h : ∀ b ∈ A, T b = 0 → f b = 0) : finrank ℂ (A.map f) ≤ finrank ℂ (A.map T)

theorem **succ_mul_finrank_map_le** : ∀ (V : Type) [AddCommGroup V] [Module ℂ V] [FiniteDimensional ℂ V] (a : Module.End ℂ V), IsNilpotent a → ∀ (x : ℕ) (A : Submodule ℂ (Module.End ℂ V)), (∀ b ∈ A, Commute b a) → (x + 1) * Module.finrank ℂ (A.map (LinearMap.mulRight ℂ (a ^ x))) ≤ Module.finrank ℂ V ^ 2

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.FixedVectorStatement

theorem **exists_least_norm_unique** {E : Type*} [NormedAddCommGroup E] [InnerProductSpace ℝ E] [CompleteSpace E] {C : Set E} (hne : C.Nonempty) (hcl : IsClosed C) (hconv : Convex ℝ C) : ∃ u ∈ C, ∀ w ∈ C, ‖u‖ ≤ ‖w‖ ∧ (‖w‖ ≤ ‖u‖ → w = u)

theorem **stFixVec** : StFixVec

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.FixedVectorStatementStatement

theorem **_root_.GlobalTheory.T.FixedVectorStatement** : StFixVec

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.FlagFacts

theorem **J_mul** (h : IsGrad ε A) : Formalism.REAL.Jm ε * A = -(A * Formalism.REAL.Jm ε)

theorem **A4** (h : A * A * A = A) : A * A * A * A * A = A * A

theorem **pM_idem** (h : A * A * A = A) : pM A * pM A = pM A

theorem **pM_mul_qM** (h : A * A * A = A) : pM A * qM A = 0

theorem **J_mul_pM** (h : IsGrad ε A) : Formalism.REAL.Jm ε * pM A = qM A * Formalism.REAL.Jm ε

theorem **trace_qM** (h : IsGrad ε A) : Matrix.trace (qM A) = Matrix.trace (pM A)

theorem **pM_conjTranspose** (h : Aᴴ = A) : (pM A)ᴴ = pM A

theorem **isProj** (h : A * A * A = A) : LinearMap.IsProj (Vm A) (Matrix.toLin' (pM A))

theorem **trace_pM** (h : A * A * A = A) : Matrix.trace (pM A) = (rk A : ℂ)

theorem **one_le_rk** (h : IsGrad ε A) : 1 ≤ rk A

theorem **isotropic** (h : IsGrad ε A) {x y : Fin n → ℂ} (hx : x ∈ Vm A) (hy : y ∈ Vm A) : star x ⬝ᵥ (Formalism.REAL.Jm ε *ᵥ y) = 0

theorem **flagFacts** : FlagFacts

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.FlagFactsStatement

theorem **_root_.GlobalTheory.T.rgExh_flagFacts** : CompanionTheory.RgExh.Dec.N.S.FlagFacts

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.FlatBoundedAcyclicBaseChange

theorem **acyclic_extendScalars_of_bounded_flat** : ∀ (R K : Type) [CommRing R] [CommRing K] (χ : R →+* K) (C : CochainComplex (ModuleCat.{0} R) ℕ) (n : ℕ) (hn : ∀ p, n < p → IsZero (C.X p)) (hflat : ∀ p, Module.Flat R (C.X p)) (hac : C.Acyclic), (((ModuleCat.extendScalars.{0, 0, 0} χ).mapHomologicalComplex _).obj C).Acyclic

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.FlatBoundedAcyclicBaseChangeStatement

theorem **_root_.GlobalTheory.T.flat_bounded_acyclic_base_change** : ∀ (R K : Type) [CommRing R] [CommRing K] (χ : R →+* K) (C : CochainComplex (ModuleCat.{0} R) ℕ) (n : ℕ) (hn : ∀ p, n < p → IsZero (C.X p)) (hflat : ∀ p, Module.Flat R (C.X p)) (hac : C.Acyclic), (((ModuleCat.extendScalars.{0, 0, 0} χ).mapHomologicalComplex _).obj C).Acyclic

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.FlatCoverBaseChangeProduct

theorem **isPullback_snd_lift_fst_snd_comp** {S : Scheme.{0}} (A X Y : Over S) (q : X ⟶ Y) : IsPullback (snd A X).left (lift (fst A X) (snd A X ≫ q)).left q.left (snd A Y).left

theorem **lift_fst_snd_comp_flat_lfp_surjective** : ∀ (S : Scheme.{0}) (A X Y : Over S) [GrpObj A], IsAbelianScheme A → ∀ q : X ⟶ Y, Flat q.left → LocallyOfFinitePresentation q.left → Surjective q.left → Flat (lift (fst A X) (snd A X ≫ q)).left ∧ LocallyOfFinitePresentation (lift (fst A X) (snd A X ≫ q)).left ∧ Surjective (lift (fst A X) (snd A X ≫ q)).left

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.FlatCoverBaseChangeProductStatement

theorem **_root_.GlobalTheory.T.flat_cover_base_change_product** : ∀ (S : Scheme.{0}) (A X Y : Over S) [GrpObj A], IsAbelianScheme A → ∀ q : X ⟶ Y, Flat q.left → LocallyOfFinitePresentation q.left → Surjective q.left → Flat (lift (fst A X) (snd A X ≫ q)).left ∧ LocallyOfFinitePresentation (lift (fst A X) (snd A X ≫ q)).left ∧ Surjective (lift (fst A X) (snd A X ≫ q)).left

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.FlatDescentOfUnits

theorem **exists_app_eq_of_lfp** {X Y : Scheme.{0}} (π : X ⟶ Y) [Flat π] [SurjectiveOfFinitePresentation π] (U : Y.Opens) (f : Γ(X, π ⁻¹ᵁ U)) (hf : ∀ (Z : Scheme.{0}) (g₁ g₂ : Z ⟶ X) (h₁ : ⊤ ≤ g₁ ⁻¹ᵁ (π ⁻¹ᵁ U)) (h₂ : ⊤ ≤ g₂ ⁻¹ᵁ (π ⁻¹ᵁ U)), g₁ ≫ π = g₂ ≫ π → (g₁.appLE _ ⊤ h₁).hom f = (g₂.appLE _ ⊤ h₂).hom f) : ∃ a, (π.app U).hom a = f

theorem **units_descend** : ∀ {P Q : Scheme.{0}} (f : P ⟶ Q), Flat f → LocallyOfFinitePresentation f → Surjective f → ∀ V : Q.Opens, Function.Injective (pullU f V) ∧ ∀ a : Γ(P, f ⁻¹ᵁ V)ˣ, (∀ (W : Scheme.{0}) (b c : W ⟶ P), b ≫ f = c ≫ f → ∀ w : W, f (b w) ∈ V → gm b w (f ⁻¹ᵁ V) a = gm c w (f ⁻¹ᵁ V) a) → ∃ s : Γ(Q, V)ˣ, pullU f V s = a

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.FlatDescentOfUnitsStatement

theorem **_root_.GlobalTheory.T.flat_descent_of_units** : ∀ {P Q : Scheme.{0}} (f : P ⟶ Q), Flat f → LocallyOfFinitePresentation f → Surjective f → ∀ V : Q.Opens, Function.Injective (pullU f V) ∧ ∀ a : Γ(P, f ⁻¹ᵁ V)ˣ, (∀ (W : Scheme.{0}) (b c : W ⟶ P), b ≫ f = c ≫ f → ∀ w : W, f (b w) ∈ V → gm b w (f ⁻¹ᵁ V) a = gm c w (f ⁻¹ᵁ V) a) → ∃ s : Γ(Q, V)ˣ, pullU f V s = a

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.FrameOfAnisotropicPlaneSplitting

theorem **herm_smul_left** (hh : IsHerm s h) (a : L) (x y : V) : h (a • x) y = s a * h x y

theorem **herm_symm_zero** (hh : IsHerm s h) {x y : V} (hxy : h x y = 0) : h y x = 0

theorem **herm_right_zero** (hh : IsHerm s h) (x : V) : h x 0 = 0

theorem **frame_cons** (hh : IsHerm s h) {δ : L} {e f : V} {U : Submodule L V} (hU : ∀ x, x ∈ U ↔ h e x = 0 ∧ h f x = 0) (hee : h e e = 0) (hff : h f f = 0) (hef : h e f = δ) (hdec : ∀ x, ∃ (a b : L), ∃ w ∈ U, x = a • e + b • f + w) {l : ℕ} {e' f' : Fin l → U} {W' : Submodule L U} (hfr : IsFrame (fun x y : U => h x y) δ e' f' W') : IsFrame h δ (Fin.cons e fun i => (e' i : V)) (Fin.cons f fun i => (f' i : V)) (W'.map U.subtype)

theorem **adapted_cons** {U : Submodule L V} {e f : V} {l : ℕ} {e' f' : Fin l → U} {W' : Submodule L U} {u0 : L} {u' : Fin l → L} {Λ : AddSubgroup V} (hsp : ∀ (a b : L), ∀ w ∈ U, (a • e + b • f + w ∈ Λ ↔ ‖a‖ ≤ ‖u0‖ ∧ ‖b‖ ≤ ‖u0‖⁻¹ ∧ w ∈ Λ)) (had : IsAdapted (fun x y : U => h x y) e' f' W' u') (Λ.comap U.subtype.toAddMonoidHom)) : IsAdapted h (Fin.cons e fun i => (e' i : V)) (Fin.cons f fun i => (f' i : V)) (Fin.cons u0 u' : Fin (l + 1) → L) Λ

theorem **exists_adapted_frame_with_unit_scaling** (hA : StAniso) (hP : StPlane) (hS : StSplit) (L : Type) [NontriviallyNormedField L] [IsUltrametricDist L] [ProperSpace L] [CharZero L] (s : L →+* L) (hs : IsInv s) (δ : L) (hδ : IsDelta s δ) (n : ℕ) : ∀ (V : Type) [AddCommGroup V] [Module L V] [FiniteDimensional L V] (h : V → V → L), Module.finrank L V = n → IsHerm s h → ∀ Λ Λ' : AddSubgroup V, IsMax h Λ → IsMax h Λ' → ∃ (l : ℕ) (e f : Fin l → V) (W : Submodule L V) (u : Fin l → L), (∀ i, u i ≠ 0) ∧ IsFrame h δ e f W ∧ IsAdapted h e f W (fun _ => 1) Λ ∧ IsAdapted h e f W u Λ'

theorem **stFrame_of** : StAniso → StPlane → StSplit → StFrame

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.FrameOfAnisotropicPlaneSplittingStatement

theorem **_root_.GlobalTheory.T.frame_of_anisotropic_plane_splitting** : StAniso → StPlane → StSplit → StFrame

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.FreeAlgebraActionOnSubquotient

theorem **descends** (C : Type) [AddCommGroup C] [Module ℂ C] (L : Type) (l : L → Module.End ℂ C) (Z B : Submodule ℂ C) (hZ : ∀ k, ∀ z ∈ Z, l k z ∈ Z) (hB : ∀ k, ∀ z ∈ B, l k z ∈ B) : ∃ ρ : FreeAlgebra ℂ L →ₐ[ℂ] Module.End ℂ (Z ⧸ B.comap Z.subtype), ∀ (y : FreeAlgebra ℂ L) (z : Z), ∃ hz : FreeAlgebra.lift ℂ l y (z : C) ∈ Z, ρ y (Submodule.Quotient.mk z) = Submodule.Quotient.mk (FreeAlgebra.lift ℂ l y (z : C), hz)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.FreeAlgebraActionOnSubquotientStatement

theorem **_root_.GlobalTheory.T.free_algebra_action_on_subquotient** : ∀ (C : Type) [AddCommGroup C] [Module ℂ C] (L : Type) (l : L → Module.End ℂ C) (Z B : Submodule ℂ C), (∀ k, ∀ z ∈ Z, l k z ∈ Z) → (∀ k, ∀ z ∈ B, l k z ∈ B) → ∃ ρ : FreeAlgebra ℂ L →ₐ[ℂ] Module.End ℂ (Z ⧸ B.comap Z.subtype), ∀ (y : FreeAlgebra ℂ L) (z : Z), ∃ hz : FreeAlgebra.lift ℂ l y (z : C) ∈ Z, ρ y (Submodule.Quotient.mk z) = Submodule.Quotient.mk (FreeAlgebra.lift ℂ l y (z : C), hz)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.FubiniOfParabolicHaarMultiple

theorem **fubini_of_parabolic_multiple** : CompanionTheory.RgExh.Dec.N.S.Fub2 → CompanionTheory.RgExh.Dec.N.S.Fub

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.FubiniOfParabolicHaarMultipleStatement

theorem **_root_.GlobalTheory.T.fubini_of_parabolic_haar_multiple** : CompanionTheory.RgExh.Dec.N.S.Fub2 → CompanionTheory.RgExh.Dec.N.S.Fub

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.FubiniParabolicMultiple

theorem **lintegral_prodMul_eq** (M : Set G) (μM μN : Measure G) [SFinite μM] [SFinite μN] (hM : μM Mᶜ = 0) (δ : G → ℝ≥0∞) (hmod : ∀ m ∈ M, μN.map (fun u => m * u * m⁻¹) = δ m • μN) (C : Set G) (hCm : MeasurableSet C) (hC0 : μN C ≠ 0) (hC1 : μN C ≠ ⊤) (F : G → ℝ≥0∞) (hF : Measurable F) : ∫⁻ x, F x ∂(GlobalTheory.ParabModulus.prodMul μM μN) = ∫⁻ u, ∫⁻ m, δ m * F (u * m) ∂μM ∂μN

theorem **muPar_facts** (A : Matrix (Fin n) (Fin n) ℂ) (ν : Measure (PSub ε A)) [ν.IsHaarMeasure] : SigmaFinite (muPar ε A ν) ∧ muPar ε A ν (PSub ε A : Set (S7.UJ ε))ᶜ = 0 ∧ (∀ s ∈ PSub ε A, (muPar ε A ν).map (fun g => s * g) = muPar ε A ν) ∧ muPar ε A ν ≠ 0

theorem **fub2** : Fub2

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.FubiniParabolicMultipleStatement

theorem **_root_.GlobalTheory.T.FubiniParabolicMultiple** : CompanionTheory.RgExh.Dec.N.S.Fub2

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.GeneratorCompatibilityCompositePullback

theorem **gm_gen_pull** {Y X : Scheme.{0}} {L : LB X} {F : Y ⟶ X} {t : LB.Gen (L.pull F)} (ht : t.IsGen) (x : Y) (i i' : L.ι) (hi : F x ∈ L.U i) (hi' : F x ∈ L.U i') : gm (𝟙 Y) x ((L.pull F).U fun _ => i) (t.s fun _ => i) = gm (𝟙 Y ≫ F) x _ (L.g i i') * gm (𝟙 Y) x ((L.pull F).U fun _ => i') (t.s fun _ => i')

theorem **gm_pull_div_g** {Z P Q : Scheme.{0}} (g : Z ⟶ P) (f : P ⟶ Q) (M : LB P) (N : LB Q) (x : Z) (i i' : N.ι) (k k' : M.ι) (hi : f (g x) ∈ N.U i) (hi' : f (g x) ∈ N.U i') (hk : g x ∈ M.U k) (hk' : g x ∈ M.U k') : gm (𝟙 Z) x (((LB.div (N.pull f) M).pull g).U (fun _ => (fun _ => i, k)) ⊓ ((LB.div (N.pull f) M).pull g).U (fun _ => (fun _ => i', k'))) (((LB.div (N.pull f) M).pull g).g (fun _ => (fun _ => i, k)) (fun _ => (fun _ => i', k'))) = gm (𝟙 Z ≫ g ≫ f) x _ (N.g i i') * (gm (𝟙 Z ≫ g) x _ (M.g k k'))⁻¹

theorem **lbIso_of_same_descent** : ∀ {Z P Q : Scheme.{0}} (g : Z ⟶ P) (f : P ⟶ Q), Flat g → LocallyOfFinitePresentation g → Surjective g → ∀ (M : LB P), M.IsLB → ∀ (N : LB Q), N.IsLB → ∀ (τ : LB.Gen (M.pull g)), τ.IsGen → ∀ (T : LB.Gen (N.pull (g ≫ f))), T.IsGen → (∀ (W : Scheme.{0}) (a b : W ⟶ Z), a ≫ g = b ≫ g → ∀ (w : W) (i : N.ι) (k : M.ι), f (g (a w)) ∈ N.U i → g (a w) ∈ M.U k → gm a w ((N.pull (g ≫ f)).U fun _ => i) (T.s fun _ => i) * (gm a w ((M.pull g).U fun _ => k) (τ.s fun _ => k))⁻¹ = gm b w ((N.pull (g ≫ f)).U fun _ => i) (T.s fun _ => i) * (gm b w ((M.pull g).U fun _ => k) (τ.s fun _ => k))⁻¹) → LBIso M (N.pull f)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.GeneratorCompatibilityCompositePullbackStatement

theorem **_root_.GlobalTheory.T.lbIso_pull_of_generator_ratio_eq** : ∀ {Z P Q : Scheme.{0}} (g : Z ⟶ P) (f : P ⟶ Q), Flat g → LocallyOfFinitePresentation g → Surjective g → ∀ (M : LB P), M.IsLB → ∀ (N : LB Q), N.IsLB → ∀ (τ : LB.Gen (M.pull g)), τ.IsGen → ∀ (T : LB.Gen (N.pull (g ≫ f))), T.IsGen → (∀ (W : Scheme.{0}) (a b : W ⟶ Z), a ≫ g = b ≫ g → ∀ (w : W) (i : N.ι) (k : M.ι), f (g (a w)) ∈ N.U i → g (a w) ∈ M.U k → gm a w ((N.pull (g ≫ f)).U fun _ => i) (T.s fun _ => i) * (gm a w ((M.pull g).U fun _ => k) (τ.s fun _ => k))⁻¹ = gm b w ((N.pull (g ≫ f)).U fun _ => i) (T.s fun _ => i) * (gm b w ((M.pull g).U fun _ => k) (τ.s fun _ => k))⁻¹) → LBIso M (N.pull f)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.GeneratorDescentUnitExists

theorem **exists_unit_of_gen** : ∀ {Z P Q : Scheme.{0}} (g : Z ⟶ P) (f : P ⟶ Q) (M : LB P), M.IsLB → ∀ (c : LB.Gen (LB.LinComb M (K

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.GeneratorDescentUnitExistsStatement

theorem **_root_.GlobalTheory.T.generator_descent_unit_exists** : ∀ {Z P Q : Scheme.{0}} (g : Z ⟶ P) (f : P ⟶ Q) (M : LB P), M.IsLB → ∀ (c : LB.Gen (LB.LinComb M (K

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.GeometricStatement

theorem **continuous_hs** : Continuous (fun M : Matrix (Fin n) (Fin n) ℂ => hs M)

theorem **hs_Pm_pos** (hA : IsGrad ε A) (x : S7.UJ ε) : 0 < hs (S7.toM x * Pm A)

theorem **prod_pos** (hA : IsGrad ε A) (x : S7.UJ ε) : 0 < hs (S7.toM x * Pm A) * (1 + hs (S7.toM x * Pz A))

theorem **tau_continuous** (hA : IsGrad ε A) : Continuous fun x : S7.UJ ε => tauG A (S7.toM x)

theorem **exp_tau** (hA : IsGrad ε A) (x : S7.UJ ε) : Real.exp (-2 * tauG A (S7.toM x)) = hs (S7.toM x * Pm A) * (1 + hs (S7.toM x * Pz A))

theorem **hs_line_Pm** (hA : IsGrad ε A) (M : Matrix (Fin n) (Fin n) ℂ) (t : ℝ) : hs (M * aT A t * Pm A) = Real.exp (-t) ^ 2 * hs (M * Pm A)

theorem **hs_line_Pp** (hA : IsGrad ε A) (M : Matrix (Fin n) (Fin n) ℂ) (t : ℝ) : hs (M * aT A t * Pp A) = Real.exp t ^ 2 * hs (M * Pp A)

theorem **hs_line_Pz** (hA : IsGrad ε A) (M : Matrix (Fin n) (Fin n) ℂ) (t : ℝ) : hs (M ∗ aT A t ∗ Pz A) = hs (M ∗ Pz A)

theorem **tau_add** (hA : IsGrad ε A) (x : S7.UJ ε) (t : ℝ) : tauG A (S7.toM x ∗ aT A t) = tauG A (S7.toM x) + t

theorem **hs_mul_unitary** (M k : Matrix (Fin n) (Fin n) ℂ) (hk : k ∗ kᴴ = 1) : hs (M ∗ k) = hs M

theorem **toMK_unitary** (k : S7.KG ε) : S7.toMK k ∗ (S7.toMK k)ᴴ = 1

theorem **toMK_unitary'** (k : S7.KG ε) : (S7.toMK k)ᴴ ∗ S7.toMK k = 1

theorem **comm_of_mem** (hA : IsGrad ε A) (k : S7.KG ε) (hk : (k : S7.UJ ε) ∈ PSub ε A) : S7.toMK k ∗ Pm A = Pm A ∗ S7.toMK k ∧ S7.toMK k ∗ Pz A = Pz A ∗ S7.toMK k

theorem **tau_right_inv** (hA : IsGrad ε A) (x y : PSub ε A) (k : S7.KG ε) (h : S7.toM (y : S7.UJ ε) = S7.toM (x : S7.UJ ε) ∗ S7.toMK k) : tauG A (S7.toM (y : S7.UJ ε)) = tauG A (S7.toM (x : S7.UJ ε))

theorem **line_mem** (hA : IsGrad ε A) (x : PSub ε A) (t : ℝ) : ∃ y : PSub ε A, S7.toM (y : S7.UJ ε) = S7.toM (x : S7.UJ ε) ∗ aT A t

theorem **hs_lines** (hA : IsGrad ε A) (x : S7.UJ ε) (t : ℝ) (h0 : 0 ≤ tauG A (S7.toM x)) (ht : 0 ≤ t) : (3 : ℝ)⁻¹ ∗ Real.exp (2 ∗ t) ∗ hs (S7.toM x) ≤ hs (S7.toM x ∗ aT A t) ∧ hs (S7.toM x ∗ aT A t) ≤ Real.exp (2 ∗ t) ∗ hs (S7.toM x)

theorem **Pm_A** (hA : IsGrad ε A) : Pm A ∗ A = −Pm A

theorem **Pp_A** (hA : IsGrad ε A) : Pp A ∗ A = Pp A

theorem **Pz_A** (hA : IsGrad ε A) : Pz A ∗ A = 0

theorem **block_zero** {P Q Z : Matrix (Fin n) (Fin n) ℂ} {a b c : ℂ} (hP : P ∗ A = a • P) (hQ : A ∗ Q = b • Q) (hZ : A ∗ Z − Z ∗ A = c • Z) (hne : a − b − c ≠ 0) : P ∗ Z ∗ Q = 0

theorem **nil_blocks** (hA : IsGrad ε A) {Z : Matrix (Fin n) (Fin n) ℂ} {c : ℂ} (hc : c = −1 ∨ c = −2) (hZ : A ∗ Z − Z ∗ A = c • Z) : Pm A ∗ Z ∗ Pm A = 0 ∧ Pz A ∗ Z ∗ Pz A = 0 ∧ Pp A ∗ Z ∗ Pp A = 0 ∧ Pz A ∗ Z ∗ Pm A = 0 ∧ Pp A ∗ Z ∗ Pz A = 0 ∧ Pp A ∗ Z ∗ Pm A = 0

theorem **nilp_decomp** (hA : IsGrad ε A) {Y : Matrix (Fin n) (Fin n) ℂ} (hY : Y ∈ nilpC A) : Y = Pm A ∗ Y ∗ Pp A + Pz A ∗ Y ∗ Pp A + Pm A ∗ Y ∗ Pz A

theorem **hilbert_schmidt_triple_mul_le** (U V W : Matrix (Fin n) (Fin n) ℂ) : hs (U ∗ V ∗ W) ≤ hs U ∗ hs V ∗ hs W

theorem **contraction** (hA : IsGrad ε A) (x : S7.UJ ε) {Y : Matrix (Fin n) (Fin n) ℂ} (hY : Y ∈ nilpC A) (h0 : 0 ≤ tauG A (S7.toM x)) : hs (S7.toM x ∗ Y ∗ (S7.toM x)⁻¹) ≤ 6 ∗ Real.exp (−2 ∗ tauG A (S7.toM x)) ∗ hs Y

theorem **gap** (g : S7.UJ ε) (hn : (n : ℝ) < hs (S7.toM g)) : ∃ (A' : Matrix (Fin n) (Fin n) ℂ) (α β : ℝ), IsGrad ε A' ∧ 1 ≤ β ∧ β ≤ α ∧ Real.log (hs (S7.toM g) / n) / (n + 1) ≤ Real.log α − Real.log β ∧ ∀ y : Fin n → ℂ, α ∗ nsq (Pm A' ∗ᵥ y) + β⁻¹ ∗ nsq (Pz A' ∗ᵥ y) ≤ nsq (S7.toM g⁻¹ ∗ᵥ y)

theorem **cover** (n : ℕ) (ε : Fin n → Bool) : ∃ (m : ℕ) (A : Fin m → Matrix (Fin n) (Fin n) ℂ) (c₁ R₀ : ℝ), 0 < c₁ ∧ (∀ j, IsGrad ε (A j)) ∧ ∀ g : S7.UJ ε, R₀ < hs (S7.toM g) → ∃ (j : Fin m) (x : PSub ε (A j)) (k : S7.KG ε), g = (x : S7.UJ ε) ∗ (k : S7.UJ ε) ∧ c₁ ∗ Real.log (hs (S7.toM (x : S7.UJ ε))) ≤ tauG (A j) (S7.toM (x : S7.UJ ε))

theorem **geo** : Geo

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.GeometricStatementStatement

theorem **_root_.GlobalTheory.T.GeometricStatement** : CompanionTheory.RgExh.Dec.L.Geo

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.GlRelOfBracket

theorem **J_mul_J** (ε : Fin n → Bool) : Jm ε ∗ Jm ε = 1

theorem **J_conj** (ε : Fin n → Bool) : (Jm ε)ᴴ = Jm ε

theorem **J_J_mul** (X : Matrix (Fin n) (Fin n) ℂ) : Jm ε ∗ (Jm ε ∗ X) = X

theorem **inG_iff** {X : Matrix (Fin n) (Fin n) ℂ} : inG ε X ↔ thetaJ ε X = X

theorem **theta_add** (X Y : Matrix (Fin n) (Fin n) ℂ) : thetaJ ε (X + Y) = thetaJ ε X + thetaJ ε Y

theorem **theta_sub** (X Y : Matrix (Fin n) (Fin n) ℂ) : thetaJ ε (X − Y) = thetaJ ε X − thetaJ ε Y

theorem **theta_smul** (c : ℂ) (X : Matrix (Fin n) (Fin n) ℂ) : thetaJ ε (c • X) = (starRingEnd ℂ c) • thetaJ ε X

theorem **theta_theta** (X : Matrix (Fin n) (Fin n) ℂ) : thetaJ ε (thetaJ ε X) = X

theorem **theta_mul** (X Y : Matrix (Fin n) (Fin n) ℂ) : thetaJ ε (X ∗ Y) = −(thetaJ ε Y ∗ thetaJ ε X)

theorem **theta_single** (a b : Fin n) : thetaJ ε (single a b 1) = (−(Formalism.REAL.sgn ε a b : ℂ)) • single b a 1

theorem **inG_add** {X Y : Matrix (Fin n) (Fin n) ℂ} (hX : inG ε X) (hY : inG ε Y) : inG ε (X + Y)

theorem **inG_smul** (s : ℝ) {X : Matrix (Fin n) (Fin n) ℂ} (hX : inG ε X) : inG ε ((s : ℂ) • X)

theorem **inG_comm** {X Y : Matrix (Fin n) (Fin n) ℂ} (hX : inG ε X) (hY : inG ε Y) : inG ε (X ∗ Y − Y ∗ X)

theorem **inv_two_I** : (1 / (2 ∗ Complex.I) : ℂ) = −(1 / 2) ∗ Complex.I

theorem **inG_re** (Y : Matrix (Fin n) (Fin n) ℂ) : inG ε (re ε Y)

theorem **inG_im** (Y : Matrix (Fin n) (Fin n) ℂ) : inG ε (im ε Y)

theorem **re_add_im** (Y : Matrix (Fin n) (Fin n) ℂ) : re ε Y + Complex.I • im ε Y = Y

theorem **theta_pair** {A B : Matrix (Fin n) (Fin n) ℂ} (hA : inG ε A) (hB : inG ε B) : thetaJ ε (A + Complex.I • B) = A − Complex.I • B

theorem **re_pair** {A B : Matrix (Fin n) (Fin n) ℂ} (hA : inG ε A) (hB : inG ε B) : re ε (A + Complex.I • B) = A

theorem **im_pair** {A B : Matrix (Fin n) (Fin n) ℂ} (hA : inG ε A) (hB : inG ε B) : im ε (A + Complex.I • B) = B

theorem **UJ_left_inv** {g : Matrix (Fin n) (Fin n) ℂ} (hg : g ∈ UJset ε) : Jm ε ∗ gᴴ ∗ Jm ε ∗ g = 1

theorem **UJ_inv** {g : Matrix (Fin n) (Fin n) ℂ} (hg : g ∈ UJset ε) : g⁻¹ = Jm ε ∗ gᴴ ∗ Jm ε

theorem **UJ_isUnit** {g : Matrix (Fin n) (Fin n) ℂ} (hg : g ∈ UJset ε) : IsUnit g

theorem **UJ_inv_mem** {g : Matrix (Fin n) (Fin n) ℂ} (hg : g ∈ UJset ε) : g⁻¹ ∈ UJset ε

theorem **UJ_mul** {g h : Matrix (Fin n) (Fin n) ℂ} (hg : g ∈ UJset ε) (hh : h ∈ UJset ε) : g ∗ h ∈ UJset ε

theorem **inG_conj** {g X : Matrix (Fin n) (Fin n) ℂ} (hg : g ∈ UJset ε) (hX : inG ε X) : inG ε (g ∗ X ∗ g⁻¹)

theorem **exp_mem** {X : Matrix (Fin n) (Fin n) ℂ} (hX : inG ε X) : NormedSpace.exp X ∈ UJset ε

theorem **conj_single** (g h : Matrix (Fin n) (Fin n) ℂ) (a b : Fin n) : g ∗ single a b 1 ∗ h = ∑ c, ∑ d, (g c a ∗ h b d) • single c d (1 : ℂ)

theorem **cext_apply** {M : Type∗} [AddCommGroup M] [Module ℂ M] (d : Matrix (Fin n) (Fin n) ℂ → M) (hadd : ∀ X Y, inG ε X → inG ε Y → d (X + Y) = d X + d Y) (hsmul : ∀ (s : ℝ) X, inG ε X → d ((s : ℂ) • X) = (s : ℂ) • d X) (Y : Matrix (Fin n) (Fin n) ℂ) : cext ε d hadd hsmul Y = d (re ε Y) + Complex.I • d (im ε Y)

theorem **cext_of_inG** {M : Type∗} [AddCommGroup M] [Module ℂ M] (d : Matrix (Fin n) (Fin n) ℂ → M) (hadd : ∀ X Y, inG ε X → inG ε Y → d (X + Y) = d X + d Y) (hsmul : ∀ (s : ℝ) X, inG ε X → d ((s : ℂ) • X) = (s : ℂ) • d X) {X : Matrix (Fin n) (Fin n) ℂ} (hX : inG ε X) : cext ε d hadd hsmul X = d X

theorem **bracket_ext** {R : Type∗} [Ring R] [Algebra ℂ R] (C : Matrix (Fin n) (Fin n) ℂ →ₗ[ℂ] R) (h : ∀ X Y, inG ε X → inG ε Y → C X ∗ C Y − C Y ∗ C X = C (X ∗ Y − Y ∗ X)) (Y Z : Matrix (Fin n) (Fin n) ℂ) : C Y ∗ C Z − C Z ∗ C Y = C (Y ∗ Z − Z ∗ Y)

theorem **glRel_of_bracket** {R : Type∗} [Ring R] [Algebra ℂ R] (C : Matrix (Fin n) (Fin n) ℂ →ₗ[ℂ] R) (h : ∀ Y Z, C Y ∗ C Z − C Z ∗ C Y = C (Y ∗ Z − Z ∗ Y)) : Formalism.REAL.GlRel fun a b => C (single a b 1)

theorem **adj_ext** {V : Type∗} [NormedAddCommGroup V] [InnerProductSpace ℂ V] (C : Matrix (Fin n) (Fin n) ℂ →ₗ[ℂ] Module.End ℂ V) (h : ∀ X, inG ε X → ∀ x y, ⟪C X x, y⟫_ℂ = −⟪x, C X y⟫_ℂ) (Y : Matrix (Fin n) (Fin n) ℂ) (x y : V) : ⟪C Y x, y⟫_ℂ = −⟪x, C (thetaJ ε Y) y⟫_ℂ

theorem **inner_rep** (hσ : IsURepOn (UJset ε) σ) {g : Matrix (Fin n) (Fin n) ℂ} (hg : g ∈ UJset ε) (v w : H) : ⟪σ g v, σ g w⟫_ℂ = ⟪v, w⟫_ℂ

theorem **skew** (hσ : IsURepOn (UJset ε) σ) {X : Matrix (Fin n) (Fin n) ℂ} (hX : inG ε X) {v w v' w' : H} (hv : HasDerivAt (fun t : ℝ => σ (NormedSpace.exp ((t : ℂ) • X)) v) v' 0) (hw : HasDerivAt (fun t : ℝ => σ (NormedSpace.exp ((t : ℂ) • X)) w) w' 0) : ⟪v', w⟫_ℂ = −⟪v, w'⟫_ℂ

theorem **conj_deriv** (hσ : IsURepOn (UJset ε) σ) {X g : Matrix (Fin n) (Fin n) ℂ} (hX : inG ε X) (hg : g ∈ UJset ε) {v v' : H} (hv : HasDerivAt (fun t : ℝ => σ (NormedSpace.exp ((t : ℂ) • X)) v) v' 0) : HasDerivAt (fun t : ℝ => σ (NormedSpace.exp ((t : ℂ) • (g ∗ X ∗ g⁻¹))) (σ g v)) (σ g v') 0

theorem **smMod** (hL : CompanionTheory.D.DSIC.SmLin) (hB : CompanionTheory.D.DSIC.SmBracket) : CompanionTheory.D.DSIC.SmMod

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.GoodPathReplicate

theorem **exists_pos_lower** {ι : Type∗} [Finite ι] (g : ι → ℝ) : ∃ e : ℝ, 0 < e ∧ ∀ i, 0 < g i → e ≤ g i

theorem **exists_sorted** (f : Fin n → ℝ) (hf : Function.Injective f) : ∃ l : List (Fin n), l.Pairwise (fun a b => f b < f a) ∧ ∀ a, a ∈ l

theorem **pathEnd_append** (χ : Fin n → ℂ) (l l' : List (Fin n)) : pathEnd χ (l ++ l') = pathEnd (pathEnd χ l) l'

theorem **goodPath_append** (χ : Fin n → ℂ) (l l' : List (Fin n)) (h : GoodPath χ l) (h' : GoodPath (pathEnd χ l) l') : GoodPath χ (l ++ l')

theorem **pathEnd_replicate** (χ : Fin n → ℂ) (s : Fin n) (k : ℕ) : pathEnd χ (List.replicate k s) = χ + Pi.single s (k : ℂ)

theorem **goodPath_replicate** (χ : Fin n → ℂ) (s : Fin n) (k : ℕ) (h : ∀ j : ℕ, j < k → ∀ r, r ≠ s → χ r ≠ χ s + (j : ℂ) + 1) : GoodPath χ (List.replicate k s)

theorem **blocks_good** (χ : Fin n → ℂ) (m : Fin n → ℕ) (f : Fin n → ℝ) (hf : ∀ a, f a = (χ a).re + m a) (hmono : ∀ a b, (χ a).re < (χ b).re → m a ≤ m b) (l : List (Fin n)) : ∀ χ' : Fin n → ℂ, l.Pairwise (fun a b => f b < f a) → (∀ r, r ∈ l → χ' r = χ r) → (∀ r, r ∉ l → χ' r = χ r + m r) → (∀ r, r ∉ l → ∀ a ∈ l, f a < f r) → GoodPath χ' (blocks m l) ∧ pathEnd χ' (blocks m l) = fun a => χ a + m a

theorem **num** (χ : Fin n → ℂ) (B : ℝ) : ∃ C δ : ℝ, 0 < δ ∧ ∀ N₀ : ℝ, ∃ (m : Fin n → ℕ) (N : ℝ), N₀ ≤ N ∧ (∀ a b, (χ a).re < (χ b).re → m a ≤ m b) ∧ (∀ a, 1 ≤ m a) ∧ 0 < N ∧ FarLabels χ (fun a => χ a + (m a : ℂ)) B ∧ (∀ a b, a ≠ b → δ * N ≤ |((χ a).re + m a) - ((χ b).re + m b)|) ∧ ∀ a, |(χ a).re + m a| ≤ C * N ∧ |(χ a).im| ≤ C

theorem **pathExists** : CompanionTheory.RgExh.Fin.PathExists

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.GrowthStatement

theorem **hs_nonneg** (h : Matrix (Fin n) (Fin n) ℂ) : 0 ≤ Formalism.REAL.hs h

theorem **row_le_hs** (h : Matrix (Fin n) (Fin n) ℂ) (j : Fin n) : ∑ i, ‖h j i‖ ^ 2 ≤ Formalism.REAL.hs h

theorem **row_cs** (h : Matrix (Fin n) (Fin n) ℂ) (j : Fin n) (a : Fin n → ℂ) : ‖∑ i, h j i * a i‖ ^ 2 ≤ Formalism.REAL.hs h * ∑ i, ‖a i‖ ^ 2

theorem **row_bound** (h : Matrix (Fin n) (Fin n) ℂ) (j : Fin n) (a : Fin n → ℂ) (m : ℕ) : ‖∑ i, h j i * a i‖ ^ 2 * Formalism.REAL.hs h ^ (-((m + 1 : ℕ) : ℝ)) ≤ ∑ i, ‖a i‖ ^ 2 * Formalism.REAL.hs h ^ (-(m : ℝ))

theorem **enorm_sq_mul** (x : ℂ) (r : ℝ) : ‖x‖ₑ ^ 2 * ENNReal.ofReal r = ENNReal.ofReal (‖x‖ ^ 2 * r)

theorem **continuous_hs** (ε : Fin n → Bool) : Continuous fun g : S7.UJ ε => Formalism.REAL.hs (S7.toM g)

theorem **measurable_term** (ε : Fin n → Bool) {f : Matrix (Fin n) (Fin n) ℂ → ℂ} (s : ℝ) (hf : Continuous fun g : S7.UJ ε => f (S7.toM g)) : Measurable fun g : S7.UJ ε => ‖f (S7.toM g)‖ₑ ^ 2 * ENNReal.ofReal (Formalism.REAL.hs (S7.toM g) ^ s)

theorem **lev_row** (ε : Fin n → Bool) (m : ℕ) (f : Fin n → Matrix (Fin n) (Fin n) ℂ → ℂ) (hf : ∀ i, Lev ε m (f i)) (j : Fin n) : Lev ε (m + 1) fun g => ∑ i, g j i * f i g

theorem **level_zero_of_memLp** (ε : Fin n → Bool) (F : Matrix (Fin n) (Fin n) ℂ → ℂ) (h : MemLp (fun g : Formalism.REAL.UJ ε => F (Formalism.MainNotions.S.umat g)) 2 (Formalism.REAL.muG ε)) : ∫⁻ g : S7.UJ ε, ‖F (S7.toM g)‖ₑ ^ 2 ∂Measure.haar < ⊤

theorem **lev_base** (ε : Fin n → Bool) {H : Type} [NormedAddCommGroup H] [InnerProductSpace ℂ H] (σ : Matrix (Fin n) (Fin n) ℂ → (H →L[ℂ] H)) (D : Submodule ℂ H) (ξ : H) (hσ : Formalism.MainNotions.S.IsURepOn (Formalism.MainNotions.S.UJset ε) σ) (hsq : Formalism.MainNotions.S.IsSqIntOn ε σ) (u : D) : Lev ε 0 (baseCoef σ D ξ u)

theorem **lev_path** (ε : Fin n → Bool) : ∀ (ss : List (Fin n)) (X : GMod n) (c : X.V →ₗ[ℂ] (Matrix (Fin n) (Fin n) ℂ → ℂ)) (χ : Fin n → ℂ) (js : List (Fin n)) (m : ℕ), (∀ u, Lev ε m (c u)) → ∀ u, Lev ε (m + ss.length) (pathCoef X c χ ss js u) | [], _, _, _, _, _, h, u => h u | s :: ss, X, c, χ, js, m, h, u => by have key

theorem **growth** : Growth

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.GrowthStatementStatement

theorem **_root_.GlobalTheory.T.GrowthStatement** : CompanionTheory.RgExh.Dec.L.Growth

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.HaarExistence

theorem **keeps_iff_forall_mem** (g : (Matrix (Fin n) (Fin n) ℂ)ˣ) (S : Submodule ℂ (Fin n → ℂ)) : Keeps (g : Matrix (Fin n) (Fin n) ℂ) S ↔ ∀ v ∈ S, (g : Matrix (Fin n) (Fin n) ℂ) *ᵥ v ∈ S

theorem **isClosed_keeps** (ε : Fin n → Bool) (S : Submodule ℂ (Fin n → ℂ)) : IsClosed {g : S7.UJ ε | Keeps (S7.toM g) S}

theorem **isClosed_pSub** (ε : Fin n → Bool) (A : Matrix (Fin n) (Fin n) ℂ) : IsClosed (PSub ε A : Set (S7.UJ ε))

theorem **haar_pSub_isHaar** (ε : Fin n → Bool) (A : Matrix (Fin n) (Fin n) ℂ) : (Measure.haar : Measure (PSub ε A)).IsHaarMeasure

theorem **haarExists** : CompanionTheory.RgExh.Dec.L.HaarExists

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.HaarExistenceStatement

theorem **_root_.GlobalTheory.T.HaarExistence** : CompanionTheory.RgExh.Dec.L.HaarExists

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.HaarMeasureNonunitsNull

theorem **null_of_overlap** {V : Type} [AddCommGroup V] [TopologicalSpace V] [IsTopologicalAddGroup V] [LocallyCompactSpace V] [SecondCountableTopology V] [T2Space V] [MeasurableSpace V] [BorelSpace V] (μ : Measure V) [μ.IsAddHaarMeasure] {Z : Set V} (hZ : MeasurableSet Z) (u : ℕ → V) (hu : Filter.Tendsto u Filter.atTop (𝓝 0)) (hov : ∀ (x : V) (s : Finset ℕ), (∀ i ∈ s, x - u i ∈ Z) → s.card ≤ 2) : μ Z = 0

theorem **card_le_two** {K : Type} [Field K] (a b q : K) (hq : q ≠ 0) (r : ℕ → K) (hr : Function.Injective r) (s : Finset ℕ) (h : ∀ i ∈ s, a + r i * b + r i ^ 2 * q = 0) : s.card ≤ 2

theorem **sigR_algebraMap** (s : R) : Formalism.MainNotions.GlobalTheoryTheory.GenusMassVolumes.sigR R σ (algebraMap R (R ⊗[F] E) s) = algebraMap R (R ⊗[F] E) s

theorem **qR_line** [Module.Free F E] [Module.Finite F E] (x y : Fin M → R ⊗[F] E) (s : R) : Formalism.MainNotions.GlobalTheoryTheory.GenusMassVolumes.qR R σ J (fun j => x j + algebraMap R (R ⊗[F] E) s * y j) = Formalism.MainNotions.GlobalTheoryTheory.GenusMassVolumes.qR R σ J x + s * (algebraMap F R (1 / 2) * Algebra.trace R (R ⊗[F] E) (Formalism.MainNotions.GlobalTheoryTheory.GenusMassVolumes.hR R σ J x y + Formalism.MainNotions.GlobalTheoryTheory.GenusMassVolumes.hR R σ J y x)) + s ^ 2 * Formalism.MainNotions.GlobalTheoryTheory.GenusMassVolumes.qR R σ J y

theorem **qR_rat** [CharZero F] (b : Module.Basis (Fin 2) F E) (x : Fin M → E) (β : F) (h : Formalism.MainNotions.GlobalTheoryTheory.GenusMassVolumes.hE σ J x x = algebraMap F E β) : Formalism.MainNotions.GlobalTheoryTheory.GenusMassVolumes.qR R σ J (fun j => Formalism.MainNotions.GlobalTheoryTheory.GenusMassVolumes.inclR (F

theorem **nhdsNE_Fv** : (𝓝[≠] (0 : Fv F v)).NeBot

theorem **exists_seq_Fv** : ∃ t : ℕ → Fv F v, Function.Injective t ∧ Filter.Tendsto t Filter.atTop (𝓝 0)

theorem **null_nonunits** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → 1 ≤ M → ∀ (ν : HeightOneSpectrum (𝒪 F)) (μ : Measure (Fin 1 → Fin M → Fv F v ⊗[F] E)) (ν : Measure (Fv F v)), μ.IsAddHaarMeasure → ν.IsAddHaarMeasure → ν {c | ¬ IsUnit c} = 0 ∧ μ {X | ¬ IsUnit ((fun X : Fin 1 → Fin M → Fv F v ⊗[F] E => Formalism.MainNotions.GlobalTheoryTheory.GenusMassVolumes.qR (Fv F v) σ J (X 0)) X)} = 0

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.HasCharOfScalChar

theorem **hasChar_path** (hS : StepChar) (ss : List (Fin n)) : ∀ (X : GMod n) (χ : Fin n → ℂ), HasChar X χ → HasChar (path X χ ss) (pathEnd χ ss)

theorem **hasChar_of_scalChar** {X : GMod n} {χ : Fin n → ℂ} (h : ScalChar X χ) : HasChar X χ

theorem **capelliPoly_eq_of_perm** {χ χ₂ : Fin n → ℂ} (τ τ₂ : Equiv.Perm (Fin n)) (h : ∀ a, χ (τ a) = χ₂ (τ₂ a)) : Formalism.QuantumDeterminants.capP χ = Formalism.QuantumDeterminants.capP χ₂

theorem **iso_of_ends** (hH : Hecke) (hI : IsoFacts) (hU : AlgUniq) {ε : Fin n → Bool} {M M' : GMod n} {χ : Fin n → ℂ} {ss : List (Fin n)} {d : Equiv.Perm (Fin n) × (Fin n → ℝ)} (hg : GoodPath χ ss) (hM : IsSimple M) (hM' : IsSimple M') (hc : HasChar M χ) (hc' : HasChar M' χ) (hs : ScalChar (path M χ ss) (pathEnd χ ss)) (e : EndOf ε M χ ss d) (e' : EndOf ε M' χ ss d) : Iso M M'

theorem **finish** : StartMod → PathFin → PathExists → PathEndNat → SimpleDistinct → ScalarCharacterOfL2Embedding → UnitarizableOfL2Embedding → IsoFacts → CharOfAlgDS → AlgUniq → DataUniq → DSOfArr → DSOfData → KModOf → ArrCount → Finish

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.HasCharSub

theorem **actM_single** {M : Type} [AddCommGroup M] [Module ℂ M] (E : Fin n → Fin n → Module.End ℂ M) (a b : Fin n) : actM E (Matrix.single a b 1) = E a b

theorem **glRel_Eb** (X : GMod n) {A : Matrix (Fin n) (Fin n) ℂ} (B : GradBasis A) : GlRel (Eb X B)

theorem **ofFn_lin** {V V' : Type} [AddCommGroup V] [Module ℂ V] [AddCommGroup V'] [Module ℂ V'] (f : V →ₗ[ℂ] V') : ∀ {t : ℕ} (g : Fin t → Module.End ℂ V) (g' : Fin t → Module.End ℂ V'), (∀ k x, f (g k x) = g' k (f x)) → ∀ x, f ((List.ofFn g).prod x) = (List.ofFn g').prod (f x) | 0, _, _, _, x => by simp | t + 1, g, g', h, x => by rw [List.ofFn_succ, List.ofFn_succ, List.prod_cons, List.prod_cons, Module.End.mul_apply, Module.End.mul_apply, h, ofFn_lin f (fun k => g k.succ) (fun k => g' k.succ) (fun k x => h k.succ x)] theorem qdet_lin {V V' : Type} [AddCommGroup V] [Module ℂ V] [AddCommGroup V'] [Module ℂ V'] (f : V →ₗ[ℂ] V') {m : ℕ} (E : Fin m → Fin m → Module.End ℂ V) (E' : Fin m → Fin m → Module.End ℂ V') (h : ∀ a b x, f (E a b x) = E' a b (f x)) (u : ℂ) (x : V) : f (qdet E u x) = qdet E' u (f x)

theorem **pow_lin** {V V' : Type} [AddCommGroup V] [Module ℂ V] [AddCommGroup V'] [Module ℂ V'] (f : V →ₗ[ℂ] V') (T : Module.End ℂ V) (T' : Module.End ℂ V') (h : ∀ x, f (T x) = T' (f x)) (k : ℕ) (x : V) : f ((T ^ k) x) = (T' ^ k) (f x)

theorem **shift_lin** {V V' : Type} [AddCommGroup V] [Module ℂ V] [AddCommGroup V'] [Module ℂ V'] (f : V →ₗ[ℂ] V') (T : Module.End ℂ V) (T' : Module.End ℂ V') (h : ∀ x, f (T x) = T' (f x)) (c : ℂ) (x : V) : f ((T - c • (1 : Module.End ℂ V)) x) = (T' - c • (1 : Module.End ℂ V')) (f x)

theorem **hasChar_sub** (Y : GMod n) (S : Submodule ℂ Y.V) (h : ∀ a b, ∀ x ∈ S, Y.E a b x ∈ S) {χ : Fin n → ℂ} (hY : HasChar Y χ) : HasChar (Y.sub S h) χ

theorem **hasChar_quo** (Y : GMod n) (S : Submodule ℂ Y.V) (h : ∀ a b, ∀ x ∈ S, Y.E a b x ∈ S) {χ : Fin n → ℂ} (hY : HasChar Y χ) : HasChar (quoMod Y S h) χ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.HasDerivAtExpMul

lemma **tor_add** (θ η : Fin n → ℝ) : tor (θ + η) = tor θ * tor η

lemma **tor_zero** : tor (0 : Fin n → ℝ) = 1

lemma **star_exp_mul** (x : ℝ) : star (Complex.exp ((x : ℂ) * Complex.I)) * Complex.exp ((x : ℂ) * Complex.I) = 1

lemma **tor_mem_UJ** (ε : Fin n → Bool) (θ : Fin n → ℝ) : tor θ ∈ UJset ε

lemma **tor_mem_K** (ε : Fin n → Bool) (θ : Fin n → ℝ) : tor θ ∈ Kset ε

lemma **diag_inG** (ε : Fin n → Bool) (θ : Fin n → ℝ) : inG ε (Matrix.diagonal fun a => (θ a : ℂ) * Complex.I)

lemma **exp_diag** (θ : Fin n → ℝ) (t : ℝ) : NormedSpace.exp ((t : ℂ) • Matrix.diagonal fun a => (θ a : ℂ) * Complex.I) = tor (t • θ)

lemma **deriv_zero** {ε : Fin n → Bool} {σ : Matrix (Fin n) (Fin n) ℂ → (H →L[ℂ] H)} {S : Submodule ℂ H} {E : Fin n → Fin n → Module.End ℂ S} (h : CompanionTheory.D.DSIC.IsSmMod ε σ S E) (θ : Fin n → ℝ) (v : S) : HasDerivAt (fun t : ℝ => σ (tor (t • θ)) v) (∑ a, ((θ a : ℂ) * Complex.I) • (E a a v : H)) 0

lemma **hasDerivAt_exp_mul** (c : ℂ) (s : ℝ) : HasDerivAt (fun t : ℝ => Complex.exp ((t : ℂ) * c)) (Complex.exp ((s : ℂ) * c) * c) s

lemma **one_param** (φ : ℝ → (H →L[ℂ] H)) (hadd : ∀ s t, φ (s + t) = φ s * φ t) (h0 : φ 0 = 1) (v : H) (c : ℂ) (hd : HasDerivAt (fun t => φ t v) (c • v) 0) : φ 1 v = Complex.exp c • v

theorem **wtLink** : CompanionTheory.D.DSIC.WtLink

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.HasDerivAtExt

theorem **fderiv_comm** {E G : Type} [NormedAddCommGroup E] [NormedSpace ℝ E] [NormedAddCommGroup G] [NormedSpace ℝ G] {F : E → G} {x : E} (hF : ContDiffAt ℝ ∞ F x) (L M : E →L[ℝ] E) {a b c : E} (ha : M x = a) (hb : L x = b) (hc : L a − M b = c) : fderiv ℝ (fun g => fderiv ℝ F g (L g)) x a − fderiv ℝ (fun g => fderiv ℝ F g (M g)) x b = fderiv ℝ F x c

theorem **lie_bracket** {J X Y : Mat n} (hX : Xᴴ * J + J * X = 0) (hY : Yᴴ * J + J * Y = 0) : (X * Y − Y * X)ᴴ * J + J * (X * Y − Y * X) = 0

theorem **hasDerivAt_ext** (h : RepOn G σ) {X g : Mat n} (hX : ∀ t, ex X t ∈ G) (hg : g ∈ G) {f F : Mat n → H} {U : Set (Mat n)} (hU : IsOpen U) (hgU : g ∈ U) (hF : ContDiffOn ℝ ∞ F U) (hfF : ∀ x ∈ U, x ∈ G → f x = F x) : HasDerivAt (fun t => f (g * ex X t)) (fderiv ℝ F g (g * X)) 0

theorem **ds_ds** (h : RepOn G σ) {X Y : Mat n} (hX : ∀ t, ex X t ∈ G) (hY : ∀ t, ex Y t ∈ G) {v : H} (hv : IsSm G σ v) {F : Mat n → H} {U : Set (Mat n)} (hU : IsOpen U) (h1 : (1 : Mat n) ∈ U) (hF : ContDiffOn ℝ ∞ F U) (hfF : ∀ x ∈ U, x ∈ G → σ x v = F x) (L : Mat n →L[ℝ] Mat n) (hL : ∀ x, L x = x * Y) : ds σ X (ds σ Y v) = fderiv ℝ (fun g => fderiv ℝ F g (L g)) 1 X

theorem **ds_bracket** (h : RepOn G σ) {X Y : Mat n} (hX : ∀ t, ex X t ∈ G) (hY : ∀ t, ex Y t ∈ G) (hXY : ∀ t, ex (X * Y − Y * X) t ∈ G) {v : H} (hv : IsSm G σ v) : ds σ X (ds σ Y v) − ds σ Y (ds σ X v) = ds σ (X * Y − Y * X) v

theorem **smBracket** : CompanionTheory.D.DSIC.SmBracket

theorem **smBracket_of** : CompanionTheory.D.DSIC.SmLin → CompanionTheory.D.DSIC.SmBracket

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.HeckeAlgebraStatement

theorem **smul_mk** (g x : G) : g • (x : G ⧸ K) = ((g * x : G) : G ⧸ K)

theorem **lam_single** (g : G) (x : G ⧸ K) (c : ℂ) : lam K g (Finsupp.single x c) = Finsupp.single (g • x) c

theorem **eps_single** (x : G ⧸ K) (c : ℂ) : eps K (Finsupp.single x c) = c

theorem **eps_lam** (g : G) (v : (G ⧸ K) →₀ ℂ) : eps K (lam K g v) = eps K v

theorem **hk_comm** (A : Hk K) (g : G) (v : (G ⧸ K) →₀ ℂ) : A.1 (lam K g v) = lam K g (A.1 v)

theorem **lam_out_dd** (y : G ⧸ K) : lam K (Quotient.out y) (dd K) = Finsupp.single y 1

theorem **lam_dd** {k : G} (hk : k ∈ K) : lam K k (dd K) = dd K

theorem **hk_fixed** (A : Hk K) {k : G} (hk : k ∈ K) : lam K k (A.1 (dd K)) = A.1 (dd K)

theorem **fixed_eq_smul** (w : (G ⧸ K) →₀ ℂ) (hw : ∀ k ∈ K, lam K k w = w) (u : G) (S : Finset (G ⧸ K)) (hS : (↑S : Set (G ⧸ K)) = MulAction.orbit K (u : G ⧸ K)) (hsupp : ∀ y, y ∉ S → w y = 0) : w = w (u : G ⧸ K) • ∑ y ∈ S, Finsupp.single y (1 : ℂ)

theorem **q_ne** (g : G) : q hp g ≠ 0

theorem **T_dd** (g : G) : (T g).1 (dd K) = ∑ y ∈ (hp g).toFinset, Finsupp.single y (1 : ℂ)

theorem **augmentation_of_hecke_operator** (g : G) : eps K ((T g).1 (dd K)) = q hp g

theorem **ext_dd** {A B : Hk K} (h : A.1 (dd K) = B.1 (dd K)) : A = B

theorem **mul_of** {g h u : G} (H : ∀ k ∈ K, ∃ a ∈ K, ∃ b ∈ K, h * k * g = a * u * b) : T g * T h = (q hp g * q hp h / q hp u) • T u

theorem **R_mul** {g h u : G} (H : ∀ k ∈ K, ∃ a ∈ K, ∃ b ∈ K, h * k * g = a * u * b) : R T hp g * R T hp h = R T hp u

theorem **T_congr** {g : G} {k k' : G} (hk : k ∈ K) (hk' : k' ∈ K) : T (k * g * k') = T g ∧ q hp (k * g * k') = q hp g

theorem **R_congr** {g : G} {k k' : G} (hk : k ∈ K) (hk' : k' ∈ K) : R T hp (k * g * k') = R T hp g

theorem **R_one** : R T hp 1 = 1

theorem **stHecke** : StHecke

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.HeckeAlgebraStatementStatement

theorem **_root_.GlobalTheory.T.HeckeAlgebraTheorem** : StHecke

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.HeckeBackward

theorem **isRes_one** : IsRes S 1 1

theorem **isRes_zero** : IsRes S 0 0

theorem **IsRes.add** {T U : Module.End ℂ M} {T' U' : Module.End ℂ S} (h : IsRes S T T') (k : IsRes S U U') : IsRes S (T + U) (T' + U')

theorem **IsRes.sub** {T U : Module.End ℂ M} {T' U' : Module.End ℂ S} (h : IsRes S T T') (k : IsRes S U U') : IsRes S (T − U) (T' − U')

theorem **IsRes.neg** {T : Module.End ℂ M} {T' : Module.End ℂ S} (h : IsRes S T T') : IsRes S (−T) (−T')

theorem **IsRes.smul** {T : Module.End ℂ M} {T' : Module.End ℂ S} (c : ℂ) (h : IsRes S T T') : IsRes S (c • T) (c • T')

theorem **IsRes.mul** {T U : Module.End ℂ M} {T' U' : Module.End ℂ S} (h : IsRes S T T') (k : IsRes S U U') : IsRes S (T * U) (T' * U')

theorem **IsRes.pow** {T : Module.End ℂ M} {T' : Module.End ℂ S} (h : IsRes S T T') (k : ℕ) : IsRes S (T ^ k) (T' ^ k)

theorem **isRes_algebraMap** (c : ℂ) : IsRes S (algebraMap ℂ (Module.End ℂ M) c) (algebraMap ℂ (Module.End ℂ S) c)

theorem **isRes_sign** (e : ℤˣ) : IsRes S (((e : ℤ) : Module.End ℂ M)) (((e : ℤ) : Module.End ℂ S))

theorem **isRes_sum** {ι : Type*} (s : Finset ι) (F : ι → Module.End ℂ M) (G : ι → Module.End ℂ S) (h : ∀ i, IsRes S (F i) (G i)) : IsRes S (∑ i ∈ s, F i) (∑ i ∈ s, G i)

theorem **isRes_ofFn** : ∀ (t : ℕ) (F : Fin t → Module.End ℂ M) (G : Fin t → Module.End ℂ S), (∀ k, IsRes S (F k) (G k)) → IsRes S (List.ofFn F).prod (List.ofFn G).prod

theorem **isRes_qdet** {n : ℕ} (E : Fin n → Fin n → Module.End ℂ M) (E' : Fin n → Fin n → Module.End ℂ S) (h : ∀ a b, IsRes S (E a b) (E' a b)) (x : ℂ) : IsRes S (qdet E x) (qdet E' x)

theorem **isPol_const** (T : Module.End ℂ M) : IsPol fun _ => T

theorem **isPol_id** : IsPol (M

theorem **IsPol.add** {F G : ℂ → Module.End ℂ M} (h : IsPol F) (k : IsPol G) : IsPol fun x => F x + G x

theorem **IsPol.mul** {F G : ℂ → Module.End ℂ M} (h : IsPol F) (k : IsPol G) : IsPol fun x => F x * G x

theorem **isPol_sum** {ι : Type*} (s : Finset ι) (F : ι → ℂ → Module.End ℂ M) (h : ∀ i, IsPol (F i)) : IsPol fun x => ∑ i ∈ s, F i x

theorem **isPol_ofFn** : ∀ (t : ℕ) (F : Fin t → ℂ → Module.End ℂ M), (∀ k, IsPol (F k)) → IsPol fun x => (List.ofFn fun k => F k x).prod

theorem **isPol_qdet** {n : ℕ} (E : Fin n → Fin n → Module.End ℂ M) : IsPol (qdet E)

theorem **IsPol.scalar** {F : ℂ → Module.End ℂ M} (h : IsPol F) (v : M) (φ : Module.Dual ℂ M) : ∃ q : ℂ[X], ∀ x, φ (F x v) = q.eval x

theorem **qdet_ext** {n : ℕ} (E : Fin n → Fin n → Module.End ℂ M) (g : ℂ[X]) (v : M) (u₀ : ℂ) (h : ∀ u, u ≠ u₀ → qdet E u v = g.eval u • v) : qdet E u₀ v = g.eval u₀ • v

theorem **commute_aeval** (T L : Module.End ℂ W) (hc : Commute T L) (p : ℂ[X]) : Commute (aeval T p) L

theorem **kill** (T L : Module.End ℂ W) (P : ℂ[X]) (hP : P ≠ 0) (hT : aeval T P = 0) (hc : Commute T L) (a : ℂ) (hL : ∀ f ∈ T.maxGenEigenspace a, L f = 0) (f : W) (hf : f ∈ T.maxGenEigenspace a) (y : W) (hy : f = L y) : f = 0

theorem **cn_def** (n : ℕ) : cn n = ((n : ℂ) - 1) / 2

theorem **capelliPoly_eq_mul_prod_erase** (χ : Fin n → ℂ) (s : Fin n) (Y : ℂ) : capP χ Y = (Y + χ s) * ∏ t ∈ univ.erase s, (Y + χ t)

theorem **capelliPoly_add_single** (χ : Fin n → ℂ) (s : Fin n) (Y : ℂ) : capP (χ + Pi.single s 1) Y = (Y + χ s + 1) * ∏ t ∈ univ.erase s, (Y + χ t)

theorem **qdet_on_step** (hS : ScalChar (step X (χ s)) (χ + Pi.single s 1)) (Y : ℂ) (f : Fin n → X.V) (hf : f ∈ stepSub X (χ s)) : qdet (tensE X) (Y + cn n) f = capP (χ + Pi.single s 1) Y • f

theorem **star** (hQ : QdetTens) (hS : ScalChar (step X (χ s)) (χ + Pi.single s 1)) (Y : ℂ) (f : Fin n → X.V) (hf : f ∈ stepSub X (χ s)) (i : Fin n) : capP (χ + Pi.single s 1) Y • ((Y + cn n) • f i + om X f i) = qdet X.E (Y + cn n) ((Y + cn n + 1) • f i + om X f i)

theorem **eig** (hQ : QdetTens) (hS : ScalChar (step X (χ s)) (χ + Pi.single s 1)) (f : Fin n → X.V) (hf : f ∈ stepSub X (χ s)) (he : om X f = (χ s - cn n) • f) (i : Fin n) (Y : ℂ) : qdet X.E (Y + cn n) (f i) = capP χ Y • f i

theorem **no_block** (hQ : QdetTens) (hS : ScalChar (step X (χ s)) (χ + Pi.single s 1)) (hfree : ∀ r, χ r ≠ χ s + 1) (f h : Fin n → X.V) (hf : f ∈ stepSub X (χ s)) (hh : h ∈ stepSub X (χ s)) (e1 : om X h = (χ s - cn n) • h) (e2 : om X f = (χ s - cn n) • f + h) : h = 0

theorem **nu_zero** (hQ : QdetTens) (hS : ScalChar (step X (χ s)) (χ + Pi.single s 1)) (hfree : ∀ r, χ r ≠ χ s + 1) (m : ℕ) (f : Fin n → X.V) (hf : f ∈ stepSub X (χ s)) (hm : ((om X - (χ s - cn n) • (1 : Module.End ℂ (Fin n → X.V))) ^ (m + 1)) f = 0) : (om X - (χ s - cn n) • (1 : Module.End ℂ (Fin n → X.V))) f = 0

theorem **om_scalar** (hQ : QdetTens) (hS : ScalChar (step X (χ s)) (χ + Pi.single s 1)) (hfree : ∀ r, χ r ≠ χ s + 1) (f : Fin n → X.V) (hf : f ∈ stepSub X (χ s)) : om X f = (χ s - cn n) • f

theorem **central** (X : GMod n) (N : Module.End ℂ X.V) (Y c : ℂ) (hN : N = qdet X.E Y - c • (1 : Module.End ℂ X.V)) (a b : Fin n) : Commute N (X.E a b)

theorem **commute_om_compLeft** (X : GMod n) (N : Module.End ℂ X.V) (hc : ∀ a b, Commute N (X.E a b)) : Commute (om X) (N.compLeft (Fin n))

theorem **scal_before** (hQ : QdetTens) (hC : CayleyHam) (hV : NonVan) (hX : HasChar X χ) (hfree : ∀ r, χ r ≠ χ s + 1) (hS : ScalChar (step X (χ s)) (χ + Pi.single s 1)) : ScalChar X χ

theorem **back** : Back

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.HeckeBackwardStable

theorem **tensVd_apply** (X : GMod n) (a b : Fin n) (f : Fin n → X.V) (i : Fin n) : tensVd X.E a b f i = X.E a b (f i) - if i = b then f a else 0

theorem **eeVd_apply** (X : GMod n) (f : Fin n → X.V) (a : Fin n) : eeVd X.E f a = -∑ c, X.E a c (f c)

theorem **commute_tensVd_eeVd** (X : GMod n) (c d : Fin n) : Commute (tensVd X.E c d) (eeVd X.E)

theorem **backStable** : BackStable

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.HeckeBackwardStableStatement

theorem **_root_.GlobalTheory.T.HeckeBackwardStable** : CompanionTheory.RgExh.Hk.BackStable

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.HeckeBackwardStatement

theorem **_root_.GlobalTheory.T.HeckeBackward** : CompanionTheory.RgExh.Hk.Back

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.HeckeDualityFacts

theorem **glRel_twE** {A : Type*} [Ring A] {E : Fin n → Fin n → A} (h : GlRel E) : GlRel (twE E)

theorem **capEntry_twE** {A : Type*} [Ring A] [Algebra ℂ A] (E : Fin n → Fin n → A) (y : ℂ) (a b : Fin n) : capEntry (twE E) y a b = -capEntry E (-y) b a

theorem **ofFn_neg_prod** {A : Type*} [Ring A] : ∀ {p : ℕ} (f : Fin p → A), (List.ofFn fun j => -f j).prod = (-1) ^ p * (List.ofFn f).prod

theorem **capelliC_twE** {A : Type*} [Ring A] [Algebra ℂ A] (E : Fin n → Fin n → A) (hE : GlRelR E) (x : ℂ) : capelliC (twE E) x = (-1) ^ n * capelliC E ((n : ℂ) - 1 - x)

theorem **capelliPoly_neg** (χ : Fin n → ℂ) (Y : ℂ) : Formalism.QuantumDeterminants.capP (-χ) Y = (-1) ^ n * Formalism.QuantumDeterminants.capP χ (-Y)

theorem **hasChar_tw** {X : GMod n} {χ : Fin n → ℂ} (h : HasChar X χ) : HasChar (tw X) (-χ)

theorem **capelliC_map** {A B : Type*} [Ring A] [Algebra ℂ A] [Ring B] [Algebra ℂ B] (φ : A →ₐ[ℂ] B) (E : Fin n → Fin n → A) (x : ℂ) : φ (capelliC E x) = capelliC (fun a b => φ (E a b)) x

theorem **hasChar_of_iso** {X X' : GMod n} {χ : Fin n → ℂ} (h : Iso X X') (hX : HasChar X χ) : HasChar X' χ

theorem **om_tw** (X : GMod n) : om (tw X) = omD X

theorem **stepSub_tw** (X : GMod n) (z : ℂ) : stepSub (tw X) (-z) = backSub X z

theorem **dual_cayley** (X : GMod n) (χ : Fin n → ℂ) (h : HasChar X χ) : ∃ K : ℕ, ((List.ofFn fun t : Fin n => omD X - (-χ t - cn n) • (1 : Module.End ℂ (Fin n → X.V))).prod) ^ K = 0

theorem **dual_nonvan** (X : GMod n) (χ : Fin n → ℂ) (h : HasChar X χ) (s : Fin n) (hne : Nontrivial X.V) (hfree : ∀ r, χ r ≠ χ s - 1) : backSub X (χ s) ≠ ⊥

theorem **tw_tens** (X : GMod n) (a b : Fin n) (y : Fin n → X.V) : -(tensE (tw X) b a y) = ReciprocityTheory.ChartRestriction.DS.Lap.Capelli.tensVd X.E a b y

theorem **iso_back** (X : GMod n) (z : ℂ) (hB : ∀ a b, ∀ x ∈ backSub X z, (tensD X).E a b x ∈ backSub X z) : Iso (tw (step (tw X) (-z))) (back X z hB)

theorem **dual_char** (X : GMod n) (χ : Fin n → ℂ) (h : HasChar X χ) (s : Fin n) (hB : ∀ a b, ∀ x ∈ backSub X (χ s), (tensD X).E a b x ∈ backSub X (χ s)) : HasChar (back X (χ s) hB) (χ - Pi.single s 1)

theorem **dualFacts** : DualFacts

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.HeckeDualityFactsStatement

theorem **_root_.GlobalTheory.T.HeckeDualityFacts** : CompanionTheory.RgExh.Hk.DualFacts

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.HeckeNonScalar

theorem **IsP.const** (a : A) : IsP (fun _ : ℂ => a)

theorem **IsP.lin** (a b : A) : IsP (fun t : ℂ => a + b * algebraMap ℂ A t)

theorem **IsP.mul** {f g : ℂ → A} (hf : IsP f) (hg : IsP g) : IsP (fun t => f t * g t)

theorem **IsP.sum** {ι : Type*} (s : Finset ι) {f : ι → ℂ → A} (hf : ∀ i, IsP (f i)) : IsP (fun t => ∑ i ∈ s, f i t)

theorem **IsP.listProd** (l : List (ℂ → A)) (h : ∀ f ∈ l, IsP f) : IsP (fun t => (l.map fun f => f t).prod)

theorem **isPolynomial_capelliEntry** {n : ℕ} (E : Fin n → Fin n → A) (x₀ : ℂ) (a b : Fin n) : IsP (fun t : ℂ => capEntry E (x₀ + t) a b)

theorem **isPolynomial_capelliMinor** {n p : ℕ} (E : Fin n → Fin n → A) (R S : Fin p → Fin n) (x₀ : ℂ) : IsP (fun t : ℂ => cminor E R S (x₀ + t))

theorem **coeff_extract** {W : Type*} [AddCommGroup W] [Module ℂ W] (T : Module.End ℂ W) (k : ℕ) (hk : 2 ≤ k) (w : ℕ → W) (P : ℂ[X]) (u : W) (hP : (X : ℂ[X]) ^ 2 ∣ P) (h : ∀ t : ℂ, T (∑ i ∈ range k, t ^ i • w i) + t • ∑ i ∈ range k, t ^ i • w i = P.eval t • u) : T (w 0) = 0 ∧ T (w 1) + w 0 = 0

theorem **om_nnPlus** {m : ℕ} (X : GMod (m + 1)) (y : Fin (m + 1) → ℂ) (x : ℂ) (v : X.V) : om X (nnPlus X.E y x v) + x • nnPlus X.E y x v = fun b => y b • capelliC X.E x v

theorem **nonScal** : NonScal

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.HeckeNonScalarStatement

theorem **_root_.GlobalTheory.T.HeckeNonScalar** : CompanionTheory.RgExh.Hk.NonScal

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.HeckeNonVanishing

theorem **eeV_eq_om** {n : ℕ} (X : GMod n) : eeV X.E = om X

theorem **exists_ker** {M : Type} [AddCommGroup M] [Module ℂ M] [Nontrivial M] (T : Module.End ℂ M) {k : ℕ} (h : T ^ k = 0) : ∃ v : M, v ≠ 0 ∧ T v = 0

theorem **eq_zero_of_nilp** {M : Type} [AddCommGroup M] [Module ℂ M] (T : Module.End ℂ M) {c : ℂ} {k : ℕ} (hc : c ≠ 0) (h : (T - c • (1 : Module.End ℂ M)) ^ k = 0) {v : M} (hv : T v = 0) : v = 0

theorem **exists_nnPlus_ne** {m : ℕ} {M : Type} [AddCommGroup M] [Module ℂ M] (E : Fin (m + 1) → Fin (m + 1) → Module.End ℂ M) (hE : GlRel E) (w : ℂ) {v : M} (hv0 : v ≠ 0) (hinj : capelliC E (w − 1) v = 0 → v = 0) : ∃ y : Fin (m + 1) → ℂ, nnPlus E y w v ≠ 0

theorem **nonVan** : NonVan

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.HeckeNonVanishingStatement

theorem **_root_.GlobalTheory.T.HeckeNonVanishing** : CompanionTheory.RgExh.Hk.NonVan

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.HeckeOperatorsSpanAndFaithful

theorem **lam_single** (g : G) (y : G ⧸ K) (c : ℂ) : lam K g (Finsupp.single y c) = Finsupp.single (g • y) c

theorem **lam_mul** (g g' : G) (u : (G ⧸ K) →₀ ℂ) : lam K (g * g') u = lam K g (lam K g' u)

theorem **lam_one** (u : (G ⧸ K) →₀ ℂ) : lam K 1 u = u

theorem **lam_apply_smul** (g : G) (u : (G ⧸ K) →₀ ℂ) (y : G ⧸ K) : lam K g u (g • y) = u y

theorem **lam_congr** {u : (G ⧸ K) →₀ ℂ} (hu : IsFixed K u) {x x' : G} (h : (x : G ⧸ K) = x') : lam K x u = lam K x' u

theorem **op_single_out** (u : (G ⧸ K) →₀ ℂ) (y : G ⧸ K) (c : ℂ) : op K u (Finsupp.single y c) = c • lam K (Quotient.out y) u

theorem **op_single** {u : (G ⧸ K) →₀ ℂ} (hu : IsFixed K u) (x : G) (c : ℂ) : op K u (Finsupp.single (x : G ⧸ K) c) = c • lam K x u

theorem **op_mem** {u : (G ⧸ K) →₀ ℂ} (hu : IsFixed K u) : op K u ∈ Hk K

theorem **lam_delta** (x : G) : lam K x (delta K) = Finsupp.single (x : G ⧸ K) 1

theorem **delta_fixed_eval** (h : Hk K) : IsFixed K ((h : Module.End ℂ ((G ⧸ K) →₀ ℂ)) (delta K))

theorem **op_delta** {u : (G ⧸ K) →₀ ℂ} (hu : IsFixed K u) : op K u (delta K) = u

theorem **eq_zero_of_delta** (h : Hk K) (h0 : (h : Module.End ℂ ((G ⧸ K) →₀ ℂ)) (delta K) = 0) : h = 0

theorem **orbitVec_apply** (hK : IsHeckePair K) (g : G) (z : G ⧸ K) : orbitVec hK g z = if z ∈ MulAction.orbit K (g : G ⧸ K) then 1 else 0

theorem **lam_apply** (k : G) (u : (G ⧸ K) →₀ ℂ) (z : G ⧸ K) : lam K k u z = u ((k⁻¹ : G) • z)

theorem **orbitVec_fixed** (hK : IsHeckePair K) (g : G) : IsFixed K (orbitVec hK g)

theorem **IsFixed.apply_of_mem_orbit** {u : (G ⧸ K) →₀ ℂ} (hu : IsFixed K u) {x z : G ⧸ K} (hz : z ∈ MulAction.orbit K x) : u z = u x

theorem **heckeOp_delta** (hK : IsHeckePair K) (g : G) : (heckeOp hK g : Module.End ℂ ((G ⧸ K) →₀ ℂ)) (delta K) = orbitVec hK g

theorem **exists_mem_span_delta** (hK : IsHeckePair K) (n : ℕ) : ∀ u : (G ⧸ K) →₀ ℂ, IsFixed K u → u.support.card ≤ n → ∃ t ∈ Submodule.span ℂ (Set.range (heckeOp hK)), (t : Module.End ℂ ((G ⧸ K) →₀ ℂ)) (delta K) = u

theorem **heckeBasis** : ∀ (G : Type) [Group G] (K : Subgroup G), IsHeckePair K → ∃ T : G → Hk K, (∀ (g : G) (S : Finset (G ⧸ K)), (↑S : Set (G ⧸ K)) = MulAction.orbit K (g : G ⧸ K) → (T g : Module.End ℂ ((G ⧸ K) →₀ ℂ)) (Finsupp.single ((1 : G) : G ⧸ K) 1) = ∑ y ∈ S, Finsupp.single y (1 : ℂ)) ∧ Submodule.span ℂ (Set.range T) = ⊤ ∧ ∀ h : Hk K, (h : Module.End ℂ ((G ⧸ K) →₀ ℂ)) (Finsupp.single ((1 : G) : G ⧸ K) 1) = 0 → h = 0

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.HeckeOperatorsSpanAndFaithfulStatement

theorem **_root_.GlobalTheory.T.hecke_operators_span_and_faithful** : ∀ (G : Type) [Group G] (K : Subgroup G), IsHeckePair K → ∃ T : G → Hk K, (∀ (g : G) (S : Finset (G ⧸ K)), (↑S : Set (G ⧸ K)) = MulAction.orbit K (g : G ⧸ K) → (T g : Module.End ℂ ((G ⧸ K) →₀ ℂ)) (Finsupp.single ((1 : G) : G ⧸ K) 1) = ∑ y ∈ S, Finsupp.single y (1 : ℂ)) ∧ Submodule.span ℂ (Set.range T) = ⊤ ∧ ∀ h : Hk K, (h : Module.End ℂ ((G ⧸ K) →₀ ℂ)) (Finsupp.single ((1 : G) : G ⧸ K) 1) = 0 → h = 0

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.HeckeRankOne

theorem **capEntry_tensE_apply** (X : GMod n) (x : ℂ) (a b : Fin n) (f : Fin n → X.V) (d : Fin n) : capEntry (tensE X) x a b f d = capEntry X.E x a b (f d) + if d = a then f b else 0

theorem **capEntry_apply_shift** {V : Type} [AddCommGroup V] [Module ℂ V] (E : Fin n → Fin n → Module.End ℂ V) (x : ℂ) (a b : Fin n) (v : V) : capEntry E x a b v = capEntry E (x − 1) a b v + if b = a then v else 0

theorem **sum_pair_swap** {M : Type*} [AddCommMonoid M] {q : ℕ} (Φ : Fin (q + 2) → Fin (q + 1) → M) : ∑ i, ∑ k, Φ i k = ∑ i : Fin (q + 2), ∑ k : Fin (q + 1), Φ (i.succAbove k) (k.predAbove i)

theorem **sign_pair_swap** {q : ℕ} (i : Fin (q + 2)) (k : Fin (q + 1)) : (−1 : ℂ) ^ (i.succAbove k : ℕ) * (−1) ^ (k.predAbove i : ℕ) = −((−1) ^ (i : ℕ) * (−1) ^ (k : ℕ))

theorem **cminor_zero** {V : Type} [AddCommGroup V] [Module ℂ V] (E : Fin n → Fin n → Module.End ℂ V) (R C : Fin 0 → Fin n) (x : ℂ) : cminor E R C x = 1

theorem **dbl_swap** (i : Fin (q + 2)) (k l : Fin (q + 1)) : dbl E R C x f (i.succAbove k) (k.predAbove i) l = dbl E R C x f i k l

theorem **minor_expand** (i : Fin (q + 2)) (l : Fin (q + 1)) : cminor E (R ∘ i.succAbove) (C ∘ l.succ.succAbove) (x − 1) (f (C l.succ)) = ∑ k : Fin (q + 1), ((−1 : ℂ) ^ (k : ℕ)) • capEntry E (x − 1) (R (i.succAbove k)) (C 0) (dbl E R C x f i k l)

theorem **key_identity** : (∑ i : Fin (q + 2), ((−1 : ℂ) ^ (i : ℕ)) • capEntry E x (R i) (C 0)) (∑ k : Fin (q + 1), ∑ l : Fin (q + 1), if d = (R ∘ i.succAbove) k then ((−1 : ℂ) ^ ((k : ℕ) + (l : ℕ))) • dbl E R C x f i k l else 0)) + (∑ i : Fin (q + 2), ((−1 : ℂ) ^ (i : ℕ)) • (if d = R i then (∑ k : Fin (q + 1), ∑ l : Fin (q + 1), if C 0 = (R ∘ i.succAbove) k then ((−1 : ℂ) ^ ((k : ℕ) + (l : ℕ))) • dbl E R C x f i k l else 0) else 0)) = ∑ i : Fin (q + 2), ∑ l : Fin (q + 1), if d = R i then ((−1 : ℂ) ^ ((i : ℕ) + ((l.succ : Fin (q + 2)) : ℕ))) • cminor E (R ∘ i.succAbove) (C ∘ l.succ.succAbove) (x − 1) (f (C l.succ)) else 0

theorem **cminor_tensE_apply** (X : GMod n) (q : ℕ) : ∀ (R C : Fin (q + 1) → Fin n) (x : ℂ) (f : Fin n → X.V) (d : Fin n), cminor (tensE X) R C x f d = cminor X.E R C x (f d) + ∑ i : Fin (q + 1), ∑ j : Fin (q + 1), if d = R i then ((−1 : ℂ) ^ ((i : ℕ) + (j : ℕ))) • cminor X.E (R ∘ i.succAbove) (C ∘ j.succAbove) (x − 1) (f (C j)) else 0

theorem **minorOp_apply** {m : ℕ} (X : GMod (m + 1)) (y : ℂ) (f : Fin (m + 1) → X.V) (a : Fin (m + 1)) : minorOp X y f a = ∑ c : Fin (m + 1), ((−1 : ℂ) ^ ((a : ℕ) + (c : ℕ))) • cminor X.E a.succAbove c.succAbove y (f c)

theorem **rankOneTens** : ∀ (m : ℕ) (X : GMod (m + 1)) (u : ℂ), qdet (tensE X) u = (qdet X.E u).compLeft (Fin (m + 1)) + minorOp X (u − 1)

theorem **rankOne** : Hk.RankOne

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.HeckeRankOneStatement

theorem **_root_.GlobalTheory.T.HeckeRankOne** : CompanionTheory.RgExh.Hk.RankOne

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.HenselStatement

theorem **stHensel_holds** : StHensel

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.HenselStatementStatement

theorem **_root_.GlobalTheory.T.HenselStatement** : StHensel

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.HermitianAdjointExists

theorem **exists_adjoint_herm** : ∀ {L V : Type} [NontriviallyNormedField L] [AddCommGroup V] [Module L V] [FiniteDimensional L V] (s : L →+* L), IsInv s → ∀ h : V → V → L, IsHerm s h → ∀ g : Module.End L V, ∃ g' : Module.End L V, ∀ x y, h (g x) y = h x (g' y)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.HermitianAdjointExistsStatement

theorem **_root_.GlobalTheory.T.hermitian_adjoint_exists** : ∀ {L V : Type} [NontriviallyNormedField L] [AddCommGroup V] [Module L V] [FiniteDimensional L V] (s : L →+* L), IsInv s → ∀ h : V → V → L, IsHerm s h → ∀ g : Module.End L V, ∃ g' : Module.End L V, ∀ x y, h (g x) y = h x (g' y)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.HighestWeightClassBoundaryAndWeight

theorem **ann_app** (i : Pos n) (x : Cn n V) (B : Finset (Pos n)) : ann i x B = if i ∈ B then 0 else CompanionTheory.D.LK.sg i B • x (insert i B)

theorem **cre_app** (i : Pos n) (x : Cn n V) (B : Finset (Pos n)) : cre i x B = if i ∈ B then CompanionTheory.D.LK.sg i B • x (B.erase i) else 0

theorem **sg_erase** (i : Pos n) (B : Finset (Pos n)) : CompanionTheory.D.LK.sg i (B.erase i) = CompanionTheory.D.LK.sg i B

theorem **sg_sq** (i : Pos n) (B : Finset (Pos n)) : CompanionTheory.D.LK.sg i B * CompanionTheory.D.LK.sg i B = 1

theorem **cre_ann** (i : Pos n) (x : Cn n V) (B : Finset (Pos n)) : cre i (ann i x) B = if i ∈ B then x B else 0

theorem **adC_self** (a : Fin n) (i k : Pos n) : adC a a i k = if k = i then ((if i.1.1 = a then 1 else 0) − (if i.1.2 = a then 1 else 0) : ℂ) else 0

theorem **adL_self_apply** (a : Fin n) (x : Cn n V) (B : Finset (Pos n)) : adL a a x B = ∑ i : Pos n, (((if i.1.1 = a then 1 else 0) − (if i.1.2 = a then 1 else 0) : ℂ)) • cre i (ann i x) B

theorem **top_cycle_weight** : ∀ (n : ℕ) (ε : Fin n → Bool) (lev : Fin n → ℕ) (V : Type) [AddCommGroup V] [Module ℂ V] (E : Fin n → Fin n → Module.End ℂ V) (γ : Fin n → ℝ) (u : V), Monotone lev → KHighest ε E γ u → bd (allT n) E (Pi.single (omU ε lev) u) = 0 ∧ ∀ a, lF E a a (Pi.single (omU ε lev) u) = (((γ a + wt (omU ε lev) a : ℝ)) : ℂ) • (Pi.single (omU ε lev) u : Cn n V)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.HighestWeightClassBoundaryAndWeightStatement

theorem **_root_.GlobalTheory.T.highest_weight_class_boundary_and_weight** : ∀ (n : ℕ) (ε : Fin n → Bool) (lev : Fin n → ℕ) (V : Type) [AddCommGroup V] [Module ℂ V] (E : Fin n → Fin n → Module.End ℂ V) (γ : Fin n → ℝ) (u : V), Monotone lev → KHighest ε E γ u → bd (allT n) E (Pi.single (omU ε lev) u) = 0 ∧ ∀ a, lF E a a (Pi.single (omU ε lev) u) = (((γ a + wt (omU ε lev) a : ℝ)) : ℂ) • (Pi.single (omU ε lev) u : Cn n V)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.HighestWeightVectorKilledByRaising

theorem **adC_mem_omU** {n : ℕ} {ε : Fin n → Bool} {lev : Fin n → ℕ} {p q : Fin n} (hpq : p < q) (hε : ε p = ε q) (hl : lev p = lev q) {i k : Pos n} (hc : adC p q i k ≠ 0) (hi : i ∈ omU ε lev) : k ∈ omU ε lev ∧ k ≠ i

theorem **lF_single_omU** : ∀ (n : ℕ) (ε : Fin n → Bool) (lev : Fin n → ℕ) (V : Type) [AddCommGroup V] [Module ℂ V] (E : Fin n → Fin n → Module.End ℂ V) (γ : Fin n → ℝ) (u : V), KHighest ε E γ u → ∀ p q : Fin n, p < q → ε p = ε q → lev p = lev q → lF E p q (Pi.single (omU ε lev) u : Cn n V) = 0

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.HighestWeightVectorKilledByRaisingStatement

theorem **_root_.GlobalTheory.T.highest_weight_vector_killed_by_raising** : ∀ (n : ℕ) (ε : Fin n → Bool) (lev : Fin n → ℕ) (V : Type) [AddCommGroup V] [Module ℂ V] (E : Fin n → Fin n → Module.End ℂ V) (γ : Fin n → ℝ) (u : V), KHighest ε E γ u → ∀ p q : Fin n, p < q → ε p = ε q → lev p = lev q → lF E p q (Pi.single (omU ε lev) u : Cn n V) = 0

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.HilbertSchmidtProjectionBound

theorem **nsq_nonneg** (y : Fin n → ℂ) : 0 ≤ nsq y

theorem **nsq_add_le** (u v : Fin n → ℂ) : nsq (u + v) ≤ 2 * nsq u + 2 * nsq v

theorem **nsq_mulVec_le** (M : Matrix (Fin n) (Fin n) ℂ) (y : Fin n → ℂ) : nsq (M *ᵥ y) ≤ Formalism.REAL.hs M * nsq y

theorem **nsq_eq_re** (y : Fin n → ℂ) : nsq y = (star y ⬝ᵥ y).re

theorem **nsq_unitary** {U : Matrix (Fin n) (Fin n) ℂ} (h : Uᴴ * U = 1) (y : Fin n → ℂ) : nsq (U *ᵥ y) = nsq y

theorem **nsq_proj** {P : Matrix (Fin n) (Fin n) ℂ} (h1 : Pᴴ = P) (h2 : P * P = P) (y : Fin n → ℂ) : nsq (P *ᵥ y) = (star y ⬝ᵥ (P *ᵥ y)).re

theorem **nsq_three** {ε : Fin n → Bool} {A : Matrix (Fin n) (Fin n) ℂ} (hA : IsGrad ε A) (y : Fin n → ℂ) : nsq y = nsq (Pm A *ᵥ y) + nsq (Pz A *ᵥ y) + nsq (Pp A *ᵥ y)

theorem **mulVec_col** (M P : Matrix (Fin n) (Fin n) ℂ) (j : Fin n) : (M *ᵥ fun i => P i j) = fun i => (M * P) i j

theorem **hs_eq_cols** (M : Matrix (Fin n) (Fin n) ℂ) : Formalism.REAL.hs M = ∑ j, nsq (fun i => M i j)

theorem **hs_mul_le_of_cols** {X P : Matrix (Fin n) (Fin n) ℂ} {c : ℝ} (h : ∀ j, nsq (X *ᵥ fun i => P i j) ≤ c * nsq (fun i => P i j)) : Formalism.REAL.hs (X * P) ≤ c * Formalism.REAL.hs P

theorem **hs_sub_comm** (M N : Matrix (Fin n) (Fin n) ℂ) : Formalism.REAL.hs (M - N) = Formalism.REAL.hs (N - M)

theorem **est** : ∀ (n : ℕ) (ε : Fin n → Bool) (A₀ A' : Matrix (Fin n) (Fin n) ℂ) (α β : ℝ) (x : PSub ε A₀) (k : S7.KG ε), IsGrad ε A₀ → IsGrad ε A' → Formalism.REAL.hs (Pm A₀ - Pm A') ≤ 1 / 4 → Formalism.REAL.hs (Pp A₀ - Pp A') ≤ 1 / 4 → 1 ≤ β → β ≤ α → (∀ y : Fin n → ℂ, α * nsq (Pm A' *ᵥ y) + β⁻¹ * nsq (Pz A' *ᵥ y) ≤ nsq (S7.toM ((x : S7.UJ ε) * (k : S7.UJ ε))⁻¹ *ᵥ y)) → Formalism.REAL.hs (S7.toM (x : S7.UJ ε) * Pm A₀) ≤ 4 * n / α ∧ Formalism.REAL.hs (S7.toM (x : S7.UJ ε) * Pz A₀) ≤ 4 * n * β

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.HilbertSchmidtProjectionBoundStatement

theorem **_root_.GlobalTheory.T.hilbert_schmidt_projection_bound** : ∀ (n : ℕ) (ε : Fin n → Bool) (A₀ A' : Matrix (Fin n) (Fin n) ℂ) (α β : ℝ) (x : PSub ε A₀) (k : S7.KG ε), IsGrad ε A₀ → IsGrad ε A' → Formalism.REAL.hs (Pm A₀ - Pm A') ≤ 1 / 4 → Formalism.REAL.hs (Pp A₀ - Pp A') ≤ 1 / 4 → 1 ≤ β → β ≤ α → (∀ y : Fin n → ℂ, α * nsq (Pm A' *ᵥ y) + β⁻¹ * nsq (Pz A' *ᵥ y) ≤ nsq (S7.toM ((x : S7.UJ ε) * (k : S7.UJ ε))⁻¹ *ᵥ y)) → Formalism.REAL.hs (S7.toM (x : S7.UJ ε) * Pm A₀) ≤ 4 * n / α ∧ Formalism.REAL.hs (S7.toM (x : S7.UJ ε) * Pz A₀) ≤ 4 * n * β

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.HomCategory

theorem **sn_nonneg** (X : Obj k r) (W : List (Mx k r)) (x : M X) : 0 ≤ sn X W x

theorem **sn_add_le** (X : Obj k r) {W : List (Mx k r)} (hW : LieWd k r W) (x y : M X) : sn X W (x + y) ≤ sn X W x + sn X W y

theorem **sn_smul** (X : Obj k r) {W : List (Mx k r)} (hW : LieWd k r W) (a : ℂ) (x : M X) : sn X W (a • x) = ‖a‖ * sn X W x

theorem **sum_nonneg'** (X : Obj k r) (Ws : List (List (Mx k r))) (x : M X) : 0 ≤ (Ws.map fun W' => sn X W' x).sum

theorem **Dom.mono** {X : Obj k r} {φ ψ : M X → ℝ} (h : Dom X ψ) (hle : ∀ x, φ x ≤ ψ x) : Dom X φ

theorem **Dom.zero** (X : Obj k r) : Dom X fun _ => 0

theorem **Dom.add** {X : Obj k r} {φ ψ : M X → ℝ} (h : Dom X φ) (h' : Dom X ψ) : Dom X fun x => φ x + ψ x

theorem **Dom.mul** {X : Obj k r} {φ : M X → ℝ} (h : Dom X φ) {a : ℝ} (ha : 0 ≤ a) : Dom X fun x => a * φ x

theorem **Dom.listSum** {X : Obj k r} {ι : Type} (φ : ι → M X → ℝ) : ∀ l : List ι, (∀ i ∈ l, Dom X (φ i)) → Dom X fun x => (l.map fun i => φ i x).sum | [], _ => by simpa using Dom.zero X | i :: l, h => by simpa using (h i List.mem_cons_self).add (Dom.listSum φ l fun j hj => h j (List.mem_cons_of_mem _ hj)) theorem homCat : HomCat

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.HomCategoryStatement

theorem **_root_.GlobalTheory.T.HomCategory** : HomCat

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.HsConjTranspose

theorem **Pp_sub_Pm** : Pp A - Pm A = A

theorem **Pp_add_Pm** : Pp A + Pm A = A * A

theorem **sum_proj** : Pm A + Pz A + Pp A = 1

theorem **Pp_conj** (hA : IsGrad ε A) : (Pp A)ᴴ = Pp A

theorem **Pz_conj** (hA : IsGrad ε A) : (Pz A)ᴴ = Pz A

theorem **four** (hA : IsGrad ε A) : A * A * (A * A) = A * A

theorem **Pm_Pp** (hA : IsGrad ε A) : Pm A * Pp A = 0

theorem **A_Pm** (hA : IsGrad ε A) : A * Pm A = -Pm A

theorem **A_Pp** (hA : IsGrad ε A) : A * Pp A = Pp A

theorem **Pm_idem** (hA : IsGrad ε A) : Pm A * Pm A = Pm A

theorem **Pp_idem** (hA : IsGrad ε A) : Pp A * Pp A = Pp A

theorem **Pz_eq** : Pz A = 1 - Pm A - Pp A

theorem **Pz_Pm** (hA : IsGrad ε A) : Pz A * Pm A = 0

theorem **Pm_Pz** (hA : IsGrad ε A) : Pm A * Pz A = 0

theorem **Pz_Pp** (hA : IsGrad ε A) : Pz A * Pp A = 0

theorem **Pp_Pz** (hA : IsGrad ε A) : Pp A * Pz A = 0

theorem **Pz_idem** (hA : IsGrad ε A) : Pz A * Pz A = Pz A

theorem **A_Pz** (hA : IsGrad ε A) : A * Pz A = 0

theorem **Pp_J** (hA : IsGrad ε A) : Pp A * Formalism.REAL.Jm ε = Formalism.REAL.Jm ε * Pm A

theorem **Pz_J** (hA : IsGrad ε A) : Pz A * Formalism.REAL.Jm ε = Formalism.REAL.Jm ε * Pz A

theorem **hs_nonneg** (M : Matrix (Fin n) (Fin n) ℂ) : 0 ≤ hs M

theorem **hs_mul_le** (M N : Matrix (Fin n) (Fin n) ℂ) : hs (M * N) ≤ hs M * hs N

theorem **hs_eq_trace** (M : Matrix (Fin n) (Fin n) ℂ) : hs M = (Matrix.trace (M * Mᴴ)).re

theorem **hs_split** {P : Matrix (Fin n) (Fin n) ℂ} (h1 : Pᴴ = P) (h2 : P * P = P) (M : Matrix (Fin n) (Fin n) ℂ) : hs M = hs (M * P) + hs (M * (1 - P))

theorem **hs_three** (hA : IsGrad ε A) (M : Matrix (Fin n) (Fin n) ℂ) : hs M = hs (M * Pm A) + hs (M * Pz A) + hs (M * Pp A)

theorem **hs_smul** (c : ℂ) (M : Matrix (Fin n) (Fin n) ℂ) : hs (c • M) = ‖c‖ ^ 2 * hs M

theorem **hs_conjTranspose** (M : Matrix (Fin n) (Fin n) ℂ) : hs Mᴴ = hs M

theorem **hs_add_le** (M N : Matrix (Fin n) (Fin n) ℂ) : hs (M + N) ≤ 2 * hs M + 2 * hs N

theorem **hs_J_mul** (M : Matrix (Fin n) (Fin n) ℂ) : hs (Formalism.REAL.Jm ε * M) = hs M

theorem **hs_mul_J** (M : Matrix (Fin n) (Fin n) ℂ) : hs (M * Formalism.REAL.Jm ε) = hs M

theorem **hs_one** : hs (1 : Matrix (Fin n) (Fin n) ℂ) = n

theorem **hs_proj_le** {P : Matrix (Fin n) (Fin n) ℂ} (h1 : Pᴴ = P) (h2 : P * P = P) : hs P ≤ n

theorem **hs_eq_zero** {M : Matrix (Fin n) (Fin n) ℂ} (h : hs M = 0) : M = 0

theorem **hs_pos** {M : Matrix (Fin n) (Fin n) ℂ} (h : M ≠ 0) : 0 < hs M

theorem **one_le_hs** {P : Matrix (Fin n) (Fin n) ℂ} (h2 : P * P = P) (h0 : P ≠ 0) : 1 ≤ hs P

theorem **Pm_ne_zero** (hA : IsGrad ε A) : Pm A ≠ 0

theorem **Pp_ne_zero** (hA : IsGrad ε A) : Pp A ≠ 0

theorem **aT_mul_of_eigen** {P : Matrix (Fin n) (Fin n) ℂ} {c : ℂ} (h : A * P = c • P) (t : ℝ) : aT A t * P = Complex.exp (t * c) • P

theorem **aT_Pm** (hA : IsGrad ε A) (t : ℝ) : aT A t * Pm A = ((Real.exp (-t) : ℝ) : ℂ) • Pm A

theorem **aT_Pp** (hA : IsGrad ε A) (t : ℝ) : aT A t * Pp A = ((Real.exp t : ℝ) : ℂ) • Pp A

theorem **aT_Pz** (hA : IsGrad ε A) (t : ℝ) : aT A t * Pz A = Pz A

theorem **toM_inv_eq** (x : S7.UJ ε) : (S7.toM x)⁻¹ = S7.toM x⁻¹

theorem **hs_Pp_inv** (hA : IsGrad ε A) (x : S7.UJ ε) : hs (Pp A * S7.toM x⁻¹) = hs (S7.toM x * Pm A)

theorem **hs_Pz_inv** (hA : IsGrad ε A) (x : S7.UJ ε) : hs (Pz A * S7.toM x⁻¹) = hs (S7.toM x * Pz A)

theorem **toM_mul_ne_zero** (x : S7.UJ ε) {P : Matrix (Fin n) (Fin n) ℂ} (hP : P ≠ 0) : S7.toM x * P ≠ 0

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.HwMapOfLowSpan

theorem **diagOp_fst** (A : Module.End ℂ T) (B : Module.End ℂ V) (x : WithLp 2 (T × V)) : (WithLp.ofLp (diagOp A B x)).1 = A (WithLp.ofLp x).1

theorem **diagOp_snd** (A : Module.End ℂ T) (B : Module.End ℂ V) (x : WithLp 2 (T × V)) : (WithLp.ofLp (diagOp A B x)).2 = B (WithLp.ofLp x).2

theorem **sum_ext** {x y : WithLp 2 (T × V)} (h1 : (WithLp.ofLp x).1 = (WithLp.ofLp y).1) (h2 : (WithLp.ofLp x).2 = (WithLp.ofLp y).2) : x = y

theorem **diag_krel** {n : ℕ} {ε : Fin n → Bool} {ET : Fin n → Fin n → Module.End ℂ T} {E : Fin n → Fin n → Module.End ℂ V} (h1 : KRel ε ET) (h2 : KRel ε E) : KRel ε (fun a b => diagOp (ET a b) (E a b))

theorem **diag_kunit** {n : ℕ} {ε : Fin n → Bool} {ET : Fin n → Fin n → Module.End ℂ T} {E : Fin n → Fin n → Module.End ℂ V} (h1 : KUnit ε ET) (h2 : KUnit ε E) : KUnit ε (fun a b => diagOp (ET a b) (E a b))

theorem **hwMap_of_lowSpan** : LowSpan → HWMap

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.ImportsAlt

theorem **actQ** : CompanionTheory.D.DSIC.ActQ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.InNOfMem

theorem **mem_NV** (E : Fin n → Fin n → Module.End ℂ V) {a b : Fin n} (h : a < b) (w : V) : E a b w ∈ NV E

theorem **inN_of_mem** (E : Fin n → Fin n → Module.End ℂ V) {y : V} (hy : y ∈ NV E) : InN (allT n) E y

theorem **JJ_mul** (E : Fin n → Fin n → Module.End ℂ V) {T : Module.End ℂ V} (h : T ∈ JJ E) (S : Module.End ℂ V) : T * S ∈ JJ E

theorem **HH_comm** {E : Fin n → Fin n → Module.End ℂ V} (hE : GlRel E) (a b : Fin n) : HH E * E a b - E a b * HH E = (((a : ℕ) : ℂ) - ((b : ℕ) : ℂ)) • E a b

theorem **adH_apply** (E : Fin n → Fin n → Module.End ℂ V) (T : Module.End ℂ V) : adH E T = HH E * T - T * HH E

theorem **adH_mul** (E : Fin n → Fin n → Module.End ℂ V) (S T : Module.End ℂ V) : adH E (S * T) = adH E S * T + S * adH E T

theorem **HH_NV** {E : Fin n → Fin n → Module.End ℂ V} (hE : GlRel E) {y : V} (hy : y ∈ NV E) : HH E y ∈ NV E

theorem **adH_JJ** {E : Fin n → Fin n → Module.End ℂ V} (hE : GlRel E) : JJ E ≤ (JJ E).comap (adH E)

theorem **congrueneCondition_mul** {E : Fin n → Fin n → Module.End ℂ V} {T S : Module.End ℂ V} (hT : T ∈ PP E) {m : ℕ} (hS : adH E S = (m : ℂ) • S) : T * S ∈ PP E

theorem **adH_evd** {E : Fin n → Fin n → Module.End ℂ V} (hE : GlRel E) (b : FreeAlgebra ℂ (Fin n)) : adH E (evd (fun c => E c c) b) = 0

theorem **mem_JJ_of** {E : Fin n → Fin n → Module.End ℂ V} (hE : GlRel E) {t p : Module.End ℂ V} (ht : adH E t = 0) (hp : p ∈ PP E) (h : t - p ∈ JJ E) : t ∈ JJ E

theorem **ev_ι** (E : Fin n → Fin n → Module.End ℂ V) (p : Fin n × Fin n) : ev E (FreeAlgebra.ι ℂ p) = E p.1 p.2

theorem **Low.zero** : Low (0 : FreeAlgebra ℂ (Fin n × Fin n))

theorem **Low.mul'** {u : FreeAlgebra ℂ (Fin n × Fin n)} (hu : Low u) : ∃ b : FreeAlgebra ℂ (Fin n), ∀ (V : Type) [AddCommGroup V] [Module ℂ V] (E : Fin n → Fin n → Module.End ℂ V), GlRel E → ev E u - evd (fun c => E c c) b ∈ PP E

theorem **Red.add** {a a' : FreeAlgebra ℂ (Fin n × Fin n)} (h : Red a) (h' : Red a') : Red (a + a')

theorem **Red.smul** (c : ℂ) {a : FreeAlgebra ℂ (Fin n × Fin n)} (h : Red a) : Red (c • a)

theorem **Red.of_low** {u : FreeAlgebra ℂ (Fin n × Fin n)} (hu : Low u) : Red u

theorem **Red.congr** {a a' : FreeAlgebra ℂ (Fin n × Fin n)} (h : Red a) (he : ∀ (V : Type) [AddCommGroup V] [Module ℂ V] (E : Fin n → Fin n → Module.End ℂ V), GlRel E → ev E a' = ev E a) : Red a'

theorem **Red.mul_low** {a : FreeAlgebra ℂ (Fin n × Fin n)} (h : Red a) (p : Fin n × Fin n) (hp : p.2 ≤ p.1) : Red (a * FreeAlgebra.ι ℂ p)

theorem **Low.red_mul** {u : FreeAlgebra ℂ (Fin n × Fin n)} (hu : Low u) : ∀ p : Fin n × Fin n, Red (u * FreeAlgebra.ι ℂ p)

theorem **Red.mul_ι** {a : FreeAlgebra ℂ (Fin n × Fin n)} (h : Red a) (p : Fin n × Fin n) : Red (a * FreeAlgebra.ι ℂ p)

theorem **red_all** (a : FreeAlgebra ℂ (Fin n × Fin n)) : Red a

theorem **hcdec** : CompanionTheory.D.LK.HCdec

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.IndOfNot

theorem **sig_anti** (a b : Fin n) : sig b a = - sig a b

theorem **sig_mul** {y : Fin n → ℝ} (hy : Antitone y) (a b : Fin n) : sig a b * (y a - y b) = |y a - y b|

theorem **pair_eq** (t : Fin n → Fin n → ℝ) (ht : ∀ a b, t b a = - t a b) (x : Fin n → ℝ) : 4 * ∑ a, x a * ((∑ b, t a b) / 2) = ∑ a, ∑ b, t a b * (x a - x b)

theorem **term_le** {s z : ℝ} (hs : |s| ≤ 1) : s * z ≤ |z|

theorem **normSq_le** (t : Fin n → Fin n → ℝ) (ht : ∀ a b, t b a = - t a b) (h1 : ∀ a b, |t a b| ≤ 1) : ∑ a, ((∑ b, t a b) / 2) ^ 2 ≤ ∑ a : Fin n, ((∑ b, sig a b) / 2) ^ 2

theorem **pair_lt** (t : Fin n → Fin n → ℝ) (ht : ∀ a b, t b a = - t a b) (h1 : ∀ a b, |t a b| ≤ 1) {lam : Fin n → ℝ} (hl : StrictAnti lam) (c d : Fin n) (hcd : c ≠ d) (hne : t c d = - sig c d) : ∑ a, lam a * ((∑ b, t a b) / 2) < ∑ a, lam a * ((∑ b, sig a b) / 2)

theorem **wtA_eq** {ε : Fin n → Bool} (A : Finset (NC ε)) (c : Fin n) : wtA A c = ∑ b, (ind A c b - ind A b c)

theorem **ind_of_not** {ε : Fin n → Bool} (A : Finset (NC ε)) {a b : Fin n} (h : ¬ (ε a = true ∧ ε b = false)) : ind A a b = 0

theorem **ind_of** {ε : Fin n → Bool} (A : Finset (NC ε)) {a b : Fin n} (h : ε a = true ∧ ε b = false) : ind A a b = if (((a, b), h) : NC ε) ∈ A then 1 else 0

theorem **tt_anti** {ε : Fin n → Bool} (A : Finset (NC ε)) (a b : Fin n) : tt A b a = - tt A a b

theorem **tt_of** {ε : Fin n → Bool} (A : Finset (NC ε)) {a b : Fin n} (h : ε a = true ∧ ε b = false) : tt A a b = if ((((a, b), h) : NC ε) ∈ A ↔ (((a, b), h) : NC ε) ∈ A1 ε) then sig a b else - sig a b

theorem **abs_sig** (a b : Fin n) : |sig a b| ≤ 1

theorem **abs_tt** {ε : Fin n → Bool} (A : Finset (NC ε)) (a b : Fin n) : |tt A a b| ≤ 1

theorem **rho_eq** (ε : Fin n → Bool) (a : Fin n) : PI.UpqRep.rhoC ε a + PI.UpqRep.rhoN ε a = (∑ b, sig a b) / 2

theorem **normLt** : CompanionTheory.D.DSIC.NormLt

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.InfinitesimalCharacterStatement

theorem **PolyFn.const** (c : A) : PolyFn (fun _ : ℂ => c)

theorem **PolyFn.alg** : PolyFn (fun u : ℂ => algebraMap ℂ A u)

theorem **PolyFn.add** {F G : ℂ → A} (hF : PolyFn F) (hG : PolyFn G) : PolyFn (fun u => F u + G u)

theorem **PolyFn.sub** {F G : ℂ → A} (hF : PolyFn F) (hG : PolyFn G) : PolyFn (fun u => F u − G u)

theorem **PolyFn.mul** {F G : ℂ → A} (hF : PolyFn F) (hG : PolyFn G) : PolyFn (fun u => F u * G u)

theorem **PolyFn.pow** {F : ℂ → A} (hF : PolyFn F) (k : ℕ) : PolyFn (fun u => F u ^ k)

theorem **PolyFn.sum** {ι : Type*} (s : Finset ι) {F : ι → ℂ → A} (h : ∀ i, PolyFn (F i)) : PolyFn (fun u => ∑ i ∈ s, F i u)

theorem **PolyFn.list** {m : ℕ} (F : Fin m → ℂ → A) (h : ∀ k, PolyFn (F k)) : PolyFn (fun u => (List.ofFn fun k => F k u).prod)

theorem **PolyFn.algProd** {ι : Type*} (s : Finset ι) {f : ι → ℂ → ℂ} (h : ∀ i, PolyFn (fun u => algebraMap ℂ A (f i u))) : PolyFn (fun u => algebraMap ℂ A (∏ i ∈ s, f i u))

theorem **polyFn_tm** {n : ℕ} (E : Fin n → Fin n → A) (c : ℂ) (a b : Fin n) : PolyFn (fun u : ℂ => Tm E (u − c) a b)

theorem **polyFn_qdet** {n : ℕ} (E : Fin n → Fin n → A) : PolyFn (fun u : ℂ => qdet E u)

theorem **commute_qdet** {n : ℕ} {E : Fin n → Fin n → A} (O : A) (h : ∀ a b, Commute O (E a b)) (u : ℂ) : Commute O (qdet E u)

theorem **PolyFn.vanish** {V : Type*} [AddCommGroup V] [Module ℂ V] {F : ℂ → Module.End ℂ V} (hF : PolyFn F) (u₀ : ℂ) (h : ∀ u, u ≠ u₀ → F u = 0) : F u₀ = 0

theorem **core** {D c ν : A} {k M : ℕ} {w r : ℂ} (hw : w ≠ 0) (hν : ν ^ M = 0) (hN : (c − algebraMap ℂ A (r * w)) ^ k = 0) (hcν : Commute c ν) (hDν : Commute D ν) (hid : D * (algebraMap ℂ A w + ν) = c * (algebraMap ℂ A (w + 1) + ν)) : (D − algebraMap ℂ A (r * (w + 1))) ^ (k + M) = 0

theorem **Rs.ext** {G G' : Module.End ℂ S} {F : Module.End ℂ V} (h : Rs G F) (h' : Rs G' F) : G = G'

theorem **Rs.mul** {G G' : Module.End ℂ S} {F F' : Module.End ℂ V} (h : Rs G F) (h' : Rs G' F') : Rs (G * G') (F * F')

theorem **Rs.add** {G G' : Module.End ℂ S} {F F' : Module.End ℂ V} (h : Rs G F) (h' : Rs G' F') : Rs (G + G') (F + F')

theorem **Rs.sub** {G G' : Module.End ℂ S} {F F' : Module.End ℂ V} (h : Rs G F) (h' : Rs G' F') : Rs (G − G') (F − F')

theorem **Rs.alg** (c : ℂ) : Rs (algebraMap ℂ (Module.End ℂ S) c) (algebraMap ℂ (Module.End ℂ V) c)

theorem **Rs.zero** : Rs (0 : Module.End ℂ S) (0 : Module.End ℂ V)

theorem **Rs.one** : Rs (1 : Module.End ℂ S) (1 : Module.End ℂ V)

theorem **Rs.pow** {G : Module.End ℂ S} {F : Module.End ℂ V} (h : Rs G F) (k : ℕ) : Rs (G ^ k) (F ^ k)

theorem **Rs.sum** {ι : Type*} (s : Finset ι) {G : ι → Module.End ℂ S} {F : ι → Module.End ℂ V} (h : ∀ i, Rs (G i) (F i)) : Rs (∑ i ∈ s, G i) (∑ i ∈ s, F i)

theorem **Rs.intCast** (z : ℤ) : Rs ((z : Module.End ℂ S)) ((z : Module.End ℂ V))

theorem **Rs.list** {m : ℕ} {G : Fin m → Module.End ℂ S} {F : Fin m → Module.End ℂ V} (h : ∀ k, Rs (G k) (F k)) : Rs (List.ofFn G).prod (List.ofFn F).prod

theorem **Rs.tm** {n : ℕ} {G : Fin n → Fin n → Module.End ℂ S} {F : Fin n → Fin n → Module.End ℂ V} (h : ∀ a b, Rs (G a b) (F a b)) (u : ℂ) (a b : Fin n) : Rs (Tm G u a b) (Tm F u a b)

theorem **Rs.qdet** {n : ℕ} {G : Fin n → Fin n → Module.End ℂ S} {F : Fin n → Fin n → Module.End ℂ V} (h : ∀ a b, Rs (G a b) (F a b)) (u : ℂ) : Rs (qdet G u) (qdet F u)

theorem **Rs.commute** {G G' : Module.End ℂ S} {F F' : Module.End ℂ V} (h : Rs G F) (h' : Rs G' F') (hc : Commute F F') : Commute G G'

theorem **nil_list** {f : Module.End ℂ V} {a : ℂ} (l : List (Module.End ℂ V)) (hl : ∀ T ∈ l, ∃ b : ℂ, T = f − b • (1 : Module.End ℂ V)) : ∀ g ∈ f.maxGenEigenspace a, l.prod g = 0 → ((f − a • (1 : Module.End ℂ V)) ^ l.length) g = 0

theorem **nil_pow** {f : Module.End ℂ V} {a : ℂ} (l : List (Module.End ℂ V)) (hl : ∀ T ∈ l, ∃ b : ℂ, T = f − b • (1 : Module.End ℂ V)) (K : ℕ) : ∀ g ∈ f.maxGenEigenspace a, (l.prod ^ K) g = 0 → ((f − a • (1 : Module.End ℂ V)) ^ (l.length * K)) g = 0

theorem **capelliPoly_eq_mul_prod_erase** (χ : Fin n → ℂ) (s : Fin n) (Y : ℂ) : capP χ Y = (∏ t ∈ univ.erase s, (Y + χ t)) * (Y + χ s)

theorem **capelliPoly_add_single** (χ : Fin n → ℂ) (s : Fin n) (Y : ℂ) : capP (χ + Pi.single s 1) Y = (∏ t ∈ univ.erase s, (Y + χ t)) * (Y + χ s + 1)

theorem **compLeft_pow_apply** {V : Type*} [AddCommGroup V] [Module ℂ V] {I : Type*} (T : Module.End ℂ V) (c : ℂ) (k : ℕ) (f : I → V) (i : I) : ((T.compLeft I − algebraMap ℂ (Module.End ℂ (I → V)) c) ^ k) f i = ((T − algebraMap ℂ (Module.End ℂ V) c) ^ k) (f i)

theorem **commute_C_om** (X : GMod n) (u : ℂ) : Commute ((qdet X.E u).compLeft (Fin n)) (om X)

theorem **commute_T_om** (X : GMod n) (u : ℂ) : Commute (qdet (tensE X) u) (om X)

theorem **stepChar_of** (hT : QdetTens) (hC : CayleyHam) : StepChar

theorem **stepChar** : CompanionTheory.RgExh.StepChar

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.InfinitesimalCharacterStatementStatement

theorem **_root_.GlobalTheory.T.InfinitesimalCharacterStatement** : CompanionTheory.RgExh.StepChar

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.InitialStatement

theorem **measurable_modA** (A : Matrix (Fin n) (Fin n) ℂ) : Measurable (modA (ε

theorem **detOn_pos_of_left_inv** {f g : Matrix (Fin n) (Fin n) ℂ} (hgf : g * f = 1) (V : Submodule ℂ (Fin n → ℂ)) (hV : ∀ y ∈ V, f *ᵥ y ∈ V) : 0 < detOn f V

theorem **modA_ne_zero** (A : Matrix (Fin n) (Fin n) ℂ) {m : Formalism.REAL.UJ ε} (hm : m ∈ flagM ε (W A)) : modA A m ≠ 0

theorem **startOf** : StartOf

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.InitialStatementStatement

theorem **_root_.GlobalTheory.T.InitialStatement** : CompanionTheory.RgExh.Dec.N.S.StartOf

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.InjROfVanKG

theorem **suppIn_sub** {P : Pos n → Prop} {x y : Cn n V} (hx : SuppIn P x) (hy : SuppIn P y) : SuppIn P (x − y)

theorem **suppIn_sum** {ι : Type} {P : Pos n → Prop} (s : Finset ι) (f : ι → Cn n V) (h : ∀ b ∈ s, SuppIn P (f b)) : SuppIn P (∑ b ∈ s, f b)

theorem **suppIn_bd** {P : Pos n → Prop} (hP : ∀ i j k : Pos n, i.1.2 = j.1.1 → k.1.1 = i.1.1 → k.1.2 = j.1.2 → P i → P j → P k) (E : Fin n → Fin n → Module.End ℂ V) {x : Cn n V} (hx : SuppIn P x) : SuppIn P (bd (allT n) E x)

theorem **inR_closed** {ε : Fin n → Bool} {lev : Fin n → ℕ} (hm : Monotone lev) (i j k : Pos n) (e1 : i.1.2 = j.1.1) (e2 : k.1.1 = i.1.1) (e3 : k.1.2 = j.1.2) (hi : inR ε lev i) (hj : inR ε lev j) : inR ε lev k

theorem **injR_of_vanKG** (hV : VanKG) : InjR

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.InjUOfInjR

theorem **injU_of_injR** : CompanionTheory.RgExh.U.InjR → CompanionTheory.RgExh.U.InjU

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.InnerEqZeroOfWeight

lemma **kword_mem** {l : List (Fin n × Fin n)} (hl : KLower ε l) : word F l w ∈ kLowSpan ε F w

lemma **kself_mem** : w ∈ kLowSpan ε F w

lemma **klower_mem** {a b : Fin n} (hab : b < a) (he : ε a = ε b) {x : U} (hx : x ∈ kLowSpan ε F w) : F a b x ∈ kLowSpan ε F w

lemma **kcomm_apply** (hF : KRel ε F) {i j a b : Fin n} (hij : ε i = ε j) (hab : ε a = ε b) (u : U) : F i j (F a b u) = F a b (F i j u) + ((if j = a then F i b u else 0) − (if b = i then F a j u else 0))

lemma **kgen_mem** (hF : KRel ε F) (hw : KHighest ε F μ w) : ∀ (l : List (Fin n × Fin n)), KLower ε l → ∀ i j, ε i = ε j → F i j (word F l w) ∈ kLowSpan ε F w

theorem **kstab_kLowSpan** (hF : KRel ε F) (hw : KHighest ε F μ w) : KStab ε F (kLowSpan ε F w)

theorem **kLowSpan_le** {S : Submodule ℂ U} (hS : KStab ε F S) (hw : w ∈ S) : kLowSpan ε F w ≤ S

lemma **kwt_nil** (μ : Fin n → ℝ) : kwt μ [] = μ

lemma **kwt_cons** (μ : Fin n → ℝ) (p : Fin n × Fin n) (l : List (Fin n × Fin n)) : kwt μ (p :: l) = kwt μ l + (Pi.single p.1 1 − Pi.single p.2 1)

lemma **isWeight_word** (hF : KRel ε F) (hw : IsWeight F μ w) : ∀ (l : List (Fin n × Fin n)), KLower ε l → IsWeight F (kwt μ l) (word F l w)

lemma **hgt_kwt** (μ : Fin n → ℝ) : ∀ l : List (Fin n × Fin n), KLower ε l → ∑ c : Fin n, ((c : ℕ) : ℝ) * μ c + l.length ≤ ∑ c : Fin n, ((c : ℕ) : ℝ) * kwt μ l c

lemma **cdom_kwt** (μ : Fin n → ℝ) : ∀ l : List (Fin n × Fin n), KLower ε l → PI.UpqRep.CDom ε (kwt μ l) μ

lemma **inner_eq_zero_of_weight** (hU : KUnit ε E) {ν ν' : Fin n → ℝ} {x y : V} (hx : IsWeight E ν x) (hy : IsWeight E ν' y) (hne : ν ≠ ν') : ⟪x, y⟫_ℂ = 0

lemma **eq_zero_of_orth** {x : V} (hx : x ∈ kLowSpan ε E ν) (h : ∀ l, KLower ε l → ⟪word E l ν, x⟫_ℂ = 0) : x = 0

lemma **cdom_of_weight** (hF : KRel ε E) (hU : KUnit ε E) (hν : IsWeight E μ ν) {ν : Fin n → ℝ} {x : V} (hx : x ∈ kLowSpan ε E ν) (hx0 : x ≠ 0) (hν : IsWeight E ν x) : PI.UpqRep.CDom ε ν μ

lemma **eq_smul_of_top** (hF : KRel ε E) (hU : KUnit ε E) (hν0 : ν ≠ 0) (hν : IsWeight E μ ν) {x : V} (hx : x ∈ kLowSpan ε E ν) (hμ : IsWeight E μ x) : ∃ c : ℂ, x = c • ν

lemma **kstab_orthogonal** (hU : KUnit ε E) {L : Submodule ℂ V} (hL : KStab ε E L) : KStab ε E L⊥

theorem **lowSpan_holds** : CompanionTheory.D.DSIC.LowSpan

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.IntegrableOfLtTop

lemma **wt_nonneg** (R : ℝ) (u : ℝ → ℂ) (t : ℝ) : 0 ≤ wt R u t

lemma **wt_cont** {u : ℝ → ℂ} (R : ℝ) (hu : Continuous u) : Continuous (wt R u)

lemma **src_cont** {u g : ℝ → ℂ} (R : ℝ) (hu : Continuous u) (hg : Continuous g) : Continuous (src R u g)

lemma **hasDerivAt_wt** {u g : ℝ → ℂ} {z : ℂ} {t : ℝ} (R : ℝ) (h : HasDerivAt u (z * u t + g t) t) : HasDerivAt (wt R u) (2 * (z.re + R) * wt R u t + src R u g t) t

lemma **abs_src_le** (R : ℝ) (u g : ℝ → ℂ) (t : ℝ) {c : ℝ} (hc : 0 < c) : |src R u g t| ≤ c * wt R u t + wt R g t / c

lemma **energy** (hu : Continuous u) (hg : Continuous g) (hd : ∀ t, 0 ≤ t → HasDerivAt u (z * u t + g t) t) {T : ℝ} (hT : 0 ≤ T) : wt R u T − wt R u 0 = ∫ t in (0 : ℝ)..T, (2 * (z.re + R) * wt R u t + src R u g t)

lemma **energy_le** (hae : z.re + R = a) (hu : Continuous u) (hg : Continuous g) (hd : ∀ t, 0 ≤ t → HasDerivAt u (z * u t + g t) t) {T : ℝ} (hT : 0 ≤ T) {c : ℝ} (hc : 0 < c) : wt R u T − wt R u 0 ≤ (2 * a + c) * (∫ t in (0 : ℝ)..T, wt R u t) + (∫ t in (0 : ℝ)..T, wt R g t) / c

lemma **energy_ge** (hae : z.re + R = a) (hu : Continuous u) (hg : Continuous g) (hd : ∀ t, 0 ≤ t → HasDerivAt u (z * u t + g t) t) {T : ℝ} (hT : 0 ≤ T) {c : ℝ} (hc : 0 < c) : (2 * a − c) * (∫ t in (0 : ℝ)..T, wt R u t) − (∫ t in (0 : ℝ)..T, wt R g t) / c ≤ wt R u T − wt R u 0

lemma **iint_le** {f : ℝ → ℝ} (hf : IntegrableOn f (Ioi 0)) (h0 : ∀ t, 0 ≤ f t) {T : ℝ} (hT : 0 ≤ T) : ∫ t in (0 : ℝ)..T, f t ≤ ∫ t in Ioi (0 : ℝ), f t

lemma **iint_mono** {f : ℝ → ℝ} (hf : Continuous f) (h0 : ∀ t, 0 ≤ f t) {S T : ℝ} (hS : 0 ≤ S) (hST : S ≤ T) : ∫ t in (0 : ℝ)..S, f t ≤ ∫ t in (0 : ℝ)..T, f t

theorem **step_pos** (hae : z.re + R = a) (ha : 0 < a) (hu : Continuous u) (hg : Continuous g) (hd : ∀ t, 0 ≤ t → HasDerivAt u (z * u t + g t) t) (iu : IntegrableOn (wt R u) (Ioi 0)) {P : ℝ} (hP : ∀ T, 0 ≤ T → ∫ t in (0 : ℝ)..T, wt R g t ≤ P) : a * ∫ t in Ioi (0 : ℝ), wt R u t ≤ P / a

theorem **step_neg** {b : ℝ} (hae : z.re + R = −b) (hb : 0 < b) (hu : Continuous u) (hg : Continuous g) (hd : ∀ t, 0 ≤ t → HasDerivAt u (z * u t + g t) t) (iu : IntegrableOn (wt R u) (Ioi 0)) {P : ℝ} (hP : ∀ T, 0 ≤ T → ∫ t in (0 : ℝ)..T, wt R g t ≤ P) : b * ∫ t in Ioi (0 : ℝ), wt R u t ≤ wt R u 0 + P / b

theorem **init_bound** {b : ℝ} (hae : z.re + R = −b) (hb : 0 < b) (hu : Continuous u) (hg : Continuous g) (hd : ∀ t, 0 ≤ t → HasDerivAt u (z * u t + g t) t) {P : ℝ} (hP : ∀ T, 0 ≤ T → ∫ t in (0 : ℝ)..T, wt R g t ≤ P) : wt R u 0 ≤ (2 + 2 * b) * (∫ t in (0 : ℝ)..1, wt R u t) + P

lemma **unit_le** (hu : Continuous u) : ∫ t in (0 : ℝ)..1, wt R u t ≤ Real.exp (2 * |R|) * ∫ t in Ioc (0 : ℝ) 1, ‖u t‖ ^ 2

theorem **step** {z : ℂ} {R : ℝ} (ha : z.re + R ≠ 0) : ∃ K : ℝ, 0 < K ∧ ∀ u g : ℝ → ℂ, Continuous u → Continuous g → (∀ t, 0 ≤ t → HasDerivAt u (z * u t + g t) t) → IntegrableOn (wt R u) (Ioi 0) → IntegrableOn (wt R g) (Ioi 0) → ∫ t in Ioi (0 : ℝ), wt R u t ≤ K * ((∫ t in Ioc (0 : ℝ) 1, ‖u t‖ ^ 2) + ∫ t in Ioi (0 : ℝ), wt R g t)

theorem **chain** (m : ℕ) (z : Fin m → ℂ) (R : ℝ) (hz : ∀ k, (z k).re + R ≠ 0) : ∃ C : ℝ, 0 < C ∧ ∀ F : Fin (m + 1) → ℝ → ℂ, (∀ (k : Fin m) (t : ℝ), 0 ≤ t → HasDerivAt (F k.succ) (z k * F k.succ t + F k.castSucc t) t) → (∀ k, Continuous (F k)) → (∀ k, IntegrableOn (wt R (F k)) (Ioi 0)) → ∫ t in Ioi (0 : ℝ), wt R (F (Fin.last m)) t ≤ C * ((∑ k, ∫ t in Ioc (0 : ℝ) 1, ‖F k t‖ ^ 2) + ∫ t in Ioi (0 : ℝ), wt R (F 0) t)

lemma **enn_wt** (S : ℝ) (u : ℝ → ℂ) (t : ℝ) : ‖u t‖ₑ ^ 2 * ENNReal.ofReal (Real.exp (2 * S * t)) = ENNReal.ofReal (wt S u t)

lemma **integrable_of_lt_top** {S : ℝ} {u : ℝ → ℂ} (hu : Continuous u) (h : ∫⁻ t in Ioi (0 : ℝ), ‖u t‖ₑ ^ 2 * ENNReal.ofReal (Real.exp (2 * S * t)) < ⊤) : IntegrableOn (wt S u) (Ioi 0)

lemma **lint_wt** {S : ℝ} {u : ℝ → ℂ} (hi : IntegrableOn (wt S u) (Ioi 0)) : ∫⁻ t in Ioi (0 : ℝ), ‖u t‖ₑ ^ 2 * ENNReal.ofReal (Real.exp (2 * S * t)) = ENNReal.ofReal (∫ t in Ioi (0 : ℝ), wt S u t)

lemma **lint_unit** {u : ℝ → ℂ} (hu : Continuous u) : ENNReal.ofReal (∫ t in Ioc (0 : ℝ) 1, ‖u t‖ ^ 2) = ∫⁻ t in Ioc (0 : ℝ) 1, ‖u t‖ₑ ^ 2

theorem **lineChain** : CompanionTheory.RgExh.Dec.L.LineChain

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.InverseGramEigenvaluesSumHs

theorem **specpos_inv_eigenvalues** (n : ℕ) (ε : Fin n → Bool) (g : S7.UJ ε) (hT : ((S7.toM g⁻¹)ᴴ * S7.toM g⁻¹).IsHermitian) : (∀ i, 0 < hT.eigenvalues i) ∧ ∑ i, hT.eigenvalues i = Formalism.REAL.hs (S7.toM g)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.InverseGramEigenvaluesSumHsStatement

theorem **_root_.GlobalTheory.T.inverse_gram_eigenvalues_sum_hs** : ∀ (n : ℕ) (ε : Fin n → Bool) (g : S7.UJ ε) (hT : ((S7.toM g⁻¹)ᴴ * S7.toM g⁻¹).IsHermitian), (∀ i, 0 < hT.eigenvalues i) ∧ ∑ i, hT.eigenvalues i = Formalism.REAL.hs (S7.toM g)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.IsDerivedOnOfIsKMod

theorem **equiv_of_graph** {H H' : Type} [NormedAddCommGroup H] [InnerProductSpace ℂ H] [CompleteSpace H] [NormedAddCommGroup H'] [InnerProductSpace ℂ H'] [CompleteSpace H'] {G : Set (Matrix (Fin n) (Fin n) ℂ)} {σ : Matrix (Fin n) (Fin n) ℂ → (H →L[ℂ] H)} {σ' : Matrix (Fin n) (Fin n) ℂ → (H' →L[ℂ] H')} (hI : IsIrredOn G σ) (hI' : IsIrredOn G σ') (Y : Submodule ℂ (WithLp 2 (H × H'))) (hc : IsClosed (Y : Set (WithLp 2 (H × H')))) (hst : ∀ g ∈ G, ∀ y ∈ Y, sσ σ σ' g y ∈ Y) (hn : ∀ y ∈ Y, ‖(ofLp y).1‖ = ‖(ofLp y).2‖) {y₀ : WithLp 2 (H × H')} (hy₀ : y₀ ∈ Y) (h0 : y₀ ≠ 0) : ∃ U : H ≃ₗᵢ[ℂ] H', ∀ g ∈ G, ∀ x, U (σ g x) = σ' g (U x)

theorem **self_mem_cyc** (v : V) : v ∈ Cyc E v

theorem **cyc_le** {v : V} {T : Submodule ℂ V} (hT : ∀ a b, ∀ x ∈ T, E a b x ∈ T) (hv : v ∈ T) : Cyc E v ≤ T

theorem **cas_comm** (ε : Fin n → Bool) (hgl : GlRel E) (a b : Fin n) (x : V) : E a b (Cas E x) = Cas E (E a b x)

theorem **cas_symm** (hU : IsUnitary ε E) (x y : V) : ⟪Cas E x, y⟫_ℂ = ⟪x, Cas E y⟫_ℂ

theorem **orth_spread** (hU : IsUnitary ε E) (J : Module.End ℂ V) (hJ : ∀ a b x, J (E a b x) = E a b (J x)) {v : V} (hv : ∀ m ∈ Cyc E v, ⟪m, J v⟫_ℂ = 0) : ∀ x ∈ Cyc E v, ∀ m ∈ Cyc E v, ⟪m, J x⟫_ℂ = 0

theorem **exists_closed_stable** {H : Type} [NormedAddCommGroup H] [InnerProductSpace ℂ H] {ε : Fin n → Bool} {σ : Matrix (Fin n) (Fin n) ℂ → (H →L[ℂ] H)} {D : Submodule ℂ H} {E : Fin n → Fin n → Module.End ℂ D} (hσ : IsURepOn (UJset ε) σ) (hD : IsDerivedOn ε σ D E) (hU : IsUnitary ε E) (hF : LocFin ε E) {v : D} {c : ℂ} (hv0 : v ≠ 0) (hc : Cas E v = c • v) : ∃ Y : Submodule ℂ H, IsClosed (Y : Set H) ∧ (∀ g ∈ UJset ε, ∀ y ∈ Y, σ g y ∈ Y) ∧ (v : H) ∈ Y ∧ Y ≤ ((Cyc E v).map D.subtype).topologicalClosure

theorem **isDerivedOn_of_isKMod** {H : Type} [NormedAddCommGroup H] [InnerProductSpace ℂ H] {ε : Fin n → Bool} {σ : Matrix (Fin n) (Fin n) ℂ → (H →L[ℂ] H)} {D : Submodule ℂ H} {E : Fin n → Fin n → Module.End ℂ D} (hK : IsKMod ε σ D E) : IsDerivedOn ε σ D E

theorem **inner_sD** (x y : sD D D') : ⟪x, y⟫_ℂ = ⟪(toP x).1, (toP y).1⟫_ℂ + ⟪(toP x).2, (toP y).2⟫_ℂ

theorem **toProd_prodMap** (a b : Fin n) (x : sD D D') : toP (sE E E' a b x) = (E a b (toP x).1, E' a b (toP x).2)

theorem **toProd_injective₂** {x y : sD D D'} (h : toP x = toP y) : x = y

theorem **isUnitary_sE** (hU : IsUnitary ε E) (hU' : IsUnitary ε E') : IsUnitary ε (sE E E')

theorem **locFin_sE** (hF : LocFin ε E) (hF' : LocFin ε E') : LocFin ε (sE E E')

theorem **kHighest_sE_iff** {μ : Fin n → ℝ} {x : sD D D'} : KHighest ε (sE E E') μ x ↔ KHighest ε E μ (toP x).1 ∧ KHighest ε E' μ (toP x).2

theorem **isMin_sE** {lam : Fin n → ℝ} (hm : IsMin ε E lam) (hm' : IsMin ε E' lam) : IsMin ε (sE E E') lam

theorem **kHighest_smul** {V : Type} [AddCommGroup V] [Module ℂ V] {E : Fin n → Fin n → Module.End ℂ V} {ε : Fin n → Bool} {μ : Fin n → ℝ} {v : V} (c : ℂ) (hv : KHighest ε E μ v) : KHighest ε E μ (c • v)

theorem **core** (hM : MultAlg) (hO : OrthCyc) {V : Type} [NormedAddCommGroup V] [InnerProductSpace ℂ V] {E : Fin n → Fin n → Module.End ℂ V} {ε : Fin n → Bool} {lam : Fin n → ℝ} (hlam : StrictAnti lam) (hgl : GlRel E) (hun : IsUnitary ε E) (hF : LocFin ε E) (hm : IsMin ε E lam) {y : V} (hy0 : y ≠ 0) (hyK : KHighest ε E (lowK ε lam) y) (J : Module.End ℂ V) (hJ : ∀ a b x, J (E a b x) = E a b (J x)) (hy : ⟪y, J y⟫_ℂ = 0) : (∃ c : ℂ, Cas E y = c • y) ∧ ∀ x ∈ Cyc E y, ⟪x, J x⟫_ℂ = 0

theorem **ds_unique** (hM : MultAlg) (hE : KModExists) (hO : OrthCyc) (n : ℕ) (ε : Fin n → Bool) (lam : Fin n → ℝ) (H H' : Type) [NormedAddCommGroup H] [InnerProductSpace ℂ H] [NormedAddCommGroup H'] [InnerProductSpace ℂ H'] (σ : Matrix (Fin n) (Fin n) ℂ → (H →L[ℂ] H)) (σ' : Matrix (Fin n) (Fin n) ℂ → (H' →L[ℂ] H')) (hlam : Formalism.QuantumDeterminants.IsHCParam lam) (hσ : IsDS ε lam σ) (hσ' : IsDS ε lam σ') : ∃ U : H ≃ₗᵢ[ℂ] H', ∀ g ∈ UJset ε, ∀ x, U (σ g x) = σ' g (U x)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.IsFibreFamilyLength

theorem **sigR_sigR** {σ : E →ₐ[F] E} (hσ : IsCMExt F E σ) (y : R ⊗[F] E) : sigR R σ (sigR R σ y) = y

theorem **hR_self_fixed** {σ : E →ₐ[F] E} (hσ : IsCMExt F E σ) {M : ℕ} {J : Matrix (Fin M) (Fin M) E} (hJ : IsHermND σ J) (x : Fin M → R ⊗[F] E) : sigR R σ (hR R σ J x x) = hR R σ J x x

theorem **qR_eq_hermHalf** {σ : E →ₐ[F] E} (hσ : IsCMExt F E σ) {M : ℕ} {J : Matrix (Fin M) (Fin M) E} (hJ : IsHermND σ J) (X : Fin 1 → Fin M → R ⊗[F] E) : Formalism.MainNotions.GlobalTheoryTheory.GenusMassVolumes.qR R σ J (X 0) = OneHerm.hermHalf R σ (gramH R hσ hJ X)

theorem **hermScal_qR** [TopologicalSpace R] [IsTopologicalRing R] {σ : E →ₐ[F] E} (hσ : IsCMExt F E σ) {M : ℕ} {J : Matrix (Fin M) (Fin M) E} (hJ : IsHermND σ J) (X : Fin 1 → Fin M → R ⊗[F] E) : ((hermScal R σ (Formalism.MainNotions.GlobalTheoryTheory.GenusMassVolumes.qR R σ J (X 0)) : HermR R σ 1) : Matrix (Fin 1) (Fin 1) (R ⊗[F] E)) = gramR R σ J X

theorem **qR_of_gram** [TopologicalSpace R] [IsTopologicalRing R] {σ : E →ₐ[F] E} (hσ : IsCMExt F E σ) {M : ℕ} {J : Matrix (Fin M) (Fin M) E} (hJ : IsHermND σ J) {X : Fin 1 → Fin M → R ⊗[F] E} {c : R} (h : gramR R σ J X = ((hermScal R σ c : HermR R σ 1) : Matrix (Fin 1) (Fin 1) (R ⊗[F] E))) : Formalism.MainNotions.GlobalTheoryTheory.GenusMassVolumes.qR R σ J (X 0) = c

theorem **isUnit_det_hermScal** (σ : E →ₐ[F] E) {c : R} (hc : IsUnit c) : IsUnit ((hermScal R σ c : HermR R σ 1) : Matrix (Fin 1) (Fin 1) (R ⊗[F] E)).det

theorem **isUnit_det_gramR** [TopologicalSpace R] [IsTopologicalRing R] {σ : E →ₐ[F] E} (hσ : IsCMExt F E σ) {M : ℕ} {J : Matrix (Fin M) (Fin M) E} (hJ : IsHermND σ J) {X : Fin 1 → Fin M → R ⊗[F] E} (h : IsUnit (Formalism.MainNotions.GlobalTheoryTheory.GenusMassVolumes.qR R σ J (X 0))) : IsUnit (gramR R σ J X).det

theorem **continuous_qR** {σ : E →ₐ[F] E} (hσ : IsCMExt F E σ) {M : ℕ} (J : Matrix (Fin M) (Fin M) E) [TopologicalSpace R] [IsTopologicalRing R] [LocallyCompactSpace R] [SecondCountableTopology R] [T2Space R] : Continuous fun X : Fin 1 → Fin M → R ⊗[F] E => Formalism.MainNotions.GlobalTheoryTheory.GenusMassVolumes.qR R σ J (X 0)

theorem **exists_bump** [TopologicalSpace R] [LocallyCompactSpace R] [T2Space R] (hU : IsOpen {c : R | IsUnit c}) {c₀ : R} (hc₀ : IsUnit c₀) : ∃ χ : R → ℝ, Continuous χ ∧ tsupport χ ⊆ {c : R | IsUnit c} ∧ ∀ᶠ c in 𝓝 c₀, χ c = 1

theorem **GlobalTheory.CompanionTheory.SW.isFibreFamily_length** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R], IsLocR R → ∀ (μ : Measure (Fin 1 → Fin M → R ⊗[F] E)) (ν : Measure R), μ.IsAddHaarMeasure → ν.IsAddHaarMeasure → ν {c | ¬ IsUnit c} = 0 → μ {X | ¬ IsUnit ((fun X : Fin 1 → Fin M → R ⊗[F] E => Formalism.MainNotions.GlobalTheoryTheory.GenusMassVolumes.qR (R) σ J (X 0)) X)} = 0 → ∀ m, IsGramFib R σ J 1 μ (ν.map (hermScal R σ)) m → Formalism.MainNotions.GlobalTheoryTheory.GenusMassVolumes.IsFibreFamily μ (fun X : Fin 1 → Fin M → R ⊗[F] E => Formalism.MainNotions.GlobalTheoryTheory.GenusMassVolumes.qR (R) σ J (X 0)) id Set.univ ν {c | IsUnit c} (fun c => m (hermScal R

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.IsIsoOfClosedImmersion

theorem **isIso_of_closedImmersion** {R : Type} [CommRing R] {Z : Scheme.{0}} (f : Z ⟶ Sp R) [IsClosedImmersion f] [Flat f] [Surjective f] : IsIso f

theorem **preDual_baseChange** {S' : Scheme.{0}} {b : S' ⟶ S} {A B : Over S} [GrpObj A] [GrpObj B] {A' B' : Over S'} [GrpObj A'] [GrpObj B'] {f : A'.left ⟶ A.left} {g : B'.left ⟶ B.left} {h : (A' ⊗ B').left ⟶ (A ⊗ B).left} (hf : IsGrpPullback A' A b f) (hg : IsGrpPullback B' B b g) (h₁ : h ≫ (fst A B).left = (fst A' B').left ≫ f) (h₂ : h ≫ (snd A B).left = (snd A' B').left ≫ g) {P : LB (A ⊗ B).left} {P' : LB (A' ⊗ B').left} (hP' : P'.IsLB) (e : LBIso (P.pull h) P') (hd : PreDual A B P) : PreDual A' B' P'

theorem **exists_point_fibre** (H3 : PointSurj) {A B : Over S} [GrpObj A] [GrpObj B] (hA : IsAbelianScheme A) (hB : IsAbelianScheme B) {P : LB (A ⊗ B).left} (hP : PreDual A B P) {m : Pic A.left} (hm : IsAddCls A m) (k : Type) [Field k] [IsAlgClosed k] (s : Sp k ⟶ S) : ∃ y : Over.mk ((𝟙_ (Over (Sp k))).hom ≫ s) ⟶ B, evc hP.isLB (fst A (Over.mk ((𝟙_ (Over (Sp k))).hom ≫ s))) (snd A (Over.mk ((𝟙_ (Over (Sp k))).hom ≫ s)) ≫ y) = CechPic.pull (fst A (Over.mk ((𝟙_ (Over (Sp k))).hom ≫ s))).left m

theorem **exists_section** (H1 : TrivLocus) (H2 : TrivLocusFlat) (H3 : PointSurj) {C : Type} [CommRing C] [IsNoetherianRing C] (A B : Over (Sp C)) [GrpObj A] [GrpObj B] (g : ℕ) (hA : IsAbelianScheme A) (hB : IsAbelianScheme B) (gA : SmoothOfRelativeDimension g A.hom) (gB : SmoothOfRelativeDimension g B.hom) {P : LB (A ⊗ B).left} (hP : PreDual A B P) {m : Pic A.left} (hm : IsAddCls A m) : ∃ x : Over.mk (𝟙 (Sp C)) ⟶ B, evc hP.isLB (fst A (Over.mk (𝟙 (Sp C)))) (snd A (Over.mk (𝟙 (Sp C))) ≫ x) = CechPic.pull (fst A (Over.mk (𝟙 (Sp C)))).left m

theorem **univNoeth** : TrivLocus → TrivLocusFlat → PointSurj → ∀ (R : Type) [CommRing R] [IsNoetherianRing R] (A B : Over (Sp R)) [GrpObj A] [GrpObj B] (g : ℕ), IsAbelianScheme A → IsAbelianScheme B → SmoothOfRelativeDimension g A.hom → SmoothOfRelativeDimension g B.hom → ∀ P : LB (A ⊗ B).left, PreDual A B P → UnivNoeth A B P

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.IsPiecesPiece

theorem **hLin_apply** (hh : IsHerm s h) (x y : V) : hLin hh x y = h x y

theorem **h_sub_right** (hh : IsHerm s h) (x y z : V) : h x (y − z) = h x y − h x z

theorem **h_zero_right** (hh : IsHerm s h) (x : V) : h x 0 = 0

theorem **eq_of_pair** (hh : IsHerm s h) {z z' : V} (H : ∀ x, h x z = h x z') : z = z'

theorem **s_ne** (hδ : δ ≠ 0) : s δ ≠ 0

theorem **projE_apply** (hh : IsHerm s h) (i : Fin l) (x : V) : projE hh δ e f i x = ((s δ)⁻¹ * h (f i) x) • e i

theorem **projF_apply** (hh : IsHerm s h) (i : Fin l) (x : V) : projF hh δ e f i x = (δ⁻¹ * h (e i) x) • f i

theorem **h_f_e** (hh : IsHerm s h) (hF : IsFrame h δ e f W) (i j : Fin l) : h (f i) (e j) = if j = i then s δ else 0

theorem **projE_e** (hh : IsHerm s h) (hδ : δ ≠ 0) (hF : IsFrame h δ e f W) (i j : Fin l) : projE hh δ e f i (e j) = if j = i then e i else 0

theorem **projE_f** (hh : IsHerm s h) (hF : IsFrame h δ e f W) (i j : Fin l) : projE hh δ e f i (f j) = 0

theorem **projE_w** (hh : IsHerm s h) (hF : IsFrame h δ e f W) (i : Fin l) {w : V} (hw : w ∈ W) : projE hh δ e f i w = 0

theorem **projF_e** (hh : IsHerm s h) (hF : IsFrame h δ e f W) (i j : Fin l) : projF hh δ e f i (e j) = 0

theorem **projF_f** (hh : IsHerm s h) (hδ : δ ≠ 0) (hF : IsFrame h δ e f W) (i j : Fin l) : projF hh δ e f i (f j) = if j = i then f i else 0

theorem **projF_w** (hh : IsHerm s h) (hF : IsFrame h δ e f W) (i : Fin l) {w : V} (hw : w ∈ W) : projF hh δ e f i w = 0

theorem **piece_e** (hh : IsHerm s h) (hδ : δ ≠ 0) (hF : IsFrame h δ e f W) (i : Idx l) (j : Fin l) : piece hh δ e f i (e j) = if i = some (Sum.inl j) then e j else 0

theorem **piece_f** (hh : IsHerm s h) (hδ : δ ≠ 0) (hF : IsFrame h δ e f W) (i : Idx l) (j : Fin l) : piece hh δ e f i (f j) = if i = some (Sum.inr j) then f j else 0

theorem **piece_w** (hh : IsHerm s h) (hF : IsFrame h δ e f W) (i : Idx l) {w : V} (hw : w ∈ W) : piece hh δ e f i w = if i = none then w else 0

theorem **end_ext** (hF : IsFrame h δ e f W) {A B : Module.End L V} (he : ∀ i, A (e i) = B (e i)) (hf : ∀ i, A (f i) = B (f i)) (hw : ∀ w ∈ W, A w = B w) : A = B

theorem **isPieces_piece** (hh : IsHerm s h) (hδ : δ ≠ 0) (hF : IsFrame h δ e f W) : IsPieces (piece hh δ e f)

theorem **lat_smul** {Λ : AddSubgroup V} (hΛ : IsLat L Λ) {c : L} (hc : ‖c‖ ≤ 1) {x : V} (hx : x ∈ Λ) : c • x ∈ Λ

theorem **e_mem** {Λ : AddSubgroup V} (hA : IsAdapted h e f W (fun _ => 1) Λ) (hh : IsHerm s h) (j : Fin l) : e j ∈ Λ

theorem **f_mem** {Λ : AddSubgroup V} (hA : IsAdapted h e f W (fun _ => 1) Λ) (hh : IsHerm s h) (j : Fin l) : f j ∈ Λ

theorem **piece_mem** {Λ : AddSubgroup V} (hΛ : IsLat L Λ) (hA : IsAdapted h e f W (fun _ => 1) Λ) (hh : IsHerm s h) (hδ : δ ≠ 0) (hF : IsFrame h δ e f W) (i : Idx l) {x : V} (hx : x ∈ Λ) : piece hh δ e f i x ∈ Λ

theorem **tor_piece** (hh : IsHerm s h) (hδ : δ ≠ 0) (hF : IsFrame h δ e f W) {u : Fin l → L} {t : V ≃ₗ[L] V} (ht : IsTor s e f W u t) (i : Idx l) : (t.toLinearMap : Module.End L V) * piece hh δ e f i = tau s u i • piece hh δ e f i ∧ piece hh δ e f i * (t.toLinearMap : Module.End L V) = tau s u i • piece hh δ e f i

theorem **wt_le** {l : ℕ} (i : Idx l) : wt i ≤ 2 * l

theorem **wt_injective** {l : ℕ} : Function.Injective (wt : Idx l → ℕ)

theorem **wt_rev** {l : ℕ} (i : Idx l) : wt (rev i) + wt i = 2 * l

theorem **rev_rev** {l : ℕ} (i : Idx l) : rev (rev i) = i

theorem **tau_mono** (hs : IsInv s) {u : Fin l → L} (hu0 : ∀ i, u i ≠ 0) (hm : Monotone fun i => ‖u i‖) (h1 : ∀ i, ‖u i‖ ≤ 1) (i j : Idx l) (hij : wt i ≤ wt j) : ‖tau s u i‖ ≤ ‖tau s u j‖

theorem **tau_ne** {u : Fin l → L} (hu0 : ∀ i, u i ≠ 0) (i : Idx l) : tau s u i ≠ 0

theorem **h_smul_left** (hh : IsHerm s h) (c : L) (x y : V) : h (c • x) y = s c * h x y

theorem **h_add_left** (hh : IsHerm s h) (x x' y : V) : h (x + x') y = h x y + h x' y

theorem **h_zero_left** (hh : IsHerm s h) (y : V) : h 0 y = 0

theorem **h_sum_left** (hh : IsHerm s h) {ι : Type} (A : Finset ι) (v : ι → V) (y : V) : h (∑ m ∈ A, v m) y = ∑ m ∈ A, h (v m) y

theorem **h_sub_left** (hh : IsHerm s h) (x x' y : V) : h (x − x') y = h x y − h x' y

theorem **projE_adj** (hs : IsInv s) (hh : IsHerm s h) (i : Fin l) (x y : V) : h (projE hh δ e f i x) y = h x (projF hh δ e f i y)

theorem **projF_adj** (hs : IsInv s) (hh : IsHerm s h) (i : Fin l) (x y : V) : h (projF hh δ e f i x) y = h x (projE hh δ e f i y)

theorem **piece_adj** (hs : IsInv s) (hh : IsHerm s h) (i : Idx l) (x y : V) : h (piece hh δ e f i x) y = h x (piece hh δ e f (rev i) y)

theorem **exists_star** (hs : IsInv s) (hh : IsHerm s h) (hadj : ∀ g : Module.End L V, ∃ g' : Module.End L V, ∀ x y, h (g x) y = h x (g' y)) : ∃ st : Module.End L V → Module.End L V, (∀ g x y, h (g x) y = h x (st g y)) ∧ (∀ a b, st (a ∗ b) = st b ∗ st a) ∧ st 1 = 1 ∧ st 0 = 0 ∧ ∀ i : Idx l, st (piece hh δ e f i) = piece hh δ e f (rev i)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.IsUnitOfTmulOne

theorem **isUnit_of_tmul_one** {F E : Type} [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] {σ : E →ₐ[F] E} (hσ : IsCMExt F E σ) {R : Type} [CommRing R] [Algebra F R] {c : R} (h : IsUnit (c ⊗ₜ[F] (1 : E))) : IsUnit c

theorem **GlobalTheory.CompanionTheory.SW.QTie.fib_eq_gramFib** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (M : ℕ) (J : Matrix (Fin M) (Fin M) E), IsHermND σ J → ∀ (R : Type) [CommRing R] [Algebra F R] [TopologicalSpace R] [MeasurableSpace R] [BorelSpace R], IsLocR R → ∀ (μ : Measure (Fin 1 → Fin M → R ⊗[F] E)) (ν : Measure R), μ.IsAddHaarMeasure → ν.IsAddHaarMeasure → ν {c | ¬ IsUnit c} = 0 → μ {X | ¬ IsUnit ((fun X : Fin 1 → Fin M → R ⊗[F] E => Formalism.MainNotions.GlobalTheoryTheory.GenusMassVolumes.qR (R) σ J (X 0)) X)} = 0 → ∀ fib, Formalism.MainNotions.GlobalTheoryTheory.GenusMassVolumes.IsFibreFamily μ (fun X : Fin 1 → Fin M → R ⊗[F] E => Formalism.MainNotions.GlobalTheoryTheory.GenusMassVolumes.qR (R) σ J (X 0)) id Set.univ ν {c | IsUnit c} fib → ∀ m, IsGramFib R σ J 1 μ (ν.map (hermScal R σ)) m → ∀ c : R, Is

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.IsWeightOfIntertwine

theorem **iso_refl** (X : GMod n) : Iso X X

theorem **intertwine_symm** {X Y : GMod n} (e : X.V ≃ₗ[ℂ] Y.V) (he : ∀ a b x, e (X.E a b x) = Y.E a b (e x)) (a b : Fin n) (y : Y.V) : e.symm (Y.E a b y) = X.E a b (e.symm y)

theorem **iso_symm** {X Y : GMod n} : Iso X Y → Iso Y X

theorem **iso_trans** {X Y Z : GMod n} : Iso X Y → Iso Y Z → Iso X Z

theorem **cj_apply** {V W : Type} [AddCommGroup V] [Module ℂ V] [AddCommGroup W] [Module ℂ W] (e : V ≃ₗ[ℂ] W) (f : Module.End ℂ V) (w : W) : cj e f w = e (f (e.symm w))

theorem **cj_smul_one** {V W : Type} [AddCommGroup V] [Module ℂ V] [AddCommGroup W] [Module ℂ W] (e : V ≃ₗ[ℂ] W) (c : ℂ) : cj e (c • (1 : Module.End ℂ V)) = c • (1 : Module.End ℂ W)

theorem **cj_algebraMap** {V W : Type} [AddCommGroup V] [Module ℂ V] [AddCommGroup W] [Module ℂ W] (e : V ≃ₗ[ℂ] W) (c : ℂ) : cj e (algebraMap ℂ (Module.End ℂ V) c) = algebraMap ℂ (Module.End ℂ W) c

theorem **cj_E** {X Y : GMod n} (e : X.V ≃ₗ[ℂ] Y.V) (he : ∀ a b x, e (X.E a b x) = Y.E a b (e x)) (a b : Fin n) : cj e (X.E a b) = Y.E a b

theorem **cj_Tm** {X Y : GMod n} (e : X.V ≃ₗ[ℂ] Y.V) (he : ∀ a b x, e (X.E a b x) = Y.E a b (e x)) (u : ℂ) (a b : Fin n) : cj e (Tm X.E u a b) = Tm Y.E u a b

theorem **cj_qdet** {X Y : GMod n} (e : X.V ≃ₗ[ℂ] Y.V) (he : ∀ a b x, e (X.E a b x) = Y.E a b (e x)) (u : ℂ) : cj e (qdet X.E u) = qdet Y.E u

theorem **scalChar_of_iso** {X Y : GMod n} (h : Iso X Y) (χ : Fin n → ℂ) (hX : ScalChar X χ) : ScalChar Y χ

theorem **hasChar_of_iso** {X Y : GMod n} (h : Iso X Y) (χ : Fin n → ℂ) (hX : HasChar X χ) : HasChar Y χ

theorem **isSimple_of_iso** {X Y : GMod n} (h : Iso X Y) (hX : IsSimple X) : IsSimple Y

theorem **locFin_of_iso** {X Y : GMod n} (ε : Fin n → Bool) (h : Iso X Y) (hX : LocFin ε X.E) : LocFin ε Y.E

theorem **isWeight_of_intertwine** {V W : Type} [AddCommGroup V] [Module ℂ V] [AddCommGroup W] [Module ℂ W] (e : V ≃ₗ[ℂ] W) (E : Fin n → Fin n → Module.End ℂ V) (F : Fin n → Fin n → Module.End ℂ W) (he : ∀ a b x, e (E a b x) = F a b (e x)) {μ : Fin n → ℝ} {x : V} (h : IsWeight E μ x) : IsWeight F μ (e x)

theorem **kHighest_of_intertwine** {V W : Type} [AddCommGroup V] [Module ℂ V] [AddCommGroup W] [Module ℂ W] (e : V ≃ₗ[ℂ] W) (ε : Fin n → Bool) (E : Fin n → Fin n → Module.End ℂ V) (F : Fin n → Fin n → Module.End ℂ W) (he : ∀ a b x, e (E a b x) = F a b (e x)) {μ : Fin n → ℝ} {x : V} (h : KHighest ε E μ x) : KHighest ε F μ (e x)

theorem **intWt_of_iso** {X Y : GMod n} (h : Iso X Y) (hX : IntWt X) : IntWt Y

theorem **algDS_of_iso** {X Y : GMod n} (ε : Fin n → Bool) (h : Iso X Y) (s : Equiv.Perm (Fin n)) (mu : Fin n → ℝ) (hX : AlgDS ε X s mu) : AlgDS ε Y s mu

theorem **coneDS_of_iso** {X Y : GMod n} (ε : Fin n → Bool) (h : Iso X Y) (s : Equiv.Perm (Fin n)) (mu : Fin n → ℝ) (hX : CompanionTheory.RgExh.Fin.ConeDS ε X s mu) : CompanionTheory.RgExh.Fin.ConeDS ε Y s mu

theorem **isoFacts** : CompanionTheory.RgExh.Fin.IsoFacts

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.IsometricLatticeCongruenceSubgroups

theorem **form_isHerm** (s : L →+* L) (J : Matrix (Fin n) (Fin n) L) (hs : ∀ x, s (s x) = x) (hJ : (J.map s).transpose = J) (hd : J.det ≠ 0) : IsHerm s (form s J)

theorem **toE_apply** (g : GL (Fin n) L) (x : Fin n → L) : toE g x = g.1.mulVec x

theorem **toE_injective** : Function.Injective (toE : GL (Fin n) L → _)

theorem **toE_ofE** (t : (Fin n → L) ≃ₗ[L] (Fin n → L)) : toE (ofE t) = t

theorem **form_toE** (s : L →+* L) (J : Matrix (Fin n) (Fin n) L) (g : GL (Fin n) L) (x y : Fin n → L) : form s J (toE g x) (toE g y) = dotProduct (s ∘ x) (((g.1.map s).transpose ∗ J ∗ g.1).mulVec y)

theorem **mem_UG_iff** (s : L →+* L) (J : Matrix (Fin n) (Fin n) L) (g : GL (Fin n) L) : g ∈ UG s J ↔ IsIso (form s J) (toE g)

theorem **toE_surj** (s : L →+* L) (J : Matrix (Fin n) (Fin n) L) (t : (Fin n → L) ≃ₗ[L] (Fin n → L)) (ht : IsIso (form s J) t) : ∃ g : UG s J, toE g.1 = t

theorem **isTor_one** (s : L →+* L) {l : ℕ} (e f : Fin l → V) (W : Submodule L V) : IsTor s e f W (fun _ => 1) (1 : V ≃ₗ[L] V)

theorem **isTor_mul** {s : L →+* L} {l : ℕ} {e f : Fin l → V} {W : Submodule L V} {u u' : Fin l → L} {t t' : V ≃ₗ[L] V} (h : IsTor s e f W u t) (h' : IsTor s e f W u' t') : IsTor s e f W (fun i => u i ∗ u' i) (t ∗ t')

theorem **exists_unif** (L : Type) [NontriviallyNormedField L] [IsUltrametricDist L] [ProperSpace L] : ∃ p : L, p ≠ 0 ∧ ‖p‖ < 1 ∧ ∀ a : L, ‖a‖ < 1 → ‖a‖ ≤ ‖p‖

theorem **exists_cover** {G : Type} [Group G] [TopologicalSpace G] [ContinuousMul G] (K K₀ : Subgroup G) (hK : IsOpen (K : Set G)) (hc : IsCompact (K₀ : Set G)) : ∃ (d : ℕ) (c : Fin d → G), (∀ i, c i ∈ K₀) ∧ ∀ g ∈ K₀, ∃ i, ∃ k ∈ K, g = c i ∗ k

theorem **ind_antitone** {l : ℕ} (i : Fin l) : Antitone (ind i)

theorem **closure_tor** {M : Type} [Monoid M] {l : ℕ} (T : (Fin l → ℕ) → M) (h0 : T 0 = 1) (hadd : ∀ a b, T (a + b) = T a ∗ T b) (g : M) : g ∈ Submonoid.closure (Set.range fun i : Fin l => T (ind i)) ↔ ∃ r : Fin l → ℕ, Antitone r ∧ g = T r

theorem **isCartan_of** {G : Type} [Group G] (K K₀ : Subgroup G) {d l : ℕ} (T : (Fin l → ℕ) → G) (h0 : T 0 = 1) (hadd : ∀ a b, T (a + b) = T a ∗ T b) (c : Fin d → G) (hc : ∀ i, c i ∈ K₀) (hcov : ∀ g ∈ K₀, ∃ i, ∃ k ∈ K, g = c i ∗ k) (hnorm : ∀ c ∈ K₀, ∀ k, k ∈ K ↔ c ∗ k ∗ c⁻¹ ∈ K) (hdec : ∀ g, ∃ k ∈ K₀, ∃ k' ∈ K₀, ∃ r : Fin l → ℕ, Antitone r ∧ g = k ∗ T r ∗ k') (hiw : ∀ k ∈ K, ∃ a b : G, k = a ∗ b ∧ ∀ r : Fin l → ℕ, Antitone r → T r ∗ a ∗ (T r)⁻¹ ∈ K ∧ (T r)⁻¹ ∗ b ∗ T r ∈ K) : IsCartan K d l

theorem **final** : StDelta → StExists → StFrame → StTor → StDecomp → StIwahori → (∀ (L : Type) [NontriviallyNormedField L] [IsUltrametricDist L] [ProperSpace L] (s : L →+* L), (∀ x, ‖s x‖ = ‖x‖) → ∀ (n : ℕ) (J : Matrix (Fin n) (Fin n) L) (Λ : AddSubgroup (Fin n → L)), IsLat L Λ → (∀ π : L, π ≠ 0 → ‖π‖ < 1 → ∃ K K₀ : Subgroup (UG s J), (∀ g, g ∈ K ↔ ∀ x ∈ Λ, ∃ y ∈ Λ, g.1.1.mulVec x − x = π • y) ∧ (∀ g, g ∈ K₀ ↔ ∀ x, g.1.1.mulVec x ∈ Λ ↔ x ∈ Λ) ∧ IsOpen (K : Set (UG s J)) ∧ IsCompact (K : Set (UG s J)) ∧ IsCompact (K₀ : Set (UG s J)) ∧ K ≤ K₀ ∧ ∀ c ∈ K₀, ∀ k, k ∈ K ↔ c ∗ k ∗ c⁻¹ ∈ K) ∧ ∀ U : Subgroup (UG s J), IsOpen (U : Set (UG s J)) → ∃ π : L, π ≠ 0 ∧ ‖π‖ < 1 ∧ ∀ g : UG s J, (∀ x ∈ Λ, ∃ y ∈ Λ, g.1.1.mulVec x − x = π • y) → g ∈ U) → StCartanU

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.IsometricLatticeCongruenceSubgroupsStatement

theorem **_root_.GlobalTheory.T.isometric_lattice_congruence_subgroups** : StDelta → StExists → StFrame → StTor → StDecomp → StIwahori → (∀ (L : Type) [NontriviallyNormedField L] [IsUltrametricDist L] [ProperSpace L] (s : L →+* L), (∀ x, ‖s x‖ = ‖x‖) → ∀ (n : ℕ) (J : Matrix (Fin n) (Fin n) L) (Λ : AddSubgroup (Fin n → L)), IsLat L Λ → (∀ π : L, π ≠ 0 → ‖π‖ < 1 → ∃ K K₀ : Subgroup (UG s J), (∀ g, g ∈ K ↔ ∀ x ∈ Λ, ∃ y ∈ Λ, g.1.1.mulVec x − x = π • y) ∧ (∀ g, g ∈ K₀ ↔ ∀ x, g.1.1.mulVec x ∈ Λ ↔ x ∈ Λ) ∧ IsOpen (K : Set (UG s J)) ∧ IsCompact (K : Set (UG s J)) ∧ IsCompact (K₀ : Set (UG s J)) ∧ K ≤ K₀ ∧ ∀ c ∈ K₀, ∀ k, k ∈ K ↔ c ∗ k ∗ c⁻¹ ∈ K) ∧ ∀ U : Subgroup (UG s J), IsOpen (U : Set (UG s J)) → ∃ π : L, π ≠ 0 ∧ ‖π‖ < 1 ∧ ∀ g : UG s J, (∀ x ∈ Λ, ∃ y ∈ Λ, g.1.1.mulVec x − x = π • y) → g ∈ U) → StCartanU

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.IsomorphismStatement

theorem **stIso** : StIso

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.IsomorphismStatementStatement

theorem **_root_.GlobalTheory.T.IsomorphismStatement** : StIso

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.IwahoriStatement

theorem **isCongIdeal_congJ** {L V : Type} [NontriviallyNormedField L] [IsUltrametricDist L] [AddCommGroup V] [Module L V] [FiniteDimensional L V] {Λ : AddSubgroup V} (hΛ : IsLat L Λ) {π : L} (hπ : ‖π‖ < 1) {ι : Type} (P : ι → Module.End L V) (hP : ∀ i, ∀ x ∈ Λ, P i x ∈ Λ) : IsCongIdeal (congJ Λ π) P

theorem **congJ_smul** {L V : Type} [NontriviallyNormedField L] [AddCommGroup V] [Module L V] {Λ : AddSubgroup V} (hΛ : IsLat L Λ) (π : L) (c : L) (hc : ‖c‖ ≤ 1) (x : Module.End L V) (hx : x ∈ congJ Λ π) : c • x ∈ congJ Λ π

theorem **isUpper_rev_iff** {E : Type} [Ring E] {l : ℕ} (P : Idx l → E) (b : E) : IsUpper (fun i => 2 ∗ l − wt i) P b ↔ IsLower wt P b

theorem **stIwahori** : StIwahori

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.IwahoriStatementStatement

theorem **_root_.GlobalTheory.T.IwahoriStatement** : StIwahori

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.IwahoriUpperLowerFactorization

theorem **eq_zero_of_mul_pieces** (hP : IsPieces P) {y : E} (h : ∀ j, y ∗ P j = 0) : y = 0

theorem **eq_zero_of_pieces_mul** (hP : IsPieces P) {y : E} (h : ∀ j, P j ∗ y = 0) : y = 0

theorem **block_mul** (hP : IsPieces P) (u x y v : E) : u ∗ (x ∗ y) ∗ v = ∑ l, (u ∗ x ∗ P l) ∗ (P l ∗ y ∗ v)

theorem **isUpper_mul** (hP : IsPieces P) {a b : E} (ha : IsUpper wt P a) (hb : IsUpper wt P b) : IsUpper wt P (a ∗ b)

theorem **isLowerUni_mul** (hwt : Function.Injective wt) (hP : IsPieces P) {a b : E} (ha : IsLowerUni wt P a) (hb : IsLowerUni wt P b) : IsLowerUni wt P (a ∗ b)

theorem **isUpper_one** (hP : IsPieces P) : IsUpper wt P 1

theorem **isLowerUni_one** (hP : IsPieces P) : IsLowerUni wt P 1

theorem **sub_one_mul_mem** {J : AddSubgroup E} (hJ : IsCongIdeal J P) {a b : E} (ha : a − 1 ∈ J) (hb : b − 1 ∈ J) : a ∗ b − 1 ∈ J

theorem **step** (hwt : Function.Injective wt) (hP : IsPieces P) {J : AddSubgroup E} (hJ : IsCongIdeal J P) (m : ℕ) (ih : ∀ k : E, k − 1 ∈ J → (∀ i j, (m ≤ wt i ∨ m ≤ wt j) → P i ∗ (k − 1) ∗ P j = 0) → ∃ a b : E, a − 1 ∈ J ∧ b − 1 ∈ J ∧ IsUpper wt P a ∧ IsLowerUni wt P b ∧ k = a ∗ b) (q : ι) (hq : wt q = m) (x : E) (hx : x ∈ J) (hH : ∀ i j, (m + 1 ≤ wt i ∨ m + 1 ≤ wt j) → P i ∗ x ∗ P j = 0) : ∃ a b : E, a − 1 ∈ J ∧ b − 1 ∈ J ∧ IsUpper wt P a ∧ IsLowerUni wt P b ∧ 1 + x = a ∗ b

theorem **exists_of_support** (hwt : Function.Injective wt) (hP : IsPieces P) {J : AddSubgroup E} (hJ : IsCongIdeal J P) (m : ℕ) : ∀ k : E, k − 1 ∈ J → (∀ i j, (m ≤ wt i ∨ m ≤ wt j) → P i ∗ (k − 1) ∗ P j = 0) → ∃ a b : E, a − 1 ∈ J ∧ b − 1 ∈ J ∧ IsUpper wt P a ∧ IsLowerUni wt P b ∧ k = a ∗ b

theorem **block_gauss** : ∀ {E ι : Type} [Ring E] [Fintype ι] (wt : ι → ℕ), Function.Injective wt → ∀ (P : ι → E), GlobalTheory.CompanionTheory.Iwahori.IsPieces P → ∀ J : AddSubgroup E, GlobalTheory.CompanionTheory.Iwahori.IsCongIdeal J P → ∀ k : E, k − 1 ∈ J → ∃ a b : E, a − 1 ∈ J ∧ b − 1 ∈ J ∧ GlobalTheory.CompanionTheory.Iwahori.IsUpper wt P a ∧ GlobalTheory.CompanionTheory.Iwahori.IsLowerUni wt P b ∧ k = a ∗ b

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.IwahoriUpperLowerFactorizationStatement

theorem **_root_.GlobalTheory.T.iwahori_upper_lower_factorization** : ∀ {E ι : Type} [Ring E] [Fintype ι] (wt : ι → ℕ), Function.Injective wt → ∀ (P : ι → E), GlobalTheory.CompanionTheory.Iwahori.IsPieces P → ∀ J : AddSubgroup E, GlobalTheory.CompanionTheory.Iwahori.IsCongIdeal J P → ∀ k : E, k − 1 ∈ J → ∃ a b : E, a − 1 ∈ J ∧ b − 1 ∈ J ∧ GlobalTheory.CompanionTheory.Iwahori.IsUpper wt P a ∧ GlobalTheory.CompanionTheory.Iwahori.IsLowerUni wt P b ∧ k = a ∗ b

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.IwasawaDecomposition

theorem **adapted_dec_one** {n : ℕ} {p : Fin n → Fin n} {b : Fin n → ℤ} {c : Fin n → ℂ} (h : Adapted p b c) (g : Matrix (Fin n) (Fin n) ℂ) (hg : gᴴ ∗ dp p 0 c ∗ g = dp p 0 c) : ∃ k B : Matrix (Fin n) (Fin n) ℂ, kᴴ ∗ k = 1 ∧ kᴴ ∗ dp p 0 c ∗ k = dp p 0 c ∧ Bᴴ ∗ dp p 0 c ∗ B = dp p 0 c ∧ g = k ∗ B ∧ B.BlockTriangular id

theorem **conj_mem** {n : ℕ} {A g : Matrix (Fin n) (Fin n) ℂ} (hA : A ∗ A = 1) (hAH : Aᴴ = A) (hg : gᴴ ∗ A ∗ g = A) : (A ∗ gᴴ ∗ A)ᴴ ∗ A ∗ (A ∗ gᴴ ∗ A) = A

theorem **conj_mul** {n : ℕ} {A g : Matrix (Fin n) (Fin n) ℂ} (hA : A ∗ A = 1) (hg : gᴴ ∗ A ∗ g = A) : (A ∗ gᴴ ∗ A) ∗ g = 1

theorem **adapted_dec_left** {n : ℕ} {p : Fin n → Fin n} {b : Fin n → ℤ} {c : Fin n → ℂ} (h : Adapted p b c) (x : Matrix (Fin n) (Fin n) ℂ) (hx : xᴴ ∗ dp p 0 c ∗ x = dp p 0 c) : ∃ s κ : Matrix (Fin n) (Fin n) ℂ, sᴴ ∗ dp p 0 c ∗ s = dp p 0 c ∧ κᴴ ∗ dp p 0 c ∗ κ = dp p 0 c ∧ s.BlockTriangular id ∧ κᴴ ∗ κ = 1 ∧ x = s ∗ κ

theorem **pq_val** (k r : ℕ) (i : Fin (2 ∗ r + k + 1)) : (pq k r i).val = if i.val ≤ r ∨ r + k ≤ i.val then 2 ∗ r + k − i.val else i.val

theorem **bq_eq** (k r : ℕ) (i : Fin (2 ∗ r + k + 1)) : bq k r i = if i.val ≤ r ∨ r + k ≤ i.val then 2 ∗ (i.val : ℤ) − (2 ∗ r + k) else 0

theorem **adapted_pq** (k r : ℕ) : Adapted (pq k r) (bq k r) (fun _ => 1) where

theorem **Hq_eq_dp** (k r : ℕ) : Hq k r = dp (pq k r) 0 (fun _ => 1)

theorem **iwaQ** : IwaQ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.IwasawaDecompositionStatement

theorem **_root_.GlobalTheory.T.IwasawaDecomposition** : IwaQ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.IwasawaParabolicCompactFactorization

theorem **mem_KG_of_mem_maxK** {n : ℕ} {ε : Fin n → Bool} {k : S7.UJ ε} (hk : k ∈ GlobalTheory.FlagMeas.maxK ε) : k ∈ S7.KG ε

theorem **exists_psub_mul_maxK** {n : ℕ} {ε : Fin n → Bool} {A : Matrix (Fin n) (Fin n) ℂ} (hA : IsGrad ε A) (g : Formalism.REAL.UJ ε) : ∃ p : Formalism.REAL.UJ ε, p ∈ PSub ε A ∧ ∃ k ∈ GlobalTheory.FlagMeas.maxK ε, g = p ∗ k

theorem **iwasawa_psub** : ∀ (n : ℕ) (ε : Fin n → Bool) (A : Matrix (Fin n) (Fin n) ℂ), IsGrad ε A → ∀ g : S7.UJ ε, ∃ (x : PSub ε A) (k : S7.KG ε), g = (x : S7.UJ ε) ∗ (k : S7.UJ ε)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.IwasawaParabolicCompactFactorizationStatement

theorem **_root_.GlobalTheory.T.iwasawa_parabolic_compact_factorization** : ∀ (n : ℕ) (ε : Fin n → Bool) (A : Matrix (Fin n) (Fin n) ℂ), IsGrad ε A → ∀ g : S7.UJ ε, ∃ (x : PSub ε A) (k : S7.KG ε), g = (x : S7.UJ ε) ∗ (k : S7.UJ ε)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.JacquetBasis

theorem **Jm_mul_self** (ε : Fin n → Bool) : Jm ε ∗ Jm ε = 1

theorem **trace_eq_zero** {ε : Fin n → Bool} {A : Matrix (Fin n) (Fin n) ℂ} (h : IsGrad ε A) : A.trace = 0

theorem **eigenvalue_cases** {A : Matrix (Fin n) (Fin n) ℂ} (hA : A.IsHermitian) (h3 : A ∗ A ∗ A = A) (i : Fin n) : hA.eigenvalues i = −1 ∨ hA.eigenvalues i = 0 ∨ hA.eigenvalues i = 1

theorem **exists_perm_blocks** {c : Fin n → Fin 3} {d : Fin 3 → ℕ} (hd : ∑ i, d i = n) (hc : ∀ i, Fintype.card {x // c x = i} = d i) : ∃ σ : Fin n ≃ Fin n, ∀ p, c (σ p) = ((stdBlocks d hd).symm p).1

theorem **exists_gradBasis** {ε : Fin n → Bool} {A : Matrix (Fin n) (Fin n) ℂ} (h : IsGrad ε A) : Nonempty (GradBasis A)

theorem **GlobalTheory.CompanionTheory.GradEig.basis** : CompanionTheory.RgExh.Dec.J.Basis

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.JacquetDegree

theorem **extract** (W' : Submodule ℂ (Module.Dual ℂ V)) (φ : Module.Dual ℂ V) {m : ℕ} (hm : 1 ≤ m) (E : Fin m → Fin m → Module.End ℂ V) (a : ℂ) (P : ℂ[X]) (hdeg : P.natDegree = m) (hP : P.Monic) (h : ∀ Y : ℂ, φ ∘ₗ qdet E (Y + a) − P.eval Y • φ ∈ W') : φ ∘ₗ (∑ k, E k k) − (P.coeff (m − 1) − (m : ℂ) ∗ a + (m : ℂ) ∗ ((m : ℂ) − 1) / 2) • φ ∈ W'

theorem **prod_coeff** {n : ℕ} (s : Finset (Fin n)) (χ : Fin n → ℂ) {m : ℕ} (hs : s.card = m) (hm : 1 ≤ m) : (∏ r ∈ s, (X + C (χ r))).coeff (m − 1) = ∑ r ∈ s, χ r

theorem **actL_apply** (E : Fin n → Fin n → Module.End ℂ M) (Y) : actL E Y = actM E Y

theorem **actL_single** (E : Fin n → Fin n → Module.End ℂ M) (a b : Fin n) (x : ℂ) : actL E (Matrix.single a b x) = x • E a b

theorem **act_single** (E : Fin n → Fin n → Module.End ℂ M) (a b : Fin n) : actM E (Matrix.single a b 1) = E a b

theorem **actL_comm** {E : Fin n → Fin n → Module.End ℂ M} (hE : GlRel E) (Y Z : Matrix (Fin n) (Fin n) ℂ) : actL E Y ∗ actL E Z − actL E Z ∗ actL E Y = actL E (Y ∗ Z − Z ∗ Y)

theorem **semiconj_aeval** {a b z : R} (h : a ∗ z = z ∗ b) (p : ℂ[X]) : aeval a p ∗ z = z ∗ aeval b p

theorem **Good.mul** {c : ℝ} {p q : ℂ[X]} (hp : Good c p) (hq : Good c q) : Good c (p ∗ q)

theorem **good_one** (c : ℝ) : Good c 1

theorem **good_lin** {c : ℝ} {z : ℂ} (h : z.re < c) : Good c (X − C z)

theorem **Good.shift** {c : ℝ} {p : ℂ[X]} (hp : Good c p) {k : ℝ} (hk : 0 ≤ k) : Good c (p.comp (X + C (k : ℂ)))

theorem **act_weight** (X : GMod n) {A Y : Matrix (Fin n) (Fin n) ℂ} {k : ℂ} (hw : A ∗ Y − Y ∗ A = k • Y) : actM X.E A ∗ actM X.E Y = actM X.E Y ∗ (actM X.E A + algebraMap ℂ _ k)

theorem **SS_le** (F : Filt X A) (c : ℝ) : SS F c ≤ W F

theorem **W_weight** (F : Filt X A) {Y : Matrix (Fin n) (Fin n) ℂ} {k : ℤ} (hw : A ∗ Y − Y ∗ A = (k : ℂ) • Y) {ψ : Module.Dual ℂ X.V} (hψ : ψ ∈ W F) : ψ ∘ₗ actM X.E Y ∈ W F

theorem **SS_weight** (F : Filt X A) (c : ℝ) {Y : Matrix (Fin n) (Fin n) ℂ} {k : ℤ} (hk : 0 ≤ k) (hw : A ∗ Y − Y ∗ A = (k : ℂ) • Y) {ψ : Module.Dual ℂ X.V} (hψ : ψ ∈ SS F c) : ψ ∘ₗ actM X.E Y ∈ SS F c

theorem **Wm_le** (F : Filt X A) (c : ℝ) : Wm F c ≤ SS F c

theorem **Wm_stab** (F : Filt X A) (c : ℝ) : Stab (Wm F c)

theorem **mat_sum** (B : GradBasis A) (M : Matrix (Fin n) (Fin n) ℂ) : M = ∑ p, ∑ q, (B.Ui * M * B.U) p q • Ym B p q

theorem **dz_mono** {a b : Fin 3} (h : a ≤ b) : dz a ≤ dz b

theorem **dv_eq** (i : Fin 3) : (![-1, 0, 1] : Fin 3 → ℂ) i = (dz i : ℂ)

theorem **conj_weight** {U Ui : Matrix (Fin n) (Fin n) ℂ} (hUi : Ui * U = 1) (d : Fin n → ℂ) (p q : Fin n) : (U * Matrix.diagonal d * Ui) * (U * Matrix.single p q 1 * Ui) - (U * Matrix.single p q 1 * Ui) * (U * Matrix.diagonal d * Ui) = (d p - d q) • (U * Matrix.single p q 1 * Ui)

theorem **Ym_weight** (B : GradBasis A) (p q : Fin n) : A * Ym B p q - Ym B p q * A = (((dz (bl B p) - dz (bl B q) : ℤ)) : ℂ) • Ym B p q

theorem **mem_Wm** (F : Filt X A) (B : GradBasis A) (c : ℝ) {φ : Module.Dual ℂ X.V} (hφ : φ ∈ SS F c) (hn : ∀ p q, bl B p < bl B q → φ ∘ₗ Eb X B p q ∈ Wm F c) : φ ∈ Wm F c

theorem **top_of_SS** (F : Filt X A) (c : ℝ) (hS : SS F c = W F) : F.N c = ⊤

theorem **tr_mem** (B : GradBasis A) (χ : Fin n → ℂ) (W' : Submodule ℂ (Module.Dual ℂ X.V)) {φ : Module.Dual ℂ X.V} {L : Fin n → Fin 3} (hL : IsLabel B.j L) (h : ∀ i Y, φ ∘ₗ Qb X B i Y - plab χ L i Y • φ ∈ W') (i : Fin 3) : φ ∘ₗ trB X B i - (∑ s ∈ univ.filter (fun s => L s = i), χ s + (dims n B.j i : ℂ) * (modExp (fun i => Fin (dims n B.j i)) i : ℂ) / 2) • φ ∈ W'

theorem **block_standard** (B : GradBasis A) (i : Fin 3) (k : Fin (dims n B.j i)) : bl B (stdBlocks (dims n B.j) B.hj (i, k)) = i

theorem **sum_blocks** (B : GradBasis A) {N : Type*} [AddCommMonoid N] (f : Fin n → N) : ∑ p, f p = ∑ k : Fin (dims n B.j 0), f (stdBlocks (dims n B.j) B.hj (0, k)) + (∑ k : Fin (dims n B.j 1), f (stdBlocks (dims n B.j) B.hj (1, k)) + ∑ k : Fin (dims n B.j 2), f (stdBlocks (dims n B.j) B.hj (2, k)))

theorem **A_eq** (B : GradBasis A) : A = ∑ p, (dz (bl B p) : ℂ) • Ym B p p

theorem **act_A** (X : GMod n) (B : GradBasis A) : actM X.E A = trB X B 2 - trB X B 0

theorem **rho_eq** (B : GradBasis A) : rhoA A = (B.j : ℝ) * ((n : ℝ) - B.j)

theorem **modExp0** (j : ℕ) (h : 2 * j ≤ n) : (modExp (fun i => Fin (dims n j i)) 0 : ℂ) = (n : ℂ) - j

theorem **modExp2** (j : ℕ) (h : 2 * j ≤ n) : (modExp (fun i => Fin (dims n j i)) 2 : ℂ) = -((n : ℂ) - j)

theorem **eig_mod** (B : GradBasis A) (χ : Fin n → ℂ) (W' : Submodule ℂ (Module.Dual ℂ X.V)) {φ : Module.Dual ℂ X.V} {L : Fin n → Fin 3} (hL : IsLabel B.j L) (h : ∀ i Y, φ ∘ₗ Qb X B i Y - plab χ L i Y • φ ∈ W') : φ ∘ₗ actM X.E A - (deg χ L - (rhoA A : ℂ)) • φ ∈ W'

theorem **degree** : CompanionTheory.RgExh.Dec.J.Degree

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.JacquetInitial

theorem **zop_mk** (S W' : Submodule ℂ (Module.Dual ℂ V)) (T : ɪ(stabD S ⊓ stabD W')) (x : ɪS) : zop S W' T (Submodule.Quotient.mk x) = Submodule.Quotient.mk ((T : Module.End ℂ V).dualMap x, (Algebra.mem_inf.1 T.2).1 _ x.2)

theorem **rho_mk** (S W' : Submodule ℂ (Module.Dual ℂ V)) (T : ɪ(stabD S ⊓ stabD W')) (x : ɪS) : rho S W' T (Submodule.Quotient.mk x) = Submodule.Quotient.mk ((T : Module.End ℂ V).dualMap x, (Algebra.mem_inf.1 T.2).1 _ x.2)

theorem **rho_mul** (S W' : Submodule ℂ (Module.Dual ℂ V)) (T T' : ɪ(stabD S ⊓ stabD W')) : rho S W' (T * T') = rho S W' T' * rho S W' T

theorem **rho_one** (S W' : Submodule ℂ (Module.Dual ℂ V)) : rho S W' 1 = 1

theorem **rho_pow** (S W' : Submodule ℂ (Module.Dual ℂ V)) (T : ɪ(stabD S ⊓ stabD W')) (m : ℕ) : rho S W' (T ^ m) = rho S W' T ^ m

theorem **rho_eq_zero** (S W' : Submodule ℂ (Module.Dual ℂ V)) (T : ɪ(stabD S ⊓ stabD W')) (h : ∀ φ ∈ S, (T : Module.End ℂ V).dualMap φ ∈ W') : rho S W' T = 0

theorem **coef_mem** {M : Type*} [AddCommGroup M] [Module ℂ M] (P : Submodule ℂ M) (l : ℕ) (c : ℕ → M) (h : ∀ t : ℂ, ∑ j ∈ Finset.range l, t ^ j • c j ∈ P) (j : ℕ) (hj : j < l) : c j ∈ P

theorem **qdet_mem** {A : Type} [Ring A] [Algebra ℂ A] (B : Subalgebra ℂ A) {m : ℕ} (E : Fin m → Fin m → A) (hE : ∀ a b, E a b ∈ B) (u : ℂ) : qdet E u ∈ B

theorem **dual_map_zero_mem** {V : Type} [AddCommGroup V] [Module ℂ V] (P : Submodule ℂ (Module.Dual ℂ V)) (φ : Module.Dual ℂ V) : (0 : Module.End ℂ V).dualMap φ ∈ P

theorem **actM_mem_stabD** (X : GMod n) (P : Submodule ℂ (Module.Dual ℂ X.V)) (hst : ∀ a b, ∀ φ ∈ P, contraE X a b φ ∈ P) (Y : Matrix (Fin n) (Fin n) ℂ) : actM X.E Y ∈ stabD P

theorem **dZ_cast** (i : Fin 3) : ((dZ i : ℤ) : ℂ) = (![-1, 0, 1] : Fin 3 → ℂ) i

theorem **dZ_lt** : ∀ i j : Fin 3, i < j → dZ i - dZ j ≤ -1

theorem **ad_unit** (B : GradBasis A) (p q : Fin n) : A * (B.U * Matrix.single p q 1 * B.Ui) - (B.U * Matrix.single p q 1 * B.Ui) * A = ((dZ (bl B p) - dZ (bl B q) : ℤ) : ℂ) • (B.U * Matrix.single p q 1 * B.Ui)

theorem **shift_Eb** (F : Filt X A) (B : GradBasis A) (p q : Fin n) (s : ℝ) (u : X.V) (hu : u ∈ F.N s) : Eb X B p q u ∈ F.N (s + ((dZ (bl B p) - dZ (bl B q) : ℤ) : ℝ))

theorem **exists_killed** (F : Filt X A) (B : GradBasis A) (W' : Submodule ℂ (Module.Dual ℂ X.V)) (hle : W' ≤ W F) (hne : W' ≠ W F) (s0 : ℝ) (hs0 : F.N s0 = ⊤) : ∃ φ ∈ W F, φ ∉ W' ∧ ∀ p q, bl B p < bl B q → (Eb X B p q).dualMap φ ∈ W'

theorem **levi_stab** (F : Filt X A) (B : GradBasis A) (W' : Submodule ℂ (Module.Dual ℂ X.V)) (hst : ∀ a b, ∀ φ ∈ W', contraE X a b φ ∈ W') (p q : Fin n) (hpq : bl B p = bl B q) : Eb X B p q ∈ stabD (Sk F B W')

theorem **nil_key** (F : Filt X A) (B : GradBasis A) (W' : Submodule ℂ (Module.Dual ℂ X.V)) (hst : ∀ a b, ∀ φ ∈ W', contraE X a b φ ∈ W') (χ : Fin n → ℂ) (k : ℕ) (Y : ℂ) (hk : (qdet X.E (Y + ((n : ℂ) - 1) / 2) - Formalism.QuantumDeterminants.capP χ Y • (1 : Module.End ℂ X.V)) ^ k = 0) : ∀ φ ∈ Sk F B W', (((List.ofFn fun i : Fin 3 => Qb X B i Y).prod - Formalism.QuantumDeterminants.capP χ Y • (1 : Module.End ℂ X.V)) ^ k).dualMap φ ∈ W'

theorem **rhoA_eq** {n : ℕ} {A : Matrix (Fin n) (Fin n) ℂ} (B : GradBasis A) : rhoA A = (B.j : ℝ) * ((n : ℝ) - B.j)

theorem **aeval_dual_mem** {V : Type} [AddCommGroup V] [Module ℂ V] (W' : Submodule ℂ (Module.Dual ℂ V)) (T : Module.End ℂ V) (hT : T ∈ stabD W') (φ : Module.Dual ℂ V) (z : ℂ) (hz : T.dualMap φ - z • φ ∈ W') (r : ℂ[X]) : (aeval T r).dualMap φ - r.eval z • φ ∈ W'

theorem **re_lt** {n : ℕ} {M : GMod n} {A : Matrix (Fin n) (Fin n) ℂ} (F : Filt M A) (W' : Submodule ℂ (Module.Dual ℂ M.V)) (φ : Module.Dual ℂ M.V) (z : ℂ) (s0 : ℝ) (hLF : LocFinA F) (hs0 : F.N s0 = ⊤) (hst : ∀ a b, ∀ ψ ∈ W', contraE M a b ψ ∈ W') (hφ : φ ∈ W F) (hn : φ ∉ W') (hz : φ ∘ₗ actM M.E A - z • φ ∈ W') : z.re < s0

theorem **label_of_prod** {ι : Type} [DecidableEq ι] (χ : ι → ℂ) (T : Finset ι) : ∀ g : Fin 3 → ℂ[X], (∀ i, (g i).Monic) → ∏ i, g i = ∏ s ∈ T, (X + C (χ s)) → ∃ L : ι → Fin 3, ∀ i, g i = ∏ s ∈ T.filter (fun s => L s = i), (X + C (χ s))

theorem **aeval_mem** (I : Submodule ℂ Z) (T : Module.End ℂ Z) (hI : ∀ x ∈ I, T x ∈ I) (p : ℂ[X]) (x : Z) (hx : x ∈ I) : aeval T p x ∈ I

theorem **exists_eig_of_poly** (I : Submodule ℂ Z) (T : Module.End ℂ Z) (hI : ∀ x ∈ I, T x ∈ I) : ∀ (m : ℕ) (q : ℂ[X]), q.natDegree = m → q ≠ 0 → ∀ x ∈ I, x ≠ 0 → aeval T q x = 0 → ∃ r : ℂ, ∃ y ∈ I, y ≠ 0 ∧ T y = r • y

theorem **exists_joint_eig** {ι : Type} [DecidableEq ι] (T : ι → Module.End ℂ Z) (hc : ∀ i j, Commute (T i) (T j)) (s : Finset ι) (hi : ∀ i ∈ s, IsIntegral ℂ (T i)) : ∀ I : Submodule ℂ Z, I ≠ ⊥ → (∀ j, ∀ x ∈ I, T j x ∈ I) → ∃ v ∈ I, v ≠ 0 ∧ ∀ i ∈ s, ∃ θ : ℂ, T i v = θ • v

theorem **natDegree_sumPoly** (θ : ℕ → ℂ) (m : ℕ) (h : θ m = 1) : (sumPoly θ m).natDegree = m

theorem **coeff_sumPoly_comp** (θ : ℕ → ℂ) (e : ℕ) (h : θ (e + 1) = 1) (c : ℂ) : ((sumPoly θ (e + 1)).comp (X + C c)).coeff e = θ e + ((e : ℂ) + 1) * c

theorem **joint_label** {n : ℕ} [Nontrivial Z] (χ : Fin n → ℂ) (d : Fin 3 → ℕ) (sh : Fin 3 → ℂ) (a : Fin 3 → ℕ → Module.End ℂ Z) (ha1 : ∀ i, a i (d i) = 1) (hcomm : ∀ i j k l, Commute (a i k) (a j l)) (hnil : ∀ Y : ℂ, ∃ m : ℕ, ((∑ k ∈ Finset.range (d 0 + 1), (Y + sh 0) ^ k • a 0 k) * (∑ k ∈ Finset.range (d 1 + 1), (Y + sh 1) ^ k • a 1 k) * (∑ k ∈ Finset.range (d 2 + 1), (Y + sh 2) ^ k • a 2 k) - (∏ s, (Y + χ s)) • (1 : Module.End ℂ Z)) ^ m = 0) : ∃ (v : Z) (L : Fin n → Fin 3), v ≠ 0 ∧ (∀ i, (Finset.univ.filter fun s => L s = i).card = d i) ∧ (∀ i (Y : ℂ), (∑ k ∈ Finset.range (d i + 1), (Y + sh i) ^ k • a i k) v = (∏ s ∈ Finset.univ.filter (fun s => L s = i), (Y + χ s)) • v) ∧ ∀ i, 1 ≤ d i → a i (d i - 1) v = (∑ s ∈ Finset.univ.filter (fun s => L s = i), χ s - (d i : ℂ) * sh i) • v

theorem **modExp_zero** (d : Fin 3 → ℕ) : modExp (fun i => Fin (d i)) 0 = (d 1 : ℤ) + d 2

theorem **modExp_two** (d : Fin 3 → ℕ) : modExp (fun i => Fin (d i)) 2 = -((d 0 : ℤ) + d 1)

theorem **degree_eq** (n j : ℕ) (hj : ∑ i, dims n j i = n) (c0 c2 : ℂ) : (-(c0 - (dims n j 0 : ℂ) * (((dims n j 0 : ℂ) - 1) / 2 - (modExp (fun i => Fin (dims n j i)) 0 : ℂ) / 2) + (dims n j 0 : ℂ) * ((dims n j 0 : ℂ) - 1) / 2) + (c2 - (dims n j 2 : ℂ) * (((dims n j 2 : ℂ) - 1) / 2 - (modExp (fun i => Fin (dims n j i)) 2 : ℂ) / 2) + (dims n j 2 : ℂ) * ((dims n j 2 : ℂ) - 1) / 2)) = c2 - c0 - (j : ℂ) * ((n : ℂ) - j)

theorem **Qb_eq** (X : GMod n) {A : Matrix (Fin n) (Fin n) ℂ} (B : GradBasis A) (i : Fin 3) (Y : ℂ) : Qb X B i Y = qdet (Xb X B i) (Y + shB n B.j i)

theorem **exists_coefs** (X : GMod n) {A : Matrix (Fin n) (Fin n) ℂ} (B : GradBasis A) : ∃ a : Fin 3 → ℕ → Module.End ℂ X.V, (∀ i, a i (dims n B.j i) = 1) ∧ (∀ i (u : ℂ), qdet (Xb X B i) u = ∑ k ∈ Finset.range (dims n B.j i + 1), u ^ k • a i k) ∧ (∀ i, 1 ≤ dims n B.j i → a i (dims n B.j i - 1) = ∑ p, Xb X B i p p - ((dims n B.j i : ℂ) * ((dims n B.j i : ℂ) - 1) / 2) • 1) ∧ ∀ i j k l, Commute (a i k) (a j l)

theorem **start1** : Start1

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theorem **aeval_mul_of_intertwine** (T Y : Module.End ℂ V) (c : ℂ) (h : T * Y = Y * (T + algebraMap ℂ _ c)) (f : ℂ[X]) : aeval T f * Y = Y * aeval (T + algebraMap ℂ _ c) f

theorem **actM_intertwine** {E : Fin n → Fin n → Module.End ℂ V} (hE : Formalism.REAL.GlRel E) {A Y : Matrix (Fin n) (Fin n) ℂ} {c : ℂ} (h : A * Y - Y * A = c • Y) : actM E A * actM E Y = actM E Y * (actM E A + algebraMap ℂ _ c)

theorem **aeval_shift** (T : Module.End ℂ V) (c d : ℂ) (h : c + d = 0) (f : ℂ[X]) : aeval (T + algebraMap ℂ _ c) (f.comp (X + C d)) = aeval T f

theorem **step_down** {M : GMod n} {A : Matrix (Fin n) (Fin n) ℂ} (F : Filt M A) (p q : ℂ[X]) (hnil : ∀ u : M.V, ∃ (m : ℕ) (Y : Fin m → Matrix (Fin n) (Fin n) ℂ) (v : Fin m → M.V), (∀ k, Y k ∈ L.nilpC A) ∧ aeval (actM M.E A) p u = ∑ k, actM M.E (Y k) (v k)) (s : ℝ) (hq : ∀ u : M.V, aeval (actM M.E A) q u ∈ F.N s) (u : M.V) : aeval (actM M.E A) (q.comp (X + C 1) * q.comp (X + C 2) * p) u ∈ F.N (s - 1)

theorem **exists_monic_descend** {M : GMod n} {A : Matrix (Fin n) (Fin n) ℂ} (F : Filt M A) (p : ℂ[X]) (hp : p.Monic) (hnil : ∀ u : M.V, ∃ (m : ℕ) (Y : Fin m → Matrix (Fin n) (Fin n) ℂ) (v : Fin m → M.V), (∀ k, Y k ∈ L.nilpC A) ∧ aeval (actM M.E A) p u = ∑ k, actM M.E (Y k) (v k)) (s : ℝ) (hs : F.N s = ⊤) (K : ℕ) : ∃ q : ℂ[X], q.Monic ∧ ∀ u : M.V, aeval (actM M.E A) q u ∈ F.N (s - K)

theorem **locFin** : LocFin

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theorem **coeff_mem_span** (G : B[X]) (i : ℕ) : G.coeff i ∈ Submodule.span ℂ (Set.range fun u : ℂ => G.eval (algebraMap ℂ B u))

theorem **isNilpotent_aeval_of_resolvent** {p f : ℂ → B} (hp : IsPolyFun p) (hf : IsPolyFun f) (Y : B) (c : ℂ[X]) (h : ∀ u, p u * (algebraMap ℂ B u - Y) = f u) (hnil : ∀ u, IsNilpotent (f u - algebraMap ℂ B (c.eval u))) (hcomm : ∀ u v, Commute (f u) (f v)) (hY : ∀ u, Commute (f u) Y) : IsNilpotent (aeval Y c)

theorem **ker_aeval_prod_le** (T : Module.End ℂ W) (S : Finset ℂ) (L : ℕ) : LinearMap.ker (aeval T (∏ z ∈ S, (X - C z) ^ L)) ≤ ⨆ z ∈ S, T.maxGenEigenspace z

theorem **iSup_maxGen_eq_top** {ι : Type*} [Fintype ι] (T : Module.End ℂ W) (a : ι → ℂ) (K : ℕ) (h : (aeval T (∏ s, (X - C (a s)))) ^ K = 0) : ⨆ s, T.maxGenEigenspace (a s) = ⊤

theorem **colOp_apply** (M : Fin n → Fin n → Module.End ℂ V) (w : Fin n → V) (i : Fin n) : colOp M w i = ∑ j, M i j (w j)

theorem **colOp_mul** (M N : Fin n → Fin n → Module.End ℂ V) : colOp M * colOp N = colOp (fun a d => ∑ b, M a b * N b d)

theorem **diagHom_apply** (T : Module.End ℂ V) (w : Fin n → V) (i : Fin n) : diagHom T w i = T (w i)

theorem **colOp_eq_sum** (M : Fin n → Fin n → Module.End ℂ V) : colOp M = ∑ a, ∑ b, diagHom (M a b) * eOp V a b

theorem **isPoly_colOp** (M : ℂ → Fin n → Fin n → Module.End ℂ V) (h : ∀ a b, IsPolyFun fun u => M u a b) : IsPolyFun fun u => colOp (M u)

theorem **colOp_diag** (T : Module.End ℂ V) : colOp (fun i l : Fin n => if i = l then T else 0) = diagHom T

theorem **glRel_negT** {A : Type*} [Ring A] {n : ℕ} {E : Fin n → Fin n → A} (hE : GlRel E) : GlRel (fun a b => -E b a)

theorem **commute_qdet** {A : Type*} [Ring A] [Algebra ℂ A] {n : ℕ} {E : Fin n → Fin n → A} {z : A} (h : ∀ a b, Commute z (E a b)) (u : ℂ) : Commute z (qdet E u)

theorem **colOp_Tm** (M : GMod n) (v : ℂ) : colOp (Tm (negT M) v) = algebraMap ℂ _ v - om M

theorem **resolvent** {m : ℕ} (M : GMod (m + 1)) (u : ℂ) : colOp (coM M (u + m)) * (algebraMap ℂ _ u - om M) = diagHom (qdet (negT M) (u + m))

theorem **commute_diag_om** (M : GMod n) (T : Module.End ℂ M.V) (h : ∀ a b, Commute T (M.E a b)) : Commute (diagHom T) (om M)

theorem **isNilpotent_om** (m : ℕ) (M : GMod (m + 1)) (χ : Fin (m + 1) → ℂ) (h : HasChar M χ) : IsNilpotent (aeval (om M) (∏ s, (X - C (χ s - (((m + 1 : ℕ) : ℂ) - 1) / 2))))

theorem **tensDecomp** : CompanionTheory.RgExh.Dec.J.TensDecomp

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theorem **plab_equiv** (ψ : Fin n → ℂ) (L : Fin n → Fin 3) (k : Fin 3) {m : ℕ} (e : Fin m ≃ {t // L t = k}) (Z : ℂ) : plab ψ L k Z = ∏ a, (Z + ψ (e a))

theorem **prod_plab** (ψ : Fin n → ℂ) (L : Fin n → Fin 3) (Y : ℂ) : ∏ i, plab ψ L i Y = Formalism.QuantumDeterminants.capP ψ Y

theorem **plab_swap** (χ : Fin n → ℂ) (L : Fin n → Fin 3) (r s : Fin n) (h : χ r = χ s) (i : Fin 3) (Y : ℂ) : plab (χ + Pi.single s 1) (L ∘ Equiv.swap r s) i Y = plab (χ + Pi.single r 1) L i Y

theorem **isLabel_swap** {j : ℕ} {L : Fin n → Fin 3} (h : IsLabel j L) (r s : Fin n) : IsLabel j (L ∘ Equiv.swap r s)

theorem **entry_eq_of_capP** (χ : Fin n → ℂ) (r s : Fin n) (h : ∀ Y, Formalism.QuantumDeterminants.capP (χ + Pi.single r 1) Y = Formalism.QuantumDeterminants.capP (χ + Pi.single s 1) Y) : χ r = χ s

theorem **glRel_inj** {A : Type*} [Ring A] {m : ℕ} {E : Fin n → Fin n → A} (hE : GlRel E) {ι : Fin m → Fin n} (hι : Function.Injective ι) : GlRel (fun a b => E (ι a) (ι b))

theorem **block_raise** (hS : StepChar) (hT : TensDecomp) (P Q : GMod n) (d : Fin 3 → ℕ) (h : ∑ i, d i = n) (θ : Fin n → P.V →ₗ[ℂ] Q.V) (hθ : ∀ a b p x, Q.E a b (θ p x) = θ p (P.E a b x) + if p = b then θ a x else 0) (χ : Fin n → ℂ) (L : Fin n → Fin 3) (hL : ∀ i, (univ.filter fun s => L s = i).card = d i) (v : P.V) (hv : v ∈ leviSub P d h χ L) (p0 : Fin n) (hne : θ p0 v ≠ 0) : ∃ w : Q.V, w ≠ 0 ∧ (∀ a b, blk (stdBlocks d h) b < blk (stdBlocks d h) a → Q.E a b w = 0) ∧ ∃ r, ∀ i Y, Qd Q d h i Y w = plab (χ + Pi.single r 1) L i Y • w

theorem **dualL_one** (V : Type) [AddCommGroup V] [Module ℂ V] : dualL V 1 = 1

theorem **dualL_mul** (V : Type) [AddCommGroup V] [Module ℂ V] (x y : Module.End ℂ V) : dualL V (x * y) = dualL V y * dualL V x

theorem **dualL_pow** (V : Type) [AddCommGroup V] [Module ℂ V] (x : Module.End ℂ V) (k : ℕ) : dualL V (x ^ k) = dualL V x ^ k

theorem **qdet_tr** (Y : GMod n) {m : ℕ} (ι : Fin m → Fin n) (hι : Function.Injective ι) (u : ℂ) : qdet (fun a b => (trMod Y).E (ι a) (ι b)) u = (qdet (fun a b => Y.E (ι a) (ι b)) u).dualMap

theorem **Qd_tr** (Y : GMod n) (d : Fin 3 → ℕ) (h : ∑ i, d i = n) (i : Fin 3) (c : ℂ) : Qd (trMod Y) d h i c = (Qd Y d h i c).dualMap

theorem **hasChar_tr** (Y : GMod n) {χ : Fin n → ℂ} (hY : HasChar Y χ) : HasChar (trMod Y) χ

theorem **sqMk_surj** : Function.Surjective (sqMk Y S1 S2 h1 h2)

theorem **sqMk_zero** (x : S1) : sqMk Y S1 S2 h1 h2 x = 0 ↔ (x : Module.Dual ℂ Y.V) ∈ S2

theorem **sq_E** (a b : Fin n) (x : S1) : (sqMod Y S1 S2 h1 h2).E a b (sqMk Y S1 S2 h1 h2 x) = sqMk Y S1 S2 h1 h2 ⟨(Y.E b a).dualMap x, h1 b a _ x.2⟩

theorem **sq_Qd** (d : Fin 3 → ℕ) (h : ∑ i, d i = n) (i : Fin 3) (c : ℂ) (x : S1) : ∃ y : S1, (y : Module.Dual ℂ Y.V) = (Qd Y d h i c).dualMap x ∧ Qd (sqMod Y S1 S2 h1 h2) d h i c (sqMk Y S1 S2 h1 h2 x) = sqMk Y S1 S2 h1 h2 y

theorem **eigen_pow** {V : Type} [AddCommGroup V] [Module ℂ V] (T : Module.End ℂ V) (w : V) (C D : ℂ) (h : T w = C • w) (k : ℕ) : ((T - D • (1 : Module.End ℂ V)) ^ k) w = (C - D) ^ k • w

theorem **sub_raise** (hS : StepChar) (hT : TensDecomp) (Y Y' : GMod n) (d : Fin 3 → ℕ) (h : ∑ i, d i = n) (S1 S2 : Submodule ℂ (Module.Dual ℂ Y.V)) (T1 T2 : Submodule ℂ (Module.Dual ℂ Y'.V)) (hS1 : ∀ a b, ∀ φ ∈ S1, (Y.E a b).dualMap φ ∈ S1) (hS2 : ∀ a b, ∀ φ ∈ S2, (Y.E a b).dualMap φ ∈ S2) (hT1 : ∀ a b, ∀ φ ∈ T1, (Y'.E a b).dualMap φ ∈ T1) (hT2 : ∀ a b, ∀ φ ∈ T2, (Y'.E a b).dualMap φ ∈ T2) (θ : Fin n → Module.Dual ℂ Y.V →ₗ[ℂ] Module.Dual ℂ Y'.V) (hθ : ∀ a b p φ, (Y'.E b a).dualMap (θ p φ) = θ p ((Y.E b a).dualMap φ) + if p = b then θ a φ else 0) (hθ1 : ∀ p, ∀ φ ∈ S1, θ p φ ∈ T1) (hθ2 : ∀ p, ∀ φ ∈ S2, θ p φ ∈ T2) (χ : Fin n → ℂ) (L : Fin n → Fin 3) (hL : ∀ i, (univ.filter fun s => L s = i).card = d i) (φ : Module.Dual ℂ Y.V) (hφ : φ ∈ S1) (hk : ∀ p q, blk (stdBlocks d h) p < blk (stdBlocks d h) q → (Y.E p q).dualMap φ ∈ S2) (hq : ∀ i c, (Qd Y d h i c).dualMap φ - plab χ L i c • φ ∈ S2) (p0 :

theorem **coordB_apply** (p : Fin n) (u : (step X z).V) : coordB X B z p u = ∑ a, B.Ui p a • comp0 X z a u

theorem **component_zero_matrix_unit** (i a b : Fin n) (u : (step X z).V) : comp0 X z i ((step X z).E a b u) = X.E a b (comp0 X z i u) + if i = a then comp0 X z b u else 0

theorem **component_zero_matrix_action** (i : Fin n) (Y : Matrix (Fin n) (Fin n) ℂ) (u : (step X z).V) : comp0 X z i (actM (step X z).E Y u) = actM X.E Y (comp0 X z i u) + ∑ b, Y i b • comp0 X z b u

theorem **coordB_Eb** (p a b : Fin n) (u : (step X z).V) : coordB X B z p (Eb (step X z) B b a u) = Eb X B b a (coordB X B z p u) + if p = b then coordB X B z a u else 0

theorem **grad_shift** {A : Matrix (Fin n) (Fin n) ℂ} (B : GradBasis A) (p q : Fin n) : A * (B.U * Matrix.single p q 1 * B.Ui) - (B.U * Matrix.single p q 1 * B.Ui) * A = ((dI (bl B p) - dI (bl B q) : ℤ) : ℂ) • (B.U * Matrix.single p q 1 * B.Ui)

theorem **W_Eb** (F : Filt X A) (B : GradBasis A) (p q : Fin n) {φ : Module.Dual ℂ X.V} (hφ : φ ∈ W F) : (Eb X B p q).dualMap φ ∈ W F

theorem **E_eq_sum** (B : GradBasis A) (a b : Fin n) : X.E a b = ∑ p, ∑ q, (B.Ui p a * B.U b q) • Eb X B p q

theorem **W_E** (F : Filt X A) (B : GradBasis A) (a b : Fin n) {φ : Module.Dual ℂ X.V} (hφ : φ ∈ W F) : (X.E a b).dualMap φ ∈ W F

theorem **stable_act** (X : GMod n) (T : Submodule ℂ (Module.Dual ℂ X.V)) (hT : ∀ a b, ∀ φ ∈ T, contraE X a b φ ∈ T) (Y : Matrix (Fin n) (Fin n) ℂ) {φ : Module.Dual ℂ X.V} (hφ : φ ∈ T) : (actM X.E Y).dualMap φ ∈ T

theorem **hasChar_eb** (X : GMod n) {A : Matrix (Fin n) (Fin n) ℂ} (B : GradBasis A) {χ : Fin n → ℂ} (h : HasChar X χ) : HasChar (ebMod X B) χ

theorem **exists_ext** {V : Type} [AddCommGroup V] [Module ℂ V] (S M : Submodule ℂ V) (ψ : Module.Dual ℂ S) (h : ∀ u : S, (u : V) ∈ M → ψ u = 0) : ∃ Ψ : Module.Dual ℂ V, (∀ u : S, Ψ u = ψ u) ∧ ∀ x ∈ M, Ψ x = 0

theorem **transport** : Transport

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theorem **kExp** : CompanionTheory.D.DSIC.KExp

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.KHighestOfPerm

theorem **kHighest_of_perm** {V : Type} [AddCommGroup V] [Module ℂ V] {ε : Fin n → Bool} {s : Equiv.Perm (Fin n)} (hs : ∀ a b, a < b → ε (s a) = ε (s b) → s a < s b) {E : Fin n → Fin n → Module.End ℂ V} {μ : Fin n → ℝ} {x : V} (h : KHighest (ε ∘ s) (fun a b => E (s a) (s b)) μ x) : KHighest ε E (μ ∘ s.symm) x

theorem **kHighest_perm** {V : Type} [AddCommGroup V] [Module ℂ V] {ε : Fin n → Bool} {s : Equiv.Perm (Fin n)} (hs : ∀ a b, a < b → ε (s a) = ε (s b) → s a < s b) {E : Fin n → Fin n → Module.End ℂ V} {μ : Fin n → ℝ} {x : V} (h : KHighest ε E (μ ∘ s.symm) x) : KHighest (ε ∘ s) (fun a b => E (s a) (s b)) μ x

theorem **dsOfData** : DSOfData

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.KHighestRenumber

theorem **kHighest_renumber** {V : Type} [AddCommGroup V] [Module ℂ V] (ε : Fin n → Bool) (E : Fin n → Fin n → Module.End ℂ V) {s s' : Equiv.Perm (Fin n)} (hs : ∀ a b, a < b → ε (s a) = ε (s b) → s a < s b) (hs' : ∀ a b, a < b → ε (s' a) = ε (s' b) → s' a < s' b) {Λ : Fin n → ℝ} {v : V} (h : KHighest (ε ∘ s') (fun a b => E (s' a) (s' b)) Λ v) : KHighest (ε ∘ s) (fun a b => E (s a) (s b)) (Λ ∘ s'.symm ∘ s) v

theorem **dataUniq** : CompanionTheory.RgExh.Fin.DataUniq

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.KModOf

theorem **kModOf** : CompanionTheory.RgExh.Fin.KModOf

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.KTrivModelOfTrivLocus

theorem **evc_lam** {L : LB G.left} (hL : L.IsLB) (hP : (abelian₂ G L).IsLB) {Y : Over S} (a c : Y ⟶ G) : evc hP a c = lam (cls L hL) a c

theorem **lam_eq** (hφ : IsGrpPullback G' G b φ) {L : LB G.left} {L' : LB G'.left} (hL : L.IsLB) (hL' : L'.IsLB) (e : LBIso L' (L.pull φ)) {X : Scheme.{0}} {s : X ⟶ S'} {s₀ : X ⟶ S} (hs : s ≫ b = s₀) (x y : Over.mk s ⟶ G') (x₀ y₀ : Over.mk s₀ ⟶ G) (hx : x.left ≫ φ = x₀.left) (hy : y.left ≫ φ = y₀.left) : lam (cls L' hL') x y = lam (cls L hL) x₀ y₀

theorem **triv_lam_iff** (hφ : IsGrpPullback G' G b φ) {L : LB G.left} {L' : LB G'.left} (hL : L.IsLB) (hL' : L'.IsLB) (e : LBIso L' (L.pull φ)) (T : Over S') (y : T ⟶ G') : Triv (ev (abelian₂ G' L') (fst G' T) (snd G' T ≫ y)) ↔ Triv (ev (abelian₂ G L) (fst G (Over.mk (T.hom ≫ b))) (snd G (Over.mk (T.hom ≫ b)) ≫ pushP hφ (s

theorem **exists_fg_fst_eq** {k V : Type} [CommRing k] [CommRing V] [Algebra k V] {X : Scheme.{0}} (p : X ⟶ Sp k) [QuasiCompact p] [LocallyOfFiniteType p] (e : Sp k ⟶ X) (he : e ≫ p = 𝟙 _) (h : pullback.fst p (spMap (algebraMap k V)) = pullback.fst p (spMap (algebraMap k V)) ≫ p ≫ e) : ∃ R : Subalgebra k V, R.FG ∧ pullback.fst p (spMap (algebraMap k R)) = pullback.fst p (spMap (algebraMap k R)) ≫ p ≫ e

theorem **kTriv_of_ker** (hA : IsAbelianScheme A) {L : LB A.left} (hL : L.IsLB) (H : ∀ (T : Over S) (y : T ⟶ A), Triv (ev (abelian₂ A L) (fst A T) (snd A T ≫ y)) → y = 1) : Out.KTriv A L

theorem **exists_model** (R : Type) [CommRing R] (A : Over (Sp R)) [GrpObj A] (g : ℕ) (hA : IsAbelianScheme A) (hg : SmoothOfRelativeDimension g A.hom) (L : LB A.left) (hL : L.IsLB) : ∃ (k : Type) (_ : CommRing k) (_ : IsNoetherianRing k) (_ : Algebra k R) (B : Over (Sp k)) (_ : GrpObj B) (φ : A.left ⟶ B.left) (M : LB B.left), IsAbelianScheme B ∧ SmoothOfRelativeDimension g B.hom ∧ IsGrpPullback A B (spMap (algebraMap k R)) φ ∧ M.IsLB ∧ LBIso L (M.pull φ)

theorem **kTrivModel_of_trivLocus** : TrivLocus → KTrivModel

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.KhwExists

theorem **gl_exists** (n : ℕ) (ν : Fin n → ℤ) (hν : Antitone ν) : ∃ (U : Type) (_ : NormedAddCommGroup U) (_ : InnerProductSpace ℂ U) (_ : FiniteDimensional ℂ U) (E : Fin n → Fin n → Module.End ℂ U) (u : U), GlRel E ∧ (∀ a b x y, ⟪E a b x, y⟫_ℂ = ⟪x, E b a y⟫_ℂ) ∧ u ≠ 0 ∧ IsHW E (fun a => (ν a : ℂ)) u

theorem **reorder** (n : ℕ) (ε : Fin n → Bool) (γ : Fin n → ℤ) (hγ : ∀ c d, c < d → ε c = ε d → γ d ≤ γ c) : ∃ (σ : Fin n ≃ Fin n) (N : ℤ), (∀ c d, c < d → ε c = ε d → σ.symm c < σ.symm d) ∧ Antitone fun i => γ (σ i) + if ε (σ i) then N else 0

theorem **khwExists** : CompanionTheory.D.DSIC.KHWExists

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.KsetSubsetUjset

theorem **kset_subset_ujset** {n : ℕ} {ε : Fin n → Bool} {g : Matrix (Fin n) (Fin n) ℂ} (hg : g ∈ Kset ε) : g ∈ UJset ε

theorem **typeOfHW** (hL : LowSpan) (hS : StabLink) (hW : WtLink) : TypeOfHW

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.KummerLiftedCocycleImageEqPicardClass

theorem **sum_nc** {R : Type} [AddCommMonoid R] (d b : Fin n) (g : Fin n → Fin n → R) : (∑ j : NC ε, if d = j.1.1 ∧ b = j.1.2 then g j.1.1 j.1.2 else 0) = if ε d = true ∧ ε b = false then g d b else 0

theorem **inner_sum** (huY : ∀ a c j, u a c * Y j - Y j * u a c = -(if a = j.1.1 ∧ c = j.1.2 then 1 else 0)) (hvY : ∀ a c j, v a c * Y j - Y j * v a c = if a = j.1.1 ∧ c = j.1.2 then 1 else 0) (i : NC ε) (c d e f : Fin n) : (∑ j : NC ε, ((u i.1.1 c * v d i.1.2) * Y j - Y j * (u i.1.1 c * v d i.1.2)) * (u j.1.1 e * v f j.1.2)) = u i.1.1 c * (if ε d = true then u d e else 0) * v f i.1.2 - v d i.1.2 * u i.1.1 e * (if ε c = false then v f c else 0)

theorem **glRel_abstract** (hYY : ∀ i j, Commute (Y i) (Y j)) (huu : ∀ a b c d, Commute (u a b) (u c d)) (huv : ∀ a b c d, Commute (u a b) (v c d)) (hvv : ∀ a b c d, Commute (v a b) (v c d)) (huY : ∀ a c j, u a c * Y j - Y j * u a c = -(if a = j.1.1 ∧ c = j.1.2 then 1 else 0)) (hvY : ∀ a c j, v a c * Y j - Y j * v a c = if a = j.1.1 ∧ c = j.1.2 then 1 else 0) (hs : ∀ d e, (if ε d = true then u d e else 0) + (if ε e = false then v d e else 0) = if d = e then 1 else 0) (c d e f : Fin n) : (∑ i : NC ε, Y i * (u i.1.1 c * v d i.1.2)) * (∑ i : NC ε, Y i * (u i.1.1 e * v f i.1.2)) - (∑ i : NC ε, Y i * (u i.1.1 e * v f i.1.2)) * (∑ i : NC ε, Y i * (u i.1.1 c * v d i.1.2)) = (if d = e then ∑ i : NC ε, Y i * (u i.1.1 c * v f i.1.2) else 0) - (if f = c then ∑ i : NC ε, Y i * (u i.1.1 e * v d i.1.2) else 0)

theorem **pderiv_comm'** {σ : Type} (i j : σ) (f : MvPolynomial σ ℂ) : pderiv i (pderiv j f) = pderiv j (pderiv i f)

theorem **dd_comm** (a b c d : Fin n) : Commute (dd ε a b) (dd ε c d)

theorem **dd_yy** (a c : Fin n) (j : NC ε) : dd ε a c * yy j - yy j * dd ε a c = if a = j.1.1 ∧ c = j.1.2 then 1 else 0

theorem **verma** : CompanionTheory.D.DSIC.Verma

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.LabelTracking

theorem **tie_step** {χ₀ χ : Fin n → ℂ} {t : Fin n} (hT : ∀ r s, r ≠ s → χ r = χ s → χ r = χ₀ r ∧ χ s = χ₀ s) (hg : ∀ r, χ r ≠ χ t + 1) : ∀ r s, r ≠ s → (χ + Pi.single t 1 : Fin n → ℂ) r = (χ + Pi.single t 1 : Fin n → ℂ) s → (χ + Pi.single t 1 : Fin n → ℂ) r = χ₀ r ∧ (χ + Pi.single t 1 : Fin n → ℂ) s = χ₀ s

theorem **apply_swap_of_eq** (χ : Fin n → ℂ) {r s : Fin n} (h : χ r = χ s) (i : Fin n) : χ i = χ (Equiv.swap r s i)

theorem **sum_filter_swap** (χ : Fin n → ℂ) (L : Fin n → Fin 3) {r s : Fin n} (h : χ r = χ s) (a : Fin 3) : ∑ i ∈ univ.filter (fun i => (L ∘ Equiv.swap r s) i = a), χ i = ∑ i ∈ univ.filter (fun i => L i = a), χ i

theorem **deg_swap** (χ : Fin n → ℂ) (L : Fin n → Fin 3) {r s : Fin n} (h : χ r = χ s) : deg χ (L ∘ Equiv.swap r s) = deg χ L

theorem **isLabel_swap** {j : ℕ} {L : Fin n → Fin 3} (h : IsLabel j L) (r s : Fin n) : IsLabel j (L ∘ Equiv.swap r s)

theorem **moved_labels** (χ₀ : Fin n → ℂ) (j : ℕ) : ∀ (ss : List (Fin n)) (χ : Fin n → ℂ) (Λ : Set (Fin n → Fin 3)), GoodPath χ ss → (∀ r s, r ≠ s → χ r = χ s → χ r = χ₀ r ∧ χ s = χ₀ s) → (∀ L ∈ Λ, IsLabel j L ∧ (deg χ₀ L).re < 0) → ∀ L ∈ moveIter χ Λ ss, IsLabel j L ∧ (deg χ₀ L).re < 0

theorem **labelTrack** : LabelTrack

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.LatticeCongruenceSubgroupsOpenCompact

theorem **norm_one_entry** (i j : Fin m) : ‖(1 : Matrix (Fin m) (Fin m) L) i j‖ ≤ 1

theorem **norm_mul_entry** {A B : Matrix (Fin m) (Fin m) L} {a b : ℝ} (ha : 0 ≤ a) (hb : 0 ≤ b) (hA : ∀ i j, ‖A i j‖ ≤ a) (hB : ∀ i j, ‖B i j‖ ≤ b) (i j : Fin m) : ‖(A * B) i j‖ ≤ a * b

theorem **norm_mulVec_entry** {B : Matrix (Fin m) (Fin m) L} {v : Fin m → L} {r : ℝ} (hr : 0 ≤ r) (hB : ∀ i j, ‖B i j‖ ≤ r) (hv : ∀ j, ‖v j‖ ≤ 1) (i : Fin m) : ‖(B *ᵥ v) i‖ ≤ r

theorem **IsNear.isInt** {r : ℝ} {A : Matrix (Fin m) (Fin m) L} (hr : r ≤ 1) (h : IsNear r A) : IsInt A

theorem **IsInt.mul** {A B : Matrix (Fin m) (Fin m) L} (hA : IsInt A) (hB : IsInt B) : IsInt (A * B)

theorem **isInt_one** : IsInt (1 : Matrix (Fin m) (Fin m) L)

theorem **IsNear.mul** {r : ℝ} {A B : Matrix (Fin m) (Fin m) L} (h0 : 0 ≤ r) (hr : r ≤ 1) (hA : IsNear r A) (hB : IsNear r B) : IsNear r (A * B)

theorem **isNear_one** {r : ℝ} (h0 : 0 ≤ r) : IsNear r (1 : Matrix (Fin m) (Fin m) L)

theorem **IsNear.inv** {r : ℝ} {A B : Matrix (Fin m) (Fin m) L} (h0 : 0 ≤ r) (hr : r < 1) (hA : IsNear r A) (hBA : B * A = 1) : IsNear r B

theorem **IsNear.conj** {r : ℝ} {C D K : Matrix (Fin m) (Fin m) L} (h0 : 0 ≤ r) (hC : IsInt C) (hD : IsInt D) (hCD : C * D = 1) (hK : IsNear r K) : IsNear r (C * K * D)

theorem **mem_intS** {g : GL (Fin m) L} : g ∈ intS ↔ IsInt g.1 ∧ IsInt (g⁻¹).1

theorem **congS_le_intS** {r : ℝ} (h0 : 0 ≤ r) (h1 : r < 1) : congS (L

theorem **congS_conj** {r : ℝ} (h0 : 0 ≤ r) (h1 : r < 1) {c : GL (Fin m) L} (hc : c ∈ intS) (k : GL (Fin m) L) : k ∈ congS r h0 h1 ↔ c * k * c⁻¹ ∈ congS r h0 h1

theorem **isOpen_near** {r : ℝ} (hr : 0 < r) : IsOpen {g : GL (Fin m) L | IsNear r g.1}

theorem **exists_near_of_mem_nhds** {c : Fin m → Fin m → L} {A : Set (Fin m → Fin m → L)} (hA : A ∈ 𝒩 c) : ∃ ε > 0, ∀ B : Fin m → Fin m → L, (∀ i j, ‖B i j − c i j‖ ≤ ε) → B ∈ A

theorem **exists_level** {W : Set (GL (Fin m) L)} (hW : W ∈ 𝒩 (1 : GL (Fin m) L)) : ∃ π : L, π ≠ 0 ∧ ‖π‖ < 1 ∧ ∀ g : GL (Fin m) L, IsNear ‖π‖ g.1 → g ∈ W

theorem **isCompact_box** (c : Matrix (Fin m) (Fin m) L) (r : ℝ) : IsCompact {g : GL (Fin m) L | (∀ i j, ‖g.1 i j − c i j‖ ≤ r) ∧ ∀ i j, ‖(g⁻¹).1 i j − c i j‖ ≤ r}

theorem **isCompact_intS** : IsCompact ((intS : Subgroup (GL (Fin m) L)) : Set (GL (Fin m) L))

theorem **isCompact_congS** {r : ℝ} (h0 : 0 ≤ r) (h1 : r < 1) : IsCompact ((congS r h0 h1 : Subgroup (GL (Fin m) L)) : Set (GL (Fin m) L))

theorem **conj_eq** (A : Matrix (Fin n) (Fin n) L) : conj b A = LinearMap.toMatrix b b (Matrix.toLin' A)

theorem **conj_apply** (A : Matrix (Fin n) (Fin n) L) (i j : Fin m) : conj b A i j = b.repr (A *ᵥ b j) i

theorem **repr_mulVec** (A : Matrix (Fin n) (Fin n) L) (x : Fin n → L) : ⇑(b.repr (A *ᵥ x)) = conj b A *ᵥ ⇑(b.repr x)

theorem **basis_mem** (hΛ : ∀ x, x ∈ Λ ↔ ∀ i, ‖b.repr x i‖ ≤ 1) (j : Fin m) : b j ∈ Λ

theorem **isInt_conj_iff** (hΛ : ∀ x, x ∈ Λ ↔ ∀ i, ‖b.repr x i‖ ≤ 1) (A : Matrix (Fin n) (Fin n) L) : IsInt (conj b A) ↔ ∀ x ∈ Λ, A *ᵥ x ∈ Λ

theorem **isNear_conj_iff** (hΛ : ∀ x, x ∈ Λ ↔ ∀ i, ‖b.repr x i‖ ≤ 1) {π : L} (hπ : π ≠ 0) (A : Matrix (Fin n) (Fin n) L) : IsNear ‖π‖ (conj b A) ↔ ∀ x ∈ Λ, ∃ y ∈ Λ, A *ᵥ x − x = π • y

theorem **continuous_conj** : Continuous (conj b)

theorem **continuous_conj_symm** : Continuous (conj b).symm

theorem **isClosed_UG** {L : Type} [NontriviallyNormedField L] (s : L →+* L) (hs : ∀ x, ‖s x‖ = ‖x‖) {n : ℕ} (J : Matrix (Fin n) (Fin n) L) : IsClosed (UG s J : Set (GL (Fin n) L))

theorem **main** : ∀ (L : Type) [NontriviallyNormedField L] [IsUltrametricDist L] [ProperSpace L] (s : L →+* L), (∀ x, ‖s x‖ = ‖x‖) → ∀ (n : ℕ) (J : Matrix (Fin n) (Fin n) L) (Λ : AddSubgroup (Fin n → L)), IsLat L Λ → (∀ π : L, π ≠ 0 → ‖π‖ < 1 → ∃ K K₀ : Subgroup (UG s J), (∀ g, g ∈ K ↔ ∀ x ∈ Λ, ∃ y ∈ Λ, g.1.1.mulVec x − x = π • y) ∧ (∀ g, g ∈ K₀ ↔ ∀ x, g.1.1.mulVec x ∈ Λ ↔ x ∈ Λ) ∧ IsOpen (K : Set (UG s J)) ∧ IsCompact (K : Set (UG s J)) ∧ IsCompact (K₀ : Set (UG s J)) ∧ K ≤ K₀ ∧ ∀ c ∈ K₀, ∀ k, k ∈ K ↔ c * k * c⁻¹ ∈ K) ∧ ∀ U : Subgroup (UG s J), IsOpen (U : Set (UG s J)) → ∃ π : L, π ≠ 0 ∧ ‖π‖ < 1 ∧ ∀ g : UG s J, (∀ x ∈ Λ, ∃ y ∈ Λ, g.1.1.mulVec x − x = π • y) → g ∈ U

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.LatticeCongruenceSubgroupsOpenCompactStatement

theorem **_root_.GlobalTheory.T.lattice_congruence_subgroups_open_compact** : ∀ (L : Type) [NontriviallyNormedField L] [IsUltrametricDist L] [ProperSpace L] (s : L →+* L), (∀ x, ‖s x‖ = ‖x‖) → ∀ (n : ℕ) (J : Matrix (Fin n) (Fin n) L) (Λ : AddSubgroup (Fin n → L)), IsLat L Λ → (∀ π : L, π ≠ 0 → ‖π‖ < 1 → ∃ K K₀ : Subgroup (UG s J), (∀ g, g ∈ K ↔ ∀ x ∈ Λ, ∃ y ∈ Λ, g.1.1.mulVec x − x = π • y) ∧ (∀ g, g ∈ K₀ ↔ ∀ x, g.1.1.mulVec x ∈ Λ ↔ x ∈ Λ) ∧ IsOpen (K : Set (UG s J)) ∧ IsCompact (K : Set (UG s J)) ∧ IsCompact (K₀ : Set (UG s J)) ∧ K ≤ K₀ ∧ ∀ c ∈ K₀, ∀ k, k ∈ K ↔ c * k * c⁻¹ ∈ K) ∧ ∀ U : Subgroup (UG s J), IsOpen (U : Set (UG s J)) → ∃ π : L, π ≠ 0 ∧ ‖π‖ < 1 ∧ ∀ g : UG s J, (∀ x ∈ Λ, ∃ y ∈ Λ, g.1.1.mulVec x − x = π • y) → g ∈ U

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.LevelWeightSumBound

theorem **card_first_le** {n : ℕ} (A : Finset (Pos n)) (c : Fin n) : (A.filter fun i => i.1.1 = c).card ≤ n

theorem **card_second_le** {n : ℕ} (A : Finset (Pos n)) (c : Fin n) : (A.filter fun i => i.1.2 = c).card ≤ n

theorem **abs_wt_le** {n : ℕ} (A : Finset (Pos n)) (c : Fin n) : |wt A c| ≤ n

theorem **abs_two_rhoC_le** {n : ℕ} (ε : Fin n → Bool) (a : Fin n) : |(2 : ℝ) * PI.UpqRep.rhoC ε a| ≤ n

theorem **abs_modExp_le** {M N : ℕ} (d : Fin M → ℕ) (h : ∑ j, d j = N) (i : Fin M) : |modExp (fun j => Fin (d j)) i| ≤ (N : ℤ)

theorem **norm_cluster_sum_le** (n M : ℕ) (ε : Fin n → Bool) (lev : Fin n → ℕ) (i : Fin M) (h : ∑ j, sz lev M j = n) : ‖∑ a ∈ univ.filter (fun a => lev a = (i : ℕ)), ((wt (omU ε lev) a − (2 : ℝ) * PI.UpqRep.rhoC ε a : ℝ) : ℂ) − ((sz lev M i : ℂ) * ((modExp (fun j => Fin (sz lev M j)) i : ℂ) / 2))‖ ≤ 3 * (n : ℝ) ^ 2

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.LevelWeightSumBoundStatement

theorem **_root_.GlobalTheory.T.level_weight_sum_bound** : ∀ (n M : ℕ) (ε : Fin n → Bool) (lev : Fin n → ℕ) (i : Fin M) (h : ∑ j, sz lev M j = n), ‖∑ a ∈ univ.filter (fun a => lev a = (i : ℕ)), ((wt (omU ε lev) a − (2 : ℝ) * PI.UpqRep.rhoC ε a : ℝ) : ℂ) − ((sz lev M i : ℂ) * ((modExp (fun j => Fin (sz lev M j)) i : ℂ) / 2))‖ ≤ 3 * (n : ℝ) ^ 2

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.LeviActionCommutesWithBoundary

theorem **cT_pos** {lev : Fin n → ℕ} {a b : Fin n} (h : tau lev a b) : (cT lev a b : Module.End ℂ (Cn n V)) = ann ⟨a, b⟩ h.1

theorem **dT_pos** {lev : Fin n → ℕ} {a b : Fin n} (h : tau lev a b) : (dT lev a b : Module.End ℂ (Cn n V)) = cre ⟨a, b⟩ h.1

theorem **cT_neg** {lev : Fin n → ℕ} {a b : Fin n} (h : ¬tau lev a b) : (cT lev a b : Module.End ℂ (Cn n V)) = 0

theorem **dT_neg** {lev : Fin n → ℕ} {a b : Fin n} (h : ¬tau lev a b) : (dT lev a b : Module.End ℂ (Cn n V)) = 0

theorem **ann_mem** (i : Pos n) (hx : x ∈ SU V lev) : ann i x ∈ SU V lev

theorem **ann_zero** {i : Pos n} (hi : ¬inU lev i) (hx : x ∈ SU V lev) : ann i x = 0

theorem **cre_mem** {i : Pos n} (hi : inU lev i) (hx : x ∈ SU V lev) : cre i x ∈ SU V lev

theorem **lift_mem** (T : Module.End ℂ V) (hx : x ∈ SU V lev) : lift n T x ∈ SU V lev

theorem **cT_mem** (a b : Fin n) (hx : x ∈ SU V lev) : cT lev a b x ∈ SU V lev

theorem **dT_mem** (a b : Fin n) (hx : x ∈ SU V lev) : dT lev a b x ∈ SU V lev

theorem **sum_pos** {M : Type*} [AddCommMonoid M] (g : Fin n → Fin n → M) (h : ∀ a b, ¬a < b → g a b = 0) : ∑ a, ∑ b, g a b = ∑ i : Pos n, g i.1.1 i.1.2

theorem **bd_allT** (E : Fin n → Fin n → Module.End ℂ V) (x : Cn n V) : bd (allT n) E x = (∑ i : Pos n, lift n (E i.1.1 i.1.2) (ann i x)) + ∑ i : Pos n, ∑ j : Pos n, ∑ k : Pos n, if i.1.2 = j.1.1 ∧ k.1.1 = i.1.1 ∧ k.1.2 = j.1.2 then cre k (ann j (ann i x)) else 0

theorem **part_one** (E : Fin n → Fin n → Module.End ℂ V) (hx : x ∈ SU V lev) : ∑ a, ∑ b, LT E a b (cT lev a b x) = ∑ i : Pos n, lift n (E i.1.1 i.1.2) (ann i x)

theorem **tri_sum** (hx : x ∈ SU V lev) (i j : Pos n) : (∑ k : Pos n, if i.1.2 = j.1.1 ∧ k.1.1 = i.1.1 ∧ k.1.2 = j.1.2 then cre k (ann j (ann i x)) else 0) = if i.1.2 = j.1.1 then dT lev i.1.1 j.1.2 (cT lev j.1.1 j.1.2 (cT lev i.1.1 i.1.2 x)) else 0

theorem **part_two** (hx : x ∈ SU V lev) : ∑ a, ∑ b, ∑ e, dT lev a e (cT lev b e (cT lev a b x)) = ∑ i : Pos n, ∑ j : Pos n, ∑ k : Pos n, if i.1.2 = j.1.1 ∧ k.1.1 = i.1.1 ∧ k.1.2 = j.1.2 then cre k (ann j (ann i x)) else 0

theorem **Dl_apply** (E : Fin n → Fin n → Module.End ℂ V) (x : Cn n V) : Dl lev E x = (∑ a, ∑ b, LT E a b (cT lev a b x)) + ∑ a, ∑ b, ∑ e, dT lev a e (cT lev b e (cT lev a b x))

theorem **Th_apply** (E : Fin n → Fin n → Module.End ℂ V) (p q : Fin n) (x : Cn n V) : Th lev E p q x = LT E p q x + (∑ r, dT lev p r (cT lev q r x) − ∑ r, dT lev r q (cT lev r p x))

theorem **bd_eq** (E : Fin n → Fin n → Module.End ℂ V) (hx : x ∈ SU V lev) : bd (allT n) E x = Dl lev E x

theorem **Dl_mem** (E : Fin n → Fin n → Module.End ℂ V) (hx : x ∈ SU V lev) : Dl lev E x ∈ SU V lev

theorem **Th_mem** (E : Fin n → Fin n → Module.End ℂ V) (p q : Fin n) (hx : x ∈ SU V lev) : Th lev E p q x ∈ SU V lev

theorem **ad_one** (hm : Monotone lev) {p q : Fin n} (hpq : lev p = lev q) (hx : x ∈ SU V lev) : ∑ i : Pos n, ∑ k : Pos n, (if q = i.1.1 ∧ k.1 = ⟨p, i.1.2⟩ then (1 : ℂ) else 0) • cre k (ann i x) = ∑ r, dT lev p r (cT lev q r x)

theorem **ad_two** (hm : Monotone lev) {p q : Fin n} (hpq : lev p = lev q) (hx : x ∈ SU V lev) : ∑ i : Pos n, ∑ k : Pos n, (if i.1.2 = p ∧ k.1 = ⟨i.1.1, q⟩ then (1 : ℂ) else 0) • cre k (ann i x) = ∑ r, dT lev r q (cT lev r p x)

theorem **lF_eq** (hm : Monotone lev) (E : Fin n → Fin n → Module.End ℂ V) {p q : Fin n} (hpq : lev p = lev q) (hx : x ∈ SU V lev) : lF E p q x = Th lev E p q x

theorem **glRel_LT** {E : Fin n → Fin n → Module.End ℂ V} (hE : GlRel E) (p q a b : Fin n) : LT E p q * LT E a b − LT E a b * LT E p q = (if q = a then LT E p b else 0) − (if b = p then LT E a q else 0)

theorem **GlobalTheory.CompanionTheory.LeviChain.lF_bd_comm** : ∀ (n : ℕ) (lev : Fin n → ℕ) (V : Type) [AddCommGroup V] [Module ℂ V] (E : Fin n → Fin n → Module.End ℂ V), Monotone lev → GlRel E → ∀ p q : Fin n, lev p = lev q → ∀ x : Cn n V, SuppIn (inU lev) x → lF E p q (bd (allT n) E x) = bd (allT n) E (lF E p q x)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.LeviActionCommutesWithBoundaryStatement

theorem **_root_.GlobalTheory.T.levi_action_commutes_with_boundary** : ∀ (n : ℕ) (lev : Fin n → ℕ) (V : Type) [AddCommGroup V] [Module ℂ V] (E : Fin n → Fin n → Module.End ℂ V), Monotone lev → GlRel E → ∀ p q : Fin n, lev p = lev q → ∀ x : Cn n V, SuppIn (inU lev) x → lF E p q (bd (allT n) E x) = bd (allT n) E (lF E p q x)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.LeviActionCommutesWithTotalBoundary

theorem **units_mul_relation_rearrange_commRing** {R : Type*} [Ring R] (a b c d : R) (h1 : b * c + c * b = d) (h2 : a * b + b * a = 0) : c * a * b - b * (c * a) = -(d * a)

theorem **term_ann** (k i l : Pos n) : (cre k * ann i * ann l - ann l * (cre k * ann i) : Module.End ℂ (Cn n V)) = -(if l = k then ann i else 0)

theorem **adL_ann** (p q : Fin n) (l : Pos n) : (adL p q * ann l - ann l * adL p q : Module.End ℂ (Cn n V)) = -∑ i : Pos n, adC p q i l • ann i

theorem **lift_mul_ann** (T : Module.End ℂ V) (i : Pos n) : ((lift n T : Module.End ℂ (Cn n V)) * ann i : Module.End ℂ (Cn n V)) = ann i * (lift n T : Module.End ℂ (Cn n V))

theorem **lift_mul_cre** (T : Module.End ℂ V) (i : Pos n) : ((lift n T : Module.End ℂ (Cn n V)) * cre i : Module.End ℂ (Cn n V)) = cre i * (lift n T : Module.End ℂ (Cn n V))

theorem **adL_mul_lift** (T : Module.End ℂ V) (p q : Fin n) : (adL p q * (lift n T : Module.End ℂ (Cn n V)) : Module.End ℂ (Cn n V)) = (lift n T : Module.End ℂ (Cn n V)) * adL p q

theorem **lift_mul** (S T : Module.End ℂ V) : (lift n (S * T) : Module.End ℂ (Cn n V)) = (lift n S : Module.End ℂ (Cn n V)) * (lift n T : Module.End ℂ (Cn n V))

theorem **lift_sub** (S T : Module.End ℂ V) : (lift n (S - T) : Module.End ℂ (Cn n V)) = (lift n S : Module.End ℂ (Cn n V)) - (lift n T : Module.End ℂ (Cn n V))

theorem **lift_sum** {ι : Type*} (s : Finset ι) (f : ι → Module.End ℂ V) : (lift n (∑ i ∈ s, f i) : Module.End ℂ (Cn n V)) = ∑ i ∈ s, (lift n (f i) : Module.End ℂ (Cn n V))

theorem **lift_smul** (c : ℂ) (S : Module.End ℂ V) : (lift n (c • S) : Module.End ℂ (Cn n V)) = c • (lift n S : Module.End ℂ (Cn n V))

theorem **lF_ann** (E : Fin n → Fin n → Module.End ℂ V) (p q : Fin n) (l : Pos n) : (lF E p q * ann l - ann l * lF E p q : Module.End ℂ (Cn n V)) = -∑ i : Pos n, adC p q i l • ann i

theorem **lF_cre'** (E : Fin n → Fin n → Module.End ℂ V) (p q : Fin n) (l : Pos n) : (lF E p q * cre l - cre l * lF E p q : Module.End ℂ (Cn n V)) = ∑ k : Pos n, adC p q l k • cre k

theorem **lF_lift** (E : Fin n → Fin n → Module.End ℂ V) (p q : Fin n) (T : Module.End ℂ V) : (lF E p q * (lift n T : Module.End ℂ (Cn n V)) - (lift n T : Module.End ℂ (Cn n V)) * lF E p q : Module.End ℂ (Cn n V)) = lift n (E p q * T - T * E p q)

theorem **bd_eq** (E : Fin n → Fin n → Module.End ℂ V) : bd (allT n) E = S1 E + S2 n V

theorem **comm3** {R : Type*} [Ring R] (L c a b dc da db : R) (hc : L * c - c * L = dc) (ha : L * a - a * L = da) (hb : L * b - b * L = db) : L * (c * a * b) - c * a * b * L = dc * a * b + c * da * b + c * a * db

theorem **term2** (E : Fin n → Fin n → Module.End ℂ V) (p q : Fin n) (i j k : Pos n) : (lF E p q * Wt3 V i j k - Wt3 V i j k * lF E p q : Module.End ℂ (Cn n V)) = ∑ m : Pos n, adC p q k m • Wt3 V i j m - ∑ l : Pos n, adC p q l j • Wt3 V i l k - ∑ l : Pos n, adC p q l i • Wt3 V l j k

theorem **reindex** {M : Type*} [AddCommGroup M] [Module ℂ M] (c : Pos n → Pos n → Pos n → ℂ) (d : Pos n → Pos n → Pos n → ℂ) (w : Pos n → Pos n → Pos n → M) : ∑ i, ∑ j, ∑ k, c i j k • (∑ m, d k m • w i j m - ∑ l, d l j • w i l k - ∑ l, d l i • w l j k) = ∑ a, ∑ b, ∑ m, (∑ k, c a b k * d k m - ∑ j, c a j m * d b j - ∑ i, c i b m * d a i) • w a b m

theorem **levi_commutator_bracket_part** (E : Fin n → Fin n → Module.End ℂ V) (p q : Fin n) : (lF E p q * S2 n V - S2 n V * lF E p q : Module.End ℂ (Cn n V)) = ∑ a : Pos n, ∑ b : Pos n, ∑ m : Pos n, Dc p q a b m • Wt3 V a b m

theorem **levi_commutator_linear_part** (E : Fin n → Fin n → Module.End ℂ V) (p q : Fin n) : (lF E p q * S1 E - S1 E * lF E p q : Module.End ℂ (Cn n V)) = ∑ l : Pos n, (lift n (E p q * E l.1.1 l.1.2 - E l.1.1 l.1.2 * E p q - ∑ k : Pos n, adC p q l k • E k.1.1 k.1.2) : Module.End ℂ (Cn n V)) * ann l

theorem **lt_of_lev** (hm : Monotone lev) {s t : Fin n} (h : lev s < lev t) : s < t

theorem **adC_eq** (p q : Fin n) (i k : Pos n) : adC p q i k = (if q = i.1.1 ∧ k.1.1 = p ∧ k.1.2 = i.1.2 then 1 else 0) - (if i.1.2 = p ∧ k.1.1 = i.1.1 ∧ k.1.2 = q then 1 else 0)

theorem **levi_bracket_sum_first** (a b m : Pos n) : ∑ k : Pos n, tc a b k * adC p q k m = cX p q a b m - cY p q a b m

theorem **levi_bracket_sum_second** (hm : Monotone lev) (hpq : lev p = lev q) {a b : Pos n} (ha : inU lev a) (hb : inU lev b) (m : Pos n) : ∑ j : Pos n, tc a j m * adC p q b j = cZ p q a b m - cY p q a b m

theorem **levi_bracket_sum_third** (hm : Monotone lev) (hpq : lev p = lev q) {a b : Pos n} (ha : inU lev a) (hb : inU lev b) (m : Pos n) : ∑ i : Pos n, tc i b m * adC p q a i = cX p q a b m - cZ p q a b m

theorem **Dc_eq_zero** (hm : Monotone lev) (hpq : lev p = lev q) {a b : Pos n} (ha : inU lev a) (hb : inU lev b) (m : Pos n) : Dc p q a b m = 0

theorem **sum_ind** {M : Type*} [AddCommGroup M] [Module ℂ M] {u v : Fin n} (h : u < v) (f : Pos n → M) : ∑ k : Pos n, (if k.1.1 = u ∧ k.1.2 = v then (1 : ℂ) else 0) • f k = f ((u, v), h)

theorem **lin_zero** (hm : Monotone lev) (hpq : lev p = lev q) {E : Fin n → Fin n → Module.End ℂ V} (hE : GlRel E) {l : Pos n} (hl : inU lev l) : E p q * E l.1.1 l.1.2 - E l.1.1 l.1.2 * E p q - ∑ k : Pos n, adC p q l k • E k.1.1 k.1.2 = 0

theorem **ann_eq_zero** {P : Pos n → Prop} {x : Cn n V} (hx : SuppIn P x) {i : Pos n} (hi : ¬P i) : ann i x = 0

theorem **ann_supp** {P : Pos n → Prop} {x : Cn n V} (hx : SuppIn P x) (i : Pos n) : SuppIn P (ann i x)

theorem **lF_bd** : ∀ (n : ℕ) (lev : Fin n → ℕ) (V : Type) [AddCommGroup V] [Module ℂ V] (E : Fin n → Fin n → Module.End ℂ V), Monotone lev → GlRel E → ∀ (x : Cn n V), SuppIn (inU lev) x → ∀ p q, lev p = lev q → lF E p q (bd (allT n) E x) = bd (allT n) E (lF E p q x)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.LeviActionCommutesWithTotalBoundaryStatement

theorem **_root_.GlobalTheory.T.levi_action_commutes_with_total_boundary** : ∀ (n : ℕ) (lev : Fin n → ℕ) (V : Type) [AddCommGroup V] [Module ℂ V] (E : Fin n → Fin n → Module.End ℂ V), Monotone lev → GlRel E → ∀ (x : Cn n V), SuppIn (inU lev) x → ∀ p q, lev p = lev q → lF E p q (bd (allT n) E x) = bd (allT n) E (lF E p q x)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.LeviActionIsGlRepresentation

theorem **lift_mul_cre** (φ : Module.End ℂ V) (g : Pos n) : (lift n φ * cre g : Module.End ℂ (Cn n V)) = cre g * lift n φ

theorem **lift_mul_ann** (φ : Module.End ℂ V) (g : Pos n) : (lift n φ * ann g : Module.End ℂ (Cn n V)) = ann g * lift n φ

theorem **lift_mul_adL** (φ : Module.End ℂ V) (p q : Fin n) : (lift n φ * adL p q : Module.End ℂ (Cn n V)) = adL p q * lift n φ

theorem **term_ann** (k i g : Pos n) : (ann g * (cre k * ann i) - cre k * ann i * ann g : Module.End ℂ (Cn n V)) = if k = g then ann i else 0

theorem **adL_ann** (p q : Fin n) (g : Pos n) : (∑ i : Pos n, adC p q i g • ann i : Module.End ℂ (Cn n V)) = ann g * adL p q - adL p q * ann g

theorem **adL_comm** (p q r v : Fin n) : (adL p q * adL r v - adL r v * adL p q : Module.End ℂ (Cn n V)) = ∑ i : Pos n, ∑ l : Pos n, (∑ k : Pos n, (adC r v i k * adC p q k l - adC p q i k * adC r v k l)) • (cre l * ann i)

theorem **ite3** (A B C : Prop) [Decidable A] [Decidable B] [Decidable C] : (if A ∧ B ∧ C then 1 else 0 : ℂ) = (if A then 1 else 0) * (if B then 1 else 0) * (if C then 1 else 0)

theorem **adC_eq** (p q : Fin n) (i k : Pos n) : adC p q i k = cc p q i.1 k.1

theorem **ite_dl** (a b : Fin n) (x : ℂ) : (if a = b then x else 0) = dl a b * x

theorem **sum_at** (f : Fin n × Fin n → ℂ) (s : Fin n × Fin n) (hs : (s.1 : ℕ) < s.2) : ∑ k : Pos n, (if k.1 = s then f k.1 else 0) = f s

theorem **sum_adC** {lev : Fin n → ℕ} (hm : Monotone lev) {p q : Fin n} (hpq : lev p = lev q) {i : Pos n} (hi : inU lev i) (f : Fin n × Fin n → ℂ) : ∑ k : Pos n, adC p q i k * f k.1 = dl q i.1.1 * f (p, i.1.2) - dl i.1.2 p * f (i.1.1, q)

theorem **mat_id** {lev : Fin n → ℕ} (hm : Monotone lev) {p q r v : Fin n} (hpq : lev p = lev q) (hrv : lev r = lev v) {i : Pos n} (hi : inU lev i) (l : Pos n) : ∑ k : Pos n, (adC r v i k * adC p q k l - adC p q i k * adC r v k l) = (if q = r then adC p v i l else 0) - (if v = p then adC r q i l else 0)

theorem **ann_eq_zero** {lev : Fin n → ℕ} {x : Cn n V} (hx : SuppIn (inU lev) x) {i : Pos n} (hi : ¬inU lev i) : ann i x = 0

theorem **adL_comm_apply** {lev : Fin n → ℕ} (hm : Monotone lev) {x : Cn n V} (hx : SuppIn (inU lev) x) {p q r v : Fin n} (hpq : lev p = lev q) (hrv : lev r = lev v) : adL p q (adL r v x) - adL r v (adL p q x) = (if q = r then adL p v x else 0) - (if v = p then adL r q x else 0)

theorem **lF_glRel** : ∀ (n : ℕ) (lev : Fin n → ℕ) (V : Type) [AddCommGroup V] [Module ℂ V] (E : Fin n → Fin n → Module.End ℂ V), Monotone lev → GlRel E → ∀ (x : Cn n V), SuppIn (inU lev) x → ∀ p q r v, lev p = lev q → lev r = lev v → lF E p q (lF E r v x) - lF E r v (lF E p q x) = (if q = r then lF E p v x else 0) - (if v = p then lF E r q x else 0)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.LeviActionIsGlRepresentationStatement

theorem **_root_.GlobalTheory.T.levi_action_is_gl_representation** : ∀ (n : ℕ) (lev : Fin n → ℕ) (V : Type) [AddCommGroup V] [Module ℂ V] (E : Fin n → Fin n → Module.End ℂ V), Monotone lev → GlRel E → ∀ (x : Cn n V), SuppIn (inU lev) x → ∀ p q r v, lev p = lev q → lev r = lev v → lF E p q (lF E r v x) - lF E r v (lF E p q x) = (if q = r then lF E p v x else 0) - (if v = p then lF E r q x else 0)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.LeviActionPreservesSupportDegree

theorem **cre_ann_ne** (i k : Pos n) (x : Cn n V) (A : Finset (Pos n)) (h : (cre k * ann i : Module.End ℂ (Cn n V)) x A ≠ 0) : k ∈ A ∧ i ∉ A.erase k ∧ x (insert i (A.erase k)) ≠ 0

theorem **lF_ne** (E : Fin n → Fin n → Module.End ℂ V) (p q : Fin n) (x : Cn n V) (A : Finset (Pos n)) (h : lF E p q x A ≠ 0) : x A ≠ 0 ∨ ∃ i k : Pos n, adC p q i k ≠ 0 ∧ (cre k ∗ ann i : Module.End ℂ (Cn n V)) x A ≠ 0

theorem **inU_of_adC** {lev : Fin n → ℕ} {p q : Fin n} (hpq : lev p = lev q) {i k : Pos n} (h : adC p q i k ≠ 0) (hi : inU lev i) : inU lev k

theorem **lF_suppIn_deg** (n : ℕ) (lev : Fin n → ℕ) (V : Type) [AddCommGroup V] [Module ℂ V] (E : Fin n → Fin n → Module.End ℂ V) (_hlev : Monotone lev) (x : Cn n V) (hx : SuppIn (inU lev) x) (p q : Fin n) (hpq : lev p = lev q) : SuppIn (inU lev) (lF E p q x) ∧ ∀ j, Deg j x → Deg j (lF E p q x)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.LeviActionPreservesSupportDegreeStatement

theorem **_root_.GlobalTheory.T.levi_action_preserves_support_degree** : ∀ (n : ℕ) (lev : Fin n → ℕ) (V : Type) [AddCommGroup V] [Module ℂ V] (E : Fin n → Fin n → Module.End ℂ V), Monotone lev → ∀ (x : Cn n V), SuppIn (inU lev) x → ∀ p q, lev p = lev q → SuppIn (inU lev) (lF E p q x) ∧ ∀ j, Deg j x → Deg j (lF E p q x)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.LeviActionUnitaryAdjoint

theorem **sg_conj** (i : Pos n) (A : Finset (Pos n)) : (starRingEnd ℂ) (sg i A) = sg i A

theorem **sg_erase_self** (i : Pos n) {A : Finset (Pos n)} (h : i ∈ A) : sg i (A.erase i) = sg i A

theorem **ip_cre** (i : Pos n) (x z : Cn n V) : ip (cre i x) z = ip x (ann i z)

theorem **ip_conj** (x z : Cn n V) : (starRingEnd ℂ) (ip x z) = ip z x

theorem **ip_ann** (i : Pos n) (x z : Cn n V) : ip (ann i x) z = ip x (cre i z)

theorem **ip_cre_ann** (i k : Pos n) (x z : Cn n V) : ip ((cre k ∗ ann i : Module.End ℂ (Cn n V)) x) z = ip x ((cre i ∗ ann k : Module.End ℂ (Cn n V)) z)

theorem **ip_add_left** (x y z : Cn n V) : ip (x + y) z = ip x z + ip y z

theorem **ip_add_right** (x y z : Cn n V) : ip x (y + z) = ip x y + ip x z

theorem **ip_smul_left** (c : ℂ) (x z : Cn n V) : ip (c • x) z = (starRingEnd ℂ) c ∗ ip x z

theorem **ip_smul_right** (c : ℂ) (x z : Cn n V) : ip x (c • z) = c ∗ ip x z

theorem **ip_sum_left** {ι : Type} (s : Finset ι) (f : ι → Cn n V) (z : Cn n V) : ip (∑ a ∈ s, f a) z = ∑ a ∈ s, ip (f a) z

theorem **ip_sum_right** {ι : Type} (s : Finset ι) (x : Cn n V) (f : ι → Cn n V) : ip x (∑ a ∈ s, f a) = ∑ a ∈ s, ip x (f a)

theorem **adC_conj** (p q : Fin n) (i k : Pos n) : (starRingEnd ℂ) (adC p q i k) = adC p q i k

theorem **adC_swap** (p q : Fin n) (i k : Pos n) : adC p q i k = adC q p k i

theorem **ip_adL** (p q : Fin n) (x z : Cn n V) : ip (adL p q x) z = ip x (adL q p z)

theorem **ip_lift** {ε : Fin n → Bool} {E : Fin n → Fin n → Module.End ℂ V} (hU : KUnit ε E) {p q : Fin n} (h : ε p = ε q) (x z : Cn n V) : ip (lift n (E p q) x) z = ip x (lift n (E q p) z)

theorem **lF_adjoint** : ∀ (n : ℕ) (ε : Fin n → Bool) (V : Type) [NormedAddCommGroup V] [InnerProductSpace ℂ V] (E : Fin n → Fin n → Module.End ℂ V), KUnit ε E → ∀ p q : Fin n, ε p = ε q → ∀ x z : Cn n V, ∑ A, ⟪lF E p q x A, z A⟫_ℂ = ∑ A, ⟪x A, lF E q p z A⟫_ℂ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.LeviActionUnitaryAdjointStatement

theorem **_root_.GlobalTheory.T.levi_action_unitary_adjoint** : ∀ (n : ℕ) (ε : Fin n → Bool) (V : Type) [NormedAddCommGroup V] [InnerProductSpace ℂ V] (E : Fin n → Fin n → Module.End ℂ V), KUnit ε E → ∀ p q : Fin n, ε p = ε q → ∀ x z : Cn n V, ∑ A, ⟪lF E p q x A, z A⟫_ℂ = ∑ A, ⟪x A, lF E q p z A⟫_ℂ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.LeviDecomposition

theorem **pr_val** (k r : ℕ) (i : Fin (2 ∗ r + k + 1)) : ((i.val ≤ r ∨ r + k ≤ i.val) ∧ (pr k r i).val + i.val = 2 ∗ r + k) ∨ (r < i.val ∧ i.val < r + k ∧ (pr k r i).val = i.val)

theorem **pr_pr** (k r : ℕ) (i : Fin (2 ∗ r + k + 1)) : pr k r (pr k r i) = i

theorem **Hq_apply** (k r : ℕ) (i j : Fin (2 ∗ r + k + 1)) : Hq k r i j = if pr k r i = j then 1 else 0

theorem **Hq_mul_apply** (k r : ℕ) (g : Matrix (Fin (2 ∗ r + k + 1)) (Fin (2 ∗ r + k + 1)) ℂ) (i j : Fin (2 ∗ r + k + 1)) : (Hq k r ∗ g) i j = g (pr k r i) j

theorem **Hq_sq** (k r : ℕ) : Hq k r ∗ Hq k r = 1

theorem **mem_iff** (k r : ℕ) (g : Matrix (Fin (2 ∗ r + k + 1)) (Fin (2 ∗ r + k + 1)) ℂ) : g ∈ UH (Hq k r) ↔ ∀ i j, ∑ x, conj (g x i) ∗ g (pr k r x) j = if pr k r i = j then 1 else 0

theorem **mul_mem** {n : ℕ} {H g h : Matrix (Fin n) (Fin n) ℂ} (hg : g ∈ UH H) (hh : h ∈ UH H) : g ∗ h ∈ UH H

theorem **diag_mem** (k r : ℕ) (d : Fin (2 ∗ r + k + 1) → ℂ) (hd : ∀ i, conj (d i) ∗ d (pr k r i) = 1) : Matrix.diagonal d ∈ UH (Hq k r)

theorem **leviQ** : LeviQ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.LeviDecompositionStatement

theorem **_root_.GlobalTheory.T.LeviDecomposition** : LeviQ

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.LeviSubspaceInvariantUnderBlocks

theorem **glRel_bops** {n : ℕ} (Q : GMod n) {c : ℕ} (d : Fin c → ℕ) (h : ∑ i, d i = n) (i : Fin c) : GlRel (bops Q d h i)

theorem **commute_E_Qd** {n : ℕ} (P : GMod n) {c : ℕ} (d : Fin c → ℕ) (h : ∑ i, d i = n) (i : Fin c) (Y : ℂ) (a b : Fin n) (hab : blk (stdBlocks d h) a = blk (stdBlocks d h) b) : Commute (P.E a b) (Qd P d h i Y)

theorem **leviSub_stable** {n : ℕ} (P : GMod n) (d : Fin 3 → ℕ) (h : ∑ i, d i = n) (χ : Fin n → ℂ) (L : Fin n → Fin 3) (a b : Fin n) (hab : blk (stdBlocks d h) a = blk (stdBlocks d h) b) : ∀ x ∈ leviSub P d h χ L, P.E a b x ∈ leviSub P d h χ L

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.LeviSubspaceInvariantUnderBlocksStatement

theorem **_root_.GlobalTheory.T.levi_subspace_invariant_under_blocks** : ∀ {n : ℕ} (P : GMod n) (d : Fin 3 → ℕ) (h : ∑ i, d i = n) (χ : Fin n → ℂ) (L : Fin n → Fin 3) (a b : Fin n), blk (stdBlocks d h) a = blk (stdBlocks d h) b → ∀ x ∈ leviSub P d h χ L, P.E a b x ∈ leviSub P d h χ L

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.LineBundleTrivialAfterFlatCover

theorem **exists_gen_of_chart** {Y P : Scheme.{0}} {M : LB P} (hM : M.IsLB) (f : Y ⟶ P) (i : M.ι) (hf : ∀ y, f y ∈ M.U i) : ∃ t : LB.Gen (M.pull f), t.IsGen

theorem **exists_flat_cover_triv** : ∀ {P : Scheme.{0}} (M : LB P), M.IsLB → ∃ (Z : Scheme.{0}) (g : Z ⟶ P), Flat g ∧ LocallyOfFinitePresentation g ∧ Surjective g ∧ Triv (M.pull g)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.LineBundleTrivialAfterFlatCoverStatement

theorem **_root_.GlobalTheory.T.line_bundle_trivial_after_flat_cover** : ∀ {P : Scheme.{0}} (M : LB P), M.IsLB → ∃ (Z : Scheme.{0}) (g : Z ⟶ P), Flat g ∧ LocallyOfFinitePresentation g ∧ Surjective g ∧ Triv (M.pull g)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.LineBundleTrivialOfDescendedGenerator

theorem **triv_of_invariant_gen** : ∀ {P Q : Scheme.{0}} (f : P ⟶ Q), Flat f → LocallyOfFinitePresentation f → Surjective f → ∀ (D : LB Q), D.IsLB → ∀ (t : LB.Gen (D.pull f)), t.IsGen → (∀ (W : Scheme.{0}) (a b : W ⟶ P), a ≫ f = b ≫ f → ∀ (w : W) (i : D.ι), f (a w) ∈ D.U i → gm a w ((D.pull f).U fun _ => i) (t.s fun _ => i) = gm b w ((D.pull f).U fun _ => i) (t.s fun _ => i)) → Triv D

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.LineBundleTrivialOfDescendedGeneratorStatement

theorem **_root_.GlobalTheory.T.line_bundle_trivial_of_descended_generator** : ∀ {P Q : Scheme.{0}} (f : P ⟶ Q), Flat f → LocallyOfFinitePresentation f → Surjective f → ∀ (D : LB Q), D.IsLB → ∀ (t : LB.Gen (D.pull f)), t.IsGen → (∀ (W : Scheme.{0}) (a b : W ⟶ P), a ≫ f = b ≫ f → ∀ (w : W) (i : D.ι), f (a w) ∈ D.U i → gm a w ((D.pull f).U fun _ => i) (t.s fun _ => i) = gm b w ((D.pull f).U fun _ => i) (t.s fun _ => i)) → Triv D

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.LineFacts

theorem **A_mul_pM** (h3 : A ∗ A ∗ A = A) : A ∗ pM A = −pM A

theorem **qM_sub_pM** : qM A − pM A = A

theorem **mem_Vm** (h3 : A ∗ A ∗ A = A) {y : Fin n → ℂ} : y ∈ Vm A ↔ pM A *ᵥ y = y

theorem **A_mulVec_of_mem** (h3 : A ∗ A ∗ A = A) {y : Fin n → ℂ} (hy : y ∈ Vm A) : A *ᵥ y = −y

theorem **pM_mul_of_keeps** (h3 : A ∗ A ∗ A = A) {g : Matrix (Fin n) (Fin n) ℂ} (hg : ∀ y ∈ Vm A, g ∗ᵥ y ∈ Vm A) : pM A ∗ (g ∗ pM A) = g ∗ pM A

theorem **commute_pM** (h3 : A ∗ A ∗ A = A) (hH : Aᴴ = A) {g : Matrix (Fin n) (Fin n) ℂ} (hg : ∀ y ∈ Vm A, g ∗ᵥ y ∈ Vm A) (hg' : ∀ y ∈ Vm A, gᴴ ∗ᵥ y ∈ Vm A) : g ∗ pM A = pM A ∗ g

theorem **J_pM_J** (hG : IsGrad ε A) : Formalism.REAL.Jm ε ∗ pM A ∗ Formalism.REAL.Jm ε = qM A

theorem **commute_A** (hG : IsGrad ε A) {m : Formalism.REAL.UJ ε} (h1 : ∀ y ∈ Vm A, uMat m ∗ᵥ y ∈ Vm A) (h2 : ∀ y ∈ Vm A, (uMat m)ᴴ ∗ᵥ y ∈ Vm A) : uMat m ∗ A = A ∗ uMat m

theorem **exp_mulVec_of_eigen** {M : Matrix (Fin n) (Fin n) ℂ} {y : Fin n → ℂ} {c : ℂ} (h : M ∗ᵥ y = c • y) : NormedSpace.exp M ∗ᵥ y = Complex.exp c • y

theorem **eT_add** (s t : ℝ) : eT A (s + t) = eT A s ∗ eT A t

theorem **eT_zero** : eT A 0 = 1

theorem **continuous_eT** : Continuous (eT A)

theorem **eT_herm** (hH : Aᴴ = A) (t : ℝ) : (eT A t)ᴴ = eT A t

theorem **eT_mulVec** (h3 : A ∗ A ∗ A = A) {y : Fin n → ℂ} (hy : y ∈ Vm A) (t : ℝ) : eT A t ∗ᵥ y = ((Real.exp (-t) : ℝ) : ℂ) • y

theorem **eT_keeps** (h3 : A ∗ A ∗ A = A) (t : ℝ) : ∀ y ∈ Vm A, eT A t ∗ᵥ y ∈ Vm A

theorem **uMat_aLine** (hG : IsGrad ε A) (t : ℝ) : uMat (aLine hG t) = eT A t

theorem **continuous_aLine** (hG : IsGrad ε A) : Continuous (aLine hG)

theorem **aLine_add** (hG : IsGrad ε A) (s t : ℝ) : aLine hG (s + t) = aLine hG s ∗ aLine hG t

theorem **aLine_mem** (hG : IsGrad ε A) (t : ℝ) : aLine hG t ∈ flagM ε (W A)

theorem **keeps_of_mem** {m : Formalism.REAL.UJ ε} (hm : m ∈ flagM ε (W A)) : ∀ y ∈ Vm A, uMat m ∗ᵥ y ∈ Vm A

theorem **keepsH_of_mem** {m : Formalism.REAL.UJ ε} (hm : m ∈ flagM ε (W A)) : ∀ y ∈ Vm A, (uMat m)ᴴ ∗ᵥ y ∈ Vm A

theorem **aLine_comm** (hG : IsGrad ε A) (t : ℝ) {m : Formalism.REAL.UJ ε} (hm : m ∈ flagM ε (W A)) : m ∗ aLine hG t = aLine hG t ∗ m

theorem **detOn_nonneg** (g : Matrix (Fin n) (Fin n) ℂ) (V : Submodule ℂ (Fin n → ℂ)) : 0 ≤ detOn g V

theorem **detOn_eT** (h3 : A ∗ A ∗ A = A) (t : ℝ) : detOn (eT A t) (Vm A) = Real.exp (-((rk A : ℝ) ∗ t))

theorem **two_rk_le** (hG : IsGrad ε A) : 2 ∗ rk A ≤ n

theorem **locSize_eq** (g : Matrix (Fin n) (Fin n) ℂ) : locSize (fun _ : Fin 1 => rk A) (W A) g 0 = detOn g (Vm A) ^ (-(1 : ℝ) / rk A)

theorem **layer_eq** (j : ℕ) (τ : Fin 1 → ℝ) : GlobalTheory.Count.layerProd n 1 (fun _ : Fin 1 => j) τ id = τ 0 ^ (2 ∗ j ∗ (n - 2 ∗ j)) ∗ (τ 0 ^ 2) ^ (j ^ 2)

theorem **detOn_mul_line** (hG : IsGrad ε A) {m : Formalism.REAL.UJ ε} (hm : m ∈ flagM ε (W A)) (t : ℝ) : detOn (uMat (m ∗ aLine hG t)) (Vm A) = detOn (uMat m) (Vm A) ∗ Real.exp (-((rk A : ℝ) ∗ t))

theorem **modA_line** (hG : IsGrad ε A) {m : Formalism.REAL.UJ ε} (hm : m ∈ flagM ε (W A)) (t : ℝ) : modA A (m ∗ aLine hG t) = modA A m ∗ ENNReal.ofReal (Real.exp (2 ∗ rhoA A ∗ t))

theorem **measurable_tau** : Measurable (tau A : Formalism.REAL.UJ ε → ℝ)

theorem **detOn_ne_zero** {m : Formalism.REAL.UJ ε} (hm : m ∈ flagM ε (W A)) : detOn (uMat m) (Vm A) ≠ 0

theorem **tau_line** (hG : IsGrad ε A) {u m : Formalism.REAL.UJ ε} (hu : u ∈ flagN ε (W A)) (hm : m ∈ flagM ε (W A)) (t : ℝ) : tau A (u ∗ m ∗ aLine hG t) = tau A m + t

theorem **lineFacts** : LineFacts

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.LineFactsStatement

theorem **_root_.GlobalTheory.T.rgExh_lineFacts** : CompanionTheory.RgExh.Dec.N.S.LineFacts

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.LineFormulaOfFubiniAndParabolic

theorem **measurable_slab** : Measurable slab

theorem **lintegral_slab** (s : ℝ) : ∫⁻ t : ℝ, slab (s - t) = 1

theorem **inner_formula** (μM : Measure G) [SFinite μM] (S P : Set G) (a : ℝ → G) (δ : G → ℝ≥0∞) (T : G → ℝ) (ρ : ℝ) (ha : Measurable a) (hδ : Measurable δ) (hT : Measurable T) (hadd : ∀ s t, a (s + t) = a s ∗ a t) (hinv : ∀ t, μM.map (· ∗ a t) = μM) (hmod : ∀ m ∈ S, ∀ t, δ (m ∗ a t) = δ m ∗ ENNReal.ofReal (Real.exp (2 ∗ ρ ∗ t))) (htime : ∀ x ∈ P, ∀ t, T (x ∗ a t) = T x + t) (F : G → ℝ≥0∞) (hF : Measurable F) (u : G) (hS : ∀ᵐ m ∂μM, m ∈ S) (hP : ∀ᵐ m ∂μM, u ∗ m ∈ P) : ∫⁻ m, δ m ∗ (slab (T (u ∗ m)) ∗ ∫⁻ t : ℝ, F (u ∗ m ∗ a t) ∗ ENNReal.ofReal (Real.exp (2 ∗ ρ ∗ t))) ∂μM = ∫⁻ m, δ m ∗ F (u ∗ m) ∂μM

theorem **ae_mul_mem** (Q : Subgroup G) (hQ : MeasurableSet (Q : Set G)) (ν : Measure Q) (μN μM : Measure G) [SFinite μN] [SFinite μM] (S : Set G) (δ : G → ℝ≥0∞) (hδ : Measurable δ) (hδ0 : ∀ m ∈ S, δ m ≠ 0) (hS : ∀ᵐ m ∂μM, m ∈ S) (c : ℝ≥0∞) (hc0 : c ≠ 0) (hc : ∀ F : G → ℝ≥0∞, Measurable F → ∫⁻ x, F x ∂(ν.map Subtype.val) = c ∗ ∫⁻ u, ∫⁻ m, δ m ∗ F (u ∗ m) ∂μM ∂μN) : ∀ᵐ u ∂μN, ∀ᵐ m ∂μM, u ∗ m ∈ Q

theorem **slab_formula** (Q : Subgroup G) (hQ : MeasurableSet (Q : Set G)) (ν : Measure Q) (μN μM : Measure G) [SFinite μN] [SFinite μM] (S : Set G) (a : ℝ → G) (δ : G → ℝ≥0∞) (T : G → ℝ) (ρ : ℝ) (ha : Measurable a) (hδ : Measurable δ) (hT : Measurable T) (hadd : ∀ s t, a (s + t) = a s ∗ a t) (hinv : ∀ t, μM.map (· ∗ a t) = μM) (hmod : ∀ m ∈ S, ∀ t, δ (m ∗ a t) = δ m ∗ ENNReal.ofReal (Real.exp (2 ∗ ρ ∗ t))) (hδ0 : ∀ m ∈ S, δ m ≠ 0) (hS : ∀ᵐ m ∂μM, m ∈ S) (htime : ∀ x ∈ Q, ∀ t, T (x ∗ a t) = T x + t) (c : ℝ≥0∞) (hc0 : c ≠ 0) (hc : ∀ F : G → ℝ≥0∞, Measurable F → ∫⁻ x, F x ∂(ν.map Subtype.val) = c ∗ ∫⁻ u, ∫⁻ m, δ m ∗ F (u ∗ m) ∂μM ∂μN) (hF : Measurable F) : ∫⁻ x in {x : Q | T x ∈ Set.Ico 0 1}, (∫⁻ t : ℝ, F ((x : G) ∗ a t) ∗ ENNReal.ofReal (Real.exp (2 ∗ ρ ∗ t))) ∂ν = c ∗ ∫⁻ u, ∫⁻ m, δ m ∗ F (u ∗ m) ∂μM ∂μN

theorem **lineFormula** : Fub2 → LineFacts → PSubEq → LineFormula

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.LineFormulaOfFubiniAndParabolicStatement

theorem **_root_.GlobalTheory.T.line_formula_of_fubini_and_parabolic** : CompanionTheory.RgExh.Dec.N.S.Fub2 → CompanionTheory.RgExh.Dec.N.S.LineFacts → CompanionTheory.RgExh.Dec.N.S.PSubEq → CompanionTheory.RgExh.Dec.L.LineFormula

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.LinearCombinationGeneratorSectionExists

theorem **exists_gen_normalised_along** : ∀ {X Y S₀ : Scheme.{0}} {M : LB X}, M.IsLB → ∀ {K K₂ : Type} [Fintype K] [Fintype K₂] (φ : K → (Y ⟶ X)) (ε : K → ℤ) (φ₂ : K₂ → (S₀ ⟶ X)) (ε₂ : K₂ → ℤ) (p : Y ⟶ S₀) (e : S₀ ⟶ Y), e ≫ p = 𝟙 S₀ → FormEq (fun k => e ≫ φ k) ε φ₂ ε₂ → ∀ (t : LB.Gen (LB.LinComb M φ ε)), t.IsGen → ∀ (R : LB.Gen (LB.LinComb M φ₂ ε₂)), R.IsGen → ∃ t' : LB.Gen (LB.LinComb M φ ε), t'.IsGen ∧ ∀ (Z : Scheme.{0}) (s : Z ⟶ S₀) (z : Z) (J : (Z ⟶ X) → M.ι), Good M z J → val t' (s ≫ e) z J = val R s z J

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.LinearCombinationGeneratorSectionExistsStatement

theorem **_root_.GlobalTheory.T.linear_combination_generator_section_exists** : ∀ {X Y S₀ : Scheme.{0}} {M : LB X}, M.IsLB → ∀ {K K₂ : Type} [Fintype K] [Fintype K₂] (φ : K → (Y ⟶ X)) (ε : K → ℤ) (φ₂ : K₂ → (S₀ ⟶ X)) (ε₂ : K₂ → ℤ) (p : Y ⟶ S₀) (e : S₀ ⟶ Y), e ≫ p = 𝟙 S₀ → FormEq (fun k => e ≫ φ k) ε φ₂ ε₂ → ∀ (t : LB.Gen (LB.LinComb M φ ε)), t.IsGen → ∀ (R : LB.Gen (LB.LinComb M φ₂ ε₂)), R.IsGen → ∃ t' : LB.Gen (LB.LinComb M φ ε), t'.IsGen ∧ ∀ (Z : Scheme.{0}) (s : Z ⟶ S₀) (z : Z) (J : (Z ⟶ X) → M.ι), Good M z J → val t' (s ≫ e) z J = val R s z J

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.LocallyFiniteLeviInvariantSubspace

theorem **mem_chainsIn** {lev : Fin n → ℕ} {d : ℕ} {S : Submodule ℂ V} {x : Cn n V} : x ∈ chainsIn lev d S ↔ SuppIn (inU lev) x ∧ Deg d x ∧ ∀ A, x A ∈ S

theorem **finiteDimensional_chainsIn** (lev : Fin n → ℕ) (d : ℕ) (S : Submodule ℂ V) [FiniteDimensional ℂ S] : FiniteDimensional ℂ (chainsIn lev d S)

theorem **lF_apply** (E : Fin n → Fin n → Module.End ℂ V) (p q : Fin n) (x : Cn n V) (A : Finset (Pos n)) : lF E p q x A = E p q (x A) + ∑ i : Pos n, ∑ k : Pos n, adC p q i k • cre k (ann i x) A

theorem **cre_ann_mem** {S : Submodule ℂ V} {x : Cn n V} (hx : ∀ A, x A ∈ S) (i k : Pos n) (A : Finset (Pos n)) : cre k (ann i x) A ∈ S

theorem **inU_of_adC** {lev : Fin n → ℕ} {p q : Fin n} (hpq : lev p = lev q) {i k : Pos n} (h : adC p q i k ≠ 0) (hi : inU lev i) : inU lev k

theorem **lF_ne** {E : Fin n → Fin n → Module.End ℂ V} {p q : Fin n} {x : Cn n V} {A : Finset (Pos n)} (h : lF E p q x A ≠ 0) : x A ≠ 0 ∨ ∃ i k, adC p q i k ≠ 0 ∧ cre k (ann i x) A ≠ 0

theorem **term_supp** {lev : Fin n → ℕ} {d : ℕ} {p q : Fin n} (hpq : lev p = lev q) {x : Cn n V} (hx : SuppIn (inU lev) x) (hd : Deg d x) {A : Finset (Pos n)} {i k : Pos n} (h0 : adC p q i k ≠ 0) (h : cre k (ann i x) A ≠ 0) : (∀ j ∈ A, inU lev j) ∧ A.card = d

theorem **lF_mem_chainsIn** {ε : Fin n → Bool} {lev : Fin n → ℕ} {d : ℕ} {E : Fin n → Fin n → Module.End ℂ V} {S : Submodule ℂ V} (hS : KStab ε E S) {p q : Fin n} (hε : ε p = ε q) (hpq : lev p = lev q) {x : Cn n V} (hx : x ∈ chainsIn lev d S) : lF E p q x ∈ chainsIn lev d S

theorem **exists_stable** : ∀ (n : ℕ) (ε : Fin n → Bool) (lev : Fin n → ℕ) (V : Type) [NormedAddCommGroup V] [InnerProductSpace ℂ V] (E : Fin n → Fin n → Module.End ℂ V) (d : ℕ) (y : Cn n V), LocFin ε E → SuppIn (inU lev) y → Deg d y → ∃ T : Submodule ℂ (Cn n V), FiniteDimensional ℂ T ∧ y ∈ T ∧ (∀ x ∈ T, SuppIn (inU lev) x ∧ Deg d x) ∧ ∀ p q : Fin n, ε p = ε q → lev p = lev q → ∀ x ∈ T, lF E p q x ∈ T

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.LocallyFiniteLeviInvariantSubspaceStatement

theorem **_root_.GlobalTheory.T.locally_finite_levi_invariant_subspace** : ∀ (n : ℕ) (ε : Fin n → Bool) (lev : Fin n → ℕ) (V : Type) [NormedAddCommGroup V] [InnerProductSpace ℂ V] (E : Fin n → Fin n → Module.End ℂ V) (d : ℕ) (y : Cn n V), LocFin ε E → SuppIn (inU lev) y → Deg d y → ∃ T : Submodule ℂ (Cn n V), FiniteDimensional ℂ T ∧ y ∈ T ∧ (∀ x ∈ T, SuppIn (inU lev) x ∧ Deg d x) ∧ ∀ p q : Fin n, ε p = ε q → lev p = lev q → ∀ x ∈ T, lF E p q x ∈ T

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.MatrixHom

theorem **mAct_coord** {ι ι' V : Type} [Fintype ι] [AddCommGroup V] [Module ℂ V] (A : Matrix ι' ι ℂ) (x : ι → V) (c : ι') : mAct A x c = ∑ d, A c d • x d

theorem **mAct_mAct** {ι ι' ι'' V : Type} [Fintype ι] [Fintype ι'] [AddCommGroup V] [Module ℂ V] (A : Matrix ι'' ι' ℂ) (B : Matrix ι' ι ℂ) (x : ι → V) : mAct A (mAct B x) = mAct (A * B) x

theorem **dOp_mAct** {ι ι' V V' : Type} [Fintype ι] [AddCommGroup V] [Module ℂ V] [AddCommGroup V'] [Module ℂ V'] (A : Matrix ι' ι ℂ) (T : V →ₗ[ℂ] V') (x : ι → V) : dOp ι' T (mAct A x) = mAct A (dOp ι T x)

theorem **dsigW_eq_prod** {k r : ℕ} (U : URep k r) : ∀ (W : List (Formalism.FourierDecayAlt.ThreeDimData.Mx k r)), LieWd k r W → ∀ v : U.R.smSub, PI.UpqRep.dsigW (k

theorem **matHom** : MatHom

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.MatrixHomStatement

theorem **_root_.GlobalTheory.T.MatrixHom** : MatHom

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.MatrixNearIdentityInCongruenceSubgroup

theorem **norm_prod_sub_prod_le** {ι : Type} (s : Finset ι) (f g : ι → L) {r : ℝ} (h0 : 0 ≤ r) (hf : ∀ i, ‖f i‖ ≤ 1) (hg : ∀ i, ‖g i‖ ≤ 1) (hfg : ∀ i, ‖f i - g i‖ ≤ r) : ‖∏ i ∈ s, f i - ∏ i ∈ s, g i‖ ≤ r

theorem **norm_det_sub_det_le** {A B : Matrix (Fin n) (Fin n) L} {r : ℝ} (h0 : 0 ≤ r) (hA : ∀ i j, ‖A i j‖ ≤ 1) (hB : ∀ i j, ‖B i j‖ ≤ 1) (hAB : ∀ i j, ‖(A - B) i j‖ ≤ r) : ‖A.det - B.det‖ ≤ r

theorem **norm_det_le_one** {A : Matrix (Fin n) (Fin n) L} (hA : ∀ i j, ‖A i j‖ ≤ 1) : ‖A.det‖ ≤ 1

theorem **norm_one_entry_le** (i j : Fin n) : ‖(1 : Matrix (Fin n) (Fin n) L) i j‖ ≤ 1

theorem **norm_adjugate_entry_le** {A : Matrix (Fin n) (Fin n) L} (hA : ∀ i j, ‖A i j‖ ≤ 1) (i j : Fin n) : ‖A.adjugate i j‖ ≤ 1

theorem **exists_mem_congK_of_entries** : ∀ (L : Type) [NontriviallyNormedField L] [IsUltrametricDist L] (n : ℕ) (r : ℝ) (h0 : 0 ≤ r) (h1 : r ≤ 1), r < 1 → ∀ A : Matrix (Fin n) (Fin n) L, (∀ i j, ‖(A - 1) i j‖ ≤ r) → ∃ k ∈ GlobalTheory.CompanionTheory.CartGL.congK L n r h0 h1, (k : Matrix (Fin n) (Fin n) L) = A

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.MatrixNearIdentityInCongruenceSubgroupStatement

theorem **_root_.GlobalTheory.T.matrix_near_identity_in_congruence_subgroup** : ∀ (L : Type) [NontriviallyNormedField L] [IsUltrametricDist L] (n : ℕ) (r : ℝ) (h0 : 0 ≤ r) (h1 : r ≤ 1), r < 1 → ∀ A : Matrix (Fin n) (Fin n) L, (∀ i j, ‖(A - 1) i j‖ ≤ r) → ∃ k ∈ GlobalTheory.CompanionTheory.CartGL.congK L n r h0 h1, (k : Matrix (Fin n) (Fin n) L) = A

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.MaximalCompactStatement

theorem **exists_riesz** (l : E →ₗ[ℂ] ℂ) : ∃ η : E, ∀ x, l x = ⟪η, x⟫_ℂ

theorem **sum_sq_eq_of_basis** {ι κ : Type*} [Fintype ι] [Fintype κ] (l : E →ₗ[ℂ] ℂ) (b : OrthonormalBasis ι ℂ E) (c : OrthonormalBasis κ ℂ E) : ∑ i, ‖l (b i)‖ ^ 2 = ∑ j, ‖l (c j)‖ ^ 2

theorem **sq_le_of_basis** {ι : Type*} [Fintype ι] (l : E →ₗ[ℂ] ℂ) (b : OrthonormalBasis ι ℂ E) (x : E) : ‖l x‖ ^ 2 ≤ ‖x‖ ^ 2 * ∑ i, ‖l (b i)‖ ^ 2

theorem **sum_enorm_sq** {ι : Type*} [Fintype ι] (z : ι → ℂ) : ∑ i, ‖z i‖ₑ ^ 2 = ENNReal.ofReal (∑ i, ‖z i‖ ^ 2)

theorem **lintegral_enorm_sq_lt_top** {X : Type*} [MeasurableSpace X] {μ : Measure X} {f : X → ℂ} (h : MemLp f 2 μ) : (∫⁻ x, ‖f x‖ₑ ^ 2 ∂μ) < ⊤

theorem **continuous_uMat** : Continuous (fun g : Formalism.REAL.UJ ε => Formalism.MainNotions.S.umat g)

theorem **continuous_coef** {H : Type} [NormedAddCommGroup H] [InnerProductSpace ℂ H] {σ : Matrix (Fin n) (Fin n) ℂ → (H →L[ℂ] H)} (hU : Formalism.MainNotions.S.IsURepOn (Formalism.MainNotions.S.UJset ε) σ) (ξ v : H) : Continuous (fun g : Formalism.REAL.UJ ε => ⟪ξ, σ (Formalism.MainNotions.S.umat g) v⟫_ℂ)

theorem **sum_sq_translate** {H : Type} [NormedAddCommGroup H] [InnerProductSpace ℂ H] {σ : Matrix (Fin n) (Fin n) ℂ → (H →L[ℂ] H)} (hU : Formalism.MainNotions.S.IsURepOn (Formalism.MainNotions.S.UJset ε) σ) {S : Submodule ℂ H} [FiniteDimensional ℂ S] (hst : Formalism.MainNotions.S.IsKStableOn ε σ S) {ι : Type*} [Fintype ι] (b : OrthonormalBasis ι ℂ S) (ξ : H) (g k : Formalism.REAL.UJ ε) (hk : k ∈ Formalism.REAL.KG ε) : ∑ i, ‖⟪ξ, σ (Formalism.MainNotions.S.umat (g * k)) (b i : H)⟫_ℂ‖ ^ 2 = ∑ i, ‖⟪ξ, σ (Formalism.MainNotions.S.umat g) (b i : H)⟫_ℂ‖ ^ 2

theorem **kMaj** : CompanionTheory.RgExh.Dec.N.S.KMaj

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.MaximalCompactStatementStatement

theorem **_root_.GlobalTheory.T.MaximalCompactStatement** : CompanionTheory.RgExh.Dec.N.S.KMaj

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.MemFiltOfWords

theorem **homotopy_ring** (A D an cr : ι → R) (h1 : ∀ i j, Commute (an i) (D j)) (h2 : ∀ i j, Commute (cr i) (A j)) (h3 : ∀ i j, D i * A j - A j * D i = if i = j then 1 else 0) (h4 : ∀ i j, an i * cr j + cr j * an i = if i = j then 1 else 0) : (∑ i, A i * an i) * (∑ j, D j * cr j) + (∑ j, D j * cr j) * (∑ i, A i * an i) = ∑ i, A i * D i + ∑ i, cr i * an i

theorem **sq_ring** (A an : ι → R) (h1 : ∀ i j, Commute (an i) (A j)) (h2 : ∀ i j, Commute (A i) (A j)) (h4 : ∀ i j, an i * an j + an j * an i = 0) : (∑ i, A i * an i) * (∑ j, A j * an j) + (∑ j, A j * an j) * (∑ i, A i * an i) = 0

theorem **euler_ring** (A D : ι → R) (h2 : ∀ i j, Commute (D i) (D j)) (h3 : ∀ i j, D i * A j - A j * D i = if i = j then 1 else 0) (i : ι) : D i * (∑ j, A j * D j) = (∑ j, A j * D j) * D i + D i

theorem **mem_filt_succ** (d : ι → Module.End ℂ Z) (m : ℕ) (z : Z) : z ∈ filt d (m + 1) ↔ ∀ i, d i z ∈ filt d m

theorem **filt_mono** (d : ι → Module.End ℂ Z) : Monotone (filt d)

theorem **mem_filt_of_words** (d : ι → Module.End ℂ Z) (m : ℕ) (z : Z) (h : ∀ l : List ι, l.length = m → (l.map d).prod z = 0) : z ∈ filt d m

theorem **euler_filt** [Fintype ι] [DecidableEq ι] (a d : ι → Module.End ℂ Z) (h2 : ∀ i j, Commute (d i) (d j)) (h3 : ∀ i j, d i * a j - a j * d i = if i = j then 1 else 0) (m : ℕ) (z : Z) (hz : z ∈ filt d (m + 1)) : (∑ j, a j * d j) z - (m : ℂ) • z ∈ filt d m

theorem **lift_mul** (S T : Module.End ℂ Z) : (lift ε (S * T) : Module.End ℂ (Ch ε Z)) = lift ε S * lift ε T

theorem **ann_lift** (i : NC ε) (T : Module.End ℂ Z) : Commute (ann i : Module.End ℂ (Ch ε Z)) (lift ε T)

theorem **cre_lift** (i : NC ε) (T : Module.End ℂ Z) : Commute (cre i : Module.End ℂ (Ch ε Z)) (lift ε T)

theorem **cre_ann_apply** (i : NC ε) (x : Ch ε Z) (A : Finset (NC ε)) : (cre i (ann i x)) A = if i ∈ A then x A else 0

theorem **number_apply** (x : Ch ε Z) (A : Finset (NC ε)) : ((∑ i : NC ε, (cre i * ann i : Module.End ℂ (Ch ε Z))) x) A = (A.card : ℂ) • x A

theorem **deg_h** (d : NC ε → Module.End ℂ Z) (j : ℕ) (x : Ch ε Z) (hx : Deg j x) : Deg (j + 1) ((∑ i : NC ε, ((lift ε (d i) : Module.End ℂ (Ch ε Z)) * cre i : Module.End ℂ (Ch ε Z))) x)

theorem **koszul** : CAR → Koszul

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.MemUJsetOfMemKset

theorem **kFinStab** : CompanionTheory.D.DSIC.KFinStab

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.ModularityStatement

theorem **cnt_congr** {a b : X} (s t : Γ) : cnt Γ a b = cnt Γ (s • a) (t • b)

theorem **finite_fib** [∀ x : X, Finite (MulAction.stabilizer Γ x)] (x y : X) : Finite {γ : Γ // γ • x = y}

theorem **cnt_self_pos** [∀ x : X, Finite (MulAction.stabilizer Γ x)] (x : X) : 0 < cnt Γ x x

theorem **support_finite** [∀ x : X, Finite (MulAction.stabilizer Γ x)] (f : X →₀ ℂ) (x : X) : (Function.support fun γ : Γ => f (γ • x)).Finite

theorem **finsum_eq** [∀ x : X, Finite (MulAction.stabilizer Γ x)] (f : X →₀ ℂ) (x : X) : ∑ᶠ γ : Γ, f (γ • x) = f.sum fun z c => c * (cnt Γ x z : ℂ)

theorem **exists_phi** [Finite (MulAction.orbitRel.Quotient Γ X)] [∀ x : X, Finite (MulAction.stabilizer Γ x)] : ∃ (N : ℕ) (φ : (X →₀ ℂ) →ₗ[ℂ] EuclideanSpace ℂ (Fin N)), Function.Surjective φ ∧ ∀ x y, ⟪φ (Finsupp.single x 1), φ (Finsupp.single y 1)⟫_ℂ = (cnt Γ x y : ℂ)

theorem **lam_single** (g : G) (x : G ⧸ K) (c : ℂ) : lam K g (Finsupp.single x c) = Finsupp.single (g • x) c

theorem **lam_apply_smul** (g : G) (f : (G ⧸ K) →₀ ℂ) (x : G ⧸ K) : lam K g f (g • x) = f x

theorem **comm_lam** (h : Hk K) (g : G) (u : (G ⧸ K) →₀ ℂ) : h.1 (lam K g u) = lam K g (h.1 u)

theorem **ent_smul** (h : Hk K) (g : G) (x y : G ⧸ K) : ent h.1 (g • x) (g • y) = ent h.1 x y

theorem **row_finite** (hp : IsHeckePair K) (h : Hk K) (x : G ⧸ K) : {z | ent h.1 x z ≠ 0}.Finite

theorem **rowF_apply** (hp : IsHeckePair K) (h : Hk K) (x z : G ⧸ K) : rowF hp h x z = conj (ent h.1 x z)

theorem **lam_rowF** (hp : IsHeckePair K) (h : Hk K) (g : G) (x : G ⧸ K) : lam K g (rowF hp h x) = rowF hp h (g • x)

theorem **starH_single** (hp : IsHeckePair K) (h : Hk K) (x : G ⧸ K) : (starH hp h).1 (Finsupp.single x 1) = rowF hp h x

theorem **inner_phi** {N : ℕ} (Γ : Subgroup G) (φ : ((G ⧸ K) →₀ ℂ) →ₗ[ℂ] EuclideanSpace ℂ (Fin N)) (hφ : ∀ x y, ⟪φ (Finsupp.single x 1), φ (Finsupp.single y 1)⟫_ℂ = (cnt Γ x y : ℂ)) (x : G ⧸ K) (f : (G ⧸ K) →₀ ℂ) : ⟪φ (Finsupp.single x 1), φ f⟫_ℂ = f.sum fun z c => c * (cnt Γ x z : ℂ)

theorem **adj_single** {N : ℕ} (Γ : Subgroup G) [∀ x : G ⧸ K, Finite (MulAction.stabilizer Γ x)] (hp : IsHeckePair K) (h : Hk K) (φ : ((G ⧸ K) →₀ ℂ) →ₗ[ℂ] EuclideanSpace ℂ (Fin N)) (hφ : ∀ x y, ⟪φ (Finsupp.single x 1), φ (Finsupp.single y 1)⟫_ℂ = (cnt Γ x y : ℂ)) (x y : G ⧸ K) : ⟪φ (Finsupp.single x 1), φ (h.1 (Finsupp.single y 1))⟫_ℂ = ⟪φ ((starH hp h).1 (Finsupp.single x 1)), φ (Finsupp.single y 1)⟫_ℂ

theorem **adj** {N : ℕ} (Γ : Subgroup G) [∀ x : G ⧸ K, Finite (MulAction.stabilizer Γ x)] (hp : IsHeckePair K) (φ : ((G ⧸ K) →₀ ℂ) →ₗ[ℂ] EuclideanSpace ℂ (Fin N)) (hφ : ∀ x y, ⟪φ (Finsupp.single x 1), φ (Finsupp.single y 1)⟫_ℂ = (cnt Γ x y : ℂ)) (h : Hk K) (u v : (G ⧸ K) →₀ ℂ) : ⟪φ u, φ (h.1 v)⟫_ℂ = ⟪φ ((starH hp h).1 u), φ v⟫_ℂ

theorem **exists_rho** {N : ℕ} (hp : IsHeckePair K) (φ : ((G ⧸ K) →₀ ℂ) →ₗ[ℂ] EuclideanSpace ℂ (Fin N)) (hs : Function.Surjective φ) (hadj : ∀ (h : Hk K) u v, ⟪φ u, φ (h.1 v)⟫_ℂ = ⟪φ ((starH hp h).1 u), φ v⟫_ℂ) : ∃ ρ : Hk K →₀ Module.End ℂ (EuclideanSpace ℂ (Fin N)), StarClosed ρ ∧ ∀ h u, ρ h (φ u) = φ (h.1 u)

theorem **stMod** : StMod

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.ModularityStatementStatement

theorem **_root_.GlobalTheory.T.ModularityStatement** : StMod

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.MovingStatement

theorem **closure_symm_eq** (e : G ≃* G') {l : ℕ} (t' : Fin l → G') : Submonoid.closure (Set.range fun j => e.symm (t' j)) = (Submonoid.closure (Set.range t')).map e.symm.toMonoidHom

theorem **apply_mem_closure** (e : G ≃* G') {l : ℕ} (t' : Fin l → G') {s : G} (hs : s ∈ Submonoid.closure (Set.range fun j => e.symm (t' j))) : e s ∈ Submonoid.closure (Set.range t')

theorem **symm_mem_closure** (e : G ≃* G') {l : ℕ} (t' : Fin l → G') {s : G'} (hs : s ∈ Submonoid.closure (Set.range t')) : e.symm s ∈ Submonoid.closure (Set.range fun j => e.symm (t' j))

theorem **isCartan_comap** (e : G ≃* G') {K' : Subgroup G'} {d l : ℕ} (h : IsCartan K' d l) : IsCartan (K'.comap e.toMonoidHom) d l

theorem **stMove** : StMove

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.MovingStatementStatement

theorem **_root_.GlobalTheory.T.MovingStatement** : StMove

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.MulMemUJset

theorem **mul_mem_UJset** {n : ℕ} (ε : Fin n → Bool) (a b : Matrix (Fin n) (Fin n) ℂ) (ha : a ∈ UJset ε) (hb : b ∈ UJset ε) : a * b ∈ UJset ε

theorem **ex_mem_UJset** {n : ℕ} {ε : Fin n → Bool} {X : Matrix (Fin n) (Fin n) ℂ} (hX : inG ε X) (t : ℝ) : ex X t ∈ UJset ε

theorem **inG_add** {n : ℕ} {ε : Fin n → Bool} {X Y : Matrix (Fin n) (Fin n) ℂ} (hX : inG ε X) (hY : inG ε Y) : inG ε (X + Y)

theorem **inG_smul** {n : ℕ} {ε : Fin n → Bool} {X : Matrix (Fin n) (Fin n) ℂ} (hX : inG ε X) (s : ℝ) : inG ε ((s : ℂ) • X)

theorem **smLin** : CompanionTheory.D.DSIC.SmLin

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.NearIdentityUpperLowerFactorization

@[simp] theorem **blk_cc** (P : Matrix (Fin n) (Fin n) L) (b c : Fin n → L) (d : L) (i j : Fin n) : blk P b c d i.castSucc j.castSucc = P i j

@[simp] theorem **blk_cl** (P : Matrix (Fin n) (Fin n) L) (b c : Fin n → L) (d : L) (i : Fin n) : blk P b c d i.castSucc (Fin.last n) = b i

@[simp] theorem **blk_lc** (P : Matrix (Fin n) (Fin n) L) (b c : Fin n → L) (d : L) (j : Fin n) : blk P b c d (Fin.last n) j.castSucc = c j

@[simp] theorem **blk_ll** (P : Matrix (Fin n) (Fin n) L) (b c : Fin n → L) (d : L) : blk P b c d (Fin.last n) (Fin.last n) = d

theorem **factor** (n : ℕ) (r : ℝ) (h0 : 0 ≤ r) (h1 : r < 1) : ∀ A : Matrix (Fin n) (Fin n) L, (∀ i j, ‖(A - 1) i j‖ ≤ r) → ∃ U W : Matrix (Fin n) (Fin n) L, (∀ i j, j < i → U i j = 0) ∧ (∀ i j, i < j → W i j = 0) ∧ (∀ i j, ‖(U - 1) i j‖ ≤ r) ∧ (∀ i j, ‖(W - 1) i j‖ ≤ r) ∧ A = U * W

theorem **iwahori_factorisation** (L : Type) [NontriviallyNormedField L] [IsUltrametricDist L] (n : ℕ) (r : ℝ) (h0 : 0 ≤ r) (h1 : r < 1) (A : Matrix (Fin n) (Fin n) L) (hA : ∀ i j, ‖(A - 1) i j‖ ≤ r) : ∃ U W : Matrix (Fin n) (Fin n) L, (∀ i j, j < i → U i j = 0) ∧ (∀ i j, i < j → W i j = 0) ∧ (∀ i j, ‖(U - 1) i j‖ ≤ r) ∧ (∀ i j, ‖(W - 1) i j‖ ≤ r) ∧ A = U * W

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.NearIdentityUpperLowerFactorizationStatement

theorem **_root_.GlobalTheory.T.near_identity_upper_lower_factorization** : ∀ (L : Type) [NontriviallyNormedField L] [IsUltrametricDist L] (n : ℕ) (r : ℝ), 0 ≤ r → r < 1 → ∀ A : Matrix (Fin n) (Fin n) L, (∀ i j, ‖(A - 1) i j‖ ≤ r) → ∃ U W : Matrix (Fin n) (Fin n) L, (∀ i j, j < i → U i j = 0) ∧ (∀ i j, i < j → W i j = 0) ∧ (∀ i j, ‖(U - 1) i j‖ ≤ r) ∧ (∀ i j, ‖(W - 1) i j‖ ≤ r) ∧ A = U * W

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.NilpotentLeftRule

theorem **sum_mapDomain_mul** {ι κ : Type} (f : κ → ι) (l : κ →₀ ℂ) (w : ι → ℂ) : ((l.mapDomain f).sum fun i r => r * w i) = l.sum fun q r => r * w (f q)

theorem **sum_weighted** {ι κ : Type} [Fintype κ] (l : ι →₀ ℂ) (w : κ → ℂ) (f : κ → ι → ℂ) : ∑ k, w k * (l.sum fun x r => r * f k x) = l.sum fun x r => r * ∑ k, w k * f k x

theorem **LeftAt.reindex** {ε : Fin n → Bool} {X : GMod n} {ι κ : Type} {c : ι → X.V →ₗ[ℂ] (Matrix (Fin n) (Fin n) ℂ → ℂ)} {c' : κ → X.V →ₗ[ℂ] (Matrix (Fin n) (Fin n) ℂ → ℂ)} (f : κ → ι) (hf : ∀ q, c (f q) = c' q) {i : ι} {k : κ} (hk : c i = c' k) (h : LeftAt ε X c' k) : LeftAt ε X c i

theorem **left_tens** {ε : Fin n → Bool} (X : GMod n) {ι : Type} (c : ι → X.V →ₗ[ℂ] (Matrix (Fin n) (Fin n) ℂ → ℂ)) (h : ∀ i, LeftAt ε X c i) (q : ι × Fin n) : LeftAt ε (tens X) (fun q : ι × Fin n => tensCoef X (c q.1) q.2) q

theorem **left_step** {ε : Fin n → Bool} (X : GMod n) (z : ℂ) {ι : Type} (c : ι → X.V →ₗ[ℂ] (Matrix (Fin n) (Fin n) ℂ → ℂ)) (h : ∀ i, LeftAt ε X c i) (q : ι × Fin n) : LeftAt ε (step X z) (fun q : ι × Fin n => stepCoef X z (c q.1) q.2) q

theorem **left_path** {ε : Fin n → Bool} (ss : List (Fin n)) : ∀ (X : GMod n) (χ : Fin n → ℂ) {ι : Type} (c : ι → X.V →ₗ[ℂ] (Matrix (Fin n) (Fin n) ℂ → ℂ)), (∀ i, LeftAt ε X c i) → ∀ i : ι × List (Fin n), LeftAt ε (path X χ ss) (fun p : ι × List (Fin n) => pathCoef X (c p.1) χ ss p.2) i

theorem **inner_deriv_skew** (hσ : IsURepOn (UJset ε) σ) {Y : Matrix (Fin n) (Fin n) ℂ} (hY : inG ε Y) {ξ ξ' u u' : H} (hξ : HasDerivAt (fun t : ℝ => σ (NormedSpace.exp ((t : ℂ) • -Y)) ξ) ξ' 0) (hu : HasDerivAt (fun t : ℝ => σ (NormedSpace.exp ((t : ℂ) • Y)) u) u' 0) : ⟪ξ, u'⟫_ℂ = ⟪ξ', u⟫_ℂ

theorem **deriv_transport** (hσ : IsURepOn (UJset ε) σ) {g X : Matrix (Fin n) (Fin n) ℂ} (hg : g ∈ UJset ε) (hX : inG ε X) {u u' : H} (hd : HasDerivAt (fun t : ℝ => σ (NormedSpace.exp ((t : ℂ) • X)) u) u' 0) : HasDerivAt (fun t : ℝ => σ (NormedSpace.exp ((t : ℂ) • (g * X * g⁻¹))) (σ g u)) (σ g u') 0

theorem **conj_entry** {Y : Matrix (Fin n) (Fin n) ℂ} (hY : inG ε Y) (p q : Fin n) : (starRingEnd ℂ) (Y p q) = -((if ε p = ε q then 1 else -1) * Y q p)

theorem **real_rule** (hσ : IsURepOn (UJset ε) σ) (hK : CompanionTheory.D.LK.IsKMod ε σ D E) (ξ u : D) {g X : Matrix (Fin n) (Fin n) ℂ} (hg : g ∈ UJset ε) (hX : inG ε X) : ⟪(ξ : H), σ g (∑ a, ∑ b, X a b • ((E a b u : D) : H))⟫_ℂ = ∑ p, ∑ q, (g * X * g⁻¹) p q * ((if ε p = ε q then 1 else -1) * ⟪((E q p ξ : D) : H), σ g u⟫_ℂ)

theorem **unit_rule** (hσ : IsURepOn (UJset ε) σ) (hK : CompanionTheory.D.LK.IsKMod ε σ D E) (ξ u : D) {g : Matrix (Fin n) (Fin n) ℂ} (hg : g ∈ UJset ε) (a b : Fin n) : ⟪(ξ : H), σ g ((E a b u : D) : H)⟫_ℂ = ∑ p, ∑ q, (g * Matrix.single a b (1 : ℂ) * g⁻¹) p q * ((if ε p = ε q then 1 else -1) * ⟪((E q p ξ : D) : H), σ g u⟫_ℂ)

theorem **left_base** (hσ : IsURepOn (UJset ε) σ) (hK : CompanionTheory.D.LK.IsKMod ε σ D E) (ξ : D) : LeftAt ε ⟨D, E, hK.gl⟩ (fun ξ : D => baseCoef σ D (ξ : H)) ξ

theorem **leftRule** : CompanionTheory.RgExh.Dec.N.LeftRule

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.NilpotentLeftRuleStatement

theorem **_root_.GlobalTheory.T.NilpotentLeftRule** : CompanionTheory.RgExh.Dec.N.LeftRule

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.NilpotentOdeFirst

theorem **energy_pointwise** (z x y : ℂ) (s e : ℝ) (hz : z.re < s) (he : 0 ≤ e) : 2 * (inner ℝ x (z * x + y)) * e + ‖x‖ ^ 2 * (e * (-2 * s)) ≤ -(s - z.re) * (‖x‖ ^ 2 * e) + (1 / (s - z.re)) * (‖y‖ ^ 2 * e)

theorem **odeA** : CompanionTheory.RgExh.Dec.N.OdeA

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.NilpotentOdeFirstStatement

theorem **_root_.GlobalTheory.T.NilpotentOdeFirst** : CompanionTheory.RgExh.Dec.N.OdeA

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.NilpotentOdeSecond

theorem **energy_lower** (z x y : ℂ) (s d e : ℝ) (hd : 0 < d) (he : 0 ≤ e) : (2 * (z.re - s) - d) * (‖x‖ ^ 2 * e) - (1 / d) * (‖y‖ ^ 2 * e) ≤ 2 * (inner ℝ x (z * x + y)) * e + ‖x‖ ^ 2 * (e * (-2 * s))

theorem **exists_small_of_lintegral** (W : ℝ → ℝ) (hW : (∫⁻ t in Ioi (0 : ℝ), ENNReal.ofReal (W t)) < ⊤) (T₀ : ℝ) (hT₀ : 0 ≤ T₀) (ε : ℝ) (hε : 0 < ε) : ∃ T, T₀ ≤ T ∧ W T < ε

theorem **young_tail** (a C : ℝ) (ha : 0 < a) (x : ℝ → ℝ) (hx : Continuous x) (t T : ℝ) (hT : t ≤ T) (hC : ∫ r in t..T, x r ^ 2 ≤ C) : ∫ r in t..T, Real.exp (-a * r) * x r ≤ Real.exp (-a * t) * (C / 2 + 1 / (4 * a))

theorem **odeB** : CompanionTheory.RgExh.Dec.N.OdeB

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.NilpotentOdeSecondStatement

theorem **_root_.GlobalTheory.T.NilpotentOdeSecond** : CompanionTheory.RgExh.Dec.N.OdeB

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.NilpotentRightRule

theorem **exp_entry** (Y : Matrix (Fin n) (Fin n) ℂ) (i j : Fin n) : HasDerivAt (fun t : ℝ => NormedSpace.exp ((t : ℂ) • Y) i j) (Y i j) 0

theorem **mul_exp_entry** (g Y : Matrix (Fin n) (Fin n) ℂ) (j i : Fin n) : HasDerivAt (fun t : ℝ => (g * NormedSpace.exp ((t : ℂ) • Y)) j i) ((g * Y) j i) 0

theorem **mul_exp_zero** (g Y : Matrix (Fin n) (Fin n) ℂ) : g * NormedSpace.exp (((0 : ℝ) : ℂ) • Y) = g

theorem **tens_action** (X : GMod n) (Y : Matrix (Fin n) (Fin n) ℂ) (u : Fin n → X.V) (i : Fin n) : (∑ a, ∑ b, Y a b • (tens X).E a b u) i = (∑ a, ∑ b, Y a b • X.E a b (u i)) + ∑ b, Y i b • u b

theorem **tens_ok** {ε : Fin n → Bool} {X : GMod n} {c : X.V →ₗ[ℂ] (Matrix (Fin n) (Fin n) ℂ → ℂ)} (h : RightOK ε X c) (j : Fin n) : RightOK ε (tens X) (tensCoef X c j)

theorem **step_ok** {ε : Fin n → Bool} {X : GMod n} {c : X.V →ₗ[ℂ] (Matrix (Fin n) (Fin n) ℂ → ℂ)} (h : RightOK ε X c) (z : ℂ) (j : Fin n) : RightOK ε (step X z) (stepCoef X z c j)

theorem **path_ok** {ε : Fin n → Bool} : ∀ (ss : List (Fin n)) (X : GMod n) (c : X.V →ₗ[ℂ] (Matrix (Fin n) (Fin n) ℂ → ℂ)) (χ : Fin n → ℂ) (js : List (Fin n)), RightOK ε X c → RightOK ε (path X χ ss) (pathCoef X c χ ss js) | [], _, _, _, _, h => h | s :: ss, X, c, χ, js, h => path_ok ss (step X (χ s)) (stepCoef X (χ s) c (js.headD s)) (χ + Pi.single s 1) js.tail (step_ok h _ _) theorem base_ok {ε : Fin n → Bool} {H : Type} [NormedAddCommGroup H] [InnerProductSpace ℂ H] {σ : Matrix (Fin n) (Fin n) ℂ → (H →L[ℂ] H)} {D : Submodule ℂ H} {E : Fin n → Fin n → Module.End ℂ D} (hK : CompanionTheory.D.LK.IsKMod ε σ D E) (hσ : Formalism.MainNotions.S.IsURepOn (Formalism.MainNotions.S.UJset ε) σ) (ξ : H) : RightOK ε (D, E, hK.gl) (baseCoef σ D ξ)

theorem **rightRule** : CompanionTheory.RgExh.Dec.N.RightRule

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.NilpotentRightRuleStatement

theorem **_root_.GlobalTheory.T.NilpotentRightRule** : CompanionTheory.RgExh.Dec.N.RightRule

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.NoOrthogonalIdempotents

theorem **word_left** (E : R) (m : ℕ) (q x : ℕ → R) (hE : ∀ i ≤ m, E * q i = q i) (hid : q m * q m = q m) (hx : ∀ i < m, q i * x i = x i) (n k : ℕ) (hk : k + n = m) : q k * (word E ((List.range' k n).map x) * q m) = word E ((List.range' k n).map x) * q m

theorem **word_cases** (E : R) (m : ℕ) (q x : ℕ → R) (hE : ∀ i ≤ m, E * q i = q i) (hid : q m * q m = q m) (hx1 : ∀ i < m, q i * x i = x i) (hx2 : ∀ i < m, x i * q (i + 1) = x i) (horth : ∀ i ≤ m, ∀ k ≤ m, i ≠ k → q i * q k = 0) (l : List ℕ) (hl : ∀ a ∈ l, a < m) : word E (l.map x) * q m = 0 ∨ ∃ k, k + l.length = m ∧ l = List.range' k l.length

theorem **exists_chain** (A S : Set R) (E : R) (m : ℕ) (q : ℕ → R) (hmul : ∀ a ∈ A, ∀ b ∈ A, a * b ∈ A) (hS : S ⊆ A) (hprime : ∀ x ∈ A, ∀ y ∈ A, y ≠ 0 → (∀ s ∈ S, x * s * y = 0) → x = 0) (hA : ∀ i ≤ m, q i ∈ A) (hne : ∀ i ≤ m, q i ≠ 0) (hid : ∀ i ≤ m, q i * q i = q i) (hE : ∀ i ≤ m, E * q i = q i) (n k : ℕ) (hk : k + n = m) : ∃ (x : ℕ → R) (y : R), y ≠ 0 ∧ y ∈ A ∧ q k * y = y ∧ (∀ i, k ≤ i → i < m → x i ∈ A ∧ q i * x i = x i ∧ x i * q (i + 1) = x i) ∧ word E ((List.range' k n).map x) * q m = y

theorem **no_chain_nat** (A S : Set R) (E : R) (m : ℕ) (q : ℕ → R) (hmul : ∀ a ∈ A, ∀ b ∈ A, a * b ∈ A) (hS : S ⊆ A) (hprime : ∀ x ∈ A, ∀ y ∈ A, y ≠ 0 → (∀ s ∈ S, x * s * y = 0) → x = 0) (hstd : ∀ a : Fin m → R, (∀ i, a i ∈ A) → stdPoly E a = 0) (hA : ∀ i ≤ m, q i ∈ A) (hne : ∀ i ≤ m, q i ≠ 0) (hid : ∀ i ≤ m, q i * q i = q i) (horth : ∀ i ≤ m, ∀ k ≤ m, i ≠ k → q i * q k = 0) (hE : ∀ i ≤ m, E * q i = q i) : False

theorem **no_orthogonal_idempotents** : ∀ (R : Type) [Ring R] (A S : Set R) (E : R) (m : ℕ) (q : Fin (m + 1) → R), (∀ a ∈ A, ∀ b ∈ A, a * b ∈ A) → S ⊆ A → (∀ x ∈ A, ∀ y ∈ A, y ≠ 0 → (∀ s ∈ S, x * s * y = 0) → x = 0) → (∀ a : Fin m → R, (∀ i, a i ∈ A) → stdPoly E a = 0) → (∀ i, q i ∈ A) → (∀ i, q i ≠ 0) → (∀ i, q i * q i = q i) → (∀ i k, i ≠ k → q i * q k = 0) → (∀ i, E * q i = q i) → False

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.NotCdomAddRoot

theorem **height_root** (a b : Fin n) : ∑ i : Fin n, ((i : ℕ) : ℝ) * root a b i = ((a : ℕ) : ℝ) - ((b : ℕ) : ℝ)

theorem **not_cdom_add_root** (ε : Fin n → Bool) (μ : Fin n → ℝ) {a b : Fin n} (hab : a < b) : ¬ CDom ε (μ + root a b) μ

theorem **isWeight_apply** {V : Type} [AddCommGroup V] [Module ℂ V] {E : Fin n → Fin n → Module.End ℂ V} (hE : GlRel E) {μ : Fin n → ℝ} {v : V} (hv : IsWeight E μ v) (a b : Fin n) : IsWeight E (μ + root a b) (E a b v)

theorem **hwOfType** : CompanionTheory.D.DSIC.StabLink → CompanionTheory.D.DSIC.WtLink → CompanionTheory.D.DSIC.HWofType

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.NumKG

theorem **numKG** : NumKG

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.OmegaMinor

theorem **capelli_entry_apply** {n : ℕ} (X : GMod n) (x : ℂ) (d b : Fin n) (v : X.V) : capEntry X.E x d b v = X.E d b v + if d = b then x • v else 0

theorem **capelli_component** {m : ℕ} (X : GMod (m + 1)) (x : ℂ) (f : Fin (m + 1) → X.V) (b : Fin (m + 1)) : ∑ c : Fin (m + 1), ∑ d : Fin (m + 1), ((-1 : ℂ) ^ ((d : ℕ) + (c : ℕ))) • capEntry X.E x d b (cminor X.E d.succAbove c.succAbove (x - 1) (f c)) = capelliC X.E x (f b)

theorem **omMinor** : OmMinor

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.OmegaMinorStatement

theorem **_root_.GlobalTheory.T.OmegaMinor** : CompanionTheory.RgExh.Hk.OmMinor

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.OmegaScalar

theorem **evAt_apply** (u : ℂ) (p : A[X]) : evAt u p = eval₂ (RingHom.id A) (algebraMap ℂ A u) p

theorem **evAt_tmPoly** {n : ℕ} (E : Fin n → Fin n → A) (v u : ℂ) (a b : Fin n) : evAt u (tmPoly E v a b) = Tm E (u - v) a b

theorem **evAt_qdetPoly** {n : ℕ} (E : Fin n → Fin n → A) (u : ℂ) : evAt u (qdetPoly E) = qdet E u

theorem **cont** {M : Type*} [AddCommGroup M] [Module ℂ M] (p : (Module.End ℂ M)[X]) (q : ℂ[X]) (h : M) (b : ℂ) (H : ∀ u, u ≠ b → evAt u p h = q.eval u • h) : evAt b p h = q.eval b • h

theorem **compLeft_smul_one** {V : Type} [AddCommGroup V] [Module ℂ V] {n : ℕ} (c : ℂ) (w : Fin n → V) : (c • (1 : Module.End ℂ V)).compLeft (Fin n) w = c • w

theorem **tens_apply** {n : ℕ} (X : GMod n) (χ : Fin n → ℂ) (hS : ScalChar X χ) (u : ℂ) (w : Fin n → X.V) : qdet (tensE X) u (u • w + om X w) = Formalism.QuantumDeterminants.capP χ (u - cn n) • ((u + 1) • w + om X w)

theorem **core** {n : ℕ} (X : GMod n) (χ : Fin n → ℂ) (s : Fin n) (hS : ScalChar X χ) (hs : ∀ r, r ≠ s → χ r ≠ χ s) (g : Fin n → X.V) (h2 : om X (om X g - (χ s - cn n) • g) = (χ s - cn n) • (om X g - (χ s - cn n) • g)) : om X g - (χ s - cn n) • g = 0

theorem **omScal** : OmScal

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.OmegaScalarStatement

theorem **_root_.GlobalTheory.T.OmegaScalar** : CompanionTheory.RgExh.Hk.OmScal

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.OperatorStatement

theorem **norm_evalHom** (hδ : IsIrrUnitary δ) (hρ : IsUnitaryRep ρ) {u : X} (hu : ‖u‖ = 1) (y : X) (T : homIn δ ρ ⊤) : ‖evalHom δ ρ y T‖ = ‖y‖ * ‖evalHom δ ρ u T‖

theorem **evalHom_injective** (hδ : IsIrrUnitary δ) (hρ : IsUnitaryRep ρ) {u : X} (hu : ‖u‖ = 1) : Function.Injective (evalHom δ ρ u)

theorem **exists_op_unit** (hδ : IsIrrUnitary δ) (hρ : IsUnitaryRep ρ) (y : X) {u : X} (hu : ‖u‖ = 1) : ∃ A : H →L[ℂ] H, ∀ T ∈ homIn δ ρ ⊤, ∀ x, A (T x) = ⟪u, x⟫_ℂ • T y

theorem **exists_op** (hδ : IsIrrUnitary δ) (hρ : IsUnitaryRep ρ) (y ξ : X) : ∃ A : H →L[ℂ] H, ∀ T ∈ homIn δ ρ ⊤, ∀ x, A (T x) = ⟪ξ, x⟫_ℂ • T y

theorem **stOp** : StOp

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.OperatorStatementStatement

theorem **_root_.GlobalTheory.T.OperatorStatement** : StOp

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.OrderedCreationCommutesPastLevi

theorem **cre_sq** (k : Pos n) : (cre k * cre k : Module.End ℂ (Cn n V)) = 0

theorem **cre_anti** (i j : Pos n) : (cre i * cre j : Module.End ℂ (Cn n V)) = -(cre j * cre i)

theorem **cre_mul_prod_of_mem** {k : Pos n} {l : List (Pos n)} (h : k ∈ l) : (cre k * (l.map cre).prod : Module.End ℂ (Cn n V)) = 0

theorem **term_comm** (k i g : Pos n) : (cre k * ann i * cre g - cre g * (cre k * ann i) : Module.End ℂ (Cn n V)) = if i = g then cre k else 0

theorem **adL_cre** (p q : Fin n) (g : Pos n) : (adL p q * cre g - cre g * adL p q : Module.End ℂ (Cn n V)) = ∑ k : Pos n, adC p q g k • cre k

theorem **lF_cre** (E : Fin n → Fin n → Module.End ℂ V) (p q : Fin n) (g : Pos n) : (lF E p q * cre g : Module.End ℂ (Cn n V)) = cre g * lF E p q + ∑ k : Pos n, adC p q g k • cre k

theorem **lF_prod** (E : Fin n → Fin n → Module.End ℂ V) (p q : Fin n) (l : List (Pos n)) (h : ∀ (pre : List (Pos n)) (f : Pos n) (post : List (Pos n)), l = pre ++ f :: post → ∀ k : Pos n, adC p q f k ≠ 0 → k ∈ post) : (lF E p q * (l.map cre).prod : Module.End ℂ (Cn n V)) = (l.map cre).prod * lF E p q

theorem **bd_prod** : ∀ (n : ℕ) (V : Type) [AddCommGroup V] [Module ℂ V] (E : Fin n → Fin n → Module.End ℂ V), (∀ e : Pos n, (bd (allT n) E * cre e + cre e * bd (allT n) E : Module.End ℂ (Cn n V)) = lF E e.1.1 e.1.2) → ∀ (l : List (Pos n)) (x : Cn n V), (∀ (pre : List (Pos n)) (f : Pos n) (post : List (Pos n)), l = pre ++ f :: post → ∀ e ∈ l, ∀ k : Pos n, adC e.1.1 e.1.2 f k ≠ 0 → k ∈ post) → (∀ e ∈ l, lF E e.1.1 e.1.2 x = 0) → bd (allT n) E (((l.map cre).prod : Module.End ℂ (Cn n V)) x) = (-1 : ℂ) ^ l.length • ((l.map cre).prod : Module.End ℂ (Cn n V)) (bd (allT n) E x)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.OrderedCreationCommutesPastLeviStatement

theorem **_root_.GlobalTheory.T.ordered_creation_commutes_past_levi** : ∀ (n : ℕ) (V : Type) [AddCommGroup V] [Module ℂ V] (E : Fin n → Fin n → Module.End ℂ V), (∀ e : Pos n, (bd (allT n) E * cre e + cre e * bd (allT n) E : Module.End ℂ (Cn n V)) = lF E e.1.1 e.1.2) → ∀ (l : List (Pos n)) (x : Cn n V), (∀ (pre : List (Pos n)) (f : Pos n) (post : List (Pos n)), l = pre ++ f :: post → ∀ e ∈ l, lF E e.1.1 e.1.2 x = 0) → bd (allT n) E (((l.map cre).prod : Module.End ℂ (Cn n V)) x) = (-1 : ℂ) ^ l.length • ((l.map cre).prod : Module.End ℂ (Cn n V)) (bd (allT n) E x)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.OrthCycCycStable

theorem **orthCyc_cyc_stable** {n : ℕ} {V : Type} [AddCommGroup V] [Module ℂ V] (E : Fin n → Fin n → Module.End ℂ V) (v : V) (a b : Fin n) {x : V} (hx : x ∈ Cyc E v) : E a b x ∈ Cyc E v

theorem **orthCyc** : CompanionTheory.D.LK.OrthCyc

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.PairingOfSandwichStatement

theorem **smul_mem_of_isLat** {Λ : AddSubgroup V} (hΛ : IsLat L Λ) {a : L} (ha : ‖a‖ ≤ 1) {x : V} (hx : x ∈ Λ) : a • x ∈ Λ

theorem **exists_smul_mem_of_isLat** {Λ : AddSubgroup V} (hΛ : IsLat L Λ) (v : V) : ∃ a : L, a ≠ 0 ∧ a • v ∈ Λ

theorem **herm_add_left** (hh : IsHerm s h) (x y z : V) : h (x + y) z = h x z + h y z

theorem **herm_smul_left** (hh : IsHerm s h) (a : L) (x y : V) : h (a • x) y = s a * h x y

theorem **herm_add_add** (hh : IsHerm s h) (u v : V) : h (u + v) (u + v) = h u u + (h u v + s (h u v)) + h v v

theorem **norm_sub_le_of_le** {a b : L} {c : ℝ} (ha : ‖a‖ ≤ c) (hb : ‖b‖ ≤ c) : ‖a - b‖ ≤ c

theorem **pairing_le** (hδ : IsDelta s δ) (hh : IsHerm s h) {Λ : AddSubgroup V} (hL : IsLat L Λ) (hint : ∀ x ∈ Λ, ‖h x x‖ ≤ 1) {x y : V} (hx : x ∈ Λ) (hy : y ∈ Λ) : ‖h x y‖ ≤ ‖δ‖

theorem **adjoin_integral** (hs : IsInv s) (hδ : IsDelta s δ) (hh : IsHerm s h) {Λ : AddSubgroup V} (hint : ∀ x ∈ Λ, ‖h x x‖ ≤ 1) {p : L} (hp0 : p ≠ 0) {x : V} (hq : ‖h x x‖ ≤ ‖p‖ ^ 2) (hsmall : ∀ y ∈ Λ, ‖h x y‖ ≤ ‖p‖ * ‖δ‖) : ∀ z ∈ adjoin L Λ (p⁻¹ • x), ‖h z z‖ ≤ 1

theorem **pairing_of_sandwich** : StSandwich → StPairing

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.PairingOfSandwichStatementStatement

theorem **_root_.GlobalTheory.T.pairing_of_sandwich_statement** : StSandwich → StPairing

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.PairingStatement

theorem **stPair** : StPair

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.PairingStatementStatement

theorem **_root_.GlobalTheory.T.PairingStatement** : StPair

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.ParabolicSubgroupEquality

theorem **mem_Vm** {x : Fin n → ℂ} : x ∈ Vm A ↔ ∃ z, Pm A *ᵥ z = x

theorem **mem_Wm** {y : Fin n → ℂ} : y ∈ Wm A ↔ Pp A *ᵥ y = 0

theorem **anti** (hA : IsGrad ε A) : Formalism.REAL.Jm ε * A = -(A * Formalism.REAL.Jm ε)

theorem **Pm_conj** (hA : IsGrad ε A) : (Pm A)ᴴ = Pm A

theorem **Pm_J** (hA : IsGrad ε A) : Pm A * Formalism.REAL.Jm ε = Formalism.REAL.Jm ε * Pp A

theorem **Pp_Pm** (hA : IsGrad ε A) : Pp A * Pm A = 0

theorem **Vm_le_Wm** (hA : IsGrad ε A) : Vm A ≤ Wm A

theorem **mem_Wm_iff** (hA : IsGrad ε A) (y : Fin n → ℂ) : y ∈ Wm A ↔ ∀ x ∈ Vm A, star x ⬝ᵥ (Formalism.REAL.Jm ε *ᵥ y) = 0

theorem **isoFlag** (hA : IsGrad ε A) : IsIsoFlag ε (fun _ : Fin 1 => rk A) (W A)

theorem **keeps_of_stable** (g : Formalism.REAL.UJ ε) {S : Submodule ℂ (Fin n → ℂ)} (h : ∀ y ∈ S, uMat g *ᵥ y ∈ S) : Keeps (uMat g) S

theorem **stable_of_keeps** {g : Matrix (Fin n) (Fin n) ℂ} {S : Submodule ℂ (Fin n → ℂ)} (h : Keeps g S) : ∀ y ∈ S, g *ᵥ y ∈ S

theorem **flagN_fix** {u : Formalism.REAL.UJ ε} (hu : u ∈ flagN ε (W A)) {y : Fin n → ℂ} (hy : y ∈ Vm A) : uMat u *ᵥ y = y

theorem **flagN_stable_Wm** (hA : IsGrad ε A) {u : Formalism.REAL.UJ ε} (hu : u ∈ flagN ε (W A)) {y : Fin n → ℂ} (hy : y ∈ Wm A) : uMat u *ᵥ y ∈ Wm A

theorem **flagM_stable_Vm** {m : Formalism.REAL.UJ ε} (hm : m ∈ flagM ε (W A)) {y : Fin n → ℂ} (hy : y ∈ Vm A) : uMat m *ᵥ y ∈ Vm A

theorem **flagM_stable_Wm** (hA : IsGrad ε A) {m : Formalism.REAL.UJ ε} (hm : m ∈ flagM ε (W A)) {y : Fin n → ℂ} (hy : y ∈ Wm A) : uMat m *ᵥ y ∈ Wm A

theorem **psubEq** : PSubEq

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.ParabolicSubgroupEqualityStatement

theorem **_root_.GlobalTheory.T.ParabolicSubgroupEquality** : CompanionTheory.RgExh.Dec.N.S.PSubEq

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.PathEndEqAddNat

theorem **pathEnd_eq_add_nat** {n : ℕ} (ss : List (_root_.Fin n)) : ∀ χ : _root_.Fin n → ℂ, ∃ m : _root_.Fin n → ℕ, pathEnd χ ss = χ + (fun a => (m a : ℂ)) ∧ ∀ a, a ∈ ss → 1 ≤ m a

theorem **pathEndNat** : CompanionTheory.RgExh.Fin.PathEndNat

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.PicardGroupOfFieldTrivial

theorem **pic_eq_zero** (k : Type) [Field k] (c : Pic (Sp k)) : c = 0

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.PicardGroupOfFieldTrivialStatement

theorem **_root_.GlobalTheory.T.picard_group_of_field_trivial** : ∀ (k : Type) [Field k] (c : Pic (Sp k)), c = 0

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.PlaneOfHenselAndPairing

theorem **exists_uniformiser** [ProperSpace L] : ∃ p : L, p ≠ 0 ∧ ‖p‖ < 1 ∧ ∀ a : L, ‖a‖ < 1 → ‖a‖ ≤ ‖p‖

theorem **lat_smul** {Λ : AddSubgroup V} (hΛ : IsLat L Λ) {a : L} (ha : ‖a‖ ≤ 1) {x : V} (hx : x ∈ Λ) : a • x ∈ Λ

theorem **lat_commens** {Λ M : AddSubgroup V} (hΛ : IsLat L Λ) (hM : IsLat L M) {p : L} (hp0 : p ≠ 0) (hp1 : ‖p‖ < 1) : ∃ r : ℕ, ∀ x ∈ M, p ^ r • x ∈ Λ

theorem **lat_primitive** {Λ : AddSubgroup V} (hΛ : IsLat L Λ) {p : L} (hp0 : p ≠ 0) (hp1 : ‖p‖ < 1) {v : V} (hv : v ≠ 0) : ∃ a : L, a ≠ 0 ∧ a • v ∈ Λ ∧ p⁻¹ • a • v ∉ Λ

theorem **herm_add_left** {s : L →+* L} {h : V → V → L} (hh : IsHerm s h) (x y z : V) : h (x + y) z = h x z + h y z

theorem **herm_smul_left** {s : L →+* L} {h : V → V → L} (hh : IsHerm s h) (a : L) (x y : V) : h (a • x) y = s a * h x y

theorem **herm_expand** {s : L →+* L} {h : V → V → L} (hh : IsHerm s h) (a c : L) (x y : V) : h (c • x + a • y) (c • x + a • y) = s c * (c * h x x) + s c * (a * h x y) + s a * (c * h y x) + s a * (a * h y y)

theorem **one_le_norm_delta** {s : L →+* L} (hs : IsInv s) {δ : L} (hδ : IsDelta s δ) : 1 ≤ ‖δ‖

theorem **isotropic_lift** [CompleteSpace L] (hH : StHensel) {s : L →+* L} (hs : IsInv s) {δ : L} (hδ : IsDelta s δ) {h : V → V → L} (hh : IsHerm s h) {x y : V} {N : L} (hN0 : N ≠ 0) (hN1 : ‖N‖ < 1) (hNs : s N = N) (hqx : ‖h x x‖ ≤ ‖N‖) (hqy : ‖h y y‖ ≤ 1) (hxy : h x y = δ) : ∃ a : L, ‖a‖ ≤ ‖N‖ ∧ h (x + a • y) (x + a • y) = 0

theorem **hyperbolic_partner** {s : L →+* L} (hs : IsInv s) {δ : L} (hδ : IsDelta s δ) {h : V → V → L} (hh : IsHerm s h) {e y : V} (he : h e e = 0) (hqy : ‖h y y‖ ≤ 1) (hey : ‖h e y‖ = ‖δ‖) : ∃ b t : L, ‖b‖ = 1 ∧ ‖t‖ ≤ 1 ∧ h (b • y + t • e) (b • y + t • e) = 0 ∧ h e (b • y + t • e) = δ

theorem **exists_pair** (hH : StHensel) (hP : StPairing) {s : L →+* L} (hs : IsInv s) {δ : L} (hδ : IsDelta s δ) {h : V → V → L} (hh : IsHerm s h) (hiso : ∃ x, x ≠ 0 ∧ h x x = 0) {Λ M : AddSubgroup V} (hΛ : IsMax h Λ) (hM : IsMax h M) {p : L} (hp0 : p ≠ 0) (hp1 : ‖p‖ < 1) (hpm : ∀ a : L, ‖a‖ < 1 → ‖a‖ ≤ ‖p‖) {r : ℕ} (hr : ∀ x ∈ M, p ^ r • x ∈ Λ) (hmin : ∀ k < r, ∃ z ∈ M, p ^ k • z ∉ Λ) (hrM : ∀ y ∈ Λ, p ^ r • y ∈ M) : ∃ e y : V, e ∈ Λ ∧ y ∈ Λ ∧ h e e = 0 ∧ ‖h e y‖ = ‖δ‖ ∧ (p ^ r)⁻¹ • e ∈ M

theorem **plane_of_pair** {s : L →+* L} (hs : IsInv s) {δ : L} (hδ : IsDelta s δ) {h : V → V → L} (hh : IsHerm s h) {Λ M : AddSubgroup V} (hΛ : IsMax h Λ) (hM : IsMax h M) {p : L} (hp0 : p ≠ 0) (hp1 : ‖p‖ < 1) {r : ℕ} (hrM : ∀ y ∈ Λ, p ^ r • y ∈ M) {e y : V} (heΛ : e ∈ Λ) (hyΛ : y ∈ Λ) (he : h e e = 0) (hey : ‖h e y‖ = ‖δ‖) (heM : (p ^ r)⁻¹ • e ∈ M) : ∃ (e f : V) (u : L), u ≠ 0 ∧ h e e = 0 ∧ h f f = 0 ∧ h e f = δ ∧ e ∈ Λ ∧ f ∈ Λ ∧ u • e ∈ M ∧ (s u)⁻¹ • f ∈ M

theorem **stPlane_of_hensel_pairing** : StHensel → StPairing → StPlane

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.PlaneOfHenselAndPairingStatement

theorem **_root_.GlobalTheory.T.plane_of_hensel_and_pairing** : StHensel → StPairing → StPlane

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.PolynomialNilpotentPath

theorem **apply_mem_upperImg** (n : Fin c → ℕ) (h : ∑ i, n i = N) (Xf : Fin N → Fin N → Module.End ℂ V) {y : Module.End ℂ V} (hy : y ∈ Submodule.span ℂ {y : Module.End ℂ V | ∃ (p q : Fin N) (w : Module.End ℂ V), blk (stdBlocks n h) p < blk (stdBlocks n h) q ∧ y = Xf p q * w}) (x : V) : y x ∈ upperImg n h Xf

theorem **exists_monic_trace** (n : Fin c → ℕ) (h : ∑ i, n i = N) (Xf : Fin N → Fin N → Module.End ℂ V) (hX : GlRel Xf) (Pc : ℂ → ℂ) (hP : ∀ Y : ℂ, ∃ k : ℕ, (qdet Xf (Y + ((N : ℂ) - 1) / 2) - Pc Y • (1 : Module.End ℂ V)) ^ k = 0) (r : Fin c → ℂ) : ∃ q : ℂ[X], q.Monic ∧ ∀ x : V, aeval (∑ i, r i • ∑ p : Fin (n i), Xf (stdBlocks n h (i, p)) (stdBlocks n h (i, p))) q x ∈ upperImg n h Xf

theorem **actL_apply** (E : Fin n → Fin n → Module.End ℂ V) (P : Matrix (Fin n) (Fin n) ℂ) : actL E P = aM E P

theorem **actM_bracket** (E : Fin n → Fin n → Module.End ℂ V) (hE : GlRel E) (P Q : Matrix (Fin n) (Fin n) ℂ) : aM E P * aM E Q - aM E Q * aM E P = aM E (P * Q - Q * P)

theorem **conj_mul** {A : Matrix (Fin n) (Fin n) ℂ} (B : GradBasis A) (M M' : Matrix (Fin n) (Fin n) ℂ) : (B.U * M * B.Ui) * (B.U * M' * B.Ui) = B.U * (M * M') * B.Ui

theorem **glRel_Eb** (X : GMod n) {A : Matrix (Fin n) (Fin n) ℂ} (B : GradBasis A) : GlRel (Eb X B)

theorem **A_eq** {A : Matrix (Fin n) (Fin n) ℂ} (B : GradBasis A) : A = B.U * Matrix.diagonal (dG B) * B.Ui

theorem **actM_grading** (X : GMod n) {A : Matrix (Fin n) (Fin n) ℂ} (B : GradBasis A) : CompanionTheory.RgExh.Dec.L.actM X A = ∑ i, (![-1, 0, 1] : Fin 3 → ℂ) i • ∑ p : Fin (dims n B.j i), Eb X B (GlobalTheory.SmGL.stdBlocks (dims n B.j) B.hj (i, p)) (GlobalTheory.SmGL.stdBlocks (dims n B.j) B.hj (i, p))

theorem **mem_nilpC** {A : Matrix (Fin n) (Fin n) ℂ} (B : GradBasis A) {p q : Fin n} (h : bl B p < bl B q) : B.U * Matrix.single p q 1 * B.Ui ∈ CompanionTheory.RgExh.Dec.L.nilpC A

theorem **exists_sum_of_mem_upperImg** (X : GMod n) {A : Matrix (Fin n) (Fin n) ℂ} (B : GradBasis A) {u : X.V} (hu : u ∈ upperImg (dims n B.j) B.hj (Eb X B)) : ∃ (m : ℕ) (Y : Fin m → Matrix (Fin n) (Fin n) ℂ) (v : Fin m → X.V), (∀ k, Y k ∈ CompanionTheory.RgExh.Dec.L.nilpC A) ∧ u = ∑ k, CompanionTheory.RgExh.Dec.L.actM X (Y k) (v k)

theorem **polyNilPath_of** (hB : Basis) (hC : ConjQdet) : CompanionTheory.RgExh.Dec.L.PolyNilPath

theorem **polyNilPath** : CompanionTheory.RgExh.Dec.L.PolyNilPath

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.PolynomialNilpotentPathStatement

theorem **_root_.GlobalTheory.T.PolynomialNilpotentPath** : CompanionTheory.RgExh.Dec.L.PolyNilPath

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.PreDualOfIsPerfect

theorem **preDual_of_isPerfect** {S : Scheme.{0}} [IsLocallyNoetherian S] {A B : Over S} [GrpObj A] [GrpObj B] (hA : IsAbelianScheme A) (hB : IsAbelianScheme B) {P : LB (A ⊗ B).left} (h : IsPerfect A B P) : PreDual A B P where

theorem **pull_lam_of_left** {S : Scheme.{0}} {G T T' : Over S} [GrpObj G] (c : Pic G.left) (u : T'.left ⟶ T.left) {a b : T ⟶ G} {a' b' : T' ⟶ G} (ha : u ≫ a.left = a'.left) (hb : u ≫ b.left = b'.left) : CechPic.pull u (lam c a b) = lam c a' b'

theorem **pull_lam_sq** {S S' : Scheme.{0}} {b : S' ⟶ S} {G : Over S} {G' : Over S'} [GrpObj G] [GrpObj G'] {φ : G'.left ⟶ G.left} (hp : IsGrpPullback G' G b φ) (c : Pic G.left) {l : (G' ⊗ G').left ⟶ (G ⊗ G).left} (l₁ : l ≫ (fst G G).left = (fst G' G').left ≫ φ) (l₂ : l ≫ (snd G G).left = (snd G' G').left ≫ φ) : CechPic.pull l (lam c (fst G G) (snd G G)) = lam (CechPic.pull φ c) (fst G' G') (snd G' G')

theorem **ktrivDual** : KTrivModel → FinDual → KTrivDual

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.PrincipalSum

theorem **prod_ofFn_sub** : ∀ (m : ℕ) (f g : Fin (m + 1) → A) (d : Fin (m + 1)), (∀ j, j ≠ d → f j = g j) → f d = g d + 1 → (List.ofFn f).prod - (List.ofFn g).prod = (List.ofFn (f ∘ d.succAbove)).prod

theorem **Tm_eq_capEntry** {n : ℕ} (E : Fin n → Fin n → A) (u : ℂ) (a b : Fin n) : Tm E u a b = capEntry E u a b

theorem **qdet_eq_colDet** {n : ℕ} (E : Fin n → Fin n → A) (u : ℂ) : qdet E u = colDet E (fun j => u - ((j : ℕ) : ℂ))

theorem **ins_self** {m : ℕ} (d : Fin (m + 1)) (τ : Equiv.Perm (Fin m)) : ins d τ d = d

theorem **ins_succAbove** {m : ℕ} (d : Fin (m + 1)) (τ : Equiv.Perm (Fin m)) (i : Fin m) : ins d τ (d.succAbove i) = d.succAbove (τ i)

theorem **sign_ins** {m : ℕ} (d : Fin (m + 1)) (τ : Equiv.Perm (Fin m)) : Equiv.Perm.sign (ins d τ) = Equiv.Perm.sign τ

theorem **ins_injective** {m : ℕ} (d : Fin (m + 1)) : Function.Injective (ins d)

theorem **exists_ins** {m : ℕ} (d : Fin (m + 1)) (σ : Equiv.Perm (Fin (m + 1))) (h : σ d = d) : ∃ τ, ins d τ = σ

theorem **colDet_sub** {m : ℕ} (E : Fin (m + 1) → Fin (m + 1) → A) (c c' : Fin (m + 1) → ℂ) (d : Fin (m + 1)) (h : ∀ j, j ≠ d → c j = c' j) (hd : c d = c' d + 1) : colDet E c - colDet E c' = ∑ τ : Equiv.Perm (Fin m), ((Equiv.Perm.sign τ : ℤˣ) : ℤ) • (List.ofFn fun i : Fin m => capEntry E (c (d.succAbove i)) (d.succAbove (τ i)) (d.succAbove i)).prod

theorem **hyb_step** {m : ℕ} (E : Fin (m + 1) → Fin (m + 1) → A) (x : ℂ) (d : Fin (m + 1)) : colDet E (hyb x (d : ℕ)) - colDet E (hyb x ((d : ℕ) + 1)) = cminor E d.succAbove d.succAbove (x - 1)

theorem **sum_principal** {m : ℕ} (E : Fin (m + 1) → Fin (m + 1) → A) (x : ℂ) : ∑ d : Fin (m + 1), cminor E d.succAbove d.succAbove (x - 1) = qdet E x - qdet E (x - 1)

theorem **prinSum** : CompanionTheory.RgExh.Hk.PrinSum

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.PrincipalSumStatement

theorem **_root_.GlobalTheory.T.PrincipalSum** : CompanionTheory.RgExh.Hk.PrinSum

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.ProjectionStatement

theorem **stProj** : StProj

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.ProjectionStatementStatement

theorem **_root_.GlobalTheory.T.ProjectionStatement** : StProj

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.QuantumDeterminantAntiHomomorphism

theorem **anti_list_prod** {A B : Type} [Ring A] [Ring B] (f : A → B) (h1 : f 1 = 1) (hm : ∀ x y, f (x * y) = f y * f x) (l : List A) : f l.prod = (l.map f).reverse.prod

theorem **anti_Tm** {A B : Type} [Ring A] [Algebra ℂ A] [Ring B] [Algebra ℂ B] (f : A →ₗ[ℂ] B) (h1 : f 1 = 1) {m : ℕ} (E : Fin m → Fin m → A) (v : ℂ) (a b : Fin m) : f (Tm E v a b) = Tm (fun a b => f (E b a)) v b a

theorem **qdet_eq_qminorRow_rev** {A : Type} [Ring A] [Algebra ℂ A] {m : ℕ} (E : Fin m → Fin m → A) (hE : GlRel E) (u : ℂ) : qdet E u = PI.qminorRow E (id ∘ Fin.revPerm) (id ∘ Fin.revPerm) u

theorem **qdet_anti** {A B : Type} [Ring A] [Algebra ℂ A] [Ring B] [Algebra ℂ B] (f : A →ₗ[ℂ] B) (h1 : f 1 = 1) (hm : ∀ x y, f (x * y) = f y * f x) {m : ℕ} (E : Fin m → Fin m → A) (hE : GlRel E) (u : ℂ) : qdet (fun a b => f (E b a)) u = f (qdet E u)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.QuantumDeterminantAntiHomomorphismStatement

theorem **_root_.GlobalTheory.T.quantum_determinant_anti_homomorphism** : ∀ {A B : Type} [Ring A] [Algebra ℂ A] [Ring B] [Algebra ℂ B] (f : A →ₗ[ℂ] B), f 1 = 1 → (∀ x y, f (x * y) = f y * f x) → ∀ {m : ℕ} (E : Fin m → Fin m → A), GlRel E → ∀ u : ℂ, qdet (fun a b => f (E b a)) u = f (qdet E u)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.QuantumDeterminantBlockEigenvalue

theorem **glRel_comp_inj** {A : Type*} [Ring A] {m n : ℕ} {E : Fin n → Fin n → A} (hE : GlRel E) (e : Fin m → Fin n) (he : Function.Injective e) : GlRel (fun a b => E (e a) (e b))

theorem **glRel_op_transpose** {A : Type*} [Ring A] {n : ℕ} {E : Fin n → Fin n → A} (hE : GlRel E) : GlRel (fun a b => MulOpposite.op (E b a))

theorem **qdet_eigen_of_killed** {n : ℕ} (Q : GMod n) (d : Fin 3 → ℕ) (h : ∑ i, d i = n) (w : Q.V) (hw : ∀ a b, blk (stdBlocks d h) b < blk (stdBlocks d h) a → Q.E a b w = 0) (c : Fin 3 → ℂ → ℂ) (hc : ∀ i Y, Qd Q d h i Y w = c i Y • w) (Y : ℂ) : qdet Q.E (Y + ((n : ℂ) - 1) / 2) w = (∏ i, c i Y) • w

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.QuantumDeterminantBlockEigenvalueStatement

theorem **_root_.GlobalTheory.T.quantum_determinant_block_eigenvalue** : ∀ {n : ℕ} (Q : GMod n) (d : Fin 3 → ℕ) (h : ∑ i, d i = n) (w : Q.V), (∀ a b, blk (stdBlocks d h) b < blk (stdBlocks d h) a → Q.E a b w = 0) → ∀ c : Fin 3 → ℂ → ℂ, (∀ i Y, Qd Q d h i Y w = c i Y • w) → ∀ Y : ℂ, qdet Q.E (Y + ((n : ℂ) - 1) / 2) w = (∏ i, c i Y) • w

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.QuantumDeterminantCoefficientsExpansion

theorem **poly_eq_zero_of_evU** (Q : A[X]) (h : ∀ u : ℂ, evU u Q = 0) : Q = 0

theorem **commute_coeff_of_evU** (P : A[X]) (a : A) (h : ∀ u : ℂ, Commute (evU u P) a) (j : ℕ) : Commute (P.coeff j) a

theorem **coeff_qdetPoly_mem** {n : ℕ} (E : Fin n → Fin n → A) (S : Subalgebra ℂ A) (hS : ∀ p q, E p q ∈ S) (j : ℕ) : (qdetPoly E).coeff j ∈ S

theorem **capelli_coef_commute** : ∀ (W : Type) [AddCommGroup W] [Module ℂ W] (M : ℕ) (d : Fin M → ℕ) (X : ∀ i, Fin (d i) → Fin (d i) → Module.End ℂ W), (∀ i, GlRel (X i)) → (∀ i j, i ≠ j → ∀ p q r v, Commute (X i p q) (X j r v)) → ∃ c : Fin M → ℕ → Module.End ℂ W, (∀ i, c i (d i) = 1) ∧ (∀ i (u : ℂ), qdet (X i) u = ∑ j ∈ Finset.range (d i + 1), u ^ j • c i j) ∧ (∀ i, 1 ≤ d i → c i (d i - 1) = ∑ p, X i p p - (((d i : ℂ) * ((d i : ℂ) - 1) / 2)) • (1 : Module.End ℂ W)) ∧ ∀ i j a b, Commute (c i a) (c j b)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.QuantumDeterminantCoefficientsExpansionStatement

theorem **_root_.GlobalTheory.T.quantum_determinant_coefficients_expansion** : ∀ (W : Type) [AddCommGroup W] [Module ℂ W] (M : ℕ) (d : Fin M → ℕ) (X : ∀ i, Fin (d i) → Fin (d i) → Module.End ℂ W), (∀ i, GlRel (X i)) → (∀ i j, i ≠ j → ∀ p q r v, Commute (X i p q) (X j r v)) → ∃ c : Fin M → ℕ → Module.End ℂ W, (∀ i, c i (d i) = 1) ∧ (∀ i (u : ℂ), qdet (X i) u = ∑ j ∈ Finset.range (d i + 1), u ^ j • c i j) ∧ (∀ i, 1 ≤ d i → c i (d i - 1) = ∑ p, X i p p - (((d i : ℂ) * ((d i : ℂ) - 1) / 2)) • (1 : Module.End ℂ W)) ∧ ∀ i j a b, Commute (c i a) (c j b)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.QuantumDeterminantEquality

theorem **capEntry_eq_Tm** {A : Type*} [Ring A] [Algebra ℂ A] {n : ℕ} (E : Fin n → Fin n → A) (x : ℂ) (a b : Fin n) : capEntry E x a b = Tm E x a b

theorem **qdet_eq_capelliC** {A : Type*} [Ring A] [Algebra ℂ A] {n : ℕ} (E : Fin n → Fin n → A) (x : ℂ) : qdet E x = capelliC E x

theorem **qdetEq** : ∀ (n : ℕ) (M : Type) [AddCommGroup M] [Module ℂ M] (E : Fin n → Fin n → Module.End ℂ M) (x : ℂ), qdet E x = capelliC E x

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.QuantumDeterminantEqualityStatement

theorem **_root_.GlobalTheory.T.QuantumDeterminantEquality** : CompanionTheory.RgExh.Hk.QdetEq

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.QuantumDeterminantTau

theorem **qdetTau** : QdetTau

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.QuantumDeterminantTauStatement

theorem **_root_.GlobalTheory.T.QuantumDeterminantTau** : CompanionTheory.RgExh.Hk.QdetTau

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.QuantumDeterminantTensor

theorem **commute_qdet_of** {A : Type*} [Ring A] [Algebra ℂ A] {n : ℕ} (E : Fin n → Fin n → A) (T : A) (h : ∀ a b, Commute (E a b) T) (u : ℂ) : Commute (qdet E u) T

theorem **commute_compLeft_om** {n : ℕ} (X : GMod n) (u : ℂ) : Commute ((qdet X.E u).compLeft (Fin n)) (om X)

theorem **qdetTens** : QdetTens

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.QuantumDeterminantTensorStatement

theorem **_root_.GlobalTheory.T.QuantumDeterminantTensor** : CompanionTheory.RgExh.Hk.QdetTens

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.QuotientBoundOfFactorReduceChart

theorem **factor_prod** {n : ℕ} {H V ι : Type*} [NormedAddCommGroup H] [NormedSpace ℂ H] [NormedAddCommGroup V] [NormedSpace ℂ V] {G : Set (Mat n)} {σ : Mat n → (H →L[ℂ] H)} (h : RepOn G σ) (B : H → V) (g : ι → Mat n) (c : ι → (V →L[ℂ] V)) (hg : ∀ j, g j ∈ G) (hc : ∀ j v, IsSm G σ v → B (σ (g j) v) = c j (B v)) (v : H) (hv : IsSm G σ v) : ∀ l : List ι, (l.map g).prod ∈ G ∧ B (σ (l.map g).prod v) = (l.map c).prod (B v)

theorem **norm_le_avg** {d : ℕ} {V : Type*} [NormedAddCommGroup V] [NormedSpace ℂ V] [CompleteSpace V] (φ : (Fin d → ℝ) → ℝ) (hφ : Continuous φ) (hc : HasCompactSupport φ) (h0 : ∀ t, 0 ≤ φ t) (h1 : ∫ t, φ t = 1) (f : (Fin d → ℝ) → V) (hf : Continuous f) (b : V) (hb : ∀ t, φ t ≠ 0 → ‖f t - b‖ ≤ ‖b‖ / 2) : ‖b‖ ≤ 2 * ‖∫ t, (φ t : ℂ) • f t‖

theorem **qBound** (hF : Factor) (hR : Reduce) (hC : ChartExists) : QBound

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.QuotientBoundOfFactorReduceChartStatement

theorem **_root_.GlobalTheory.T.quotient_bound_of_factor_reduce_chart** : Factor → Reduce → ChartExists → QBound

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.ReductionOfDerivedSeries

theorem **contDiff_of_smOn_univ** {E V : Type*} [NormedAddCommGroup E] [NormedSpace ℝ E] [NormedAddCommGroup V] [NormedSpace ℝ V] {f : E → V} (hf : SmOn Set.univ f) : ContDiff ℝ ∞ f

theorem **cut_mul** {E : Type*} [NormedAddCommGroup E] [NormedSpace ℝ E] {U : Set E} (hU : IsOpen U) {φ a : E → ℝ} (hφ : ContDiff ℝ ∞ φ) (hc : HasCompactSupport φ) (hs : tsupport φ ⊆ U) (ha : ContDiffOn ℝ ∞ a U) : ContDiff ℝ ∞ (fun t => φ t * a t) ∧ HasCompactSupport (fun t => φ t * a t) ∧ tsupport (fun t => φ t * a t) ⊆ U

theorem **cut_fderiv** {E : Type*} [NormedAddCommGroup E] [NormedSpace ℝ E] {U : Set E} {φ : E → ℝ} (hφ : ContDiff ℝ ∞ φ) (hc : HasCompactSupport φ) (hs : tsupport φ ⊆ U) (v : E) : ContDiff ℝ ∞ (fun t => fderiv ℝ φ t v) ∧ HasCompactSupport (fun t => fderiv ℝ φ t v) ∧ tsupport (fun t => fderiv ℝ φ t v) ⊆ U

theorem **contDiff_orbit** {E : Type*} [NormedAddCommGroup E] [NormedSpace ℝ E] (h : RepOn G σ) {w : H} (hw : IsSm G σ w) {W : List (Mat n)} (hW : LieW G W) {s : E → Mat n} (hs : ContDiff ℝ ∞ s) (hm : ∀ t, s t ∈ G) : ContDiff ℝ ∞ (fun t => dsW σ W (σ (s t) w))

theorem **dsW_sum** (h : RepOn G σ) {W : List (Mat n)} (hW : LieW G W) {ι : Type*} (c : ι → ℂ) (v : ι → H) (hv : ∀ i, IsSm G σ (v i)) (S : Finset ι) : IsSm G σ (∑ i ∈ S, c i • v i) ∧ dsW σ W (∑ i ∈ S, c i • v i) = ∑ i ∈ S, c i • dsW σ W (v i)

theorem **lieW_of** {Hm : Mat n} {W : List (Mat n)} (hW : LieWd Hm W) : LieW (UH Hm) W

theorem **isSm_ds** {Hm : Mat n} (h : RepOn (UH Hm) σ) {X : Mat n} (hX : Lie Hm X) {u : H} (hu : IsSm (UH Hm) σ u) : IsSm (UH Hm) σ (ds σ X u)

theorem **dsW_append** (W W' : List (Mat n)) (v : H) : dsW σ (W ++ W') v = dsW σ W (dsW σ W' v)

theorem **last_letter** [CompleteSpace H] (hlin : DsLin) (had : DsAd) {Hm : Mat n} (hσ : URep Hm σ) {W : List (Mat n)} (hW : LieWd Hm W) {Y y : Mat n} (hY : Lie Hm Y) (hy : y ∈ UH Hm) {d e : ℕ} {Z : Fin d → Mat n} {Q : Fin e → Mat n} (hZ : ∀ j, Lie Hm (Z j)) (hQ : ∀ i, Lie Hm (Q i)) {a : Fin d → ℝ} {b : Fin e → ℝ} (hab : y⁻¹ * Y * y = ∑ j, (a j : ℂ) • Z j + ∑ i, (b i : ℂ) • Q i) {u : H} (hu : IsSm (UH Hm) σ u) : dsW σ (W ++ [Y]) (σ y u) = ∑ j, a j • dsW σ W (σ y (ds σ (Z j) u)) + ∑ i, b i • dsW σ W (σ y (ds σ (Q i) u))

theorem **fderiv_orbit** [CompleteSpace H] (hder : DsDer) {Hm : Mat n} (hσ : URep Hm σ) {W : List (Mat n)} (hW : LieWd Hm W) {E : Type*} [NormedAddCommGroup E] [NormedSpace ℝ E] {s : E → Mat n} (hs : ContDiff ℝ ∞ s) (hm : ∀ t, s t ∈ UH Hm) {t v : E} {Z : Mat n} (hZ : Lie Hm Z) (hfl : ∀ τ : ℝ, s (t + τ • v) = s t * ex Z τ) {u : H} (hu : IsSm (UH Hm) σ u) : fderiv ℝ (fun t => dsW σ W (σ (s t) u)) t v = dsW σ W (σ (s t) (ds σ Z u))

theorem **integrable_cut** {d : ℕ} {ψ : (Fin d → ℝ) → ℝ} (hψ : Continuous ψ) (hc : HasCompactSupport ψ) {f : (Fin d → ℝ) → H} (hf : Continuous f) : Integrable (fun t => ψ t • f t)

theorem **parts** {d : ℕ} {ψ : (Fin d → ℝ) → ℝ} (hψ : ContDiff ℝ ∞ ψ) (hc : HasCompactSupport ψ) {f : (Fin d → ℝ) → H} (hf : ContDiff ℝ ∞ f) (v : Fin d → ℝ) : ∫ t, ψ t • fderiv ℝ f t v = - ∫ t, fderiv ℝ ψ t v • f t

theorem **reduce** : DsLin → DsAd → DsDer → Reduce

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.ReductionOfDerivedSeriesStatement

theorem **_root_.GlobalTheory.T.reduction_of_derived_series** : DsLin → DsAd → DsDer → Reduce

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.RigDescent

theorem **mul_mid** {G : Type*} [CommGroup G] (x y z p q : G) : x * (p * q) * z⁻¹ = (x * p * y⁻¹) * (y * q * z⁻¹)

theorem **mul_fix** {G : Type*} [CommGroup G] (x y t : G) : (x * y⁻¹ * t) * x⁻¹ = t * y⁻¹

theorem **gs_appTop** {W Y : Scheme.{0}} (φ : W ⟶ Y) (u : Γ(Y, ⊤)ˣ) (x : W) : gs (𝟙 W) x ⊤ (φ.appTop.hom u.val) = (gm φ x ⊤ u).val

theorem **lbDescent** (hH : H90) {P Q : Scheme.{0}} (f : P ⟶ Q) (hf : Flat f) (hl : LocallyOfFinitePresentation f) (hs : Surjective f) (M : LB P) (hM : M.IsLB) (c : LB.Gen (LB.LinComb M (K

theorem **rigDescent** : H90 → RigDescent

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.RootVectors

theorem **mul_single_mul_entry** {n : ℕ} (A B : Mat n) (a b i l : Fin n) (c : ℂ) : (A * single a b c * B) i l = A i a * c * B b l

theorem **ex_of_mul_self_eq_zero** {n : ℕ} (Y : Mat n) (hY : Y * Y = 0) (τ : ℝ) : ex Y τ = 1 + (τ : ℂ) • Y

theorem **Hq_row** {i i' : Fin (2 * r + k + 1)} (hi : outer k r i) (h : i.val + i'.val = 2 * r + k) (l : Fin (2 * r + k + 1)) : Hq k r i l = if l = i' then 1 else 0

theorem **Hq_col** {i i' : Fin (2 * r + k + 1)} (hi : outer k r i) (h : i.val + i'.val = 2 * r + k) (l : Fin (2 * r + k + 1)) : Hq k r l i = if l = i' then 1 else 0

theorem **Hq_mul_single** {i i' : Fin (2 * r + k + 1)} (hi : outer k r i) (h : i.val + i'.val = 2 * r + k) (l : Fin (2 * r + k + 1)) (c : ℂ) : Hq k r * single i l c = single i' l c

theorem **single_mul_Hq** {i i' : Fin (2 * r + k + 1)} (hi : outer k r i) (h : i.val + i'.val = 2 * r + k) (l : Fin (2 * r + k + 1)) (c : ℂ) : single l i c * Hq k r = single l i' c

theorem **dg_Hq** {d : Fin (2 * r + k + 1) → ℝ} (hd : IsTor k r d) : (dg d)ᴴ * Hq k r * dg d = Hq k r

theorem **rootE_lie** (a : Fin r) (κ : ℂ) : Lie (Hq k r) (rootE k r a κ)

theorem **rootE_sq** (hk : 1 ≤ k) (a : Fin r) (κ : ℂ) : rootE k r a κ * rootE k r a κ = 0

theorem **dg_rootE** {d : Fin (2 * r + k + 1) → ℝ} (hd : IsTor k r d) (a : Fin r) (κ : ℂ) : dg d * rootE k r a κ = ((d (eI k r a) / d (nxt k r a) : ℝ) : ℂ) • (rootE k r a κ * dg d)

theorem **norm_conj_rootE** {R : Mat (2 * r + k + 1)} (hR : Rᴴ * R = 1) (a : Fin r) (κ : ℂ) : ‖Rᴴ * rootE k r a κ * R‖ ≤ 2 * ‖κ‖

theorem **col_of_mulVec** {n : ℕ} {A : Mat n} {j : Fin n} (h : A *ᵥ bvec j = bvec j) (i : Fin n) : A i j = if i = j then 1 else 0

theorem **frame_row** {Γ : Mat (2 * r + k + 1)} (hΓ : Γᴴ * Hpk k r * Γ = Hq k r) {i i' : Fin (2 * r + k + 1)} (hi : i.val < r ∨ r + k < i.val) (h : i.val + i'.val = 2 * r + k) (hf : Γ *ᵥ bvec i = bvec i) (l : Fin (2 * r + k + 1)) : Γ i' l = if l = i' then 1 else 0

theorem **inv_row** {n : ℕ} {Γ : Mat n} (hu : IsUnit Γ.det) {i : Fin n} (h : ∀ l, Γ i l = if l = i then 1 else 0) (l : Fin n) : Γ⁻¹ i l = if l = i then 1 else 0

theorem **inv_mulVec_fix** {n : ℕ} {Γ : Mat n} (hu : IsUnit Γ.det) (v : Fin n → ℂ) : Γ⁻¹ *ᵥ (Γ *ᵥ v) = v

theorem **rootVec** : RootVec

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.RootVectorsStatement

theorem **_root_.GlobalTheory.T.RootVectors** : RootVec

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.SandwichStatement

theorem **exists_common_denom** {L ι : Type} [Field L] (O : Subring L) (hfrac : ∀ x : L, ∃ a : O, (a : L) ≠ 0 ∧ (a : L) * x ∈ O) (s : Finset ι) (g : ι → L) : ∃ d : O, (d : L) ≠ 0 ∧ ∀ i ∈ s, (d : L) * g i ∈ O

theorem **exists_basis_of_pid** {L V : Type} [Field L] [AddCommGroup V] [Module L V] (O : Subring L) [IsPrincipalIdealRing O] (hfrac : ∀ x : L, ∃ a : O, (a : L) ≠ 0 ∧ (a : L) * x ∈ O) {m : ℕ} (b : Module.Basis (Fin m) L V) (Λ : AddSubgroup V) (c : L) (hc : c ≠ 0) (h1 : ∀ x ∈ Λ, ∀ i, b.repr (c • x) i ∈ O) (hspan : ∀ x : V, ∃ a : L, a ≠ 0 ∧ a • x ∈ Λ) (hO : ∀ a ∈ O, ∀ x ∈ Λ, a • x ∈ Λ) : ∃ (n : ℕ) (B : Module.Basis (Fin n) L V), ∀ x, x ∈ Λ ↔ ∀ i, B.repr x i ∈ O

theorem **stSandwich** : StSandwich

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.SandwichStatementStatement

theorem **_root_.GlobalTheory.T.SandwichStatement** : StSandwich

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.ScalarOfCommute

theorem **monic_eval_capP** : ∀ (n : ℕ) (p : ℂ[X]), p.Monic → p.natDegree = n → ∃ χ : Fin n → ℂ, ∀ Y : ℂ, p.eval Y = Formalism.QuantumDeterminants.capP χ Y | 0, p, hm, hd => (Fin.elim0, fun Y => by rw [eq_one_of_monic_natDegree_zero hm hd] simp [Formalism.QuantumDeterminants.capP]) | n + 1, p, hm, hd => by have hdeg : p.degree ≠ 0

theorem **exists_char** {n : ℕ} {V : Type} [AddCommGroup V] [Module ℂ V] (E : Fin n → Fin n → Module.End ℂ V) (h : ∀ u, ∃ c : ℂ, qdet E u = c • (1 : Module.End ℂ V)) : ∃ χ : Fin n → ℂ, ∀ Y : ℂ, qdet E (Y + ((n : ℂ) - 1) / 2) = Formalism.QuantumDeterminants.capP χ Y • (1 : Module.End ℂ V)

theorem **exists_emb** (hσ : Formalism.MainNotions.S.IsURepOn (Formalism.MainNotions.S.UJset ε) σ) (hirr : Formalism.MainNotions.S.IsIrredOn (Formalism.MainNotions.S.UJset ε) σ) (hK : IsKMod ε σ D E) : ∃ ι : D →₁[ℂ] ReciprocityTheory.ChartRestriction.DS.smD ε σ, Function.Injective ι ∧ (∀ x, ι x ∈ CompanionTheory.D.gk.HC.KF ε σ) ∧ ∀ a b x, ReciprocityTheory.ChartRestriction.DS.smE ε σ a b (ι x) = ι (E a b x)

theorem **exists_kType** [Nontrivial H] (hσ : Formalism.MainNotions.S.IsURepOn (Formalism.MainNotions.S.UJset ε) σ) : ∃ (μ : Fin n → ℝ) (S : Submodule ℂ H), Formalism.MainNotions.S.IsKTypeOf ε σ μ S

theorem **scalar_of_commute** [Nontrivial H] (hσ : Formalism.MainNotions.S.IsURepOn (Formalism.MainNotions.S.UJset ε) σ) (hirr : Formalism.MainNotions.S.IsIrredOn (Formalism.MainNotions.S.UJset ε) σ) (hK : IsKMod ε σ D E) (T : Module.End ℂ D) (hT : ∀ a b, T * E a b = E a b * T) : ∃ c : ℂ, T = c • (1 : Module.End ℂ D)

theorem **intWt** (hσ : Formalism.MainNotions.S.IsURepOn (Formalism.MainNotions.S.UJset ε) σ) (hirr : Formalism.MainNotions.S.IsIrredOn (Formalism.MainNotions.S.UJset ε) σ) (hK : IsKMod ε σ D E) : IntWt (D, E, hK.gl)

theorem **startMod** : CompanionTheory.RgExh.Fin.StartMod

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.ScalarOfSeparatingSmoothL2Maps

theorem **exists_adj_sum** {ε : Fin n → Bool} {E : Fin n → Fin n → Module.End ℂ M} (hU : IsInfUnitary ε E) (l : List (ℂ × List (Fin n × Fin n))) : ∃ B : Module.End ℂ M, ∀ x y : M, inner ℂ ((l.map fun c => c.1 • (c.2.map fun p => E p.1 p.2).prod).sum x) y = inner ℂ x (B y)

theorem **exists_adj_qdet** {ε : Fin n → Bool} {E : Fin n → Fin n → Module.End ℂ M} (hU : IsInfUnitary ε E) (u : ℂ) : ∃ B : Module.End ℂ M, ∀ x y : M, inner ℂ (qdet E u x) y = inner ℂ x (B y)

theorem **adjSign_mul_self** (ε : Fin n → Bool) (a b : Fin n) : adjSign ε a b * adjSign ε a b = 1

theorem **commute_adj** {ε : Fin n → Bool} {E : Fin n → Fin n → Module.End ℂ M} (hU : IsInfUnitary ε E) {A B : Module.End ℂ M} (hA : ∀ a b, Commute A (E a b)) (hB : ∀ x y : M, inner ℂ (A x) y = inner ℂ x (B y)) (c d : Fin n) : Commute B (E c d)

theorem **normal_sq** {A B : Module.End ℂ M} (hB : ∀ x y : M, inner ℂ (A x) y = inner ℂ x (B y)) (hc : Commute A B) (y : M) (h : A (A y) = 0) : A y = 0

theorem **normal_pow** {A B : Module.End ℂ M} (hB : ∀ x y : M, inner ℂ (A x) y = inner ℂ x (B y)) (hc : Commute A B) (k : ℕ) (y : M) (h : (A ^ k) y = 0) : A y = 0

theorem **qdet_shift_pow** {ε : Fin n → Bool} {E : Fin n → Fin n → Module.End ℂ M} (hE : GlRel E) (hU : IsInfUnitary ε E) (w s : ℂ) (k : ℕ) (y : M) (h : ((qdet E w − s • (1 : Module.End ℂ M)) ^ k) y = 0) : (qdet E w − s • (1 : Module.End ℂ M)) y = 0

theorem **scalar_of_separating_smooth_l2_maps** : ScalarCharacterOfL2Embedding

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.SchurOnMinimalSubspace

theorem **schur_on_minimal_subspace** : ∀ (H : Type) [NormedAddCommGroup H] [InnerProductSpace ℂ H] [CompleteSpace H] (N : Set (H →L[ℂ] H)) (Q : Submodule ℂ H), (∀ T ∈ N, star T ∈ N) → IsClosed (Q : Set H) → (∀ T ∈ N, ∀ v ∈ Q, T v ∈ Q) → (∀ U : Submodule ℂ H, IsClosed (U : Set H) → U ≤ Q → (∀ T ∈ N, ∀ v ∈ U, T v ∈ U) → U = ⊥ ∨ U = Q) → ∀ A : H →L[ℂ] H, (∀ T ∈ N, T ∗ A = A ∗ T) → ∃ c : ℂ, ∀ v ∈ Q, A v − c • v ∈ Q⊥

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.SeesawTrivialityAbelianScheme

theorem **val_lift_eq_of_comp_eq** (hA : IsAbelianScheme A) (q : X ⟶ Y) {N : LB (A ⊗ Y).left} (hN : N.IsLB) (r : LB.Gen (ev N (1 : Y ⟶ A) (𝟙 Y))) (t : LB.Gen (ev N (fst A X) (snd A X ≫ q))) (ht : t.IsGen) (hn : ∀ (Z : Scheme.{0}) (s : Z ⟶ X.left) (z : Z) (J : (Z ⟶ (A ⊗ Y).left) → N.ι), Good N z J → val t (s ≫ (lift (1 : X ⟶ A) (𝟙 X)).left) z J = val r (s ≫ q.left) z J) {T : Over S} (α : T ⟶ A) (x x' : T ⟶ X) (h : x ≫ q = x' ≫ q) (z : T.left) (J : (T.left ⟶ (A ⊗ Y).left) → N.ι) (hJ : Good N z J) : val t (lift α x).left z J = val t (lift α x').left z J

theorem **triv_of_triv_of_flat_cover** : ∀ (S : Scheme.{0}) (A X Y : Over S) [GrpObj A], IsAbelianScheme A → ∀ q : X ⟶ Y, Flat q.left → LocallyOfFinitePresentation q.left → Surjective q.left → ∀ N : LB (A ⊗ Y).left, N.IsLB → Triv (ev N (1 : Y ⟶ A) (𝟙 Y)) → Triv (ev N (fst A X) (snd A X ≫ q)) → Triv N

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.SeesawTrivialityAbelianSchemeStatement

theorem **_root_.GlobalTheory.T.seesaw_triviality_abelian_scheme** : ∀ (S : Scheme.{0}) (A X Y : Over S) [GrpObj A], IsAbelianScheme A → ∀ q : X ⟶ Y, Flat q.left → LocallyOfFinitePresentation q.left → Surjective q.left → ∀ N : LB (A ⊗ Y).left, N.IsLB → Triv (ev N (1 : Y ⟶ A) (𝟙 Y)) → Triv (ev N (fst A X) (snd A X ≫ q)) → Triv N

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.SelfDualMap

theorem **selfDual_map** {V W : Type} [AddCommGroup V] [TopologicalSpace V] [IsTopologicalAddGroup V] [MeasurableSpace V] [BorelSpace V] [AddCommGroup W] [TopologicalSpace W] [IsTopologicalAddGroup W] [MeasurableSpace W] [BorelSpace W] (e : V ≃ₜ+ W) (μ : Measure V) (χ : V → V → ℂ) (h : SelfDual μ χ) : SelfDual (μ.map e) (fun x y => χ (e.symm x) (e.symm y))

theorem **iff_of_unit** (ψR : R → ℂ) (lam : Measure R) [SigmaFinite lam] (c : ℝ≥0) (B : Matrix Unit Unit R) (hB : B = 1) (h : ∀ μ : Measure (Unit → R), SelfDual μ (fun x y => ψR (x ⬝ᵥ (B *ᵥ y))) ↔ μ = c • Measure.pi fun _ => lam) (ν : Measure R) : SelfDual ν (fun s t => ψR (s ∗ t)) ↔ ν = c • lam

theorem **selfDual_any** : ∀ (F E : Type) [Field F] [NumberField F] [Field E] [NumberField E] [Algebra F E] (σ : E →ₐ[F] E), IsCMExt F E σ → ∀ (laminf : Measure (Ainf F)) (lam : ∀ v : HeightOneSpectrum (𝒪 F), Measure (Fv F v)), IsStdInf laminf → (∀ v, IsStdV v (lam v)) → ∀ ψ, IsCharAF F ψ → ∃ (cinf : ℝ≥0) (c : HeightOneSpectrum (𝒪 F) → ℝ≥0), IsUnitProd cinf c ∧ (∀ v : Measure (Ainf F), SelfDual ν (fun s t => psiInf F ψ (s ∗ t)) ↔ ν = cinf • laminf) ∧ ∀ v : HeightOneSpectrum (𝒪 F), ∀ ν : Measure (Fv F v), SelfDual ν (fun s t => psiV F ψ v (s ∗ t)) ↔ ν = c v • lam v

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.SeparatedMovingStatement

lemma **cosetEquiv_smul** (e : G ≃* G') (K : Subgroup G) (g : G) (x : G ⧸ K) : cosetEquiv e K (g • x) = e g • cosetEquiv e K x

theorem **stSepMove** : StSepMove

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.SeparatedMovingStatementStatement

theorem **_root_.GlobalTheory.T.SeparatedMovingStatement** : StSepMove

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.SimpleDistinct

private theorem **simplePath_cons_iff** {n : ℕ} (χ : Fin n → ℂ) (a : Fin n) (l : List (Fin n)) : SimplePath χ (a :: l) ↔ (∀ r, r ≠ a → χ r ≠ χ a) ∧ SimplePath (χ + Pi.single a 1) l

theorem **simplePath_ne** {n : ℕ} (ss : List (Fin n)) : ∀ χ : Fin n → ℂ, SimplePath χ ss → ∀ r s, r ≠ s → (r ∈ ss ∨ s ∈ ss) → χ r ≠ χ s

theorem **simpleDistinct** : CompanionTheory.RgExh.Fin.SimpleDistinct

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.SimpleOfCommutativeStatement

theorem **finrank_sq_le_of_span** {A V : Type} [Ring A] [Algebra ℂ A] [AddCommGroup V] [Module ℂ V] [FiniteDimensional ℂ V] (ρ : A →ₐ[ℂ] Module.End ℂ V) (hρ : Function.Surjective ρ) {d l : ℕ} (x y : Fin d → A) (a : Fin l → A) (hs : Submodule.span ℂ {z | ∃ i j, ∃ c ∈ Algebra.adjoin ℂ (Set.range a), z = x i ∗ c ∗ y j} = ⊤) : Module.finrank ℂ V ^ 2 ≤ d ^ 2 ∗ Module.finrank ℂ (Algebra.adjoin ℂ (Set.range fun i => ρ (a i)))

theorem **stSimple_of_stComm** : StComm → StSimple

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.SimpleOfCommutativeStatementStatement

theorem **_root_.GlobalTheory.T.simple_of_commutative_statement** : StComm → StSimple

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.SlabStatement

lemma **wt_add** (c s t : ℝ) : wt c (s + t) = wt c s ∗ wt c t

lemma **wt_ne_zero** (c t : ℝ) : wt c t ≠ 0

lemma **measurable_wt** (c : ℝ) : Measurable (wt c)

lemma **measurable_unitInd** : Measurable unitInd

lemma **lintegral_unitInd** (x : ℝ) : ∫⁻ t, unitInd (x − t) = 1

lemma **measurable_line** {F : G → ℝ≥0∞} {a : ℝ → G} (c : ℝ) (hF : Measurable F) (ha : Measurable a) : Measurable (line F a c)

lemma **line_shift** {F : G → ℝ≥0∞} {a : ℝ → G} {c : ℝ} (hadd : ∀ s t, a (s + t) = a s ∗ a t) (u m : G) (s : ℝ) (hF : Measurable F) (ha : Measurable a) : line F a c (u, m ∗ a s) = wt c (−s) ∗ line F a c (u, m)

lemma **slab_identity** (μM : Measure G) [SFinite μM] {S : Set G} {a : ℝ → G} {δ : G → ℝ≥0∞} {τ : G → ℝ} {c : ℝ} (hSc : μM Sᶜ = 0) (ha : Measurable a) (hδ : Measurable δ) (hτ : Measurable τ) (hinv : ∀ t, μM.map (· ∗ a t) = μM) (hδa : ∀ m ∈ S, ∀ t, δ (m ∗ a t) = δ m ∗ wt c t) (hτa : ∀ m ∈ S, ∀ t, τ (m ∗ a t) = τ m + t) {H : G → ℝ≥0∞} (hH : Measurable H) : ∫⁻ m, unitInd (τ m) ∗ δ m ∗ ∫⁻ t, H (m ∗ a t) ∗ wt c t ∂volume ∂μM = ∫⁻ m, δ m ∗ H m ∂μM

theorem **GlobalTheory.CompanionTheory.SlabLemma.slab** : CompanionTheory.RgExh.Dec.N.S.Slab

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.SlabStatementStatement

theorem **_root_.GlobalTheory.T.SlabStatement** : CompanionTheory.RgExh.Dec.N.S.Slab

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.SnTens2

theorem **sum_list_comm** {α β : Type} [Fintype β] (l : List α) (a : α → β → ℝ) : ∑ e, (l.map fun w => a w e).sum = (l.map fun w => ∑ e, a w e).sum

theorem **act_apply** (X : Obj k r) (g : Formalism.FourierDecayAlt.ThreeDimData.Mx k r) (x : M X) (c : X.1.ι) : CompanionTheory.Zuck.act X g x c = ∑ d, X.1.ρ g c d • X.2.sig g (x d)

theorem **act_tens1** (F E : FRep k r) (U : URep k r) (g : Formalism.FourierDecayAlt.ThreeDimData.Mx k r) (x : M (F.tens E, U)) (e : E.ι) : (fun c => CompanionTheory.Zuck.act (F.tens E, U) g x (c, e) : M (F, U)) = ∑ e', E.ρ g e e' • CompanionTheory.Zuck.act (F, U) g (fun c' => x (c', e'))

theorem **act_tens2** (F E : FRep k r) (U : URep k r) (g : Formalism.FourierDecayAlt.ThreeDimData.Mx k r) (x : M (E.tens F, U)) (e : E.ι) : (fun c => CompanionTheory.Zuck.act (E.tens F, U) g x (e, c) : M (F, U)) = ∑ e', E.ρ g e e' • CompanionTheory.Zuck.act (F, U) g (fun c' => x (e', c'))

theorem **sn_tens1** (F E : FRep k r) (U : URep k r) (W : List (Formalism.FourierDecayAlt.ThreeDimData.Mx k r)) (x : M (F.tens E, U)) : sn (F.tens E, U) W x = ∑ e, sn (F, U) W (fun c => x (c, e))

theorem **sn_tens2** (F E : FRep k r) (U : URep k r) (W : List (Formalism.FourierDecayAlt.ThreeDimData.Mx k r)) (x : M (E.tens F, U)) : sn (E.tens F, U) W x = ∑ e, sn (F, U) W (fun c => x (e, c))

theorem **tensHom** : TensHom

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.SnTens2Statement

theorem **_root_.GlobalTheory.T.tensHom'** : TensHom

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.SpanGenerators

theorem **lie_conj** {A B P Q X : Matrix (Fin n) (Fin n) ℂ} (hQ : Qᴴ * B * Q = A) (hQP : Q * P = 1) (hX : Lie B X) : Lie A (P * X * Q)

theorem **lie_sub** {A X Y : Matrix (Fin n) (Fin n) ℂ} (hX : Lie A X) (hY : Lie A Y) : Lie A (X - Y)

theorem **lie_add** {A X Y : Matrix (Fin n) (Fin n) ℂ} (hX : Lie A X) (hY : Lie A Y) : Lie A (X + Y)

theorem **pr_val** (i : Fin (2 * r + k + 1)) : (pr k r i).val = if i.val ≤ r ∨ r + k ≤ i.val then 2 * r + k - i.val else i.val

theorem **pr_pr** (i : Fin (2 * r + k + 1)) : pr k r (pr k r i) = i

theorem **hq_apply** (i j : Fin (2 * r + k + 1)) : Hq k r i j = if j = pr k r i then 1 else 0

theorem **hq_apply'** (i j : Fin (2 * r + k + 1)) : Hq k r i j = if i = pr k r j then 1 else 0

theorem **lie_hq_iff** (X : Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ) : Lie (Hq k r) X ↔ ∀ i j, star (X (pr k r j) i) + X (pr k r i) j = 0

theorem **mask_pos** {n : ℕ} {p : Fin n → Fin n → Prop} {X : Matrix (Fin n) (Fin n) ℂ} {i j : Fin n} (h : p i j) : mask p X i j = X i j

theorem **mask_neg** {n : ℕ} {p : Fin n → Fin n → Prop} {X : Matrix (Fin n) (Fin n) ℂ} {i j : Fin n} (h : ¬ p i j) : mask p X i j = 0

theorem **lie_mask** {p : Fin (2 * r + k + 1) → Fin (2 * r + k + 1) → Prop} (hp : ∀ i j, p (pr k r j) i → p (pr k r i) j) {X : Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ} (hX : Lie (Hq k r) X) : Lie (Hq k r) (mask p X)

theorem **lie_swap** {X : Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ} (hX : Lie (Hq k r) X) : Lie (Hq k r) (X.submatrix (pr k r) (pr k r))

theorem **lie_scalar** {a : ℂ} (ha : star a + a = 0) : Lie (Hq k r) (a • (1 : Matrix (Fin (2 * r + k + 1)) _ ℂ))

theorem **up_pr** (i j : Fin (2 * r + k + 1)) (h : up k r (pr k r j) i) : up k r (pr k r i) j

theorem **blk_pr** (i j : Fin (2 * r + k + 1)) : blk k r (pr k r j) i ↔ blk k r (pr k r i) j

theorem **span_hq** (hk : 1 ≤ k) {Y : Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ} (hY : Lie (Hq k r) Y) : ∃ B A C : Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ, Lie (Hq k r) B ∧ Lie (Hq k r) A ∧ Lie (Hq k r) C ∧ (∀ i j, ¬ blk k r i j → B i j = 0) ∧ (∀ i, B i (wI k r 0) = B i (wI k r (Fin.last k))) ∧ (∀ i j, ¬ (up k r i j ∧ ¬ blk k r i j) → A i j = 0) ∧ (∀ i j, up k r i j → C i j = 0) ∧ Y = B + A + C

theorem **spanG** : SpanG

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.SpanGeneratorsStatement

theorem **_root_.GlobalTheory.T.SpanGenerators** : SpanG

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.SpecMapAppFromSpec

theorem **factor_iff** {Y Z : Scheme.{0}} (i : Z ⟶ Y) [IsClosedImmersion i] {V : Y.Opens} (hV : IsAffineOpen V) (D : Type) [CommRing D] (φ : Γ(Y, V) →+* D) : (∃ z : Sp D ⟶ Z, z ≫ i = Spec.map (CommRingCat.ofHom φ) ≫ hV.fromSpec) ↔ RingHom.ker (i.app V).hom ≤ RingHom.ker φ

theorem **spread** {C : Type} [CommRing C] [IsNoetherianRing C] (I p : Ideal C) [p.IsPrime] (g : ℕ) (f : Fin g → Localization.AtPrime p) (hf : Ideal.span (Set.range f) = I.map (algebraMap C (Localization.AtPrime p))) : ∃ h : C, h ∉ p ∧ ∀ (C' : Type) [CommRing C'] [Algebra C C'] [IsLocalization.Away h C'], ∃ f' : Fin g → C', Ideal.span (Set.range f') = I.map (algebraMap C C')

theorem **triv_pull_iff** {P Q X : Scheme.{0}} {L : LB X} (hL : L.IsLB) (e : P ≅ Q) (a : Q ⟶ X) (b : P ⟶ X) (h : e.hom ≫ a = b) : Triv (L.pull b) ↔ Triv (L.pull a)

theorem **flat_piece** (hCI : CIFlat) {S Y Z : Scheme.{0}} (s : Y ⟶ S) (i : Z ⟶ Y) [IsClosedImmersion i] [LocallyQuasiFinite (i ≫ s)] {U : S.Opens} (hU : IsAffineOpen U) [IsNoetherianRing Γ(S, U)] {V : Y.Opens} (hV : IsAffineOpen V) (e : V ≤ s ⁻¹ᵁ U) (e' : i ⁻¹ᵁ V ≤ (i ≫ s) ⁻¹ᵁ U) (g : ℕ) (hs : RingHom.IsStandardSmoothOfRelativeDimension g (s.appLE U V e).hom) (f : Fin g → Γ(Y, V)) (hf : Ideal.span (Set.range f) = RingHom.ker (i.app V).hom) : ((i ≫ s).appLE U (i ⁻¹ᵁ V) e').hom.Flat

theorem **SpecMap_app_fromSpec** {Y Z : Scheme.{0}} (i : Z ⟶ Y) [IsClosedImmersion i] {V : Y.Opens} (hV : IsAffineOpen V) (hZV : IsAffineOpen (i ⁻¹ᵁ V)) : Spec.map (i.app V) ≫ hV.fromSpec = hZV.fromSpec ≫ i

theorem **ker_le_prime** {Y Z : Scheme.{0}} (i : Z ⟶ Y) [IsClosedImmersion i] {V : Y.Opens} (hV : IsAffineOpen V) (z : Z) (hy : i z ∈ V) : RingHom.ker (i.app V).hom ≤ (hV.primeIdealOf (i z, hy)).asIdeal

theorem **flat_of_gens** (hCI : CIFlat) {S Y Z : Scheme.{0}} [IsLocallyNoetherian S] (s : Y ⟶ S) (i : Z ⟶ Y) [IsClosedImmersion i] (g : ℕ) [SmoothOfRelativeDimension g s] [LocallyQuasiFinite (i ≫ s)] (H : ∀ (V : Y.Opens), IsAffineOpen V → ∀ (O : Type) [CommRing O] [IsLocalRing O] [IsNoetherianRing O] (φ : Γ(Y, V) →+* O), (RingHom.ker (i.app V).hom).map φ ≠ ⊤ → ∃ f : Fin g → O, Ideal.span (Set.range f) = (RingHom.ker (i.app V).hom).map φ) : Flat (i ≫ s)

theorem **triv_pull_pull** {P Q X : Scheme.{0}} {L : LB X} (hL : L.IsLB) (a : Q ⟶ X) (b : P ⟶ Q) : Triv ((L.pull a).pull b) ↔ Triv (L.pull (b ≫ a))

theorem **few_gens** (hF : FewEquations) {S : Scheme.{0}} (A Y Z : Over S) [GrpObj A] (g : ℕ) (hab : IsAbelianScheme A) (hsm : SmoothOfRelativeDimension g A.hom) (N : LB (A ⊗ Y).left) (hN : N.IsLB) (i : Z ⟶ Y) (hi : IsTrivLocus A N i) {V : Y.left.Opens} (hV : IsAffineOpen V) (O : Type) [CommRing O] [IsLocalRing O] [IsNoetherianRing O] (φ : Γ(Y.left, V) →+* O) (hne : (RingHom.ker (i.left.app V).hom).map φ ≠ ⊤) : ∃ f : Fin g → O, Ideal.span (Set.range f) = (RingHom.ker (i.left.app V).hom).map φ

theorem **trivLocusFlat** : FewEquations → CIFlat → TrivLocusFlat

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.SplittingOfSandwichAndPairing

theorem **lat_smul_mem** {Λ : AddSubgroup V} (hΛ : IsLat L Λ) {a : L} (ha : ‖a‖ ≤ 1) {x : V} (hx : x ∈ Λ) : a • x ∈ Λ

theorem **lat_absorb** {Λ : AddSubgroup V} (hΛ : IsLat L Λ) {ι : Type} [Fintype ι] (v : ι → V) : ∃ c : L, c ≠ 0 ∧ ‖c‖ ≤ 1 ∧ ∀ i, c • v i ∈ Λ

theorem **lat_bdd** {V' : Type} [AddCommGroup V'] [Module L V'] {Λ : AddSubgroup V} {Λ₀ : AddSubgroup V'} (hΛ : IsLat L Λ) (hΛ₀ : IsLat L Λ₀) (P : V →ₗ[L] V') : ∃ c : L, c ≠ 0 ∧ ‖c‖ ≤ 1 ∧ ∀ x ∈ Λ, c • P x ∈ Λ₀

theorem **latOf_isLat** {m : ℕ} (b : Module.Basis (Fin m) L V) : IsLat L (latOf b)

theorem **herm_add_left** (hh : IsHerm s h) (x y z : V) : h (x + y) z = h x z + h y z

theorem **herm_smul_left** (hh : IsHerm s h) (a : L) (x y : V) : h (a • x) y = s a * h x y

@[simp] theorem **hlin_apply** (hh : IsHerm s h) (x y : V) : hlin hh x y = h x y

theorem **ca_apply** (hh : IsHerm s h) (δ : L) (f x : V) : ca hh δ f x = (s δ)⁻¹ * h f x

theorem **cb_apply** (hh : IsHerm s h) (δ : L) (e x : V) : cb hh δ e x = δ⁻¹ * h e x

theorem **pr_apply** (hh : IsHerm s h) (δ : L) (e f x : V) : pr hh δ e f x = x - ca hh δ f x • e - cb hh δ e x • f

theorem **decomp** (hh : IsHerm s h) (δ : L) (e f x : V) : x = ca hh δ f x • e + cb hh δ e x • f + pr hh δ e f x

theorem **h_fe** (hh : IsHerm s h) (hef : h e f = δ) : h f e = s δ

theorem **pr_orth** (hh : IsHerm s h) (hδ : δ ≠ 0) (he : h e e = 0) (hf : h f f = 0) (hef : h e f = δ) (x : V) : h e (pr hh δ e f x) = 0 ∧ h f (pr hh δ e f x) = 0

theorem **coef_eq** (hh : IsHerm s h) (hδ : δ ≠ 0) (he : h e e = 0) (hf : h f f = 0) (hef : h e f = δ) (a b : L) {w : V} (hew : h e w = 0) (hfw : h f w = 0) : ca hh δ f (a • e + b • f + w) = a ∧ cb hh δ e (a • e + b • f + w) = b ∧ pr hh δ e f (a • e + b • f + w) = w

theorem **norm_form** (hs : IsInv s) (hh : IsHerm s h) (he : h e e = 0) (hf : h f f = 0) (hef : h e f = δ) (a b : L) {w : V} (hew : h e w = 0) (hfw : h f w = 0) : h (a • e + b • f + w) (a • e + b • f + w) = δ * (s a * b) + s (δ * (s a * b)) + h w w

theorem **coef_int** (hs : IsInv s) (hδ : δ ≠ 0) (hh : IsHerm s h) {Λ : AddSubgroup V} (hΛ : IsLat L Λ) (hpair : ∀ x ∈ Λ, ∀ y ∈ Λ, ‖h x y‖ ≤ ‖δ‖) (heΛ : e ∈ Λ) (hfΛ : f ∈ Λ) {x : V} (hx : x ∈ Λ) : ‖ca hh δ f x‖ ≤ 1 ∧ ‖cb hh δ e x‖ ≤ 1 ∧ pr hh δ e f x ∈ Λ

theorem **herm_restrict** (hh : IsHerm s h) (hδ : δ ≠ 0) (he : h e e = 0) (hf : h f f = 0) (hef : h e f = δ) (hU : ∀ x, x ∈ U ↔ h e x = 0 ∧ h f x = 0) : IsHerm s (fun x y : U => h x y)

theorem **restrict_isLat** (hS : StSandwich) [ProperSpace L] [FiniteDimensional L V] (hh : IsHerm s h) (hδ : δ ≠ 0) (he : h e e = 0) (hf : h f f = 0) (hef : h e f = δ) (hU : ∀ x, x ∈ U ↔ h e x = 0 ∧ h f x = 0) {Λ : AddSubgroup V} (hΛ : IsLat L Λ) : IsLat L (Λ.comap U.subtype.toAddMonoidHom)

theorem **restrict_isMax** (hS : StSandwich) [ProperSpace L] [FiniteDimensional L V] (hs : IsInv s) (hδ : IsDelta s δ) (hh : IsHerm s h) (he : h e e = 0) (hf : h f f = 0) (hef : h e f = δ) (hU : ∀ x, x ∈ U ↔ h e x = 0 ∧ h f x = 0) {Λ : AddSubgroup V} (hΛ : IsMax h Λ) (hpair : ∀ x ∈ Λ, ∀ y ∈ Λ, ‖h x y‖ ≤ ‖δ‖) (heΛ : e ∈ Λ) (hfΛ : f ∈ Λ) : IsMax (fun x y : U => h x y) (Λ.comap U.subtype.toAddMonoidHom)

theorem **split_mem_iff** (hs : IsInv s) (hδ : δ ≠ 0) (hh : IsHerm s h) (he : h e e = 0) (hf : h f f = 0) (hef : h e f = δ) {Λ : AddSubgroup V} (hΛ : IsLat L Λ) (hpair : ∀ x ∈ Λ, ∀ y ∈ Λ, ‖h x y‖ ≤ ‖δ‖) (heΛ : e ∈ Λ) (hfΛ : f ∈ Λ) (a b : L) {w : V} (hew : h e w = 0) (hfw : h f w = 0) : a • e + b • f + w ∈ Λ ↔ ‖a‖ ≤ 1 ∧ ‖b‖ ≤ 1 ∧ w ∈ Λ

theorem **stSplit** : StSandwich → StPairing → StSplit

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.SplittingOfSandwichAndPairingStatement

theorem **_root_.GlobalTheory.T.splitting_of_sandwich_and_pairing** : StSandwich → StPairing → StSplit

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.StTensor

theorem **exists_good** {G X H : Type} [Group G] [TopologicalSpace G] [NormedAddCommGroup X] [InnerProductSpace ℂ X] [NormedAddCommGroup H] [InnerProductSpace ℂ H] [CompleteSpace H] {δ : G →* (X →L[ℂ] X)} {ρ : G →* (H →L[ℂ] H)} {A : X → X → (H →L[ℂ] H)} (hA : IsOpFam δ ρ A) (hi : IsIsotypic δ ρ) (hδ : IsIrrUnitary δ) (hρ : IsUnitaryRep ρ) (K : Subgroup G) (hl : ∀ x : X, x ≠ 0 → (∀ k ∈ K, δ k x = x) → ∀ y, (∀ k ∈ K, δ k y = y) → ∃ c : ℂ, y = c • x) {w : H} (hw : w ≠ 0) (hK : ∀ k ∈ K, ρ k w = w) : ∃ ξ : X, ‖ξ‖ = 1 ∧ (∀ k ∈ K, δ k ξ = ξ) ∧ ∀ h : H, (∀ k ∈ K, ρ k h = h) → A ξ ξ h = h

theorem **ext_projPl** {a b : GAd σ J} (h : ∀ u, projPl σ J u a = projPl σ J u b) : a = b

theorem **commute_embPl** {v : Formalism.MainNotions.SchurIrreducible.Pl F} {g : GAd σ J} (h : projPl σ J v g = 1) (x : GPl σ J v) : Commute g (embPl σ J v x)

theorem **exists_levelOff_fixed** (hFix : StFixVec) (L : Submodule (𝒪 E) (Fin n → E)) [Submodule.IsLattice E L] {H : Type} [NormedAddCommGroup H] [InnerProductSpace ℂ H] {π : GAd σ J →* (H →L[ℂ] H)} (hπ : IsIrrUnitary π) : ∃ (S : Finset (HeightOneSpectrum (𝒪 F))) (w : H), w ≠ 0 ∧ ∀ k ∈ levelOff σ J L S, π k w = w

theorem **eventually_line** (hAdm : GlobalTheory.Published.UnitaryIrreducibleAdmissible) (hc : IsCMExt F E σ) (hJ : IsHermND σ J) (L : Submodule (𝒪 E) (Fin n → E)) [hL : Submodule.IsLattice E L] : ∀ᶠ v in cofinite, ∀ (X : Type) [NormedAddCommGroup X] [InnerProductSpace ℂ X] (δ : GLoc σ J v →* (X →L[ℂ] X)), IsIrrUnitary δ → ∀ x : X, x ≠ 0 → (∀ k ∈ stabLoc σ J L v, δ k x = x) → ∀ y, (∀ k ∈ stabLoc σ J L v, δ k y = y) → ∃ c : ℂ, y = c • x

theorem **exists_finset_projPl_mem** (L : Submodule (𝒪 E) (Fin n → E)) [Submodule.IsLattice E L] (g : GAd σ J) : ∃ t : Finset (Formalism.MainNotions.SchurIrreducible.Pl F), ∀ u ∉ t, projPl σ J u g ∈ plK σ J L u

theorem **projPl_eq_one_of_mem_levelOff** {L : Submodule (𝒪 E) (Fin n → E)} {S : Finset (HeightOneSpectrum (𝒪 F))} {k : GAd σ J} (hk : k ∈ levelOff σ J L S) {u : Formalism.MainNotions.SchurIrreducible.Pl F} (hu : ∀ v, u = .inr v → v ∈ S) : projPl σ J u k = 1

theorem **embPl_mem_levelOff** {L : Submodule (𝒪 E) (Fin n → E)} {S : Finset (HeightOneSpectrum (𝒪 F))} {v : HeightOneSpectrum (𝒪 F)} (hv : v ∉ S) {k : GLoc σ J v} (hk : k ∈ stabLoc σ J L v) : emb σ J v k ∈ levelOff σ J L S

theorem **stTensor** : StTensor

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.StTensorStatement

theorem **_root_.GlobalTheory.T.stTensor'** : StTensor

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.StandardBlocksLevelSum

theorem **card_lev_lt_succ** {n : ℕ} (lev : Fin n → ℕ) (k : ℕ) : (univ.filter fun a => lev a < k + 1).card = (univ.filter fun a => lev a < k).card + (univ.filter fun a => lev a = k).card

theorem **sum_sz_castLE** {n M : ℕ} (lev : Fin n → ℕ) : ∀ (k : ℕ) (hk : k ≤ M), ∑ b : Fin k, sz lev M (Fin.castLE hk b) = (univ.filter fun a => lev a < k).card

theorem **lt_card_iff** {n : ℕ} {lev : Fin n → ℕ} (hm : Monotone lev) (k : ℕ) (a : Fin n) : (a : ℕ) < (univ.filter fun b => lev b < k).card ↔ lev a < k

theorem **lev_stdBlocks** {n M : ℕ} (lev : Fin n → ℕ) (h : ∑ i, sz lev M i = n) (hm : Monotone lev) (i : Fin M) (p : Fin (sz lev M i)) : lev (stdBlocks (sz lev M) h ⟨i, p⟩) = (i : ℕ)

theorem **stdBlocks_clusters** : ∀ (n M : ℕ) (lev : Fin n → ℕ) (h : ∑ i, sz lev M i = n), Monotone lev → (∀ a, lev a < M) → (∀ (i : Fin M) (p : Fin (sz lev M i)), lev (stdBlocks (sz lev M) h ⟨i, p⟩) = (i : ℕ)) ∧ ∀ (f : Fin n → ℂ) (i : Fin M), ∑ p : Fin (sz lev M i), f (stdBlocks (sz lev M) h ⟨i, p⟩) = ∑ a ∈ univ.filter (fun a => lev a = (i : ℕ)), f a

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.StandardBlocksLevelSumStatement

theorem **_root_.GlobalTheory.T.standard_blocks_level_sum** : ∀ (n M : ℕ) (lev : Fin n → ℕ) (h : ∑ i, sz lev M i = n), Monotone lev → (∀ a, lev a < M) → (∀ (i : Fin M) (p : Fin (sz lev M i)), lev (stdBlocks (sz lev M) h ⟨i, p⟩) = (i : ℕ)) ∧ ∀ (f : Fin n → ℂ) (i : Fin M), ∑ p : Fin (sz lev M i), f (stdBlocks (sz lev M) h ⟨i, p⟩) = ∑ a ∈ univ.filter (fun a => lev a = (i : ℕ)), f a

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.StandardDifferential

theorem **dR_std** (k r : ℕ) (X : Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ) : (vr k r false).dR X = X

theorem **dR_dual** (k r : ℕ) (X : Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ) : (vr k r true).dR X = -Xᵀ

theorem **reH_add_imH** (k r : ℕ) (Y : Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ) : reH k r Y + Complex.I • imH k r Y = Y

theorem **stdD** : StdD

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.StandardDifferentialStatement

theorem **_root_.GlobalTheory.T.StandardDifferential** : StdD

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.StandardPolynomialVanishesOnBicommutant

theorem **diag_mem_of_bicomm** (σ : G →* (H →L[ℂ] H)) (hσ : ∀ g v, ‖σ g v‖ = ‖v‖) (C : Submodule ℂ (PiLp 2 (fun _ : ι => H))) (hcl : IsClosed (C : Set (PiLp 2 (fun _ : ι => H)))) (hC : ∀ g, ∀ z ∈ C, diag (σ g) z ∈ C) (T : H →L[ℂ] H) (hT : ∀ S : H →L[ℂ] H, (∀ g, S * σ g = σ g * S) → S * T = T * S) (z : PiLp 2 (fun _ : ι => H)) (hz : z ∈ C) : diag T z ∈ C

theorem **sum_eq_zero_of_bicomm** (σ : G →* (H →L[ℂ] H)) (hσ : ∀ g v, ‖σ g v‖ = ‖v‖) (L : ι → H →L[ℂ] H) (y : ι → H) (h : ∀ g, ∑ k, L k (σ g (y k)) = 0) (T : H →L[ℂ] H) (hT : ∀ S : H →L[ℂ] H, (∀ g, S * σ g = σ g * S) → S * T = T * S) : ∑ k, L k (T (y k)) = 0

theorem **standard_polynomial_vanishes_on_bicommutant** (σ : G →* (H →L[ℂ] H)) (hσ : ∀ g v, ‖σ g v‖ = ‖v‖) (E : H →L[ℂ] H) (M : ℕ) (h : ∀ g : Fin M → G, stdPoly E (fun i => σ (g i)) = 0) (a : Fin M → H →L[ℂ] H) (ha : ∀ i, ∀ T : H →L[ℂ] H, (∀ g, T * σ g = σ g * T) → T * a i = a i * T) : stdPoly E a = 0

theorem **stdPoly_bicomm** : ∀ (G H : Type) [Group G] [NormedAddCommGroup H] [InnerProductSpace ℂ H] [CompleteSpace H] (σ : G →* (H →L[ℂ] H)), (∀ g v, ‖σ g v‖ = ‖v‖) → ∀ (E : H →L[ℂ] H) (m : ℕ), (∀ g : Fin m → G, stdPoly E (fun i => σ (g i)) = 0) → ∀ a : Fin m → (H →L[ℂ] H), (∀ i, ∀ T : H →L[ℂ] H, (∀ g, T * σ g = σ g * T) → T * a i = a i * T) → stdPoly E a = 0

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.StarProjectionCommute

theorem **inner_map_map** {G H : Type} [Group G] [NormedAddCommGroup H] [InnerProductSpace ℂ H] (σ : G →* (H →L[ℂ] H)) (hσ : ∀ g v, ‖σ g v‖ = ‖v‖) (g : G) (x y : H) : ⟪σ g x, σ g y⟫_ℂ = ⟪x, y⟫_ℂ

theorem **map_inv_map** {G H : Type} [Group G] [NormedAddCommGroup H] [InnerProductSpace ℂ H] (σ : G →* (H →L[ℂ] H)) (g : G) (x : H) : σ g (σ g⁻¹ x) = x

theorem **orthogonal_stable** {G H : Type} [Group G] [NormedAddCommGroup H] [InnerProductSpace ℂ H] (σ : G →* (H →L[ℂ] H)) (hσ : ∀ g v, ‖σ g v‖ = ‖v‖) (U : Submodule ℂ H) (hU : ∀ g, ∀ v ∈ U, σ g v ∈ U) (g : G) (v : H) (hv : v ∈ Uᗮ) : σ g v ∈ Uᗮ

theorem **starProjection_commute** {G H : Type} [Group G] [NormedAddCommGroup H] [InnerProductSpace ℂ H] (σ : G →* (H →L[ℂ] H)) (hσ : ∀ g v, ‖σ g v‖ = ‖v‖) (U : Submodule ℂ H) [U.HasOrthogonalProjection] (hU : ∀ g, ∀ v ∈ U, σ g v ∈ U) (g : G) : U.starProjection ∘ σ g = σ g ∘ U.starProjection

theorem **exists_isIrredSubrep** : ∀ (G H : Type) [Group G] [NormedAddCommGroup H] [InnerProductSpace ℂ H] [CompleteSpace H] (σ : G →* (H →L[ℂ] H)), (∀ g v, ‖σ g v‖ = ‖v‖) → ∀ (Q : Submodule ℂ H) (ξ : H), IsClosed (Q : Set H) → ξ ∈ Q → ξ ≠ 0 → (∀ T : H →L[ℂ] H, (∀ g, T * σ g = σ g * T) → ∀ v ∈ Q, T v ∈ Q) → (∀ g, ∃ c : ℂ, σ g ξ - c • ξ ∈ Qᗮ) → ∃ W, IsIrredSubrep σ W

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.StatementCommutative

theorem **arith_nil** (N D D' E m K : ℕ) (hN : 0 < N) (h1 : D ≤ (2 * N - 1) * D' + E) (h2 : (2 * N - 1 + 1) * E ≤ m ^ 2) (h3 : 4 * N ^ 2 * D' ≤ m ^ 2 + K * m) : N * D ≤ m ^ 2 + K * m

theorem **adjoin_shift** {V : Type} [AddCommGroup V] [Module ℂ V] {l : ℕ} (a : Fin (l + 1) → Module.End ℂ V) (μ : ℂ) : Algebra.adjoin ℂ (Set.range (Fin.cons (a 0 - algebraMap ℂ _ μ) (fun i : Fin l => a i.succ) : Fin (l + 1) → Module.End ℂ V)) = Algebra.adjoin ℂ (Set.range a)

theorem **positionOne_invariantIntegral_exchange_step** (l N K : ℕ) (hN : 0 < N) (hK : ∀ (V : Type) [AddCommGroup V] [Module ℂ V] [FiniteDimensional ℂ V] (a : Fin l → Module.End ℂ V), (∀ i j, Commute (a i) (a j)) → 4 * N ^ 2 * Module.finrank ℂ (Algebra.adjoin ℂ (Set.range a)) ≤ Module.finrank ℂ V ^ 2 + K * Module.finrank ℂ V) : ∀ (n : ℕ) (V : Type) [AddCommGroup V] [Module ℂ V] [FiniteDimensional ℂ V] (a : Fin (l + 1) → Module.End ℂ V), Module.finrank ℂ V = n → (∀ i j, Commute (a i) (a j)) → N * Module.finrank ℂ (Algebra.adjoin ℂ (Set.range a)) ≤ Module.finrank ℂ V ^ 2 + K * Module.finrank ℂ V

theorem **stComm** : StComm

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.StatementCommutativeStatement

theorem **_root_.GlobalTheory.T.StatementCommutative** : StComm

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.StatementStar

theorem **restrictAlg_coe** [AddCommGroup V] [Module ℂ V] (ρ : A →ₐ[ℂ] Module.End ℂ V) (W : Submodule ℂ V) (hW : ∀ h, ∀ w ∈ W, ρ h w ∈ W) (h : A) (x : W) : ((restrictAlg ρ W hW h x : W) : V) = ρ h x

theorem **inf_orthogonal_invariant** [NormedAddCommGroup V] [InnerProductSpace ℂ V] (ρ : A →ₐ[ℂ] Module.End ℂ V) (hs : StarClosed ρ) {U W : Submodule ℂ V} (hU : ∀ h, ∀ u ∈ U, ρ h u ∈ U) (hW : ∀ h, ∀ w ∈ W, ρ h w ∈ W) : ∀ h, ∀ x ∈ Uᗮ ⊓ W, ρ h x ∈ Uᗮ ⊓ W

private theorem **map_word_one** {A B : Type} [Ring A] [Ring B] [Algebra ℂ A] [Algebra ℂ B] (f : A →ₐ[ℂ] B) (l : List A) : f (word 1 l) = word 1 (l.map f)

private theorem **map_stdPoly_one** {A B : Type} [Ring A] [Ring B] [Algebra ℂ A] [Algebra ℂ B] (f : A →ₐ[ℂ] B) {M : ℕ} (b : Fin M → A) : f (stdPoly 1 b) = stdPoly 1 (fun i => f (b i))

theorem **stStar** : StStar

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.StatementStarStatement

theorem **_root_.GlobalTheory.T.StatementStar** : StStar

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.SubChar

theorem **sub_char** {W : Type} [AddCommGroup W] [Module ℂ W] (R : Subalgebra ℂ (Module.End ℂ W)) (hc : ∀ x ∈ R, ∀ y ∈ R, x * y = y * x) (hfg : R.FG) (t : W) (ht : t ≠ 0) : ∃ f : Module.End ℂ W → ℂ, f 1 = 1 ∧ (∀ x ∈ R, ∀ y ∈ R, f (x + y) = f x + f y ∧ f (x * y) = f x * f y) ∧ (∀ x ∈ R, ∀ c : ℂ, f (c • x) = c * f x) ∧ ∀ x ∈ R, ∀ c : ℂ, x t = c • t → f x = c

theorem **L_core** : ∀ (W : Type) [AddCommGroup W] [Module ℂ W] (t : W) (M : ℕ) (d : Fin M → ℕ) (B : ∀ i, Fin (d i) → Fin (d i) → Module.End ℂ W) (s : Fin M → ℂ) (n : ℕ) (χ : Fin n → ℂ) (τ : Fin M → ℂ), t ≠ 0 → (∀ i, GlRel (B i)) → (∀ i j, i ≠ j → ∀ p q r v, Commute (B i p q) (B j r v)) → (∀ Y : ℂ, (List.ofFn fun i => qdet (B i) (Y + s i)).prod = Formalism.QuantumDeterminants.capP χ Y • (1 : Module.End ℂ W)) → (∀ i, (∑ p, B i p p) t = τ i • t) → ∃ P : Fin M → ℂ[X], (∀ i, (P i).Monic ∧ (P i).natDegree = d i ∧ (P i).nextCoeff = τ i - (d i : ℂ) * ((d i : ℂ) - 1) / 2) ∧ ∀ Y : ℂ, ∏ i, (P i).eval (Y + s i) = Formalism.QuantumDeterminants.capP χ Y

theorem **L_lF** : ∀ (n : ℕ) (lev : Fin n → ℕ) (V : Type) [AddCommGroup V] [Module ℂ V] (E : Fin n → Fin n → Module.End ℂ V), Monotone lev → GlRel E → ∀ (x : Cn n V), SuppIn (inU lev) x → (∀ p q, lev p = lev q → SuppIn (inU lev) (lF E p q x)) ∧ (∀ j, Deg j x → Deg j (lF E p q x)) ∧ lF E p q (bd (allT n) E x) = bd (allT n) E (lF E p q x)) ∧ ∀ p q r v, lev p = lev q → lev r = lev v → lF E p q (lF E r v x) - lF E r v (lF E p q x) = (if q = r then lF E p v x else 0) - (if v = p then lF E r q x else 0)

theorem **coBlock** : InjU → GenCOU → COBlock

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.SumInsertErase

theorem **sg_insert** (i : NC ε) (A : Finset (NC ε)) : sg i (insert i A) = sg i A

theorem **conj_sg** (i : NC ε) (A : Finset (NC ε)) : (starRingEnd ℂ) (sg i A) = sg i A

theorem **sum_insert_erase** (i : NC ε) (f : Finset (NC ε) → Finset (NC ε) → ℂ) : (∑ A : Finset (NC ε), if i ∈ A then 0 else f (insert i A) A) = ∑ B : Finset (NC ε), if i ∈ B then f B (B.erase i) else 0

theorem **term_adj** (i : NC ε) (T T' : Module.End ℂ V) (h : ∀ x y : V, ⟪T x, y⟫_ℂ = -⟪x, T' y⟫_ℂ) (w x : Ch ε V) : ip (lift ε T (ann i w)) x = -ip w (lift ε T' (cre i x))

theorem **ip_sum_left** {ι : Type} (s : Finset ι) (f : ι → Ch ε V) (x : Ch ε V) : ip (∑ i ∈ s, f i) x = ∑ i ∈ s, ip (f i) x

theorem **ip_sum_right** {ι : Type} (s : Finset ι) (f : ι → Ch ε V) (w : Ch ε V) : ip w (∑ i ∈ s, f i) = ∑ i ∈ s, ip w (f i)

theorem **adj** : CompanionTheory.D.DSIC.Adj

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.SumOfLevelSizesEq

theorem **sum_sz_eq_card** : ∀ (n M : ℕ) (lev : Fin n → ℕ), (∀ a, lev a < M) → ∑ i, sz lev M i = n

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.SumOfLevelSizesEqStatement

theorem **_root_.GlobalTheory.T.sum_of_level_sizes_eq** : ∀ (n M : ℕ) (lev : Fin n → ℕ), (∀ a, lev a < M) → ∑ i, sz lev M i = n

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.TensorPowerIntegral

theorem **okTens** : OKTens

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.TensorPowerIntegralStatement

theorem **_root_.GlobalTheory.T.TensorPowerIntegral** : OKTens

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.TensorQuotientQuantumDeterminantEigen

theorem **exists_capelliPoly_eq_sum** {m : ℕ} (χ : Fin m → ℂ) : ∃ a : ℕ → ℂ, ∀ Y : ℂ, Formalism.QuantumDeterminants.capP χ Y = ∑ j ∈ Finset.range (m + 1), Y ^ j * a j

theorem **exists_raised** : StepChar → TensDecomp → ∀ {m : ℕ} (M : GMod m) (χ : Fin m → ℂ), HasChar M χ → ∀ {V : Type} [AddCommGroup V] [Module ℂ V] (E : Fin m → Fin m → Module.End ℂ V), GlRel E → ∀ Φ : (Fin m → M.V) →ₗ[ℂ] V, (∀ a b u, Φ (tensE M a b u) = E a b (Φ u)) → Φ ≠ 0 → ∃ r, ∃ w ∈ LinearMap.range Φ, w ≠ 0 ∧ ∀ Y : ℂ, qdet E (Y + ((m : ℂ) - 1) / 2) w = Formalism.QuantumDeterminants.capP (χ + Pi.single r 1) Y • w

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.TensorQuotientQuantumDeterminantEigenStatement

theorem **_root_.GlobalTheory.T.tensor_quotient_quantum_determinant_eigen** : StepChar → TensDecomp → ∀ {m : ℕ} (M : GMod m) (χ : Fin m → ℂ), HasChar M χ → ∀ {V : Type} [AddCommGroup V] [Module ℂ V] (E : Fin m → Fin m → Module.End ℂ V), GlRel E → ∀ Φ : (Fin m → M.V) →ₗ[ℂ] V, (∀ a b u, Φ (tensE M a b u) = E a b (Φ u)) → Φ ≠ 0 → ∃ r, ∃ w ∈ LinearMap.range Φ, w ≠ 0 ∧ ∀ Y : ℂ, qdet E (Y + ((m : ℂ) - 1) / 2) w = Formalism.QuantumDeterminants.capP (χ + Pi.single r 1) Y • w

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.TheoremOfCubeGeneratorExists

theorem **aH_comp** {T T' : Over S} (v : T' ⟶ T) (x : T ⟶ X) : aH A v ≫ aH A x = aH A (v ≫ x)

theorem **zS_aH** [GrpObj A] {T : Over S} (x : T ⟶ X) : zS A T ≫ aH A x = x ≫ zS A X

theorem **zS_left_snd** [GrpObj A] (T : Over S) : (zS A T).left ≫ (snd A T).left = 𝟙 T.left

theorem **exists_normed** (hM : M.IsLB) (hR : R.IsGen) {T : Over S} (x x' : T ⟶ X) (h : LBIso (ev M (fst A T) (snd A T ≫ x)) (ev M (fst A T) (snd A T ≫ x'))) : ∃ l, Normed M R x x' l

theorem **normed_mul** (hG : ReciprocityTheory.ChartRestriction.G.Out.GlobalFnsSt) (hA : IsAbelianScheme A) (hM : M.IsLB) {T : Over S} {x x' x'' : T ⟶ X} {l₁ l₂ l₃} (h₁ : Normed M R x x' l₁) (h₂ : Normed M R x' x'' l₂) (h₃ : Normed M R x x'' l₃) (Z : Scheme.{0}) (r : Z ⟶ (A ⊗ T).left) (z : Z) (J) (hJ : Good M z J) : val l₁ r z J * val l₂ r z J = val l₃ r z J

theorem **normed_pull** (hG : ReciprocityTheory.ChartRestriction.G.Out.GlobalFnsSt) (hA : IsAbelianScheme A) (hM : M.IsLB) {T T' : Over S} {x x' : T ⟶ X} {y y' : T' ⟶ X} (v : T' ⟶ T) (e : v ≫ x = y) (e' : v ≫ x' = y') {l l'} (h : Normed M R x x' l) (h' : Normed M R y y' l') (Z : Scheme.{0}) (r : Z ⟶ (A ⊗ T').left) (z : Z) (J) (hJ : Good M z J) : val l' r z J = val l (r ≫ (aH A v).left) z J

theorem **fst_eq** {W : Over S} {a b : W ⟶ A ⊗ X} (h : a ≫ aH A q = b ≫ aH A q) : a ≫ fst A X = b ≫ fst A X

theorem **snd_eq** {W : Over S} {a b : W ⟶ A ⊗ X} (h : a ≫ aH A q = b ≫ aH A q) : (a ≫ snd A X) ≫ q = (b ≫ snd A X) ≫ q

theorem **hom_eq** {W : Scheme.{0}} {a b : W ⟶ (A ⊗ X).left} (h : a ≫ (aH A q).left = b ≫ (aH A q).left) : b ≫ (A ⊗ X).hom = a ≫ (A ⊗ X).hom

theorem **charpoly_eq_of_isIso** : P₁ A q ≫ aH A q = P₂ A q ≫ aH A q

theorem **sg_one** : sg A q ≫ aH A (P₁ A q ≫ snd A X) = P₁ A q

theorem **sg_two** : sg A q ≫ aH A (P₂ A q ≫ snd A X) = P₂ A q

theorem **pr_one** {W : Over S} (a b : W ⟶ A ⊗ X) (h) : pr q a b h ≫ P₁ A q = a

theorem **pr_two** {W : Over S} (a b : W ⟶ A ⊗ X) (h) : pr q a b h ≫ P₂ A q = b

theorem **pr_sg** {W : Over S} (a b : W ⟶ A ⊗ X) (h) : lift (a ≫ fst A X) (𝟙 W) ≫ aH A (pr q a b h) = pr q a b h ≫ sg A q

theorem **exists_c** (hM : M.IsLB) {l : LB.Gen (LB.LinComb M (K

theorem **val_pr** (hG : ReciprocityTheory.ChartRestriction.G.Out.GlobalFnsSt) (hA : IsAbelianScheme A) (hM : M.IsLB) {l : LB.Gen (LB.LinComb M (K

theorem **descent_datum** : ∀ (S : Scheme.{0}) (A X Y : Over S) [GrpObj A], IsAbelianScheme A → ∀ (q : X ⟶ Y) (M : LB (A ⊗ X).left), M.IsLB → Triv (ev M (1 : X ⟶ A) (𝟙 X)) → (∀ (T : Over S) (x x' : T ⟶ X), x ≫ q = x' ≫ q → LBIso (ev M (fst A T) (snd A T ≫ x)) (ev M (fst A T) (snd A T ≫ x'))) → ∃ c : LB.Gen (LB.LinComb M (K

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.TheoremOfCubeGeneratorExistsStatement

theorem **_root_.GlobalTheory.T.theorem_of_cube_generator_exists** : ∀ (S : Scheme.{0}) (A X Y : Over S) [GrpObj A], IsAbelianScheme A → ∀ (q : X ⟶ Y) (M : LB (A ⊗ X).left), M.IsLB → Triv (ev M (1 : X ⟶ A) (𝟙 X)) → (∀ (T : Over S) (x x' : T ⟶ X), x ≫ q = x' ≫ q → LBIso (ev M (fst A T) (snd A T ≫ x)) (ev M (fst A T) (snd A T ≫ x'))) → ∃ c : LB.Gen (LB.LinComb M (K

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.ToL2CongrOnUnitaryBis

theorem **tens_cont** {X : GMod n} {c : X.V →ₗ[ℂ] (Matrix (Fin n) (Fin n) ℂ → ℂ)} (h : ContOK ε X c) (j : Fin n) : ContOK ε (tens X)

theorem **step_cont** {X : GMod n} {c : X.V →ₗ[ℂ] (Matrix (Fin n) (Fin n) ℂ → ℂ)} (h : ContOK ε X c) (z : ℂ) (j : Fin n) : ContOK ε (step X z)

theorem **path_cont** : ∀ (ss : List (Fin n)) (X : GMod n) (c : X.V →ₗ[ℂ] (Matrix (Fin n) (Fin n) ℂ → ℂ)) (χ : Fin n → ℂ) (js : List (Fin n)), ContOK ε X c → ContOK ε (path X χ ss) (pathCoef X c χ ss js) | [], _, _, _, _, h => h | s :: ss, X, c, χ, js, h => path_cont ss (step X (χ s)) (stepCoef X (χ s) c (js.headD s)) (χ + Pi.single s 1) js.tail (step_cont h _ _) theorem base_cont {H : Type} [NormedAddCommGroup H] [InnerProductSpace ℂ H] {σ : Matrix (Fin n) (Fin n) ℂ → (H →L[ℂ] H)} {D : Submodule ℂ H} {E : Fin n → Fin n → Module.End ℂ D} (hE : Formalism.REAL.GlRel E) (hσ : Formalism.MainNotions.S.IsURepOn (Formalism.MainNotions.S.UJset ε) σ) (ξ : H) : ContOK ε (D, E, hE) (baseCoef σ D ξ)

theorem **path_sep** : ∀ (ss : List (Fin n)) (X : GMod n) (χ : Fin n → ℂ) (C : Set (X.V →ₗ[ℂ] (Matrix (Fin n) (Fin n) ℂ → ℂ))), (∀ x : X.V, x ≠ 0 → ∃ c ∈ C, c x 1 ≠ 0) → ∀ u : (path X χ ss).V, u ≠ 0 → ∃ c ∈ C, ∃ js, pathCoef X c χ ss js u 1 ≠ 0 | [], X, χ, C, h, u, hu => by obtain ⟨c, hc, h1⟩

theorem **eab_of_rightOK** {X : GMod n} {c : X.V →ₗ[ℂ] (Matrix (Fin n) (Fin n) ℂ → ℂ)} (hR : RightOK ε X c) (a b : Fin n) (u : X.V) {g : Matrix (Fin n) (Fin n) ℂ} (hg : Formalism.REAL.inUJ ε g) : Formalism.REAL.Eab ε a b (c u) g = c (X.E a b u) g

theorem **exp_cont** : Continuous (NormedSpace.exp : Matrix (Fin n) (Fin n) ℂ → Matrix (Fin n) (Fin n) ℂ)

theorem **exp_flip** {Y m : Matrix (Fin n) (Fin n) ℂ} (hr : NormedSpace.exp Y = m) (hu : Formalism.REAL.inUJ ε m) : NormedSpace.exp (-(Formalism.REAL.Jm ε * Yᴴ * Formalism.REAL.Jm ε)) = m

theorem **logcurve** (n : ℕ) (ε : Fin n → Bool) (m : ℝ → Matrix (Fin n) (Fin n) ℂ) (Y₀ : Matrix (Fin n) (Fin n) ℂ) (s₀ : ℝ) (hm : ∀ i j, HasDerivAt (fun s => m s i j) (Y₀ i j) s₀) (h1 : m s₀ = 1) (hU : ∀ᶠ s in nhds s₀, Formalism.REAL.inUJ ε (m s)) : ∃ Y : ℝ → Matrix (Fin n) (Fin n) ℂ, (∀ i j, HasDerivAt (fun s => Y s i j) (Y₀ i j) s₀) ∧ Y s₀ = 0 ∧ ∀ᶠ s in nhds s₀, Formalism.REAL.inG ε (Y s) ∧ NormedSpace.exp (Y s) = m s

theorem **frechet** (n : ℕ) (ε : Fin n → Bool) (X : GMod n) (c : X.V →ₗ[ℂ] (Matrix (Fin n) (Fin n) ℂ → ℂ)) (hR : RightOK ε X c) (hC : ContOK ε X c) (u : X.V) (g : Matrix (Fin n) (Fin n) ℂ) (hg : Formalism.REAL.inUJ ε g) (η : ℝ) (hη : 0 < η) : ∃ δ : ℝ, 0 < δ ∧ ∀ Y : Matrix (Fin n) (Fin n) ℂ, Formalism.REAL.inG ε Y → (∀ i j, ‖Y i j‖ < δ) → ‖c u (g * NormedSpace.exp Y) - c u g - c (∑ a, ∑ b, Y a b • X.E a b u) g‖ ≤ η * ∑ a, ∑ b, ‖Y a b‖

theorem **chain** {X : GMod n} {c : X.V →ₗ[ℂ] (Matrix (Fin n) (Fin n) ℂ → ℂ)} (hR : RightOK ε X c) (hC : ContOK ε X c) (u : X.V) {cv : ℝ → Matrix (Fin n) (Fin n) ℂ} {cv' : Matrix (Fin n) (Fin n) ℂ} {s0 : ℝ} (hc : HasDerivAt cv cv' s0) (hcU : ∀ᶠ s in 𝓝 s0, Formalism.REAL.inUJ ε (cv s)) : HasDerivAt (fun s => c u (cv s)) (∑ a, ∑ b, (slmJi ε (cv s0) * cv') a b * c (X.E a b u) (cv s0)) s0

theorem **smooth_of_rightOK** {X : GMod n} {c : X.V →ₗ[ℂ] (Matrix (Fin n) (Fin n) ℂ → ℂ)} (hR : RightOK ε X c) (hC : ContOK ε X c) (u : X.V) : Formalism.REAL.SmoothOnG ε (c u)

theorem **word_of_rightOK** {X : GMod n} {c : X.V →ₗ[ℂ] (Matrix (Fin n) (Fin n) ℂ → ℂ)} (hR : RightOK ε X c) : ∀ (w : List (Fin n × Fin n)) (u : X.V), ∃ v : X.V, ∀ g, Formalism.REAL.inUJ ε g → Formalism.REAL.Eword ε w (c u) g = c v g | [], u => ⟨u, fun _ _ => rfl⟩ | p :: w, u => by obtain ⟨v, hv⟩

theorem **memLp_of_level** {F : Matrix (Fin n) (Fin n) ℂ → ℂ} (hc : ContinuousOn F {g | Formalism.REAL.inUJ ε g}) (hL : LevelF ε F 0) : MemLp (fun g : Formalism.REAL.UJ ε => F (Formalism.MainNotions.S.umat g)) 2 (Formalism.REAL.muG ε)

theorem **smooth_l2_of_right_continuous_level** {X : GMod n} {c : X.V →ₗ[ℂ] (Matrix (Fin n) (Fin n) ℂ → ℂ)} (hR : RightOK ε X c) (hC : ContOK ε X c) (hL : ∀ u, LevelF ε (c u) 0) (u : X.V) : IsSmoothL2 ε (c u)

theorem **toL2_congr_on_unitary_bis** {F G : Matrix (Fin n) (Fin n) ℂ → ℂ} (h : ∀ g, Formalism.REAL.inUJ ε g → F g = G g) : toL2 ε F = toL2 ε G

theorem **emb_of** {X : GMod n} {c : X.V →ₗ[ℂ] (Matrix (Fin n) (Fin n) ℂ → ℂ)} (hR : RightOK ε X c) (hC : ContOK ε X c) (hL : ∀ u, LevelF ε (c u) 0) (u : X.V) (h1 : c u 1 ≠ 0) : ∃ c' : X.V →ₗ[ℂ] smD ε (rreg ε), (∀ a b x, c' (X.E a b x) = smE ε (rreg ε) a b (c' x)) ∧ c' u ≠ 0

theorem **pack** : CompanionTheory.RgExh.Dec.L.Pack

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.ToL2CongrOnUnitaryBisStatement

theorem **_root_.GlobalTheory.T.rgExh_pack** : CompanionTheory.RgExh.Dec.L.Pack

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.ToRawComplex

theorem **dsigW_zero'** (R : Rep k r) (W : List (Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ)) : PI.UpqRep.dsigW (k

theorem **extZ_coe** {R R' : Rep k r} (f : R.smSub →ₗ[ℂ] R'.smSub) (y : R.smSub) : extZ f (y : R.H) = (f y : R'.H)

theorem **smMap_extZ** {R R' : Rep k r} (f : R.smSub →ₗ[ℂ] R'.smSub) (hf : ∀ W, LieWd k r W → ∃ (C : ℝ) (Ws : List (List (Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ))), (∀ W' ∈ Ws, LieWd k r W') ∧ ∀ y : R.smSub, ‖PI.UpqRep.dsigW (k

theorem **sn_single** (X : Obj k r) (W : List (Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ)) (c : X.1.ι) (v : X.2.R.smSub) : sn X W (Pi.single c v) = ‖PI.UpqRep.dsigW (k

theorem **le_sn** (X : Obj k r) (W : List (Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ)) (c : X.1.ι) (x : M X) : ‖PI.UpqRep.dsigW (k

theorem **coord_apply** {X Y : Obj k r} (f : M X →ₗ[ℂ] M Y) (d : Y.1.ι) (c : X.1.ι) (v : X.2.R.smSub) : coord f d c v = f (Pi.single c v) d

theorem **smMap_coord** {X Y : Obj k r} {f : M X →ₗ[ℂ] M Y} (hf : IsHom X Y f) (d : Y.1.ι) (c : X.1.ι) : SmMap X.2.R Y.2.R (extZ (coord f d c))

theorem **sum_coord** {X Y : Obj k r} (f : M X →ₗ[ℂ] M Y) (d : Y.1.ι) (x : M X) : ∑ c, extZ (coord f d c) (x c : X.2.R.H) = (f x d : Y.2.R.H)

theorem **act_coe** (X : Obj k r) {g : Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ} (hg : g ∈ GG k r) (x : M X) (c : X.1.ι) : (CompanionTheory.Zuck.act X g x c : X.2.R.H) = BranchingSupport.act X.1.ρ X.2.R g (fun d => (x d : X.2.R.H)) c

theorem **one_coord** {U : URep k r} {Y : Obj k r} (f : M (one U) →ₗ[ℂ] M Y) (d : Y.1.ι) (x : M (one U)) : extZ (coord f d ()) (x () : U.R.H) = (f x d : Y.2.R.H)

theorem **act_one** (U : URep k r) {g : Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ} (hg : g ∈ GG k r) (x : M (one U)) : (CompanionTheory.Zuck.act (one U) g x () : U.R.H) = U.R.σ g (x ())

theorem **toRaw** : ToRaw

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.ToRawComplexStatement

theorem **_root_.GlobalTheory.T.ToRawComplex** : ToRaw

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.TorBound

theorem **step_le** (k r : ℕ) (d : Fin (2 * r + k + 1) → ℝ) (M : ℝ) (hd : IsTor k r d) (hM : 1 ≤ M) (ha : ∀ a : Fin r, d (eI k r a) ≤ M * d (nxt k r a)) (j : ℕ) (hj : j + 1 < 2 * r + k + 1) : d (j, by omega) ≤ M * d (j + 1, hj)

theorem **chain_le** {n : ℕ} (d : Fin n → ℝ) (M : ℝ) (hM : 1 ≤ M) (hs : ∀ (j : ℕ) (hj : j + 1 < n), d (j, by omega) ≤ M * d (j + 1, hj)) (j m : ℕ) (h : j + m < n) : d (j, by omega) ≤ M ^ m * d (j + m, h)

theorem **ratio_le** (k r : ℕ) (d : Fin (2 * r + k + 1) → ℝ) (M : ℝ) (hd : IsTor k r d) (hM : 1 ≤ M) (ha : ∀ a : Fin r, d (eI k r a) ≤ M * d (nxt k r a)) (i j : Fin (2 * r + k + 1)) (h : ¬ up k r i j) : d j ≤ M ^ (2 * r + k + 1) * d i

theorem **unitary_conj_le** {n : ℕ} (R B : Mat n) (hR : Rᴴ * R = 1) : ‖Rᴴ * B * R‖ ≤ (n : ℝ) ^ 2 * ‖B‖

theorem **torBound** : TorBound

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.TorBoundStatement

theorem **_root_.GlobalTheory.T.torBound'** : TorBound

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.TorusConjugacy

theorem **cStd_ne** : cStd ≠ 0

theorem **exists_scale** (r : ℕ) (c : Fin r → ℂ) (hc : ∀ a, c a ≠ 0) : ∃ T : ℕ → ℂ, (∀ n, T n ≠ 0) ∧ (∀ n, r ≤ n → T n = 1) ∧ ∀ a : Fin r, c a * T a.val = cStd * T (a.val + 1)

theorem **hpk_row** (k r : ℕ) (p q : Fin (2 * r + k + 1)) (hpq : p.val + q.val = 2 * r + k) (h : p.val < r ∨ q.val < r) (j : Fin (2 * r + k + 1)) : Hpk k r p j = if j = q then 1 else 0

theorem **uw_col** (k r : ℕ) (z : Fin (2 * r + k + 1) → ℂ) (h : Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ) (hh : h ∈ UWset k r z) (p : Fin (2 * r + k + 1)) (hp : p.val < r ∨ r + k < p.val) (i : Fin (2 * r + k + 1)) : h i p = if i = p then 1 else 0

theorem **uw_row** (k r : ℕ) (z : Fin (2 * r + k + 1) → ℂ) (h : Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ) (hh : h ∈ UWset k r z) (q : Fin (2 * r + k + 1)) (hq : q.val < r ∨ r + k < q.val) (j : Fin (2 * r + k + 1)) : h q j = if j = q then 1 else 0

theorem **torConj** : CompanionTheory.Bmu.TorConj

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.TorusConjugacyStatement

theorem **_root_.GlobalTheory.T.TorusConjugacy** : TorConj

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.TorusConjugationPreservesCongruence

theorem **conj_sub_one_mem** {L E ι : Type} [NormedField L] [Ring E] [Algebra L E] [Fintype ι] {P : ι → E} (hP : GlobalTheory.CompanionTheory.Iwahori.IsPieces P) {J : AddSubgroup E} (hJ : GlobalTheory.CompanionTheory.Iwahori.IsCongIdeal J P) (hc : ∀ c : L, ‖c‖ ≤ 1 → ∀ x ∈ J, c • x ∈ J) {S S' : E} {σ ρ : ι → L} (hS : ∀ i, P i * S = σ i • P i) (hS' : ∀ j, S' * P j = ρ j • P j) (hSS : S * S' = 1) {a : E} (ha : a - 1 ∈ J) (h : ∀ i j, ‖σ i * ρ j‖ ≤ 1 ∨ (i ≠ j ∧ P i * a * P j = 0)) : S * a * S' - 1 ∈ J

theorem **conj_congruence** {L E ι : Type} [NormedField L] [Ring E] [Algebra L E] [Fintype ι] (wt : ι → ℕ) (P : ι → E) (hP : GlobalTheory.CompanionTheory.Iwahori.IsPieces P) (J : AddSubgroup E) (hJ : GlobalTheory.CompanionTheory.Iwahori.IsCongIdeal J P) (hc : ∀ c : L, ‖c‖ ≤ 1 → ∀ x ∈ J, c • x ∈ J) (T T' : E) (τ : ι → L) (hτ : ∀ i, τ i ≠ 0) (hTT : T * T' = 1) (hTT' : T' * T = 1) (hl : ∀ i, T * P i = τ i • P i) (hr : ∀ i, P i * T = τ i • P i) (hw : ∀ i j, wt i ≤ wt j → ‖τ i‖ ≤ ‖τ j‖) (a b : E) (ha : a - 1 ∈ J) (hb : b - 1 ∈ J) (hU : GlobalTheory.CompanionTheory.Iwahori.IsUpper wt P a) (hL : GlobalTheory.CompanionTheory.Iwahori.IsLower wt P b) : T * a * T' - 1 ∈ J ∧ T' * b * T - 1 ∈ J

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.TorusConjugationPreservesCongruenceStatement

theorem **_root_.GlobalTheory.T.torus_conjugation_preserves_congruence** : ∀ {L E ι : Type} [NormedField L] [Ring E] [Algebra L E] [Fintype ι] (wt : ι → ℕ) (P : ι → E), GlobalTheory.CompanionTheory.Iwahori.IsPieces P → ∀ J : AddSubgroup E, GlobalTheory.CompanionTheory.Iwahori.IsCongIdeal J P → (∀ c : L, ‖c‖ ≤ 1 → ∀ x ∈ J, c • x ∈ J) → ∀ (T T' : E) (τ : ι → L), (∀ i, τ i ≠ 0) → T * T' = 1 → T' * T = 1 → (∀ i, T * P i = τ i • P i) → (∀ i, P i * T = τ i • P i) → (∀ i j, wt i ≤ wt j → ‖τ i‖ ≤ ‖τ j‖) → ∀ a b : E, a - 1 ∈ J → b - 1 ∈ J → GlobalTheory.CompanionTheory.Iwahori.IsUpper wt P a → GlobalTheory.CompanionTheory.Iwahori.IsLower wt P b → T * a * T' - 1 ∈ J ∧ T' * b * T - 1 ∈ J

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.TorusStatement

theorem **span_gens** {h : V → V → L} {δ : L} {l : ℕ} {e f : Fin l → V} {W : Submodule L V} (hF : IsFrame h δ e f W) : Submodule.span L (gens e f W) = ⊤

theorem **gens_induction** {h : V → V → L} {δ : L} {l : ℕ} {e f : Fin l → V} {W : Submodule L V} (hF : IsFrame h δ e f W) {P : V → Prop} (hg : ∀ x ∈ gens e f W, P x) (h0 : P 0) (ha : ∀ x y, P x → P y → P (x + y)) (hm : ∀ (a : L) x, P x → P (a • x)) (x : V) : P x

@[simp] theorem **pair_apply** {s : L →+* L} {h : V → V → L} (hH : IsHerm s h) (x y : V) : pair hH x y = h x y

theorem **add_left** {s : L →+* L} {h : V → V → L} (hH : IsHerm s h) (x x' y : V) : h (x + x') y = h x y + h x' y

theorem **smul_left** {s : L →+* L} {h : V → V → L} (hH : IsHerm s h) (a : L) (x y : V) : h (a • x) y = s a * h x y

theorem **tor_apply** {s : L →+* L} {h : V → V → L} (hH : IsHerm s h) (δ : L) {l : ℕ} (e f : Fin l → V) (u : Fin l → L) (x : V) : tor hH δ e f u x = x + (∑ i, ((u i - 1) * (s δ)⁻¹) • (h (f i) x • e i)) + ∑ i, (((s (u i))⁻¹ - 1) * δ⁻¹) • (h (e i) x • f i)

theorem **pair_fe** (hH : IsHerm s h) (hF : IsFrame h δ e f W) (i j : Fin l) : h (f i) (e j) = if i = j then s δ else 0

theorem **tor_e** (hH : IsHerm s h) (hF : IsFrame h δ e f W) (hδ : δ ≠ 0) (u : Fin l → L) (j : Fin l) : tor hH δ e f u (e j) = u j • e j

theorem **tor_f** (hH : IsHerm s h) (hF : IsFrame h δ e f W) (hδ : δ ≠ 0) (u : Fin l → L) (j : Fin l) : tor hH δ e f u (f j) = (s (u j))⁻¹ • f j

theorem **tor_w** (hH : IsHerm s h) (hF : IsFrame h δ e f W) (u : Fin l → L) {w : V} (hw : w ∈ W) : tor hH δ e f u w = w

theorem **tor_inv** (hH : IsHerm s h) (hF : IsFrame h δ e f W) (hδ : δ ≠ 0) (u : Fin l → L) (hu : ∀ i, u i ≠ 0) : (tor hH δ e f u).comp (tor hH δ e f fun i => (u i)⁻¹) = LinearMap.id

theorem **tor_iso_gens** (hs : ∀ x, s (s x) = x) (hH : IsHerm s h) (hF : IsFrame h δ e f W) (hδ : δ ≠ 0) (u : Fin l → L) (hu : ∀ i, u i ≠ 0) {x y : V} (hx : x ∈ gens e f W) (hy : y ∈ gens e f W) : h (tor hH δ e f u x) (tor hH δ e f u y) = h x y

theorem **tor_iso** (hs : ∀ x, s (s x) = x) (hH : IsHerm s h) (hF : IsFrame h δ e f W) (hδ : δ ≠ 0) (u : Fin l → L) (hu : ∀ i, u i ≠ 0) (x y : V) : h (tor hH δ e f u x) (tor hH δ e f u y) = h x y

theorem **stTor** : StTor

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.TorusStatementStatement

theorem **_root_.GlobalTheory.T.TorusStatement** : StTor

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.TotalBound

theorem **x_bound** {ι : Type*} (s : Finset ι) (x y : ι → ℝ) (a0 : ι) (h0 : a0 ∈ s) (B M K : ℝ) (hK0 : 0 ≤ K) (hy : ∀ a ∈ s, |y a| ≤ B) (hM : ∀ a ∈ s, |x a0 - x a| ≤ M) (hK : |∑ a ∈ s, (y a - x a)| ≤ K) : |x a0| ≤ B + M + K

theorem **fiber_bound** {ι : Type*} (s : Finset ι) (x y : ι → ℝ) (a0 : ι) (M X K : ℝ) (hX0 : 0 ≤ X) (hX : |x a0| ≤ X) (hM : ∀ a ∈ s, |x a - x a0| ≤ M) (hK : |∑ a ∈ s, (y a - x a)| ≤ K) : ∑ a ∈ s, ((y a - x a) ^ 2 / 2 - 2 * M ^ 2) - 2 * X * K ≤ ∑ a ∈ s, (y a ^ 2 - x a ^ 2)

theorem **total_bound** {n : ℕ} (x y : Fin n → ℝ) (lev : Fin n → ℕ) (M X K : ℝ) (hX0 : 0 ≤ X) (hK0 : 0 ≤ K) (hX : ∀ a, |x a| ≤ X) (hM : ∀ a b, lev a = lev b → |x a - x b| ≤ M) (hK : ∀ c, |∑ a ∈ univ.filter (fun a => lev a = c), (y a - x a)| ≤ K) : ∑ a, ((y a - x a) ^ 2 / 2 - 2 * M ^ 2) - n * (2 * X * K) ≤ ∑ a, (y a ^ 2 - x a ^ 2)

theorem **two_bound** {n : ℕ} (d : Fin n → ℝ) (a b : Fin n) (hab : a ≠ b) (D : ℝ) (hD : 0 ≤ D) (h : D ≤ |d a - d b|) : D ^ 2 / 2 ≤ ∑ c, d c ^ 2

theorem **farNum** : FarNum

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.TransferStatementBis

theorem **word_one** (l : List R) : word (1 : R) l = l.prod

theorem **word_mul_idem** {E : R} (hE : E * E = E) (l : List R) : word E l * E = word E l

theorem **sum_reverse_eq_zero** {M : ℕ} (b : Fin M → R) (h : stdPoly (1 : R) b = 0) : ∑ π : Equiv.Perm (Fin M), (Equiv.Perm.sign π : ℤ) • (List.ofFn fun i => b (π i)).reverse.prod = 0

theorem **phi_single** (hv : ∀ k ∈ K, σ k v = v) (x : G) (c : ℂ) : phi K σ v (Finsupp.single (x : G ⧸ K) c) = c • σ x v

theorem **lam_single** (g x : G) (c : ℂ) : lam K g (Finsupp.single (x : G ⧸ K) c) = Finsupp.single ((g * x : G) : G ⧸ K) c

theorem **phi_lam** (hv : ∀ k ∈ K, σ k v = v) (g : G) (f : (G ⧸ K) →₀ ℂ) : phi K σ v (lam K g f) = σ g (phi K σ v f)

theorem **lam_dlt** {k : G} (hk : k ∈ K) : lam K k (dlt K) = dlt K

theorem **hk_comm** (h : Hk K) (g : G) (f : (G ⧸ K) →₀ ℂ) : lam K g ((h : Module.End ℂ ((G ⧸ K) →₀ ℂ)) f) = (h : Module.End ℂ ((G ⧸ K) →₀ ℂ)) (lam K g f)

theorem **phi_fixed** (hv : ∀ k ∈ K, σ k v = v) (h : Hk K) : ∀ k ∈ K, σ k (phi K σ v ((h : Module.End ℂ ((G ⧸ K) →₀ ℂ)) (dlt K))) = phi K σ v ((h : Module.End ℂ ((G ⧸ K) →₀ ℂ)) (dlt K))

theorem **phi_phi** (hv : ∀ k ∈ K, σ k v = v) (h : Hk K) (f : (G ⧸ K) →₀ ℂ) : phi K σ (phi K σ v ((h : Module.End ℂ ((G ⧸ K) →₀ ℂ)) (dlt K))) f = phi K σ v ((h : Module.End ℂ ((G ⧸ K) →₀ ℂ)) f)

theorem **avg_sigma** {E : H →L[ℂ] H} (hA : IsAvg σ K E) (hv : ∀ k ∈ K, σ k v = v) (g : G) (T : Hk K) (S : Finset (G ⧸ K)) (hS : (↑S : Set (G ⧸ K)) = MulAction.orbit K (g : G ⧸ K)) (hT : (T : Module.End ℂ ((G ⧸ K) →₀ ℂ)) (dlt K) = ∑ y ∈ S, Finsupp.single y (1 : ℂ)) : E (σ g v) = phi K σ v ((((S.card : ℂ)⁻¹ • T : Hk K) : Module.End ℂ ((G ⧸ K) →₀ ℂ)) (dlt K))

theorem **word_apply** {E : H →L[ℂ] H} (hA : IsAvg σ K E) (b : G → Hk K) (hb : ∀ (u : H), (∀ k ∈ K, σ k u = u) → ∀ g, E (σ g u) = phi K σ u ((b g : Module.End ℂ ((G ⧸ K) →₀ ℂ)) (dlt K))) (hv : ∀ k ∈ K, σ k v = v) (l : List G) : word E (l.map fun g => σ g) v = phi K σ v ((((l.map b).reverse.prod : Hk K) : Module.End ℂ ((G ⧸ K) →₀ ℂ)) (dlt K))

theorem **stTransfer** : StTransfer

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.TransferStatementBisStatement

theorem **_root_.GlobalTheory.T.TransferStatementBis** : StTransfer

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.TruncatedDerivation

theorem **continuous_of** : Continuous fun y : PI.MtM n => (Matrix.of y : Mat n)

theorem **continuous_scale** (c : ℝ) : Continuous fun y : PI.MtM n => (c • Matrix.of y : Mat n)

theorem **notMem_tsupport** {ρ : Mat n → ℝ} {R : ℝ} (hR : 0 < R) (X : Mat n) {T : Set ℝ} {t : ℝ} (ht : t ∈ T) {y : PI.MtM n} (hy : y ∉ (fun p : ℝ × Mat n => (Matrix.of.symm (R • (ex X p.1 * p.2)) : PI.MtM n)) '' (T ×ˢ tsupport ρ)) : ex X (-t) * (R⁻¹ • Matrix.of y) ∉ tsupport ρ

theorem **isCompact_img** {ρ : Mat n → ℝ} (hc : HasCompactSupport ρ) (R : ℝ) (X : Mat n) {T : Set ℝ} (hT : IsCompact T) : IsCompact ((fun p : ℝ × Mat n => (Matrix.of.symm (R • (ex X p.1 ∗ p.2)) : PI.MtM n)) '' (T ×ˢ tsupport ρ))

theorem **hasCompactSupport_coef** {ρ : Mat n → ℝ} (hc : HasCompactSupport ρ) {R : ℝ} (hR : 0 < R) (φ : PI.MtM n → ℂ) : HasCompactSupport fun y : PI.MtM n => (ρ (R⁻¹ • Matrix.of y) : ℂ) ∗ φ y

theorem **continuous_coef** {ρ : Mat n → ℝ} (hρ : Continuous ρ) (R : ℝ) {F : Mat n → H} (hF : Continuous F) (v : H) : Continuous fun y : PI.MtM n => (ρ (R⁻¹ • Matrix.of y) : ℂ) ∗ ⟪F (Matrix.of y), v⟫_ℂ

theorem **cutI_eq** (hμ : PI.IsHaarOnM (UH Hm) μ) (hFw : ∀ a ∈ UH Hm, F a = σ a w) (R : ℝ) (ρ : Mat n → ℝ) (v : H) : cutI σ μ w R ρ v = ∫ y : PI.MtM n, ((ρ (R⁻¹ • Matrix.of y) : ℂ) ∗ ⟪F (Matrix.of y), v⟫_ℂ) • F (Matrix.of y) ∂μ

theorem **inner_iso** (h : URep Hm σ) {g : Mat n} (hg : g ∈ UH Hm) (x y : H) : ⟪σ g x, σ g y⟫_ℂ = ⟪x, y⟫_ℂ

theorem **apply_cutI** (h : URep Hm σ) (hμ : PI.IsHaarOnM (UH Hm) μ) (hF : Continuous F) (hFw : ∀ a ∈ UH Hm, F a = σ a w) {R : ℝ} (hR : 0 < R) {ρ : Mat n → ℝ} (hρ : Continuous ρ) (hc : HasCompactSupport ρ) (v : H) {X : Mat n} (hX : ∀ t, ex X t ∈ UH Hm) (t : ℝ) : σ (ex X t) (cutI σ μ w R ρ v) = ∫ y : PI.MtM n, ((ρ (ex X (-t) ∗ (R⁻¹ • Matrix.of y)) : ℂ) ∗ ⟪F (Matrix.of y), σ (ex X t) v⟫_ℂ) • F (Matrix.of y) ∂μ

theorem **cutDer** : CutDer

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.TruncatedDerivationLimit

theorem **tendsto_cut** {E : Type∗} [NormedAddCommGroup E] [NormedSpace ℂ E] {μ : Measure (PI.MtM n)} {f : PI.MtM n → E} (hf : Integrable f μ) {ρ : Mat n → ℝ} (hρ : Continuous ρ) (hc : HasCompactSupport ρ) : Tendsto (fun N : ℕ => ∫ y : PI.MtM n, (ρ (((N : ℝ) + 1)⁻¹ • Matrix.of y) : ℂ) • f y ∂μ) atTop (𝓝 ((ρ 0 : ℂ) • ∫ y, f y ∂μ))

theorem **integrable_conj_smul** {E : Type∗} [NormedAddCommGroup E] [NormedSpace ℂ E] {G : Set (Mat n)} (hG : IsClosed G) {μ : Measure (PI.MtM n)} (hae : ∀ᵐ y ∂μ, (Matrix.of y : Mat n) ∈ G) {c : PI.MtM n → ℂ} (hc : Integrable c μ) {g : Mat n → E} (hg : ContinuousOn g G) {K : ℝ} (hK : ∀ x ∈ G, ‖g x‖ ≤ K) : Integrable (fun y => conj (c y) • g (Matrix.of y)) μ

theorem **lieW_of** {Hm : Mat n} {W : List (Mat n)} (hW : LieWd Hm W) : LieW (UH Hm) W

theorem **tendsto_dsW** (hD : CutDer) (hσ : URep Hm σ) {μ : Measure (PI.MtM n)} (hμ : PI.IsHaarOnM (UH Hm) μ) {w : H} (hw : IsSm (UH Hm) σ w) (hi : ∀ v, IsSm (UH Hm) σ v → Integrable (fun y : PI.MtM n => ⟪v, σ (Matrix.of y) w⟫_ℂ) μ) {κ : ℂ} (hκ : ∀ v, IsSm (UH Hm) σ v → ∫ y : PI.MtM n, conj ⟪v, σ (Matrix.of y) w⟫_ℂ • σ (Matrix.of y) w ∂μ = κ • v) (W : List (Mat n)) : LieWd Hm W → ∀ ρ : Mat n → ℝ, ContDiff ℝ ∞ ρ → HasCompactSupport ρ → ∀ v, IsSm (UH Hm) σ v → Tendsto (fun N : ℕ => dsW σ W (cutI σ μ w ((N : ℝ) + 1) ρ v)) atTop (𝓝 ((ρ 0 : ℂ) • κ • dsW σ W v))

theorem **dsW_cutI** (hσ : URep Hm σ) {μ : Measure (PI.MtM n)} (hμ : PI.IsHaarOnM (UH Hm) μ) {w : H} (hw : IsSm (UH Hm) σ w) {R : ℝ} (hR : 0 < R) {ρ : Mat n → ℝ} (hρ : Continuous ρ) (hc : HasCompactSupport ρ) (v : H) {W : List (Mat n)} (hW : LieWd Hm W) : Integrable (fun y : PI.MtM n => ((ρ (R⁻¹ • Matrix.of y) : ℂ) ∗ conj ⟪v, σ (Matrix.of y) w⟫_ℂ) • dsW σ W (σ (Matrix.of y) w)) μ ∧ dsW σ W (cutI σ μ w R ρ v) = ∫ y : PI.MtM n, ((ρ (R⁻¹ • Matrix.of y) : ℂ) ∗ conj ⟪v, σ (Matrix.of y) w⟫_ℂ) • dsW σ W (σ (Matrix.of y) w) ∂μ

theorem **cutLim_of** (hF : Factor) (hD : CutDer) : CutLim

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.TruncatedDerivationLimitStatement

theorem **_root_.GlobalTheory.T.truncated_derivation_limit** : Factor → CutDer → CutLim

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.TruncatedDerivationStatement

theorem **_root_.GlobalTheory.T.TruncatedDerivation** : CutDer

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.UnitarizableOfSeparatingSmoothL2Maps

theorem **unitarizable_of_separating_smooth_l2_maps** : CompanionTheory.RgExh.Fin.UnitarizableOfL2Embedding

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.UnitaryIso

theorem **isSm_map** (e : H →L[ℂ] H') (he : ∀ g ∈ G, ∀ x, e (σ g x) = σ' g (e x)) {v : H} (hv : IsSm G σ v) : IsSm G σ' (e v)

theorem **ds_map** (h : RepOn G σ) (e : H →L[ℂ] H') (he : ∀ g ∈ G, ∀ x, e (σ g x) = σ' g (e x)) {X : Mat n} (hX : ∀ t, ex X t ∈ G) {v : H} (hv : IsSm G σ v) : ds σ' X (e v) = e (ds σ X v)

theorem **dsW_map** (h : RepOn G σ) (e : H →L[ℂ] H') (he : ∀ g ∈ G, ∀ x, e (σ g x) = σ' g (e x)) {v : H} (hv : IsSm G σ v) : ∀ {W : List (Mat n)}, LieW G W → dsW σ' W (e v) = e (dsW σ W v) | [], _ => rfl | X :: W, hW => by have hW' : LieW G W

theorem **res_coe** (U U' : URep k r) (e : U.R.H →L[ℂ] U'.R.H) (he : ∀ g ∈ GG k r, ∀ x, e (U.R.σ g x) = U'.R.σ g (e x)) (v : U.R.smSub) : (res U U' e he v : U'.R.H) = e v

theorem **isHom_res** (F : FRep k r) (U U' : URep k r) (e : U.R.H →L[ℂ] U'.R.H) (he : ∀ g ∈ GG k r, ∀ x, e (U.R.σ g x) = U'.R.σ g (e x)) (hn : ∀ x, ‖e x‖ = ‖x‖) : IsHom (F, U) (F, U') (dOp F.ι (res U U' e he)) where

theorem **uIso** : UIso

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.UnitaryIsoStatement

theorem **_root_.GlobalTheory.T.UnitaryIso** : UIso

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.UnitaryMaximalCompactIsCompact

private theorem **norm_apply_le_one_of_conjTranspose_mul_self** {n : ℕ} {U : Matrix (Fin n) (Fin n) ℂ} (hU : Uᴴ ∗ U = 1) (i j : Fin n) : ‖U i j‖ ≤ 1

private theorem **isCompact_unitary_commute_Jm_inUJ** (n : ℕ) (ε : Fin n → Bool) : IsCompact {M : Matrix (Fin n) (Fin n) ℂ | M ∈ Matrix.unitaryGroup (Fin n) ℂ ∧ M ∗ Jm ε = Jm ε ∗ M ∧ inUJ ε M}

theorem **isCompact_KG_set** : ∀ (n : ℕ) (ε : Fin n → Bool), IsCompact ((S7.KG ε : Subgroup (S7.UJ ε)) : Set (S7.UJ ε))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.UnitaryMaximalCompactIsCompactStatement

theorem **_root_.GlobalTheory.T.unitary_maximal_compact_is_compact** : ∀ (n : ℕ) (ε : Fin n → Bool), IsCompact ((S7.KG ε : Subgroup (S7.UJ ε)) : Set (S7.UJ ε))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.UpperTriangularInverseIsUpper

theorem **sum_pieces** {P : ι → Module.End L V} (hP : IsPieces P) (x : V) : ∑ j, P j x = x

theorem **map_low_le** {wt : ι → ℕ} {P : ι → Module.End L V} (hP : IsPieces P) {a : Module.End L V} (ha : IsUpper wt P a) (m : ℕ) : (low wt P m).map a ≤ low wt P m

theorem **map_low_eq** [FiniteDimensional L V] {wt : ι → ℕ} {P : ι → Module.End L V} (hP : IsPieces P) {a a' : Module.End L V} (h1 : a ∗ a' = 1) (h2 : a' ∗ a = 1) (ha : IsUpper wt P a) (m : ℕ) : (low wt P m).map a = low wt P m

theorem **inv_mem_low** [FiniteDimensional L V] {wt : ι → ℕ} {P : ι → Module.End L V} (hP : IsPieces P) {a a' : Module.End L V} (h1 : a ∗ a' = 1) (h2 : a' ∗ a = 1) (ha : IsUpper wt P a) {m : ℕ} {x : V} (hx : x ∈ low wt P m) : a' x ∈ low wt P m

theorem **piece_mem_low** {wt : ι → ℕ} {P : ι → Module.End L V} (hP : IsPieces P) (j : ι) (x : V) : P j x ∈ low wt P (wt j + 1)

theorem **isUpper_inv** : ∀ {L V ι : Type} [Field L] [AddCommGroup V] [Module L V] [FiniteDimensional L V] [Fintype ι] (wt : ι → ℕ) (P : ι → Module.End L V), GlobalTheory.CompanionTheory.Iwahori.IsPieces P → ∀ a a' : Module.End L V, a ∗ a' = 1 → a' ∗ a = 1 → GlobalTheory.CompanionTheory.Iwahori.IsUpper wt P a → GlobalTheory.CompanionTheory.Iwahori.IsUpper wt P a'

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.UpperTriangularInverseIsUpperStatement

theorem **_root_.GlobalTheory.T.upper_triangular_inverse_is_upper** : ∀ {L V ι : Type} [Field L] [AddCommGroup V] [Module L V] [FiniteDimensional L V] [Fintype ι] (wt : ι → ℕ) (P : ι → Module.End L V), GlobalTheory.CompanionTheory.Iwahori.IsPieces P → ∀ a a' : Module.End L V, a ∗ a' = 1 → a' ∗ a = 1 → GlobalTheory.CompanionTheory.Iwahori.IsUpper wt P a → GlobalTheory.CompanionTheory.Iwahori.IsUpper wt P a'

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.VNormSqRenum

theorem **glRel_renum** {A : Type∗} [Ring A] {E : Fin n → Fin n → A} (hE : GlRel E) (s : Equiv.Perm (Fin n)) : GlRel (fun a b => E (s a) (s b))

theorem **tm_renum** {A : Type∗} [Ring A] [Algebra ℂ A] (E : Fin n → Fin n → A) (s : Equiv.Perm (Fin n)) (u : ℂ) (a b : Fin n) : Tm (fun a b => E (s a) (s b)) u a b = Tm E u (s a) (s b)

theorem **qminor_renum** {A : Type∗} [Ring A] [Algebra ℂ A] {t : ℕ} (E : Fin n → Fin n → A) (s : Equiv.Perm (Fin n)) (a b : Fin t → Fin n) (u : ℂ) : qminor (fun a b => E (s a) (s b)) a b u = qminor E (s ∘ a) (s ∘ b) u

theorem **qdet_renum** {A : Type} [Ring A] [Algebra ℂ A] {E : Fin n → Fin n → A} (hE : GlRel E) (s : Equiv.Perm (Fin n)) (u : ℂ) : qdet (fun a b => E (s a) (s b)) u = qdet E u

theorem **symm_lt** {ε : Fin n → Bool} {s : Equiv.Perm (Fin n)} (hs : ∀ a b, a < b → ε (s a) = ε (s b) → s a < s b) {a b : Fin n} (hab : a < b) (he : ε a = ε b) : s.symm a < s.symm b

theorem **kHighest_iff** {V : Type} [AddCommGroup V] [Module ℂ V] {ε : Fin n → Bool} (E : Fin n → Fin n → Module.End ℂ V) {s : Equiv.Perm (Fin n)} (hs : ∀ a b, a < b → ε (s a) = ε (s b) → s a < s b) (γ : Fin n → ℝ) (u : V) : KHighest ε E γ u ↔ KHighest (ε ∘ s) (fun a b => E (s a) (s b)) (γ ∘ s) u

theorem **rhoC_renum** {ε : Fin n → Bool} {s : Equiv.Perm (Fin n)} (hs : ∀ a b, a < b → ε (s a) = ε (s b) → s a < s b) : rhoC (ε ∘ s) = rhoC ε ∘ s

theorem **vNormSq_renum** {ε : Fin n → Bool} {s : Equiv.Perm (Fin n)} (hs : ∀ a b, a < b → ε (s a) = ε (s b) → s a < s b) (γ : Fin n → ℝ) : vNormSq (ε ∘ s) (γ ∘ s) = vNormSq ε γ

theorem **isMinG_iff** {V : Type} [AddCommGroup V] [Module ℂ V] {ε : Fin n → Bool} (E : Fin n → Fin n → Module.End ℂ V) {s : Equiv.Perm (Fin n)} (hs : ∀ a b, a < b → ε (s a) = ε (s b) → s a < s b) (γ : Fin n → ℝ) : IsMinG ε E γ ↔ IsMinG (ε ∘ s) (fun a b => E (s a) (s b)) (γ ∘ s)

theorem **renumber** : CompanionTheory.RgExh.U.Renumber

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.VanKGOf

theorem **vanKG_of** (hN : NumKG) : VanKG

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.WittStatement

theorem **h_zero_right** (hh : IsHerm s h) (x : V) : h x 0 = 0

theorem **h_zero_left** (hh : IsHerm s h) (x : V) : h 0 x = 0

theorem **h_sum_right** (hh : IsHerm s h) {ι : Type} (t : Finset ι) (x : V) (c : ι → L) (v : ι → V) : h x (∑ i ∈ t, c i • v i) = ∑ i ∈ t, c i * h x (v i)

theorem **pairMap_apply** (e f : Fin l → V) (p : (Fin l → L) × (Fin l → L)) : pairMap e f p = ∑ i, p.1 i • e i + ∑ i, p.2 i • f i

theorem **pair_e** (hh : IsHerm s h) {e f : Fin l → V} (he : ∀ i j, h (e i) (e j) = 0) (hef : ∀ i j, h (e i) (f j) = if i = j then δ else 0) (j : Fin l) (p : (Fin l → L) × (Fin l → L)) : h (e j) (pairMap e f p) = p.2 j * δ

theorem **pair_f** (hh : IsHerm s h) {e f : Fin l → V} (hf : ∀ i j, h (f i) (f j) = 0) (hef : ∀ i j, h (e i) (f j) = if i = j then δ else 0) (j : Fin l) (p : (Fin l → L) × (Fin l → L)) : h (f j) (pairMap e f p) = p.1 j * s δ

theorem **pair_form** (hh : IsHerm s h) {e f : Fin l → V} (he : ∀ i j, h (e i) (e j) = 0) (hf : ∀ i j, h (f i) (f j) = 0) (hef : ∀ i j, h (e i) (f j) = if i = j then δ else 0) (p r : (Fin l → L) × (Fin l → L)) : h (pairMap e f p) (pairMap e f r) = ∑ j, r.1 j * s (p.2 j * δ) + ∑ j, r.2 j * s (p.1 j * s δ)

theorem **pairMap_injective** (hh : IsHerm s h) (hd : δ ≠ 0) {e f : Fin l → V} (he : ∀ i j, h (e i) (e j) = 0) (hf : ∀ i j, h (f i) (f j) = 0) (hef : ∀ i j, h (e i) (f j) = if i = j then δ else 0) : Function.Injective (pairMap e f : ((Fin l → L) × (Fin l → L)) →ₗ[L] V)

theorem **mem_of_perp** (hh : IsHerm s h) (hd : δ ≠ 0) {e f : Fin l → V} {W : Submodule L V} (F : IsFrame h δ e f W) (x : V) (h1 : ∀ i, h (e i) x = 0) (h2 : ∀ i, h (f i) x = 0) : x ∈ W

theorem **extend_isom** [CharZero L] [FiniteDimensional L V] (hs : ∀ x, s (s x) = x) (hh : IsHerm s h) (U : Submodule L V) (φ : U →ₗ[L] V) (hinj : Function.Injective φ) (hiso : ∀ x y : U, h (φ x) (φ y) = h x y) : ∃ g : V ≃ₗ[L] V, IsIso h g ∧ ∀ x : U, g x = φ x

theorem **exists_isom** [CharZero L] [FiniteDimensional L V] (hs : ∀ x, s (s x) = x) (hh : IsHerm s h) (hd : δ ≠ 0) {l' : ℕ} (hl : l ≤ l') {e f : Fin l → V} {W : Submodule L V} {e' f' : Fin l' → V} {W' : Submodule L V} (F : IsFrame h δ e f W) (F' : IsFrame h δ e' f' W') : ∃ g : V ≃ₗ[L] V, IsIso h g ∧ (∀ i, g (e i) = e' (Fin.castLE hl i)) ∧ ∀ i, g (f i) = f' (Fin.castLE hl i)

theorem **frame_core** [CharZero L] [FiniteDimensional L V] (hs : ∀ x, s (s x) = x) (hh : IsHerm s h) (hd : δ ≠ 0) {l' : ℕ} (hl : l ≤ l') {e f : Fin l → V} {W : Submodule L V} {e' f' : Fin l' → V} {W' : Submodule L V} (F : IsFrame h δ e f W) (F' : IsFrame h δ e' f' W') : ∃ (q : Fin l ≃ Fin l') (g : V ≃ₗ[L] V), IsIso h g ∧ (∀ i, g (e i) = e' (q i)) ∧ (∀ i, g (f i) = f' (q i)) ∧ ∀ w, w ∈ W ↔ g w ∈ W'

theorem **stWitt** : StWitt

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.WittStatementStatement

theorem **_root_.GlobalTheory.T.WittStatement** : StWitt

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.WlemOfGfacts

theorem **ht_add_root** {n : ℕ} (w : Fin n → ℝ) (c d : Fin n) (h : c < d) : ht w < ht (w + Pi.single c 1 − Pi.single d 1)

theorem **wlem_of_gfacts** (hG : GFacts) : WLem

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.WordLengthBoundOfLinear

theorem **exists_basis_sub** {E : Type*} [NormedAddCommGroup E] [NormedSpace ℝ E] [FiniteDimensional ℝ E] (S : Submodule ℝ E) : ∃ (N : ℕ) (b : Fin N → E) (κ : ℝ), 0 ≤ κ ∧ (∀ i, b i ∈ S) ∧ ∀ X, X ∈ S → ∃ c : Fin N → ℝ, X = ∑ i, c i • b i ∧ ∀ i, |c i| ≤ κ * ‖X‖

theorem **exists_basis** {n : ℕ} (Hm : Mat n) : ∃ (N : ℕ) (b : Fin N → Mat n) (κ : ℝ), 0 ≤ κ ∧ (∀ i, Lie Hm (b i)) ∧ ∀ X, Lie Hm X → ∃ c : Fin N → ℝ, X = ∑ i, c i • b i ∧ ∀ i, |c i| ≤ κ * ‖X‖

theorem **dsW_sum** {n : ℕ} {H : Type*} [NormedAddCommGroup H] [NormedSpace ℂ H] {G : Set (Mat n)} {σ : Mat n → (H →L[ℂ] H)} (h : RepOn G σ) {ι : Type*} (s : Finset ι) (v : ι → H) (hv : ∀ i, IsSm G σ (v i)) {W : List (Mat n)} (hW : LieW G W) : IsSm G σ (∑ i ∈ s, v i) ∧ dsW σ W (∑ i ∈ s, v i) = ∑ i ∈ s, dsW σ W (v i)

private theorem **ex_mem_UH_of_lie** {n : ℕ} {Hm X : Mat n} (hsq : Hm * Hm = 1) (hX : Lie Hm X) (t : ℝ) : ex X t ∈ Formalism.QuantumDeterminants.UH Hm

private theorem **dsW_append'** {n : ℕ} {H : Type} [NormedAddCommGroup H] [NormedSpace ℂ H] (σ : Mat n → (H →L[ℂ] H)) (P W : List (Mat n)) (v : H) : dsW σ (P ++ W) v = dsW σ P (dsW σ W v)

theorem **wordBd_of_dsLin** : DsLin → WordBd

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.WordLengthBoundOfLinearStatement

theorem **_root_.GlobalTheory.T.word_length_bound_of_linear** : DsLin → WordBd

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.WordMonomial

theorem **glRel_add** {A : Type} [Ring A] {n : ℕ} {E F : Fin n → Fin n → A} (hE : GlRel E) (hF : GlRel F) (h : ∀ a b c d, E a b * F c d = F c d * E a b) : GlRel fun a b => E a b + F a b

theorem **glRel_map** {A B : Type} [Ring A] [Ring B] (φ : A →+* B) {n : ℕ} {E : Fin n → Fin n → A} (hE : GlRel E) : GlRel fun a b => φ (E a b)

theorem **pd_apply** (i : σ) (f : MvPolynomial σ ℂ) : pd i f = pderiv i f

theorem **pd_comm** (i j : σ) : pd i * pd j = pd j * pd i

theorem **word_monomial** (l : List σ) : ∀ (s : σ →₀ ℕ) (c : ℂ), s.degree < l.length → (l.map pd).prod (monomial s c) = 0

theorem **word_pol** (f : MvPolynomial σ ℂ) : ∃ m : ℕ, ∀ l : List σ, m ≤ l.length → (l.map pd).prod f = 0

theorem **word_tmul** (l : List σ) (v : V) (f : MvPolynomial σ ℂ) : (l.map fun i => ((pd i).lTensor V : Module.End ℂ (V ⊗[ℂ] MvPolynomial σ ℂ))).prod (v ⊗ₜ f) = v ⊗ₜ (l.map pd).prod f

theorem **word_tens** (z : V ⊗[ℂ] MvPolynomial σ ℂ) : ∃ m : ℕ, ∀ l : List σ, m ≤ l.length → (l.map fun i => ((pd i).lTensor V : Module.End ℂ (V ⊗[ℂ] MvPolynomial σ ℂ))).prod z = 0

theorem **cover** : Koszul → Verma → Cover

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.ZetaDegZero

theorem **pres_mul** {R : Submodule ℂ (V × W)} {S S' : Module.End ℂ V} {T T' : Module.End ℂ W} (h : Pres R S T) (h' : Pres R S' T') : Pres R (S * S') (T * T')

theorem **pres_ofFn** {R : Submodule ℂ (V × W)} : ∀ {t : ℕ} (S : Fin t → Module.End ℂ V) (T : Fin t → Module.End ℂ W), (∀ k, Pres R (S k) (T k)) → Pres R (List.ofFn S).prod (List.ofFn T).prod | 0, _, _, _ => by intro x y hxy simpa using hxy | t + 1, S, T, h => by rw [List.ofFn_succ, List.ofFn_succ, List.prod_cons, List.prod_cons] exact pres_mul (h 0) (pres_ofFn _ _ fun k => h k.succ) theorem pres_qminor {n t : ℕ} {R : Submodule ℂ (V × W)} (E : Fin n → Fin n → Module.End ℂ V) (F : Fin n → Fin n → Module.End ℂ W) (a b : Fin t → Fin n) (h : ∀ i j, Pres R (E (a i) (b j)) (F (a i) (b j))) (u : ℂ) : Pres R (qminor E a b u) (qminor F a b u)

theorem **blk_eq** {n : ℕ} (ε : Fin n → Bool) (s : Bool) (i : Fin (nb ε s)) : ε (blk ε s i) = s

theorem **pres_zeta** {n : ℕ} {R : Submodule ℂ (V × W)} (ε : Fin n → Bool) (E : Fin n → Fin n → Module.End ℂ V) (F : Fin n → Fin n → Module.End ℂ W) (h : ∀ c d, ε c = ε d → Pres R (E c d) (F c d)) (u : ℂ) : Pres R (zeta ε E u) (zeta ε F u)

theorem **pres_graph** (φ : V →ₗ[ℂ] W) (S : Module.End ℂ V) (T : Module.End ℂ W) : Pres φ.graph S T ↔ ∀ x, φ (S x) = T (φ x)

theorem **deg_lift** {j : ℕ} (φ : V →ₗ[ℂ] W) {x : Ch ε V} (hx : Deg j x) : Deg j (lift ε φ x)

theorem **bd_apply** (E : Fin n → Fin n → Module.End ℂ V) (x : Ch ε V) (A : Finset (NC ε)) : bd ε E x A = ∑ i : NC ε, E i.1.1 i.1.2 (if i ∈ A then 0 else sg i A • x (insert i A))

theorem **deg_bd** {j : ℕ} (E : Fin n → Fin n → Module.End ℂ V) {x : Ch ε V} (hx : Deg (j + 1) x) : Deg j (bd ε E x)

theorem **lift_bd** (φ : V →ₗ[ℂ] W) (E : Fin n → Fin n → Module.End ℂ V) (F : Fin n → Fin n → Module.End ℂ W) (h : ∀ a b x, φ (E a b x) = F a b (φ x)) (x : Ch ε V) : lift ε φ (bd ε E x) = bd ε F (lift ε φ x)

theorem **lift_qdet** (φ : V →ₗ[ℂ] W) (E : Fin n → Fin n → Module.End ℂ V) (F : Fin n → Fin n → Module.End ℂ W) (h : ∀ a b x, φ (E a b x) = F a b (φ x)) (u : ℂ) (x : Ch ε V) : lift ε φ (lift ε (qdet E u) x) = lift ε (qdet F u) (lift ε φ x)

theorem **lift_zeta** (hG : GFacts) (φ : V →ₗ[ℂ] W) (E : Fin n → Fin n → Module.End ℂ V) (F : Fin n → Fin n → Module.End ℂ W) (h : ∀ a b x, φ (E a b x) = F a b (φ x)) (u : ℂ) (x : Ch ε V) : lift ε φ (zeta ε (Fop ε E) u x) = zeta ε (Fop ε F) u (lift ε φ x)

theorem **deg_Fop** (hG : GFacts) {j : ℕ} (E : Fin n → Fin n → Module.End ℂ V) (c d : Fin n) {x : Ch ε V} (hx : Deg j x) : Deg j (Fop ε E c d x)

theorem **deg_zeta** (hG : GFacts) {j : ℕ} (E : Fin n → Fin n → Module.End ℂ V) (u : ℂ) {x : Ch ε V} (hx : Deg j x) : Deg j (zeta ε (Fop ε E) u x)

theorem **Gop_deg_zero** (c d : Fin n) {x : Ch ε V} (hx : Deg 0 x) : Gop ε c d x = 0

theorem **zeta_deg_zero** (E : Fin n → Fin n → Module.End ℂ V) (u : ℂ) {x : Ch ε V} (hx : Deg 0 x) : zeta ε (Fop ε E) u x ∅ = zeta ε E u (x ∅)

theorem **bd_zeta** (hB : BdFacts) (E : Fin n → Fin n → Module.End ℂ V) (hE : GlRel E) (u : ℂ) (x : Ch ε V) : bd ε E (zeta ε (Fop ε E) u x) = zeta ε (Fop ε E) u (bd ε E x)

theorem **bd_qdet** (E : Fin n → Fin n → Module.End ℂ V) (hE : GlRel E) (u : ℂ) (x : Ch ε V) : bd ε E (lift ε (qdet E u) x) = lift ε (qdet E u) (bd ε E x)

theorem **co_of** (hB : BdFacts) (hG : GFacts) (hC : Cover) (h0 : CapelliDeterminantBlockProduct) : CO

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.ZuckermanHypothesis

theorem **dot_conj** {n : ℕ} (P a Q : Matrix (Fin n) (Fin n) ℂ) (u x : Fin n → ℂ) : u ⬝ᵥ ((P * a * Q) *ᵥ x) = (u ᵥ* P) ⬝ᵥ (a *ᵥ (Q *ᵥ x))

theorem **dot_expand** {n : ℕ} (a : Matrix (Fin n) (Fin n) ℂ) (u x : Fin n → ℂ) : u ⬝ᵥ (a *ᵥ x) = ∑ p : Fin n × Fin n, u p.1 * x p.2 * a p.1 p.2

theorem **kfin_smul** {H : Type} [NormedAddCommGroup H] [InnerProductSpace ℂ H] {σ : Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ → (H →L[ℂ] H)} {v : H} (c : ℂ) (hv : IsKFinHpk (k

theorem **transport** (U : URep k r) {P : Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ} (hP : IsCayleyU k r ε P) (q : Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ → Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ) (hq : ∀ g, q (P * g * P⁻¹) = P * q g * P⁻¹) (v : Fin (2 * r + k + 1) → Fin (2 * r + k + 1) → ℝ) (Sm : Finset (Fin (2 * r + k + 1))) (Hs : Fin (2 * r + k + 1) → Type) [∀ s, NormedAddCommGroup (Hs s)] [∀ s, InnerProductSpace ℂ (Hs s)] [∀ s, CompleteSpace (Hs s)] (σs : ∀ s, Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ → (Hs s →L[ℂ] Hs s)) (hSm : ∀ s, s ∈ Sm ↔ IsHCParam (ν s)) (ds : ∀ s ∈ Sm, IsHCParam (ν s) ∧ IsDS ε (ν s) (σs s)) (toSum : ∀ (v w : U.R.H) (x y : Fin (2 * r + k + 1) → ℂ), IsKFin ε (fun g => U.R.σ (P⁻¹ * g * P)) v → IsKFin ε (fun g => U.R.σ (P⁻¹ * g * P)) w → ∃ (ι : Type) (_ : Fintype ι) (sI : ι → Fin (2 * r + k + 1)) (vI wI : ∀ i, Hs (sI i))

theorem **zuckerman_hypothesis_holds** : ZuckH

CompanionTheory.SpectralGeometry.LocalGlobalProofs.StableLattices.ZuckermanHypothesisStatement

theorem **_root_.GlobalTheory.T.zuckerman_hypothesis** : ZuckH

CompanionTheory.SpectralGeometry.LocalGlobalProofs.WeylGroupActions.CapelliParamsDiffInt

theorem **capelli_params_diff_int** : ∀ (V : Type) [AddCommGroup V] [Module ℂ V] [FiniteDimensional ℂ V] [Nontrivial V] (n : ℕ) (E : Fin n → Fin n → Module.End ℂ V) (m : Fin n → ℂ), GlRel E → (∀ w v, qdet E w v = Qr.cpol m w • v) → ∀ i j, ∃ N : ℤ, m i − m j = (N : ℂ)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.WeylGroupActions.ExistsIsAlgebraicRatFuncFractionRingDedekind

theorem **glRel_perm** {E : Fin n → Fin n → A} (hE : GlRel E) (σ : Equiv.Perm (Fin n)) : GlRel fun a b => E (σ a) (σ b)

theorem **qdet_perm** {E : Fin n → Fin n → A} (hE : GlRel E) (σ : Equiv.Perm (Fin n)) (u : ℂ) : qdet (fun a b => E (σ a) (σ b)) u = qdet E u

CompanionTheory.SpectralGeometry.LocalGlobalProofs.WeylGroupActions.HasDerivAtExpReal

theorem **isOpen_cone** : IsOpen cone

theorem **coef_one** (p N : ℕ) : coef p N 1 = 1

theorem **hasDerivAt_exp_real** (X : Matrix (Fin n) (Fin n) ℂ) : HasDerivAt (fun t : ℝ => NormedSpace.exp ((t : ℂ) • X)) X 0

theorem **rdV_eq_fderiv** (X g : Matrix (Fin n) (Fin n) ℂ) (f : Matrix (Fin n) (Fin n) ℂ → E) (hf : DifferentiableAt ℂ f g) : RdV X f g = fderiv ℂ f g (g * X)

theorem **ldV_eq_fderiv** {k r : ℕ} (X g : B.Mat k r) (f : B.Mat k r → E) (hf : DifferentiableAt ℂ f g) : B.LdV X f g = fderiv ℂ f g (−(X * g))

theorem **eabH_eq_fderiv** (H g : Matrix (Fin n) (Fin n) ℂ) (a b : Fin n) (f : Matrix (Fin n) (Fin n) ℂ → E) (hf : DifferentiableAt ℂ f g) : EabH H a b f g = fderiv ℂ f g (g * Matrix.single a b 1)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.WeylGroupActions.OmgLeftRight

lemma **one_mem_UWset** (k r : ℕ) (z : Fin (2 * r + k + 1) → ℂ) : (1 : Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ) ∈ UWset k r z

theorem **omg_left_right** : ∀ (k r : ℕ) (z z' : (Fin (2 * r + k + 1) → ℂ)), inWb k r z → 0 < Tz k r z → inWb k r z' → 0 < Tz k r z' → ∀ g ∈ B.GV k r, (∀ h ∈ UWset k r z, GlobalTheory.WigSh.omg k r z z' (h * g) = GlobalTheory.WigSh.omg k r z z' (g)) ∧ ∀ h' ∈ UWset k r z', GlobalTheory.WigSh.omg k r z z' (g * h') = h'⁻¹ *ᵥ GlobalTheory.WigSh.omg k r z z' (g)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.WeylGroupActions.OmgNMulN

theorem **omg_N_mul_N** : ∀ (k r : ℕ) (z z' : (Fin (2 * r + k + 1) → ℂ)), inWb k r z → inWb k r z' → ∀ g ∈ B.GV k r, ∀ n ∈ Nset k r, ∀ n' ∈ Nset k r, GlobalTheory.WigSh.omg k r z z' (n * g * n') = GlobalTheory.WigSh.omg k r z z' (g)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.WeylGroupActions.TorusMovesCharacter

theorem **torusTail_ne** {r : ℕ} {c : Fin r → ℂ} (hc : ∀ a, c a ≠ 0) (n : ℕ) : torusTail c n ≠ 0

theorem **torusTail_ge** {r : ℕ} (c : Fin r → ℂ) {n : ℕ} (hn : r ≤ n) : torusTail c n = 1

theorem **torusTail_succ** {r : ℕ} (c : Fin r → ℂ) (a : Fin r) : torusTail c a.val = c a * torusTail c (a.val + 1)

theorem **torusDiag_ne** (k : ℕ) {r : ℕ} {c : Fin r → ℂ} (hc : ∀ a, c a ≠ 0) (i : Fin (2 * r + k + 1)) : torusDiag k c i ≠ 0

theorem **torusDiag_mid** (k : ℕ) {r : ℕ} (c : Fin r → ℂ) (i : Fin (2 * r + k + 1)) (h1 : r ≤ i.val) (h2 : i.val ≤ r + k) : torusDiag k c i = 1

theorem **torusDiag_tau** (k : ℕ) {r : ℕ} {c : Fin r → ℂ} (hc : ∀ a, c a ≠ 0) (i : Fin (2 * r + k + 1)) : star (torusDiag k c i) * torusDiag k c (tau k r i) = 1

theorem **torusDiag_uh** (k : ℕ) {r : ℕ} {c : Fin r → ℂ} (hc : ∀ a, c a ≠ 0) : Matrix.diagonal (torusDiag k c) ∈ UH (Hpk k r)

theorem **torus_diagonal_inv** {n : ℕ} {d : Fin n → ℂ} (hd : ∀ i, d i ≠ 0) : (Matrix.diagonal d)⁻¹ = Matrix.diagonal (fun i => (d i)⁻¹)

theorem **torus_col_conj** {k r : ℕ} {d e : Fin (2 * r + k + 1) → ℂ} (hde : ∀ i, d i * e i = 1) {m : Matrix (Fin (2 * r + k + 1)) (Fin (2 * r + k + 1)) ℂ} (hm : col k r m) : col k r (Matrix.diagonal d * m * Matrix.diagonal e)

theorem **torus_moves_character** : ∀ (k r : ℕ) (z : Fin (2 * r + k + 1) → ℂ) (c : Fin r → ℂ), Formalism.QuantumDeterminants.inWb k r z → (∀ a, c a ≠ 0) → ∃ a ∈ Formalism.QuantumDeterminants.UH (Formalism.QuantumDeterminants.Hpk k r), (∀ h ∈ Formalism.QuantumDeterminants.UWset k r z, a * h = h * a) ∧ (∀ m ∈ Formalism.QuantumDeterminants.Nset k r, a * m * a⁻¹ ∈ Formalism.QuantumDeterminants.Nset k r ∧ a⁻¹ * m * a ∈ Formalism.QuantumDeterminants.Nset k r ∧ PI.BesselL2FixAtZero.psiGen k r z (fun _ => 1) (a * m * a⁻¹) = PI.BesselL2FixAtZero.psiGen k r z c m)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.WeylGroupActions.TypeExponentsConsecutiveGap

theorem **gap_eq_nat_add_one** : ∀ (Wq : Type) [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wq] (k r : ℕ) (z : (Fin (2 * r + k + 1) → ℂ)) (π : B.Mat k r → (Wq →L[ℂ] Wq)) (u : ℕ → ℝ), inWb k r z → 0 < Tz k r z → IsType k r z π u → (∀ i ∈ Finset.Ico 1 k, u (i + 1) + 1 ≤ u i) → ∀ i ∈ Finset.Icc 1 (k − 1), ∃ N : ℕ, u i − u (i + 1) = (N : ℝ) + 1

CompanionTheory.SpectralGeometry.LocalGlobalProofs.WeylGroupActions.TypeExponentsConsecutiveGapStatement

theorem **_root_.GlobalTheory.T.type_exponents_consecutive_gap** : ∀ (Wq : Type) [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wq] (k r : ℕ) (z : (Fin (2 * r + k + 1) → ℂ)) (π : B.Mat k r → (Wq →L[ℂ] Wq)) (u : ℕ → ℝ), inWb k r z → 0 < Tz k r z → IsType k r z π u → (∀ i ∈ Finset.Ico 1 k, u (i + 1) + 1 ≤ u i) → ∀ i ∈ Finset.Icc 1 (k − 1), ∃ N : ℕ, u i − u (i + 1) = (N : ℝ) + 1

CompanionTheory.SpectralGeometry.LocalGlobalProofs.WeylGroupActions.TypeExponentsDifferByIntegers

theorem **type_param_diff_int** : ∀ (Wq : Type) [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wq] (k r : ℕ) (z : (Fin (2 * r + k + 1) → ℂ)) (π : B.Mat k r → (Wq →L[ℂ] Wq)) (u : ℕ → ℝ), inWb k r z → 0 < Tz k r z → IsType k r z π u → ∀ i ∈ Finset.Icc 1 k, ∀ j ∈ Finset.Icc 1 k, ∃ N : ℤ, u i - u j = (N : ℝ)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.WeylGroupActions.TypeExponentsDifferByIntegersStatement

theorem **_root_.GlobalTheory.T.type_exponents_differ_by_integers** : ∀ (Wq : Type) [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wq] (k r : ℕ) (z : (Fin (2 * r + k + 1) → ℂ)) (π : B.Mat k r → (Wq →L[ℂ] Wq)) (u : ℕ → ℝ), inWb k r z → 0 < Tz k r z → IsType k r z π u → ∀ i ∈ Finset.Icc 1 k, ∀ j ∈ Finset.Icc 1 k, ∃ N : ℤ, u i - u j = (N : ℝ)

CompanionTheory.SpectralGeometry.LocalGlobalProofs.WeylGroupActions.UpdateEqUMinus

theorem **update_eq_uPlus** (u : ℕ → ℝ) (l n₀ : ℕ) : Function.update u (l + 1) (u (l + 1) + (n₀ : ℝ)) = uPlus u l n₀

theorem **update_eq_uMinus** (u : ℕ → ℝ) (l n₀ : ℕ) : Function.update u l (u l - (n₀ : ℝ)) = uMinus u l n₀

theorem **wigExists** : WigExists

CompanionTheory.SpectralGeometry.LocalGlobalProofs.WeylGroupActions.WignerMapContinuousRep

theorem **wigEquiv_omg** (Wq : Type) [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] (Wq' : Type) [NormedAddCommGroup Wq'] [InnerProductSpace ℂ Wq'] (k r : ℕ) (z z' : (Fin (2 * r + k + 1) → ℂ)) (π' : B.Mat k r → (Wq →L[ℂ] Wq)) (π'' : B.Mat k r → (Wq' →L[ℂ] Wq')) (d : ℕ) (A : (Fin (2 * r + k + 1) → ℂ) → (Wq' →L[ℂ] Wq)) (hz : inWb k r z) (hT : 0 < Tz k r z) (hz' : inWb k r z') (hT' : 0 < Tz k r z') (hA : GlobalTheory.WigSh.WigMap k r z' π'' π' d A) (hπ : IsCRepOn (UWset k r z') π'') : WigEquiv z z' π' π'' (fun g => A (GlobalTheory.WigSh.omg k r z z' (g)))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.WeylGroupActions.WignerMapContinuousRepStatement

theorem **_root_.GlobalTheory.T.wigEquiv_of_wigMap_of_isCRepOn** : ∀ (Wq : Type) [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] (Wq' : Type) [NormedAddCommGroup Wq'] [InnerProductSpace ℂ Wq'] (k r : ℕ) (z z' : (Fin (2 * r + k + 1) → ℂ)) (π' : B.Mat k r → (Wq →L[ℂ] Wq)) (π'' : B.Mat k r → (Wq' →L[ℂ] Wq')) (d : ℕ) (A : (Fin (2 * r + k + 1) → ℂ) → (Wq' →L[ℂ] Wq)), inWb k r z → 0 < Tz k r z → inWb k r z' → 0 < Tz k r z' → GlobalTheory.WigSh.WigMap k r z' π'' π' d A → IsCRepOn (UWset k r z') π'' → WigEquiv z z' π' π'' (fun g => A (GlobalTheory.WigSh.omg k r z z' (g)))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.WeylGroupActions.WignerMapPolynomialOperator

theorem **polyOp_wigMap_omg** : ∀ (Wq : Type) [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] (Wq' : Type) [NormedAddCommGroup Wq'] [InnerProductSpace ℂ Wq'] (k r : ℕ) (z z' : (Fin (2 * r + k + 1) → ℂ)) (π' : B.Mat k r → (Wq →L[ℂ] Wq)) (π'' : B.Mat k r → (Wq' →L[ℂ] Wq')) (d : ℕ) (A : (Fin (2 * r + k + 1) → ℂ) → (Wq' →L[ℂ] Wq)), inWb k r z → 0 < Tz k r z → inWb k r z' → 0 < Tz k r z' → GlobalTheory.WigSh.WigMap k r z' π'' π' d A → PolyOp (d * (r + 1)) (fun g : B.Mat k r => A (GlobalTheory.WigSh.omg k r z z' (g)))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.WeylGroupActions.WignerMapPolynomialOperatorStatement

theorem **_root_.GlobalTheory.T.wigner_map_polynomial_operator** : ∀ (Wq : Type) [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] (Wq' : Type) [NormedAddCommGroup Wq'] [InnerProductSpace ℂ Wq'] (k r : ℕ) (z z' : (Fin (2 * r + k + 1) → ℂ)) (π' : B.Mat k r → (Wq →L[ℂ] Wq)) (π'' : B.Mat k r → (Wq' →L[ℂ] Wq')) (d : ℕ) (A : (Fin (2 * r + k + 1) → ℂ) → (Wq' →L[ℂ] Wq)), inWb k r z → 0 < Tz k r z → inWb k r z' → 0 < Tz k r z' → GlobalTheory.WigSh.WigMap k r z' π'' π' d A → PolyOp (d * (r + 1)) (fun g : B.Mat k r => A (GlobalTheory.WigSh.omg k r z z' (g)))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.WeylGroupActions.WignerMapUpperBound

theorem **wigUpper_omg** : ∀ (Wq : Type) [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] (Wq' : Type) [NormedAddCommGroup Wq'] [InnerProductSpace ℂ Wq'] (k r : ℕ) (z z' : (Fin (2 * r + k + 1) → ℂ)) (π' : B.Mat k r → (Wq →L[ℂ] Wq)) (π'' : B.Mat k r → (Wq' →L[ℂ] Wq')) (d : ℕ) (A : (Fin (2 * r + k + 1) → ℂ) → (Wq' →L[ℂ] Wq)), inWb k r z → 0 < Tz k r z → inWb k r z' → 0 < Tz k r z' → GlobalTheory.WigSh.WigMap k r z' π'' π' d A → WigUpper z z' d (fun g : B.Mat k r => A (GlobalTheory.WigSh.omg k r z z' (g)))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.WeylGroupActions.WignerMapUpperBoundStatement

theorem **_root_.GlobalTheory.T.wigner_map_upper_bound** : ∀ (Wq : Type) [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] (Wq' : Type) [NormedAddCommGroup Wq'] [InnerProductSpace ℂ Wq'] (k r : ℕ) (z z' : (Fin (2 * r + k + 1) → ℂ)) (π' : B.Mat k r → (Wq →L[ℂ] Wq)) (π'' : B.Mat k r → (Wq' →L[ℂ] Wq')) (d : ℕ) (A : (Fin (2 * r + k + 1) → ℂ) → (Wq' →L[ℂ] Wq)), inWb k r z → 0 < Tz k r z → inWb k r z' → 0 < Tz k r z' → GlobalTheory.WigSh.WigMap k r z' π'' π' d A → WigUpper z z' d (fun g : B.Mat k r => A (GlobalTheory.WigSh.omg k r z z' (g)))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.WeylGroupActions.WignerPlusMinusDecomposition

theorem **wigLower_omg** : ∀ (Wq : Type) [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wq] (Wqp : Type) [NormedAddCommGroup Wqp] [InnerProductSpace ℂ Wqp] (Wqm : Type) [NormedAddCommGroup Wqm] [InnerProductSpace ℂ Wqm] [FiniteDimensional ℂ Wqm] (k r : ℕ) (z z' : (Fin (2 * r + k + 1) → ℂ)) (b : ℝ) (n₀ : ℕ) (Ap : (Fin (2 * r + k + 1) → ℂ) → (Wqp →L[ℂ] Wq)) (Am : (Fin (2 * r + k + 1) → ℂ) → (Wqm →L[ℂ] Wq)), inWb k r z → 0 < Tz k r z → inWb k r z' → 0 < Tz k r z' → (∀ ω, inWb k r ω → herm (Hpk k r) ω z' = 0 → ∀ v : Wq, (herm (Hpk k r) ω ω).re ^ n₀ * ‖v‖ ^ 2 ≤ (3 * b) ^ (2 * n₀) * (‖ContinuousLinearMap.adjoint (Ap ω) v‖ ^ 2 + ‖ContinuousLinearMap.adjoint (Am ω) v‖ ^ 2)) → WigLower z z' b n₀ (fun g : B.Mat k r => Ap (GlobalTheory.WigSh.omg k r z z' (g))) (fun g : B.Mat k r => Am (GlobalTheory.WigSh.omg k r z z' (g)))

CompanionTheory.SpectralGeometry.LocalGlobalProofs.WeylGroupActions.WignerPlusMinusDecompositionStatement

theorem **_root_.GlobalTheory.T.wigLower_of_adjoint_norm_sq_lower_bound** : ∀ (Wq : Type) [NormedAddCommGroup Wq] [InnerProductSpace ℂ Wq] [FiniteDimensional ℂ Wq] (Wqp : Type) [NormedAddCommGroup Wqp] [InnerProductSpace ℂ Wqp] [FiniteDimensional ℂ Wqp] (Wqm : Type) [NormedAddCommGroup Wqm] [InnerProductSpace ℂ Wqm] [FiniteDimensional ℂ Wqm] (k r : ℕ) (z z' : (Fin (2 * r + k + 1) → ℂ)) (b : ℝ) (n₀ : ℕ) (Ap : (Fin (2 * r + k + 1) → ℂ) → (Wqp →L[ℂ] Wq)) (Am : (Fin (2 * r + k + 1) → ℂ) → (Wqm →L[ℂ] Wq)), inWb k r z → 0 < Tz k r z → inWb k r z' → 0 < Tz k r z' → (∀ ω, inWb k r ω → herm (Hpk k r) ω z' = 0 → ∀ v : Wq, (herm (Hpk k r) ω ω).re ^ n₀ * ‖v‖ ^ 2 ≤ (3 * b) ^ (2 * n₀) * (‖ContinuousLinearMap.adjoint (Ap ω) v‖ ^ 2 + ‖ContinuousLinearMap.adjoint (Am ω) v‖ ^ 2)) → WigLower z z' b n₀ (fun g : B.Mat k r => Ap (GlobalTheory.WigSh.omg k r z z' (g))) (fun g : B.Mat k r => Am (GlobalTheory.WigSh.o

CompanionTheory.ThickeningEstimates.InnerProductFactorization.EqOfInner

theorem **exists_exp** {A : Type*} [NormedRing A] [NormedAlgebra ℝ A] [CompleteSpace A] (m : ℝ → A) (hc : Continuous m) (h0 : m 0 = 1) (hm : ∀ s t, m (s + t) = m s * m t) : ∃ X : A, ∀ t, m t = NormedSpace.exp (t • X)

theorem **Impl.inner** {s} {S : Sch l →ₗ[ℂ] Sch l} (h : Impl s S) (f g : Sch l) : inner ℂ ((S f).toLp 2 volume) ((S g).toLp 2 volume) = inner ℂ (f.toLp 2 volume) (g.toLp 2 volume)

theorem **exists_rho** (w : l ⊕ l → ℝ) (f : Sch l) : ∃ g : Sch l, ⇑g = rhoFun w f

theorem **toLp_rho** {w : l ⊕ l → ℝ} {f g : Sch l} (h : ⇑g = rhoFun w f) : g.toLp 2 volume = rho w (f.toLp 2 volume)

theorem **eq_of_inner** {u v : Sch l} (h : ∀ φ : Sch l, inner ℂ (φ.toLp 2 volume) (u.toLp 2 volume) = inner ℂ (φ.toLp 2 volume) (v.toLp 2 volume)) : u = v

theorem **Impl.dRho** {s : (l ⊕ l → ℝ) →ₗ[ℝ] (l ⊕ l → ℝ)} {S : Sch l →ₗ[ℂ] Sch l} (h : Impl s S) (hs : Function.Surjective S) (w : l ⊕ l → ℝ) (f : Sch l) : S (dRho w f) = dRho (s w) (S f)

theorem **Impl.continuous** {s : (l ⊕ l → ℝ) ≃ₗ[ℝ] (l ⊕ l → ℝ)} {S : Sch l →ₗ[ℂ] Sch l} (h : Impl s S) (hs : Function.Surjective S) : Continuous S

theorem **exists_ne_zero** (x : EuclideanSpace ℝ l) : ∃ φ : Sch l, φ x ≠ 0

theorem **Impl.scalar** {S : Sch l →ₗ[ℂ] Sch l} (h : Impl id S) (hs : Function.Surjective S) : ∃ c : ℂ, ∀ f, S f = c • f

theorem **quad_mem** (X : Matrix (l ⊕ l) (l ⊕ l) ℝ) : quad X ∈ weylAlg l

theorem **sympl_eq** (w u : l ⊕ l → ℝ) : sympl w u = -(w ·ᵥ Matrix.J l ℝ *ᵥ u)

theorem **exists_lift** (hP : GlobalTheory.Published.RealMetaplecticRepresentation) {X : Matrix (l ⊕ l) (l ⊕ l) ℝ} (hX : X ∈ LieAlgebra.Symplectic.sp l ℝ) : ∃ (U : ℝ → Sch l →L[ℂ] Sch l) (Q : Sch l →L[ℂ] Sch l), Q ∈ weylAlg l ∧ (∀ f, U 0 f = f) ∧ (∀ s t f, U (s + t) f = U s (U t f)) ∧ (∀ t, Impl (fun w => NormedSpace.exp (t • X) *ᵥ w) (U t).toLinearMap) ∧ (∀ f, Continuous fun t => (U t f).toLp 2 volume) ∧ ∀ f (lam : Sch l →L[ℂ] ℂ), HasDerivAt (fun t => lam (U t f)) (lam (Q f)) 0

theorem **lie** (hP : GlobalTheory.Published.RealMetaplecticRepresentation) (s : ℝ → (l ⊕ l → ℝ) ≃ₗ[ℝ] (l ⊕ l → ℝ)) (sc : ∀ w, Continuous fun t => s t w) (s0 : ∀ w, s 0 w = w) (sm : ∀ t u w, s (t + u) w = s t (s u w)) (ss : ∀ t w w', sympl (s t w) (s t w') = sympl w w') (S : ℝ → Sch l →ₗ[ℂ] Sch l) (hS : ∀ t, Impl (s t) (S t)) (S0 : ∀ f, S 0 f = f) (Sm : ∀ t u f, S (t + u) f = S t (S u f)) (Sc : ∀ f (lam : Sch l →L[ℂ] ℂ), Continuous fun t => lam (S t f)) : ∃ Q ∈ weylAlg l, ∀ f (lam : Sch l →L[ℂ] ℂ), HasDerivAt (fun t => lam (S t f)) (lam (Q f)) 0

CompanionTheory.ThickeningEstimates.LieGroupReduction.HasDerivAtSchwartzToL2

theorem **exists_decay** {F : Type} [NormedAddCommGroup F] [NormedSpace ℝ F] (g : SchwartzMap (EuclideanSpace ℝ l) F) (k : ℕ) : ∃ C : ℝ, 0 ≤ C ∧ ∀ x, (1 + ‖x‖) ^ k * ‖g x‖ ≤ C

theorem **csmul** (r : ℝ) (x : L2 l) : (r : ℂ) • x = r • x

theorem **hasDerivAt_schwartz_to_L2** {F : ℝ → Sch l} {G : Sch l} {t₀ : ℝ} (hp : ∀ x, HasDerivAt (fun t => F t x) (G x) t₀) {C : ℝ} (hb : ∀ᶠ t in 𝓝 t₀, ∀ x, (1 + ‖x‖) ^ (Module.finrank ℝ (EuclideanSpace ℝ l) + 1) * ‖F t x − F t₀ x‖ ≤ C * |t − t₀|) : HasDerivAt (fun t => schwartzToL2 l (F t)) (schwartzToL2 l G) t₀

theorem **LD.const** (U : Sch l →L[ℂ] Sch l) (t₀ : ℝ) : LD (fun _ => U) (fun _ => 0) t₀

theorem **LD.smul** {U : ℝ → Sch l →L[ℂ] Sch l} {Q : Sch l → Sch l} {t₀ : ℝ} (h : LD U Q t₀) {c : ℝ → ℂ} {c' : ℂ} (hc : HasDerivAt c c' t₀) : LD (fun t => c t • U t) (fun f => c t₀ • Q f + c' • U t₀ f) t₀

theorem **LD.mul** {U W : ℝ → Sch l →L[ℂ] Sch l} {P Q : Sch l → Sch l} {t₀ : ℝ} (hU : LD U P t₀) (hW : LD W Q t₀) (hn : ∀ᶠ t in 𝓝 t₀, ∀ f, ‖schwartzToL2 l (U t f)‖ = ‖schwartzToL2 l f‖) : LD (fun t => U t ∘ W t) (fun f => P (W t₀ f) + U t₀ (Q f)) t₀

theorem **weyl_step** {V : Sch l →L[ℂ] Sch l} {w v : l ⊕ l → ℝ} (h : ∀ f, dRho w (V f) = V (dRho v f)) (A : Sch l →L[ℂ] Sch l) (f : Sch l) : schwartzToL2 l ((A ∗ dRho w) (V f)) = ∑ a, v a • schwartzToL2 l (A (V (dRho (Pi.single a 1) f)))

theorem **LD.differentiableAt** {V : ℝ → Sch l →L[ℂ] Sch l} {Q : Sch l → Sch l} {t₀ : ℝ} (hV : LD V Q t₀) (v : ℝ → (l ⊕ l → ℝ) → (l ⊕ l → ℝ)) (hv : ∀ w a, DifferentiableAt ℝ (fun t => v t w a) t₀) (h : ∀ᶠ t in 𝓝 t₀, ∀ w f, dRho w (V t f) = V t (dRho (v t w) f)) (f : Sch l) (lam : Sch l →L[ℂ] ℂ) : DifferentiableAt ℝ (fun t => lam (V t f)) t₀

theorem **LD.hasDerivAt** {V : ℝ → Sch l →L[ℂ] Sch l} {Q : Sch l →L[ℂ] Sch l} (hV : LD V Q 0) (h0 : ∀ f, V 0 f = f) (v : ℝ → (l ⊕ l → ℝ) → (l ⊕ l → ℝ)) (v' : (l ⊕ l → ℝ) → (l ⊕ l → ℝ)) (hv0 : ∀ w, v 0 w = w) (hv : ∀ w a, HasDerivAt (fun t => v t w a) (−v' w a) 0) (hQ : ∀ w, Q ∗ dRho w − dRho w ∗ Q = dRho (v' w)) (h : ∀ᶠ t in 𝓝 (0 : ℝ), ∀ w f, dRho w (V t f) = V t (dRho (v t w) f)) (f : Sch l) (lam : Sch l →L[ℂ] ℂ) : HasDerivAt (fun t => lam (V t f)) (lam (Q f)) 0

CompanionTheory.ThickeningEstimates.LieGroupReduction.IsoOfIntegral

theorem **rhoFun_eq** (w : l ⊕ l → ℝ) (f : Eu l → ℂ) : rhoFun w f = rf (pPart w) (qPart w) f

theorem **Intw.mul** {S T : Sch l →L[ℂ] Sch l} {p q p' q' p'' q'' : Eu l} (hS : Intw S p' q' p'' q'') (hT : Intw T p q p' q') : Intw (S ∗ T) p q p'' q''

theorem **Intw.smul** {S : Sch l →L[ℂ] Sch l} {p q p' q' : Eu l} (hS : Intw S p q p' q') (c : ℂ) : Intw (c • S) p q p' q'

theorem **Iso.mul** {S T : Sch l →L[ℂ] Sch l} (hS : Iso S) (hT : Iso T) : Iso (S ∗ T)

theorem **iso_of_integral** {S : Sch l →L[ℂ] Sch l} (h : ∀ f : Sch l, ∫ x, ‖S f x‖ ^ (2 : ℝ) = ∫ x, ‖f x‖ ^ (2 : ℝ)) : Iso S

theorem **temperate_quad** (S : Eu l →L[ℝ] Eu l) : Function.HasTemperateGrowth fun x : Eu l => ⟪x, S x⟫

theorem **ph_apply** (S : Eu l →L[ℝ] Eu l) (f : Sch l) (x : Eu l) : ph S f x = exp (((−π * ⟪x, S x⟫ : ℝ) : ℂ) * I) * f x

theorem **ph_mul** (S T : Eu l →L[ℝ] Eu l) : ph S ∗ ph T = ph (S + T)

theorem **ph_zero** : ph (0 : Eu l →L[ℝ] Eu l) = 1

theorem **isUnit_ph** (S : Eu l →L[ℝ] Eu l) : IsUnit (ph S)

theorem **iso_ph** (S : Eu l →L[ℝ] Eu l) : Iso (ph S)

theorem **intw_ph** {S : Eu l →L[ℝ] Eu l} (hS : ∀ x y, ⟪x, S y⟫ = ⟪y, S x⟫) (p q : Eu l) : Intw (ph S) p q p (q + S p)

theorem **dil_apply** {B : Eu l →L[ℝ] Eu l} (h : IsUnit B) (f : Sch l) (x : Eu l) : dil B f x = f (B x)

theorem **dil_one** : dil (1 : Eu l →L[ℝ] Eu l) = 1

theorem **dil_mul** {A B : Eu l →L[ℝ] Eu l} (hA : IsUnit A) (hB : IsUnit B) : dil A ∗ dil B = dil (B ∗ A)

theorem **isUnit_dil** {B : Eu l →L[ℝ] Eu l} (h : IsUnit B) : IsUnit (dil B)

theorem **intw_dil** {B : Eu l →L[ℝ] Eu l} (hB : IsUnit B) {p q p' q' : Eu l} (hp : B p' = p) (hq : ∀ x, ⟪q', x⟫ = ⟪q, B x⟫) : Intw (dil B) p q p' q'

theorem **iso_dil** {B : Eu l →L[ℝ] Eu l} (hB : IsUnit B) {c : ℝ} (hc : c ^ 2 = |B.det|) : Iso ((c : ℂ) • dil B)

theorem **fou_apply** (f : Sch l) : fou l f = 𝓕 f

theorem **fouInv_apply** (f : Sch l) : fouInv l f = 𝓕⁻ f

theorem **fou_mul_fouInv** : fou l ∗ fouInv l = 1

theorem **fouInv_mul_fou** : fouInv l ∗ fou l = 1

theorem **isUnit_fou** : IsUnit (fou l)

theorem **isUnit_fouInv** : IsUnit (fouInv l)

theorem **iso_fou** : Iso (fou l)

theorem **iso_fouInv** : Iso (fouInv l)

theorem **intw_fou** (p q : Eu l) : Intw (fou l) p q (−q) p

theorem **intw_fouInv** (p q : Eu l) : Intw (fouInv l) p q q (−p)

theorem **isUnit_big** {Ai : Eu l →L[ℝ] Eu l} (h : IsUnit Ai) (B C : Eu l →L[ℝ] Eu l) {c : ℝ} (hc : c ≠ 0) : IsUnit (big Ai B C c)

theorem **iso_big** {Ai : Eu l →L[ℝ] Eu l} (h : IsUnit Ai) (B C : Eu l →L[ℝ] Eu l) {c : ℝ} (hc : c ^ 2 = |Ai.det|) : Iso (big Ai B C c)

theorem **big_one** : big (1 : Eu l →L[ℝ] Eu l) 0 0 1 = 1

theorem **intw_big** {A Ai B C D : Eu l →L[ℝ] Eu l} (h : IsUnit Ai) (h1 : ∀ x, A (Ai x) = x) (h2 : ∀ x, Ai (A x) = x) (hAC : ∀ y z, ⟪A y, C z⟫ = ⟪C y, A z⟫) (hBD : ∀ y z, ⟪B y, D z⟫ = ⟪D y, B z⟫) (hAD : ∀ y z, ⟪A y, D z⟫ − ⟪C y, B z⟫ = ⟪y, z⟫) (c : ℝ) (p q : Eu l) : Intw (big Ai B C c) p q (A p + B q) (C p + D q)

CompanionTheory.ThickeningEstimates.LieGroupReduction.LocalLift

theorem **bl_apply** (i j : l → l ⊕ l) (g : Matrix (l ⊕ l) (l ⊕ l) ℝ) (x : Eu l) (a : l) : bl i j g x a = ∑ b, g (i a) (j b) * x b

theorem **pPart_mulVec** (g : Matrix (l ⊕ l) (l ⊕ l) ℝ) (w : l ⊕ l → ℝ) : pPart (g *ᵥ w) = bl Sum.inl Sum.inl g (pPart w) + bl Sum.inl Sum.inr g (qPart w)

theorem **qPart_mulVec** (g : Matrix (l ⊕ l) (l ⊕ l) ℝ) (w : l ⊕ l → ℝ) : qPart (g *ᵥ w) = bl Sum.inr Sum.inl g (pPart w) + bl Sum.inr Sum.inr g (qPart w)

theorem **det_bl** (i j : l → l ⊕ l) (g : Matrix (l ⊕ l) (l ⊕ l) ℝ) : (bl i j g).det = (g.submatrix i j).det

theorem **block_inl_inl_one** : bl Sum.inl Sum.inl (1 : Matrix (l ⊕ l) (l ⊕ l) ℝ) = 1

theorem **block_inl_inr_one** : bl Sum.inl Sum.inr (1 : Matrix (l ⊕ l) (l ⊕ l) ℝ) = 0

theorem **block_inr_inl_one** : bl Sum.inr Sum.inl (1 : Matrix (l ⊕ l) (l ⊕ l) ℝ) = 0

theorem **hasDerivAt_lin_exp** {F : Type} [NormedAddCommGroup F] [NormedSpace ℝ F] (L : Matrix (l ⊕ l) (l ⊕ l) ℝ →ₗ[ℝ] F) (X : Matrix (l ⊕ l) (l ⊕ l) ℝ) (t : ℝ) : HasDerivAt (fun t : ℝ => L (NormedSpace.exp (t • X))) (L (NormedSpace.exp (t • X) ∗ X)) t

theorem **exp_mul_exp_neg** (X : Matrix (l ⊕ l) (l ⊕ l) ℝ) : NormedSpace.exp X ∗ NormedSpace.exp (−X) = 1

theorem **sympl_exp** {X : Matrix (l ⊕ l) (l ⊕ l) ℝ} (hX : X ∈ LieAlgebra.Symplectic.sp l ℝ) (w w' : l ⊕ l → ℝ) : sympl (NormedSpace.exp X *ᵥ w) (NormedSpace.exp X *ᵥ w') = sympl w w'

theorem **intw_of_sympl** {g : Matrix (l ⊕ l) (l ⊕ l) ℝ} (hg : ∀ w w', sympl (g *ᵥ w) (g *ᵥ w') = sympl w w') (h : IsUnit (bl Sum.inl Sum.inl g)) (c : ℝ) (w : l ⊕ l → ℝ) : Intw (big (Ring.inverse (bl Sum.inl Sum.inl g)) (bl Sum.inl Sum.inr g) (bl Sum.inr Sum.inl g) c) (pPart w) (qPart w) (pPart (g *ᵥ w)) (qPart (g *ᵥ w))

theorem **fou_mq_fouInv** (B : Matrix l l ℝ) (f : Sch l) : fou l (mq (−(Matrix.toEuclideanCLM (𝕜

theorem **quad_eq** {X : Matrix (l ⊕ l) (l ⊕ l) ℝ} (hX : X ∈ LieAlgebra.Symplectic.sp l ℝ) (f : Sch l) : quad X f = mq (bl Sum.inr Sum.inl X) f + vf (−bl Sum.inl Sum.inl X) f + fou l (mq (−bl Sum.inl Sum.inr X) (fouInv l f)) − ((X.toBlocks₁₁.trace / 2 : ℝ) : ℂ) • f

theorem **differentiableAt_det** {M : ℝ → Matrix l l ℝ} {t : ℝ} (h : ∀ i j, DifferentiableAt ℝ (fun t => M t i j) t) : DifferentiableAt ℝ (fun t => (M t).det) t

theorem **localLift_holds** : LocalLift

CompanionTheory.ThickeningEstimates.LieGroupReduction.MetaplecticRepresentation.ContinuousOm

theorem **Ann.map** {s s' : Mat l} {U : Sch l →L[ℂ] Sch l} (h : ∀ w f, U (dRho w f) = dRho (s *ᵥ w) (U f)) {f : Sch l} (hf : Ann s' f) : Ann (s ∗ s') (U f)

theorem **mem** (g : G l) : Mem (Subtype.val (p

theorem **om_sch** (g : G l) (f : Sch l) : om g (schwartzToL2 l f) = schwartzToL2 l (omS g f)

theorem **om_rho** (g : G l) (w : l ⊕ l → ℝ) (f : L2 l) : om g (rho w f) = rho ((pr g).1 *ᵥ w) (om g f)

theorem **omS_dRho** (g : G l) (w : l ⊕ l → ℝ) (f : Sch l) : omS g (dRho w f) = dRho ((pr g).1 *ᵥ w) (omS g f)

theorem **omS_nrm** (g : G l) : Nrm (pr g).1 (omS g)

theorem **continuous_pair** : Continuous fun g : G l => ((pr g, fun f => om g f) : Sp l × (L2 l → L2 l))

theorem **continuous_pr** : Continuous (pr : G l → Sp l)

theorem **continuous_om** (f : L2 l) : Continuous fun g : G l => om g f

theorem **CompanionTheory.ThKR.Mp.realMetaplectic_of_forall** (h : ∀ (l : Type) [Fintype l] [DecidableEq l] [Nonempty l], Mp.TopG l ∧ Mp.Cover l ∧ Mp.Surj l ∧ Mp.Ker l ∧ Mp.Smooth l ∧ Mp.OnePar l) : GlobalTheory.Published.RealMetaplecticRepresentation

CompanionTheory.ThickeningEstimates.LieGroupReduction.MetaplecticRepresentation.ContinuousTo

theorem **dense_sch** : DenseRange (schwartzToL2 l)

theorem **ext_sch** (V : (Sch l →L[ℂ] Sch l)ˣ) (hn : ∀ f, ‖schwartzToL2 l (V.1 f)‖ = ‖schwartzToL2 l f‖) (f : Sch l) : ext V hn (schwartzToL2 l f) = schwartzToL2 l (V.1 f)

theorem **G_ext** {g h : G l} (h1 : pr g = pr h) (h2 : om g = om h) (h3 : omS g = omS h) : g = h

theorem **continuous_to** {X : Type} [TopologicalSpace X] {F : X → G l} (h1 : Continuous fun x => pr (F x)) (h2 : ∀ f, Continuous fun x => om (F x) f) : Continuous F

theorem **sp_mul** {X : Type} [TopologicalSpace X] {a b : X → Sp l} (ha : Continuous a) (hb : Continuous b) : Continuous fun x => a x * b x

theorem **sp_inv** {X : Type} [TopologicalSpace X] {a : X → Sp l} (ha : Continuous a) : Continuous fun x => (a x)⁻¹

theorem **om_cont2** : Continuous fun p : G l × L2 l => om p.1 p.2

theorem **om_mul** (g h : G l) (f : L2 l) : om (g * h) f = om g (om h f)

theorem **fderiv_gs** (j : l) (x : EuclideanSpace ℝ l) : fderiv ℝ (⇑(gs l)) x (EuclideanSpace.single j 1) + 2 * Real.pi * (x j : ℂ) * gs l x = 0

theorem **ann_gs** : Ann (1 : Sp l).1 (gs l)

theorem **gs_zero** : gs l 0 = 1

theorem **dd_one** : dd (1 : Mat l) = 1

theorem **eps_mem** : Mem ((1, LinearIsometryEquiv.neg ℂ, -1) : Amb l)

theorem **om_eps** (f : L2 l) : om (eps : G l) f = -f

theorem **omS_eps** (f : Sch l) : omS (eps : G l) f = -f

theorem **pr_eps** : pr (eps : G l) = 1

theorem **eps_ne** : (eps : G l) ≠ 1

theorem **om_of_omS** {g : G l} {c : ℂ} (hc : ∀ f, omS g f = c • f) (F : L2 l) : om g F = c • F

theorem **mkElt** : MkElt l

theorem **topG** : TopG l

theorem **ker** : Ker l

CompanionTheory.ThickeningEstimates.LieGroupReduction.MetaplecticRepresentation.HasFDerivAtOfCurves

theorem **hasFDerivAt_of_curves** {E F : Type} [NormedAddCommGroup E] [NormedSpace ℝ E] [ProperSpace E] [NormedAddCommGroup F] [NormedSpace ℝ F] {Φ : E → F} {L : E →L[ℝ] F} {x₀ : E} (h : ∀ (c : ℝ → E) (v : E), c 0 = x₀ → HasDerivAt c v 0 → HasDerivAt (fun t => Φ (c t)) (L v) 0) : HasFDerivAt Φ L x₀

theorem **hasDerivAt_det_one** {M : ℝ → l → l → ℝ} {X : l → l → ℝ} (h : HasDerivAt M X 0) (h0 : Matrix.of (M 0) = 1) : HasDerivAt (fun t => (Matrix.of (M t)).det) (Matrix.of X).trace 0

theorem **curve** {g : ℝ → (l ⊕ l) → (l ⊕ l) → ℝ} {X : (l ⊕ l) → (l ⊕ l) → ℝ} (g0 : Matrix.of (g 0) = 1) (hg : HasDerivAt g X 0) (f : Sch l) : HasDerivAt (fun t => schwartzToL2 l (sec (Matrix.of (g t)) f)) (schwartzToL2 l (Qd (Matrix.of X) f)) 0

theorem **secDer** : SecDer l

CompanionTheory.ThickeningEstimates.LieGroupReduction.MetaplecticRepresentation.HasFDerivAtOrb

theorem **hasFDerivAt_orb** (hT : TopG l) (hN : NearOne l) (hD : SecDer l) {k : ℕ} (γ : (Fin k → ℝ) → G l) (hγ : Continuous γ) (hs : ∀ a b, Differentiable ℝ fun t => (pr (γ t) : Mat l) a b) (f : Sch l) (t₀ : Fin k → ℝ) : HasFDerivAt (orb γ f) (∑ a, ∑ b, (dm γ t₀ a b).smulRight (orb γ (Qd (Matrix.single a b 1) f) t₀)) t₀

theorem **contDiff_dm** {k : ℕ} (γ : (Fin k → ℝ) → G l) (hs : ∀ a b, ContDiff ℝ ∞ fun t => (pr (γ t) : Mat l) a b) (a b : l ⊕ l) (n : ℕ) : ContDiff ℝ n fun t => dm γ t a b

theorem **smooth_of** (hT : TopG l) (hN : NearOne l) (hD : SecDer l) : Smooth l

CompanionTheory.ThickeningEstimates.LieGroupReduction.MetaplecticRepresentation.MapOfRealMul

theorem **upperPart_mul** (s s' : Mat l) : upperParabolic (s * s') = s.toBlocks₁₁.map Complex.ofReal * upperParabolic s' + s.toBlocks₁₂.map Complex.ofReal * loQ s'

theorem **map_ofReal_mul** (M N : Matrix l l ℝ) : (M * N).map Complex.ofReal = M.map Complex.ofReal * N.map Complex.ofReal

theorem **det_map_ofReal** (M : Matrix l l ℝ) : (M.map Complex.ofReal).det = (M.det : ℂ)

theorem **dd_mul_eq** (s s' : Mat l) (hd : s.toBlocks₁₁.det ≠ 0) : dd (s * s') = (s.toBlocks₁₁.det : ℂ) * (upperParabolic s' + (s.toBlocks₁₁⁻¹ * s.toBlocks₁₂).map Complex.ofReal * loQ s').det

theorem **blocks_symm** {A B C D : Matrix l l ℝ} (h : fromBlocks A B C D * Matrix.J l ℝ * (fromBlocks A B C D)ᵀ = Matrix.J l ℝ) : B * Aᵀ = A * Bᵀ

theorem **isSymm_inv_mul** (s : Sp l) (hd : s.1.toBlocks₁₁.det ≠ 0) : (s.1.toBlocks₁₁⁻¹ * s.1.toBlocks₁₂).IsSymm

theorem **sympl_sp** (s : Sp l) (w w' : l ⊕ l → ℝ) : sympl (s.1 *ᵥ w) (s.1 *ᵥ w') = sympl w w'

theorem **sec_apply_zero** (s : Mat l) (hA : IsUnit (bl Sum.inl Sum.inl s)) (f : Sch l) : sec s f 0 = (((√|s.toBlocks₁₁.det|)⁻¹ : ℝ) : ℂ) * fou l (ph (-(Ring.inverse (bl Sum.inl Sum.inl s) * bl Sum.inl Sum.inr s)) (fouInv l f)) 0

theorem **secEx_of** (h1 : MkElt l) (h2 : NrmUp l) : SecEx l

theorem **lo_mem** (t : ℝ) : lo (l

theorem **lo_mul** (t : ℝ) : lo (l

theorem **toBlocks₁₁_mul_lo** (s : Mat l) (t : ℝ) : (s * lo t).toBlocks₁₁ = s.toBlocks₁₁ + t • s.toBlocks₁₂

theorem **exists_det_ne** (s : Mat l) (h : dd s ≠ 0) : ∃ t : ℝ, (s.toBlocks₁₁ + t • s.toBlocks₁₂).det ≠ 0

theorem **surj_of** (h1 : DdNe l) (h2 : SecEx l) : Surj l

CompanionTheory.ThickeningEstimates.LieGroupReduction.MetaplecticRepresentation.MetaplecticGroupT2Space

theorem **toBlocks₁₁_one'** : (1 : Mat l).toBlocks₁₁ = 1

theorem **sec_one** : sec (1 : Mat l) = 1

theorem **eventually_det_pos** : ∀ᶠ s in 𝒩 (1 : Sp l), 0 < s.1.toBlocks₁₁.det

theorem **eventually_det_pos_G** : ∀ᶠ g in 𝒩 (1 : G l), 0 < (pr g).1.toBlocks₁₁.det

theorem **tendsto_sec** (hD : SecDer l) (f : Sch l) : Tendsto (fun s : Sp l => schwartzToL2 l (sec s.1 f)) (𝒩 1) (𝒩 (schwartzToL2 l f))

theorem **exists_sch_ne** : ∃ f : Sch l, schwartzToL2 l f ≠ 0

theorem **nearOne_of** (hS : SecEx l) (hK : Ker l) (hD : SecDer l) : NearOne l

theorem **isInducing_F** : IsInducing (fun g : G l => ((pr g, fun f => om g f) : Sp l × (L2 l → L2 l)))

theorem **metaplectic_group_t2Space** : T2Space (G l)

theorem **cover_of** (hT : TopG l) (hO : Surj l) (hK : Ker l) (hS : SecEx l) (hD : SecDer l) : Cover l

CompanionTheory.ThickeningEstimates.LieGroupReduction.MetaplecticRepresentation.MetaplecticUnipotentBlockSecondDiag

theorem **J_mul_transpose_inl** (s : Mat l) (a : l) (j : l ⊕ l) : (Matrix.J l ℝ * sᵀ) (.inl a) j = -s j (.inr a)

theorem **J_mul_transpose_inr** (s : Mat l) (a : l) (j : l ⊕ l) : (Matrix.J l ℝ * sᵀ) (.inr a) j = s j (.inl a)

theorem **sp_rel** (s : Sp l) (i j : l ⊕ l) : ∑ a, (s.1 i (.inr a) * s.1 j (.inl a) - s.1 i (.inl a) * s.1 j (.inr a)) = Matrix.J l ℝ i j

theorem **ddNe** : DdNe l

theorem **nn_mem** {S : Matrix l l ℝ} (hS : S.IsSymm) (t : ℝ) : nn S t ∈ Matrix.symplecticGroup l ℝ

theorem **upperPart_unipotent_mul** (S : Matrix l l ℝ) (t : ℝ) (s : Mat l) : upperParabolic (nn S t * s) = upperParabolic s + (t : ℂ) • (S.map ofReal * loQ s)

theorem **loQ_nn** (S : Matrix l l ℝ) (t : ℝ) (s : Mat l) : loQ (nn S t * s) = loQ s

theorem **tc_apply** (M : Matrix l l ℝ) (x : Eu l) (a : l) : Matrix.toEuclideanCLM (𝕜

theorem **metaplectic_unipotent_block_first_diag** (S : Matrix l l ℝ) (t : ℝ) : bl Sum.inl Sum.inl (nn S t) = 1

theorem **metaplectic_unipotent_block_upper** (S : Matrix l l ℝ) (t : ℝ) : bl Sum.inl Sum.inr (nn S t) = t • Matrix.toEuclideanCLM (𝕜

theorem **metaplectic_unipotent_block_lower** (S : Matrix l l ℝ) (t : ℝ) : bl Sum.inr Sum.inl (nn S t) = 0

theorem **metaplectic_unipotent_block_second_diag** (S : Matrix l l ℝ) (t : ℝ) : bl Sum.inr Sum.inr (nn S t) = 1

theorem **inner_sym** {S : Matrix l l ℝ} (hS : S.IsSymm) (t : ℝ) (x y : Eu l) : ⟪x, (-(t • Matrix.toEuclideanCLM (𝕜

theorem **intw_UU** {S : Matrix l l ℝ} (hS : S.IsSymm) (t : ℝ) (p q : Eu l) : Intw (UU S t) p q (p + (t • Matrix.toEuclideanCLM (𝕜

theorem **impl_UU** {S : Matrix l l ℝ} (hS : S.IsSymm) (t : ℝ) : Impl (fun w => nn S t *ᵥ w) (UU S t).toLinearMap

theorem **isUnit_UU** (S : Matrix l l ℝ) (t : ℝ) : IsUnit (UU S t)

theorem **UU_dRho** {S : Matrix l l ℝ} (hS : S.IsSymm) (t : ℝ) (w : l ⊕ l → ℝ) (f : Sch l) : UU S t (dRho w f) = dRho (nn S t *ᵥ w) (UU S t f)

theorem **UU_zero** (S : Matrix l l ℝ) : UU S 0 = 1

theorem **fou_zero** (h : Sch l) : fou l h 0 = ∫ x, h x

theorem **hasDerivAt_UU** (S : Matrix l l ℝ) (f : Sch l) (t₀ : ℝ) : HasDerivAt (fun t => UU S t f 0) (fou l (mq (-(Matrix.toEuclideanCLM (𝕜

theorem **dRho_eval_zero** (w : l ⊕ l → ℝ) (h : Sch l) : dRho w h 0 = ∑ a, (w (.inl a) : ℂ) * (∂_{(EuclideanSpace.single a 1 : Eu l)} h) 0

theorem **hess** {s : Mat l} {H : Sch l} (hA : Ann s H) (j k : l) : ∑ a, upperParabolic s a j * (∂_{(EuclideanSpace.single a 1 : Eu l)} (∂_{(EuclideanSpace.single k 1 : Eu l)} H)) 0 = -(2 * π * I) * loQ s k j * H 0

theorem **trace_eq** {P Q : Matrix l l ℂ} (hP : P.det ≠ 0) (S M : Matrix l l ℂ) (c : ℂ) (h : ∀ j k, ∑ a, P a j * M a k = c * Q k j) : ∑ j, ∑ k, S j k * M j k = c * (P⁻¹ * (S * Q)).trace

theorem **hasDerivAt_det** (P K : Matrix l l ℂ) (t₀ : ℝ) (hP : (P + (t₀ : ℂ) • K).det ≠ 0) : HasDerivAt (fun t : ℝ => (P + (t : ℂ) • K).det) ((P + (t₀ : ℂ) • K).det * ((P + (t₀ : ℂ) • K)⁻¹ * K).trace) t₀

theorem **nrmUp** : NrmUp l

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theorem **mq_add'** (S T : Eu l →L[ℝ] Eu l) (g : Sch l) : mq (S + T) g = mq S g + mq T g

theorem **mq_smul'** (c : ℝ) (S : Eu l →L[ℝ] Eu l) (g : Sch l) : mq (c • S) g = c • mq S g

theorem **vf_add'** (H K : Eu l →L[ℝ] Eu l) (g : Sch l) : vf (H + K) g = vf H g + vf K g

theorem **vf_smul'** (c : ℝ) (H : Eu l →L[ℝ] Eu l) (g : Sch l) : vf (c • H) g = c • vf H g

theorem **qd_sum** (X : Mat l) (f : Sch l) : ∑ a, ∑ b, X a b • Qd (Matrix.single a b 1) f = Qd X f

theorem **exp_mem_sp** {X : Mat l} (hX : X ∈ LieAlgebra.Symplectic.sp l ℝ) : NormedSpace.exp X ∈ Matrix.symplecticGroup l ℝ

theorem **onePar_of** (h1 : TopG l) (h2 : Cover l) (h3 : NearOne l) (h4 : SecDer l) : OnePar l

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theorem **CompanionTheory.ThKR.realMetaplecticRepresentation_holds** : GlobalTheory.Published.RealMetaplecticRepresentation

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theorem **mq_apply** (S : Eu l →L[ℝ] Eu l) (f : Sch l) (x : Eu l) : mq S f x = ((-π * ⟪x, S x⟫ : ℝ) : ℂ) * I * f x

theorem **norm_exp_sub_le** (a b : ℝ) : ‖exp ((a : ℂ) * I) - exp ((b : ℂ) * I)‖ ≤ |a - b|

theorem **exists_lip** {F : Type} [NormedAddCommGroup F] [NormedSpace ℝ F] {S : ℝ → F} {S' : F} {t₀ : ℝ} (h : HasDerivAt S S' t₀) : ∃ K : ℝ, 0 ≤ K ∧ ∀ᶠ t in 𝓝 t₀, ‖S t - S t₀‖ ≤ K * |t - t₀|

theorem **ld_ph** {S : ℝ → Eu l →L[ℝ] Eu l} {S' : Eu l →L[ℝ] Eu l} {t₀ : ℝ} (h : HasDerivAt S S' t₀) : LD (fun t => ph (S t)) (fun f => mq S' (ph (S t₀) f)) t₀

theorem **vf_apply** (H : Eu l →L[ℝ] Eu l) (f : Sch l) (x : Eu l) : vf H f x = fderiv ℝ f x (H x)

theorem **ld_dil_one** {H : ℝ → Eu l →L[ℝ] Eu l} {H' : Eu l →L[ℝ] Eu l} {t₀ : ℝ} (h : HasDerivAt H H' t₀) (h0 : H t₀ = 1) (f : Sch l) {F : ℝ → Sch l} (hF : ∀ᶠ t in 𝓝 t₀, ∀ x, F t x = f (H t x)) : HasDerivAt (fun t => schwartzToL2 l (F t)) (schwartzToL2 l (vf H' f)) t₀

theorem **ld_dil** {B : ℝ → Eu l →L[ℝ] Eu l} {B' : Eu l →L[ℝ] Eu l} {t₀ : ℝ} (h : HasDerivAt B B' t₀) (h0 : IsUnit (B t₀)) : LD (fun t => dil (B t)) (fun f => vf (Ring.inverse (B t₀) * B') (dil (B t₀) f)) t₀

theorem **ld_big** {Ai B C : ℝ → Eu l →L[ℝ] Eu l} {Ai' B' C' : Eu l →L[ℝ] Eu l} {c : ℝ → ℝ} {c' t₀ : ℝ} (hA : HasDerivAt Ai Ai' t₀) (hB : HasDerivAt B B' t₀) (hC : HasDerivAt C C' t₀) (hc : HasDerivAt c c' t₀) (h : ∀ᶠ t in 𝓝 t₀, IsUnit (Ai t) ∧ c t ^ 2 = |(Ai t).det|) : ∃ Q, LD (fun t => big (Ai t) (B t) (C t) (c t)) Q t₀ ∧ (Ai t₀ = 1 → B t₀ = 0 → C t₀ = 0 → c t₀ = 1 → ∀ f, Q f = mq C' f + vf Ai' f + fou l (mq (-B') (fouInv l f)) + (c' : ℂ) • f)

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theorem **val_rIdU** [IsTotallyReal F] (c : ℝ) : (rIdU F E c : AF F ⊗[F] E) = algebraMap (AF F) (AF F ⊗[F] E) (rId F c)

theorem **inv_rIdU** [IsTotallyReal F] (c : ℝ) : (↑(rIdU F E c)⁻¹ : AF F ⊗[F] E) = algebraMap (AF F) (AF F ⊗[F] E) (rId F (-c))

theorem **card_infinitePlace** [IsTotallyReal F] : Fintype.card (InfinitePlace F) = Module.finrank ℚ F

theorem **norm_evInf_rId** [IsTotallyReal F] (w : InfinitePlace F) (c : ℝ) : ‖evInf F w (rId F c)‖ = Real.exp c

theorem **absE_rIdU** [IsTotallyReal F] (h2 : Module.finrank F E = 2) (c : ℝ) : (GlobalTheory.Doubling.absE F E (rIdU F E c) : ℝ) = Real.exp (2 * Module.finrank ℚ F * c)

theorem **pos_hom'** {H : Type} [Group H] (δ : H →* ℝ≥0) (p : H) : 0 < δ p

theorem **sigRA_sigRA** (h : IsCMExt F E σ) (x : AF F ⊗[F] E) : Formalism.MainNotions.GlobalTheoryTheory.GenusMassVolumes.sigR (AF F) σ (Formalism.MainNotions.GlobalTheoryTheory.GenusMassVolumes.sigR (AF F) σ x) = x

theorem **sigRA_inclR** (e : E) : Formalism.MainNotions.GlobalTheoryTheory.GenusMassVolumes.sigR (AF F) σ (inclR (F

theorem **cT_map_inclR** {a b : Type} (P : Matrix a b E) : (Formalism.UG.cT (σ : E →+* E) P).map (inclR (F

theorem **PFr.cc** (hF : PFr s Jm Xm Ym) {a b : Type} (P : Matrix a b R) : cT s (cT s P) = P

theorem **PFr.yx** (hF : PFr s Jm Xm Ym) : cT s Ym * Jm * Xm = 1

theorem **pgram_unitary** {g : Matrix (Fin n) (Fin n) R} (hg : cT s g * Jm * g = Jm) {a b : Type} (P : Matrix (Fin n) a R) (Q : Matrix (Fin n) b R) : cT s (g * P) * Jm * (g * Q) = cT s P * Jm * Q

theorem **PFr.inv_coef** (hF : PFr s Jm Xm Ym) {g : Matrix (Fin n) (Fin n) R} (hg : cT s g * Jm * g = Jm) {B : Matrix (Fin r) (Fin r) R} (hB : g * Xm = Xm * B) : cT s (cT s Xm * Jm * (g * Ym)) * B = 1

theorem **pfrA** (h : IsCMExt F E σ) (hJ : IsHermND σ J) (hW : IsWittFrame σ J X Y) : PFr (Formalism.MainNotions.GlobalTheoryTheory.GenusMassVolumes.sigR (AF F) σ) (Formalism.MainNotions.GlobalTheoryTheory.GenusMassVolumes.JR (AF F) J) (GlobalTheory.SiegelSet.adMat F X) (GlobalTheory.SiegelSet.adMat F Y)

theorem **unitaryA_matHom** (p : GAd σ J) : Formalism.UG.cT (Formalism.MainNotions.GlobalTheoryTheory.GenusMassVolumes.sigR (AF F) σ) (matHom _ σ J p).1 * Formalism.MainNotions.GlobalTheoryTheory.GenusMassVolumes.JR (AF F) J * (matHom _ σ J p).1 = Formalism.MainNotions.GlobalTheoryTheory.GenusMassVolumes.JR (AF F) J

theorem **Bm_eq** {p : GAd σ J} {B : Matrix (Fin r) (Fin r) (AF F ⊗[F] E)} (e : (matHom _ σ J p).1 * GlobalTheory.SiegelSet.adMat F X = GlobalTheory.SiegelSet.adMat F X * B) : Bm σ J X Y p = B

theorem **mem_minPar_iff** {p : GAd σ J} : p ∈ minPar σ J X ↔ (Bm σ J X Y p).BlockTriangular id ∧ (matHom _ σ J p).1 * GlobalTheory.SiegelSet.adMat F X = GlobalTheory.SiegelSet.adMat F X * Bm σ J X Y p

theorem **Bm_mul** {p q : GAd σ J} (hp : p ∈ minPar σ J X) (hq : q ∈ minPar σ J X) : Bm σ J X Y (p * q) = Bm σ J X Y p * Bm σ J X Y q

theorem **Bm_one** : Bm σ J X Y (1 : GAd σ J) = 1

theorem **diag_mul_bt** {R : Type} [CommRing R] {B C : Matrix (Fin r) (Fin r) R} (hB : B.BlockTriangular id) (hC : C.BlockTriangular id) (i : Fin r) : (B * C) i i = B i i * C i i

theorem **mul_mem_minimal_parabolic** {p q : GAd σ J} (hp : p ∈ minPar σ J X) (hq : q ∈ minPar σ J X) : p * q ∈ minPar σ J X

theorem **inv_mem_minimal_parabolic** (h : IsCMExt F E σ) (hJ : IsHermND σ J) (hW : IsWittFrame σ J X Y) {p : GAd σ J} (hp : p ∈ minPar σ J X) : p⁻¹ ∈ minPar σ J X

theorem **continuous_matHom_val** : Continuous fun p : GAd σ J => (matHom _ σ J p).1

theorem **continuous_Bm** : Continuous (Bm σ J X Y)

theorem **isClosed_minimal_parabolic** (h : IsCMExt F E σ) (hJ : IsHermND σ J) (hW : IsWittFrame σ J X Y) : IsClosed (minPar σ J X)

theorem **val_uU** (h : IsCMExt F E σ) (hJ : IsHermND σ J) (hW : IsWittFrame σ J X Y) {p : GAd σ J} (hp : p ∈ minPar σ J X) (hp' : p⁻¹ ∈ minPar σ J X) (i : Fin r) : (uU σ J X Y h hJ hW hp hp' i : AF F ⊗[F] E) = Bm σ J X Y p i i

theorem **minParOne_sub** {p : GAd σ J} (hp : p ∈ minParOne σ J X) : p ∈ minPar σ J X

theorem **val_uP** (h : IsCMExt F E σ) (hJ : IsHermND σ J) (hW : IsWittFrame σ J X Y) (P : Subgroup (GAd σ J)) (hP : ∀ p, p ∈ P ↔ p ∈ minPar σ J X) (p : P) (i : Fin r) : (uP σ J X Y h hJ hW P hP p i : AF F ⊗[F] E) = Bm σ J X Y p.1 i i

theorem **exists_torus_section** (h : IsCMExt F E σ) (hJ : IsHermND σ J) (hW : IsWittFrame σ J X Y) (P : Subgroup (GAd σ J)) (hP : ∀ p, p ∈ P ↔ p ∈ minPar σ J X) {a : (Fin r → ℝ) → GAd σ J} (hadd : ∀ s t, a (s + t) = a s * a t) (honX : ∀ t, (matHom _ σ J (a t)).1 * GlobalTheory.SiegelSet.adMat F X = GlobalTheory.SiegelSet.adMat F X * Matrix.diagonal (fun i => algebraMap (AF F) (AF F ⊗[F] E) (rId F (t i)))) (ta : (Fin r → ℝ) → P) (hta : ∀ t, (ta t : GAd σ J) = a t) : ∃ τ : P → Fin r → ℝ, Continuous τ ∧ (∀ p q, τ (p * q) = τ p + τ q) ∧ (∀ t, τ (ta t) = t) ∧ (∀ p (hp : p ∈ minParOne σ J X), τ (p, (hP p).2 (minParOne_sub hp)) = 0) ∧ ∀ p : P, p.1 * (a (τ p))⁻¹ ∈ minParOne σ J X

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theorem **one_mem_minPar** (X : Matrix (Fin n) (Fin r) E) : (1 : GAd σ J) ∈ minPar σ J X

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theorem **char_eq_of_retract** (ι : N →* H) (hι : Continuous ι) (π : H → N) (hπ : Continuous π) (hmul : ∀ x p, π (ι x * p) = x * π p) (φ : H ≃ₜ* H) (ψ : N ≃ₜ* N) (h1 : ∀ p, π (φ p) = ψ (π p)) (h2 : ∀ p, (ι (π (φ p)))⁻¹ * φ p = (ι (π p))⁻¹ * p) : mulEquivHaarChar φ = mulEquivHaarChar ψ

theorem **Heis.mul_def** (x y : Heis β) : x * y = (x.1 + y.1, x.2 + y.2 + β x.1 y.1)

theorem **Heis.inv_def** (x : Heis β) : x⁻¹ = (-x.1, -x.2 + β x.1 x.1)

theorem **Heis.isTopologicalGroup** (hβ : Continuous fun p : A × A => β p.1 p.2) : IsTopologicalGroup (Heis β) where

theorem **Heis.char_eq** [IsTopologicalGroup (Heis β)] (hβ : Continuous fun p : A × A => β p.1 p.2) (ψ : Heis β ≃ₜ* Heis β) (θ₁ : A ≃ₜ+ A) (θ₂ : B ≃ₜ+ B) (h : ∀ x, ψ x = (θ₁ x.1, θ₂ x.2)) : mulEquivHaarChar ψ = addEquivAddHaarChar θ₁ * addEquivAddHaarChar θ₂

theorem **isClosed_image_section** {X : Type} [TopologicalSpace X] [T2Space X] (τ : X → Fin r → ℝ) (hτ : Continuous τ) (a : (Fin r → ℝ) → X) (ha : Continuous a) (h : ∀ t, τ (a t) = t) {D : Set (Fin r → ℝ)} (hD : IsClosed D) : IsClosed (a '' D)

theorem **lintegral_tau** (ν : Measure G) [ν.IsHaarMeasure] (τ : G → Fin r → ℝ) (hτ : Continuous τ) (hτm : ∀ p q, τ (p * q) = τ p + τ q) (a : (Fin r → ℝ) → G) (ha : Continuous a) (hadd : ∀ s t, a (s + t) = a s * a t) (hτa : ∀ t, τ (a t) = t) (ρ : (Fin r → ℝ) → ℝ) (hρ : Continuous ρ) (hpa : ∀ s t, ρ (s + t) = ρ s + ρ t) (hmod : ∀ s, ν.map (· * a s) = ENNReal.ofReal (Real.exp (ρ s)) • ν) {Ω : Set G} (hΩ : IsCompact Ω) (hΩτ : ∀ p ∈ Ω, τ p = 0) : ∃ C : ℝ≥0, ∀ h : (Fin r → ℝ) → ℝ≥0∞, Measurable h → ∫⁻ p in Ω * range a, h (τ p) ∂ν = C * ∫⁻ t, h t * ENNReal.ofReal (Real.exp (-ρ t))

theorem **map_mul_right_of_char** (ν : Measure G) [ν.IsHaarMeasure] (b : G) (φ : G ≃ₜ* G) (hφ : ∀ p, φ p = b⁻¹ * p * b) : ν.map (· * b) = ((mulEquivHaarChar φ)⁻¹ : ℝ≥0) • ν

theorem **lintegral_siegel_le** (μ : Measure G) [μ.IsHaarMeasure] (P K : Subgroup G) (hP : IsClosed (P : Set G)) (hK : IsCompact (K : Set G)) (hPK : ∀ g, ∃ p ∈ P, ∃ k ∈ K, g = p * k) [LocallyCompactSpace P] (τ : P → Fin r → ℝ) (hτ : Continuous τ) (hτm : ∀ p q, τ (p * q) = τ p + τ q) (a : (Fin r → ℝ) → P) (ha : Continuous a) (hadd : ∀ s t, a (s + t) = a s * a t) (hτa : ∀ t, τ (a t) = t) (ρ : (Fin r → ℝ) → ℝ) (hρ : Continuous ρ) (hpa : ∀ s t, ρ (s + t) = ρ s + ρ t) (hmod : ∀ s, (haar : Measure P).map (· * a s) = ENNReal.ofReal (Real.exp (ρ s)) • haar) {Ω : Set G} (hΩ : IsCompact Ω) (hΩτ : ∀ p ∈ Ω, ∃ h : p ∈ P, τ (p, h) = 0) (hc : ∀ m : P, m.1 ∈ K → ∀ t, a t * m = m * a t) : ∃ C : ℝ≥0, ∀ (D : Set (Fin r → ℝ)) (S : Set G), MeasurableSet D → MeasurableSet S → (∀ g ∈ S, ∃ p ∈ Ω, ∃ t ∈ D, ∃ k ∈ K, g = p * a t * k) → ∀ (f : G → ℝ≥0∞) (h : (Fin r → ℝ) → ℝ≥0∞), Measurable h → (∀ p ∈ Ω, ∀ t ∈ D, ∀ k ∈

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theorem **cT_add** {a b : Type} (P Q : Matrix a b R) : cT s (P + Q) = cT s P + cT s Q

theorem **cT_sub** {a b : Type} (P Q : Matrix a b R) : cT s (P - Q) = cT s P - cT s Q

theorem **cT_neg** {a b : Type} (P : Matrix a b R) : cT s (-P) = -cT s P

theorem **cT_zero** {a b : Type} : cT s (0 : Matrix a b R) = 0

theorem **cT_smul** {a b : Type} (c : R) (P : Matrix a b R) : cT s (c • P) = s c • cT s P

theorem **Fr.cc** (h : Fr s J X Y) {a b : Type} (P : Matrix a b R) : cT s (cT s P) = P

theorem **Fr.swap** (h : Fr s J X Y) {a b : Type} (P : Matrix (Fin n) a R) (Q : Matrix (Fin n) b R) : cT s (cT s P * J * Q) = cT s Q * J * P

theorem **Fr.yx** (h : Fr s J X Y) : cT s Y * J * X = 1

theorem **Fr.zero** (h : Fr s J X Y) {a b : Type} {P : Matrix (Fin n) a R} {Q : Matrix (Fin n) b R} (e : cT s P * J * Q = 0) : cT s Q * J * P = 0

theorem **gram_unitary** {g : Matrix (Fin n) (Fin n) R} (hg : cT s g * J * g = J) {a b : Type} (P : Matrix (Fin n) a R) (Q : Matrix (Fin n) b R) : cT s (g * P) * J * (g * Q) = cT s P * J * Q

theorem **Fr.par** (h : Fr s J X Y) {g : Matrix (Fin n) (Fin n) R} (hg : cT s g * J * g = J) {B : Matrix (Fin k) (Fin k) R} (hB : g * X = X * B) : cT s Y * J * (g * X) = B ∧ cT s B * (cT s X * J * (g * Y)) = 1 ∧ cT s X * J * (g * Y) * cT s B = 1 ∧ cT s X * J * g = cT s X * J * (g * Y) * (cT s X * J)

theorem **Fr.symm** (h : Fr s J X Y) : Fr s J Y X

theorem **Fr.ext** (h : Fr s J X Y) {g g' : Matrix (Fin n) (Fin n) R} (h1 : g * X = g' * X) (h2 : g * Y = g' * Y) (h3 : ∀ Z : Matrix (Fin n) (Fin n) R, cT s X * J * Z = 0 → cT s Y * J * Z = 0 → g * Z = g' * Z) : g = g'

theorem **Fr.comm** (h : Fr s J X Y) {l b : Matrix (Fin n) (Fin n) R} (hl : cT s l * J * l = J) {B D B' D' : Matrix (Fin k) (Fin k) R} (hB : l * X = X * B) (hD : l * Y = Y * D) (hB' : b * X = X * B') (hD' : b * Y = Y * D') (e1 : B * B' = B' * B) (e2 : D * D' = D' * D) (hb : ∀ Z : Matrix (Fin n) (Fin n) R, cT s X * J * Z = 0 → cT s Y * J * Z = 0 → b * Z = Z) : b * l = l * b

theorem **Fr.eich_mul_of** (h : Fr s J X Y) {Z : Matrix (Fin n) (Fin k) R} (c : Matrix (Fin k) (Fin k) R) {a : Type} {U : Matrix (Fin n) a R} (hU : cT s X * J * U = 0) : eich s J X Z c * U = U - X * (cT s Z * J * U)

theorem **Fr.eich_X** (h : Fr s J X Y) {Z : Matrix (Fin n) (Fin k) R} (hZ : cT s X * J * Z = 0) (c : Matrix (Fin k) (Fin k) R) : eich s J X Z c * X = X

theorem **Fr.eich_Y** (h : Fr s J X Y) {Z : Matrix (Fin n) (Fin k) R} (hZ : cT s Y * J * Z = 0) (c : Matrix (Fin k) (Fin k) R) : eich s J X Z c * Y = Y + Z + X * c

theorem **Fr.eich_mul** (h : Fr s J X Y) {Z₀ Z : Matrix (Fin n) (Fin k) R} (h₀ : cT s X * J * Z₀ = 0) (hZ : cT s X * J * Z = 0) (c₀ c : Matrix (Fin k) (Fin k) R) : eich s J X Z₀ c₀ * eich s J X Z c = eich s J X (Z₀ + Z) (c₀ + c - cT s Z₀ * J * Z)

theorem **eich_zero** : eich s J X 0 0 = 1

theorem **Fr.eich_unitary** (h : Fr s J X Y) {Z : Matrix (Fin n) (Fin k) R} (hZ : cT s X * J * Z = 0) {c : Matrix (Fin k) (Fin k) R} (hc : cT s c + c + cT s Z * J * Z = 0) : cT s (eich s J X Z c) * J * eich s J X Z c = J

theorem **s_half** : s ⅟2 = ⅟2

theorem **half_add** {a b : Type} (P : Matrix a b R) : (⅟2 : R) • P + (⅟2 : R) • P = P

theorem **Fr.β_val** (h : Fr s J X Y) (Z₀ Z : W1 s J X Y) : (h.β Z₀ Z).1 = (⅟2 : R) • (cT s Z.1 * J * Z₀.1 - cT s Z₀.1 * J * Z.1)

theorem **Fr.nMat_apply** (h : Fr s J X Y) (x : Heis h.β) : h.nMat x = eich s J X x.1.1 (x.2.1 - (⅟2 : R) • (cT s x.1.1 * J * x.1.1))

theorem **Fr.nMat_unitary** (h : Fr s J X Y) (x : Heis h.β) : cT s (h.nMat x) * J * h.nMat x = J

theorem **Fr.decomp_inj** (h : Fr s J X Y) {Z Z' : Matrix (Fin n) (Fin k) R} (hZ : cT s Y * J * Z = 0) (hZ' : cT s Y * J * Z' = 0) {c c' : Matrix (Fin k) (Fin k) R} (e : Y + Z + X * c = Y + Z' + X * c') : Z = Z' ∧ c = c'

theorem **Fr.nMat_inj** (h : Fr s J X Y) {x y : Heis h.β} (e : h.nMat x * Y = h.nMat y * Y) : x = y

theorem **Fr.nMat_pt** (h : Fr s J X Y) (W : Matrix (Fin n) (Fin k) R) (h1 : cT s X * J * W = 1) (h2 : cT s W * J * W = 0) : h.nMat (h.pt W h1 h2) * Y = W

theorem **continuous_cT** (hs : Continuous s) {a b T : Type} [TopologicalSpace T] {f : T → Matrix a b R} (hf : Continuous f) : Continuous fun x => cT s (f x)

theorem **isClosed_frame_annihilator** : IsClosed (W1 s J X Y : Set (Matrix (Fin n) (Fin k) R))

theorem **isClosed_skew_hermitian** (hs : Continuous s) : IsClosed (W2 s k : Set (Matrix (Fin k) (Fin k) R))

theorem **smul_mem_frame_annihilator** (c : R) (x) (hx : x ∈ W1 s J X Y) : c • x ∈ W1 s J X Y

theorem **smul_mem_skew_hermitian** {c : R} (hc : s c = c) (x) (hx : x ∈ W2 s k) : c • x ∈ W2 s k

theorem **s_inv** {u : Rˣ} (hu : s u = u) : s u⁻¹.1 = u⁻¹.1

theorem **haar_character_frame_annihilator** (h : Fr s J X Y) [LocallyCompactSpace (W1 s J X Y)] (u : Rˣ) : addEquivAddHaarChar (scalM n k u) = addEquivAddHaarChar (scal (W1 s J X Y) u (smul_mem_frame_annihilator _) (smul_mem_frame_annihilator _)) * (addEquivAddHaarChar (scalM k k u) * addEquivAddHaarChar (scalM k k u))

theorem **haar_character_skew_hermitian** [Invertible (2 : R)] (hs : Continuous s) (hss : ∀ x, s (s x) = x) [LocallyCompactSpace (W2 s k)] (ω : Rˣ) (hω : s ω = -ω) {u : Rˣ} (hu : s u = u) : addEquivAddHaarChar (scalM k k u) = addEquivAddHaarChar (scal (W2 s k) u (smul_mem_skew_hermitian hu) (smul_mem_skew_hermitian (s_inv hu))) *

addEquivAddHaarChar (scal (W2 s k) u (smul_mem_skew_hermitian hu) (smul_mem_skew_hermitian (s_inv hu)))

theorem **char_conj** [Invertible (2 : R)] (h : Fr s J X Y) (hs : Continuous s) (ω : Rˣ) (hω : s ω = -ω) {u : Rˣ} (hu : s u = u) {H : Type} [Group H] [TopologicalSpace H] [IsTopologicalGroup H] [LocallyCompactSpace H] [SecondCountableTopology H] [T2Space H] [MeasurableSpace H] [BorelSpace H] (j : H →* GL (Fin n) R) (hj : IsEmbedding j) (hU : ∀ p, cT s (j p).1 * J * (j p).1 = J) (hP : ∀ p, ∃ B : Matrix (Fin k) (Fin k) R, (j p).1 * X = X * B) (hN : ∀ x, ∃ p, (j p).1 = h.nMat x) (b : H) (bX : (j b).1 * X = u.1 • X) (bY : (j b).1 * Y = u⁻¹.1 • Y) (bZ : ∀ (m : ℕ) (Z : Matrix (Fin n) (Fin m) R), cT s X * J * Z = 0 → cT s Y * J * Z = 0 → (j b).1 * Z = Z) (φ : H ≃ₜ* H) (hφ : ∀ p, φ p = b * p * b⁻¹) : mulEquivHaarChar φ * addEquivAddHaarChar (scalM k k u) = addEquivAddHaarChar (scalM n k u)

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theorem **mul_mem_minPar** {p q : GAd σ J} (hp : p ∈ minPar σ J X) (hq : q ∈ minPar σ J X) : p * q ∈ minPar σ J X

theorem **inv_mem_minPar** (h : IsCMExt F E σ) (hJ : IsHermND σ J) (hW : IsWittFrame σ J X Y) {p : GAd σ J} (hp : p ∈ minPar σ J X) : p⁻¹ ∈ minPar σ J X

theorem **IsTorus.mem** {a : (Fin r → ℝ) → GAd σ J} (ha : IsTorus σ J X Y a) (t : Fin r → ℝ) : a t ∈ minPar σ J X

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theorem **realAd_inf** [IsTotallyReal F] (w : InfinitePlace F) (t : ℝ) : InfinitePlace.Completion.extensionEmbedding w ((realAd F t).1 w) = t

theorem **rId_snd** (t : ℝ) : (rId F t).2 = 1

theorem **rId_inf** [IsTotallyReal F] (w : InfinitePlace F) (t : ℝ) : InfinitePlace.Completion.extensionEmbedding w ((rId F t).1 w) = Real.exp t

theorem **rId_add** [IsTotallyReal F] (s t : ℝ) : rId F (s + t) = rId F s * rId F t

theorem **rId_zero'** : rId F 0 = 1